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Page 1259, Eq. 30 should read $T_o = \dfrac{E\,c\,t_o\,t}{\left(1 - \dfrac{K}{2}\right)\left(0.64\,\dfrac{f^2}{t^2} + 1.20\right)}$.

Transactions, Vol. 116, 1951

Page ix, among the discussers of Paper No. 2470, insert "George J. Soffer (page) 1374" between Messrs. Jackson and Aloff.

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Page 250, line 14, change "sketon" to "skelton."

On pages 1047 and 1048, Eqs. 109 and 112, change "$(L - R)\ e^{(L-x)/L}$" to "$L\ e^{(L-R)/L}$."

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SOME ASPECTS OF ELECTRONIC SURVEYING

By Carl I. Aslakson,[1] M. ASCE

With Discussion by Messrs. John A. O'Keefe, and Carl I. Aslakson.

Synopsis

Although the earliest attempts to test electronic instruments for use in surveying and mapping began as recently as the latter stages of World War II, much has been accomplished in a brief time. Electronic instruments have been used or tested in the following surveying fields:

1. Hydrographic surveying;
2. Airborne magnetometer surveys;
3. Measurement of long distances for geodetic control purposes;
4. Photogrammetry, in which their application is twofold:
   - *a.* Accurate flight-line navigation, and
   - *b.* Geodetic control of individual photographs;
5. Measurement of short distances—that is, distances ordinarily associated with conventional ground triangulation;
6. Topographic surveying for both distance and direction measurements; and
7. Vertical control in photogrammetry.

Clearly all branches of surveying and mapping are being influenced by these new developments. The purpose of this paper is to review these aspects of electronic surveying practices.

Hydrographic Surveying

Electronic surveying equipment was adopted in hydrographic surveying with scarcely the formality of a test period in that field. Shoran, for example, was tested by the United States Coast and Geodetic Survey, and the first field trial developed into a well-controlled hydrographic survey. Originally a blind-bombing instrument, it was used for hydrography without modification.

Note.—Published in January, 1951, as *Proceedings-Separate No. 52*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Comdr., U. S. Coast and Geodetic Survey, Washington, D. C.

Because of its transmission frequency of from 220 millicycles per sec to 350 millicycles per sec, shoran is limited in range. Electromagnetic waves of this frequency are propagated in nearly the same manner as light, and hence the range is limited to "line of sight." With moderate shore antenna heights of from 50 ft to 100 ft, the effective shoran range of the survey vessel is from about 30 miles to 40 miles. The necessity of extending this range led the Coast and Geodetic Survey to develop the "Electronic Position Indicator," commonly known as the EPI. Designed to use a frequency of 1,850 kilocycles per sec, the signals can be propagated for long distances by the "surface wave." The British Decca was designed for a frequency of 100 kilocycles per sec and is also a long-range hydrographic instrument. Some accuracy is sacrificed when lower frequencies are used, but hydrographic surveys can be extended hundreds of miles beyond the limits of shoran with a smaller positional error than is inherent in any previously known method.

Therefore it may be stated that electronic surveying methods have already revolutionized hydrographic surveying and that advances are continuing.

Airborne Magnetometer Surveys

Soon after the end of World War II, commercial organizations secured shoran equipment for controlling airborne magnetometer surveys. Previously no method of plotting flight lines over water or unmapped areas with the accuracy required for these magnetometer surveys was known. Up to the present time, numerous successful magnetic surveys have been completed, and valuable geophysical information has been compiled. Continued use of electronic positioning equipment for geophysical surveying is assured.

Measurement of Long Distances

Because shoran and other similar instruments were designed for positioning bombers over their targets, the possible application of these methods to long-distance measurement immediately attracted the geodesist. The possibilities of making intercontinental and interisland ties where distances were too great for conventional optical methods, and of making measurements over inaccessible terrain, inspired geodesists to undertake several test projects. The most important question to be answered was: "Can long distances be measured with geodetic accuracy by electronic instruments?"

The most extensive of these test programs was begun in 1944 by the United States Army Air Forces under the direction of the writer. Two comprehensive projects were undertaken which were concluded in 1947. Considering the fact that these tests were based on using a modified blind-bombing instrument instead of one designed for surveying, the results obtained were remarkable. Because certain cyclic or "repeating" errors inherent in the system could not be completely eliminated by design modifications, it was necessary to design observational techniques to minimize them. This approach was highly successful, and in the latter phase of the project forty-seven distances which varied between 67 miles and 367 miles were measured in the Caribbean area.

A complete discussion of the technique is beyond the limits of this paper, but the problems that faced the project staff were so extensive that the methods designed to overcome them will be described briefly.

As previously stated, the high frequencies necessary for greater accuracy are propagated on line-of-sight paths. Range is therefore limited by curvature of the earth and local obstructions. However, the equipment is adapted for installation in aircraft, making the range a function of airplane altitude when no local obstructions exist near the ground stations. When an airplane is flying at an altitude of 40,000 ft, shoran or similar instruments can measure a distance of more than 250 miles between the aircraft transceiver and ground station transponder. Using two ground transponders the effective measuring range is doubled. This result is accomplished by flying the airplane across the line between the stations, recording simultaneously at short intervals the distances to both ground stations and determining the minimum sum distance graphically or analytically. The minimum sum distance can be corrected to the geodetic distance by geometrical corrections involving plane altitude, ground station altitudes, and the radius of the earth.

Using long distances measured in the foregoing manner, a trilateration scheme can be executed by which geographic positions can be carried forward on a known datum.

The Caribbean tests proved that the method was practical. A board of experts appointed to evaluate the results concluded that:

(1) Accuracy approaching second order was attained; and

(2) Evidence of a possible error in the accepted value of the velocity of propagation of radio waves was present. If true, this would increase the estimated accuracy.

In accordance with the second comment by the board, the writer made an investigation of the velocity of radio waves as indicated by the shoran results. The deduced velocity of electromagnetic waves in vacuo was found to be 299,792 km per sec or 16 km per sec greater than the previously accepted value.[2] Further evidence that this uncertainty exists is shown by the results of two other observers, L. Essen[3] in England and Erik A. Bergstrand[4] in Sweden, both of whom have obtained substantially the same value by independent methods. Obviously, further research in determining this important constant is necessary before electronic systems of measuring long distances can be evaluated properly.

The operational procedure and computational methods of shoran trilateration at first appear so laborious as to be impractical, but experience has proved otherwise. The flight maneuvers are operationally practicable and the necessary computational steps can be arranged in an orderly manner. The computations can be so simplified that relatively inexperienced computers can perform the mass calculations. Extensive use of tables expedites the computations. The following fundamental steps are required to determine the geodetic distances and the final geographic positions:

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[2] "Can the Velocity of Propagation of Radio Waves Be Mesaured by Shoran?" by Carl I. Aslakson, *Transactions*, Am. Geophysical Union, August, 1949, p. 475.

[3] "Velocity of Electronic Waves," by L. Essen, *Letter in Nature*, London, England, Vol. 195, 1947, No. 4044, p. 611.

[4] "A Preliminary Determination of the Velocity of Light," by Erik A. Bergstrand, *Kunglige Svenska Vetenskapsakadmien*, Vol. 36A, Stockholm, Sweden, No. 20, 1948.

(*a*) Determination of minimum sum distances and altitudes of aircraft and ground stations;
(*b*) Application of instrumental calibration corrections;
(*c*) Determination of velocity corrections;
(*d*) Application of geometrical corrections;
(*e*) Least-squares adjustment of the net work of trilateration; and
(*f*) Computation of the geographic positions.

DETERMINATION OF MINIMUM SUM DISTANCES

In practice the equation of the time intervals (or frame numbers) versus sum distances is considered as a parabola. This assumption is not exactly true but it can be shown that for the conditions under which the lines are flown no appreciable error accrues from making it. The observed distances, as read from the film of the automatic recorder, are added and a parabola is fitted to the observations by a least-squares adjustment. Extensive use is made of precomputed constants to shorten the operation. The minimum value on the vertex of the computed parabola is the desired sum distance, representing all observations made during the line crossing. A graphical plot of observed sum distance versus frame number, and the corresponding fitted parabola, is shown in Fig. 1.

Simultaneously with the line-crossing measurements the aircraft altitude is determined by one or more of several methods involving the use of radio altimeters, barometric altimeters, and meteorological observations. The ground station elevations are determined by conventional methods.

The line-crossing procedure, known as the "figure-eight method" is shown in Fig. 2. The purpose of adopting this flight procedure is to eliminate or minimize observer's errors and certain instrumental errors of a repeating nature. Approximately 1 mile of distance change for each station is used for the computation, the reason being that the instrument has a cyclic error that repeats itself each mile. The observing procedure also eliminates errors caused by slackness in certain gear trains and minimizes the personal errors of observers in tending to lead or lag as they make coincidences between moving "pips" on the oscilloscope.

INSTRUMENTAL CALIBRATION CORRECTIONS

Certain instrumental corrections must be determined by calibration and applied to the minimum distances. The most important of these are crystal frequency calibrations, measurements of some fixed delays in component parts of the equipment, a zero or an index correction, and an empirical correction for an error which results from varying signal intensities.

VELOCITY CORRECTIONS

Shoran equipment must be designed for a constant velocity of propagation which is fixed in the instrument by a crystal of a given frequency. However, the radio waves travel through air of varying pressure, temperature, and moisture content which change the velocity along the path. Therefore it is necessary to adjust each distance by a correction which is a function of the difference between the assumed and actual conditions. The lapse rates of pressure, temperature, and relative humidity along the ray path can be determined in a number of ways, but the best method is to use a second airplane to make the meteorological measurements. It can be shown that the velocity correction can be expressed satisfactorily by the following relationship:

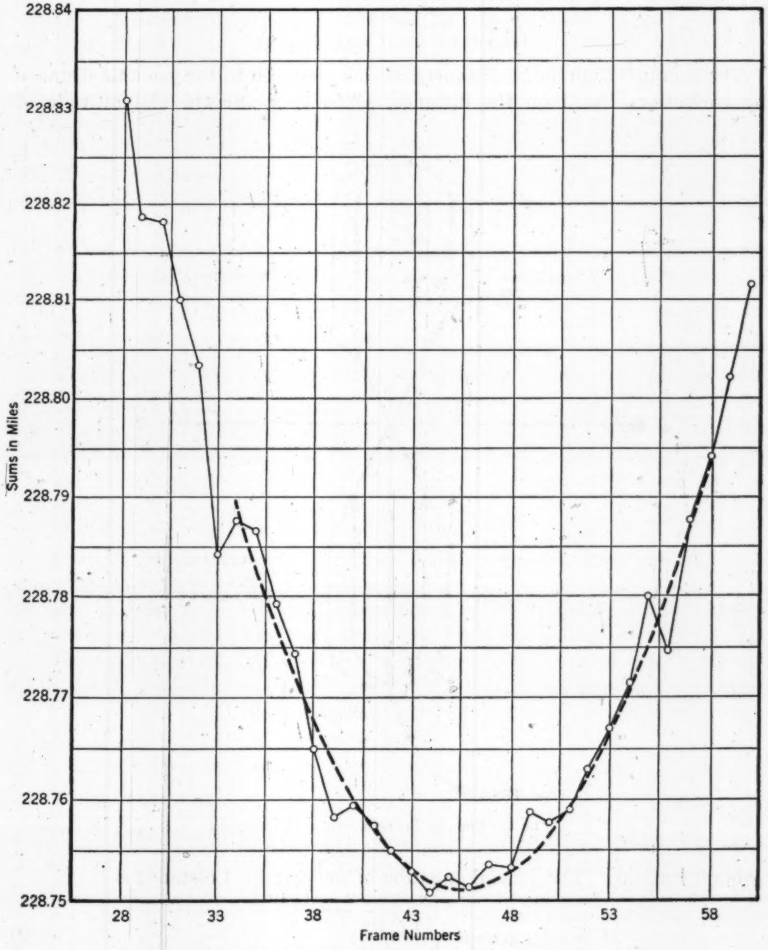

FIG. 1.—GRAPHICAL PLOT OF OBSERVED SUM DISTANCES VERSUS FRAME NUMBERS, AND THE CORRESPONDING FITTED PARABOLA

$$\Delta S = \int_0^S \Delta n \, ds \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which ΔS is the velocity correction; S is the shoran distance; and Δn is the difference between the index of refraction assumed in the instrument design and the index of refraction at a point on the ray path corresponding to the distance s. In practice the integration is made numerically.

GEOMETRICAL CORRECTIONS

The computed minimum distances must be reduced to the geodetic distance by corrections based on the distance, altitude of aircraft, and altitude of ground stations. The general equation of the corrected distance is

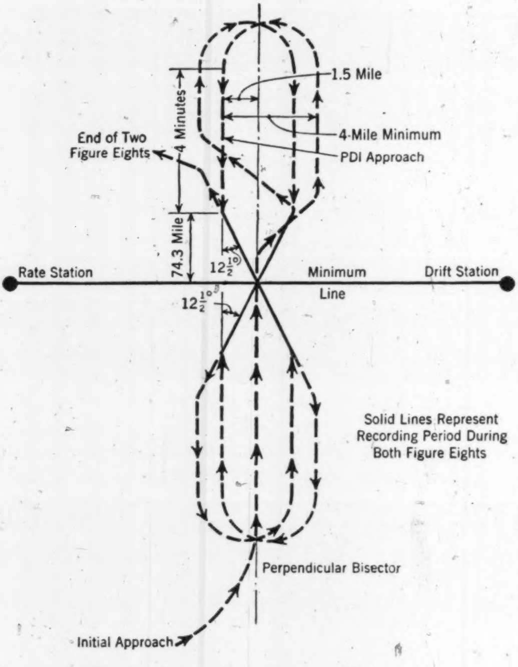
1.5 Mile
4 Minutes
4-Mile Minimum
End of Two Figure Eights
PDI Approach
74.3 Mile
12½°
Rate Station
Minimum Line
Drift Station
12½°
Solid Lines Represent Recording Period During Both Figure Eights
Perpendicular Bisector
Initial Approach

FIG. 2.—SHORAN LINE-CROSSING PROCEDURE

$$M = 2\,R_\alpha \arcsin \sqrt{\frac{4\,r^2 \sin^2 (S/2\,r) - (H - K)^2}{4\,(R_\alpha + H)\,(R_\alpha + K)}} \quad \text{. . . . .} \quad (2)$$

in which M is the geodetic distance; R_α is the radius of the earth at the point of the line crossing along the azimuth of the line; r is the mean radius of curvature of the shoran ray; S is the shoran distance as observed; H is the altitude of the aircraft; and K is the altitude of the ground station. The assumption that the ray path is a circular curve causes a small error which can be removed by a correction table since the actual ray path can be determined with moderate accuracy from the meteorological observations. The error resulting from the assumption that the length of the "geodesic" is equal to the length of the plane curve is negligible.

In actual practice the use of the rigid formula (Eq. 2) is not justified. Several simpler relationships are currently in use and are sufficiently accurate for all practical purposes.

Cedric W. Kroll[5] has developed a rigorous method of computing the velocity correction and geometrical correction simultaneously by numerical integration of the basic differential equation of refraction along the ray path. This method is too laborious to use unless electronic computing instruments are available.

ADJUSTMENT OF SHORAN TRILATERATION

A network of shoran-measured distances can be adjusted easily by the method of least squares if observation equations are used. The general form of the observation equation is

$$S_{pq} + V_{pq} = -X_p \sin \alpha_{pq} + Y_p \cos \alpha_{pq} - X_q \sin \alpha_{qp} + Y_q \cos \alpha_{qp} + i_{pq} + K \quad \ldots (3)$$

in which S_{pq} is the observed shoran distance; V_{pq} is the most probable change in S_{pq}; X_p and Y_p are the changes in the abscissa and the ordinate of P; X_q and Y_q are the corresponding changes in Q; α_{pq} and α_{qp} are the geodetic azimuths at P and Q, respectively; i_{pq} is the inverse distance computed from the assumed positions of P and Q; and K is a constant which represents a constant error in each measurement. The constant K need not be introduced unless the presence of such an error is suspected.

The assumed positions can be determined by computing geographic positions through a single chain of triangles using first-order geographic position forms. The next step consists of computing rigid inverse distances between all points over which shoran distances are measured to obtain the values of i_{pq}.

The general equation may be rearranged as necessary to fit specific conditions such as those when certain points, distances, and azimuths have to be held fixed. The observation equations are then reduced to normal equations and are solved by conventional methods.

COMPUTATION OF GEOGRAPHIC POSITIONS

The rigorous computation of geographic positions is difficult when long lines are involved. When the aforementioned adjustment method is used, the position computation is simple. After the adjustment is completed, it merely becomes necessary to change the assumed positions by the X-value and the Y-value as derived from the adjustment. Obviously this method does not furnish azimuths; but they can be computed by inverse computations if required. In practice, azimuths will seldom be required in long-line trilateration.

ADVANTAGES OF SHORAN TRILATERATION

Long-distance electronic trilateration has certain advantages that may assume great importance in surveying and mapping:

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[5] "A Rigorous Method for Computing Geodetic Distance from Shoran Observations," by Cedric W. Kroll, *Transactions*, Am. Geophysical Union, February, 1949, p. 1.

(1) Distances of more than 500 miles can be measured—greatly exceeding distances which can be measured directly by conventional methods.

(2) When the time factor is important, the rapidity with which shoran surveys can be made may be the controlling factor in determining its adoption. It has been demonstrated that large areas can be surveyed in a few months which could not be surveyed in decades using conventional methods.

(3) The functions of geodetic control and aerial photography can be combined efficiently in a single organization, and these operations can be performed simultaneously on the same project.

(4) Unit area costs are far lower than those of conventional methods of control. A satisfactory estimate of the cost of shoran geodetic control is about $1.00 per sq mile. This estimate is derived from a single large shoran project. The cost per station is large, but few stations are required for comprehensive surveys.

Electronic Distance Measurement in Photogrammetry

There are two important applications of electronic distance-measuring equipment in photogrammetry:

(*a*) Shoran or similar instruments provide the most efficient method of flight-line navigation that has been developed; and

(*b*) Shoran distances can be recorded for every aerial photograph, providing the third-order control simultaneously with aerial photography.

Flight-Line Navigation

Conventional navigation of photogrammetric airplanes is always difficult even when good maps are available. Over water or uncharted areas excessive side lap is required, and even then many gaps occur which necessitate costly reflights. When shoran is available, flight-line navigational instruments can be used to simplify airplane navigation greatly. These instruments, operating directly from the shoran mileage counters, indicate the position of the aircraft with respect to a given flight line at all times. In fact, a pilot-direction indicator shows this relationship in unit changes of about 25 ft. Several designs of these flight-line indicators have been operated successfully with large savings resulting from reduction in overlap, avoidance of gaps, and reduced cost of compilation.

Control of Aerial Photographs

The British have tested their electronic instruments Oboe, G-H, and Decca for control of aerial photographs. An official British report has stated:

> "Although maps at 1–25,000 to 1–50,000 can be made to a reasonable standard of accuracy without direct reference to the ground, mapping to full peacetime specifications at present requires a small amount of ground control when radar fixes are employed but on a very much reduced scale."

The only published estimate of the accuracy of shoran is based on early tests. The Engineer Research and Development Laboratories, at Fort Belvoir, Va., estimate that:

1. Random errors amounting to 120 ft are encountered in individual sheets of 1:50,000 compilations; and
2. In addition, systematic errors of about 40 ft may occur between sheets. Studies of recent projects, particularly those where the flight-line navigator was used, may reduce these estimates.

Clearly, electronic control of aerial photographs approaches the accuracy afforded by the slow and costly conventional ground control. Certainly the present accuracy is adequate for many areas which are now unmapped. The advantages of mapping entirely by electronic control are very great in that:

a. Less time is lost due to weather;
b. Unit costs are low;
c. Inaccessible terrain can be mapped; and
d. Large areas can be mapped very quickly.

Efficient Use of Weather

Continuously good photographic weather is rare. Frequently photographic airplanes remain on the ground for long periods when flying conditions are good but cloud coverage is too great for photography. All this time can be used in line measurement for the geodetic control. When, at last, photographic weather prevails, the mission can be changed to photography. Clouds, haze, smoke, and fog do not prevent shoran-distance measurement. Obviously a combination of both these functions contributes toward efficiency.

Unit Costs of Combined Operations

Aside from the reduction in cost of geodetic control, shoran photography should prove to be cheaper than photography flown with conventional navigation because of the efficiency of the flight-line navigation system. No appreciable gaps requiring reflights have thus far been encountered on any shoran-navigated project. At the same time the side lap can be reduced to the minimum amount required by the compiler. On numerous occasions, flight lines abandoned because of the presence of cloud banks have been continued at a later date exactly where the break occurred. Logistical costs are decreased because location of control station is very flexible. Airfields often provide good sites for shoran ground stations. Observers who have seen shoran-controlled photography in use are enthusiastic supporters.

Mapping of Inaccessible Terrain

The facility with which inaccessible mountains, jungle, tundra, or swamps can be mapped by shoran control is most important. For example, frequently a mountain range may be mapped without establishing a single shoran station on a mountain peak. Fig. 3 shows a typical hypothetical shoran scheme in the Appalachian Mountains. Examination of Fig. 3. will show that the distance measurements can be made on or near the centers of each line and that careful location of the stations on local prominences outside the mountain range will provide all needed control. Furthermore, the same stations provide shoran control for a large area on both sides of the trilateration scheme.

Examination of Fig. 3 will show a number of long thin triangles. The deliberate incorporation of these slim triangles into a scheme is an interesting feature of shoran trilateration. Their purpose is to reveal the possible presence of constant errors of measurement. This may be clarified by stating that, if a line is measured as a whole by shoran and if a second measurement is derived from projection of the two shorter sides, a constant error occurring in each single measurement would equal the difference between the long measurement and the sum of the two short measurements. Consequently, the presence of a constant error in each shoran measurement would be revealed in the adjustment when a number of thin triangles is included in the network.

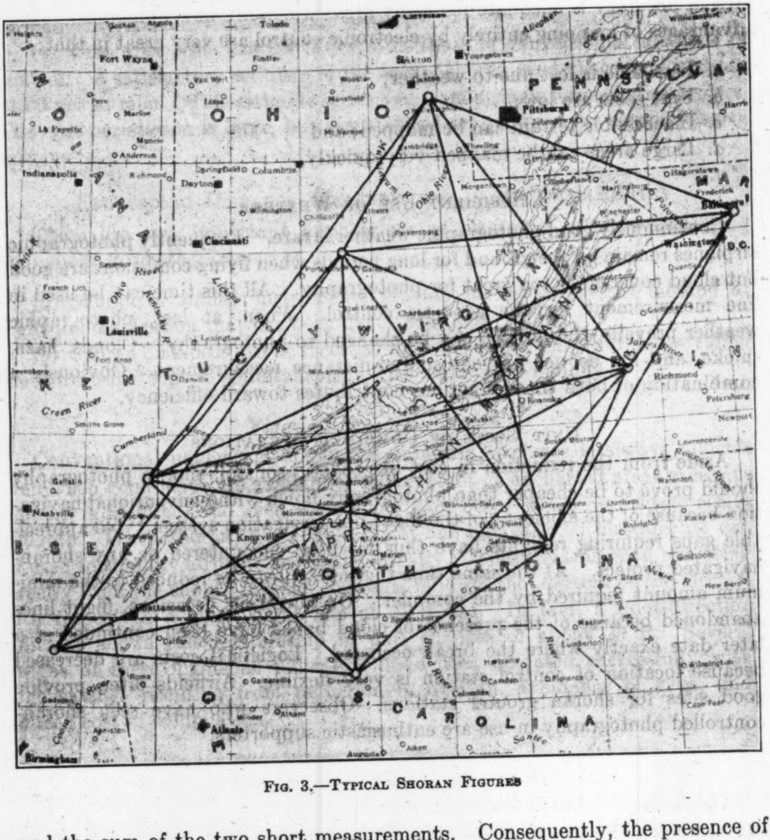
Toledo
Fort Wayne
Akron
Youngstown
PENNSYLVANIA
OHIO
Pittsburgh
Indianapolis
Dayton
Columbus
Cincinnati
Washington D.C.
Louisville
KENTUCKY
W. VIRGINIA
VIRGINIA
Richmond
Roanoke
Nashville
Knoxville
NORTH CAROLINA
Chattanooga
Greenville
Atlanta
S. CAROLINA
Birmingham
Augusta
Ohio River
Cumberland River
Tennessee River
Green River
James River
APPALACHIAN MOUNTAINS

Fig. 3.—Typical Shoran Figures

Speed of Shoran Surveys

The rapidity with which shoran-controlled photography can be executed has been proved repeatedly in practice. In one case, an area of 36,000 sq miles was photographed which had previously remained unmapped in spite of the efforts of several years to photograph it with conventional navigation. No time is lost in beginning flight lines. The shoran straight-line navigator enables the photographic plane to be flown directly to the starting point of each flight line. Small open areas free from cloud coverage can be photographed efficiently even though the flight lines are short. Time lost because of weather is therefore reduced to a miminum.

Frequently, the time element is more important than all other considerations. Mineral discoveries, economic development, or military requirements may demand rapid mapping of large areas where no control exists. At present, makeshift reconnaissance maps or uncontrolled mosaics must suffice. In such areas electronically-controlled aerial photography will quickly provide unified and controlled maps with reasonable accuracy.

SHORT-DISTANCE MEASUREMENT BY ELECTRONIC METHODS

The field of ordinary ground surveying is also being invaded by electronic surveying methods. Here competition is keen because one advantage of the electronic long-range equipment is its capacity to obtain results where conventional methods fail. However, even in ground surveying it is now safe to predict that many electronic surveying instruments will soon be in use. At present many are being developed and a few descriptions have been published.

One electronic distance measuring instrument which is very promising has been developed in Sweden. The "Bergstrand geodimeter" uses light reflected from a plane mirror target. The light is modulated in precisely controlled waves which are picked up by a photo cell after reflection from the target. Space limitations prevent a detailed description of this instrument, but published reports claim that accuracy greater than 1 part in 300,000 can be attained with this instrument over distances up to 30 km. The time required for an observation is reported to be two hours, with two additional hours required for computation of the results. The instrument is reported to be portable, and adaptable to field use. Observing conditions required are similar to those needed for conventional triangulation.

Several other portable field instruments using visible light or high radio frequencies have been designed or are under development. The principles differ widely but many of them are very promising.

As an example of an efficient use of instruments of this type reference is made to Figs. 4 and 5. Fig. 4 shows a conventional scheme of river triangulation used by the United States Coast and Geodetic Survey. Between line A and line B there is a total of fifty-one stations. Fig. 5 shows how this scheme can be reduced to twenty-five stations and great strength of figure can be maintained by measuring the heavy lines with an instrument similar to the geodimeter. Azimuth checks can be carried accurately through the scheme of triangles and, at the same time, checks on the electronically-measured bases are possible from the thin triangles. Certainly the surveying engineer must soon learn to revise his conception of "strength of figure."

TOPOGRAPHIC SURVEYING

At least one electronic system promises to be useful in topographic surveying. The Engineers Research and Development Laboratories have published a description[6] of an electro-optical system that measures distance by light pulses

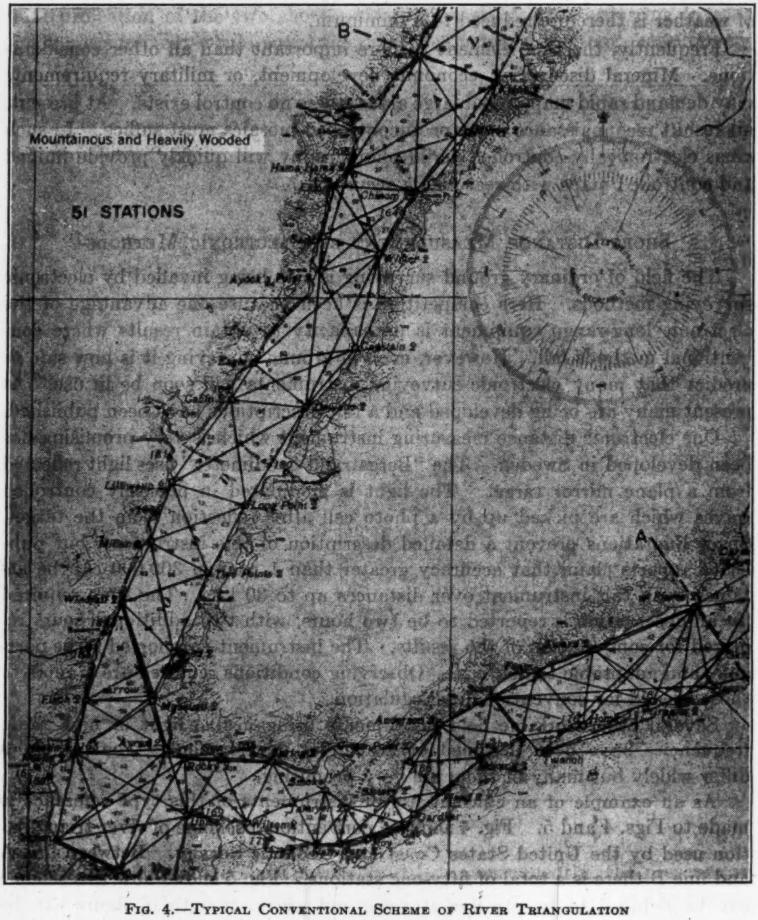

FIG. 4.—TYPICAL CONVENTIONAL SCHEME OF RIVER TRIANGULATION

with an accuracy of about 1 part in 2,500. This instrument also measures both horizontal and vertical angles to about 1 min of arc. The system is portable, easy to use, and has a range of about 3 miles. The target is a corner cube reflector. The use of such an instrument may be important in small topographic surveys, aerial photograph control, and low-order triangulation schemes.

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[6] "Development of an Optical Radar System," by Charles A. Hoover, *Transactions*, Am. Geophysical Union, August, 1950.

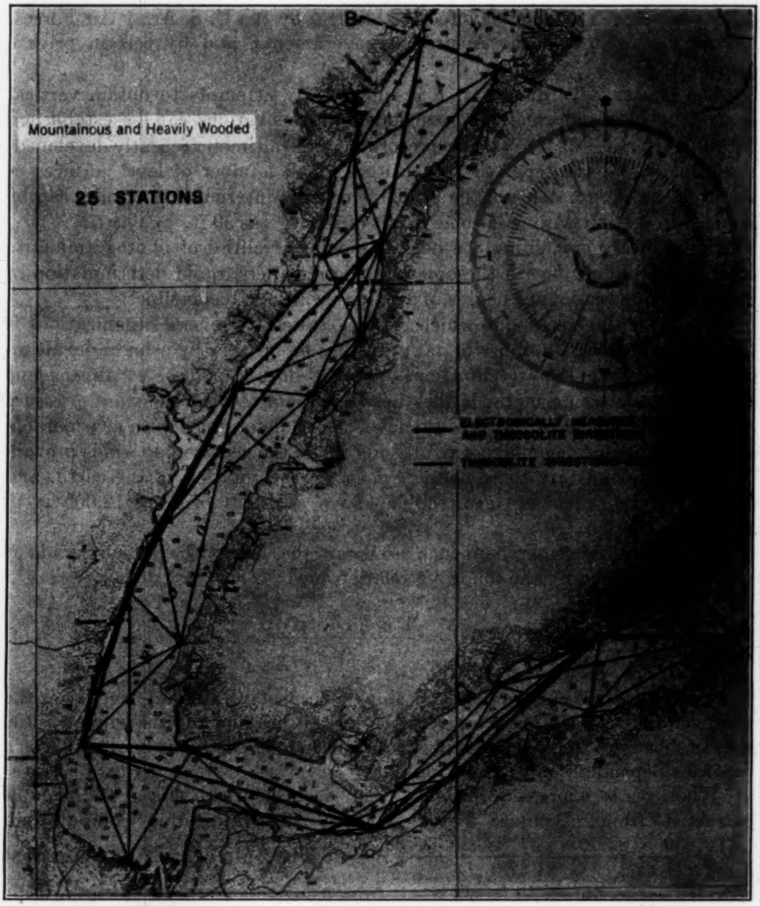
B
Mountainous and Heavily Wooded
25 STATIONS
ELECTRONICALLY MEASURED AND THEODOLITE DIRECTIONS
THEODOLITE DIRECTIONS

FIG. 5.—TYPICAL CONVENTIONAL SCHEME OF RIVER TRIANGULATION REDUCED BY USING ELECTRONICALLY MEASURED DISTANCES

THE DETERMINATION OF ELEVATIONS BY ELECTRONIC METHODS

Radio altimeter development, including that of narrow-beam instruments, has been advancing. Recording instruments or profile recorders for use in airplanes are also being designed. To the present time (1950) an approach to the accuracy provided by closely spaced ground control has not been attained, but progress is being made. When the upper air is mapped by meteorological observations from balloons or airplanes simultaneously with the airplane altimeter observations, a moderate degree of accuracy can be obtained. Considerable research is being devoted to this field by the U. S. Army Air Forces, by the Canadian and the Australian governments, and by certain private organizations.

The British government, as a result of early attempts to obtain vertical control by electronic methods, concluded that elevations can be established within ± 10 m (± 33 ft). Using a recording radio altimeter recently developed, the Canadians have reported that, with a small number of level surfaces of known elevation for control, the probable error of intermediate profiles can be reduced to ± 10 ft with occasional errors of as much as 50 ft.

Certainly by combining the height-bridging qualities of photogrammetric compiling equipment with these new profile recorders, rapid determination of elevations of a reasonable degree of accuracy will soon be possible.

One promising technique which is being tested by several organizations is that of establishing a number of datum planes or surfaces by using radio altimeter ties between known elevations and the datum planes, as an airplane located by electronic methods flies between them along a constant pressure level. The gradient of this pressure level can be determined with considerable accuracy if the airplane flies at an altitude where the geostrophic winds prevail. Geostrophic winds are those caused solely by the rotation of the earth, and frequently the winds are geostrophic above altitudes of from 12,000 ft to 15,000 ft.

Using electronic equipment such as shoran, the slope of the pressure gradient can be determined by the following relationship:

$$\Delta H = \frac{V\,S \sin \alpha \sin \phi}{K} \quad \text{.....................} (4)$$

in which ΔH is the change in height along the constant pressure level; V is the wind velocity; α is the angle between the aircraft heading and the wind; ϕ is the mean latitude of the line; S is the air distance along the flight line; and K is a constant depending on the units used.

When V is expressed in knots and S in nautical miles, an example extracted from a paper by J. Warner[7] illustrates this computation: $\Delta H = \dfrac{30 \times 82 \times \sin 47° \times \sin 33° 30'}{21.47} = 46.3$ ft. Mr. Warner estimates that a probable error of ± 35 ft will result from using this method. Clearly, using shoran, the necessary data needed in the computation can be observed to the number of significant places needed for reasonable accuracy. Multiple observations under varying conditions, ties between datum planes, and adjustment of the differences between these datum planes to obtain a consistent series, should reduce the probable error considerably. The method was first tested by the writer, during the Caribbean project of the U. S. Army Air Forces, and additional tests were recommended in the final report. The Canadian government is also using this technique, but no estimate of its accuracy is available in official reports.

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[7] "A Method of Reduction of Line-Crossing Observations in Radar Surveying," by J. Warner, *RPR90*, Council for Scientific & Industrial Research, Div. of Geophysics, Commonwealth of Australia, Radio Laboratory, Sydney, Australia, March, 1950.

Further research, using the pressure-gradient technique, will certainly provide vertical control in many areas where conventional leveling may not be feasible or where it is possible to relax present specifications.

FUTURE RESEARCH REQUIRED IN ELECTRONIC SURVEYING

The accomplishments in electronic surveying, to 1950, have been discussed. However, there is need for further research to obtain:

(1) Better determinations of the velocity of propagation of radio waves;

(2) Better knowledge of the relationship between the index of refraction of moist air and changes in pressure, temperature, and relative humidity; and

(3) Design of instruments specifically for surveying purposes.

VELOCITY OF RADIO WAVES

The velocity of electromagnetic waves in vacuo is the fundamental yardstick used in electronic distance measurement. At present there is considerable doubt as to the correct value of this constant. The velocity of electromagnetic waves in vacuo has been measured by many observers. The results that have been published since the observation reported by A. A. Michelson in 1879 are listed in Table 1.

TABLE 1.—MEASUREMENTS OF VELOCITY OF ELECTROMAGNETIC WAVES IN VACUO

Year published	Observer	Length (m)	Velocity in vacuo (km per sec)
1879	Michelson[a]	600	299,910 ±50
1882	Newcomb[a]	4,000	299,860 ±30
1902	Perrotin[b]	46,000	299,901 ±84
1906	Rosa and Dorsey[c]	....	299,781 ±10
1924	Michelson[a]	35,000	299,802 ±30
1926	Michelson[e]	35,000	299,796 ± 4
1929	Mittlestaedt[d]	250	299,778 ±20
1935	Michelson, Pease, and Pearson[e]	15,000	299,774 ±11
1937	Anderson[d]	170	299,771 ±15
1940	Huttel[d]	48 to 118	299,768 ±10
1941	Anderson[d]	170	299,776 ±14
1948	Essen[f]	....	299,792 ± 9
1949	Aslakson[g]	....[h]	299,792 ± 1
1950[i]	Bergstrand[d]	9,000	299,792.8 ±0.25

[a] Using rotating mirrors. [b] Using a toothed wheel. [c] By computing the ratio of electrostatic units to electromagnetic units. [d] Using the Kerr cell-photo cell method. [e] Using rotating mirrors in vacuo. [f] Using the resonance in wave guide. [g] Using the shoran method. [h] Forty-seven distances varying from 67 miles to 367 miles. [i] From correspondence between Mr. Bergstrand and the writer.

Obviously this important constant may be uncertain by about 1 part in 20,000. Until greater accuracy is assured in its determination, electronic distance measurement will have proportional uncertainty. However, it is very probable that electronic surveying instruments may prove to offer the best method of determining the velocity of radio waves by comparison with accurately determined distances of varying length. This is particularly true if the long-range instruments such as shoran are improved in accuracy. Shoran errors remain nearly the same irrespective of the distance; therefore the proportional error decreases with an increase in distance.

INDEX OF REFRACTION OF AIR

Although the velocity of propagation of radio waves is considered to be constant in vacuo, and independent of the frequency, surveying measurements are made under varying atmospheric conditions. This procedure causes changes in the velocity which must be evaluated for corrective purposes. The relationship between velocity in air, which has a permeability very nearly unity, and in vacuo is expressed by

$$V = V_0/n \qquad (5)$$

in which V is the velocity in air; V_0 is the velocity in a vacuum; and n is the index of refraction.

Various relationships for the variation of the index of refraction with changes in pressure, temperature, and relative humidity are in use. In general, these expressions take the following form with small differences in the constants:

$$(n - 1)\,10^6 = \frac{79.5\,(P - e)}{T} + 3.80 \times 10^5 \frac{e}{T^2} \qquad (6)$$

in which P is the total pressure, in millibars; e is the partial vapor pressure, in millibars; and T is the absolute temperature, or 273° plus the centigrade temperature. In extreme cases, the uncertainties in Eq. 6, and in the methods of measuring the quantities involved, may affect the fifth significant place in distance determinations. Thus the evaluation of present long-distance measurements as "approaching the second order" may be improved by a better knowledge of Eq. 6 and by instrumental improvements.

A further uncertainty exists because of a discontinuity in this relationship in the frequency band width which lies between the higher radio frequencies and the optical band. The discontinuity is caused by a change in the dielectric constant of water vapor with respect to frequency in the transition range. Both experimental results and mathematical studies confirm the existence of this discontinuity. The boundary limits have not been closely defined although it is known that the dielectric constant of water vapor remains constant down to wave lengths of the order of 1 mm. A transition then takes place through the infrared region until a new optical value is reached. Therefore, Eq. 6 is valid only for wave lengths longer than 1 mm. Extensive research on the dielectric constant of mixtures of water vapor and air over widely varying frequencies, as well as the improvement of automatic recording meteorological instruments, is of paramount importance to electronic surveying.

INSTRUMENTAL DESIGN

Considering the fact that, except for the Swedish geodimeter, all research on precise distance measurement has been done with adaptations of instruments designed for military purposes, the results have been remarkable. The shoran instrument used in the Caribbean project contained numerous circuits useless to the surveyor as, for instance, antijamming circuits. It was designed for more liberal tolerances; and, although field modifications were made on that project to increase the accuracy, these modifications did not always eliminate the errors completely. The principal source of error in existing pulse-type radar instruments arises from a signal intensity error which is a function of the "rise time" of the signal. Unless the time required for the signals to increase from zero to a maximum is reduced to a very small amount, some form of empirical signal-strength correction is needed. This method of correction is unsatisfactory, since many variables difficult to assess are involved. In current studies of shoran an attempt is being made to overcome this difficulty by maintaining voltage gains at a constant level, but at best this is a makeshift solution. The real solution lies in the design of electronic surveying instruments with the "rise time" of the signal greatly decreased. Such studies are currently in progress.

CONCLUSION

All research in electronic surveying is done either by government organizations or by government development contracts. The success that has attended these efforts is causing the surveying and mapping engineer to realize that his field is undergoing a revolutionary change. Commercial organizations are becoming interested in production, and it is only a matter of time before new electronic surveying equipment will be available for civilian use. The short-distance measuring equipment will no doubt have the wider application, and instruments of this type will effect profound changes in present surveying techniques. Methods of making local precision surveys will be greatly altered. For example, city control surveys will be executed more rapidly and with greater accuracy. The word "trilateration" will become a commonly used surveying term.

The members of the surveying and mapping professions are evincing greater and greater interest in these developments. It is to be hoped that this interest will increase, and that popular support for the research programs in electronic surveying will continue.

DISCUSSION

John A. O'Keefe[8].—It is useful to notice that the signal strength correction referred to by the author under the heading "Instrumental Calibration Corrections" is of the opposite sign and much the same magnitude and form as the correction to improve the velocity of light. This accounts for the fact that in the early experiments with shoran, accuracies of the order of 1 part in 60,000 were frequently—even generally—obtained. The signal strength correction was discovered while everyone still believed in the Anderson value of the velocity of light. The author courageously inserted this correction into the calculations even though it made all of the checks with ground measurements worse and reduced the apparent accuracy to 1:20,000. He reaped the reward of his courage when he made the discrepancy the basis for a new value of the velocity of light.

The writer was one of those who saw that the systematic 1:20,000 discrepancy might mean that the velocity of light was in error, but he hesitated to say so, especially after consultation with the recognized authorities in the field. Examination of the list of determinations for the velocity of light from Mittelstaedt's 1929 value to Anderson's 1941 values, all of which are within a few kilometers per second of the value 299,776 km per sec, strengthened this hesitation. How many others since 1930 have seen evidence that this value was too small and have been afraid to say so?

There are several scientific papers[9] in existence noting the fact that the values found for the velocity of light in any given decade seem to group around the same value more consistently than would be expected on the basis of accidental error. This has been attributed to a gradual diminution of the velocity of light or to periodic changes in its velocity, but it could also be nothing more than the human unwillingness to fly in the face of constituted authority.

The author has since justified his faith in the reality of the signal intensity correction and the consequent change required in the velocity of light by flying over the same area with apparatus in which the gain is changed in such a way as to have constant signal strength presented to the measuring apparatus. Thus he has been able to confirm his ideas about the velocity of light without making use of the signal intensity correction.

Some users of the shoran apparatus continue to use Anderson's values of the velocity of light and to omit the signal intensity correction with moderately good practical results.

Carl I. Aslakson,[10] M. ASCE.—The writer has stated that developments in the field of electronic surveying were so rapid that publication dates could not keep pace with them. On a project undertaken in 1951, it can be stated with assurance that long distances were measured with first-order accuracy. In this project the Bergstrand[11] value of the velocity of light (299793.1 km per sec) was used as the design value in the instrument. However, during the course of the project, a large number of surveyed distances will also be measured by high precision shoran, utilizing the "gain-riding" technique metioned by Mr. O'Keefe, and therefore not requiring a correction for signal strength. Thus, in scaling to these fixed lengths, the exact value of the true velocity is not of prime importance inasmuch as the scale ratio of the shoran distances to the surveyed distances will provide another velocity determination. In a test project completed in 1950, the validity of the new, higher velocity has again been closely verified. A value for the velocity of light of 299794.2 ± 1.6 km per sec was determined.

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[8] Chf. Research and Analysis Branch, Geodetic Div., Army Map Service, Washington, D. C.

[9] "Velocity of Light and Radio Waves," by L. Essen, *Nature*, Vol. 165, 1950, p. 583.

[10] Comdr., U. S. Coast and Geodetic Survey, Washington, D. C.

[11] "Proposed New Value for the Velocity of Light," by L. Essen, *Nature*, Vol. 167, 1951, p. 258.

Mr. O'Keefe's remarks concerning the courage of the author in publishing this value are gratifying. However, it appears to the writer that the real exhibition of courage was shown by Mr. Essen, who has insisted since 1947 that the Birge statistical value of 299776 km per sec was too low. It must give him great satisfaction to see so many recent determinations confirm his early cavity resonator results. Certainly the writer, faced with the necessity of applying an empirical correction for signal strength would never have had the temerity to propose a new value for the velocity of light had not Mr. Essen and Mr. Bergstrand previously published their results.

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Paper No. 2477

HIGHWAY PLANNING IN TURKEY

By H. E. Hilts,[1] A. M. ASCE

Synopsis

It was apparent to the Turkish people in 1947 that progress in highway construction and maintenance was seriously lagging, and in the public interest it was imperative from commercial, agricultural, and security viewpoints that steps should be taken to develop an over-all transport policy consistent with national needs. An opportunity to benefit by American assistance was first presented by *Public Law No. 75* (Eightieth United States Congress), popularly known as the "Aid to Greece and Turkey Program."

On December 1, 1947, a group of experienced highway engineers was sent to Turkey by the United States Public Roads Administration (later the Bureau of Public Roads, United States Department of Commerce). Although American grants or loans have paid for this technical assistance and for purchase of modern road equipment, the Turkish government itself has financed its road building and maintenance program.

The reaction of the Turkish people to this active program for highways has been surprisingly like that of the average American and the Turks are somewhat astounded by the rapidity with which their own engineers and workmen have been able to translate words into action. There is promise that the Turkish government will, within a period of a few years, develop a highway department that will compare favorably with highway departments in the United States.

Introduction

Modern Turkey is a tight little republic in the bridge of Asia Minor which joins the Middle East with Western Europe and which in the past has witnessed the passage of many peoples, both in terms of world trade and of conquests. It is the legendary land of the Hittites, the Persians, the Greeks, the Romans,

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[1] Deputy Commr., Bureau of Public Roads, U. S. Dept. of Commerce, Washington, D. C.

the Genoese, the Crusaders, and finally the Ottoman Turks of Central Asia. It was the land of Helen of Troy, the land which interested Jason in his search for the Golden Fleece. Paul of Tarsus traveled much in southern Turkey and the descendants of the Amazons still live in the northeastern section near the Russian border.

Turkey in Asia Minor, about 1,000 miles long from east to west and about 300 miles wide, is a little larger than Texas. It became a republic in 1923. With one eighth of the population and one tenth of the area of the United States, it has all-weather roads aggregating less than 1% of the mileage of similar roads in the United States. It has a population of 18,000,000—about 54 people per sq mile as compared to 44 people per sq mile in the United States.

Predominantly it is an agricultural nation. As lack of adequate highway facilities is no reflection on the ability of the Turkish nation to visualize and determine transportation needs, the first objective in transportation was to finance and build a network of 6,900 km of railroad to handle mass transport. In the short space of twenty-three years this was accomplished, except for a few extensions to be added in due course.

The determination of the Turkish nation to proceed in fulfilling its highway needs in an orderly manner is both rational and timely. In such a country of relatively short distances, in modern terms, highways can be built, bettered, and maintained at low cost. Increased mobility is an economic necessity; expensive animal transport can no longer be afforded. Agricultural and industrial development must not be retarded by lack of modern highway transportation. Fertile land too far removed from railroads to make products marketable should be brought into agricultural production by tapping these regions with farm-to-market highways. Improved highway transportation will make it possible for farmers to grow wider varieties of products, marketable in new trade centers farther from home, and thus will reduce the risks involved in relying on a single crop. Losses in weight of livestock caused by driving the animals to market could be conserved by trucking, as in the United States. Just as in the case of agriculture, improvements would result in all economic fields—mining, manufacturing, and fishing—as markets and sources of the products are brought closer together by better transportation.

Another ruling factor is that the terrain is unusually difficult. The approaches to the Anatolian plateau, at an elevation of about 3,000 ft above sea level, have made railroad location difficult and costs high.

The following units may be useful in converting from the metric to the English system of units:

Metric	English
1 kilometer	0.62137 miles
1 meter	3.28083 ft
1 metric ton	2,204.62 lb
1 kilogram	2.20462 lb

THE PROBLEM

The 1947 rural road mileage of Turkey was estimated as 41,800 km. This total is divided into a proposed national system of 20,000 km and 21,800 km of provincial roads. There are also about 100,000 km of city and village streets and other rural roads. Although 11,000 km are classified as having macadam surfaces, inspections have shown that the roads are deficient in width and are not structurally sound enough to carry the increasing motor traffic. Likewise most of the bridges are deficient in both width and strength. These conditions lend themselves to revisions of both line and grade on practically all the main routes of the system.

Total motor vehicle registration in Turkey in 1947 was 12,913 of which two thirds represented trucks and buses. In 1947 traffic counts showed that vehicles were distributed as follows:

Type	%
Animal drawn	70.7
Passenger automobiles	10.9
Trucks	18.4
Total	100.0

The total traffic per day exceeded 1,000 vehicles at only seven stations out of one hundred and ninety, whereas the total motor traffic exceeded 500 vehicles per day at only five stations.

Since 1932, the expenditures made on the national and provincial system averaged about 18,000,000 Turkish liras per year ($6,500,000 at 2.8 liras per dollar). Experience has shown that this rate of expenditure was not sufficient to maintain the roads already constructed and the new sections completed each year.

The revenues that were used for national highways prior to 1948 consisted largely of appropriations by the General Assembly and 15% of a compulsory road contribution known as a "Poll Tax." The latter was levied on males from eighteen years to sixty years of age, at an annual rate of from 8 liras to 12 liras. These revenues were budgeted and the amount unspent at the end of the year was returned to the general fund for rebudgeting in the following year.

A certain volume of contract construction was completed yearly by Turkish road contractors between specified termini. These improvements were financed by bonds issued in accordance with acts of the General Assembly. Usually these bonds were discounted at the state banks and were matured in succeeding years.

CIRCUMSTANCES WHICH LED TO AMERICAN AID

As was apparent to the Turkish people in 1947, progress in highway construction and maintenance was seriously lagging and in the public interest it was imperative from commercial, agricultural, and security viewpoints that steps should be taken to develop an over-all transport policy consistent with national needs. With highway, railroad, reclamation, and port construction under the jurisdiction of the Ministry of Public Works, the time was opportune to initiate and carry forward studies aimed at coordinating the development of transportation facilities.

Public Law No. 75 (Eightieth United States Congress, First Session, 61 Statute 103), popularly known as the Aid to Greece and Turkey Program, was in keeping with the spirit of the United Nations. Pursuant to this act, an agreement was signed on July 12, 1947, between the American and Turkish governments on aid to Turkey. Subsequently, it was decided that, of the $100,000,000 granted to Turkey, $5,000,000 should be used for highway purposes. It was further decided in October, 1947, by an understanding with the Turkish government and a Memorandum Agreement between the State Department and the Public Roads Administration (PRA) that the PRA would arrange promptly to send a group of experienced highway engineers to Turkey.

EARLY ACTIVITIES OF THE PUBLIC ROADS GROUP

The PRA Group arrived in Ankara, Turkey, on December 1, 1947. Prior to its arrival, a receiving center for equipment had been established by a detail of civilian employees of the United States Army at Iskenderun and the training of equipment operators had begun. This activity was transferred to the PRA Group prior to its arrival.

The cooperative activity of the PRA Group, in the words of the agreement between the American Mission for Aid to Turkey and the Ministry of Public Works of Turkey, was—

> "The [PRA Group] * * * will assist the Ministry of Public Works of Turkey in establishing a long-range highway improvement program and in establishing a pattern for highway administration on a national scale based upon:
>
> "A. Inspection and studies of the topography, present condition of the roads, and other physical aspects, such as soil types and available local materials;
> "B. Economic studies of present and potential kinds and uses of improved highways, particularly with reference to the national security and the national economy;
> "C. Preparation of estimates of costs of proposed improvements;
> "D. Methods and types of construction and maintenance and their application to the actual operations;
> "E. A study of equipment and equipment-maintenance shop requirements to carry out the proposed program;
> "F. A study and establishment of required highway laboratory facilities;
> "G. Training of Turkish personnel in highway construction, maintenance and administration."

The American aid funds were used for the purchase of: (1) Highway engineering and construction equipment in the United States and its transportation cost to Turkey; and (2) payment of salaries, office supplies, and other expenses of the chief of the PRA Group and his staff. The budget of the Ministry of Public Works provided for financing the road programs and included such items as purchase of materials, supplies, and rights of way, as well as all salaries, wages, and expenses of Turkish personnel engaged on the program.

No time was lost in establishing direct office contacts between the individual members of the PRA Group and their counterparts in the Department of Roads and Bridges of the Ministry. Fortunately, the department was able to detail English-speaking Turkish engineers as liaison men to reduce the language barrier.

The members of the PRA Group were selected from the experienced personnel of the Public Roads Administration and state highway departments in the United States. The initial staff sectional assignments, on arrival in Ankara, were: (1) Administration; (2) personnel, finance, and accounting; (3) planning and programming; (4) surveys and plans; (5) bridges; (6) materials; (7) construction; (8) equipment; and (9) maintenance. The characteristic cordiality of the Turkish technicians eased a difficult first-stage development of mutual understanding by frequent conferences; and, at the end of the first two weeks, a well-defined outline had been prepared looking forward to the opening, in April, of the 1948 maintenance, betterment, and construction season.

THE PUBLIC ROADS ANALYSIS OF THE HIGHWAY SITUATION

The major objective in administration was to prepare a well-rounded report on the Turkish highway situation. As a result of inspection trips in the field covering some 2,000 km of the primary highways and through conferences with other ministries and with the Public Works Committee of the Chamber of Deputies, facts were arranged so that by late February, 1948, a report on the highway situation was completed by the PRA Group and transmitted to Turkish officials. The facts developed by the coöperative study were sufficiently explicit to permit the suggestion of an action program which would gradually lead to an accelerated schedule.

The report recommended the creation of an autonomous Department of Highways within the Ministry of Public Works to be directed by a commissioner reporting directly to the Minister of Public Works. It called for three staff divisions at headquarters—(1) design and research, (2) finance and control, and (3) maintenance and construction—and recommended the establishment of ten division field offices.

In addition, the report recommended the creation of a highway fund in the Ministry of Finance to receive all funds available for the use of the Department of Highways. It analyzed all present and potential sources of revenue, both for a national system and for a provincial system of highways, and suggested policies for the control of the use of funds from such sources. Appropriations by the General Assembly were to be earmarked for routine maintenance of the national system and for its betterment and construction. All customs duties collected on motor vehicles and their accessories, fuels, and lubricants would be allocated to the highway fund as would a corresponding tax imposed on equivalent items produced in Turkey. The establishment of national fees was recommended for the registration and operation of motor vehicles and the creation of a ton-mile tax for commercial haulers registered as such with the Ministry of Finance.

The report showed that, if its recommendations had been in force in 1946, there would have been available more than 55,000,000 Turkish liras for the national highways and 15,500,000 liras for the provincial systems, which could have been invested for the highway transport users to reduce operating costs. It recommended the establishment of a national system of licensing of operators and chauffeurs at reasonable fees, to be collected by the Ministry of Finance for the Ministry of Police and to be used for the development of an adequate patrol for policing traffic on the national system. This fund, if collected in 1946, would have made available 90,000 liras.

Of the 20,366-km network, about 6,395 km were to be considered for immediate improvement by betterment and routine maintenance, largely in accordance with traffic requirements. There were about 800 km of asphalt-surfaced roads in good condition and 7,800 km of passable macadam roads. On much of the passable macadam, a speed of only about 25 km per hr could be maintained. Thus, it could be concluded that the national system should consist ultimately of almost 35,000 km of arterial routes connecting all major centers of population.

Stage construction was recommended to secure, at reasonable cost, the maximum mileage of two-lane, all-weather roads with adequate bridges. Traffic needs indicated that stabilized soil, gravel, or macadam surfaces should be preferred and that more detailed attention should be given to the methods used in all earthwork and grading operations, particularly in consolidating fills and in using stabilized subbases. After consolidation by traffic, and as the need developed, existing surfaces should be given bituminous-surface treatments, to be followed later by plant or road-mixed bituminous surfaces. To lessen right-of-way difficulties in future years, it was recommended that the available controlled width of rights of way on all rural national highways be at least 35 m, that at entrances to municipalities it should be at least 55 m, and that no new buildings should be permitted within those limits.

The activities of the Planning and Programming Section of the Design and Research Division of the Department of Roads and Bridges were initiated at once. The cardinal principle adopted was that all work should be pointed toward an analysis of the economics of the rail, highway, water, and air transportation in Turkey, with the ultimate aim of logically coordinating these four systems from both civil and national security viewpoints. Other studies included:

(*a*) A physical field inventory of all national highways and bridges, including all adjacent culture, started and finished in 1948;

(*b*) A traffic survey, planned and completed in 1948, to determine volume, classification, weight, physical dimensions, and origin and destination of all vehicles using the national system of highways; and

(*c*) Financial studies, amplified to include all sources of highway revenue and the probable impact of these revenues on the entire Turkish economy.

In arranging the programs for 1948 on the national system, the policy determined was that the major part be on a pay-as-you-go basis with emphasis on the extension of routine maintenance and betterments, on the extension of two-lane all-weather roads, and on the construction of essential bridges by the Department of Roads and Bridges. This program was to be expanded as rapidly as finances would permit.

The Department of Roads and Bridges had always contracted for its surveys and plans for roads, although bridge plans were prepared by the staff. It was recommended that department forces should make all surveys and plans and that they should proceed aggressively with the preparation of a shelf of plans for roads and bridges on a priority programming basis, all this field work to be based on the principles of highway transport economics as used in the United States.

In recommendations regarding the new Division of Finance and Control, particular emphasis was given to strengthening personnel policies and to establishing a classified list of positions to be filled by candidates on a career basis, thus preventing the former excessive turnover in employment of technical personnel. It was suggested that a new accounting procedure be adopted, that the national highway system be divided into control sections about 100 km long for accounting and administrative purposes, and that the accounting system be designed to permit the accumulation of cost and road life data for use in future budgeting and programming. The system was to be arranged for easy distribution of expenditures, by control sections and by purpose of expenditure.

From information supplied by the Ministry of Public Works during the exploratory survey it was evident that the engineering personnel had received careful training in Turkish, European, and American universities, but practically nothing was being done, by laboratory and field tests, to study and control the construction materials used. A complete physical and chemical testing laboratory was ordered and immediate steps were taken to establish standards for studying, testing, and coordinating the use of materials.

Recommendations for roadway design were based on the fundamental condition that road standards, vehicle registration, and vehicle taxation must be in fair balance. Taxation should be at reasonable rates because the returns would result in transportation at lower cost and thus would warrant more material expenditures for maintenance, betterments, and new construction as the improved mileage increased.

To meet the national highway system requirements for the foreseeable future, designs for alinement, grade, cross section, and surfacing should be premised on fast-moving motorized traffic. The minimum standards recommended were based on estimated operating speeds for flat, rolling, and mountainous topography. Design speeds were 75 km per hr, 60 km per hr, and 45 km per hr. The sharpest curvatures were to be 6°, 10°, and 20° and the maximum grades were to be 4%, 8%, and 10% (for a maximum distance of 300 m in one continuous section). Minimum sight distances were to be 125 m, 100 m, and 70 m for both vertical and horizontal curves. The minimum width of all-weather surfacing was to be 6 m, with suitable increases at curves and on the more heavily traveled routes, and the minimum width of roadbed was to be 9 m.

Bridges were to have a clear width of roadway of at least 7 m and a minimum vertical clearance of 4.25 m. The standard loading was to be 13.6-metric-ton trucks suitably spaced. For design purposes, 4,100-kg pneumatic-tired wheel loads were to be used. The wheel load was to be considered as half the axle load.

It was recommended that continuing studies be made of all engineering standards that apply to various phases of highway design, maintenance, and construction and that coordination be established in all operating divisions by the continued issuance of memoranda covering engineering standards.

In developing standards for contract work, definite recommendations were made, largely in consonance with American highway practice: For control by the engineering section, for prequalification of contractors, for filing of job plans by contractors indicating staff and equipment, and for verification of financial responsibility.

In the interest of highway safety, it was recommended that a Committee on Safety be formed with the Commissioner of Highways as chairman and including as members representatives of the Ministry of Finance, Ministry of Education, and Ministry of Police. This committee should establish a traffic bureau to be operated by the Ministry of Police and staffed by specially trained personnel, and should immediately sponsor the preparation of a manual on uniform traffic control devices.

MATERIAL ACCOMPLISHMENTS FOLLOW

This general preview of the cooperative activities prompts the query as to what was accomplished by the sixteen men engaged in administrative and engineering duties and by the eleven equipment instructors and operators during 1948.

Administratively, the chief of the Department of Roads and Bridges and three deputies organized the department along the lines suggested in the report. A basic highway law was prepared and approved by the Council of Ministers of the Turkish government, for submission to the General Assembly in November, 1949. This law creates an autonomous department with continuity of financing as suggested in the report.

A further detailed and enlarged economic report was prepared, the agenda being kept up to date by the Planning Division. This report was helpful in giving the detailed information needed by the Turkish government when it applied to the Economic Cooperation Administration of the United States for a loan of $5,000,000 which would make possible the purchase of additional American equipment and pay administrative expenses. This loan was approved.

The highway budget for 1948 was 38,000,000 liras. The budget approved for 1949 was for 50,000,000 liras.

Study of the personnel, financial, and accounting procedures of the Department of Highways has been continued. Changes were made gradually but the complete readjustments could not be put into effect until after the approval of the basic highway law.

Late in 1949 the Planning and Programming Division consisted of about twenty engineers and analysts. A study was completed and approved covering a national highway system of more than 20,000 km. Maps were made for a suggested integrated system of provincial roads of about 21,800 km, to be submitted to the provincial governments for their consideration. It is estimated that the national system ultimately will be about 35,000 km long.

A physical inventory of the national system was completed and a preliminary traffic survey was made in 1948 to determine volume, classification, weight, origin and destination, and commodities carried. The commodity classification used was that of the Turkish Railway Administration. During the fall of 1949, three inventory parties and four traffic survey parties were in the field working on predetermined schedules.

These diverse field and office studies should be helpful in solving the economics of highway location and relocation which are many and varied in a country of such rough topography with such diverse living standards and pursuits.

The PRA Group received prompt and cordial cooperation from all departments of the ministries of Public Works, Agriculture, Finance, Economics, and Transportation. Many technical studies were available in the universities and in the technical schools of Ankara and Istanbul. Such fundamental and reliable facts as population trends, rainfall distribution, production and consumption of commodities, and classes and types of agriculture were quickly available. Many complete studies were furnished on present and potential mining developments, prospective irrigation and flood control projects, port development, and power development resources.

The work of the Division of Surveys and Plans was organized in accordance with the general practice of the Bureau of Public Roads and of state highway departments in the United States. Both field and office procedures were new to the Turks and courses of instruction were necessary. Late in 1949, the Department of Roads and Bridges had twenty-one survey parties in the field and twenty engineers in the central office section of the Division of Surveys and Plans. Modern American equipment is used. During 1948, complete field surveys were made on 450 km, center lines were established on 694 km scheduled for betterment work, and contract drawings were made or checked on 228 km for immediate construction by Turkish contractors.

The Bridge Division largely concerned itself with designs for the most important structures. It was reorganized to deal with all types with emphasis on relatively simple bridges such as steel pile bents, with creosoted timber decks, and on designs for masonry and concrete culverts. The preliminary estimates from the road inventory indicated that more than 80 km of bridges should be built within a decade.

During 1948, designs were made for steel pile bent structures, and local materials assembled for the first 35-m bridge. The steel came from the Turkish Karabuk Steel Mills, the timber from the Turkish Forest Service, and the creosote from the steel mills.

A crawler-type crane was equipped as a pile driver, the leads, hammers, and jigs being designed and assembled locally by the Turks under the advice of the bridge engineer of the PRA Group. The first bridge was completed in seven weeks by a trained Turkish bridge gang. Other bridge gangs then were organized. All cranes, compressors, welders, and miscellaneous items were of American manufacture but the pile leads, hammers, boom extensions, jigs, and tools were fashioned in Turkey.

The first reinforced-concrete culverts, new to Turkey, presented a problem. All manufactured materials for construction are in limited supply. Portland cement, for instance, is rationed on a priority basis and crushed stone is difficult to obtain.

The physical testing laboratory furnished under the program was installed in the Ministry Building at Ankara. A training program in laboratory and field procedures was inaugurated in December, 1947. The soils and bituminous and concrete sections, using Turkish personnel entirely, perform all necessary routine functions and tests. Local gravel and quarry sites are investigated, the bituminous materials of Eastern Turkey are tested, soils are studied, and standard tests are initiated on other materials of construction.

The Construction Division, as reorganized, directed the construction of three central machine shops during 1948 and had the added responsibility of revamping specifications and contract forms and methods, with particular emphasis on cost records. The division, for the first time, handled field engineering and inspection on contract work.

During 1949, the construction of nineteen additional division and district shops and of three division office buildings was completed in time to receive and install new equipment purchased under Economic Cooperation Administration financing. For the first time, winter overhaul of all construction and maintenance equipment is possible. Many capable Turkish contractors, experienced in railroad construction, have become interested in the highway field. Because of world conditions, they have been forced to use hand methods. Now construction equipment can be imported to expedite and thus make possible more work —at lower prices. There has been excellent cooperation with Turkish contractors on the work already undertaken. One Turkish engineer reported that the cost of projects built with American equipment was from 40% to 55% lower than the cost of similar projects built by the old hand-labor and team methods.

In the Maintenance Division (office and field) the addition of mechanical equipment has forced a complete reorientation of procedures. Any change, however, must be effected gradually because in a primitive country training of engineers and operators is essential prior to the arrival and the allocation of equipment. It has been found that the rate of absorption of equipment has to be given careful consideration so that it can be assigned for work immediately on arrival. For instance, it was necessary to train twelve complete units of men for the operation and maintenance of rock crushers and quarry layouts to make prompt use of those units on arrival. The same detailed training has been necessary for other equipment, such as compressors, drills, jackhammers, concrete mixers, bituminous distributors, road graders, steam shovels, cranes, bulldozers, and carryalls, not to mention automobiles and trucks of various kinds.

The adaptability of the Turkish worker and mechanic has been an essential aid in the development of the program. Road drags and jigs of various kinds have been built in quantity from designs made on the ground by the PRA Group; and foremen and mechanics have been trained and have accepted their new responsibilities.

During the first year, much of the operator training was done at the receiving center at Iskenderun on the Mediterranean. More than six hundred and fifty operators completed the training course of about six weeks. During the first year, the trainees built a 6-km distribution route around the City of Iskenderun.

The receiving center includes shops, warehouses, servicing stalls, storage facilities, and a spare-parts warehouse. The layout is well arranged for prompt assembly and dispatch of equipment—for instance, six trucks can be serviced, oiled, and greased at once. The receipt, storage in bins, and forwarding of spare parts is no easy task. It is necessary to translate the names of all items into an old or a new Turkish word that will become standard nomenclature and that will be readily understood both by the sender and by the receiver. The best American practices were taught and used.

SUMMARY

In this short paper, an attempt has been made to describe the progress made on a "Point Four" project initiated in late 1947. The Turks are a home-loving and a country-loving people. Their reactions on an active program for highways has been surprisingly like that of the average American and they are somewhat astounded by the rapidity with which their own engineers and workmen have been able to translate words into action. This assignment has proved that, through close cooperation, the PRA Group and its Turkish associates in the Ministry of Public Works have proceeded to plan and carry out in the field a highway program fitted to the Turkish economy, which in the short time of nine years should open up a complementary form of motor transport that will benefit the economy of all parts of the Turkish Republic.

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Paper No. 2478

HUMAN ASPECTS OF MEXICAN IRRIGATION

By Antonio Rodríguez Langoné[1]

Synopsis

When more than 90% of a country is deficient in natural water for agriculture, when arable land is badly needed, and when private financing is inadequate or undesirable, the state must shoulder the national burden. Mexico is doing just this, making great progress, especially since 1926. Legislation to meet local needs has been continually revised, particularly to benefit the smaller landowner, but at the same time to eliminate the uneconomical farm of less than 5 hectares. The government runs the large projects; and the water users, the smaller ones, adjusting water distribution and canal maintenance to meet local needs. Continuous repayment of development costs by landowners is a basic policy. Industry as well as agriculture has been greatly benefited, and living conditions have been materially improved.

Why Irrigation Is Essential in Mexico

Most of Mexico needs irrigation. About 93% of its surface is arid or semiarid—that is, rainfall is not sufficient for the growth of crops. The agricultural and physical aspects can be divided into three great regions—

(*a*) The North Zone: In this zone soils and topography are favorable for irrigation but rainfall is scarce and its monthly distribution is inadequate for the growth of crops. Rivers are mostly torrential in character;

(*b*) The Central Plateau: This area is not suitable for extensive agriculture since the valleys are small with rolling hills and most of the rivers and creeks are torrential. Irregular and insufficient rainfall results in uncertain crop yields;

(*c*) The Atlantic and Pacific Coastal Plains: These plains have excellent soil and are very attractive for agricultural development. However, sanitary conditions need to be greatly improved in the southern part before serious farming can be undertaken.

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Although irrigation has been practiced in Mexico since the time of the Aztecs, not until 1926 was a serious irrigation program initiated by the government. Mexican lands can be roughly classified as follows (1 hectare = 0.404687 acres):

Classification	Million hectares	%
Arable land	23.4	11.9
Pasture (in flat or rolling country)	66.5	33.9
Pasture (in mountains or hilly country)	30.0	15.3
Forests	45.0	22.9
Deserts	31.5	16.0
Total	196.4	100.0

GENERAL POSSIBILITIES

From hydrological studies made by the Ministry of Hydraulic Resources of Mexico it appears that approximately 6,800,000 hectares (16,800,000 acres) can be irrigated through storage dams assuming that the dams can utilize about 80% of the mean stream flow. By adequate use of underground water, 1,000,000 hectares (2,500,000 acres) more could be added to the irrigable area. The addition of an approximate area of 2,000,000 hectares (5,000,000 acres) of land in the humid region where irrigation is not necessary gives a total of 10,000,000 hectares (25,000,000 acres) of land with adequate water supply. As arable lands in Mexico are limited to about 23,400,000 hectares (57,800,000 acres), 13,400,000 hectares (33,100,000 acres) would remain "temporal" (dry farming) lands with the water supply relying on uncertain rainfall.

The area with assured water supply means an area of 0.4 hectare (1 acre) per inhabitant for a population of 25,000,000. A cropping area of 23,400,000 hectares would mean a surface of 0.93 hectare per inhabitant (2.29) acres which is lower than the area of 1.2 hectares (2.9 acres) per inhabitant with adequate water supply in the United States.

AIMS OF MEXICAN IRRIGATION POLICY

Irrigation is necessary in Mexico not only because physical conditions require it, but also because it meets the need for a better distribution of the rural population. In addition, it is one of the best methods of improving the national economy.

Since water is scarce, very careful planning of irrigation projects is imperative. Every drop of water available will be utilized in the proper irrigation, hydroelectric, and multiple-use developments for the benefit of agriculture and industry. The Mexican government believes that irrigation works constitute the "means toward the end" of improving its agriculture and the economy of the rural masses but that the correct interpretation and management of the physical and human aspects involved and the adequate organization and operation of the resulting irrigation districts constitute the final purpose of its irrigation policy.

To this end irrigation projects should be conceived and planned from a sociological viewpoint. From the beginning every step concerning the problem of supplying water to irrigate the land should be coordinated with the problem of organizing the community that is to settle the land.

ASPECTS OF SETTLING

Mexican Experience Under 1926 Irrigation Law.—To obtain a fair understanding of Mexican experience in relation to settling irrigation districts, two periods should be considered: The initial period covered by the 1926 Irrigation Law and the present (1949) period operating under the law of December, 1946.

On some of the irrigation projects constructed and developed during the initial period, such as the Don Martin project (Fig. 1(*a*)) in the states of Coahuila and Nuevo León and the Conchos River project in Chihuahua, the land was in the hands of a few large owners; and therefore it was possible to design an adequate system of secondary canals, drains, and roads. On other projects where the land was greatly subdivided, like the Tula project in the State of Hidalgo, the secondary system of water distribution had to be adapted to very irregular and small holdings.

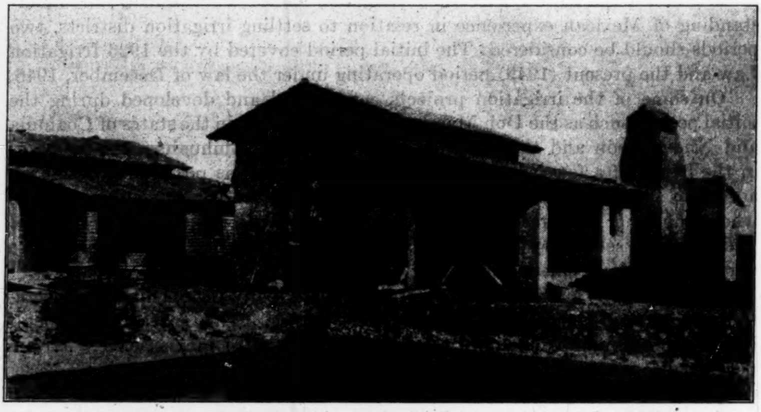
By 1930 many of the irrigation projects were ready for opening and operation. The problem of settling immediately arose. This enterprise, difficult in itself, was made even more troublesome because of inexperience. It was necessary to introduce measures which had not been tried and to issue a series of special rules. Some of these rules had to be tested in the field and adapted to the social and economic conditions prevailing in the region—in every case efforts were made to obtain the best results. All these measures had, as was to be expected, aspects of both success and failure.

Early settling at the Don Martin and Conchos projects was rather slow; applicants seemed loath to settle the new irrigated lands. Later, however, in 2 years or 3 years, the situation changed entirely; and, ever since, the demand for irrigated land has been so great that it cannot be met.

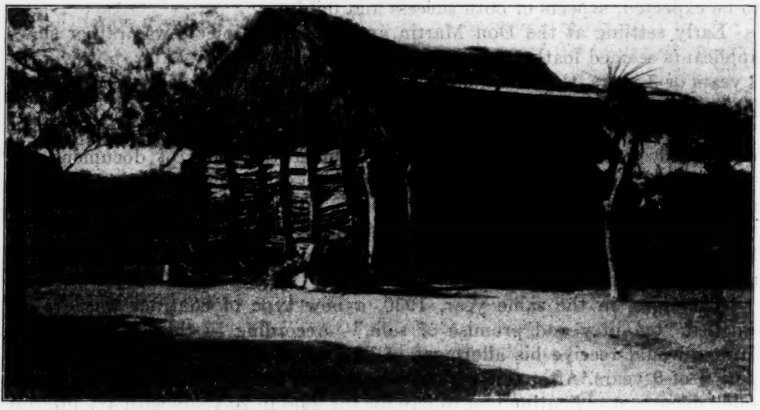
Several Forms of Contract.—The first type of contract drawn for settlers was called "promise of sale" contract. Under this kind of document the settler had to pay in advance 5% of the total value of the land and the remainder in twenty-five annuities with 4% interest to be computed on the unpaid balance. Original lack of demand for the irrigated land led the National Irrigation Commission to think that poor prospective settlers could not afford the initial 5% payment.

Therefore, in the same year, 1930, a new type of contract was drawn, entitled "tenantry and promise of sale." According to this agreement the farmer would receive his allotment of land in the character of tenant for a period of 3 years. After this period the contract would become a "promise of sale" document obligating him to pay for the land in 20 years with 4% interest on the unpaid balance. The results of this type of contract seemed satisfactory at the start and a great number of settlers was established in irrigation districts. In other irrigation districts which were already settled by original owners, like the Tula District in the State of Hidalgo, no settling problems had to be solved.

Experience showed, by 1932, that this policy for obtaining very rapid settling, together with the lack of an adequate set of regulations, had resulted in serious abuses by applicants and settlers. Drastic changes were necessary and strict regulations had to be issued specifying prerequisites for applicants, settlers' rights and duties, and various other measures to obtain discipline and to avoid the award of land to people not living within the district who might try to cultivate the property through tenants.



(a) Settlers' New Homes in Don Martin Irrigation District, Coahuila and Nuevo León States



(b) Old Cottage in Lower Rio Bravo District, State of Tamaulipas

FIG. 1.—FEDERAL IRRIGATION HAS IMPROVED LIVING CONDITIONS IN MEXICO

During 1932 three types of contract were established: The "cash down sale," the "sale type A," and the "sale type B" forms. A maximum area of 100 hectares could be brought under the first type mentioned. Also a maximum of 100 hectares could be purchased under the sale type A contract with 5% of the total value of the land required as a down payment and the remainder to be liquidated in 25 years, with 4% interest to be computed on the unpaid balance. Only 25 hectares could be acquired under the sale type B contract which required no down payment; but the settler had to turn over enough crops to pay for 4% of the total land value every year plus operation charges. In both types A and B contracts no tenancy was permitted.

Later on, after the operation of the irrigation districts was turned over to the National Agricultural Bank, a new type of contract was drawn up under the title of "sale contract with mortgage security."

All these later types of contract have been fairly satisfactory. Still, tenancy has not been entirely abolished and difficulties and delays occur when it becomes necessary to eliminate unsatisfactory settlers through legal procedures.

Settling Experience Under the 1946 Irrigation Law.—In the short period since the 1946 law was issued, the results can hardly be analyzed accurately. The law provides that original landowners with more than 100 hectares have to subdivide the part of their property that exceeds this amount within a year of the official establishing of the district. In case they fail to comply they have to sell the excess area to the federal government who will arrange for settlers. Although the construction of many irrigation districts—for example, the Laguna District in the states of Coahuila and Durango, the Culiacan District in Sinaloa, the Lower San Juan River District, and the Lower Rio Bravo District—was begun under the 1926 law, operation, settling, and repayment charges were established under the 1946 law. Some of the irrigation districts do not have settling problems since all the irrigated land is in the hands of either agrarians or original owners in small holdings.

The Lower San Juan River and the Lower Rio Bravo districts are fully occupied. Settlers have been established on lands even before construction was finished, with permission to clear their farms and risk a cotton "temporal" crop. The Agrarian Department has also settled some of the land in the last two mentioned districts with farmers from overcrowded regions like the State of Tlaxcala. In spite of original difficulties, abuses, and some speculation, land is substantially subdivided, as shown by the following data: Out of the 574,127 hectares under cultivation in 1943, 336,113 hectares or 58.6% is in the hands of agrarian interests or settlers with holdings up to 10 hectares; 196,098 hectares or 34.1% belongs to settlers or owners of from 10 hectares to 20 hectares; 32,739 hectares or 5.70% belongs to settlers or owners of from 20 hectares to 50 hectares and only 9,176 hectares or 1.6% represents holdings of more than 50 hectares. It is to be noted that 92.7% of the irrigated land is owned by holders of less than 20 hectares.

It has been stated that it takes three generations of farmers to settle irrigated land. This might even be true in Mexico, although, in various districts that were settled from 1930 to 1935, a good many original settlers still hold the land.

To Improve Settling Policy.—Settling regulations for new regions or districts where integral development is contemplated, like the Papaloapan Project, should be improved so that:

1. Speculation of original landowners who resell at very high prices and thus reduce the settlers' capacity to pay for construction charges can be avoided.
2. True farmers are established as landowners.
3. Uneconomical holdings of less than 5 hectares are readjusted.

General Operation and Maintenance Aspects

The success of irrigation districts generally depends on adequate operation. The term "operation" is used in Mexico in a very broad sense to mean the general managing of the irrigation district—including settling; distribution of water; maintenance of channels, drains, service roads, and structures; as well as agronomical control. Operation in its true sense should include only water distribution, maintenance of irrigation systems, and operation charges.

The National Irrigation Commission was in charge of operation until 1935 when it was turned over to the National Agricultural Bank. By 1937, the commission began to operate some districts where "ejidos" predominated and in 1944 operation by the Commission was reassumed in all irrigation districts. (The King of Spain granted lands called "ejidos" to Indian villages and towns in Mexico. These lands could be used but could not be sold. The Mexican Revolution released these lands to the poor farmer and, in addition, granted other lands to him on similar terms.) Under the 1946 Irrigation Law, operation is entrusted to the Ministry of Agriculture. The Ministry of Hydraulic Resources (formerly the National Irrigation Commission) is supposed to turn over operation to the Ministry of Agriculture as soon as a unit is ready for water delivery. Operation by water users, who eventually will take complete charge, is regulated by law.

Large irrigation districts are operated by the government, and water users are being organized in such a way as to make it possible to turn over to them lateral systems and eventually the whole district. Important structures like multiple-use reservoirs, diversion dams in some cases, and main canals in special cases, will be excepted and will always be controlled by the government. Since small irrigation projects are so numerous, it is economically impossible for the government to operate them; and therefore they are turned over for operation to water users, with some special exceptions, as soon as construction is finished. Regulations are established for water distribution, and periodical inspections check on the operation and maintenance of the district.

Water Distribution.—Water distribution is a very important item of operation. Rules for adequate water distribution have been issued since the beginning of the irrigation program. These regulations are modified as indicated by need or by experience. Since irrigation districts were established in 1930, regulations have set up water users' associations that, among other activities, assist the management to enforce water distribution rules.

As a new development, administrative councils have been organized on which water users and the proper authorities are represented. These councils formulate the annual irrigation schedule and discuss operation charges and general matters in connection therewith.

Different methods of water distribution have been used, such as "continuous service," "rotation among users," "rotation among distributing canals" (especially used in torrential streams where no storage has been constructed), and "free demand." Service on demand was established at the beginning, the only restrictions being that the district was authorized to deliver water in accordance with crop needs and that a number of the water users had to ask for water in any lateral to obtain service.

The demand method resulted in serious inconvenience. Water was wasted because it was necessary to maintain high head on delivery canals. Many people have to be employed to meet the demand requests; and, of course, simultaneous demands are hard to fulfill. Although it was planned as a transitory system, the demand method has been very difficult to change because there always seems to be a reaction against any radical revision of customary procedures.

It has been much easier to enforce good water distribution rules in irrigation districts like Don Martin (Fig. 1(*a*)), Conchos, or Lower San Juan where the former National Irrigation Commission had designed the subdivision. On the other hand, where land was extensively subdivided into small and irregular holdings and where distribution laterals had to be laid out in connection with property lines or property layout, water distribution became very wasteful and difficult to control.

Maintenance of Canals and Structures.—Good maintenance is of course essential for efficient operation of irrigation districts. Carelessness can result in serious crop losses or in endangering irrigation works or even in failures of canals and structures. A few aspects of this problem will serve as illustrations.

Conduction losses in irrigation ditches have been observed in various districts. Preliminary data show percentage losses of from 1.71% in the Tijuana District (Fig. 2) to 50.08% in the Conchos District as observed during 1939 and 1938, respectively—the Tijuana District being provided with concrete-lined canals. The losses per kilometer of canal run from 0.22% in Tijuana to 1.89% in the east canal of Rio Mante District.

Several methods have been followed for clearing irrigation ditches or drains—drag lines, scrapers, "fresnos," and hand shovels all have been used depending on the size of the canals and the regional cost of labor. Another problem is the vegetation growth in the Conchos main canal. The capacity of the Conchos main canal has been reduced sometimes as much as 65%. This canal was designed with a roughness coefficient n of 0.0225 and the laterals have coefficients of 0.026. Because of the great growth of water weeds the coefficient has been as great as 0.131. Chains have been used to fight the weeds but results have not been entirely satisfactory; disk harrows combined with chains have also been tried as was muddling the water with clay (although not extensively, since salts in the water made the clay coagulate). Commercial chemicals have been found too expensive; and naphtha emulsion has been advised but has not been tested.

Operation Charges.—Operation charges are collected on several bases: Per volume of water used, per irrigated hectare (annually), and per hectare per times of irrigation. Although the volumetric charge is perhaps the best method for collections since it promotes an economic use of water, it is not very widely used because equipment is required for accurate measurement of the water delivered as well as the good cooperation of water users.

Social, economic, and disciplinary circumstances made it impossible for some time to fix operation charges that would balance expenses. All efforts made by the operating institutions have been aimed at gradually balancing the operation budget. Therefore, the National Irrigation Commission and the Ministry of Agriculture have gradually increased charges, to the point that the government's subsidy for operation is very small.

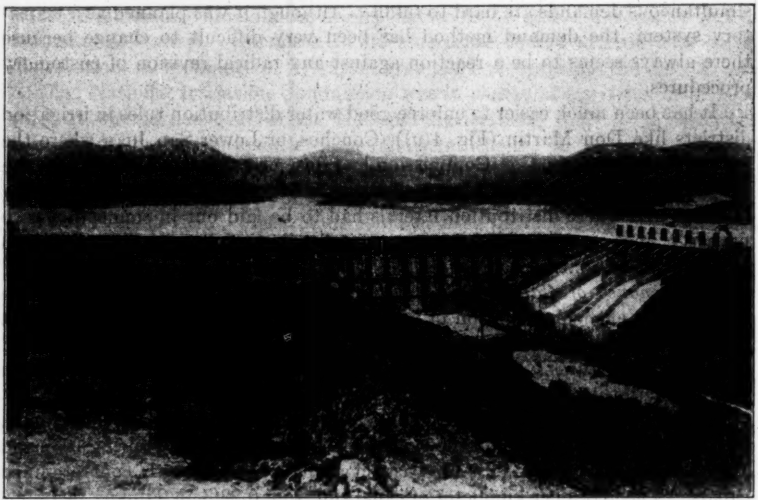


Fig. 2.—Main Unit of a Successful Irrigation Project, Abelardo Rodriguez Dam, Tijuana District, Lower California

From experience thus far it can be concluded that all irrigation canals and structures should be very carefully observed, particularly for 2 years or 3 years after operation is begun. A special sum should be set aside for maintainance during this period and probably should be charged as "operation during construction." Close coordination should be established between the builders of irrigation systems and the people in charge of operation.

Economic Aspects

Repayment of Cost.—If world-wide experience in irrigation, particularly in the United States and Mexico, is analyzed, it is realized that early irrigation represented the efforts of a community to subsist without any idea of business or gain. Later on, private enterprise tried, without success, to develop irrigation for business purposes. Nowadays irrigation is generally considered as a national project whose purpose is to improve the agricultural and economical conditions of rural masses and consequently to benefit the nation as a whole. Several reasons account for this:

1. Irrigation requires the investment of enormous amounts of money. Direct total repayment is almost impossible, with the exception of some projects where a great quantity of hydroelectric power can be developed.

2. Capital invested in irrigation works does not produce returns for a long period depending on the time required for construction, settling, and organization of production.

3. Private enterprise would have to plan irrigation systems to fit definite purposes whereas government projects are planned for multiple and integral uses.

From world-wide experience, as related to Mexican demographic, social, land ownership, and economic conditions, it can be concluded:

a. Irrigation and drainage works in Mexico have to be developed mainly by the federal government.

b. Conclusion *a* does not hamper private initiative—especially in the cases of hydroelectric plants, individual pumping projects, cooperative irrigation developments, rural credit companies, industry, and so forth.

c. Private enterprises should be supervised by the state so that all hydroelectric or irrigation developments fit the national plan and also for the purpose of avoiding undue speculation.

For comparative purposes some items of American experience on repayment of cost of irrigation should be mentioned. Originally it was thought in the United States that a 10-year term would be sufficient for repayment of charges assigned to water users. Soon it became evident that such a period was very short. In 1914 terms were extended to 20 years; in 1924 terms were again extended and other rules were issued to ease payment conditions; and in 1926 the period for repayment was made 40 years. According to John A. Widsoe, by 1922 only 10.9% of construction cost had been paid, although the punctuality index was 85.8%

Conditions seem to have improved according to a 1947 publication of the Bureau of Reclamation,[2] United States Department of the Interior, which indicates that 94% of the total investment can be repaid; and of this percentage 21.6% corresponds to contracts signed by water users of irrigation projects and 49.4% corresponds to contracts made with users of electrical energy.

Again, in Mexico, experience concerning repayment or irrigation costs is divided into two periods—that under the 1926 Irrigation Law and that under the 1946 law. The 1926 law provided that landowners of areas with more than 150 hectares should pay the government for the irrigation charge with a part of their land in such a way that they would retain an amount of irrigated land which would have the same value as that of their original total property previous to irrigation. For owners of less than 150 hectares the same law provided that they should pay in the way that would be fixed for each case (generally in annual payments).

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[2] "How Reclamation Pays," Bureau of Reclamation, U. S. Dept. of the Interior, Washington, D. C., 1947.

Because of a number of circumstances, especially the practical impossibility of complying with the process established by the 1926 law (since strict enforcement of procedures would have meant a serious delay), only in exceptional cases was it possible to follow the legal procedure. Also much speculation took place, particularly by original owners. Therefore it was necessary to buy land and, in extraordinary instances, to expropriate it. As soon as lands were ready for settling, a selling price was fixed. The price ran from 60.00 pesos per hectare to 260.00 pesos per hectare to be paid annually with 4% interest on unpaid capital. Terms were from 20 years to 25 years.

During this period irrigation cost per hectare averaged from 500.00 pesos to 700.00 pesos and therefore repayments assigned represented:

Cost per hectare, in pesos:	%
500.00	14 to 52
700.00	10 to 37.2

Prices for irrigated land were lower than actual cost but were graded so that settlers would be able to meet obligations and also so that the increase in public wealth would result in indirect payment of part of the investment.

A general idea of settlers' capability to meet obligations during the early period of operation is presented in Tables 1 and 2. From the study of Table 1 it can be concluded that, for this earlier period:

1. It was almost impossible for settlers to pay 100% of the cost for holdings of less than 20 hectares.
2. It was possible for settlers to pay charges of from 200.00 pesos per hectare to 300.00 pesos per hectare since annual costs would be from 12.80 pesos per hectare to 19.20 pesos per hectare.

By 1945, the 1935 charges were really low if the increase in crop values, shown by Table 2, is considered. Prices for irrigated land were fixed for the Don Martin, Conchos, Palestina, Pabellon, and Tijuana districts during the first period. By 1943, 41.9% of amounts due for contracted lands had been repaid. The 1946 law radically changed procedures in relation to repayment of cost. It provided that owners of areas greater than 100 hectares must subdivide the excess within a year of the official opening of the irrigation district. If they failed, their excess land must be sold to the government.

TABLE 1.—SETTLERS' RETURNS FOR IRRIGATION IN MEXICO IN 1935

Size of holding, in hectares	Apparent profit,[a] in pesos	ANNUAL PAYMENT,[b] IN PESOS	
		With 4% interest	Without interest
4	417.48	179.25	112.00
10	1,043.70	448.08	250.00
20	2,087.40	896.17	560.00

[a] 50% cotton; 50% wheat. [b] 25-year terms for a cost of 700 pesos per hectare.

There is an irrigation fee generally smaller than the actual irrigation cost on lands remaining in the hands of original owners and on those sold to settlers either by the government or by the original owners. This fee is determined on the basis that commercially irrigated land prices in the neighboring regions increase through irrigation, because of added productive value of land, and especially because of the farmers' capacity to meet obligations. Before the irrigation charge is officially fixed, the Ministry of Hydraulic Resources hears the opinions of the water users.

This procedure has been established since 1947 and by April, 1949, the irrigation repayment fee was fixed for new units of the Conchos, Rio Culiacan, Bajo Rio Bravo, and Bajo Rio San Juan districts. Legal action has been begun on many others.

The 1948 irrigation charges average from 500.00 pesos per hectare to 700.00 pesos per hectare. The average cost of irrigation per hectare is about 1,500.00 pesos. The charge represents from 33% to 46% of present average cost. The 1948 value of crops is given in Table 2.

A study to be completed is concerned with whether or not the irrigation repayment charge should be considered together with the water charge—that is, whether the volumetric water charge would cover operation cost and construction charges over a certain period of years.

TABLE 2.—INCREASES IN THE VALUE OF IRRIGATED LANDS IN MEXICO

Year (1)	Irrigated area[a] (2)	VALUE OF CROPS, IN PESOS	
		Annual (3)	Unit[b] (4)
1935...	162,266	31,301,549	192.50
1945...	554,214	349,024,533	630
1948...	759,966	637,936,634	840

[a] Land in national irrigation, in hectares (1 hectare = 0.404687 acre). [b] Gross value in pesos per hectare (1.00 peso per hectare = 2.471 pesos per acre).

Because farmers recognize the necessity for storage works and because public funds are limited, it has been possible to finance parts of irrigation systems like main canals or distribution systems by direct cooperation from the farmers. In these cases water users either sign contracts for repayment of cost on 2-year of 3-year terms which can be negotiated with some credit institution, or they pledge themselves to buy bonds issued for the special purpose. Water users can use these bonds to pay for contracted obligations of repayment for irrigation works.

Rural Credit in Irrigation Districts.—Although rural credit in irrigation districts is very important, the size and scope of this paper will only allow a general presentation of the problem.

As soon as an irrigation project is opened for settling or for cultivation and when the lands are already subdivided, agricultural, economical, and social work really starts, and complex problems begin to appear. Selection of settlers, rural population redistribution, organization of agricultural production, and choice of crops are among the important problems. Another is rural credit.

According to the special characteristics of the agrarian problem, official banks have two different activities: One strictly as a financial institution, although without profit, and the other as manager and supervisor of agricultural plans. "Ejido" lands are attended to by the National Agrarian Credit Bank and the small holdings are served by the National Agricultural Bank. Credit for special crops like sugar cane and cotton is extended in many cases by private enterprises and banks. Private capital is also appropriated through the official agricultural banks. In some cases like the Culiacan, Rio Yaqui (Fig. 3), Rio Mayo, Rio Colorado, and Lower California districts, regional credit unions have been organized, in which the farmers are stockholders. Such credit unions, if properly managed, should finally solve the rural credit problem in the irrigation districts.

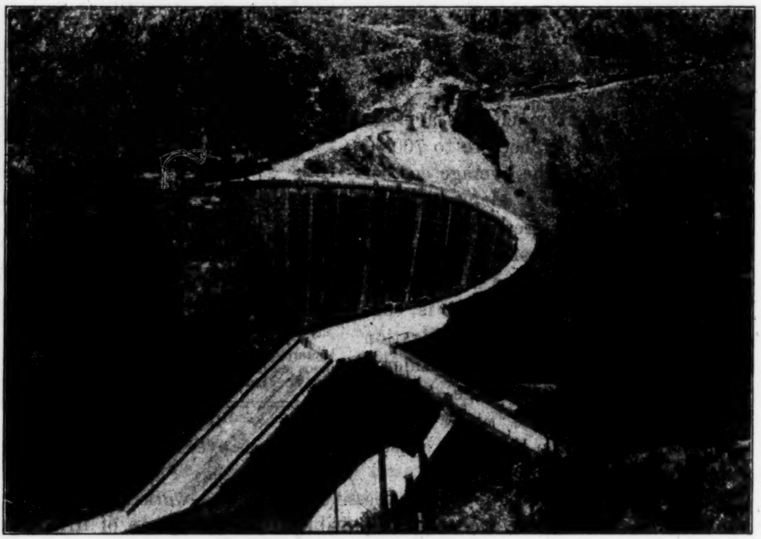

Fig. 3.—Picturesque Structure Impounds Main Reservoir; Angostura Dam, on Rio Yaqui Irrigation District, State of Sonora

General Results of Mexican Irrigation

From an agricultural point of view the 1949 increase of more than 1,000,000 hectares and the addition of another 1,000,000 hectares within the succeeding few years should cover the demand for agricultural products in Mexico. Farm production values have been increased from 50% to perhaps 100%. This planned irrigation advance, besides settling some half a million people, thus far has promoted the creation of industries (especially those related to agricultural products) and has encouraged commercial activities. Socially it has improved the living conditions of many thousands of people and has helped redistribute some of the rural population.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 2479

SOUNDING METHODS FOR SEDIMENTATION STUDIES

A SYMPOSIUM

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SUPERSONIC SOUNDING INSTRUMENTS AND METHODS

By Joseph M. Caldwell,[1] A. M. ASCE

Synopsis

A brief history of the development of echo sounders is given. The distinguishing features of various basic types of sounders are discussed, together with the elements of echo sounder operation. The inherent limitations of echo sounding are pointed out with certain supporting data to enable some of the limitations to be determined on a quantitative basis. The use of echo sounders over firm sand bottoms and over soft mud bottoms is discussed, and factual data are introduced to show the "average" error to be expected in repeated soundings of the same profile. A comparison is made between soundings of a selected profile as obtained by echo sounders and by lead line, and the comparative results from the two methods are given in considerable detail.

Introduction

The desire to increase the rate of making soundings in hydrographic survey work has probably existed since men first began to chart the navigable waters of the earth. The first recorded suggestion that underwater sound waves might be used for such a purpose was made by a French physicist, Dominique François Jean Arago, in 1807. Although numerous attempts were made during the remainder of the century to evolve a method of measuring depths by sound waves, 100 years elapsed from the time of Arago before anyone was successful in perfecting even a crude echo sounder. These first successful developments were made, strangely enough, not as echo sounders but as iceberg detectors following the sinking of the *S. S. Titanic* in 1912.

With the incidence of World War I and submarine warfare, knowledge of underwater acoustics was increasing rapidly and by 1925 several forms of echo sounders had been put into use. One of the most promising developments, in 1925, was a device that enabled extremely short time intervals to be measured accurately, and to be converted automatically to a depth measurement. This development adapted the echo sounder to shallow water hydrographic studies, and the feature was incorporated in Fathometer model No. 312 by the United States Coast and Geodetic Survey (USCGS). The No. 312 Fathometer was still used on some ships of the USCGS as late as 1941.

Two of the principal users of echo sounders for hydrographic survey work in the United States are the Corps of Engineers, United States Department of the

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Note.—Published in December, 1950, as *Proceedings-Separate No. 50*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Hydr. Engr., Beach Erosion Bd., Washington, D. C.

Army (USED), and the USCGS, the former being concerned chiefly with inland waters and the latter, with coastal waters. The remainder of this paper will be devoted, for the most part, to the experiences of these two organizations with echo sounders.

TYPES OF SOUNDERS

There are at least twenty-five or thirty features of construction and operation which make one type of sounder different from another. However, three of the most important features that identify a particular sounder are: (*1*) The portable or fixed instrument; (*2*) the visually indicating or recording instrument; and (*3*) the frequency of the signal, or note used as the sound signal.

1. Portable or Fixed Instrument.—Generally, the portable instruments weigh about 200 lb and can be moved from craft to craft with little difficulty. The fixed installations are much more massive (approximately 800 lb) and are more or less permanently mounted aboard a suitable survey craft.

2. Visually Indicating or Recording Instrument.—The indicating instruments give only a transient indication of depth and therefore require an attendant to record the data. The recording instruments, on the other hand, furnish a permanent record which is obviously advantageous in hydrographic survey work. Most of the instruments manufactured in the United States are of the recording type.

3. Frequency of the Signal or Note Used As the Sound Signal.—If the sound note is in the ordinary, audible, musical range the instrument is considered to be a sonic sounder. If the note is near the top or above the audible sound range (14,000 cycles per sec or more), it is considered to be a supersonic instrument. The advantages of the two frequency ranges are discussed subsequently in this paper.

PRINCIPLES OF OPERATION

Briefly stated, the essential components of an echo sounder are a signal generator, a signal transmitter, an echo receiver, and an amplifier-recorder. The first and last items are frequently put in the same box and are referred to simply as the sounder. The other two items are usually mounted together or near each other, and the combination is called "the transceiver." The sequence of operations in obtaining a sounding is as follows:

a. The signal generator initiates by electronic methods a short, but strong, electrical impulse which is sent to the signal transmitter.

b. The signal transmitter, which is mounted underwater, converts the electrical impulse to a mechanical vibration or sound wave which is directed toward the bottom.

c. The sound wave reaches the bottom and is reflected as an echo.

d. This echo is detected by the signal receiver which is also mounted underwater. The receiver converts the echo into an electrical signal which is sent on to an electronic amplifier.

e. The electronic amplifier receives the signal and amplifies it sufficiently to make it discernible on a suitable indicator or recorder.

f. The indicator or recorder receives the signal from the amplifier, measures the time elapsed from the generation of the sound signal to the return of the echo, converts this time interval to equivalent depth of water, and furnishes an indication or record of this depth.

It should be realized that the foregoing steps are repeated automatically by the instrument so that, in shallow water, soundings may be taken at the rate of 100 per min or more.

Limitations in Echo Sounders

Several factors limit the application and accuracy of the instruments. For instance, the speed of the survey craft must be slow enough to avoid entraining air under the bow which will find its way under the vessel. Air bubbles will form a sound block and may cause the record to show erroneous readings, or to be lost entirely.

TABLE 1.—Effect of Frequency on the Directional Character of a Sound Beam

Frequency, in cycles per second	Dimensions of Sound Cone	
	Angle, a	Ratio, diameter to depth
(1)	(2)	(3)
8,000[a]	94°	2.14
16,000[b]	43°	0.79
32,000[b]	21°	0.37
64,000[b]	10½°	0.18

[a] Sonic. [b] Supersonic.

Another limitation is the angular size of the sound cone. Generally, the surveyor would like to measure the water depth directly under the hull. However, the sound signal sent out by the sound transmitter is in the form of a cone whose central angle, a, varies with the radius, r, of the transmitter plate and the frequency of the sound signal. Its value can be calculated from the formula:

$$\sin \tfrac{1}{2} a = 0.61 \frac{L}{r} \quad \ldots\ldots\ldots (1)$$

in which L is the wave length in water of the sound signal used. The significance of the frequency is illustrated in Table 1, which is based on computations for a sound transmitter plate 12 in. in diameter ($r = 6$) in sea water at 37°F, or in fresh water at 65°F.

One common type of sounder for which $r = 0.5$ ft operates with a frequency of about 16,000 cycles per sec for the sound signal, giving a wave length of about 0.30 ft in fresh water at 65°F. If these values are substituted in Eq. 1, the sound cone angle is computed to be about 43°. With this size of sound cone, the area scanned or covered by the cone at any instant has a diameter of about eight tenths of the depth of the water below the sound transmitter. For instance, if the depth below the transmitter is 20 ft, the area being scanned by the sound cone has a diameter of about 16 ft, and the instrument will indicate the distance to the object within the sound cone nearest the transceiver. Where the hydrography is relatively flat, this fact is of no great concern, but it may introduce considerable error where steep underwater gradients are encountered. In fact it can be shown that the aforementioned sounder will indicate a depth approximately 6% less than the true depth at the junction of a flat bottom and a 1:2½ slope; under these conditions this difference amounts to an error of about 1 ft in 20-ft depth of water. This 6% error would be evident at all points over the 1:2½ slope and, of course, would become greater for steeper slopes, being in the order of 12% for a 1:1½ slope. For higher sound signal frequencies, this inherent error would tend to become less, whereas for lower frequencies it would tend to become somewhat greater. As an example, consider Fig. 1, in which point E denotes an echo sounder having a frequency of 16,000 cycles per sec, and in which $r = 0.5$ ft. Although the true depth under the echo sounder is line EB and the shortest distance from the echo sounder to the bottom is ED, the depth recorded by the echo sounder will be line EC, since this is the shortest distance to the bottom within the sound cone.

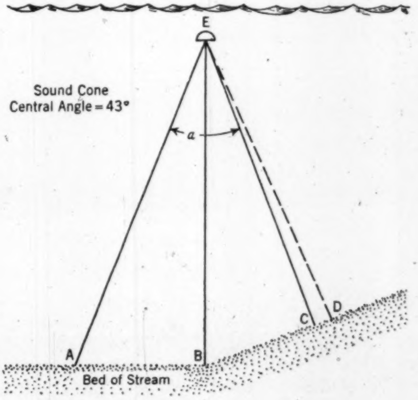

FIG. 1.—EFFECT OF BOTTOM SLOPE ON THE ACCURACY OF RECORDING BY ECHO SOUNDER

Another limitation in the use of echo sounders is the strength of the echo necessary to enable the amplifier-recorder to detect the echo and give a usable record. When working in shallow water, say, less than 200 ft, and over a clean rock or sand bottom, a sharp, well-defined echo is returned to the surface. However, the top layers of an unconsolidated mud bottom may return such a faint echo that the sounder may not detect this initial echo; instead the sounder may detect, as the bottom, a point down in the material which has become sufficiently consolidated to return a usable echo. To complicate this picture further, this discernible echo point can be shifted up or down, in some cases, by increasing or decreasing the gain on the amplifier. In such cases the recorder chart can be made to show a noticeable spread of depth when operating over a single point simply by changing the amplifier gain control.

Another basic limitation in the operation of echo sounders is the absorption of sound energy by the water. As a result of this absorption there is a limit to the depth to which the sounder can operate because of the loss of strength of the sound signal and the echo. This factor is of little concern in inland and coastal work, however, as practically all sounders will work satisfactorily to depths of 500 ft or less. It is of real concern, of course, in charting the great ocean basins. In this connection, it should be noted that the higher the frequency is, the greater will be the absorption of the sound by the water. This characteristic, in effect, controls the upper limit of the frequency that is practicable to use in various depths.

Generally, the supersonic sounders are preferred because of the directional character of the sound beam and because of their relative freedom from the influence of random engine noises and similar disturbances.

Adoption of Echo Sounders

The USCGS first used echo sounders in 1923 when the survey ship *Guide* was equipped with an early model for a trip from Norfolk, Va., to San Diego, Calif. Reliance on these instruments has increased steadily since that time and all USCGS survey ships are either equipped with fixed sounders or are adapted to utilize portable sounders. It is the basic sounding method used at present by that organization. Until 1939, all instruments used by the USCGS had been of the indicating type. In 1939, a recording sounder of British manufacture was installed on one of its ships, and the popularity of the recording type has increased rapidly since that time.

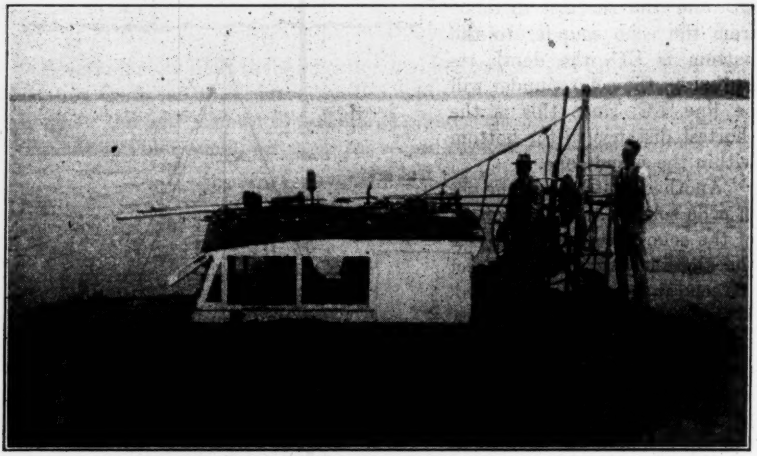

Fig. 2.—"Transceiver" for Portable Echo Sounder Mounted on a Survey Boat of the Vicksburg (Miss.) District, Corps of Engineers

The USED began to use echo sounders in the early 1930's and their use by the Corps has increased steadily since that time. At present (1950) the greater part of the hydrographic work of the USED is conducted with the aid of supersonic echo sounders operating at frequencies of the order of 15,000 cycles per sec. The portable recording sounder is in general use in the USED because it is best adapted to the wide variety of hydrographic work to which any survey crew may be assigned from time to time. Fig. 2 shows an installation of a portable sounder transceiver on a USED craft; the transceiver is submerged during actual sounding operations.

It should not be inferred from the foregoing statements that the echo sounder has met with immediate acceptance by those engaged in hydrographic survey work. Like most new instruments and methods, the sounder has had to earn its way into widespread acceptance. Some opposition was justified, principally because of various imperfections and limitations in the earlier models; yet, part of the opposition was more of a personal reluctance to adopt the new, rather than any coldly logical objection. When it is remembered that, before the introduction of the echo sounder, the techniques of hydrographic survey crews had been more or less stabilized for many years, some of the objections that arose to the use of the sounder are understandable. These older techniques of the sounding line and the sounding pole were simple and well established, the survey crews understood them, and the members of the crew felt well qualified and relatively secure in their jobs. When suddenly confronted with a complicated, electrical device with vacuum tubes, pigtail resistances, and condensers the members of the survey crew suddenly felt themselves unqualified and insecure. Thus, when the new echo sounder broke down or showed evidence of giving an erroneous set of soundings, the crew was all too ready to condemn it as a waste of time and to return to using the lead line.

Another objection to the echo sounder has come from the contractors engaged by the USED. In a significant majority of dredging contracts made since 1935 the contractors have insisted that payment for yardage removed be made on the basis of lead line soundings as opposed to echo soundings. This prejudice is gradually being overcome as experience shows that a properly operated sounder will give a fair and representative picture of the hydrography.

FIELD EXPERIENCE WITH SOUNDERS

The echo sounder is used as a navigation aid, for hydrographic surveying purposes, and by deep-sea fishing fleets for locating schools of fish. Its use as a navigation and fishing aid requires a lower degree of accuracy than does the hydrographic survey work. Since commercial craft interested in the sounder from a fishing and navigational standpoint are much more numerous than hydrographic survey parties, it is only natural that the manufacturers have been inclined to build an instrument adapted primarily to commercial needs and secondarily to surveying needs. Nevertheless, the sustained interest and encouragement of the USCGS, the United States Navy, and the USED, combined with the inherent impulse of the manufacturers to do a good job, have resulted in the production of sounders very well adapted to hydrographic surveying.

Assuming that a fairly skilled operator is employed, the supersonic sounders presently being used by the USED are rated as having an accuracy of ± 6 in. at a 50-ft depth for the portable model, and ± 3 in. at a 50-ft depth for the nonportable or fixed model. It should be noted that the accuracy is stated in inches at 50 ft and not as a percentage, because some errors in the sounder vary with the depth whereas others are independent of the depth. Thus, the error could be expected to be less at 10 ft than at 50 ft but not as much as the ratio of depths might indicate.

CONDITIONS OF OPERATION

In discussing the accuracy and dependability of echo sounder equipment as used by the USED, it appears advisable to divide the conditions of operation into three separate classifications, according to whether the sounder is operated over: (1) Sandy bottoms; (2) sandy beaches in wave action; or (3) soft mud bottoms.

1. Operation Over Sandy Bottoms.—Ordinary sand bottoms present an excellent reflecting surface to the sound wave in that they send a sharp, clear echo back to the receiver. The accuracy of echo soundings under these conditions is limited only by the stability of the speed control on the echo sounder and the proper assessment of the speed of sound through the underlying water. Tables are available which show the variation of sound speed with temperature and salinity which enables the proper adjustment or correction to be made for these variables. The speed controls on the instruments are fairly reliable although some improvement could probably be made on this feature.

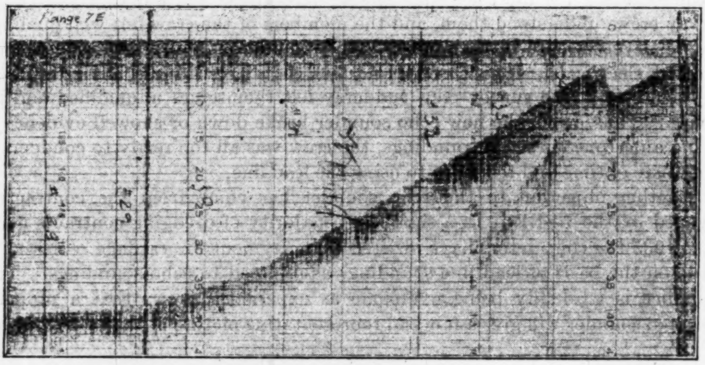

FIG. 3.—ECHO SOUNDER RECORD OVER SAND BOTTOM, WITH NO WAVE ACTION; GULF OF MEXICO NEAR PENSACOLA, FLA.

In practice, the accuracy of an instrument is checked periodically during the day by a check bar suspended at a known depth below the sounder or by a carefully obtained lead line sounding on a firm bottom. As stated previously, the portable sounders used by the USED are rated at ± 6 in. in 50 ft of water whereas the more expensive fixed types are rated at ± 3 in. in 50 ft of water. Fig. 3 is an example of a record taken over a firm, sand bottom with no wave action.

For the most part, the accuracy of the echo sounder under these conditions is accepted by all users. The USCGS has used them almost exclusively since before 1935 and uses lead line soundings only to detect any gross misadjustment in the echo sounder.

2. Operation Over Sandy Beaches in Wave Action.—In connection with a hydrographic study at Long Branch, N. J., the Beach Erosion Board sounded a series of points from an amphibious truck using both a lead line and a portable echo sounder. The soundings were taken with the truck moving into the waves with minimum headway, and a "fix" was burned on the chart at the instant the lead line was read for depth. The waves were running about 3 ft high at the time. A total of 37 soundings in depths of from 15 ft to 46 ft were taken; a comparison of the soundings by the two methods is given in Table 2. This table indicates that differences of approximately 0.3 ft can be expected between lead line and echo sounder records at each cast of the line; however, the plus and minus differences so nearly balanced over the set of observations that the average difference was found to be only 0.04 ft, with the lead line averaging deeper than the sounder.

TABLE 2.—COMPARISON OF ECHO SOUNDER AND LEAD LINE SOUNDING OVER SANDY BEACH (PORTABLE SOUNDER AND ORDINARY LEAD LINE)

Description	15 FT TO 46 FT			20 FT TO 29 FT			30 FT TO 39 FT			40 FT TO 49 FT		
	No. of Soundings	Total of Differences	Average of Differences	No. of Soundings	Total of Differences	Average of Differences	No. of Soundings	Total of Differences	Average of Differences	No. of Soundings	Total of Differences	Average of Differences
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Echo Sounder Depths:												
Deeper than by lead line..	12	+5.1	+0.42	3	+1.0	0.3	2	+0.8	+0.4	7	+3.3	+0.5
Shallower than by lead line.	16	−6.5	−0.41	5	+2.1	0.4	3	−1.2	−0.4	7	−2.9	−0.4
Identical with lead line...	9	....	....	3	0	....	1	0	....	3	0	....
Sum of Differences:												
Numerical...............	37	11.6	0.32	11	3.1	0.3	6	2.0	0.3	17	6.2	0.4
Algebraic...............	37	−1.4	−0.04	11	−1.1	−0.10	6	−0.4	−0.07	17	0.4	+0.02

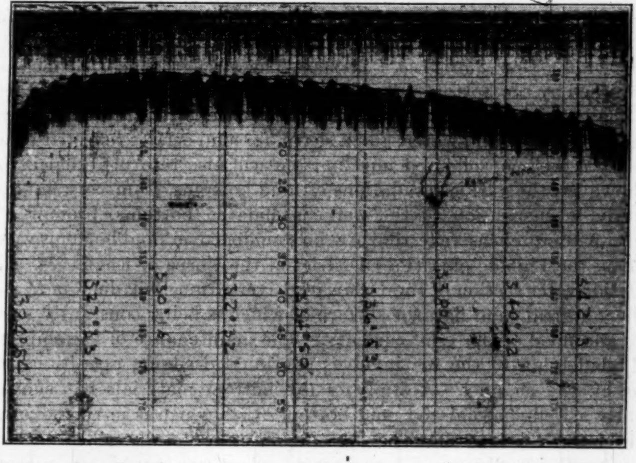

FIG. 4.—ECHO SOUNDER RECORD OVER SAND BOTTOM, WITH 3-FT SURFACE WAVES; GULF OF MEXICO NEAR PENSACOLA, FLA.

To study the effect of depth on the comparison, the data in Table 2 also show a comparison of these soundings in certain ranges of depth (from 20 ft to 30 ft, from 30 ft to 40 ft, and from 40 ft to 50 ft). The breakdown showed no significant difference from the over-all picture discussed in the preceding paragraph, the expected difference on each cast being in the order of from 0.3 ft to 0.4 ft, the average difference being in the order of from 0.1 ft or less.

It is recognized that this analysis was based on only 37 soundings, approximately half of which were made on one day and the other half on the next day. However, as no extraordinary care was taken in making the simultaneous soundings, it is believed that they are representative of the results that might be expected. In interpreting the sounder records, the Beach Erosion Board draws a line one half or one third the way between the troughs and crests, depending on the wave conditions; this line is then used as the bottom profile.

It is to be noted that the descrepancies between the lead line and echo sounder results have been designated as differences and not errors. It is difficult to assess the absolute accuracy of the two under such conditions, but it is believed that with the average difference so slight the echo sounder is probably the more consistently accurate of the two. In any event, it will permit more accurate readings with the survey boat traveling at higher speeds because the problem of plumbing the line does not enter the picture. Also, the record itself can be interpreted easily to compensate for the rise and fall of the survey boat with the passage of the wave crests; this is not possible with isolated lead line soundings.

An echo sounder record taken from an amphibious truck operating over a firm sand bottom, in about 3-ft waves, is shown in Fig. 4.

3. Operation Over Soft Mud Bottoms.—The strength and sharpness of the echo returned by the bottom depend on the relative speed of sound in the water and in the bottom material. This speed difference is relatively great between rock, sand or gravel, and water; and a sharp, forceful echo is reflected from such bottoms. A well-consolidated mud bottom will also return a sharp, forceful echo; however, the various gradations of mud bottoms, from very liquid muds practically in suspension to the well-consolidated bottoms, present another picture. A comprehensive testing of echo sounders over mud bottoms was made in 1941 by the Philadelphia (Pa.) District of the Corps of Engineers.

The tidewater section of the Delaware River (from Philadelphia to the sea) is in effect a great mud flat, the surface of which is being constantly swept by the ebb and flood of the tidal currents. The maintenance of the ship channel from Philadelphia to the sea involves continual dredging operations in, and surveys of, the channel. A study of the bottom materials has shown that frequently a well-consolidated mud bottom exists which is in turn overlain by several feet of less dense mud in a semiliquid state, locally called "fluff." In determining the dredging requirements and quantities moved, an accurate and consistent method of determining the bottom elevation is very desirable.

The test made on March 26, 1941, by the Philadelphia District involved among other things a set of simultaneous soundings by lead line and echo sounder and a study of the density of samples of the mud bottom taken at various elevations in the same area.

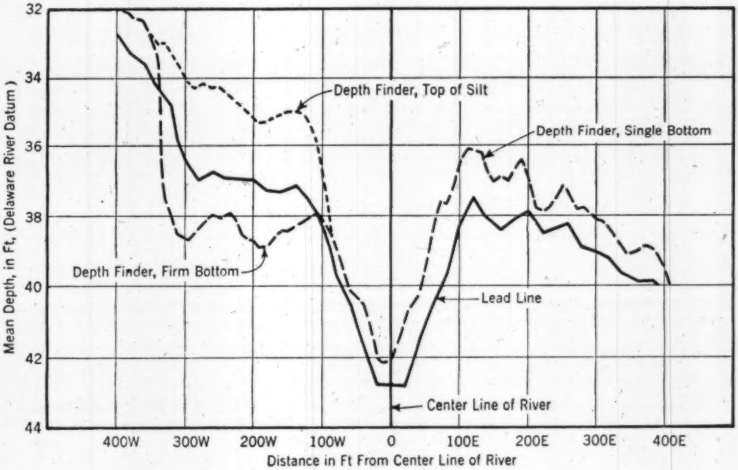

Fig. 5.—Comparison of Profiles by Lead Line and Echo Sounder; Station 193+108, Deepwater Point Range, Delaware River

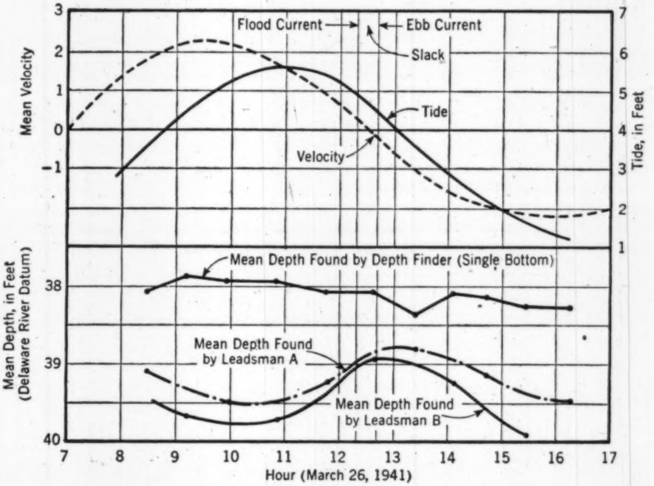

Fig. 6.—Effect of the Velocity of Tidal Currents on the Average Soundings Over Firm Mud Bottom

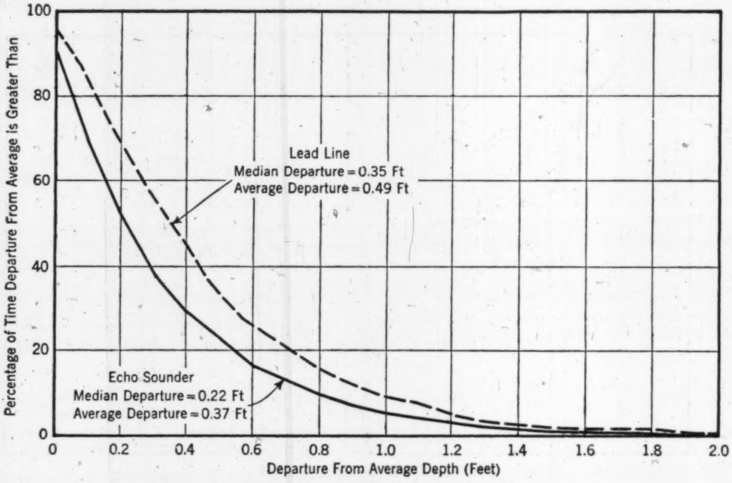

FIG. 7.—FREQUENCY-MAGNITUDE STUDY OF DEPARTURE OF INDIVIDUAL SOUNDINGS FROM THE AVERAGE OF ALL COMPARABLE SOUNDINGS OVER MUD BOTTOM

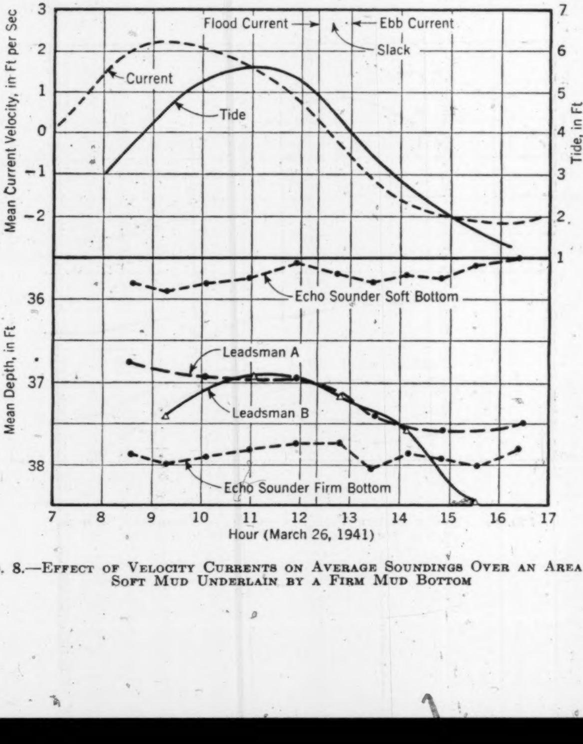

FIG. 8.—EFFECT OF VELOCITY CURRENTS ON AVERAGE SOUNDINGS OVER AN AREA OF SOFT MUD UNDERLAIN BY A FIRM MUD BOTTOM

In taking the lead line soundings, two experienced leadsmen using a 10-lb lead were employed, and they alternately sounded the same range (one, 5 times and one, 6 times) over a period of 8½ hours. Concurrently, the same range was sounded by an echo sounder 33 times. The results of the soundings were averaged and the profiles resulting from the two methods were compared; this comparison is shown in Fig. 5. It is to be noted that over part of the area the echo sounder indicated a double bottom, the two being approximately 4 ft apart. In other sections of the profile, only one bottom was indicated by the echo sounder.

A study of Fig. 5 shows that the lead line was not consistent in its indication of depth to firm bottom. Over the east side of the range, where the echo sounder indicated a single, fairly firm bottom, the lead sank into the bottom on an average of about 1 ft. On the west side of the channel where a double bottom was indicated by the echo sounder, the lead sank into the upper bottom generally from 1½ ft to 2 ft and was still 1 ft or 2 ft above the lower and(firmer) bottom.

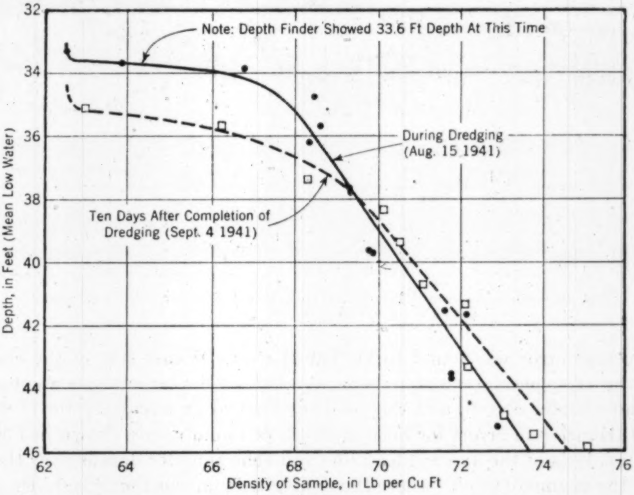

FIG. 9.—CHARACTERISTICS OF THE BOTTOM MATERIAL AT DEEPWATER POINT (STATION 187+650), DELAWARE RIVER

The analysis of the lead line soundings over the east half of the channel where a fairly firm mud bottom was found shows a personal factor to be in evidence in that leadsman A consistently found less depth than leadsman B; over the entire period the average difference was about 0.65 ft, the difference being about 0.3 ft at slack water and 1.0 ft at strength of current. This variation with current strength indicated that the bowing of the sounding line was definitely a factor in the accuracy of the lead line soundings. On the other hand, the echo sounder is free from this influence. This is shown in Fig. 6, in which the tide level, current strength, lead line soundings, and echo soundings are plotted. At each point the soundings represent the average depth over the easterly 400 ft of the profile. Each plotted point on the curve, "Mean Depth Found by Depth Finder," represents the average of three runs.

Assuming the average profile indicated by the lead line soundings as a base, the error for each individual lead line sounding was calculated and summarized.

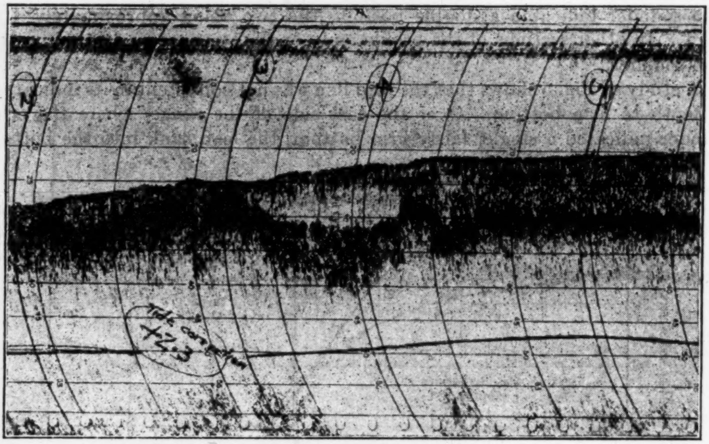

FIG. 10.—ECHO SOUNDER RECORD, RARITAN RIVER CHANNEL IN NEW JERSEY, TAKEN ABOUT 3 MONTHS AFTER DREDGING

The average error was found to be 0.49 ft over the east side of the channel. Using the average echo sounder profile as a base, a similar analysis was made of the echo sounder record, and this analysis showed an average error of 0.37 ft. The distribution of errors for both methods of sounding are shown in Fig. 7.

An analysis of the average lead line and echo sounder depths over the west half of the channel (400 ft) where the double bottom was found is shown in Fig. 8. The soft bottom area occurred from 80 ft to 350 ft west of the center line. The findings are in general similar to those shown in Fig. 6.

The project width of the Delaware Ship Channel is 800 ft. Over a mile of channel, uncompensated errors of as low an order as 0.1 ft will introduce an error in yardage computations of approximately 15,000 cu yd of material. Thus, uncompensated errors of as little as 0.1 ft assume some importance in calculations relative to dredging activities.

The lead line appears to have at least three inherent sources of error which might introduce uncompensated errors of 0.1 ft or even much greater: (*a*) The bowing of the lead line due to the pressure of the currents; (*b*) an error in judging the instant the lead line becomes plumb; and (*c*) the sinking of the lead by varying amounts in the soft bottom.

The echo sounder record is free from the influence of errors *a* and *b* but may be subject to error in interpretation due to variations in the consistency of the bottom material. A study of the bottom material in the Delaware River at a point near the test sounding range, however, indicates that the error in the sounder record should not be excessive from this cause in this area. In studying the bottom materials a set of bottom material samples was taken at progressively increasing depths from the relatively clear top water, down through the fluff, and on down to the firm bottom, the fluff being about 10 ft thick in this area. A plot of the analysis of these samples is shown in Fig. 9. This plate shows two curves, one taken during dredging operations in the area and the other 10 days after dredging had been completed in the area. Both curves show a rather pronounced upper level of the fluff where the density changes abruptly from that of clear water at 62.4 lb per cu ft to about 66.5 lb per cu ft in about a 0.4-ft change in depth. The increase in density over the next 10 ft or so is essentially uniform, reaching about 74 lb per cu ft immediately above the firm bottom. The firm bottom material below 46 ft was found to have a density of approximately 92 lb per cu ft. These well-defined increases in density at the interface between the clear water and the top of the fluff and at the interface between the bottom of the fluff and the relatively firm bottom, of course, are

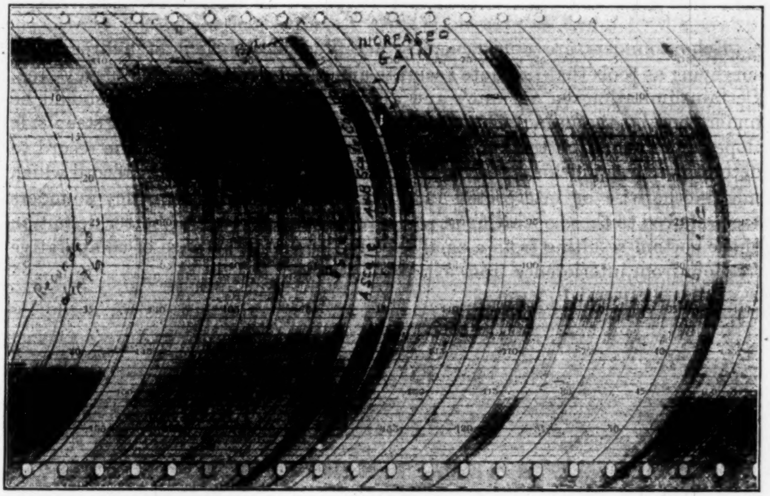

FIG. 11.—ECHO SOUNDER RECORD, CLEAR LAKE, LAKE COUNTY, CALIFORNIA, SHOWING THREE BOTTOM LAYERS

responsible for the well-defined record of these interfaces which are found on the echo sounder recorder.

Although the foregoing discussion of soft bottom areas concerns specifically conditions in the Delaware River in the area in which the observations were made, it is believed reasonable to assume that the findings might have a degree of application to other areas with soft bottoms. However, under conditions that depart significantly from the conditions mentioned, similar studies might well be justified to determine the true significance of the echo sounder record. For instance, silting resulting from flocculation at the interface between fresh water and sea water might exhibit significantly different density characteristics, as might silt settling in a large reservoir where it was relatively undisturbed by waves and strong currents. The observations at Deepwater Point were taken in the tidal part of the Delaware but upstream from the influence of the salt water wedge.

Echo sounder records from the Raritan River, New Jersey, near New York Harbor, are shown in Fig. 10. The silt in this area was freshly deposited over a recent dredge cut and the line of demarcation can be clearly seen. The solid bottom of the dredged channel is shown at a depth of about 30 ft, overlain by the more recent deposit of silt. Fig. 11 shows a record furnished by the USCGS taken in Clear Lake, California. Here the silt has accumulated rather slowly, although two definite layers are indicated. Fig. 11 also indicates the effect of amplifier-gain setting on a determination of the bottom from echo sounder charts in this type of silt; in this case the topmost layer of silt was detected only with the high gain setting on the amplifier.

SUMMARY

Echo sounders are now generally accepted in the field of hydrographic surveying as tools that promote accurate and rapid coverage of the survey area. Certain limitations in accuracy and application are present which should be understood by the user. The use of echo sounders over areas with steep side slopes or soft mud bottoms may require careful interpretation of the record to determine the significance and magnitude of certain features of the record which might otherwise be interpreted incorrectly. Supersonic frequency sounders are preferred to those operating at sonic frequencies because of better definition of the bottom resulting from a smaller sound cone and because of comparative freedom from interference from extraneous noises.

MEASUREMENT OF SEDIMENTATION IN SMALL RESERVOIRS

By L. C. Gottschalk[1]

With Discussion by Messrs. Earl E. Sanderson and L. C. Gottschalk

Synopsis

Measurements of the volume of sediment in reservoirs provide a basis for determining the extent of sedimentation damages, for periodically correcting the capacity curve to assure more efficient operation, for evaluating the effects of watershed and climatic factors on rates of sediment production from watersheds, and for developing sediment production indexes useful in the design of reservoirs. Sediment volumes may be determined by comparison of original contour maps with maps showing the present topography of the sediment surface or by direct measurement of sediment thicknesses. Two methods of survey are used by the Soil Conservation Service (SCS), United States Department of Agriculture—the contour method and the range method. Water depths are measured by use of a lead line, sounding pole, or echo sounder. Sediment thicknesses are determined by use of a spud, sounding pole, or soil auger. The specific weight of sediment is determined in conjunction with reservoir sedimentation surveys. Water capacities and sediment volumes are computed for contour surveys by a modified prismoidal formula and for range surveys by the Dobson prismoidal formula.

Introduction

The need for sedimentation data in the design of water control and utilization projects has been so prominently emphasized in the past few years that it needs no further discussion in this paper. Slowly, but surely, much needed data are accumulating on rates of sediment production from drainage basins. The time has come when the emphasis should not be on why surveys should be made but on how and where such surveys are to be made. Several scientific organizations have established committees to study the problem of reservoir sedimentation and to prepare recommendations on survey methods

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and techniques. The ASCE Joint Committee on Sedimentation in Reservoirs is an example. In general, there is a need for standardizing methods of making reservoir sedimentation surveys so that the results obtained by various federal and state agencies are comparable and useful to all. The methods described in this paper are those developed and used by the SCS for surveying small reservoirs.

In general, the purpose of a reservoir sedimentation survey is to measure the accumulated volume of sediment in a reservoir during the period of storage record. This information is useful for:

1. Determining the prevailing and probable future sedimentation damages to a particular reservoir;
2. Periodically correcting the capacity curve to assure more efficient operation;
3. In combination with similar data from other reservoirs and suspended load measurements, evaluating the effects of watershed and climatic factors on the rate of sedimentation; and
4. Preparing regional sediment production indexes for developing design data.

The procedures for measuring the accumulated volume of sediment in a reservoir, established by the SCS in 1934, in connection with a nationwide study of reservoir sedimentation, with minor variations, are rather widely used today (1950). These procedures have been adopted by various other federal and state agencies engaged in reservoir sedimentation studies. Therefore, a discussion of the methods developed by the SCS, to a large extent, covers the methods generally used throughout the United States, with the exception of special equipment such as echo sounders. The latter are of recent development and have been adopted as standard equipment by other federal agencies, particularly in surveying major reservoirs.

Equipment for Measuring Water Depths and Silt Thicknesses

General.—The volume of sediment in a reservoir may be determined by one of two methods:

a. By determining the present capacity and assuming that the difference between it and the original capacity is a measure of the volume of sediment; and

b. By direct measurement of the volume of sediment deposits.

The first method is satisfactory only where an accurate map of the original reservoir basin was made at or before closing of the dam. Such maps are usually few and far between, particularly on smaller and older reservoirs. On the newer major reservoirs, where modern methods of mapping have been used, such as multiplex, more accurate maps are available and the original capacity data are reliable enough for estimating sediment volume from the measured present capacity. It must be emphasized that any error in the original capacity of the reservoir imparts an equal error in the computed volume of sediment. Although the difference between the estimated original capacity and the true original capacity may be small in terms of the total capacity, it may represent a sizable error in the estimated volume of sediment in a reservoir if the sediment volume is small in comparison with the total original capacity.

Sounding.—No matter what method is used for measuring the sediment volume in a reservoir, the present capacity must be determined. This is done by measuring the present water depths at representative points in the reservoir and computing the volume in respect to the surface area. Water depths may be measured by lead-line sounding, by use of a sounding pole, or by echo sounding. Only lead-line sounding and a sounding pole are used by the SCS for surveying small reservoirs. Both wire and rope sounding lines are used. Wire lines are wound on manually-operated reels with registering sheaves to measure the length of line "paid out." Depths can be determined accurately to 0.1 ft by reading a counter on the apparatus, but the boat must be stopped to obtain each depth reading. Rope sounding lines have been found more convenient to use than wire lines, mainly because soundings can be taken faster and because the SCS does not have boats of its own on which sounding reels can be mounted permanently. The SCS has found cotton-covered, bronze core tiller rope, $\frac{1}{4}$ in. in diameter, to be the most satisfactory for sounding lines because of the durability and the capability of such lines to maintain a constant length. Rope sounding lines are marked at 0.5-ft intervals, using a convenient color system of markings for identifying the 0.5-ft, 1.0-ft, and 5.0-ft marks, and different 10-ft intervals up to 150 ft. Water depths are measured accurately to 0.5 ft and estimated to the nearest 0.1 ft. In hand-line sounding, observations can be made for depths up to about 75 ft without stopping the boat (see Fig. 1). The sounding weight is cast ahead of the boat and allowed to sink to the bottom. The depth reading is taken when the boat is directly over the

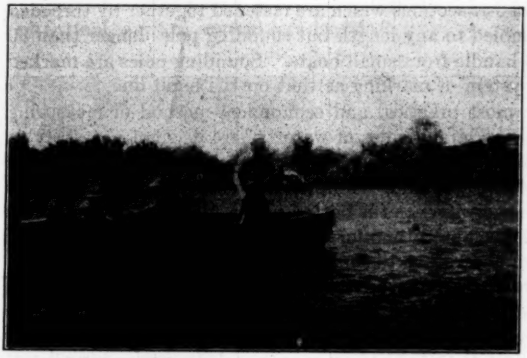

FIG. 1.—SOUNDING LINE BEING CAST WITH BOAT IN MOTION

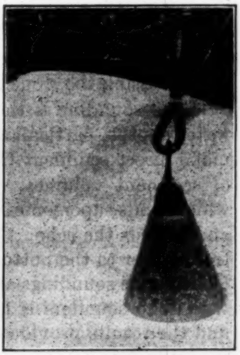

FIG. 2.—SOUNDING WEIGHT

sounding weight and the line is vertical. A bell-shaped cast aluminum sounding weight (see Fig. 2) is used for both wire-line sounding and hand-line sounding. A 5-lb all-aluminum weight is used for depths to 100 ft and a 9-lb weight equal in size to the 5-lb weight, but filled with lead, is used for greater depths. A bell-shaped weight permits rapid descent through the water and the wide, flat bottom prevents the weight from sinking too deeply into soft sediment deposits.

The sounding pole is a convenient tool for measuring shallow depths in small reservoirs and has been used with considerable success in determining sediment thicknesses as well. The use of a sounding pole is limited to depths not exceeding 30 ft. An ordinary $1\frac{1}{2}$-in.-diameter closet rod, obtainable in most any lumber yard, has been found to be satisfactory. A closet rod may be obtained in 20-ft lengths or, more conveniently from the standpoint of transportability, in 10-ft lengths that can be spliced together. Recently the SCS has developed a lightweight steel sounding pole constructed from 1-in., thin-wall electrical conduit. It is made up in 5-ft sections which are fastened together by threaded dowels. It may be assembled to any length but sounding poles longer than 30 ft are too cumbersome to handle from small boats. Sounding poles are marked in color using the same system of marking as that on the hand line.

Echo sounding is the most practical and economical method of measuring water depths, particularly on large bodies of water where it is not necessary to make direct sediment thickness measurements for the accurate determination of sediment volumes. Echo sounding equipment consists of a transmitter which sends supersonic sound waves to the bottom and a receiver which receives and records the echo. The elapsed time for the sound wave to travel from the transmitter to the bottom and return to the receiver is a measure of the depth. Continuous soundings are made as the boat travels along the water. Sounding with echo sounders is faster and more accurate than sounding with lead line and the results provide a continuous profile of the bottom. A skilled operator is required to maintain and operate the equipment and its use requires boats larger than those ordinarily found on small reservoirs. Echo sounders are not economical to use for surveys which require direct sediment thickness measurements because the boat must be stopped for each measurement. Thus, ample time is available for measuring water depth with a lead line and the need for specialized equipment and personnel is eliminated. Since sediment thickness measurements are usually required on the smaller and older reservoirs surveyed by the SCS, this equipment is not used by this agency.

Sediment Thicknesses.—Where no maps or where maps of questionable accuracy are available on a reservoir, it is necessary to determine the volume of sediment in the reservoir by direct measurement of sediment thicknesses. This may be done with a spud, sounding pole, or auger. A spud is an instrument developed by the SCS specifically for this work. Several refinements have been made of this instrument since it was originally developed, the most recent being a sectional spud that can be packed in a carrying case for easy transportation. The spud (see Figs. 3 and 4) is a case-hardened or well-tempered steel rod, $1\frac{1}{2}$ in. in diameter, into which encircling triangular grooves have been machined at intervals of 0.1 ft. Each groove tapers outward from a maximum depth of ¼ in. to 0 at the rim of the groove 0.1 ft above. In appearance the spud resembles a series of inverted frustrums of cones with a maximum diameter of 1½ in., a minimum diameter of 1 in., and an altitude of 0.1 ft, set vertically one on another. The base of each cone is machined to a depth of ⅛ in. to form a cup in which to catch and hold the sediment. The spud is made up in 3-ft sections which can be assembled to any length up to 18 ft by nickel alloy steel dowel pins. The spud is attached to a ⅜-in. or ½-in (outside diameter) bronze core tiller rope and thrown downward from a boat with sufficient force to carry it through the sediment and into the original bottom materials of the reservoir. The spud is withdrawn from the original bottom materials and sediment by snubbing the line over the side of the boat and rocking the boat until the spud is jerked loose. The spud is pulled slowly through the water to the boat to prevent the sediment from washing off. The thickness of sediment is determined directly by inspection of the deposit and original bottom materials adhering to the spud. The spud has been used and found satisfactory for determining sediment thicknesses, in water depths up to several hundred feet.

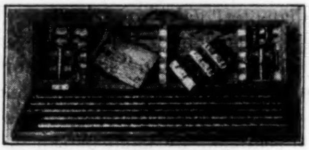

FIG. 3.—SECTIONAL SPUD PACKED IN CARRYING CASE

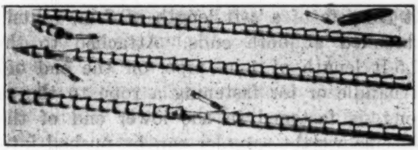

FIG. 4.—PARTLY ASSEMBLED SPUD

In shallow water, where depths to original reservoir bottom do not exceed 30 ft, a sounding pole has been found convenient for determining sediment thicknesses. Sediment often can be distinguished from original bottom materials by differences in compaction alone, even after sediment has been subjected to extended periods of aeration. The use of a sounding pole depends on this criterion for distinguishing the difference between original soil and sediment. The depth of water to the top of the sediment is measured by the sounding pole. The pole is then pushed through the sediment to the more resistant original bottom. The depth is again noted and the difference is a measure of the sediment thickness. The sounding pole is most useful for measuring sediment depths usually not exceeding 10 ft, particularly deposits having a high percentage of silt and clay fractions. The latter deposits usually are less compact than the bottom materials below.

Often the difference between sediment and bottom materials cannot be detected by difference in compaction alone. Then the thickness of sediment must be determined by inspection. In submerged deposits this can be done by the use of a spud if the deposits are not too compact and do not exceed 18 ft in thickness. If the deposits are too thick or too compact for measurement with a spud, a soil auger is used, consisting of a 1½-in. bit fastened to a 1½-in. (outside diameter) pipe. It is provided with 5-ft extensions so that it can be assembled to any desired length. Deposits of as deep as 40 ft have been measured by use of this auger. The soil auger is particularly useful in measuring the thickness of exposed deposits such as may be found in the delta areas of reservoirs.

Under particularly favorable conditions, sound waves from echo sounders sometimes penetrate soft sediment to be reflected from a hard bottom as well as from the surface of the soft sediment. Under these conditions sediment thicknesses can be determined from the fathograms of echo sounders. Fathograms of Chesapeake Bay often show from 15 ft to 20 ft of soft sediment deposited in dredged navigation channels. Although this appears to be the exception rather than the rule, it does indicate the possibility that echo sounders could be developed which would record sediment thicknesses as well as water depths. Such instruments would probably require the use of sonic frequencies rather than the supersonic frequencies used in present day echo sounders because sonic frequencies have higher penetrating power and lower absorption than supersonic frequencies.

Specific Weight of Sediment.—A core type sampler has been developed by the SCS to obtain undisturbed samples of sediment for determining specific weight. It consists of a 4-ft length of 2-in. (outside diameter) galvanized iron pipe threaded at both ends. Attached to the upper end by a reducer is a 4.5-ft length of 1-in. pipe, on the end of which is a tee joint suitable for a handle or for fastening a rope to the sampler. A sampling nipple, 4 in. long, is fastened to the lower end of the 2-in. pipe by a coupling. The specific weight sampler can be pushed into the sediment by hand or can be thrown or withdrawn in the same manner as the spud. A flap valve, contained in the reducer, opens to permit the outflow of air and water as the specific weight sampler descends. When the sampler is withdrawn, the valve closes, creating a suction which holds the sediment in the sampler. The nipple is removed from the sampler, capped at both ends, and sent to the laboratory for specific weight analysis.

THE FIELD SURVEY

Survey Methods.—Two general methods of determining the capacity of a reservoir are in use today—the contour method and the range method. The contour method requires an accurate original contour map of the reservoir basin, and both the range method and contour method require an accurate shore line map. If no maps, or if maps of questionable accuracy, exist on a reservoir, a shore line map is made by standard surveying methods. This consists of establishing primary control by a triangulation system extended upstream from a base line near, or on, the dam. If there are existing maps, and if they check well with this primary system, the maps may be used for the survey with minor corrections for the shore line contour if necessary. If no maps are available, or if existing maps are not good, and if no aerial photographs are available, the shore line is mapped with plane table and telescopic alidade.

Range ends, instrument stations, and triangulation stations are permanently monumented, using iron pipe set in concrete or reinforced precast concrete posts with bronze caps. Station numbers are marked directly on the pipe or on the bronze caps with steel dies. Various scales are used for mapping, depending on the size of the reservoir. For small reservoirs and ponds a scale of 100 ft equal to 1 in. is used, whereas a scale of 1,000 ft equal to 1 in. is used for major reservoirs.

Shore line mapping is a time consuming job in the field. Its primary purpose is to locate survey stations for future reference and to determine the surface area needed to compute water and sediment volumes. The SCS has found it convenient to use aerial photographs for this purpose whenever possible. These may be "blown up" to any convenient scale, the scale depending on the size of reservoir surveyed. On larger reservoirs 500 ft equal to 1 in. is used and on smaller reservoirs 200 ft equal to 1 in. Because of inaccuracies on the outside borders of aerial photographs caused by parallax, only the center areas of the photographs are used. These are blown up to the desired scale, cut to plane-table size, and used on the plane table in the same manner as a regular plane-table sheet. If the water surface level of the reservoir was at crest elevation on the day the photograph was made, the shore line can be determined directly from the photograph. If not, the shore line contour must be determined by multiplex methods. In all cases the scale of each photograph is checked in the field by chaining the distance between two identifiable objects on the photographs.

Contour Survey.—Of the two methods of survey, the contour method gives more reliable results, provides a better picture of the vertical and horizontal distribution of sediment, and allows plotting of capacity curves. However, it requires more widely distributed observations than the range method and generally necessitates a longer period of survey. The contour method of survey consists of establishing elevations of the present sediment surface and sketching in the contours. The area enclosed by each contour is planimetered and the volume increment computed on the basis of the contour interval. The contour method is used by the SCS only for surveys of reservoirs which have original contour maps with a high standard of accuracy, where sediment cannot be penetrated by ordinary methods, and where sediment thicknesses are large enough to eliminate the possibility of sizable errors due to inaccuracies in original maps. In a contour survey an adequate number of accurately located soundings must be taken to afford a sure basis of interpolating contours on an invisible surface (the more irregular the surface, the more soundings required). Conversely, the more gentle and regular is the slope, the fewer are the soundings required. Several methods of locating observations may be used depending on the discretion of the instrumentman. Thus, in shallow water the direction and distance to an observation may be determined from a single instrument station using a transit or properly orientated plane table and stadia board. In deeper water, locations may be determined by plotting on the plane-table sheet the point of intersection of two direction lines from adjacent stations. Several patterns of soundings have been used, including radial, grid, and closely spaced ranges.

Range Survey.—The range method of survey is used more often than the contour method of survey by the SCS because much of its work is on small and older reservoirs where good original maps are not available. A range survey can be made with much less field time than a contour survey. In a range survey, sediment thicknesses are measured directly. The range method consists of laying out a system of representative ranges (see Fig. 5) and determining the present water and sediment depths at regular intervals along these ranges. The boat is directed along this range and kept on line by a transit and a signalman on one end of the range or, if the range is short, by use of a backsight flag behind one of the range ends. The plane table is set up at a cut-in station, and the location of the soundings is predetermined and indicated lightly in pencil on the plane-table sheet. As the boat travels along the range, the instrumentman signals the boat crew when the boat is over the point where an observation is made. Simultaneous soundings and sediment-thickness measurements are made along the ranges at regularly spaced intervals of 25 ft to 50 ft on small reservoirs and 75 ft to 100 ft on large reservoirs. These are plotted on cross-sectional paper and the volume is computed on the basis of the segment areas between the ranges. The accuracy of this method depends on the variability of the topography of the reservoir and the proper selection of ranges to sample this variability adequately. Where the cross sections represent a good sampling of topography of the reservoir basin, a high degree of accuracy is attained. Results of surveys of reservoirs with regular shape are usually within 2% of the true value, whereas reservoirs of irregular shapes, with many embayments, variable valley characteristics, and long, winding arms, give results well within 10%.

COMPUTATION OF WATER CAPACITY AND SEDIMENT VOLUME

Contour Method of Survey.—For computing the sediment volume when the contour method of survey is used, water depths are plotted on the reservoir map and the present contours of the reservoir basin are drawn in. The area enclosed by each contour of original capacity and sediment deposits is then determined in acres from the reservoir map by planimeter. The original capacity and sediment volume are computed by the following modified prismoidal formula:

$$V = \frac{L}{3}\left(A_l + \sqrt{A_l A_u} + A_u\right) \qquad \text{(1)}$$

in which

V = the capacity, in acre-feet;

L = the contour interval, in feet (in the lowest prismoid, L, is the vertical distance between the lowest contour and the lowest point in the bottom of the reservoir);

A_l = the area, in acres, of the original lower contour for original capacity, or the difference between the areas of the original lower contour and the present lower contour for sediment volume (in the lowest prismoid, $A_l = 0$, for both original capacity and sediment volume); and

A_u = the area, in acres, of the original upper contour for original capacity, or difference between the areas of the original upper contour and the present upper contour for sediment volume (in the uppermost prismoid, A_u is equal to the area enclosed by the crest contour).

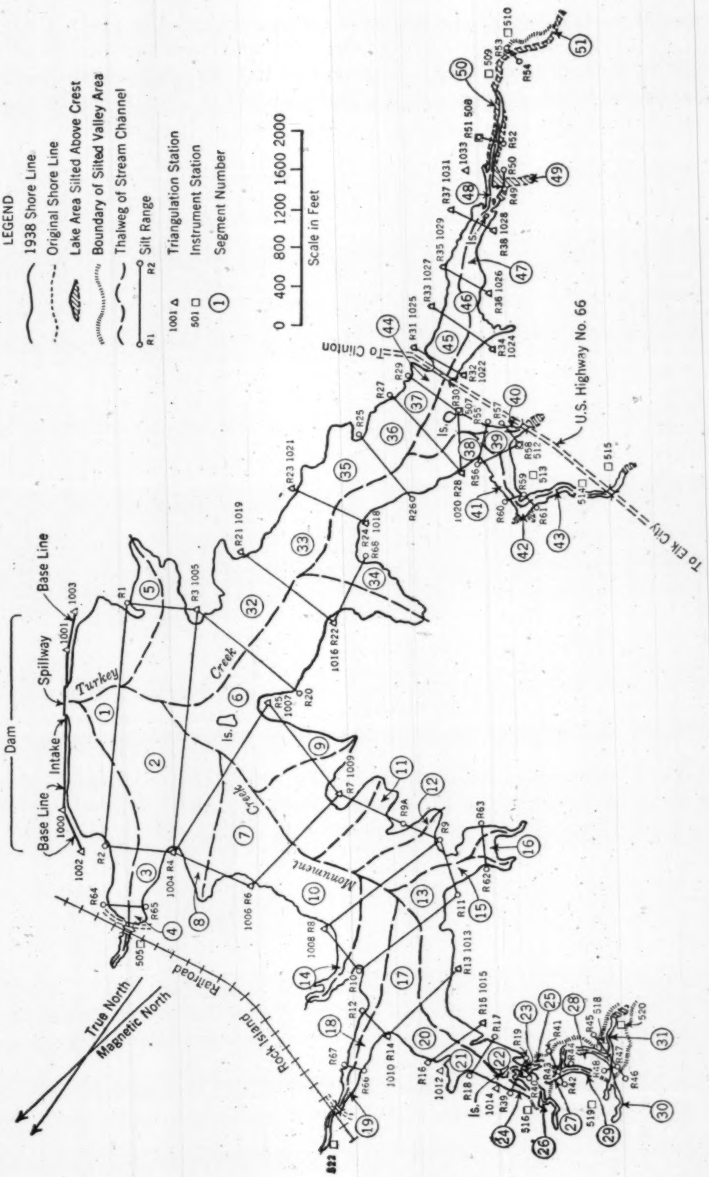
LEGEND
1938 Shore Line
Original Shore Line
Lake Area Silted Above Crest
Boundary of Silted Valley Area
Thalweg of Stream Channel
R1 R2 Silt Range
1001 △ Triangulation Station
501 □ Instrument Station
① Segment Number
0 400 800 1200 1600 2000
Scale in Feet
Dam
Base Line
Intake
Spillway
Turkey Creek
Monument Creek
Is.
True North
Magnetic North
Rock Island Railroad
To Clinton
U.S. Highway No. 66
To Elk City

FIG. 5.—A MAP OF LAKE CLINTON, WASHITA COUNTY, OKLAHOMA, SHOWING TYPICAL RANGE LAYOUT

Range Method of Survey.—In the range method of survey, ranges are laid out parallel to each other, as far as possible. The measured water depth and sediment thicknesses along these ranges are plotted on cross-section paper and the cross-sectional areas of original capacity and silt volume are determined in square feet, by planimeter (see Fig. 6). The segment areas, or those areas bounded by a set of ranges and crest-line contour, are determined in acres from a map or aerial photograph of the lake, using a planimeter. The volume of water or sediment in each segment is determined by the Dobson prismoidal formula:[2]

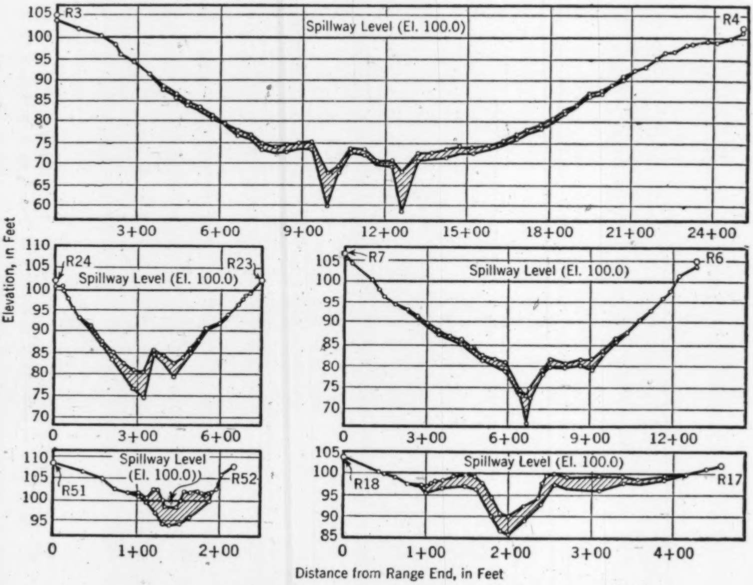

FIG. 6.—REPRESENTATIVE CROSS SECTIONS OF LAKE CLINTON, WASHITA COUNTY, OKLAHOMA

$$V_s = \frac{A'}{3}\left(\frac{E_1 + E_2}{W_1 + W_2}\right) + \frac{A}{3}\left(\frac{E_1}{W_1} + \frac{E_2}{W_2}\right) + \frac{h_3 E_3 + h_4 E_4}{130{,}680} + \cdots \quad ..(2)$$

in which

V_s = the original capacity or sediment volume, in acre-feet;

A' = the quadrilateral area formed on two sides by the range ends and on the other two sides by lines drawn from the intersection of the range lines and crest contour, in acres;

A = the segment area of the reservoir, in acres;

E = the cross-sectional area of water or sediment along the range, in square feet;

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[2] "Silting of Reservoirs," by Henry M. Eakin, *Technical Bulletin No. 524*, U.S.D.A., revised by Carl B. Brown, U. S. Govt. Printing Office, Washington, D. C., 1939.

W = the width (length of bounding range) at crest elevation, in feet; and
h = the perpendicular distance from the range on a tributary to the junction of the tributary to the main stream; or, if this junction is outside the segment, to the point where the thalweg of the tributary intersects the downstream range.

Eq. 2 applies to segments with irregular shore line and is applicable only for ranges that are essentially parallel. In curved segments with regular shore line, the following average end-area formula gives more reliable results:

$$V = \frac{A}{2}\left(\frac{E_1}{W_1} + \frac{E_2}{W_2}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The volumes of water and sediment are computed separately for each segment of the reservoir, with proper correction for the water and sediment displacement by the dam in the segment immediately upstream from the dam, and the individual segment volumes are totaled to obtain the entire water and sediment volume in the reservoir.

Presentation of Data.—It has been a set policy of the SCS to prepare reports on all detailed sedimentation surveys so that the data can be made available to all who have use for them. One comprehensive printed report[2] was issued in 1936 (revised in 1939) which includes information on all the detailed reservoir sedimentation surveys made by the SCS to that date. Numerous reports have been prepared on surveys of individual reservoirs in mimeographed or multilithed form. These reports give detailed information on the dam and the reservoir, methods of survey, sediment characteristics, and watershed characteristics, as well as a summary of sedimentation data in terms of storage loss and rate of sediment production per unit of drainage area on both a volumetric basis and a weight basis.

Summary

The methods developed by the SCS to determine the volume of sediment in reservoirs give results which are well within 10% of the true values. The results, therefore, are reliable for estimating sediment damages to individual reservoirs, for correcting capacity curves, for evaluating the effects of watershed and climatic factors on sedimentation, and for developing sediment production indexes for specific problem areas. The methods described have been adopted to a large degree by the various federal, state, and local agencies engaged in reservoir sedimentation investigations. The results of such investigations, therefore, are comparable in accuracy and are useful to all concerned with the problem of reservoir sedimentation.

DISCUSSION

Earl E. Sanderson.[3]—The Division of Water, Ohio Department of Natural Resources, has made detailed sedimentation surveys of more than thirty small, shallow lakes and reservoirs ranging in area from 2 acres to 400 acres. Reconnaisance surveys have been made of reservoirs as large as 2,000 acres in area. The principal objective of the surveys has been to establish criteria useful in the design of reservoirs by determining the sediment production rates for typical watersheds. For this purpose the state has been divided into three general areas: (1) level corn belt; (2) rolling areas, general and dairy farming; and (3) hilly woodland areas, general farming. Annual sediment production is found to range from 0.30 acre-ft per sq mile on 100% wooded hill land to 2.56 acre-ft per sq mile on gently rolling corn belt land.

The methods and techniques developed by the Soil Conservation Service (SCS), United States Department of Agriculture, have been used for both field and office work. The range method of survey and the Dobson prismoidal formula for computation have proved highly satisfactory for the type of reservoirs found in Ohio. They provide a simple, workable, and expeditious means for determining sediment volumes with an accuracy sufficient for practical purposes. In some instances in which the sediments are being deposited in large amounts in localized delta areas, supplemental range lines and random shots have been found to be useful and essential. In the computation of the sectional area on the range lines it is not necessary to plot the field date on cross-sectional paper and planimeter the areas. The segments of the areas between sounding points are trapezoids, and the areas are easily computed without plotting and planimetering.

Adequate maps and aerial photos have not been available for most of the reservoirs surveyed so it has been necessary to make plane-table, shore line surveys. About 60% of the survey time has been spent on shore line surveys. Soundings are located on the range lines by means of a 1,500-ft tag line that is mounted on the survey boat. After the range lines have been located by plane table, the tag line is anchored on shore and unreeled from a boat powered by an outboard motor. The tag line is then reeled in, pulling the boat back along the range line. Soundings are taken at uniform intervals along the range, generally not greater than 20 ft. The use of uniform intervals greatly facilitates the computation of the cross-sectional areas. A drag anchor is used to keep the boat from drifting and coasting, and to keep the tag line taut. Ranges up to 3,000 ft in length are sounded in this manner by marking the center of the range with a buoy and working out to the buoy from each shore line. This method has been found to be faster and more satisfactory than the method of locating soundings by the plane-table method. A smaller crew is required, and the crew works more efficiently.

Sediment samples are obtained with a piston-type sampling tube. The sampler consists of a brass tube, $1\frac{1}{2}$-in. inside diameter, attached to a $\frac{3}{4}$-in.

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[3] Principal Hydr. Engr., Div. of Water, Ohio Dept. of Natural Resources, Columbus, Ohio.

standard iron pipe. A standard double acting water-pump piston attached to a $\frac{7}{16}$-in. rod is placed in the tube and a driving weight slides on the section of $\frac{3}{4}$-in. pipe. Light tiller cables are attached to the driving weight and to the end of the piston rod. The sampler is lowered to the surface of the sediment deposit with the piston flush with the end of the sampling tube. The cable attached to the piston rod is then held in a fixed vertical position as the sampling tube is driven into the sediment by raising the driving weight and permitting it to fall as many times as may be necessary to get the penetration desired. Then the sampler is removed by pulling on both cables simultaneously. The sample is ejected by means of the piston and rod. Samples up to 4 ft in depth, in any depth of water, can be obtained. By using a locking mechanism, the sample tube can be driven some distance into the sediment before the piston is released making it possible to obtain volumetric samples at some distance below the sediment surface with a relatively short sample tube.

L. C. Gottschalk[4].—Several refinements in methods of measuring and computing reservoir sedimentation are indicated in the discussion and provide a useful addition to the paper. The use of tag lines and the computation of cross-sectional areas are timesavers in both field and office work.

The specific-weight sampler developed by Mr. Sanderson is an improvement over the sampler used by the SCS. With it, samples of less dense material may be obtained, and samples may be taken at greater sediment depths. The specific-weight sampler stands alone as the only instrument used in sedimentation work that does not have behind it the benefit of years of experimentation and development to reach the degree of refinement and precision found in other types of surveying equipment. It took many years for hydrographers to progress from lead-line sounding to echo sounding. It may take many years to develop a truly scientific and precise specific-weight measuring device.

In 1950, the Subcommittee on Sedimentation, Inter-Agency River Basin Committee, initiated a project in cooperation with the Atomic Energy Commission to investigate the feasiblity of using radioactive isotopes in the design of an instrument for measuring specific weight of sediment. Work has progressed steadily on this project. A pilot model of an instrument was completed in 1951, and laboratory tests are currently underway (1951). Preliminary tests to date indicate the practical possibility of this instrument, and the next few years may witness the development of a truly scientific and precise instrument that will render established volume-weight samplers obsolete.

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[4] Head, Sedimentation Section, Soil Conservation Service, U. S. Dept. of Agri., Washington, D. C.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 2480

OPERATION AND MAINTENANCE OF IRRIGATION SYSTEMS

By Raymond A. Hill,[1] M. ASCE

With Discussion by Messrs. George W. Howard, Lyman R. Flook, Jr., Rodolfo E. Ballester, Alfred R. Golzé, and Raymond A. Hill.

Synopsis

A description of the problems of operation and maintenance of irrigation systems, particularly in the United States, is presented in this paper. Operating procedures based on deliveries of water in accordance with natural flow rights, on demand, in rotation, and on advance orders are discussed. Causes of operating waste are described and the need for measurement of water is emphasized. It is pointed out that efficient operation of irrigation projects depends on the maintenance of all storage, diversion, distribution, and drainage works in good condition at all times. Various maintenance problems are described, and attention is called to the need for more efficient machinery and equipment. Certain records of costs of operation and maintenance of irrigation projects are presented and the importance of better cost accounting is emphasized.

Introduction

Most ancient civilizations were dependent on irrigation because they developed in regions where rainfall was insufficient to support agricultural economy. Hence, it is not strange that clay tablets from the ruins of Babylon, hieroglyphics cut in rocks along the Nile, and the literature of many other races describe problems of operation and maintenance of irrigation systems. If the prehistoric peoples who built irrigation works in the Americas had left a written record, they too would probably have described such problems. In the twentieth century of the Christian era there are only better tools with which to maintain irrigation systems in good condition so that water may be delivered to each farm as needed for beneficial use without unnecessary waste.

No sharp line can be drawn between the functions of operation and maintenance. It may be stated that operation includes everything which relates directly to the delivery of water to farms, whereas maintenance embraces the

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Note.—Published in December, 1950, as *Proceedings-Separate No. 47*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Cons. Engr., Leeds, Hill and Jewett, Los Angeles, Calif.

repair of structures and cleaning of canals and drains. On small projects the distinction is largely academic because the works are usually kept in repair by the same forces who handle the diversion and delivery of water. Even on large projects where the irrigation season is short, the system may be maintained in large part by operating personnel. However, for administrative and cost accounting purposes, it is advisable to distinguish between the functions of operation and maintenance.

OPERATING PROCEDURES

Although the basic requirements of efficient operation are the same everywhere, many problems are peculiar to a single irrigation system or at least to projects having similar characteristics.

In northern regions, the growing season is short; in other areas, water is used for irrigation throughout all the months of the year. On some projects, the entire supply is controlled by storage dams; on others, a reservoir on one branch of a river must be operated to compensate for fluctuations in the discharge of another; and in a number of cases, there are no storage works. In certain instances, such as on the Rio Grande Project in New Mexico and Texas, waste water from one unit is rediverted for irrigation use in the next valley downstream, whereas in others, such as the Imperial Valley in southeastern California and the Closed Basin in San Luis Valley, Colorado, no part of the water diverted returns to the stream system.

Differences in rights to the use of water must also be taken into account in the operation of irrigation systems. Generally these have different priorities depending on the time when the particular parcels of land were first irrigated, because water rights can be acquired under the law of most western states only by appropriation and subsequent beneficial use. Rights to divert from a stream are not changed when the supply is regulated; water must be delivered on demand in the order of priorities as if no storage dam had been built. There are sometimes rights of different priority to the use of stored water, usually where the area of a project has been enlarged coincident with the construction of supplemental reservoirs. In the Salt River Valley of Arizona the situation is further complicated by the fact that underground water supplies have been developed partly on behalf of all the lands and partly for the benefit of particular lands.

Obviously, no set rules or operating procedure could be made generally applicable, even where a single agency, such as the Bureau of Reclamation (USBR), United States Department of the Interior, administers a number of irrigation systems. On each project special procedures must be developed to meet local physical and legal conditions without violation of the fundamental principles of efficient operation.

In some areas, where the need is not yet evident, little progress has been made in this regard; in others, such as the Salt River Project, where no waste of water can be countenanced, there is rigid control of all releases from storage, diversions to canals, and deliveries to farms.

CONTINUOUS FLOW SYSTEM

There are a few instances where "natural flow" rights must be complied with, literally, and water must be delivered continuously to farms at the rates and for the periods of time fixed by such rights. However, because this procedure is so wasteful of water and so conducive to improper practices, it has been supplanted generally by a system of temporary exchanges among holders of rights of approximately equal priority. In such cases one farmer on a ditch uses two or three times his quota for a few days and then takes no water while his neighbors are irrigating their land.

On some projects where the stream flow is now regulated, the system of continuous deliveries has been abandoned completely, and the quantities of water due under "natural flow" rights are accured for a few days and then delivered later on demand. For example, on the Salt River Project the water to which these rights apply is accrued for eight days, but must be used during the ensuing eight-day period.

Demand System

It is the natural desire of farmers that water be delivered to them on demand—that is, in the quantities and at the times deemed most desirable by each individual. This is a convenient system of operation and, if costs of construction and waste of water could be disregarded, no other operating procedure would be necessary. It is commonly followed during the early life of irrigation systems, but there is probably no fully developed project in the world where the collective demands of farmers could be met without overtaxing the capacity of the irrigation works and without causing excessive waste of water.

Even where it is practicable to make deliveries to farms on demand, advance notice should be required and other reasonable limitations on the time and rate of delivery of water should be enforced from the beginning. Otherwise, the imposition of controls, when the necessity arises, will be resisted as an infringement of a vested right.

Rotation System

When the Reclamation Act of 1902 was enacted and the United States undertook the construction of irrigation projects, it was with the primary intention of developing land then owned by the federal government. Farmers who settled on these raw lands had to clear, level, and otherwise prepare them for irrigation, in addition to assuming the burden of repaying the costs of construction of the irrigation works. Consequently, economy of construction was a governing consideration on these public projects.

It was recognized by all that, if water were delivered to farms in rotation at regular intervals rather than on demand, economies could be effected because smaller canals could be used. Accordingly, several projects were designed and built to be operated in this manner. In some cases rotation was to be limited to the farms served from each lateral, maintaining at least one irrigation head in every lateral; in others, laterals were to be served in rotation; and occasionally canals were to be operated intermittently, particularly during the spring and fall when water is used less than at the peak of the irrigation season. Similar principles have been followed in the design of many nonfederal irrigation systems to minimize capital costs.

In practice, however, the delivery of water in accordance with a predetermined schedule of rotation was not effective on most projects. When these systems were first put in operation, only a small part of the land was ready to receive water and many years elapsed before full development.

In the meantime, there was no need to regulate either the quantity of water delivered to a farm or the time at which it was delivered. Farmers naturally found it more convenient to irrigate in the daytime and during the middle of the week than to conform to a schedule requiring irrigation at night and occasionally on Sundays and holidays. Hence, it became customary to supply water as requested by the farmers. They naturally were unwilling to forego this privilege when the time came that their collective demands for water exceeded the capacity of the canals. Then, of course, the engineers were blamed for designing inadequate works, and pressure was brought to have the system enlarged.

WATER ORDERS

In most well-established projects a compromise procedure of water deliveries has been developed which retains many of the advantages of the rotation system, while recognizing that the need for water on individual farms varies considerably, depending on climatic conditions, the nature of the soil, and the character and stage of development of the crops planted. In principle, each farmer is required to anticipate his needs so that there will be time enough to schedule deliveries in rotation.

Normally, requests for the delivery of specified quantities of water are placed by individual farmers with the zanjero, or ditch rider, three or four days in advance. The zanjero assembles similar water orders from other farmers on the laterals operated by him and adjusts conflicts as to time of delivery. He then transmits to his watermaster a request for the delivery into these laterals of the quantity of water which he will need two or three days later to fulfil these demands.

The watermaster, who may have control of a canal serving a dozen laterals, receives similar requests from each of the zanjeros under his jurisdiction. He assembles these and schedules the diversion of water into the several laterals to minimize fluctuations in the feeder canal. After allowance for losses in transmission and other factors, he then transmits to his superior an order for the diversion into his canal of the water necessary to supply the laterals and, in turn, the farms needing water. The senior watermaster (or superintendent as the case may be) compiles these canal orders and determines when and how much the release of water from storage, or the rate of diversion into the canal systems, must be increased or decreased so that just enough water will be available as needed for diversion into the canals and from these into laterals and from the laterals to the farms.

Even under most favorable conditions, good judgment and long experience are needed to fulfil such water orders without undue waste. Consideration must be given to the time required for water to flow from a reservoir to points of diversion and from there through the canals and laterals to the farm headgates. Such times of transit vary markedly depending on the quantity that was flowing in the stream and canals prior to the change and also on the degree of the change in flow. Channel storage also must be taken into account, because a part of any increased outflow from storage will be absorbed and a decrease will be offset in part by changes in the level of the water in streams and canals.

OPERATING WASTE

Irrespective of whether the operating procedure involves the delivery of water on demand, in rotation, or in accordance with scheduled water orders, sudden climatic changes almost invariably cause operating waste of water.

On the Rio Grande Project, for example, it requires about four days for water released from storage to reach the lowest point of diversion. This region is characterized by sudden changes in temperature and humidity, frequently accompanied by thunderstorms, which cannot be predicted with any certainty four days in advance. When such storms occur in the valleys above El Paso, Tex., the use of water is immediately curtailed, because the farmers naturally refuse to take delivery of water when their ground is already wet. This reaction, of course, results in an increase in the water wasted from canals serving this part of the project. As the normal volume of operating waste from the upper units of the project was taken into account in scheduling the supply for El Paso Valley, the superintendent of the lower unit suddenly finds himself with more water at his diversion point than he ordered, and he has no choice but to permit it to waste downstream. At other times, operating waste from this part of the project becomes unavoidable, because an inch or more of rain falls in El Paso Valley just as the water previously ordered reaches the diversion point.

Conversely, when there is a sudden rise in temperature and a decrease in humidity, transpiration of plants and evaporation from land and water surfaces are increased; less water then reaches El Paso than was expected. The natural answer would be to guard against a shortage and to have always enough water in the river to meet all possible diversion demands. Unfortunately, this cannot be done because the total supply available to the Rio Grande Project is limited and excess operating waste in one year can be overcome only by rationing deliveries of water in subsequent years.

Most other large irrigation projects have similar problems because of climatic changes. Only occasionally would it be feasible to reduce the operating waste of water to the extent that has been done on the Salt River Project where about one third of the total supply of water is obtained from wells. These wells are so spread throughout the irrigated valley that, when any sudden decrease in the demand for water occurs, the well pumps can be shut off and the flow of water in the distribution system can be made to match the lessened demand. Seldom must the rate of outflow from the reservoirs be changed, except to conform to scheduled changes in the use of water.

On the other projects it has been found practicable to provide storage basins at the ends of primary canals in which the excess water may be impounded, until the canal flow can be adjusted to the demand, and from which this water can later be withdrawn for irrigation use.

MEASUREMENT OF WATER

The release, diversion, and delivery of water must be controlled if the reasonable demands of farmers are to be met without undue waste. Such controls cannot be imposed intelligently without good measurements of both the quantities of water in storage and the quantities released from storage and then diverted from streams, passed from canals into laterals, delivered to farms, and wasted at the ends of the system. On some projects, particularly those where water rights are in dispute, the inflow to reservoirs, the volume in storage, the release from storage, and the quantities of water diverted are measured consistently and with reasonable accuracy. On a few exceptional projects, passable records are kept of the water diverted from canals into laterals; but, with the exception of those systems where water is so valuable as to require distribution through pipe lines, measurements of water delivered to farm headgates are generally little better than guesses.

More complete and accurate hydrographic records are kept on the Salt River Project than on any other large project in the United States. The inflow to and the outflow from each reservoir are measured, and a record is kept of all fluctuations in storage. Gaging stations are operated on both the Salt River and the Verde River below all reservoirs and on each of the canals just below the diversion dam. Where these canals branch, enough measurements are made so that the quantity of water diverted into laterals can be estimated with reasonable accuracy. Records are also kept of the water pumped into canals. However, in spite of the completeness of such hydrographic data, an unreasonable proportion of the total supply remains unaccounted for by charges for water delivered to farms.

The practice of delivering more water to farms than is actually charged to water users is accepted almost everywhere as being reasonable and proper. Only recently has it been recognized that water rights are thereby being jeopardized. Recently (1950), for example, the State Water Board of Texas granted an application by the City of El Paso to appropriate certain waters of the Rio Grande. This application was based in considerable part on the apparent availability of waste waters. Actually most of the presumed excess supply of water was used beneficially for irrigation but it was not accounted for in the records of deliveries to farms in the El Paso Valley.

If the quantity of water delivered to farms were measured accurately, the magnitude of wastes and seepage losses from canals and laterals would not be exaggerated and the quantity of water required for beneficial use in the irrigation of land could be evaluated accurately. Better methods of measuring the flow of water from laterals through farm headgates are needed.

MAINTENANCE OF WORKS

Efficient operation of irrigation projects depends on all the works being kept in a good state of repair. As stated in a manual of the USBR:

> "The maintenance of irrigation projects is a highly important function, and essentially a technical job requiring a competent and experienced organization. Sufficient equipment and machines to accomplish the necessary

> work shall be available at all times, commensurate with cost. Shop facilities for the repair and overhaul of machinery and equipment must also be provided. The water users are entitled to water delivery service from year to year with but minor interruptions."

Wherever water is used to irrigate land, there is the problem of removal and disposition of the portion not consumed by the plants or evaporated from the surface of the ground. This may be an operating function, or it may be a maintenance problem, depending on the manner in which the area is drained. In the Salt River Valley, the water that seeps out of canals and percolates downward from irrigated fields is removed by pumping from wells, there being very few open drainage channels. In most other projects, waterlogging of the land is prevented by a network of open drains. Their operation is automatic, but maintenance of these channels is a major problem because of the tendency for them to become clogged with silt or aquatic vegetation.

The problems of maintenance are too varied to permit more than reference to some of them—such as repair of frost damage, removal of silt deposits, and control of weeds and aquatic plants.

Frost Damage.—There are few irrigation projects in the United States where some structures are not subject to damage by alternate freezing and thawing, such as the spalling of concrete on the face of a dam and the heaving of pavement slabs in spillways and other concrete-lined channels. Some of the effects of ice and of alternate freezing and thawing can be obviated in the design of structures; others can be minimized by care in construction. However, repair of damage from such causes is a substantial part of the maintenance expense on most projects in northern climates.

Removal of Silt.—On many irrigation systems, the removal of silt deposits constitutes the greatest item of maintenance. For example, before Hoover Dam was constructed on the Colorado River, the removal of silt from the headworks and canals in Imperial Valley was a never-ending battle. The water diverted from this river carried with it in suspension many millions of tons of silt and sand each year. Some of this was caught in settling basins just downstream from the headworks and removed by dredges which operated continuously; the greater part, however, passed on down the canals, and it was only by keeping these full and by wasting the excess water into channels which flowed to Salton Sea that the deposition of silt could be controlled reasonably. Even so, it was necessary to keep draglines, "Ruth" dredges, and other excavating equipment in operation on every canal. The maintenance expense was so great as to jeopardize the economic stability of the project.

In the Middle Rio Grande Valley of New Mexico it has not been possible to keep the irrigation system in good operating condition because of the volume of sand and silt carried by the river. Large reservoirs to intercept these sediments must be constructed if the rate of deposition is to be kept within reasonable bounds. Elsewhere the problem is generally less acute.

Control of Vegetation.—Except for the few irrigation systems where water is obtained from wells and distributed in closed conduits, the control of weeds and aquatic vegetation is a universal maintenance problem.

Some grass or other vegetation is desirable on ditch banks and other earth slopes to prevent erosion, but it cannot be left to grow uncontrolled. Sooner or later objectionable grasses, weeds, willows, or other plant pests too numerous to mention take root and persist in spite of all preventive efforts. The best that can be done is to keep such growths reasonably under control.

Aquatic plants, such as tules, cattails, and water hyacinth, are not normally much of a problem in the maintenance of canals, but they thrive in the clear water of drainage channels. Drains can become almost nonoperative in a surprisingly short time if maintenance is deferred.

Moss will frequently form and almost clog canals where the water is clear. Under favorable conditions of temperature and sunlight the growth of moss is rapid; streamers 20 ft to 30 ft long are not uncommon. Raking or cutting this moss free is only part of the problem because it comes to the surface and floats downstream where it tends to clog gate openings, etc. Hence, while one crew is cutting it free, another maintenance crew must be stationed downstream to remove it from the canal.

Occasionally, and for no apparent reason, some new type of growth will make its appearance, requiring almost heroic measures to keep it under control. This was the case in the Imperial Valley shortly after the silt problem was solved by the construction of Hoover Dam. A species of bamboo began to grow on the canal banks and to send out roots for great distances, from which other clumps of bamboo would sprout.

The growth of tamarisk, or salt cedar, is another example of plant infestation which has added to the maintenance problems of many irrigation systems. Before 1920 it was almost unknown; now it grows so thickly in many places as to be almost impenetrable. The consumption of water by tamarisk is about the same as the evaporation loss from a free water surface. Consequently, the eradication (or at least the control of it) constitutes a substantial maintenance problem for which no definite remedy has yet been found.

Methods of controlling weeds and other undesirable vegetation along canals and around reservoirs differ widely depending on conditions peculiar to the area. On some projects, particularly those where the removal of silt deposits is necessary, canals are generally cleaned by mechanical methods. On others, burning with flame throwers has been found successful and reasonably economical. Chemical spraying is favored in some areas and probably will become more generally adopted as better sprays are developed and better equipment is manufactured.

MACHINERY AND EQUIPMENT

In this connection, the need for machinery and equipment designed for cleaning canals and drains has been unrecognized, although the amount of money spent annually in the maintenance of canal systems is great enough to justify the development of special machines. Tremendous strides have been made in the development of excavating equipment for use in construction because of contractors who, under the whip of competition, demand greater speed, efficiency, and ease of control. Those who maintain irrigation systems still struggle along with makeshift adaptations of equipment designed for other purposes. Perhaps this is because irrigation projects are separate entities, among which there is little competition, and the cost of maintenance is looked upon as a burden to be borne with resignation.

COSTS AND COST ACCOUNTING

Little effort is generally made to allocate costs of operation and maintenance. Even the manuals of the USBR are substantially silent in this respect, although many millions of dollars are spent each year on federal reclamation projects alone for such purposes.

This apparent lack of interest in cost accounting presumably arises from the fact that irrigation projects are operated in effect as cooperatives. Under ordinary circumstances, the cost of labor of all classes is estimated for the year, and to this is added the probable cost of materials and supplies and equipment to be purchased. Estimates are also made of the probable revenue from water sales, if any, and of other incidental revenues. The difference between the total estimated cost and the estimated direct revenues is then charged to the water users in the form of assessments per acre or taxes assessed against all property within the project. Such budgetary accounting is not sufficient for proper control of costs.

In 1940, when economic conditions were reasonably stable, the combined cost of operation and maintenance on typical federal reclamation projects was as follows:

Project	Area, in acres	Cost per acre
Milk River, Montana	117,400	$0.69
Belle Fourche, North Dakota	61,000	0.85
Minidoka, Idaho	68,900	1.48
Boise, Idaho	166,400	1.20
Yakima, Washington	165,500	1.87
Rio Grande, New Mexico and Texas	142,800	2.40
Yuma, California and Arizona	67,400	2.26
Salt River, Arizona	243,000	1.75

These costs are no measure of what must be spent today to operate and maintain an irrigation project in good condition. For example, the total cost for such functions was reported to have been only $414,350 on the Salt River Project in 1940, whereas the total reported for 1948 was $2,433,302 (see Table 1).

This total cost, equal to practically $10.00 per acre, is exclusive of $393,393.89 set up on the books as depreciation accrued in that year on irrigation properties in service. For comparison with the reported cost in 1940, the charge made for power used in pumping should be deducted in Table 1. This was $728,866 computed at the same rates as for power sold to others. Even on this basis, the direct costs of operation and maintenance of the Salt River Project quadrupled from 1940 to 1948.

The impact of changed economic conditions has been less on most other projects up to the present time, principally because they have not had to conform to federal labor laws. As these become applicable to such other projects, which should be anticipated, their costs of operation and maintenance must likewise increase tremendously. In this connection the Supreme Court of the United States on June 27, 1949, held that a mutual water company was subject to the provisions of the wage and hour acts (the Farmers Reservoir and Irrigation Company versus William R. McComb, Administrator of the Wage and Hour Division, 17 LW 4688).

TABLE 1.—OPERATING EXPENSES OF THE IRRIGATION DEPARTMENT, SALT RIVER PROJECT, FOR THE YEAR ENDING DECEMBER 31, 1948

Item (1)	Class of expense (2)	Operation (3)	Maintenance (4)	Total (5)
1	Surface water production	$ 40,853.16	$ 15,099.88	$ 55,953.04
2	Ground water production	775,837.08	58,260.62	834,097.70
3	Purchased water	10,440.81	....	10,440.81
4	Transmission expense	52,570.38	71,151.76	123,722.14
5	Distribution expense	461,519.43	426,129.47	887,648.90
6	Drainage expense	43,167.24	126,915.56	170,082.80
7	Subtotal	$1,384,388.10	$697,557.29	$2,081,945.39
8	General expenses not allocated	....	....	351,356.84
9	Total: Irrigation, operation, and maintenance	....	....	$2,433,302.23

SUMMARY

Inasmuch as the foregoing has been little more than a summary of the problems of operation and maintenance of irrigation systems, it seems appropriate to close with only a restatement of the basic principles that govern their solution: Efficient operation is defined as the delivery of enough water to each farm at the right time to satisfy all beneficial needs without waste. It depends on the maintenance of all storage, diversion, distribution, and drainage works in good condition at all times.

DISCUSSION

George W. Howard,[2] M. ASCE.—This excellent paper presents in a straightforward manner the numerous problems confronting the operators of irrigation systems.

To those individuals having a direct interest in the costs of operation and maintenance, the involved system of government bookkeeping frequently results in confusion. Perhaps this type of accounting procedure causes the apparent lack of interest in cost accounting referred to by Mr. Hill; and when control of the district is taken over from the Bureau of Reclamation (USBR), United States Department of the Interior, the water users employ the absolute minimum of bookkeeping required to keep their accounts legible to the members of the using association. Certainly it is not economically feasible for an independent organization to adopt the involved accounting system used by the government, for the expenses connected with the cost accounting would probably exceed their benefits. As a result of the many legal restrictions placed on the USBR, however, it is necessary that the more elaborate system be used when a project is operated by the USBR.

TABLE 2.—Operating Expenses of the Reservation Division, Yuma Project, Arizona–California (in Dollars).

Item	1946	1947	1948	1949	1950
Delivery system	$1,873	$3,253	$2,811	$9,709	$4,228
Distribution system	39,398	41,008	50,063	39,631	86,097
Drainage system	200	5,589	1,318	1,821	15,597
Total irrigation operation and maintenance	50,397	62,595	69,966	57,925	119,696
Replacements and improvements	....	....	....	....	18,598
Total irrigation cost	$50,397	$62,595	$69,966	$57,925	$138,294
Minimum water rate, in dollars per acre[a]	3.00	4.00	6.00	6.00	6.00

[a] Based on a minimum usage of 4 acre-ft.

The Bard Irrigation District, Imperial County, is the California part of the Yuma (Calif.) and Arizona Reclamation Project of 67,400 acres. The Reservation Division of this project, of which the Bard Irrigation District is a part, has 7,743 acres of Indian lands and 6,287 acres in the Bard Irrigation District. It is still operated by the USBR and efforts to understand the financial outlay chargeable against this district result in many headaches for the board of directors. Although the USBR operates the district, the directors represent the people who are the actual water users and who pay the expenses incurred in operating the project. Excellent cooperation is obtained from the USBR, but the board of directors of the Bard Irrigation District is placed in the awkward position of lending approval to expenditures after-the-fact and of having no voice in the policies connected with administration of the district.

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[2] Chf., Technical Service Dept., Eng. Research and Development Laboratories, Fort Belvoir, Va.; Director, Bard Irrigation District, 1949.

Operating expenses of the reservation division of the Yuma project for the years 1946–1950 are shown in Table 2, which gives an indication of the increase in operation costs for the last five calendar years. This represents a considerable increase over the $2.26 listed by the author as the cost per acre in 1940. From inspection of the total irrigation cost figure in this table, it is evident that the $6.00 per acre minimum water rate will not continue long.

The basic principle indicated in this paper is met in the Bard Irrigation District; that is, that enough water is delivered to each farm at the right time to satisfy all beneficial needs without waste. It is still a very evident fact and a cause of great concern to all water users that unless the minimum water rate per acre is kept as low as possible, the resultant operating expenses of irrigation projects are going to cause real financial hardship.

Lyman R. Flook, Jr.,[3] Jun. ASCE.—A discussion of the operational water wastes in irrigation systems is given in this paper, with an accurate description of how waste originates during thunderstorm activity. A large quantity of water is wasted in this manner, but a more complete list of operating waste for irrigation projects in the Southwest would include: (*a*) Reservoir losses; (*b*) evaporation and transpiration losses along the distribution system; (*c*) excessive water orders; (*d*) climatic changes; (*e*) unwillingness to irrigate at night; (*f*) structure failures; (*g*) communications breakdown; and (*h*) excessive releases to speed delivery.

(*a*) *Reservoir Losses.*—Evaporation and transpiration for the storage reservoirs and contiguous vegetation may be more than 30% of the irrigation release in a year.

(*b*) *Evaporation and Transpiration Losses Along the Distribution System.*—Water is lost from the free water surfaces of canals, laterals, and ponds to rank vegetation growing along the distribution system. Good maintenance practice reduces these losses to a minimum.

(*c*) *Excessive Water Orders.*—Unrealistic and often excessive water orders are placed by farmers for a variety of reasons. The average farmer on irrigated land cannot be expected to place consistently accurate water orders. When water charges are relatively low compared to his other operating costs, and particularly when he is charged only for water used and not for water ordered, excessive water orders are to be expected. Farmers will actually divert and pay for excessive volumes of water, much of which is wasted. Then, when a high demand period occurs, the farmer can obtain an immediate increase in his individual water supply by simply reducing his normal waste. Since wastes are frequently rediverted to meet the needs of lower users, this action further exaggerates water delivery problems during a high demand period.

(*d*) *Climatic Changes.*—These changes are thoroughly discussed by the author in the section on "Operating Wastes."

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[3] Engr., International Boundary & Water Commission, El Paso, Tex.

(*e*) *Unwillingness to Irrigate at Night.*—Refusal to use available water at night or other inconvenient times results in wastes from the lower division of a project. Wastes from the upper division are often usable since they are redivertable. It is impractical to attempt to schedule reservoir releases several days in advance to eliminate arrival of irrigation water at night.

(*f*) *Structure Failures.*—Canal breaks and other structure failures, that usually occur during periods of peak water demand, cause water to be available at diversion points without sufficient notice and at unusable rates.

(*g*) *Communications Breakdown.*—During thunderstorm activity, the long telephone lines from a headquarters office to the point of irrigation release may go out of service. This results in delay in ordering changes in irrigation release and aggravates water loss caused by storms.

(*h*) *Excessive Releases to Speed Delivery.*—Occasionally, during periods of hot weather, the water demand exceeds the capacity of the system to supply it. Then it becomes necessary to release excessive amounts of water from storage to reduce the travel time of the release through the system to the lower water users. The excess releases are wasted below the lower end of the project.

Any enumeration of causes of water loss on irrigation projects is of little value unless it points the way to reducing unnecessary losses. Obviously, the best time to create an efficiently operating irrigation system is in the original planning and design of the system. As full stream-flow utilization is approached, an excellent opportunity exists for designing irrigation systems that can be operated with minimum waste. Those charged with the responsibility of laying out irrigation systems should be fully aware of all important operating problems likely to arise.

One of the best methods of obtaining flexibility in a system, a method often overlooked, is mentioned by the author in the section on "Operating Waste." This is to provide short-term reservoir storage capacity near the end of the main canals. The capacity need not be large compared to the long-term storage capacity supplying the system, but wherever the point of initial irrigation water release is far from the point of use, waste is inevitable unless some storage capacity is provided near the point of use.

Given a well designed and constructed system, adequate funds and competent personnel must be obtained to insure adequate protection of the original investment and continuous efficient operation. Too often a good irrigation system is allowed to degenerate, and ultimatly to require heavy expenditures, in the interest of short term economy.

Rodolfo E. Ballester[4].—A very interesting problem is called to the attention of engineers in this paper. "The maintenance of irrigation projects is a highly important function, and essentially a technical job * * *" as stated in the section "Maintenance of Works." Engineers have at their disposal an extensive and varied literature covering structures from dams to canal linings, but very few or no references concerning the technique and methods of operation and maintenance of irrigation structures. It would seem that engineers prefer to pass the burden of this type of job to the water users.

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[4] Prof. of Applied Hydraulics, Univ. of Buenos Aires, Buenos Aires, Argentina.

The author calls attention to some additional points: Measurement of water, delivery of water for beneficial use, and rigid control of all releases from dams to the farm laterals. The writer thinks that the last two points cannot be accomplished if the first is not realized. Under the heading "Measurement of Water," the author states that "* * * measurements of water delivered to farm headgates are generally little better than guesses."

The writer has had a very interesting experience on one of the irrigation systems of Argentina. The "Centenario" ("Neuquen") system is operated in rotation from laterals. The discharge for each lateral was measured at the headgates, with all the complications of measuring the opening, upstream head, downstream head, and fixing the coefficient of discharge for free or submerged orifice. There were always differences with the water users about the true quantity delivered. On three laterals, just below the headgates, standing wave flumes were installed with scales graduated in liters per second. The first reaction of the water users was that water delivery would be diminished. They said that the reduction of section of the lateral caused by the flume would reduce the discharge. However, after a year of operation, instead of asking for delivery in terms of centimeters of opening of the headgate, they asked for a quantity of liters per second. The most illiterate farmers started to think in terms of volume of discharge instead of gate openings. A water user even built a flume at his own farm to control the general delivery.

To the watermaster or superintendent, the record of the flume discharge was very useful for the quantitative consideration of the requirements of the lands served by the laterals and for the preparation of the program of deliveries for the whole system.

Of the various systems of water distribution the rotation system is the most rational. In the Centenario system water was delivered at the start of cultivation using the continuous flow system. The headgate of each farm had a padlock under the "zanjero" control for keeping a fixed opening. Later, the system was changed to rotation with a schedule of deliveries to each farm fixed at the beginning of the water year. Padlocks were removed. Now each water user opens his headgate at the time fixed in the schedule. The rotation system is not applicable to smaller canals, as the author states, because larger sections are necessary to carry the accumulated discharge that occurs in a shorter time.

The chief problem in the maintenance of canals and open drains is the control of weeds. The writer has observed that irrigation canals with velocities equal to or exceeding the critical velocity given by R. G. Kennedy (neither silting nor scouring) are almost always free of weeds. In open drains the problem is most serious, for it is not possible to maintain a high velocity for weed control. In an irrigation system in which the water is used with high efficiency, the open drains must discharge a very small quantity of water, and this condition favors the growth of weeds.

An irrigation system is comparable to a railroad. Once built, technical progress of every kind must be applied for betterment of service. The maximum beneficial use of water is the ideal of an irrigation system, and to arrive at this goal, cooperation is necessary between water users and engineers.

This paper shows that there are many opportunities for engineers to apply their technical skill in the operation and maintenance of irrigation systems, but they must never neglect the farmers' viewpoint and capacity.

Alfred R. Golzé,[5] M. ASCE.—In the section devoted to "Costs and Cost Accounting," the opening paragraph states:

> "Little effort is generally made to allocate costs of operation and maintenance. Even the manuals of the USBR are substantially silent in this respect, although many millions of dollars are spent each year on federal reclamation projects alone for such purposes."

It is not known what manual the author had reference to. The USBR in 1949 concluded a revision of its accounting system that makes full provision for the operation and maintenance activities on the Federal Reclamation Projects both for irrigation and for power. For irrigation, accounts are provided to record separately the expenses for storage systems, carriage systems, distribution systems, drainage systems, and general administrative expenses. For the first four elements, costs are separated between operation and maintenance. Operation is further subdivided between labor, supplies, and expenses. Maintenance is subdivided to show cost for physical features, such as reservoirs, dams and waterways; structures, and improvements; and so forth. For general and administrative expenses costs are recorded for office salaries, office supplies and expenses, office furniture, etc.

Expenses are related to operating income in the accounts so that the net income for irrigation can be determined. Cost reports of income and expenses and the relation of the two are made each month for each project operated by the USBR. In addition to maintaining costs under the breakdown cited, each irrigation project operated by the USBR is required to program its anticipated expenditures over a seven-year period. The components of these programs correspond to and are arranged in the same order as the expense (cost) accounts and constitute the support for the annual budget preparation. The purpose of preparing expenditure programs over a relatively long period is to aid the project personnel in the development of an orderly schedule for replacement of worn-out or obsolete structures and for the replacement of operating equipment such as drag lines. It is obvious that replacement expenditures must not be allowed to "peak" in any one year, as they could be too heavy a burden for the water users.

The USBR secures its funds for the operation and maintenance of its irrigation projects from two sources. The Congress annually appropriates part of the money and the remainder is advanced by the water users as trust funds. At the present time (1951) Congress is appropriating about $3,000,000, and the water users are advancing about $2,500,000. It should be understood that 48 of the 65 completed federal projects or divisions of projects are operated by water users themselves at their own expense without reference to the federal government. An example of this is the Salt River Valley Project in Arizona. The principal works operated by the government at this time are the backbone irrigation facilities of the Central Valley Project in California and reserved works of reservoirs serving both federal and private lands on a number of projects. The only completed irrigation projects wholly operated by the government are the Rio Grande Project in New Mexico and Texas, which the water users have consistently refused to take over, and the Klamath Project in California and Oregon.

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[5] Director, Programs and Finance, Bureau of Reclamation, U. S. Dept. of the Interior, Washington, D. C.

For those projects for which appropriations are made by Congress, the benefiting water users are obligated to return the cost annually as determined by the Bureau's accounting system. As a general rule the government recovers the cost of operation and maintenance chargeable to appropriated funds within twelve months from the time in which the expenditures are made.[6]

RAYMOND A. HILL,[7] M. ASCE.—The discussion by Mr. Flook is largely an amplification of the causes for operating waste. It is not believed that the first two of the eight causes listed by him (reservoir losses and evaporation and transpiration losses along the distribution system) should be classified as operating waste. Such evapo-transpiration losses cannot be reduced materially by improvements in operating procedure. Although failure of structures and breakdown of communications may occasionally contribute to the volume of operating waste and provision should be made to prevent such losses by design of adequate works, it hardly seems appropriate to include these in the classification of operating waste. Except for these matters of terminology, the writer agrees fully with Mr. Flook.

The comments of Mr. Ballester should be particularly informative to engineers in this country because they are prone to think that their methods are the most advanced. The writer was surprised, however, to note that the rotation system of deliveries had superseded the demand system (with apparent approval of the farmers) in the irrigation system in Argentina. It may be that the quantity of water available there for deliveries in rotation is sufficient to meet the maximum requirements on individual farms regardless of climatic changes. Generally, however, it is believed that the delivery of enough water to meet crop requirements cannot be made under a rigid time schedule, fixed at the beginning of the water year, without substantial operating waste.

The remaining two discussions of the original paper have to do with costs of operation and maintenance. In amplification of the tabulation given by Mr. Howard it is to be noted that the unit cost per acre on the Reservation Division of the Yuma Project has increased from $2.26 in 1940 to $3.60 in 1946 and to $8.50 in 1950, exclusive of costs of replacements and improvements.

The objection made by Mr. Golzé to the reference by the writer to manuals of the USBR is proper, and the original statement warrants explanation. The manuals and other data obtained from the USBR at the time of preparation of the paper in 1949 did not include the edition of the manual to which Mr. Golzé refers. Neither was it intended to give the impression that many millions of dollars are spent each year by the USBR for operation and maintenance, as it was realized that many federal projects or divisions of projects are operated by the water users themselves. The fact remains, however, that the federal government can and does exercise considerable control over the operations of all projects built by the USBR.

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[6] "Operation and Maintenance Costs and Gross Crop Values per Irrigated Acre—1949," Bureau of Reclamation, U. S. Dept. of the Interior, Washington, D. C., pp. 1–3.

[7] Cons. Engr., Leeds, Hill, and Jewett, Los Angeles, Calif.

It appears from the description of the irrigation accounts now in use on federal reclamation projects that these accounts are fundamentally the same as those that have been used on the Salt River Project since 1943. These were patterned on the standard accounts prescribed by the Federal Power Commission for utilities coming under its jurisdiction. This system of accounts has the great advantage of being usable by any operating entity, regardless of size. Such detailed accounts as those described by Mr. Golzé can be kept wherever such detail is warrented by the magnitude of the costs; on the other hand, even a very small project can maintain the control accounts listed in Table 1.

There is a disposition on the part of most organizations of water users on irrigation projects to keep no accounts beyond a record of income and disbursements. On the other hand, every privately owned domestic water company must report its costs in considerable detail to its state regulatory body, and most municipal water companies make similar reports. The costs of operation and maintenance of domestic water companies are in the same order of magnitude as those incurred in the operation and maintenance of irrigation systems. It is believed that the same benefits would accrue to irrigation districts and water users' associations if their records of account were kept in sufficient detail to permit effective management control of such costs.

Mr. Golzé refers to the development of an orderly schedule for replacement of worn-out or obsolete structures and for the replacement of operating equipment. So great has the need for replacements become on many federal reclamation projects that the cost of such replacements cannot be met now on a "pay-as-you-go" basis. Accordingly, the USBR is entering into contracts with the water users on many projects that provide for repayments over an extended period of years of the costs of replacements that can no longer be deferred.

In the opinion of the writer, reserves for depreciation should be established on every irrigation project, and the charges to depreciation expense should be sufficient to provide for all replacements of property. All public utilities are required to take depreciation into account; there is every reason why those responsible for the operation of irrigation projects should do likewise. As stated by the United States Supreme Court,[8]

> "Broadly speaking, depreciation is the loss, not restored by current maintenance, which is due to all the factors causing the ultimate retirement of the property. These factors embrace wear and tear, decay, inadequacy, and obsolescence. Annual depreciation is the loss which takes place in a year. In determining reasonable rates for public service, it is proper to include in the operating expense, that is, in the cost of producing the service, an allowance for consumption of capital in order to maintain the integrity of the investment in the service rendered."

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[8] Lindheimer versus Illinois Bell Telephone Company, 292 U.S. 167, 1934.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DESIGN OF PRESTRESSED TANKS

By J. M. Crom[1]

With Discussion by Messrs. L. J. Mensch, Herbert A. Sawyer, Jr., and J. M. Crom.

Synopsis

In the past, the safe design of circular structures has been extremely difficult, mostly because exact knowledge concerning shrinkage and plastic flow of concrete has been lacking. This paper contains new data for evaluating these phenomena and describes methods that make practicable the use of the high strength of cold-drawn steel wire. Such developments have made it possible to design tanks and other large circular structures on a rational basis, with the assurance that an adequate prestress will be maintained to eliminate cracking of concrete and that important reductions will be made in the weight and in the quantities of critical raw materials required for their construction.

Historical

The first important experimentation on methods of prestressing concrete began in France and Germany during the last two decades of the nineteenth century, principally in connection with the reinforcement of beams and girders. Patents covering these methods were taken out during this period in the United States and in the principal nations of Europe. For many years, however, results were highly discouraging because the prestress in the steel reinforcement was released by shrinkage and plastic flow of the concrete. The early experimenters, C. E. W. Doehring, in Germany (1888), J. G. F. Lund in Norway (1907), M. Koenen,[2] in Germany (1907), E. Freyssinet,[3,4,5,6,7] in France (1928

Note.—Published in October, 1950, as *Proceedings-Separate No. 37*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Vice-Pres., The Preload Enterprises, Inc., New York, N. Y.

[2] "Wie kann die Anwendung des Eisenbetons in der Eisenbahnverwaltung gefördert werden?" by M. Koenen, *Zentralblatt der Bauverwaltung*, 1907.

[3] "Idées et Voies Nouvelles," by E. Freyssinet, *Science et Industrie*, Construction et Travaux Publics Ed., January, 1933, p. 3.

[4] "Progrès Pratiques des Méthodes de Traitement Mécanique des Bétons," by E. Freyssinet, *ibid.*, Travaux Ed., June 30, 1935, pp. 199–210.

[5] "Une Révolution dans les Techniques du Béton," by E. Freyssinet, Librairie d'Eyrolles et l'Enseignement Technique, Paris, 1936.

[6] "Developments in Concrete Making," by E. Freyssinet, *Concrete and Constructional Engineering*, April, 1936, pp. 209–220.

[7] "Une Révolution dans l'Art de Bâtir les Constructions Précontraintes," by E. Freyssinet, *Science et Industrie*, Travaux Ed., November, 1941.

to 1950), Ewald Hoyer,[8] in Germany (1939), P. H. Jackson, in the United States (1888), J. Mandl,[9] in Austria (1896), G. R. Steiner, in the United States (1908), and others discovered that they would not be able to prestress a structure successfully until they could use concretes and steel of higher elastic values and higher strength values than those available at that time.

Large sums were spent to improve the quality of both materials. Mr. Freyssinet, for instance, produced concrete that hardened rapidly, with compressive values in excess of 14,000 lb per sq in. and used reinforcing steel that had an ultimate strength of about 200,000 lb per sq in. With these new materials he was able to use steel stresses sufficiently high so that the shrinkage and flow of the concrete eliminated only a small percentage of the prestress. As a result, many important structures containing prestressed beams and arches were built in Europe during the years preceding World War II.

In the United States, most of the prestressing work has been done in connection with circular structures such as storage tanks, silos, and large-diameter pipe, although some small precast members such as joists and roof slabs have been built. In the early 1920's, in Minnesota and other locations, A. L. Hewett[10] designed and built a number of water tanks whose walls were prestressed with steel rods of ordinary structural grade. Mr. Hewett and many other engineers believed at that time that shrinkage and plastic flow of concrete were almost negligible, and that it was not necessary to consider these factors seriously in tank design.

Although many successful tanks were built under this assumption, it gradually became apparent that there were defects either in the designs or in the methods of construction. Occasionally there were serious cracking and leaking which could not be explained and costly repairs had to be made. Pneumatic mortar or gunite, to use the more common term, was used for the walls and domes of some of these early tanks, since it had greater density and imperviousness than ordinary concrete and because its use practically eliminated construction joints. About 1941, however, it was rumored that the shrinkage rate of gunite was several times that of conventional concrete. These rumors could not be refuted, because of the lack of authentic data.

Accordingly, an interested commercial organization, in collaboration with the Massachusetts Institute of Technology (in Cambridge) and other institutions, sponsored a series of tests to determine not only the comparative shrinkage rates of concrete and gunite, but also the characteristics of these materials with respect to shrinkage, plastic flow, and other factors that might affect prestressing operations.

As the results of these tests became available, it was apparent that shrinkage and flow were principally responsible for the failures that had occurred. It was also obvious that new methods for maintaining stresses in steel would have to be devised if prestressed tank structures were to remain consistently stable under normal operating conditions.

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[8] "Der Stalsaitenbeton," by Ewald Hoyer, *Verlagsgesellschaft*, Otto Elsner, Berlin, 1939.

[9] "Zur Theorie der Cement-Eisen-Constructionen," by J. Mandl, *Zeitschrift*, Oesterreichischer Ingenieur-und Architekten-Verein, November 6, 1896, pp. 593–596; November 13, 1896, pp. 605–609.

[10] "Water Tanks of Reinforced Concrete," by A. L. Hewett, *Journal*, A.W.W.A., Vol. 28, 1936, p. 43.

SHRINKAGE AND PLASTIC FLOW

Concrete.—Various testing laboratories, and writers in the technical press,[11] have repeatedly demonstrated that the shrinkage of concrete is due principally to drying out or dehydration. Curing has little effect on the ultimate amount of shrinkage. Franklin R. McMillan has shown that concrete, if kept saturated with water, will not shrink, but shrinkage will progress rapidly as soon as concrete begins to dry.[12] Resaturation will cause concrete to expand, but will not return it to its original dimensions when shrinkage has been under way for any appreciable length of time. Concretes with low water-cement ratios shrink less than concretes with higher water-cement ratios. Concrete that has been dried in a baking oven is subjected to maximum shrinkage, having a coefficient of 0.0010 and more.

Tests reported in 1946 show[13] that, over a 5-year period, concrete under a constant load approximating 500 lb per sq in. may attain a deformation defined by a coefficient of 0.0006. Thus maximum shrinkage and plastic flow may express a total coefficient of 0.0016, which approaches a release in steel stress of 48,000 lb per sq in., with E_s at 30,000,000 lb per sq in.

Mr. Freyssinet[3,4,5,6,7] has cited conditions under which the combined losses from plastic flow and shrinkage in both steel and concrete have reached totals equivalent to a coefficient of 0.0024, or of 72,000 lb per sq in. in steel. The data in this paper, however, are based on the use of steels and concretes of high unit values; and therefore the coefficients used for the computations reported herein are considered conservative.

Gunite or Pneumatic Mortar.—Loaded and unloaded specimens of concrete and pneumatic mortar were under observation in the laboratories of the Massachusetts Institute of Technology for more than 1,600 days. The results at the end of 400 days were published in 1946.[14] These tests demonstrated that: (*a*) The concrete specimens shrank about one third more than the pneumatic mortar specimens; and (*b*) the extent of plastic flow in the concrete specimens was greater than that in the pneumatic mortar specimens.

Reinforced Concrete and Pneumatic Mortar Under Stress.—The most significant tests made, however, were those designed to measure the loss of steel stress in reinforced concrete and pneumatic mortar under conditions that paralleled as closely as possible those that would occur in the walls of an empty tank in the field.

The specimens for the tests were 4-in. by 4-in. bars of concrete and pneumatic mortar, 24 in. long. The mortar mix was 1:3 and the concrete mix was 1:2.1:3.2. Reinforcing bars were $\frac{3}{8}$-in. and $\frac{1}{2}$-in. round rods, intermediate grade. Each specimen was made under standard testing laboratory conditions except that the pneumatic mortar bars were shot in the field by operators with experience in handling this material.

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[11] "Effect of Plastic Flow and Volume Changes on Design," by C. T. Morris, Rept. of Committee 313, *Journal*, A.C.I., November–December, 1936, pp. 123–128.

[12] "Shrinkage and Time Effects in Reinforced Concrete," by Franklin R. McMillan, *Bulletin No. 3*, Studies in Engineering, Univ. of Minnesota, Minneapolis, Minn., p. 29, Fig. 13.

[13] "Theory of Plates and Shells," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1st Ed., 1940.

[14] "Shrinkage and Plastic Flow of Pre-Stressed Concrete," by Howard R. Staley and Dean Peabody, Jr., *Journal*, A.C.I., January, 1946, p. 232, Fig. 2.

The specimens were loaded by using external plates and threaded rods so as to simulate, as nearly as possible, the prestressing operations then in use. The sectional areas of the rods were adjusted to provide high, intermediate, and low loadings for both the concrete and the pneumatic mortar specimens. These values (in pounds per square inch) were, respectively, 2,390, 1,550, and 920 to 935. Readings were also taken on parallel sets of unloaded bars to determine the deformation that could be attributed only to shrinkage, and thus, by deduction, to measure the amount of plastic flow.

TABLE 1.—STUDY OF STRESS LOSSES RESULTING FROM SHRINKAGE AND PLASTIC FLOW

Item	Description	STALEY-PEABODY TESTS[a]				AS ASSUMED FOR HIGH-STRENGTH STEEL			
		Steel Stresses,[b] f_s		Concrete Stresses, f_c		Steel Stresses,[b] f_s		Concrete Stresses, f_c	
		Gunite	Concrete	Gunite	Concrete	Gunite	Concrete	Gunite	Concrete
(1)	(2)	(3)	(4)	(5)	(6)	(3)	(4)	(5)	(6)
	(a) HIGH LOAD[c]								
	Areas, in Square Inches:								
1	Concrete section, A_c	17	16	17	16	17	16	17	16
2	Steel section, A_s	1.18	1.18	1.18	1.18	0.271	0.255	0.271	0.255
	Stress, in Pounds per Square Inch:								
3	Initial unit stress	34,330	32,515	2,390	2,390	150,000	150,000	2,390	2,390
4	Stress at 400 days	13,700	8,150	950	600	129,370	125,650	2,060	2,000
	Loss of Stress (Line 3 − Line 4):								
5	In pounds per square inch	20,630	24,355	1,440	1,790	20,630	24,350	330	390
6	Percentage (line 5/line 3)	60	75	60	75	13.7	16.2	13.7	16.2
	(b) LOW LOAD[c]								
	Area, in Square Inches:								
7	Concrete section, A_c	17	16	17	16	17	16	17	16
8	Steel section, A_s	0.44	0.44	0.44	0.44	0.104	0.100	0.104	0.100
	Stress, in Pounds per Square Inch:								
9	Initial unit stress	35,545	34,000	920	935	150,000	150,000	920	935
10	Stress at 400 days	15,100	10,000	390	275	129,555	126,000	795	780
	Loss of Stress (Line 9 − Line 10):								
11	In pounds per square inch	20,445	24,000	530	660	20,445	24,000	125	150
12	Percentage (line 11/line 9)	57	71	57	71	13.6	16.0	13.6	16.0

[a] "Shrinkage and Plastic Flow of Pre-Stressed Concrete," by H. R. Staley and Dean Peabody, Jr., *Journal*, A.C.I., January, 1946, p. 229. [b] Unit Stress in steel, $f_s = \frac{f_c A_c}{A_s}$. [c] Computations for medium loads have been omitted to conserve space.

Table 1 gives the stress losses in the loaded bars at the end of 400 days as reported by Messrs. Howard R. Staley, A. M. ASCE, and Dean Peabody, Jr., M. ASCE. These losses were the principal data which the tests were designed to determine. The losses in steel stress shown in Cols. 3 and 4 were caused by the shortening of the concrete and pneumatic mortar sections as the result of shrinkage and plastic flow. The bars were shortened by approximately the same amount irrespective of the load that had been imposed on the steel. The variation in the length of the respective bars was used to measure the losses of stress in the rods.

Before the results of these tests could be applied to the construction of tank walls in the field, it was necessary to make allowance for a number of additional factors which experience has shown also may be responsible for losses of stress.

The Staley-Peabody tests in Table 1, for instance, do not include losses incurred during the beginning of the tests, because the rods were retightened at the end of the second day. The amount of loss was determined by examining the unloaded specimens. There must be included, therefore, an additional loss of about 3,000 lb per sq in. due to initial shrinkage (as determined by examining the unloaded specimens). A loss of this magnitude would certainly take place if concrete was kept saturated with water to the time when it was stressed.

In addition, the tests were made under a constant temperature of 70° F and a humidity of 50%. Under some field conditions the humidity is considerably lower, and an additional 4,000 lb per sq in. should be included to take care of this contingency.

During the supervision of numerous rod-stressing operations in the field, the writer has repeatedly noted variations in stress as high as 10,000 lb per sq in. Lever arms, rusted threads, temperature changes, sun heat, inaccurate stressometer measurements, and other factors can easily account for an error of this magnitude.

Thus, the maximum losses, in pounds per square inch, that might be expected when using band rods and turnbuckles are as follows:

Description	Pneumatic mortar	Concrete
Shrinkage and flow (from Table 1)	21,000	24,000
Initial shrinkage loss	3,000	3,000
Allowance for low humidity, from laboratory oven tests and field tests	4,000	4,000
Possible variation in rod stressing	10,000	10,000
Total	38,000	41,000

With losses of this magnitude, it can readily be seen that, under the low initial prestresses, the tank walls reverted rapidly to an unprestressed state. Serious cracking and trouble due to tension in the concrete should not therefore be surprising. The designers had not contemplated deformations of concrete so great as those which actually occurred.

Little or nothing could be done to reduce appreciably the shrinkage and flow of the concrete. In these respects the poorer concretes do not differ very much from the better ones. There is a remedy. The use of higher strength steels with much higher prestresses will for practical purposes completely counteract losses of stress due to these deformations of the concrete.

To illustrate, there has been included in Table 1 a second set of values based on the use of the high-strength steel. The steel stresses are assumed to have been increased with relative reductions in steel areas. The loadings on the concrete are left as before and the shrinkage and flow are assumed to be the same. The amount of loss in unit stress of the reinforcing will be the same as before but the percentage loss is vastly different and this is the interesting feature. The unit stresses lost in both concrete and steel are now about 16% or less whereas with the use of the structural grade steel and comparatively low stresses they averaged around 67%.

Furthermore, the use of the high-strength steel permits an overstress to take care of the small percentage loss so that the designed prestress is always within the limits intended. For instance, if the reinforcement available is strong enough to take an initial stress of 140,000 lb per sq in. and losses of 35,000 lb per sq in. in the steel are possible, although improbable, then a working stress of 105,000 lb per sq in. (140,000 lb per sq in. − 35,000 lb per sq in.) is practical when it can be produced in the field. Under such conditions there is always a residual compressive stress in the concrete. Tension of concrete results merely in the release of compression.

CIRCUMFERENTIAL PRESTRESSING OF TANK WALLS

To obtain the initial prestress of approximately 140,000 lb per sq in. required for such a design, it was first necessary to find a reinforcing material of high strength and elasticity. Cold-drawn, high-carbon wire possesses these characteristics. The ultimate strength of this wire varies from 200,000 lb per sq in. to 240,000 lb per sq in., and it has a yield point of about 85% (0.2% by the offset method) of the ultimate. There remained the problem of applying the wire, under the desired stress, around the walls of circular structures. This was accomplished through the development of a traction machine that would simultaneously place and stress the wire to the desired specifications without any friction between the wire and the concrete. It thus became practicable to design and build prestressed reinforced-concrete tanks that would be stable and free from cracking and leaking. The largest circular concrete tanks in existence now contain wire reinforcement stressed by these machines.

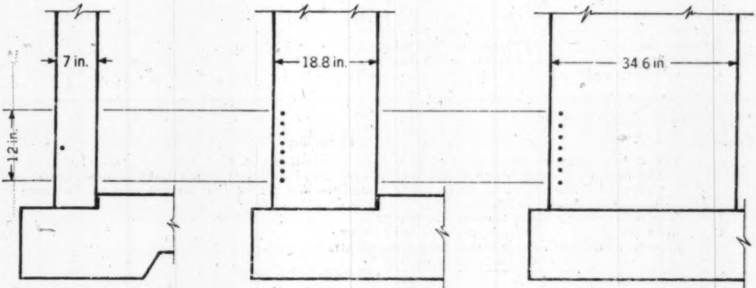
Where several layers of the wire are used, the inner layers act with the supporting concrete, and, in effect, take some compression. There is little loss of stress on this account since, under live load, the inner wires will expand with the concrete and act in unison with the outer wires to take their full part of the load. The load resistance of the steel ($R = f_s A_s$) will be in accordance with its total sectional area (A_s) and the average stress is (f_s) $= R/A_s$, as would be the case with a cable composed of multiple strands.

The use of the wire also has the advantage that its modulus of elasticity averages approximately 26,000,000 lb per sq in. against 30,000,000 lb per sq in. for steel rods. Therefore, the wire will stretch further (in inches per pound of stress) than rods, within the elastic range of the material. Since plastic deformation of the supporting concrete thus eliminates a smaller percentage of the stretch, the losses in stress when wire is used are less than those when rods are used.

In addition, the machine-controlled wire-stressing operation is highly accurate. Where threaded rods are used, variation in stress, due to friction between the rod and the concrete surface, may amount to 25% when the rods are stressed to 40,000 lb per sq in. Attempts have been made in Europe to stress wires around a circular wall by jacking the wire at one point; but it was found that, unless elaborate means were employed to reduce this friction, it was impossible to obtain sufficient stress in the wire on the side opposite the stressing point to make the structure dependable. In the case of the machine-controlled wire stressing, where the wire is already stressed before it comes in contact with the wall, variations in stress are less than 3% of the initial stress. For this reason, 35,000 lb per sq in. is an adequate allowance for stress losses with machine-controlled wire stressing; but at least 40,000 lb per sq in. must be allowed for prestressed rods with turnbuckles, and more than 70,000 lb per sq in. if wire is used which is stressed after it is in contact with the surface, depending on the friction coefficient and methods used to overcome it.

Comparative Tank Designs

It may be of interest to compare the band stresses in the walls of a concrete water tank under designs using high-carbon wire, prestressed rods, and unstressed rods. Assume that: (1) The tank is to have a capacity of 1,000,000 gal, with an inside diameter of 90 ft and a water depth of 21.4 ft; (2) the joint between the floor and the side wall is free; and (3) the maximum allowable stress in the concrete at the bottom of the walls, due to initial stressing, is 1,000 lb per sq in. The moduli of elasticity (in pounds per square inch) for the wire will be 26,000,000; for the rods, 30,000,000; and, for the concrete and pneumatic mortar, 3,000,000. The minimum yield point stress for wire will be 185,000 lb per sq in. and for the rods it will be 60,000 lb per sq in. (In the past, many rod prestressed tanks were designed using an initial stress of approximately 28,000 lb per sq in. and a working stress of about 25,000 lb per sq in. Such stresses proved insufficient. By making use of the highest practicable initial stresses, and low working stresses, it is possible to design a fairly safe rod tank such as that shown in Col. 2, Table 2. Under extreme conditions, however, circumferential tension due to plastic deformation and temperature changes may be sufficient to cause the walls of this tank to crack.)



(a) PRE-STRESSED WIRE (b) PRE-STRESSED RODS (c) CONVENTIONAL RODS

FIG. 1.—DESIGNS SELECTED FOR COMPARISON

To facilitate comparison, the designs for these three types of tanks are presented in Table 2. Figs. 1(*a*), 1(*b*), and 1(*c*) show the thicknesses of the respective shells in lines 1, 2, and 3, Table 2. The condition for design is based on a tensile stress not to exceed 300 lb per sq in. for 3,000-lb concrete.[15] In line 3, Col. 20, the maximum stress is 299 lb per sq in.

The design of the tank in line 3 is based on a free sliding base, and is thus comparable to the tank designs in lines 1 and 2. If a fixed or hinged base had been used, the wall thickness would have less because the cantilever action between the floor and the walls would have absorbed some of the band tension over the bottom of the wall. The Portland Cement Association,[15] however, uses the coefficient of shrinkage of 0.0003, whereas for these computations, based on field observations and laboratory tests, the coefficient of shrinkage was set at 0.0006. In any large tank, it would seem highly dangerous to use less than the full amount of band steel required by the hydrostatic load. Therefore no consideration has been given to possible assistance due to cantilever action between base and wall.

TABLE 2.—COMPARISON OF DESIGN METHODS (STORAGE TANK: INSIDE

Line	Design (see Fig. 1)	Design Criteria				Design Dimensions					
		f_s, in lb per sq in.	f_{si}, in lb per sq in.	f_{ci}, in lb per sq in.	$n = \frac{E_s}{E_c}$	[a] A_s	A_c (prestress)	$t = \frac{A_c}{12}$	Conventional		$P = \frac{A_s}{A_c}$
									t	A_c	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	Prestressed wire.....	−100,000	−140,000	+1,000	8.7	0.6	84	7	...	...	0.007
2	Prestressed rods.....	− 12,000	− 45,000	+1,000	10	5.0	225	18.8	...	...	0.022
3	Conventional rods...	− 14,000	....	− 300	10	4.28	...	...	34.6	415	0.0103

[a] Definition of column headings:

Column	Definition
1	The cross sections corresponding to lines 1, 2, 3, are shown in Figs. 1(*a*), 1(*b*), and 1(*c*), respectively. The formula for the design of line 3 (Fig. 1(*c*)) was based partly on equations presented in "Circular Concrete Tanks Without Prestressing," *Bulletin No. ST-57*, Structural Bureau, Portland Cement Assn., Chicago, Ill., October, 1947.
3	f_s = unit working stress used in the design.
4	f_{si} = maximum allowable initial unit steel stress.
5	f_{ci} = maximum allowable initial unit concrete stress.
7	$A_s = \frac{\gamma h r}{f_s} = \frac{62.5 \times 21.4 \times 45}{f_s} = \frac{60{,}000}{f_s}$, in square inches.
10	t = thickness of wall, in inches $= \frac{C E_s + f_s - n f_c}{12 f_c f_s} \times \gamma h r$ (in which $C = 0.0006$).
11	$A_c = 12 t$ in the conventional tank (see Fig. 1(*c*)).
13	f_{sL} = stress in steel (with tank empty), after a loss of 35,000 lb per sq in. for line 1; and 40,000 lb per sq in. for line 2.
14	f_{cL} = stress in concrete $= \frac{f_{sL} A_s}{A_c}$ (with tank empty), after a loss of 35,000 lb per sq in. for line 1, and 40,000 lb per sq in. for line 2.

The most significant data in Table 2 are contained in Col. 20, which sets down the band stresses of the full tanks after anticipated losses have been taken into account. It will be noted that concrete of the wire-wound tank still retains 78 lb per sq in. of circumferential compression. The data in Cols. 15 and 16 agree with data reported by Torata Matsumoto in 1921.[16]

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[15] "Circular Concrete Tanks Without Prestressing," *Bulletin No. ST-57*, Structural Bureau, Portland Cement Assn., Chicago, Ill., October, 1947, p. 3.

[16] "A Study of the Effect of Moisture Content Upon the Expansion and Contraction of Plain and Reinforced Concrete," by Torata Matsumoto, *Bulletin No. 126*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., p. 27, Fig. 12.

Cols. 22 and 23 are also interesting, since they show what would happen if general cracking in a vertical direction should take place. With the unstressed tank, leakage due to an increase in the circumference of the walls would be substantial. With the prestressed rod tank, insurance against vertical cracking would be partly dependent on the tensile values of the concrete. A vertical crack in the wire-wound tank would not leak; and the steel unit stress would not be increased on this account. This has been proved repeatedly by actual conditions in the field. Before being banded with wire, shrinkage cracks, as wide as ¼ in., have frequently been observed in tank walls. After the prestressing had been completed, these cracks were not only leakproof, but they could be detected only under close scrutiny.

DIAMETER, 90.0 FT; HEIGHT, 21.4 FT; PLUS INDICATES COMPRESSION)[a]

ANALYSIS AFTER ASSUMED LOSSES FOR PLASTIC DEFORMATION								VERTICAL CRACKING[b]			QUANTITIES[c]		Line
f_{sL}	f_{cL}	f_{ss}	f_{cs}	$f_c\Delta$	$f_s\Delta$	f'_s	f'_c	$f_s\Delta$	f_{ss}	$\frac{\text{Col. 22}}{E_s}$	Concrete (cu yd)	Reinforcement (lb)	
In Pounds per Square Inch													
(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(1)
−105,000	+750	....	....	−672	−5,820	−110,820	+ 78	−5,820	0	0	145	6,800	1
− 5,000	+110	....	....	−218	−2,180	− 7,180	−107	−2,180	− 4,820	0.54	395	58,500	2
....	....	+16,300	−167	−132	−1,320	+ 14,980	−299	−1,320	−28,980	3.25	660	51,200	3

Column Definition

15 f_{ss} = unstressed compression rods $= \frac{C\,E_s}{1 + p\,n}$, in which $C = 0.0006$.

16 f_{cs} = unstressed tension rods $= \frac{C\,E_s}{1 + p\,n}$, as in "Circular Concrete Tanks Without Prestressing," *Bulletin ST–57*, Structural Bureau, Portland Cement Assn., Chicago, Ill., October, 1947, p. 42.

17 $f_c\Delta$ = concrete stress caused by filling the tank $= \frac{\gamma\,h\,r}{A_c + n\,A_s}$.

18 $f_s\Delta$ = steel stress caused by filling the tank $= n\,f_c\Delta$.

19 f'_s = Col. 13 (or Col. 15) plus Col. 18.

20 f'_c = Col. 14 (or Col. 16) plus Col. 17 (full tank).

21 $f_s\Delta$ = change in steel stress caused by filling the tank.

22 f_{se} = change in steel stress in the case of cracks from any cause (Cols. 3 to 19).

23 E_s = maximum increase in circumference of tank $= 90 \times 12 \times \pi \times \frac{\text{Col. 22}}{E_s}$.

24 = volume of concrete in the straight wall, not footings.

[b] Stress analysis for the case of vertical cracking. [c] Band reinforcement and concrete.

It is difficult to fix a factor of safety for a tank structure because, almost invariably, overflow pipes limit the head to that of the design. Should the tank be covered with a dome, it is usual practice to place a hatch near the dome ring which will be pushed open by the liquid if the overflow pipe should become stopped up; therefore, the principal factor desired is safety against cracking. In the case of prestressed wire walls there is a real factor of safety in this respect. With unstressed tanks, this factor does not apply because, by virtue of the foregoing calculations, they must crack unless the tensile strength of the concrete is sufficient to carry all loads. In other words, the concrete walls of the conventionally designed tank must carry not only the hydrostatic load but also the expansive stress of the rods, which become compressed by the shrinkage of the concrete to about 16,300 lb per sq in. (Col. 15, Table 2). This condition is illogical and unsafe.

VERTICAL REINFORCEMENT

In designing the reinforcement for the walls of a prestressed tank, it must be kept in mind that the conditions are different from those for the reinforcement of a conventional tank. With the latter, aside from shrinkage, the principal factor is the radial force exerted by the liquid, and its tendency to bend the walls outwardly.

In prestressed tank design, however, the bending forces are reversed. If shrinkage and plastic flow did not have to be taken into account, the radial inward forces of the prestressed reinforcement would be designed to counterbalance exactly the radial outward forces of the liquid load. Obviously, when the tank is empty, the radial forces exerted by the prestressed reinforcement are unopposed except by the concrete. Consequently, moments are developed in the walls, and vertical reinforcement must be provided to take care of them.

The computations of such moments are complex. In effect, a circular wall is a cylinder or a deep ring. It is not correct, therefore, to consider a unit vertical section of the wall and assume that it is acted upon as a simple beam supported at the ends. Circumferential ring stresses, as well as the moments due to bottom and top connections, must also be taken into consideration.

S. Timoshenko[13] and others have analyzed these conditions. Excellent tables for use in the computation of such moments, based on Mr. Timoshenko's work, are available.[15] The steel sectional area in the tension faces of the tank wall, required to take the stresses as computed from the moment tables, is determined as follows:[15]

$$A_s = \frac{12\,M}{f_s\,j\,d} \quad \text{.........................} \quad (1)$$

in which d is the effective depth of beam and j is the ratio of the distance between the resultants of the compressive and tensile stresses to the effective depth. However, in using the tables, it must always be remembered that the tension faces of the walls of a prestressed tank are exactly opposite to those of a conventionally designed reinforced tank, because the stressing of the horizontal reinforcement causes the walls to bend inward. When the tank is filled, the liquid load will serve only to reduce this inward bending.

In addition, the tables are predicated on the use of hinged and fixed walls. Thus they do not quite fit the conditions assumed for the designs in Table 2, which, for purposes of comparison, were based on the use of a free sliding joint. This joint, however, cannot be considered free in the sense that it slides easily since, even with good lubrication such as asphalt or greased paper, it will not (except by special provision) move to a position at which all lateral restraint is eliminated.

New computations were therefore made on the assumption that a free sliding joint has a friction coefficient of 0.5. This ratio, based on the writer's observations during many field construction operations, seems sufficiently accurate for practical purposes and is on the side of safety. Moment curves based on the use of this kind of joint and applicable to the wire-wound tank

(Fig. 1(*a*)) are shown in Fig. 2. The maximum moment in the side walls resulting from the stressing of the band reinforcement is approximately 2,850 ft-lb and it exists about 3.8 ft above the floor.

Using Eq. 1, the steel required for the inside face of the wall will be $A_s = \frac{12\,M}{f_s\,j\,d} = \frac{12 \times 2{,}850}{20{,}000 \times 0.88 \times 5.5} = 0.35$ sq in.—which ordinarily indicates the use of nonprestressed $\frac{5}{8}$-in. round rods on 10-in. centers. This type of reinforcement is illogical. It would resist shrinkage of the concrete and set up adverse stresses in both steel and concrete, as follows: $f_{cs} = \frac{E_c\,C \times p\,n}{1 + p\,n}$ $= \frac{3{,}000{,}000 \times 0.0007 \times 0.0042 \times 10}{1 + 0.042} = 85$ lb per sq in. The comparative stress in the steel resisting this tension in the concrete is, theoretically: $\frac{A_c\,f_{cs}}{A_s}$ $= \frac{84 \times 85}{0.35} = +\,20{,}000$ lb per sq in.

Bond failure might reduce this stress to some extent; but, in any event, the steel could not resist any part of the moment until after the concrete had ruptured and the section had elongated sufficiently for the rods to act in tension. Furthermore, the prestressing imposes a continuous load on the concrete of the walls. When the tank is filled, the circumferential stresses will be partly released by the liquid load, thus reducing the effects of plastic flow. There will be no release vertically. Therefore, to maintain actual resistance to wall moments in large tanks, it is necessary to include a large amount of prestress in the vertical reinforcement.

Many designers have believed that no useful purpose would be served by prestressing tank walls in a vertical direction. The writer has inspected more than 100 conventionally designed reinforced concrete tanks and silos, and has found horizontal cracking in the walls almost invariably. Such cracks not only disfigure the structure, but also result in leakage which may cause early loss of the tank due to freezing and thawing of water in the fracture planes. The writer's experience with tanks that have been prestressed circumferentially also strongly indicates the need for vertical prestressing. Since about 1942 approximately 50 tanks have been built under designs using vertical prestressed steel as described herein. Results have been most satisfactory and no repairs have been necessary.

The prestressing of the vertical reinforcement has been difficult and expensive. Since 1942 several methods have been employed with varying degrees of success, but it is only recently that fully satisfactory results have been achieved. For small tanks whose side walls are less than 15 ft high, conventional reinforcement has sometimes been used. A number of larger tanks has been built with prestressed rods as the vertical reinforcement. Although this method was expensive, the results were sometimes satisfactory when the rods were stressed as much as possible and then bonded into the concrete. The use of slip rods, that is, rods covered with a plastic, was very unsatisfactory. This is illustrated by the history of a 2,000,000-gal tank built during World War II for the United States Navy Department at its Great Lakes Training

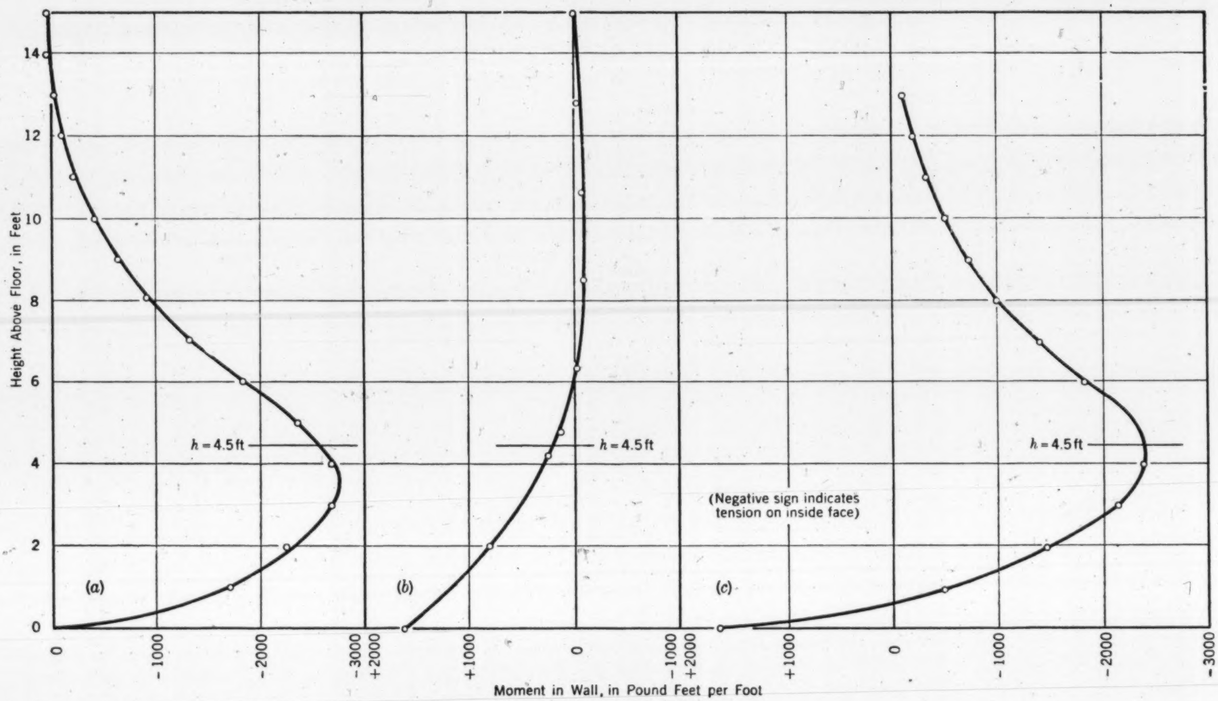

FIG. 2.—MOMENTS IN THE WALL DUE TO:

(*a*) Horizontal Prestressed Wire
(*b*) Eccentric Loading of Vertical Steel
(*c*) Horizontal Prestressed Wire and Eccentric Loading of Vertical Steel

Station. Soon after completion, horizontal cracks developed in the walls. Leakage was substantial, and attempts to control it by patching and grouting the concrete wall were unsuccessful. After several months it was found, by examining the nuts on the top of the rods, that the stress of from 35,000 lb per sq in. to 40,000 lb per sq in. which had originally been imparted to the rods had virtually disappeared. The rods were retightened; and leakage was eliminated, only to reappear after a short interval, indicating that the rods were again loose. Results were similar when slip rods were used in other tanks. Large cracks, concentrated in a few locations, invariably appeared.

In later tanks smaller rods of intermediate grade were regularly used. Vertical slots were left in the concrete of the walls into which the rods were placed with anchors at the ends. They were then stressed to about 70,000 lb per sq in. after which the slots were filled with gunite thus bonding the rods into the wall concrete. For a time this seemed to be good practice. Later however horizontal cracks appeared in some of the larger tanks indicating that shrinkage and plastic flow were eliminating the prestress of the rods to a degree that it was no longer effective.

As a result it became evident that vertical shrinkage and flow in the concrete of tank walls are more dangerous than those taking place circumferentially and that at least an equal amount of overstress must be included to maintain continued resistance to wall moments. The use of the high-strength, cold-drawn wire was again indicated but from a practical standpoint it was not feasible until recently when machines were developed which would economically place it to the desired stresses. With the aid of these machines the vertical wire reinforcement is now regularly used at stresses slightly under the yield point and in quantity to provide ample additional unit compressive stresses in the walls, not only to take care of shrinkage and flow losses, but also to guard against temperature stresses which are incurred by seasonable weather changes.

The prestressing of the vertical wires alters, somewhat, the moment conditions that will exist after the band reinforcement has been placed. In Fig. 2(*c*), the combination curve shows that the point of maximum moment is at an approximate elevation of 4.5 ft. At this elevation, the moment induced by the band reinforcement is 2,600 ft-lb and that due to eccentric loading of the vertical wires is 200 ft-lb (Fig. 2(*b*)).

Using the conventional formula for the determination of the sectional area of the vertical reinforcement: $M = f_s A_s j d$; or $A_s = \frac{2{,}600 \times 12}{145{,}000 \times 0.9 \times 6.8}$ $= 0.035$ sq in. In this example, $d = 6.8$ in. instead of 5.5 in. as in the unstressed examples, and f_s is 145,000 lb per sq in. The difference in depth explains the reduced area A_s.

The wall of the empty tank has no vertical stress from shrinkage but will have compressive and tensile stresses due to the

(*a*) Weight of dome and wall;

(*b*) Direct compression caused by the stressing of vertical wires;

(*c*) Stresses in the inside and outside faces due to eccentricity of prestressed vertical wires;

(*d*) Radial load of band reinforcement;

(*e*) Radial forces of shrinkage in concrete (partly eliminated by sliding joint); and

(*f*) Stresses of opposite sign in outside and inside faces due to temperature differences.

For this type of vertical reinforcement the summary in Table 3(*a*) shows an excellent condition with regard to shrinkage and flow. Both faces of the wall have a substantial surplus of compressive stress. However stresses due to variations in temperature must be added. Experience dictates that the sectional areas of the vertical wires must be considerably increased to provide insurance compression in the walls to prevent leakage even though, through some circumstance during construction, a horizontal fracture or weak plane has been left in the walls. Until the development of the machines for the vertical prestressing, the cost of the extra reinforcement was such that the engineer, unless he was very sure that it was needed, hesitated to require it in specification. The extra amounts required might vary with the locality but in general it would seem good practice to maintain a sufficient amount of residual stress horizontally across the wall section to exceed the unit pressure caused by the liquid by a large factor of safety.

The values shown in Table 3(*a*) are for the conditions that will exist at the instant that wire winding is completed. Subsequent losses of stress due to shrinkage and plastic flow will be substantial. After 4 or 5 years, however, most of the shrinkage and flow will have taken place. Table 3(*b*) has been prepared to cover this condition, and shows the vertical stresses remaining after all anticipated losses due to shrinkage and flow. The loss of stress in the vertical wires is assumed to be 35,000 lb per sq in., and thus the stress remaining in these wires is 145,000 lb per sq in. The respective columns in Table 3(*b*) show the approximate stresses in the concrete resulting from the vertical wire prestressing and the vertical and circumferential loads.

Despite the large amount of shrinkage and flow assumed to have taken place, there is little change in the vertical stresses from those shown in Table 3(*a*). The tank walls are stable and will not crack as has been repeatedly demonstrated by tanks reinforced in this manner.

THE CONSTRUCTION OF PRESTRESSED CIRCULAR STRUCTURES

Experience gained in the design and construction of more than 500 tanks and other large annular structures leads to the following observations.

Concrete.—For best results, the concrete throughout the tank must be dense and free from voids and pockets. This can be accomplished through careful workmanship. An absorption factor less than 8% or 10% should prevent the concrete from disintegrating because of freezing, thawing, or general weathering.

Pneumatic Mortar or Gunite.—An encasement of pneumatic mortar has been proved to be the best protection for high-carbon wire after it has been stressed. Steel wire and rods coated with pneumatic mortar have been free from rust after exposure to tidal water for more than 25 years. There is no case on record of rusting having taken place where steel has been covered with a coating of good pneumatic mortar half an inch or more in thickness. This is confirmed by studies on the corrosion of steel covered with pneumatic mortar conducted by the University of Toronto (Toronto, Ont., Canada) and reported in 1926.[17]

The bond between pneumatic mortar and wire is excellent. It can be readily demonstrated that a through cut in any of the wires encased in pneumatic mortar will result in negligible loss of stress. The increased diameter of the ends of the bonded wire, due to the release of stress, will prevent such loss except at the point of rupture. Should more than a ¾-in. thickness of pneumatic mortar be applied, a layer of light wire mesh should be included to prevent sagging and reactions caused by temperature differences.

TABLE 3.—DESIGN OF VERTICAL REINFORCEMENT
(STRESSES IN POUNDS PER SQUARE INCH)

Stress	Description	(a) BEGINNING OF PERIOD[a]			(b) AFTER FOUR OR FIVE YEARS		
		Computation	Inside face	Outside face	Computation	Inside face	Outside face
(1)	(2)	(3)	(4)	(5)	(3)	(4)	(5)
(a)	Weight of dome and wall......	$13+(h\times1.0)$	+ 30	+ 30	....	+ 30	+ 30
(b)	Direct compression, vertical wires; $f_c=\frac{A_s f_s}{A_c}$.............	$\frac{0.035\times180{,}000}{84}$	+ 75	+ 75	$\frac{0.035\times145{,}000}{84}$	+ 61	+ 61
(c)	Eccentricity, prestressed wires; $f_e=\frac{6\ M}{b\ d^2}$..................	$\frac{6\times200}{1\times7\times7}$	+ 25	− 25	$\times\frac{145{,}000}{180{,}000}$	+ 20	− 20
(d)	Radial load, band reinforcement; $f_c=\frac{6\ M}{b\ d^2}$..................	$\frac{6\times2{,}600}{1\times7\times7}$	−318	+318	$\times\frac{100{,}000}{140{,}000}$	−227	+227
(e)	Radial forces of shrinkage......	....	....	....	....	....	....
...	Empty tank; total stress (a) to (e)......................	....	−188	+398	....	−116	+298
...	Add full tank, liquid load; $f=\frac{6\ M}{b\ d^2}$	$\frac{6}{49}\times2{,}600\times\frac{100{,}000}{140{,}000}$	+227	−227	....[b]	+227[b]	−227[b]
...	Summation of stresses, full tank	....	+ 39[c]	+171[c]	....	+111	+ 71

[a] The instant that wire winding is completed. [b] Same as for stress (d) but with opposite sign. [c] Assuming no plastic losses.

High-Carbon Cold-Drawn Wire.—The high-carbon, cold-drawn wire used for the horizontal and vertical prestressing of tank walls is one of the most reliable of all building materials because of the repeated working and reworking under high stresses to which it is subjected during manufacture. The wire is packaged in coils at the factory and delivered to the site wrapped in water-repellent paper to prevent rusting prior to use. During the wire-winding operation, each coil is jointed to the next coil by special, patented, spring-loaded splices. Tests made under the supervision of the writer demonstrate that less than 5% of the stress originally imparted to the wire will be lost from creep or plastic flow of the wire through the passage of time. Fig. 3 illustrates a self-propelled machine that winds wire around a tank at the uniform unit stress of the design, thus placing the walls in permanent circumferential compression. The operators on the ground are applying a pneumatic mortar coat that bonds the wire to the structure and protects it against corrosion.

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[17] "Gunite as a Protection for Steel Structures," by Peter Gillespie and P. J. Culleton, *Bulletin No. 4*, Univ. of Toronto, Toronto, Ont., Canada, 1926, p. 109.

Floors.—It is of the utmost importance that the foundation for the structure be stable, especially where the joint between the floor and the side wall is free. Until recently dowels have been included to tie floor and side walls rigidly together. Where this practice has been followed, there have been no difficulties from leakage at the joint. Where the side wall is tied to the floor, the wall serves as a deep beam and supports the edge of the floor against settlement, although the edge of the floor must be relatively thick to take the increased shear stress. In general, a fixed joint is used with tanks having diameters less than 100 ft. If free joints are desired, however, they can be made bottle tight without too much difficulty.

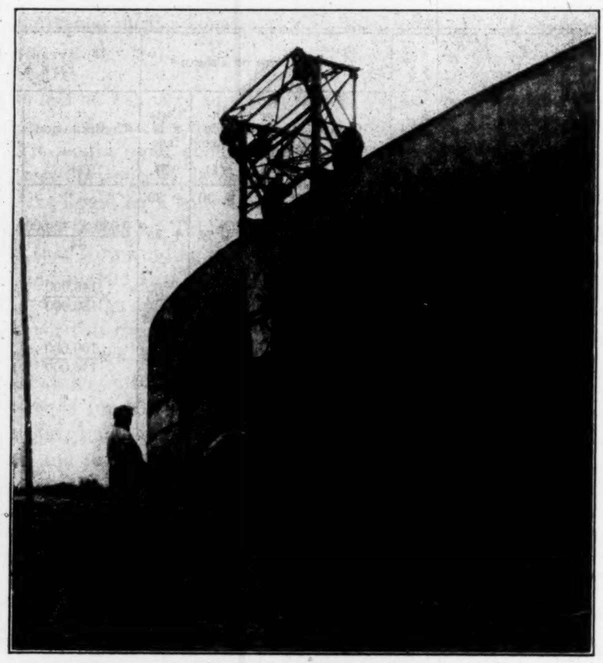

FIG. 3.—SELF-PROPELLED MACHINE THAT WINDS WIRE AROUND A TANK AT A UNIFORM STRESS

Experience indicates that relatively thin floors with a high percentage of reinforcing steel will be the most watertight and permanent. If pneumatic mortar is used, a floor approximately 2 in. thick, reinforced with wire mesh having a sectional area of 0.5%, will conform best to earth movements with less leakage. It will also prevent the passage of any free water under heads of 100 ft and more. This is in accordance with the history of such floors and may be demonstrated by the use of hydraulic testing machines.

Where a poured concrete floor is used, a 4-in. thickness with a $\frac{5}{8}$-in. topping of pneumatic mortar is satisfactory if the sectional area of the reinforcement in each direction is equal to 0.5% of that of the concrete. The pour, however, should be continuous so that construction joints are eliminated. The topping of pneumatic mortar, which should not be applied until just before the tank is to be filled, will effectively seal any shrinkage cracks that may occur during the initial curing.

After the floor has been poured or shot, it must be kept saturated with water until the walls have been constructed to prevent it from shrinking until that time. By using this procedure the walls and the floor will shrink together to some extent, and thus prevent excessive moment in the walls. This procedure is mandatory when floors and walls are doweled together.

Thicker concrete floors with smaller percentages of reinforcement cannot be expected to bend or conform to earth movements without cracking and leaking. In addition, they require expansion joints, which almost always leak.

Walls.—Sufficient surplus stress to offset maximum shrinkage and plastic flow of concrete must be imposed on both horizontal and vertical wall reinforcement. The minimum surplus allowance for successful circumferential prestressing should not be less than 40,000 lb per sq in. for rod reinforcement or 35,000 lb per sq in. for high-carbon, cold-drawn wire. The apparatus for circumferential prestressing with wire (see Fig. 3) is a self-propelled traction machine, hung from the top of the tank, which travels continuously around the walls.[18] A sprocket operated by a gasoline engine engages an endless linked steel belt which is passed about the perimeter of the structure. A spring-loaded take-up maintains the tension necessary for adequate traction. An automatic power drive controls the vertical movement of the machine insuring accurate spacing of the wire (Fig. 4), which is taken from a reel generally mounted on the working platform of the machine.

The desired stress is imparted to the wire by drawing it through an aperture of smaller diameter. For the convenience of the operator, a dial gage on the machine registers the load being imparted to the wire. Wire winding may start at the bottom or the top of the wall from an anchor set in the concrete. It takes only about 2 days to wind the walls of an average million-gallon water tank. With larger tanks, the walls may require several layers of wire; if so there is no added difficulty. Each layer is covered with a minimum thickness of pneumatic mortar and then the succeeding layer is applied.

In view of the large number of wires required for the vertical prestressing of large tanks it has been necessary to develop special techniques and equipment to place and stress the wire in groups of six or more. For large tanks special anchors are placed in the concrete at the top and bottom of vertical keys on the inside face of the wall. An assembly of wires is then affixed to the top anchor, and is stressed at the floor level of the tank by a machine that stretches the strands simultaneously and forces "home" a coupling pin to the bottom anchor when the wires have reached the proper stress. Sometimes, the wires are anchored in the keys at the bottom of the wall of small tanks and are stressed individually from the top of the wall by hydraulic jacks. As each wire is brought to the desired stress, a special anchorage device is affixed to the wire to hold it in the stressed condition. In both large and small tanks, after the wire is fully stressed, the keys are filled with pneumatic mortar to bond the prestressed wire into the main body of the structure, and to protect the wire permanently from corrosion. Recently in order to provide more reinforcement for very thick walls, cable assemblies with cast anchor plates at the top of the wall were used. These cables are assembled as "hairpins" and are sheathed to provide the slippage which occurs during the stressing operation. The units are placed in the walls during construction.

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[18] "High Stressed Wire in Concrete Tanks," by J. M. Crom, *Engineering News-Record*, Vol. 131, 1943, p. 947.

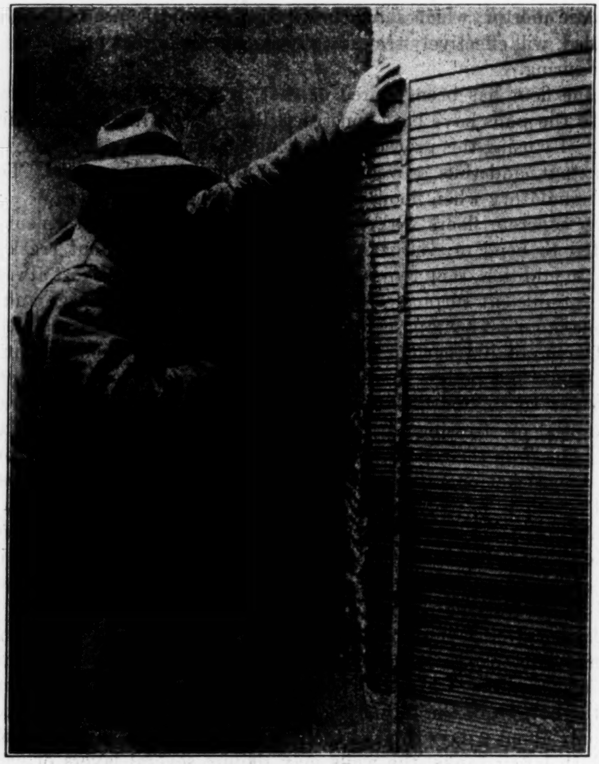

FIG. 4.—SPACING AND COVERING OF PRESTRESSED WIRE

Domes.—It is not within the scope of this paper to discuss the theory and design of prestressed domes, which have been thoroughly examined by M. F.

Fornerod,[19] M. ASCE. It is sufficient to state that for large domes, with proportionately high band loads, multiple layers of high-carbon wire may be concentrated economically and efficiently around the points of maximum reaction by the previously described machines. The cost of such domes is substantially below that of any other type of permanent covering, including rectangular truss and precast roof construction. These domes, which are only a few inches in thickness and eliminate the need for supporting columns, are fireproof and need little maintenance.

Although dome roofs of this type were developed principally in connection with tanks, they have been adapted to other structures such as sports arenas, aircraft hangars, shopping centers, and motion picture theaters where an unobstructed roof surface which permits good natural or artificial light diffusion was desired.

SUMMARY AND CONCLUSIONS

One of the principal purposes of this paper has been to discuss and analyze the results of tests conducted by the Massachusetts Institute of Technology to measure the shrinkage and plastic flow of concrete under stress, and to call attention to their importance in connection with the design of prestressed concrete structures. These tests have proved that shrinkage and flow can be sufficiently great to nullify most of the prestress which designers formerly attempted to impart to the wall reinforcement of tanks and other circular structures. The failure of many tanks is thus accounted for.

On the basis of these tests and other tests it is disclosed that, using unit steel stresses, developed without friction, considerably in excess of 100,000 lb per sq in., a balance of prestress, ample for working purposes, can be maintained in the reinforcement after all losses due to shrinkage, plastic flow, and other factors have taken place.

In the case of tanks and other large circular structures the required steel stresses for the walls can be attained through the use of machines that simultaneously place and stress without friction high-carbon, cold-drawn wire to the desired specifications, thus making possible the economical construction of tanks of very large diameter and capacity, and, when desired, of large-span, thin-shelled dome covers, unsupported except at the rings.

Most of the designed dimensions for prestressed circular structures are based on actual tests and simple, direct computations that do not require the complicated assumptions necessary for conventional reinforced concrete.

This paper also presents certain observations from experience on the construction of prestressed circular structures. When all factors are considered, it seems conclusive that large annular concrete structures, reinforced with highly prestressed wire are more rational and safer than those built of conventionally-designed reinforced concrete. Fortunately, because of a saving in labor and materials, together with low first cost and maintenance, prestressed tanks are also more economical.

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[19] "Prestressed Concrete Shell Roof Construction," by M. F. Fornerod, *Publications*, International Assn. for Bridge and Structural Eng., Vol. 8, 1947, pp. 91–103.

DISCUSSION

L. J. MENSCH,[20] M. ASCE.—Tanks and pipes are the most logical applications of prestressed concrete. As early as 1912 the city of Bologna, Italy, had a high-pressure prestressed pipe line built on the Siegwart system. Other early pipe lines have been reported from Switzerland and France.

Prestressed cast iron water pipes and guns have been in use since 1850 in France, England, and the United States. Wire-wound guns had been built in various countries since 1890, using wire with a strength of 225,000 lb per sq in., long before such wires had been used for prestressed concrete. High strength concrete of 10,000 lb per sq in. for concrete pipes was being made before 1920. Wire-wound wood stave pipes, tanks, and barrels have been used very extensively in the United States and seldom have failed by tension in the steel. Failure usually was due to shrinkage when empty or partly filled, or to connection of staves with bottom, or to frost. Prestressed or conventional concrete tanks are no exception to these effects.

From an experience with several hundred tanks the writer can mention the following reasons for unsatisfactory performance of some early concrete tanks:

1. The earliest tanks and pipes were of a concrete mixture of 1:3:6; later a mixture of 1:2:4 was thought rich enough. Experienced builders used a richer mixture, especially for elevated tanks.
2. No adequate waterproofing was used. European practice regarded three coats of rich mortar linings, each about $\frac{1}{8}$ in. thick, as absolutely necessary. Here waterproofing was provided only after it was discovered that the tanks leaked and the face of the tank was disfigured. This gave the waterproofing contractors a chance to charge a high price for their work. A two- or three-coat membrane lining of high grade asphalt did a good job; but, if such a lining were specified in the beginning, a great deal less concrete and steel could have been used, and the resulting saving would have paid for the waterproofing.
3. The joint of walls and bottom was poorly designed, and the joint broke when the tank was filled. This was emphasized in the past.[21,22]
4. There was always a leak where a sliding joint was used. To the writer's knowledge a sliding joint was first used for a concrete standpipe in the city of Fulton, N. Y., in 1913 and proved unsatisfactory. A very large and high prestressed standpipe was built about 10 years ago in Cincinnati, Ohio; it had a sliding joint and leaked badly.
5. No adequate waterstop was used in the joints of tanks when concreting was interrupted.

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[20] Civ. Engr. and Contr., Evanston, Ill.

[21] Discussion by L. J. Mensch of "The Relation Between Deflections and Stresses in Arch Dams," by F. A. Noetzli, *Transactions*, ASCE, Vol. LXXXV, 1922, p. 309.

[22] Discussion by L. J. Mensch of "Design of Circular Concrete Tanks," by George S. Salter, *ibid.*, Vol. 105, 1940, p. 525.

6. The walls of tanks must be liberally reinforced vertically. One-half-inch bars placed 2 ft on centers will not do. As the author showed, there are serious moments acting in the walls, tending to produce horizontal cracks whether the tank is loose or fixed at the bottom. In addition, cracks are caused by the sun shining on one side of the tank.

7. The tanks were not built of exact circular shape.

The writer quite agrees with the figures in line 1 of Table 2 for a prestressed tank. However, the deduction of 35,000 lb per sq in. for secondary stresses might not be sufficient for reasons (not mentioned by the author) such as one-sided sunshine, large temperature variation, and ice formation in the winter.

The author considers the wall resting loosely on the bottom and assumes a coefficient of friction of one half. Serious cracking can be observed in concrete structures with sliding joints in which no sliding steel plates well covered with graphite were provided. It is nearly impossible to produce an even plane surface with a radius of 45 ft for the tank under discussion. Variations of ⅛ in. to ¼ in. in both directions would be considered excellent work. Hence the friction may be much greater in some spots than in others. The author should explain how the coefficient of friction was arrived at. Assume that the movement during prestressing at the bottom was measured. Then, by a very difficult chain of reasoning we can guess at a formula for this movement containing the unknown modulus of elasticity that may vary from 3,000,000 to 5,000,000 lb per sq in., and then the coefficient will vary in proportion.

Another effect is not mentioned. The concrete wall does not remain vertical but has a slope of about 1 to 1,000 at the bottom that will cause breakage of the inside corner in sliding. In addition, the formulas offered by the Portland Cement Association[15] are valid only for walls of uniform thickness, and as the author's tanks have variable thickness from bottom to top, use of these formulas is of doubtful value. If proper sliding cannot be obtained, then the moments in the tank walls may be three times as great as those shown in Fig. 2(*a*) which may explain the cracks found by the author in some tanks.

The use of 2-in. domes in tanks 100 ft in diameter is to be condemned. While the compression in the concrete under full snow load is only 95 lb per sq in., there are many secondary stresses which will crack the dome in time, such as stresses due to prestressing, sunshine, moisture variation, and deviation of shape from a true sphere.

The choice of the unusual wall thickness of 34.6 in. for a conventional concrete tank 90 ft in diameter and 21.4 ft deep was not wise. A great part of the water pressure will be taken up by the cantilever action of the thick wall and transferred to the base slab. This base slab will certainly crack even if it is made four times as thick as the paper recommends. It is assumed further that all circular reinforcing steel be placed near the outside of the tank wall, but this plan has two serious disadvantages:

a. The shrinkage stresses are doubled; and even without shrinkage stresses the tank will fail at 33% less water pressure than if the horizontal rings are placed near both sides of the wall.[23]

b. Successful practice shows that such wall thicknesses are not necessary. In 1918 a 3,000,000-gal tank was designed for the city of Lansing, Mich. It was 152 ft in diameter and 21 ft high, and according to the author such a tank should have had a wall thickness of 58 in. Actually, it was made 24 in. thick at the bottom and 12 in. thick at the top and had a well-designed connection with a thickened base slab. It never leaked in its 33 years of existence. Another tank, designed in 1910 by the same firm for the town of Norway, Mich., was 35 ft in diameter and 40 ft deep. According to the author the wall should have been 25 in. thick. It was actually 12 in. thick and was so satisfactory that the city built another tank on the identical plans a few years ago. This tank was subjected to unusually severe trials. Sub-zero temperatures of 20° to 40° are common in that part of the country, and every year an ice crust was formed 5 ft thick at the walls and top. Only the fact that the concrete was 1:1:2 mixture saved the tank.

The author claims that ordinary standard reinforcement cracks the concrete when reinforcing is stressed to 20,000 lb per sq in. On this basis all rectangular tanks ought to crack and give trouble. Very long tanks crack on account of shrinkage but can be easily repaired. The writer has built many rectangular tanks and has not found that they leaked because of flexural stress.

The writer wishes to point out that in Table 2, Col. 15, the steel stress due to a shrinkage of 0.0006 should be obtained by the formula $\frac{C\,E_s}{1 + 4\,p\,n}$ as this is a very slow action and E_c becomes $\frac{1}{4}$ or less of the initial modulus. Good practice requires that not only the tank bottom but also the tank walls be kept wet for a long time, thus resulting in a much smaller coefficient of shrinkage. There is no question that prestressing saves a great deal of steel in tanks, high pressure pipes, and culverts, when properly designed.

Herbert A. Sawyer, Jr.,[24] **A. M. ASCE.**—The experimental evidence presented in this paper for the behavior of high-strength steel reinforcement is an extrapolation of the data presented in 1946 by Howard R. Staley, A. M. ASCE, and Dean Peabody, Jr.,[25] M. ASCE, involving much lower strength steels. An analytical extrapolation seems to indicate that the loss in stress for high-strength steel is significantly higher than shown in Table 1 especially for case (*a*) High Load. The total stress loss from all causes is important in prestressed tank design, as the author clearly demonstrates, so it will be worthwhile to consider these losses carefully.

The following six factors should be considered in the extrapolation of the Staley-Peabody data to obtain the stress loss in high-strength reinforcement: (1) The plastic flow and stress in concrete; (2) the elastic strain of concrete;

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[23] *Bericht No. 27*, Laboratoire Federal d'Essai, Zurich, Switzerland, 1927, p. 8.

[24] Associate Prof. of Civ. Eng., Univ. of Connecticut, Storrs, Conn.

[25] "Shrinkage and Plastic Flow of Pre-Stressed Concrete," by Howard R. Staley and Dean Peabody, Jr., *Proceedings*, A.C.I., Vol. 42, 1946, pp. 229–244.

(3) the time effect; (4) the winding effect; (5) the modulus of elasticity of the steel; and (6) the creep of the wire.

(1) *Plastic Flow and Stress in Concrete.*—Since the initial concrete stress is better maintined by the high-strength steel than it is by the low-strength steel, plastic flow of the concrete with high-strength steel will be greater than with low-strength steel. J. R. Shank[26] has shown that the rate of plastic flow is approximately proportional to the stress for stresses of up to 50% of the ultimate, and the Staley-Peabody tests fall in this range. For this discussion the plastic flow is assumed to be proportional to the average of the initial and final stress in a member. This assumption is accurate enough for present purposes. A more accurate solution, for which there is not sufficient data, would involve the solution of a very complex expression with time, plastic flow, shrinkage, stress, and creep in the steel as variables.

(2) *Elastic Strain of Concrete.*—Concrete which is gradually losing compressive stress expands elastically, and this expansional strain component partly cancels out the contractive, stress-reducing, strain components from plastic flow and shrinkage. This cancelling-out effect is smaller with high-strength steel because less concrete stress is lost with high-strength steel.

(3) *Time Effect.*—Loss of stress from shrinkage and plastic flow should be based on a period much longer than the 400-day losses of the Staley-Peabody tests. According to tests at the University of California,[27] in Berkeley, roughly 80% of the total deformation from plastic flow occurred in 1 year for specimens under constant load. Of course, a correction must be made for the decrease in load on prestressed concrete. Also, according to tests reported by Raymond E. Davis,[28] roughly 75% of the shrinkage in 10 years takes place in the first year. This percentage, based on tests performed about 1910, seems low. In computations that follow, the shrinkage at 400 days is arbitrarily assumed to be five sixths of the shrinkage at 25 years. Of course, changing moisture conditions would change these figures. The values given have been estimated for conditions similar to the Staley-Peabody test conditions, which, in terms of humidity and loading, are severe and conservative and correspond to those pertaining to an unfilled tank. If the tank were always full, the concrete stress, and hence the plastic flow, would be insignificant, and the high humidity on one side of the tank wall would retard shrinkage. Obviously few, if any, tanks could be designed for this assumption. Incidently it follows that the water-tightness of prestressed tanks under all conditions cannot be judged by the behavior of tanks that have very rarely been less than two thirds full. Keep them busy, and they stay out of trouble.

The tests of Messrs. Staley and Peabody were conducted in the Department of Building, Engineering, and Construction at Massachusetts Institute of Technology in Cambridge. This department has continued observation of the specimens of these tests and has kindly made available the more recent data on these specimens. The data are of some help in estimating, for wire-reinforced concrete, the time effect of shrinkage and plastic flow subsequent to a 400-day age. Data of the unstressed control specimens indicate that the shrinkage at 400 days is about nine tenths of the shrinkage at 1,800 days for both gunite and concrete. The forgoing assumption as to shrinkage at 25 years seems to agree with these results. Data of the specimens prestressed with rods indicate that, for final stresses of from 25% to 34% of the initial stresses, the calculated plastic flow at 400 days averages about 95% of the calculated plastic flow at 1,800 days. This finding probably does not disagree with the foregoing assumption as to total plastic flow with a final stress maintained (with wire reinforcement) at a much higher percentage of initial stress.

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[26] "The Mechanics of Plastic Flow of Concrete," by J. R. Shank, *Proceedings*, A.C.I., Vol. 32, 1936, pp. 149–180.

[27] "Plastic Flow and Volume Changes of Concrete," by Raymond E. Davis, Harmer E. Davis, and Elwood H. Brown, *Proceedings*, Am. Soc. for Testing Materials, Vol. 37, Pt. II, 1937, pp. 319–320.

[28] "Volume Changes in Concrete," by Raymond E. Davis, *Proceedings*, A.C.I., Vol. 26, 1930, p. 429.

(4) *Winding Effect.*—Of course it is the winding of the wires in tension around the initially unstressed concrete shell which gradually compresses the shell. On the average, when and where the wire makes contact with the shell, only half this compression exists in the shell since the tensioned wires have been applied to only one side of the point of contact. Further compression tends to unload the wire. The approximate ratio to which the initial steel tension must be increased to cancel this loss is

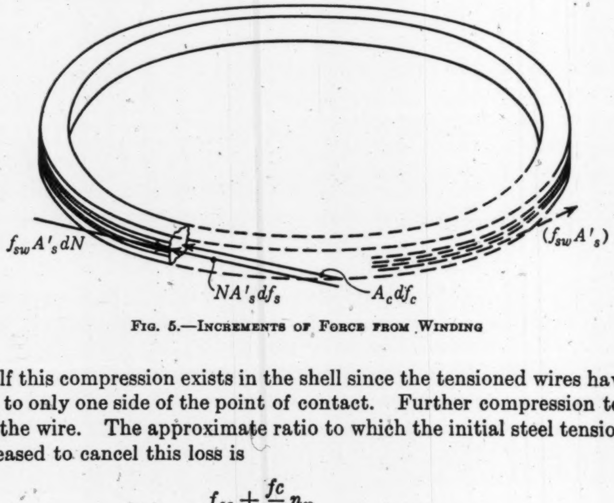

FIG. 5.—INCREMENTS OF FORCE FROM WINDING

$$\frac{f_{sw}}{f_{sa}} = \frac{f_{sa} + \frac{fc}{2} n_w}{f_{sa}} = 1 + \frac{p_w n_w}{2} \qquad \text{(2)}$$

in which f_{sw} is the steel stress in winding; f_{sa} is the average steel stress immediately after winding; p_w is the ratio of steel area to the concrete area immediately after winding; and n_w is the ratio E_s/E_c at the time of winding.

The more exact solution for this loss should probably be included here, if for no other reason than to prove that Eq. 2 is on the safe side and is accurate enough for all except extremely high values of $p\,n$. Consider an initially unstressed thin ring of cross-sectional area A_c, about which many turns of wire (each turn of cross-sectional area A'_s and winding unit tension f_{sw}) are being wound (Fig. 5). Increments of tension introduced by the wire being wound must be balanced (by statics) by equal increments of compression in the ring and previously wound wire:

$$f_{sw} A'_s \, dN = A_c \, df_c + N A'_s \, df_s \quad \text{. . . . . (3)}$$

in which N, f_c, and f_s are variables, N being the number of turns of wire at any time during the winding process. Also, if the materials are assumed elastic during the winding process,

$$df_s = n_w \, df_c \quad \text{. . . . . (4)}$$

Solving these two differential equations simultaneously, the ratio of Eq. 2 becomes

$$\frac{f_{sw}}{f_{sa}} = \frac{p_w n_w}{\log_e (1 + p_w n_w)} \quad \text{. (5)}$$

A 10% error would not be out of line with the accuracy of the assumptions underlying this analysis, and it is only for $p_w n_w > 1$ that the error of the stress increment of Eq. 2 compared to that of Eq. 5 exceeds 10%. Hence, for practical design purposes, Eq. 2 is accurate enough.

(5) *Modulus of Elasticity of Steel.*—The author states that this value for the wire "averages" 26,000,000 lb per sq in. This is about the same as the secant modulus for the wire commonly used by Gustave Magnel.[29] The ratio of stress to strain for a decrease of a high stress in steel is more accurately given by the low-stress tangent modulus. This modulus, as Mr. Magnel shows, is about 29,000,000 lb per sq in. which will be used in the following computations for relaxation of steel stresses.

(6) *Creep of the Wire.*—Experimental data for this phenomenon at non-elevated temperature are very sparse. The author's value of a maximum of 5% stress loss from this effect was evidently not included in Table 1 or in other calculations of the paper. The effect of creep may be neglected for the low stresses in low-carbon steel of the Staley-Peabody tests but not for high stresses in high-strength steel, especially for wire that receives tension by cold drawing at the time of winding. Mr. Magnel found strains in Belgian wire from creep of from 1% to 16% of the initial strains. This variation depended, primarily, on the rate of application of the initial stress, magnitude of initial stress relative to strength of wire, type of steel, and condition of restraint. In a constant-length test of the wire (mentioned herein under factor (5)) stressed in 2.5 min to 123,000 lb per sq in. (57% of its ultimate tensile strength), the loss in stress was 12% in 14 days. An arbitrary loss of 8% of the initial stress after winding has been assumed for cold-drawn wire initially stressed to about 75% of its ultimate in the computations that follow. There are many reasons why this may or may not be conservative enough, but, even if more data on creep were on hand, an accurate quantitative estimate of it would be almost impossible because of the interaction of creep strains with all the factors that cause strains in the concrete. Further investigations of this potentially important factor are very much needed. Perhaps the author can provide some more information on creep from his own experience in his closure.

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[29] "Creep of Steel and Concrete in Relation to Prestressed Concrete," by Gustave Magnel, *Proceedings*, A.C.I., Vol. 44, 1948, pp. 485–500.

The loss of stress in the steel wire has been computed in Table 4 for the same cases considered by the author in his Table 1. In this table, the six factors, (1) to (6), have been taken into account. The claim of great accuracy cannot be made for these computations. It is not possible for such computations to be very accurate, and for this reason only two significant figures are carried. This writer believes, however, that the inclusion of each of these six factors has greatly improved the accuracy of the result. Also, even if the maximum error per factor were ± 25%, the probable error for the summation would be ± 10% to 15%.

TABLE 4.—REVISED STUDY OF STRESS LOSSES

Item	Description[a]	Reference	HIGH LOAD		LOW LOAD	
			Gunite	Concrete	Gunite	Concrete
(1)	(2)	(3)	(4)	(5)	(6)	(7)
a	Average E_c at the time of winding	238[b]	3.34×10^6	3.30×10^6	3.34×10^6	3.30×10^6
	Winding stress f_{ws}—					
b	for f_{as} = 150,000 lb per sq in.	Factor 4[c]	160,400	160,500	154,000	154,100
c	for 3% error = 1.03b	Crom[d]	165,200	165,300	158,600	158,700
d	Average[e] f_c	236[b]	1,710	1,540	660	610
e	Total concrete strain after reload[e]	236[b]	−0.00086	−0.00112	−0.00081	−0.00096
f	Average[e] E_c	242[b]	3.67×10^6	4.35×10^6	3.67×10^6	4.35×10^6
g	Change in stress[e] f_c	236[b]	1,360	1,710	510	640
h	Elastic strain, concrete[e] = g/f	Factor 2[c]	+0.00037	+0.00039	+0.00014	+0.00015
i	Shrinkage strains after reload[e]	232[b]	−0.00045	−0.00061	−0.00061	−0.00079
j	Plastic flow after reload[e] = e − h − i	....	−0.00078	−0.00090	−0.00034	−0.00032
k	Initial f_c (second reload)	236[b]	2,390	2,390	920	935
l	Assumed loss in f_c-high tensile wire; see item y	....	1,000	1,200	320	370
m	Assumed average f_c, high tensile wire = k − l/2	....	1,900	1,800	760	750
n	Plastic flow, high tensile wire, 400 days = m j/d	Factor 1[c]	−0.00087	−0.00105	−0.00039	−0.00039
o	Plastic flow, at 25 yr = n $\left(1 + 0.25 \frac{k - l}{m}\right)$	Factor 3[c]	−0.00103	−0.00122	−0.00047	−0.00046
p	Elastic strain, concrete = l/f	Factor 2[c]	+0.00027	+0.00028	+0.00009	+0.00009
q	Shrinkage strain 6 to 400 days	232[b]	−0.00071	−0.00093	−0.00071	−0.00093
r	Shrinkage strain at 25 yr = 1.2q	Factor 3[c]	−0.00085	−0.00112	−0.00085	−0.00112
s	Total strain = o + p + r	....	−0.00161	−0.00206	−0.00123	−0.00149
t	Partial loss of stress in steel = $E_s s$	Factor 5[c]	46,600	59,800	35,700	43,200
u	Loss in steel from creep = 0.08 × 150,000	Factor 6[c]	12,000	12,000	12,000	12,000
v	Low humidity loss	Crom[d]	3,000	3,000	3,000	3,000
w	Maximum total loss of steel stress = c − 150,000 + t + u + v	....	76,800	90,100	59,300	66,900
x	Maximum percentage loss in f_{ws} = 100 w/c	....	46.5	54.5	37.5	42
y	Loss of stress in concrete = $\frac{t + u + v}{150{,}000}$ k	....	980	1,190	310	360
z	Percentage loss, concrete stress = 100 y/k	....	41	50	34	39

[a] In Col. 2, stresses *f* and moduli *E* are pounds per square inch; strains and plastic flow are inches per inch. The nonitalic letters a, b, c, . . ., z denote the numerical values recorded opposite the corresponding letters in Col. 1. [b] "Shrinkage and Plastic Flow of Pre-Stressed Concrete," by Howard R. Staley and Dean Peabody, Jr., *Proceedings*, A.C.I., Vol. 42, 1946, at the page cited in Col. 3. [c] Factors 1 to 6, defined in this discussion. [d] As given in the paper under discussion. [e] Computations necessary to evaluate the approximate plastic flow of the concrete for the Staley-Peabody tests are listed in lines d to j, inclusive.

The percentage of stress lost by the steel, item x of Table 4, is not equal to the percentage of stress lost by the concrete, item z, because the steel loses some stress before the concrete has reached its maximum initial stress. Items d through j of Table 4 outline the computations necessary to evaluate the approximate plastic flow of the concrete for the Staley-Peabody tests. The most important conclusions to be drawn from Table 4 are:

(*a*) The stress loss from shrinkage and plastic flow is not a constant, as indicated in Table 1, but a variable depending mainly on the wire-winding stress and to a lesser extent on the factor $p\ n$.

(*b*) This variation is so important that actual stress losses for high-tension reinforcement are two or three times the losses indicated in Table 1, assuming full concrete shrinkage but neglecting the 3% tension-error loss and the low humidity loss. Stress loss allowances for high-carbon wire should be greater than for rods, not less as indicated in Table 2.

(c) Discrepancies of this magnitude indicate the need for careful reconsideration of stress losses for other types of prestressed concrete construction.

On the other hand, several of the most important stress-reducing factors could be partly eliminated if special measures were taken:

1. The loss from shrinkage of the concrete would be reduced if the winding operation were delayed until much of the shrinkage had taken place. For Staley-Peabody shrinkage conditions, winding 21 days instead of 6 days after placement would reduce stress loss by about 10,000 lb per sq in. Such a delay could possibly disrupt the construction schedule, making it economically unfeasible for the average single-tank job.
2. Mr. Magnel's experiments[29] indicate that special steel wires (perhaps heat-treated after drawing) may be developed for which the magnitude of creep is almost negligible—another potential 10,000-lb-per-sq-in. reduction in the stress loss of Table 4. Whether a stress-saving of this magnitude would be sufficient to balance the extra cost of such a wire is a question the writer cannot answer.
3. As stated before, stress losses from both shrinkage and, to a lesser extent, from plastic flow, would be greatly reduced from those of Table 4 if the tank were filled to near-capacity most of the time.

The author did not specify any of these special measures for the prestressed-wire design of Table 2; thus this design roughly corresponds to the low-load condition of Tables 1 and 4, the winding stress being slightly lower and $p\ n$ being roughly twice as high. If the assumed stress loss of this design were to be changed from 35,000 lb per sq in. to 60,000 lb per sq in. to correspond with item w, Table 4, the final stress in the concrete for full tank, Col. 20, Table 2, would be 100 lb per sq in. in tension instead of 78 lb per sq in. in compression. The change in steel stress in case of cracks, Col. 22, Table 2, would be 14,000 lb per sq in., and the maximum increase in the circumference of tank, Col. 23, Table 2, would be 1.82 in.

How serious are these changes in stresses for this design? The usual engineering design guards against a failure that would involve loss of life, wealth, and the structure itself. Thus, of necessity, the usual engineering design is extremely conservative. This conservativeness is still satisfied by the storage tank in Table 2 because even with the concrete cracked the stress in the wire is only about 100,000 lb per sq in., whereas its minimum ultimate is 200,000 lb per sq in.

Prestressed tank design is different from the usual design in that it also involves a calculated guarding against cracking failure, a failure that, except for the corrosion danger, involves only leakage losses and unsightly stains. Traditional engineering conservativeness is certainly uncalled for here, especially when this leakage would occur only on the occasional tank, which would be empty part of the time. (Corrosion would remain a problem, especially since the wire is so small and since the tensioning method used for prestressed tanks precludes the use of galvanized wire.)

Therefore, in the writer's opinion, these changes in stresses are not serious, except for corrosion, and the prestressed wire design is still superior to the prestressed rod design in saving materials.

The revisions involved in Table 4 would be of more vital importance for tanks with higher hoop tensions, for which there would be a tendency to use higher quality materials and higher working stresses.

As the author states, an overload is extremely unlikely, but it should be mentioned that the crack resistance of a wired tank to overload is much inferior to that of a rodded tank. The ultimate tensile resistance of the concrete would be approached much more rapidly in the wired tank with an increasing overload. After this tensile resistance had been exceeded, cracks would open up about eight times as fast as those in a prestressed rod tank (eight being the ratio of the steel areas). This might be an advantage; the overload would soon leak out of a wired tank! Of course, with removal of the overload, the cracks would disappear, a phenomenon typical of prestressed concrete.

Computations similar to those of Table 4 could be used to determine approximate stress losses for any prestressed tank or pipe design. It would be possible also to develop an algebraic expression that would give an approximate stress loss for any design almost automatically.

J. M. Crom[30].—The discussion by Mr. Mensch was most interesting since he is one of the pioneers in concrete tank construction and his account of tanks built during the past are valuable. His comments relative to sliding joints are of particular interest since any uneven settlement of the tank foundation will allow leakage if the wall and foundation are not doweled together or if there is not some kind of flexible key connection between them. However, experiences with more than 200 tanks having sliding joints indicate that leakage develops only occasionally and that it usually can be sealed at small expense. The writer has on several occasions made careful studies of the Fulton, N. Y., tank mentioned by Mr. Mensch, before it was demolished as a failure. As would be expected, the base of the wall moved outward. However, when the tank was in the empty condition, the walls did not return to the original position, perhaps partially because of friction, but chiefly on account of the chemical deposits that had formed in the vertical cracks, thus increasing the circumference and in effect simulating the conditions of a prestressed body. The percolation of the water through the walls caused rusting of the band rods, and the expansive force of the rust and the freezing of the water caused the spalling of large areas of the concrete outside of the reinforcement. The design of this tank is described and illustrated elsewhere.[31]

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[30] Vice-Pres., The Preload Enterprises, Inc., New York, N. Y.

Regarding the resistance to movement between wall and foundation slab because of friction, an accurate satisfactory coefficient that would apply to all jobs cannot be fixed. However, experience with tanks constructed has resulted in some data that are of help. For instance, during the wire winding operations when the first movement of the wall occurs, notes are kept showing the number of bands in place. These data, together with a knowledge of the condition of the joint prior to pouring, indicate a figure that can be used by the designer. In some instances this coefficient has been low, indicating that with proper care it can be kept within practicable limits.

Designs are under consideration that include the tying of the floor to the sidewalls by means of the vertical prestressing units after the specified amount of inward movement of the wall has occurred. Experiments having to do with heavy rubber dumbbell waterstops that extend from the wall through the joint and into the foundation slab have also been undertaken.

Mr. Mensch discusses a tank built for the City of Lansing, Mich., during the year 1918, and also another built in 1910 for the Town of Norway, Mich. It would be interesting to know whether or not these tanks were ever exposed to severe drying conditions such as would occur during a long, hot, dry period while the tank was empty and whether or not the tanks were covered partially or wholly with earth. If the walls were continuously damp it is doubtful that the shrinkage exceeded a coefficient of 0.0003, in which case the unit tensile band stresses in the concrete of the walls never exceeded 400 lb per sq in., which should be within the strength of the type of concrete used. The band steel would still have been in compression and thus impose an extra tensile load on the concrete.

The thickness to specify for dome shells is largely a matter of conscience. There are scores of tanks now in regular use that have shell domes 2 in. in thickness. Some of these shells show cracking, but this is also the case with domes of much greater thicknesses. All are structurally strong if the form work over which they are cast is true. The meridional and circumferential stresses are under 200 lb per sq in.

Sometime prior to 1920 an 83-ft-diameter shell was built over an oil tank in California. This shell was still giving good service when it was inspected by the writer during 1948. It had a thickness of 1.5 at the crown and about 2 in. at the ring. It contained a number of cracks that seemed to be causing no damage.

Concerning a reduced value of the factor E_c after a lapse of time, as pointed out in the last paragraph of Mr. Mensch's discussion, it is not believed that this condition will occur in all cases in which the shrinkage coefficient has reached a value of $C = 0.0006$. In several instances within the experience of the writer this value was reached very soon after the completion of the construction work.

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[31] "Concrete Engineers Handbook," George A. Hool and Nathan C. Johnson, editors, McGraw-Hill Book Co., Inc., New York, N. Y., 1918, p. 769.

Mr. Sawyer discusses the losses of stress caused by the creep of the wire under high stress. There is no question but that the creep is considerable when the stresses are very high. Luckily, however, a large percentage of it takes place during the first periods of the stressing, and since the initial stresses are not unduly high, it is believed, and it is reasonably well verified, that the losses because of this fact do not exceed 3%.

The results of tests made by the writer showed that with a constant length, and at the stresses used in tank work, the total creep over a period of several months did not exceed 6%. Since about ⅔ of this creep took place during the first, second, and third minutes of the stressed period, a similar phenomenon could be expected in the wire-winding operation. For example, in a winding operation the pull on the wire is constant as it comes from the machine. Since some of the creep takes place almost instantly, the release of slack is taken up over a length of wire to the point at which the friction between wall and wire prevents further take-up. Also, the wire is somewhat increased in temperature by the stressing mechanism. The return of the wire to normal temperature fully takes up any loss incurred by creep.

Many critics have deprecated the assumptions of stress losses of the magnitude of 40,000 lb per sq in., and, since Mr. Sawyer would have them at 60,000 lb per sq in. in the case of the usual method of prestressing, there is comfort in his statements. He also supplies further data in connection with the Staley-Peabody tests as well as the development of formulas to simplify the computations involved in the designs of prestressed structures.

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PLANNING THE NATIONAL CAPITAL: OBJECTIVES AND PROBLEMS OF ATTAINMENT

By Ulysses S. Grant III,[1] A. M. ASCE

With Discussion by Mr. A. P. Greensfelder.

Synopsis

The nation's capital is faced with problems of growth and expansion that are typical of other urban areas; but the city planners of Washington, D. C., face one special problem that is predominantly political in nature. The District of Columbia is under federal control, whereas the suburbs on which the capital is encroaching are under the jurisdiction of the neighboring states of Maryland and Virginia.

Legislation enacted on a federal and state level to deal with the problem is discussed in this paper. The shortcomings of this legislation are also presented, with proposed new laws designed to overcome such limitations. Cooperation between the existing authorities has been generally good, but progress is slow, and many urgent needs are faced by the city authorities. Some difficulties encountered under the existing plan of multiple authority are presented.

An over-all financial and construction agency is suggested as a means of expediting the execution of plans, financing and construction of needed public works for the capital; and the organization and operation of this agency are outlined.

Introduction

At the annual convention of the American Institute of Architects in Washington, D. C., in 1926, James Monroe Hewlett said in referring to the Washington Monument:

"There is in Washington a construction designed by engineers. The wind piles up masses of sculptural clouds behind it. The rising sun paints it

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[1] Maj. Gen., United States Army (Retired), Vice Pres., George Washington Univ., Washington, D. C.

> silver and the setting sun paints it gold, and it is, I venture to think, the most beautiful single object in the world today * * *."

This was more than just a gracious and noble tribute from a member of a sister profession. It carries an implication that calls for serious consideration—the important part played by engineers in the proper development of the nation's capital. George Washington himself was an engineer, and the remains of the largest construction work that he supervised (the Potomack Canal) are still extant and slowly succumbing to decay and neglect on the west bank of the Potomac River a few miles from the city. Major Pierre Charles L'Enfant, who planned the new federal city for President Washington, was an engineer.

From 1869 to 1933 the parks were developed by a succession of engineers. The water supply system was designed and built by engineers. The harbor developments and the reclamation that transformed the Potomac and Anacostia flats into beautiful river parks were designed and constructed by engineers. The municipal public works have been the responsibility of an officer of the Corps of Engineers, United States Department of the Army, since 1874, free from political or financial scandal. It was Brigadier General Theodore A. Bingham who revived interest in the original L'Enfant plan which had been reproduced by the United States Coast & Geodetic Survey (USCGS) in 1887. It was he who initiated the campaign to stop legislation that would have turned the Mall into a railroad yard.

The Library of Congress, the Lincoln Memorial, the Arlington Memorial Amphitheater, and the Arlington Memorial Bridge are projects planned and supervised by engineers. Since 1927 there have been three engineers on the Planning Commission, and John Nolen, Jr., Assoc. M. ASCE, is present Director of Planning.

Of course, other professional men—the architects, sculptors, landscape architects, and city planners—have played appropriate parts in the development of the city; but the contribution of the engineer has been greater here than elsewhere. The engineering profession has a special responsibility to maintain this contribution in the future on the same high level.

The nation's capital has outgrown, far and wide, the district "ten miles square" that was prescribed by the Constitution[2] for the "seat of Government of the United States"; it is no longer contained within the District of Columbia. The resulting problem of securing coordination among different political jurisdictions is of interest to city planners in so far as similar questions recur, in some form or other, in nearly every large metropolitan area.

The special problem in the Washington metropolitan region arises from the fact that one political entity is a federal district with the Congress of the United States for its city council, and the other five are local jurisdictions within two states. To illustrate the importance of these local communities to their states, it is only necessary to point out that Silver Spring is the second most populous community in Maryland, and that Arlington County is the fourth most populous county in Virginia. Therefore, any discussion of extending the boundaries of the District of Columbia to annex the suburbs seems quite futile. Neither state will ever consent to cede to the federal government some of its most prosperous territory, and the other states will certainly not join in passing an amendment to the Constitution that would force such action on their sister states. This is the problem that faces those who would develop a comprehensive plan for the national capital.

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[2] **Constitution of the United States, Article I, Section 8.**

History of Legislative Action

The Federal Government.—In 1924 those who had the interests of the national capital at heart, both in and out of Congress, were distressed that little or no progress had been made toward activating the inspiring plans and recommendations of the 1901 McMillan Park Commission. The natural beauties of the district were being destroyed rapidly by building operations to conform to the grades called for by the 1898 Highway Plan. Their united efforts produced the federal act approved June 6, 1924, establishing a National Capital Park Commission (hereinafter called "the Commission") which was authorized to buy land and to plan a comprehensive and coordinated park, parkway, and playground system. This is just one more case in which Congress acted with wisdom and foresight in regard to local matters, when public opinion, responding to sound leadership, was fairly united in stating the need for legislation.

The newly established Commission found that parks and playgrounds could not be planned, bought, and established intelligently without being fitted into an over-all city plan. There was no such adequate plan, since the part of the district not yet fully developed was nearly all outside the L'Enfant plan. The inadequacies of the 1898 Highway Plan were only too apparent. Under the same leadership, therefore, an amendment by the act approved April 30, 1926, was obtained, which added to the Commission's name the words "and Planning" and to its duties the preparation of a comprehensive plan of the national capital and its environs and the making of recommendations to appropriate executive authorities on all elements of city planning. To this end, it added to the ex-officio membership of the Commission "four eminent citizens, well qualified and experienced in city planning," to be appointed by the president.

Congress is so often blamed for everything that goes wrong and is so seldom given credit for the things it has done well that it is important to emphasize the wisdom and foresight with which this effort to make "a comprehensive, coordinated and consistent plan" for the national capital was extended explicitly to cover the suburban areas. Of course, Congress could not give to this Commission, or to any federal agency, power to control the states and their local authorities within their own borders; but at least it could and did set up a federal agency charged with planning for the suburbs and with undertaking such coordination of effort as was possible on a friendly, advisory, and cooperative basis. Under this act, the Commission has indeed accomplished much in helping with advance planning and in bringing about closer cooperation between federal, district, state, and local authorities.

The Neighboring States.—A primary effort that was entirely successful was the securing of state legislation setting up local planning commissions. On the east side of the Potomac River a single Maryland-National Capital Park and

Planning Commission was established, with definite planning and control powers and limited taxing authority in Montgomery and Prince George's Counties, Maryland. Cooperation of the National Capital Park Commission with the state commission for twenty years has been close, cordial, and successful. An agreement for extension of the national capital park and parkway system into the Maryland suburbs under the Capper-Cramton Act of 1930 has been administered successfully throughout the years, and may be of interest to other cities as an example of how a metropolitan parkway and recreation system can be carried out.

In Virginia the single planning commission was not acceptable at first, but state legislation was passed in 1936 authorizing cities and counties to set up their own planning commissions and to cooperate with the National Capital Park and Planning Commission. Again valuable cooperation has been operative through the years with these local planning commissions. In 1949 the local authorities of northeastern Virginia united in establishing a Northern Virginia Regional Planning Commission to correlate their interests and coordinate the planning of the local commissions.

INTERAUTHORITY COOPERATION

In general excellent understanding and cooperation have existed with the local planning commissions and their staffs, and cordial and friendly relations have been established and maintained with the local executive and legislative authorities. At times the latter have failed to understand and, on occasion, have acted contrary to the recommendations of the federal commission and of their own planning commission, especially on zoning questions. It must be confessed that even the Zoning Commission of the District of Columbia has sometimes ignored the best advice and has violated sound zoning principles.

The types of inconsistencies and incongruities in planning that have thus been authorized are to be found in the following: Higher height limits in some suburban counties than in the district, commercial zoning on one side of the district boundary facing a residential zone on the other side, and intensive apartment house development along the Mount Vernon Memorial Highway, threatening to impair its memorial and expressway characteristics. The only remedy would seem to be a better public understanding of the values and principles of good planning and perhaps a requirement that a three-fourths or four-fifths vote of the legislative authority be required to overrule the Planning Commission's recommendation. The writer does not favor giving to any planning commission complete veto power without appeal to the elected local authorities; but there is much to be said for making it harder for the latter to make mistakes affecting the general welfare of the community.

The point to be emphasized is that the act of 1926 set up a single planning commission for the Washington metropolitan area and that local and state legislation have acknowledged its leadership and provided suitable suburban coordinated agencies. Therefore, no new agency is required to ensure coordinated regional planning. Legislation is needed to strengthen the present Commission's leadership and power and to draw closer the bonds with the Maryland and Virginia authorities.

From 1942 to 1944 the Bureau of the Budget made an exhaustive and thorough investigation of the entire planning problem for the Washington metropolitan region. Legislation under consideration by the 80th Congress (S. 1931 and H. R. 4848) was approved by the President as adequate to carry out the recommendations of the Director of the Budget.

Essentially this legislation will result in seven items of improvement, as follows:

1. The membership of the Commission will be increased by the inclusion of the Architect of the Capitol and a representative of the Federal Works Agency. These two members are much needed to bring around the same board, ex-officio, two important constructing agencies. In the interest of keeping the membership to a practicable minimum the representative of the Forest Service is to leave the Commission in spite of his valuable contributions in the past to the work.
2. Because the chairmen of the Senate and House district committees are very busy men and can rarely attend Commission meetings, they will be authorized to name another member of their committee to act in their stead, thus again making closer the connection with the national legislature.
3. The lay membership will be increased by three—a representative each from Maryland, Virginia, and the District of Columbia—recommended by the chief executive of each of these. This will give the Commission a member, presumably fully acquainted with its policies, who can make closer the liaison with the local authorities, speaking to them in the name of the Commission and, on the other hand, representing their ideas and needs to the Commission.
4. The control exercised by the Commission over federal building projects in the District of Columbia will be extended to such projects in the suburbs and also to District of Columbia projects. This is very important for the protection of the suburbs against incongruous or badly located federal and district projects.
5. The appropriation for planning staff and studies, heretofore included in the local district budget, will be wholly from the federal treasury. It is hoped that in this way a staff can be secured adequate to do the Commission's work, which has heretofore been inadequate because of the limitation on the funds obtainable.
6. New procedures are prescribed to inform the Commission, from the very beginning, of new projects proposed by federal or district authorities. These safeguards should prevent argument about projects that have sometimes progressed without the Commission's knowledge to a point where it is difficult or expensive to modify them in conformity with a comprehensive plan. Experience shows that this consideration is truly most important.
7. Provision is also made for a coordinated six-year schedule of public works and for the adoption of a coordinated thoroughfare and highway plan. Such a plan has been recommended by the Commission for years, but never was definitely adopted by the executive authorities. In several other minor matters the law is improved or made more specific.

From the foregoing it will be evident that, although the 1926 federal act was surprisingly foresighted and was sound legislation, more than twenty years of experience in administering it and subsequent supplementary legislation have shown some ways in which the original act should be revised and improved.

PLANNING NEEDS OF THE CAPITAL AREA

The coordinated planning having thus been provided for in the best way possible under the United States form of government, the problem remains to find a way to construct the projects planned. Projects located wholly within one local jurisdiction will evidently have to depend on whether the local authorities are convinced of their soundness and whether the funds can be found to finance them. It may be presumed that they will be adopted as the need for them becomes apparent and as public opinion demands them.

In such a large metropolitan region, however, there are projects needed by the area as a whole, which are not within the responsibility of any one local jurisdiction. These require coordinated or cooperative financing, which is something difficult to obtain when there is a crying need in each jurisdiction for all available funds to pay for strictly local projects. Examples of this type of project, which are so badly needed by the metropolis for its sound development but for which it is difficult to obtain funds by local appropriation, are: (*a*) A bridge across the Potomac River from the southerly part of the district to a point north of Alexandria, Va.; (*b*) coordinated water supply and sewage disposal systems for the metropolitan region; (*c*) coordinated and progressive action to secure the step-by-step reduction of pollution in both the Potomac and the Anacostia Rivers; and (*d*) a coordinated waterfront development.

(*a*) *A Bridge Across the Potomac River.*—At present (1951) all traffic from the south must come to a point opposite 14th Street, west of the center of the district and, crossing there, must pour into the most congested central business area. For years there has been general agreement as to the necessity and justification for this bridge, but no action has been taken to finance it, while the needed approaches on the Virginia side are being bought up and encroached upon by building operations.

(*b*) *Coordinated Water Supply and Sewage Disposal Systems for the Metropolitan Region.*—Heretofore, provision for local needs has been made by localities individually. With the increased population and the future growth that must be foreseen, the need for such a metropolitan system and a schedule for its development is obvious.

(*c*) *Coordinated and Progressive Action to Secure the Step-by-Step Reduction of Pollution.*—Although the Interstate Commission on the Potomac River Basin and the Federal Commission are both ready to lend leadership and assistance, the solution of the problem evidently depends upon works that require funds, and the cost of which may be disproportionate to the interest of the particular community in which they can best be located.

(*d*) *A Coordinated Waterfront Development.*—Plans for this development exist, thanks to work done by the United States District Engineers' office in cooperation with the Federal Commission, the National Park Service, and the other authorities affected. However, assurance that the plans will be carried out is needed. A very important alternative route for the Mount Vernon Memorial Highway around Alexandria is today being occupied in part by building operations financed by the Federal Housing Administration against the federal government's own interests. Other important projects are constantly being threatened.

With the evident lack of interest of the local political authorities in regional problems, some kind of over-all financing and constructing agency seems called for. The New York-New Jersey Port Authority offers an example of a possible type of solution; the newly established Missouri-Illinois Interstate Development Agency is a more recent one. It would be particularly appropriate for American engineers to develop the right kind of agency structure (for instance, a "National Capital Metropolitan Works Agency") and to espouse its being set up with necessary legislative sanction.

Of course, the metropolitan highway system and the national capital park and parkway system are examples of real practical accomplishment in such metropolitan projects with federal aid and leadership. They can be mentioned with considerable pride and satisfaction to show what can be done by diplomatic approach, cooperation, persistence, and a fair distribution of costs. However, today the new problems are too urgent and too important to allow for this slow-going procedure. A National Capital Metropolitan Works Agency is needed, with funds contributed to such extent as practicable by the taxing authorities and itself, with borrowing power and the right of eminent domain, and with a reasonably free hand to procede with approved projects without having to go to Congress and state legislatures for legislation in each particular case.

Problems Encountered.—The outstanding and only important frustration suffered by the Commission to date is the bogging down of urban redevelopment. On the Commission's initiative and after eight months of hearings in the Senate and four months in the House, a sound urban redevelopment act for the District of Columbia was passed and approved August 2, 1946. This act provided for the selection of blighted areas and their replanning by the Federal Commission; when approved by the district commissioners after a public hearing, the project plan is then certified to the Redevelopment Land Agency, which acquires the land and disposes of it after resubdivision by sale or lease for redevelopment according to the plan. The only project for which planning funds were obtainable has encountered opposition on the part of the local inhabitants, who have been misled and misinformed as to the effect on their holdings. Sadly erroneous allegations that this scheme will put Negroes in ghettos and that it will be mostly a public housing project have been raised. Actually the law gives every advantage and opportunity to private enterprise, and the policy adopted and published gives priority in new housing to present occupants.

Such investigation as the author has been able to make indicates that redevelopment projects throughout the United States are bogged down because of local opposition and lack of funds; but perhaps nowhere does the possibility of early progress seem gloomier than in the Washington area. Apparently, while billions are being spent to revive a stunned world and to help other nations in need, the few hundred thousands annually needed and authorized by law to save the nation's capital from the further spread of slums will be refused. It can only be hoped that a better understanding of urban redevelopment may be achieved in time and that public opinion will eventually rout the present opposition.

Summary and Conclusions

The basic objectives of the Federal Commission can be summarized as follows: By farsighted planning, based on careful study of trends in the national capital region and on the experience of other cities in the world, to guide the growth and development of the metropolitan area so as to keep it a suitable and appropriate capital for the Government of the United States. At the same time, the Commission aims to provide for the needs of the inhabitants and the retention of the amenities of civilized life, so as to insure the happiness and prosperity of the people and to strengthen their conviction that it is a good place in which to live, to work, and to bring up a family.

At the Citizens' Planning Conference in Oklahoma City, Okla., it was stated very clearly that industry expects, primarily, a city to be so planned and so developed as to ensure contented employees, satisfied with their living conditions and free from the worries caused by shortage of housing, poor schools, inadequate recreation facilities, poor public transportation, daily traffic jams on the way to work, and so forth. The federal government being a major and dominant industry in Washington, the entire country has an interest in having such favorable conditions provided.

The basic plan of what is now the central part of the city, prepared by Major L'Enfant under President Washington's personal supervision, is fixed—a monument to their foresight and wisdom. Although departures from the L'Enfant plan and its principles have been made in the past to plague the city planners of today, and, although some minor features create problems that call for study to find a solution, the general central plan must be accepted (as it was by the 1901 McMillan Park Commission) as a living heritage marking a special achievement in American history—a planned capital in a federal district.

The present generation and generations that follow can only modify, revise, and supplement, providing for new needs and applying new techniques as they arise. With due regard to the best interests of the capital as a whole, with care to avoid satisfying one interest entirely at the expense of another, and with a readiness on the part of all authorities to cooperate and adjust their individual demands to the general interest, much can be accomplished to fulfill this ideal. Complete realization of it can hardly be expected, human nature being what it is and funds not always being forthcoming. As Emerson exclaimed a century ago: "Can anybody remember when times were not hard and money not scarce?"

An immediate problem having bearing on the region's future is whether or not Congress will be willing to limit the number of federal employees in Washington to the minimum needed to do the federal government's immediate work of administration, efficiently and economically, and to permit the kind of decentralization of federal employment recommended to industry by the National Security Resources Board. Washington has but one war industry and is not an inviting target for such physical destruction of war resources as might be effected by a sneak air raid. Nevertheless, as the capital of the nation, it may have political significance in the mind of a potential enemy. To rebuild the city to meet such a potential danger seems out of the question; it would cost too much and take too long. However, it would seem only the least demand of common sense to make arrangements now, so that such a raid shall have the least disturbing or shock effect on the federal administration in case of war. The Planning Commission has naturally given study and thought to that subject.

If the hands of the National Capital Park and Planning Commission are strengthened by the passage of suitable legislation, if appropriations are forthcoming to make its staff adequate, and if such a metropolitan works agency is set up, as has been suggested herein, the nation can look forward confidently to the growth of the capital to 2,000,000 inhabitants by 1980, to its development in a manner worthy of this country, and to its being a credit to the present generation—like Zion of old, "Beautiful for situation, the joy of the whole earth."

DISCUSSION

A. P. GREENSFELDER,[3] HON. M. ASCE.—Numerous objectives and various problems of planning the national capital at Washington, D. C., have been presented in this paper in a very informative and constructive manner. As a citizen planning member of the National Capital Park and Planning Commission the writer knows what an outstanding chairman the author was during 1948 and 1949 when he retired from the Commission. America is very fortunate that, during his various official duties for many years in Washington, he was connected with the improvement and development of the national capital. Few engineers were either in the position or had the disposition to strive as diligently in the interests of the finest possible plan for Washington.

Outstanding experience and constructive viewpoints as well as judicial qualities made the author a distinguished chairman of the Commission. It was for these reasons he was elected Chairman Emeritus of the Commission upon his retirement.

Having been interested in city and regional planning for many years in the St. Louis metropolitan district, the writer recognized various legal and area limitations of the Commission. Being both metropolitan-minded and regional-minded, the writer felt deep concern about the lack of coordination of the District of Columbia and its neighboring states (Virginia and Maryland), in the Washington metropolitan area. The comprehensive plan that the Commission issued in 1951 tried to point out the great need for cooperation in developing the environs of the national capital. Forecasts indicate that there will be approximately 1,000,000 people in the district and another 1,000,000 in its environs by 1980.

The present congestion around the government triangle, caused by occupants of both permanent and wartime buildings, is one of the most serious problems of the capital. When L'Enfant envisioned our national capital a century and a half ago he could not possibly have foreseen the effect of railway, automobile, or airplane congestion. The original swampy and light rolling terrain of Washington lends itself to the diagonal street plan that L'Enfant developed. Fortunately for the city, the wide streets then envisioned are the lifesaving but now congested thoroughfares of the captial. Without such wide streets it is difficult to imagine how the capital, as presently constructed, could provide circulation at all. To alleviate the present congestion is one of the prime necessities. It can be relieved by prohibiting individual passenger vehicles from entering certain congested areas and distributing public employees by means of shuttle busses. Such buses have been quite effective in the central St. Louis retail district. Although the shuttle bus system can be very helpful, it cannot entirely relieve congestion created by through traffic in the morning and night periods when government employees are coming from or returning to their homes. Thus, the dire need is evident for the proposed circumferential drives.

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[3] Cons. Engr., Fruin-Colnon Contracting Co., St. Louis 3, Mo.

A National Capital Suitable for a Great Nation.—The writer's greatest concern, however, is to achieve a national capital befitting a great nation, a goal which should be uppermost in all planning endeavors. It is an obligation as well as an opportunity; eventually adequate Congressional, state, and local legislation and appropriations may be obtained to insure its accomplishment. On several occasions the writer has inspected many of the national capitals of Europe and is firm in the belief that Washington is and can continue to be the most outstanding national capital in the world.

However, it is evident that all the planning intelligence of the world is not on one side of the ocean. At an international convention of planning and housing in Stockholm, Sweden, in 1939 some of the most outstanding city plans that have been conceived were on display. Washington, however, has the distinctive characteristic of being solely a national capital and does not, like London or Paris, contain large commercial and industrial areas.

Therefore, to suit the needs of a great nation, such as the United States, Americans should envision not only a magnificent capital but also an area so practical as to truly fulfill its purposes. To accomplish this result will require understanding from civil engineers and architects and from thousands of civic-minded American citizens who can express their views to their Congressmen who guide and appropriate the funds for the capital. This paper can be very helpful in this particular, and the ASCE should continue to take an active interest in attaining the objectives and solving the problems outlined.

International Airport.—While traveling in the Mediterranean countries last year, the writer was particularly impressed by the international airports of Cairo, Athens, Rome, Paris, and London. Washington lacks such a large facility. International guests are received in New York City, miles away from the Washington Monument. Recently the Commission visited Friendship International Airport, near Baltimore, Md. As soon as one of the planned projects (the Baltimore-Washington Parkway through Fort Mead) is completed, this airport could be developed readily as the International Airport for the capital. The airport should not be any closer to the Potomac than it is. The writer has stressed on more than one occasion that, if our nation in a century and a half has outgrown the District of Columbia, what will become of the headquarters of the United Nations now confined to a few city blocks? Men should learn from history, instead of frequently having to correct past mistakes. A great international airport would certainly seem to be only one of the major necessities of the capital.

Larger Parks and Parkways.—In spite of the fact that Washington itself is mostly on low ground, it is surrounded fortunately by fine hills and rugged terrain. In the years past the author has been one of the foremost advocators of a metropolitan park and parkway system along the valleys of the streams entering the Potomac. The George Washington National Parkway, extended from Mt. Vernon, Va., to Great Falls, Va., on the Potomac should prove to be so desirable as to induce the extension of the valley parkway system. One might be forgiven for overlooking Sugar Loaf Mountain in near-by Maryland or the historical mountains at Harpers Ferry if automobiles or airplanes were not available with which to view them. Certainly, the planning and development of a great system of fine parks, parkways, and recreation areas should be an immediate undertaking. Such a beginning is shown in the proposed Fort Drive, Green Belt Outer Park, and various other projected areas nurtured by the acting chairman of the Commission, Arthur E. Demaray, director of the National Park Service. But, unless plans are made and realized genuine satisfaction cannot be felt with the present development of the capital.

Reurbanization.—The needs of all large cities throughout the world were obvious at the Stockholm meeting. The capital of the United States is no exception to the fact that every large city must be reurbanized by the following steps:

1. Replanning;
2. Rezoning;
3. Removing obsolete structures;
4. Reconstruction of old structures; and
5. Recreating an adequate modern transportation system.

Presumably planners will have to struggle continuously in the future to achieve these five objectives. No city is ever finished, and therefore no plan for that city can ever be complete. Every plan must develop continuously and should precede urban and suburban development. Such planning must have sufficient interest so that it will arouse the enthusiasm not only of the City Planning Division and the entire membership of ASCE but of the community that is being planned. When planners recognize the need of adequate and timely publicity, their public relations will improve, and their planning will be made more readily attainable.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2483

LARGE HYPERBOLIC FUNCTIONS COMPUTED BY FISSION

By F. T. Llewellyn,[1] M. ASCE

With Discussion by Messrs. John Sherman, K. L. Cooper,
and F. T. Llewellyn.

Synopsis

A rapid "fission" method is presented for computing the hyperbolic sines and cosines of fractional radians larger than are listed in current reference tables. Such functions are often needed by structural engineers when solving cubic and quartic equations (notably in problems requiring the determination of maxima and minima) by a direct procedure such as those of Cardan,[2] Ferrari,[3] or Euler.[4] The primary objective is to simplify the work of practical designers and computers.

Within the range of widely spaced arguments larger than about six radians, the precision of the fission method is much greater than that of interpolation by proportional parts, and adequate precision is attained much more readily than by academic methods. The scope of several current sets of reference tables is surveyed and analyzed. Examples illustrate the preferred application of the slide rule method.

Introduction

Important phases of engineering design can often be expedited by the use of "hypes" (hyperbolic sines and cosines) larger than those listed in current tables of reference. The nonlisted functions at issue are the hypes of fractional arguments greater than about six radians. Their lack may explain in part why the advantages of a direct procedure, notably in solving cubic and quartic equations, have not been accorded fuller recognition by civil engineers.

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[1] Baton Rouge, La.

[2] "Smithsonian Mathematical Tables," by G. F. Becker and C. E. Van Orstrand, Smithsonian Institution, Washington, D. C., 1909, p. 26.

[3] "Transmission Circuits for Telephonic Communication, Methods of Analysis and Design," by K. S. Johnson, D. Van Nostrand Co., Inc., New York, N. Y., 1925, p. 249.

[4] "Higher Algebra," by G. E. Fisher and I. J. Schwatt, Macmillan Co., New York, N. Y., 1924, p. 614.

This study seeks to simplify direct procedure in such solutions by presenting an efficient method for computing the functions of any large radian in probable demand, and certainly any that may be expected in structural and allied problems. In so doing certain limitations that mark the standard process of interpolation are analyzed with what is thought to be a new approach. As far as the writer can find, the predictability of degrees of precision, as developed in this paper, has not hitherto been evaluated. The study is intended primarily for designers who cannot afford to interrupt the solution of a specific problem to "brush up" on their higher mathematics. Its objective is adequate precision combined with minimum labor, rather than unnecessarily high precision. Emphasis is therefore laid on the adaptation of natural functions and a slide rule, thereby obviating the use of logarithms.

BACKGROUND

About 1926 the late O. E. Hovey, Hon. M. ASCE, while preparing the manuscript for his classic treatise on the design of bridge machinery, found that certain fourth-power examples,[5] if solved by a direct and complete method (such as those of Ferrari or Euler), would entail the meticulous computation, by log logs, of the hypes of large unlisted radians. (The "radian," of course, is a unit of circular measure equal to $180/\pi$, or 57.29578°; but it is also used as a general term for the argument of a function. For example, "large" radians are radian arguments whose sinh and cosh hypes are substantially equal. Radians greater than 6.0, or the hypes of such radians, are considered "large.") Such a procedure was then regarded by the average designer as too formidable for everyday use. Mr. Hovey told the writer that this was one of several reasons why he had substituted an indirect and partial "trial-and-error" method of solution. The episode aroused in the writer a continuing interest in the direct solution of such problems and, hence, in hypes. (The term, "hype," denotes any hyperbolic function, although in this paper only sinh hypes and cosh hypes are treated.)

For instance, shortly thereafter he completed the direct solution[6] of Mr. Hovey's examples with the aid of large hypes; but the fission method of computing them was not then realized. (The fission method, described in this paper, is a process of computing any large unlisted hype by splitting its radian into two listed parts—or vice versa—and then combining their listed *e*-functions by multiplication or division. The convenient and accurate term "fission" was not borrowed from recent accounts of atomic reaction. Its use had long been anticipated elsewhere, as in the fields of botany and geology.) In 1935 he privately circulated 100 copies of 8 cubics and quartics, similarly solved; and in the succeeding fifteen years (in industry and research) he solved more than fifty such problems wherein the radians ranged from 0.0017 to 16.473, with eleven radians being classed as "large." He had previously observed that large unlisted hypes arise in the Cardan solution of many third-power equations, and elsewhere.

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[5] "Movable Bridges," by O. E. Hovey, John Wiley & Sons, Inc., New York, N. Y., 1926–1927, Vol. II, Chapter 2, Examples B2 and B3.

[6] "Solving Cubic and Quartic Equations," by F. T. Llewellyn, *Engineering News-Record*, August 14, 1930, pp. 254–256.

SURVEY OF TABLES

A logical study of large hypes cannot disregard the behavior of small ones, and, hence, the entire scope of a number of available reference tables was surveyed and analyzed. Table 1 is a semi-idealized frame in which are summarized the relevant data from six sets of "current" tables containing natural functions. (In the sense used in this paper, "current" tables are the mathematical tables commonly accessible to civil engineers.) Some of their groupings overlap, so that the combined scope of a plurality of tables is depicted. Complete references, and a discussion of the six sets of tables, are given in Appendix I; and the mathematical derivations are presented in Appendix II. Referring to Table 1, let: e be the base of natural (Naperian) logarithms = 2.71828 +; M be the magnitude of hype y; M_P be the value of M found by proportional parts (PP), or;

TABLE 1.—SURVEY OF CURRENT HYPERBOLIC TABLES

Group	SCOPE OF TABLES				INTERPOLATION[a] OF HYPE y	
	Range of Radians		Listed intervals,	Nature of functions	Maximum error,[b] R_E (%)	Critical[c] factor p_c
	x_{min}	x_{max}	v_{min}			
(1)	(2)	(3)	(4)	(5)	(6)	(7)
(a) SMALL RADIANS; SINH x AND COSH x ARE SUBSTANTIALLY DIFFERENT[d]						
1	0.0	0.1	0.0001	e^x and hypes	12.5 (10^{-8})	0.500
2	0.1	3.0	0.001	e^x and hypes	12.6 (10^{-6})	0.499[c]
3	3.0	6.0	0.01	e^x and hypes	12.7 (10^{-4})	0.497
(b) LARGE RADIANS; SINH x AND COSH x ARE SUBSTANTIALLY EQUAL[e]						
4	6.0	10.0	0.1	e^x and hypes	12.9 (10^{-2})	0.492
5	10.0	100	1.0	e^x only	13.1 (10^0)	0.418
(c) HYPOTHETICAL RADIANS; EXTENSIONS BY EQS. 5 AND 6						
6	none	none	5 (assumed)	neither e^x nor hypes	10 (10^2)	0.193
7	none	none	10 (assumed)	neither e^x nor hypes	8 (10^4)	0.100

[a] The values in Cols. 6 and 7 are approximate, being determined by the interpolation of hype y by proportional parts (PP). [b] In groups 2 to 5, Col. 6, each R_E-ratio is approximately one hundred times that of the next smaller group. [c] The critical p-curve (Fig. 1) appears to approach $p = 0.5$ as x and v approach zero. As v approaches infinity, the curve is definitely asymptotic to $p = 0$. [d] Both logarithms and naturals in each set. [e] Naturals in two sets and logarithms in one set.

$$M_P = \text{hype}\,x + p\,[\text{hype}\,(x + v) - \text{hype}\,x] \qquad (1a)$$

Let M_T denote the true value of M; that is:

$$M_T = \text{hype}\,y = \text{hype}\,(x + p\,v) \qquad (1b)$$

Furthermore let p equal a nondimensional factor of v in the part $p\,v$ to be interpolated: and p_c equal the critical value per group of p corresponding to $R_{\max}$. Then, R equals a general ratio of M_P/M_T, as expressed by the basic formula:

$$\frac{\text{Eq. } 1a}{\text{Eq. } 1b} = \frac{\text{hype } x + p\,[\text{hype } (x + v) - \text{hype } x]}{\text{hype } (x + p\,v)} \quad \ldots\ldots\ldots (1c)$$

in which:

R_E = the maximum ratio of the proportional parts (PP) error ($R_{\max} - 1$); multiplied by 100, it is expressed as a percentage;
v = the interval between any two listings of x, in radians;
x = any listed radian, in circular measure; and
y = any unlisted radian ($x + p\,v$) whose hypes are sought.

Col. 6, Table 1, shows the approximate percentages of maximum PP-error by groups. In certain respects group 4 in Table 1(b) may be classed as "small," but the upper limit of small radians was held at $x = 6.0$ because the more popular tables detail no hypes beyond that limit. Groups 6 and 7 (Table 1(c)) do not appear as such in current tables. They are included in Table 1 because their trends were helpful in establishing the critical p-curve plotted in Fig. 1 and discussed in Appendix II.

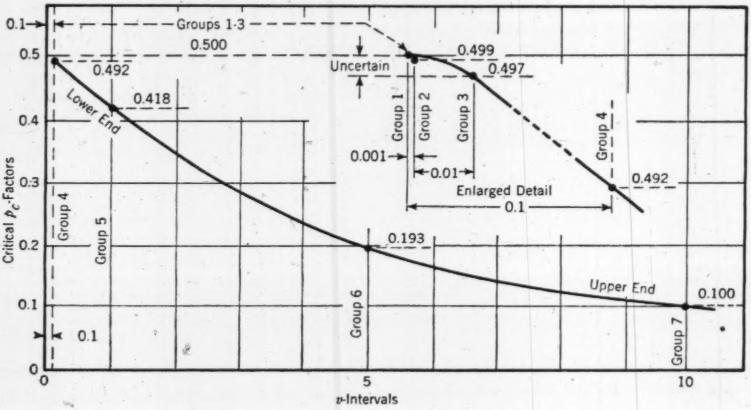

FIG. 1.—GRAPH OF CRITICAL p_c-CURVE (INVERTED)

In two sets of tables (discussed subsequently under the heading, "Fission Method" and in Appendix I) special cognizance was taken of a series of natural e^x-entries against radians ranging up to 100, which is much greater than any argument the writer has met in practice. In group 5 their listed interval ($v = 1.0$) would be too large for PP-interpolation, but it was correctly surmised that even these widely spaced entries might suffice to implement some method of computation yet to be developed. However, before considering any new methods, existing procedures were subjected to two lines of scrutiny, suggested by the questions:

1. Is there any practical substitute for hyperbolic tables? and
2. To what extent do current tables provide for nonlisted functions?

The first question does not warrant long discussion. A convergent series of the $y^{f(n)}$-type might indeed be carried (generally by the often puzzling and always irksome use of log logs) to any desired number of terms $(1 + r)$. (This type of convergent series was used to find the maximum PP-error in group 1.) Again, the magnitude of any hype could be found by a single evaluation of the $e^{f(v)}$-type, which sums up another series and also entails log logs; but academic procedures of this kind would tend to distract the attention of busy designers from the design in hand. They are doubtless useful to mathematicians when compiling permanent tables, and in research work, but they are much too prolix to replace published tables of reference for everyday use.

The second question could not be dismissed so summarily. To be sure, it suggested the time-honored process of interpolation by proportional parts, which is fairly simple to apply, but its reliability is not uniform. Its R_E-ratios increase sharply by groups as the v-interval increases, whereas (within each group) its R-ratios rise and fall in waves whose crest changes, both in position and size, as the p-factor increases. One rigorous characteristic is common to all groups; when $p = 0.5$, $R = \cosh p\,v$ (see Appendix II). The method followed in approximating the critical value of the other ratios calls for some particulars and a frank admission.

In the general case, the writer had been unable to manipulate rigorously the basic R-formula, Eq. 1*c*, with its three variables (see Appendix II) so as to locate the critical value of p_c corresponding to $R_{\max}$ and hence for R_E. Therefore, the writer started with the special case of large radians and intervals, and in that range he found that concise formulas, having ample precision, were derivable, and that they would fix the upper part of the critical p-curve (Fig. 1). In their development the following identity was assumed:

$$2 \sinh x \doteq e^x \doteq 2 \cosh x \qquad (2)$$

At $x = 6.0$ the error in this assumption is less than 0.001%, and it diminishes as x increases. By treating the assumption as exact it is readily seen that, when large hypes are interpolated by PP, $f(x)$ vanishes, and the ensuing relations reduce quite simply from Eq. 1*c* to terms of e, v, and p, with only two variables. For example (see Appendix II), the general formula for any large group becomes

$$R = \frac{1 + p\,(e^v - 1)}{e^{pv}} \qquad (3)$$

A characteristic common to all groups is that, when $p = 0.5$, Eq. 3 becomes

$$R = \cosh\,(p\,v) \qquad (4)$$

For the critical factor—

$$p_c = \frac{1}{v} - \frac{1}{e^v - 1} \qquad (5)$$

—the maximum value of Eq. 3 is

$$R_{\max} = \frac{e^{v} - 1}{v\,e^{pv}} \quad \text{(6)}$$

and, in group 5, with—

$$p_c = \frac{e - 2}{e - 1} \quad \text{(7)}$$

—the maximum value of Eq. 3 becomes

$$R_{\max} = \frac{e - 1}{e^{p}} \quad \text{(8)}$$

Eqs. 3 and 6 apply very closely for both sinh hypes and cosh hypes in group 5 (the principal range at issue); they apply quite well in group 4; and, to some extent, even in group 3. The R-ratios as thus forecast can be checked most easily near the boundaries between these three groups, and by the check for nonconsecutive entries suggested subsequently. It will be noted from Col. 6, Table 1, that, in group 5, the possible error ($R_E = 13.1\%$) is much greater than safe practices would tolerate.

In the case of very small radians, however, $f(x)$, instead of vanishing, becomes so dominant that Eqs. 3 and 6 no longer apply, except at $p = 0.5$. Since the basic R-equation (Eq. 1c) seemed here to be unworkable, recourse was taken to a semi-empirical procedure. By Eq. 6, and by trial, the critical p_c-curve (Fig. 1) had straightened out for groups 4 and 3. It was also noted that p_c seemed to approach 0.5 as x and v approach zero, and this value was assumed for entire group 1. The simplest reverse curve was then "faired" by eye, tangential to the curve at group 3, and to the horizontal at group 1. This curve gave $p_c = 0.499 +$ as the nearest three-digit value (less than 0.5) for group 2. The respective R_E-ratios were finally adjusted to correspond, and are so entered in Table 1.

These ratios show tenable trends at the lower end of the curve but they are too small for close checking, even by seven-place log tables, and their true values are uncertain. The topic is intriguing but purely academic because the amount of the PP-error here becomes microscopic. To summarize, the group values in Fig. 1 were plotted as follows:

Groups	Plotted by
3 to 7	Eq. 6
1	analogy
2	eye

Groups 3 to 7 were checked by trial, group 2 was checked partly by trial, and group 1 was too small to be checked.

Of more practical significance as applied to Eqs. 3 and 6 is the fact that v is not restricted to the intervals that characterize groups 3 to 7 in Table 1; nor is p restricted to factors less than unity. Thus, these equations satisfy any interpolation, or indeed any upward extrapolation, within or outside any pair of large or semilarge radians. The validity of Eq. 3 and of Eq. 6 can thus be checked against nonconsecutive entries.

When the main problem is reversed, as in computing radians from hypes, R is subject to considerable reduction because those terms that might have adverse effects can then be thrown into a relatively small addendum, $f(pv)$. This truism is applicable to any approximate method. Subsequently, under the heading, "Fission Method," Example 2 illustrates the closer precision attainable by both PP and fission in this type of problem.

In either type the survey, as summarized in Table 1, definitely indicated that the PP-process of interpolation, although entirely adequate for small radians, might lead to excessive error for the large ones at issue. In the former ranges the exact amount of error was theoretically elusive but is practically negligible, whereas in the latter it was readily ascertainable and is important. It was concluded that either more comprehensive tables would be necessary, or a better method must be sought for handling large radians and hypes.

FISSION METHOD

Further study of this dilemma led to the development of a procedure that is based on well-known theory, but whose practical application may be new to many engineers. It asks for no elaboration of existing tables. By this process the unlisted hypes of any large radian (up to $y = 100$) can be interpolated very rapidly (but not by PP) from current natural tables that list both large and small e^x-values and are otherwise in line with Table 1. With the aid of an ordinary slide rule, such tables permit results that are correct to at least four significant digits when hypes are to be computed from radians, or to three places, and sometimes four places, when radians are to be found from hypes. The process entails much less labor than computation by $e^{f(y)}$, and somewhat less than by the (here) unsuitable PP-method.

The writer has named the suggested procedure a "fission" method, because it splits the nonlisted radian (y) into two listed parts (a and b). When $y = a + b$, current textbooks show that

$$\cosh y = \cosh a \cosh b + \sinh a \sinh b \ldots\ldots\ldots\ldots\ldots (9)$$

Also, when $a > 6.0$,

$$\sinh a \doteq \cosh a \doteq \frac{e^a}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Therefore, when a (and hence y) is large, Eq. 9 simplifies to

$$\text{hype } y \doteq e^b \text{ hype } a \doteq \frac{e^b e^a}{2} \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

in which hype denotes either sinh or cosh. Eq. 11 works also inversely, when it takes the form

$$e^b \doteq \frac{e^y}{e^a} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Eq. 9 is a general expression for splitting any cosh hype.

If logs were to be used, a and b could have any listed values. However, when natural tables and the slide rule are used, b should be kept small enough to fall into such tables as give the most decimals. Under the following rule, b is thus thrown into group 1 or 2 (which give radians to four decimals or three decimals), whereas a will remain in group 4 or group 5, thus: a (listed) $= x$ = integral part of y; and b (listed) $= p\,v$ = fractional part of y.

The handiest tables list values of e^b alongside hypes of b. In case such tables are inaccessible the value sought can be found, but less readily, from the following identity in which, for small radians, sinh b and cosh b are markedly unequal:

$$e^b = \sinh b + \cosh b \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Two rather extreme examples may suffice to illustrate the precision and brevity of the fission method. Its trivial aberrations are caused almost entirely by slide rule "roundings." No problem needs more than two tabular references and one slide rule setting.

Example 1 (Direct); Compute Sinh 28.49 (Unlisted).—By the rule for a and b, $a = 28$ (listed) and $b = 0.49$ (listed). Eq. 11, then, yields (by slide rule): $\sinh y = \frac{1}{2} e^{0.49} e^{28} = 1.632 \times 723 \times 10^9 = 11{,}800 \times 10^8$.

Example 2 (Reverse); Compute y (Unlisted), from Cosh y = 713 × 10⁴.—In this case $a = 16$, which is the integer-radian x tabulated next preceding y, in which (compare Eq. 2) $e^y = 2 \cosh y = 1{,}426 \times 10^4$. Consequently, e^a (listed) $= 8{,}886 \times 10^3$. By Eq. 12 (using slide rule), $e^b = e^y/e^a = 14{,}260/8{,}886 = 1.605$. Then b (listed) $= 0.473$ and $y = 16.473$.

TABLE 2.—RELATIVE PRECISION OF METHODS

Method	EXAMPLE 1		EXAMPLE 2	
	sinh y	Error (%)	Radian y	Error (%)
Logarithmic (seven place)	11,804 (10^8)	Norm	16.473	Norm
Fission (10-in. slide rule)	11,800 (10^8)	0.034	16.473	0.00
Proportional parts (10-in. slide rule)	13,320 (10^8)	12.8	16.352	0.73

Table 2 compares three of the methods herein discussed as applied to the two examples. The method marked "logarithmic" furnishes norms of precision against which the other two methods are rated. These norms were calculated by the meticulous $e^{f(y)}$-procedure and checked by seven-place log logs with Eqs. 11 and 12. The negligibility of fissional errors is apparent immediately. The importance of adequate precision is properly emphasized because even a small error in hype magnitudes may have a considerable effect on the final design, especially in those rather frequent cases in which the critical difference between two large quantities proves to be relatively small.

SUMMARY

The large functions to whose computation the fission method is directed are the hypes of fractional radians lying between 6 and 100. Within this range the entries listed in current tables can be combined by slide rule at a single step to find any intermediate unlisted hypes with a precision substantially that of their listed sources. In its labor-saving slide rule work the method calls for no special skill to meet the ordinary needs of civil engineers. Those who prefer to use log logs can apply the fission concept as a quick independent check.

The fission method is not applicable to cases in which radians are much less than 6; but in those cases the old process of interpolation by proportional parts is fortunately available, and is adequate.

Thus, the unlisted, as well as the listed, hypes of all radians in normal demand become readily determinable. In the field of cubic and quartic equations having real coefficients, the writer has shown, elsewhere,[6] the universal feasibility of a direct method of solution by hypes (or in one type by trigs), and has emphasized its general advantages. (By printer's error, a radical $\sqrt{3A + y_1}$ appeared as $\sqrt{3A - y_1}$ in that work.[7]) To these advantages the special relief afforded by fission may now be added. The writer has not investigated the extent to which the fission method may be desirable in other fields, but its practical scope appears to be quite broad.

APPENDIX I. DESCRIPTION OF CURRENT SETS OF NATURAL HYPERBOLIC TABLES

The six sets of tables containing natural functions that were examined in detail are listed chronologically in Table 3. Each one appeared to be excellent in its respective fields, but not equally well suited to the objectives of the fission method. In sets 3 to 6 (Tables 3(*c*), 3(*d*), 3(*e*), and 3(*f*)) some entries were either too widely spaced or too limited in range to meet all cases readily. This was notably true in the otherwise meticulous British set, set 3 (fifteen decimal places), which was found only in very large scientific libraries.

However, it was the writer's experience that sets 1 and 2 (Tables 3(*a*) and 3(*b*)) were fully suited to the fission method. Their hype entry and *f*(*e*)-entry comply essentially with the ideals of Table 1, and they were the only sets found containing group 5 (Table 1), the principal range at issue.

For general purposes the Smithsonian tables are the more comprehensive. They are best suited to logarithmic computation. For fission by slide rule they may entail reference to three dissociated pages of the book.

The Hayashi tables[8] contain only natural functions and are thus restricted to slide rule computation. (Note: There is an error in the Hayashi tables[8] where the factor for e^{50} should read "10^{13}" and not "10^{14}.") Most of the information needed for a given problem is assembled on a single pair of facing pages, only one additional crossreference being required. This handy little book, although published abroad, is classed as "current" because copies were on general sale in New York, N. Y., prior to World War II, and are doubtless still available in the United States.

The only known previous literature on the practical objectives of the present study consists of brief statements in the prefaces of some of the tables of reference, none of which, however, suggest fission.

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[7] "Solving Cubic and Quartic Equations," by F. T. Llewellyn, *Engineering News-Record*, August 14, 1930, p. 254, step 10.

[8] "Fünfstellige Tafeln der Kreis, und Hyperbelfunktionen," by K. Hayashi, Walter de Gruyter and Co., Berlin, Germany, 1921, p. 175.

TABLE 3.—Pertinent Details of Six Current Sets of Natural Hyperbolic Tables[a]

Pages (inclusive)	Radians		Minimum interval	Functions		Decimal places
	Range			Hype (=h) e^x (=e)	Natural (=N) or logs (=L)	
	Minimum	Maximum				
(1)	(2)	(3)	(4)	(5)	(6)	(7)
(a) Set 1.—"Smithsonian Mathematical Tables," (Hyperbolic Functions) by G. F. Becker and C. E. Van Orstrand, Smithsonian Institution, Washington, D. C., 1909.						
2 to 21	0	0.1	0.0001	h	L	Five
22 to 79	0.1	3	0.001	h	L	Five
80 to 85	3	6	0.01	h	L	Five
88 to 171	[b]	[b]	[b]	h	N	Five
226 to 255	0	3	0.001	e	L, N	Seven and six
256 to 258	3	6	0.01	e	L, N	Seven and six
259 to 260	1	100	1	e	L, N	Integers
261 to 262	0	0.5	0.001	e	L	Auxiliary
(b) Set 2.—"Fünfstellige Tafeln der Kreis, und Hyperbelfunktionen," by K. Hayashi, Walter de Gruyter and Co., Berlin, Germany, 1928.						
2 to 41	0	0.1	0.0001	h, e	N	Five and six
42 to 157	0.1	3	0.001	h, e	N	Five and six
158 to 171	3	6.3	0.01	h, e	N	Five and six
171 to 173	6.3	10	0.1	h, e	N	Five and six
174	0	1	varies	e	N	Twelve, ten, and eight
175	1	10	1	e	N	Eight
175 to 176	11	100	1	e	N	Integers
(c) Set 3.—"Mathematical Tables," British Assn. for the Advancement of Science, University Press, Cambridge, England, 1931, Vol. 1.						
30	0	10	0.1	h[c]	N[c]	Fifteen[c]
(d) Set 4.—"Tables of Circular and Hyperbolic Sines and Cosines," by a Committee of the Works Progress Administration, National Bureau of Standards, Washington, D. C., 1937.						
1 to 400	0	2	0.0001	h	N	Nine
402 to 403	0	10	0.1	h	N	Nine
(e) Set 5.—"Mathematical Tables of Elementary and Some Higher Mathematical Functions Including Trigonometric Functions of Decimals of Degrees and Logarithms," by H. B. Dwight, McGraw-Hill Book Co., Inc., New York, N. Y., 1941.						
128 to 147	0	5	0.001	e	N	Nine
148 to 165	0	3	0.001	h	N	Nine
(f) Set 6.—"Six-Place Tables," by Edward S. Allen, McGraw-Hill Book Co., Inc., New York, N. Y., 1947.						
172 to 191	0	1	0.001	h, e	N	Six
192 to 201	1	2	0.002	h, e	N	Six
202 to 205	2	3	0.005	h, e	N	Six
206 to 208	3	6	0.01	h, e	N	Six

[a] General reference: "An Index of Mathematical Tables," by A. Fletcher, J. C. P. Miller, and L. Rosenhead, McGraw-Hill Cook Co., Inc., New York, N. Y., 1946 (an analytical summary including hyperbolic tables). [b] Same as the foregoing three groups. [c] Other functions not pertinent.

APPENDIX II. MATHEMATICS

General Relations at $p = 0.5$.—By trigonometry,

$$\sinh(x + v) = \sinh x \cosh v + \cosh x \sinh v \tag{14}$$

$$\cosh(x + v) = \cosh x \cosh v + \sinh x \sinh v \tag{15}$$

$$\tfrac{1}{2}(1 + \cosh v) = \cosh^2 \frac{v}{2} \tag{16}$$

and

$$\sinh v = 2 \sinh \frac{v}{2} \cosh \frac{v}{2} \tag{17}$$

The basic R-formula Eq. 1c is then written—

$$R\,(\sinh) = \frac{M_P}{M_T} = \frac{\sinh x + p\,[\sinh(x + v) - \sinh x\,]}{\sinh(x + p\,v)} \tag{18}$$

At $p = 0.5$—

$$\left.\begin{aligned} M_P &= \frac{\sinh x}{2} + \frac{\sinh x \cosh v}{2} + \frac{\cosh x \sinh v}{2} \\ M_P &= \sinh x \frac{1 + \cosh v}{2} + \frac{\cosh x}{2}\, 2 \sinh \frac{v}{2} \cosh \frac{v}{2} \\ M_P &= \sinh x \cosh^2 \frac{v}{2} + \cosh x \sinh \frac{v}{2} \cosh \frac{v}{2} \\ M_P &= \cosh \frac{v}{2} \left[\sinh x \cosh \frac{v}{2} + \cosh x \sinh \frac{v}{2} \right] \\ M_P &= \cosh \frac{v}{2} \sinh \left(x + \frac{v}{2} \right) = \cosh \frac{v}{2}\, M_T \end{aligned}\right\} \tag{19}$$

Then $R\,(\sinh) = \cosh \frac{v}{2} = \cosh p\,v$ (see Eq. 4). Similarly,

$$\left.\begin{aligned} M_P\,(\cosh) &= \cosh \frac{v}{2} \left(\cosh x \cosh \frac{v}{2} + \sinh x \sinh \frac{v}{2} \right) \\ M_P\,(\cosh) &= \cosh \frac{v}{2} \cosh \left(x + \frac{v}{2} \right) = \cosh \frac{v}{2}\, M_T \end{aligned}\right\} \tag{20}$$

Then $R\,(\cosh) = \cosh \frac{v}{2} = \cosh p\,v$ (see Eq. 4). Thus, in all groups, and for both sinh hypes and cosh hypes at $p = 0.5$, $R = \cosh p\,v$. At $p = 0.5$, however, R is not (Table 1) generally a maximum. It is almost a maximum in groups 1 to 4; but in group 6 it drops to 56%, and in group 7 to only 9.2% of the maximum.

Eq. 3 (x Large).—When x is large Eq. 1c, as simplified by Eq. 2, becomes

$$R \doteq \frac{e^x + p\,(e^{x+v} - e^x)}{e^{x+pv}} \tag{21}$$

Clearing e^x, $R = \dfrac{1 + p\,(e^v - 1)}{e^{pv}}$ (see Eq. 3).

Eq. 6 (x Large).—In Eq. 3, e is constant, and v is constant per group, and thus R and p are the only variables. By calculus, R is a maximum (or minimum) when its first derivative (with respect to p) is zero. Here dR gives

$$e^{pv}\,d\,[1 + p\,(e^v - 1)] = [1 + p\,(e^v - 1)]\,d\,e^{pv} \dots\dots\dots\dots (22)$$

Hence, $e^{pv}\,(e^v - 1) = [1 + p\,(e^v - 1)]\,v\,e^{pv}$. Dividing by the left-hand member,

$$1 = \frac{v}{e^v - 1} + p\,v \dots\dots\dots\dots\dots\dots\dots\dots (23)$$

and, dividing by v, the critical value of $p_c = \dfrac{1}{v} - \dfrac{1}{e^v - 1}$ (see Eq. 5), in which p_c is positive in interpolation and in upward extrapolation

Since the second derivative of R here becomes negative, this point of inflection denotes a maximum. (By inspection, minimum R (= 1) occurs at two points ($p = 0$ and 1), but they are not points of inflection.) Then, by Eqs. 3 and 5 (see Eq. 6),

$$R_{\max}\ (x \text{ large}) = \frac{1 + \dfrac{e^v - 1}{v} - 1}{e^{pv}} = \frac{e^v - 1}{v\,e^{pv}}$$

Lower End of p_c-Curve.—The statement in the text that Eq. 1*c* seemed to be unworkable for the general case does not mean, of course, that it could not be differentiated, but rather that the resulting expression became too complex to justify its rigorous solution. It definitely indicated, however, that for small radians p_c would vary with each entry, and not per group.

Since trial had shown that R_E was here quite negligible, it seemed sufficient to indicate reasonable trends at the lower end of the curve as in Fig. 1. An alternative hypothesis might be that $p_c \doteq 1.0$ instead of 0.5 for group 1, Table 1, which would change the lower end of the curve, but the PP-error for small radians would remain negligible.

DISCUSSION

John Sherman,[9] M. ASCE.—It is correct that available tables for large hypes are arranged with intervals too large to allow interpolation to be made for intermediate values. The method to overcome this deficiency in the tables is based on the use of a trigonometric equation that has been in use for many years. Although the use of the word "fission" to designate this method is grammatically correct, it is open to question because it is based on the use of a long-established trigonometric equation. The writer objects to the author's use of the word "radian" to define a hyperbolic angle because radians are the only circular functions that are lacking from the close analogy between circular and hyperbolic functions.

The expression $\sinh v = \frac{1}{2}e^v$ is correct[10] to within one decimal place for values of v from 5.09 to 6.908 and to within two decimal places for values of v from 6.908 to 9.210. Therefore Eq. 10 gives precise results when v is larger than 5.09. This equation will give results correct to within 1% when v is larger than 3.00.

As a matter of fact, there is a suitable substitute for tables of large hypes. Every engineer has access to a log-log slide rule or to a four-place table of Naperian logarithms, either of which is sufficient equipment for the calculation of large hypes. The writer will present and demonstrate a method for testing the accuracy of values obtained by interpolating in tables.

Using inverse hyperbolic functions and denoting $\sinh v$ by u, then,

$$\sinh^{-1} u = \log_e (u + \sqrt{u^2 - 1}) \ldots\ldots (24a)$$

When the hyperbolic angle v is large, Eq. $24a$ is approximated by

$$\sinh^{-1} u = \log_e (2\,u) \ldots\ldots (24b)$$

which can be written in the form:

$$v = \log_e 2 \sinh v \ldots\ldots (24c)$$

Let v be a small angle; then, for large angles Eq. $24c$ becomes

$$v \times 10^n = \log_e 2 \sinh v + \log_e 10^n \ldots\ldots (24d)$$

Recalling that $\log_e 10 = 2.302585$ Eq. $24d$ becomes

$$v \times 10^n = \log_e 2 \sinh v + 2.302585 \times n \ldots\ldots (25)$$

Conversely

$$\log_e 2 \sinh v = v \times 10^n - 2.302585 \times n \ldots\ldots (26)$$

Since $\log_e 2 \sinh v$ must be a positive number, it follows that the multiplier of 2.302585 in Eq. 26 must be the largest whole number of which its product

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[9] Designing Engr., Virginia Bridge Co., Roanoke, Va.

[10] *Hütte, Des Ingenieurs Taschenbuch*, Academischen Verein Hütte, Wilhelm Ernst und Sohn, Berlin, Germany, Vol. 1, 1902, p. 37.

2.302585, when subtracted from $v \times 10^n$, will leave a remainder. Since v is either known or to be determined, the multiplier 10^n can be omitted in Eqs. 25 and 26. Using the data from Example 1, a comparison of two solutions of Eq. 26 is as follows:

Description	Naperian log table	Log-log scale of slide rule
$v =$	28.49	28.49
$-\ 12 \times 2.302585 =$	-27.631	-27.63
$\log_e 2 \sinh v =$	0.859	0.86
$2 \sinh 28.49 =$	2.36072×10^{12}	2.360×10^{12}
$\sinh 28.49 =$	$11{,}803.6 \times 10^{8}$	$11{,}800 \times 10^{8}$
Percentage error	0.0023	0.034

Using the data from Example 2, a similar comparison of two solutions of Eq. 25 is as follows:

Description	Naperian log table	Log-log scale of slide rule
$\log_e 1.426 =$	0.3549	0.355
$7 \times 2.302585 =$	$+16.1181$	$+16.118$
$v =$	16.4730	16.473

The foregoing results are the same as those obtained by the author.

There is no need for interpolation in the case of large hypes when Eqs. 12, 25, and 26 are used. Interpolation for large hypes is difficult for the following reasons: Consider an ordinate to the x-axis produced until it intersects the upper and lower branches of an equilateral hyperbola, which has its origin at the center of co-ordinates and its major axis on the x-axis. Draw a line from each point of intersection of the ordinate and of the curve to the origin. The area enclosed between these two radial lines and the curve is defined as the hyperbolic angle v. The hyperbolic sine, $\sinh v$, is equal to the length measured from the x-axis along the ordinate to the upper branch of the curve; and the hyperbolic cosine, $\cosh v$, is equal to the distance measured along the x-axis from the ordinate to the origin. The asymptotes of the hyperbola make an angle of 45° with the x-axis, and the $\sinh v$ and $\cosh v$ are equal when v is infinite. As v increases, the curve approaches the asymptote rapidly until, for successive values of $\cosh v$, the increase in area becomes very small. For very large values of v an infinitesimal increase in v causes the increase in $\sinh v$ and $\cosh v$ to become infinite. Therefore, if tables are to be adaptable to interpolation, the interval between successive tabulated values should diminish as v increases. Since this is not the case in any of the tables described by the author, these tables cannot be used for interpolation between large values of v.

Consider four successive terms, having equal intervals, taken from a table of hypes, and denote the first term by a. Let the successive differences between the four terms be d_1, d'_1, and d''_1; then, to find the value between the first two terms at the $\frac{1}{n}$th of the interval, $n\, d_1$ is added to a. This is the same as direct interpolation by proportional parts. Now let d_2 and d'_2 be the successive differences between d_1, d'_1, and d''_1, and let d_3 be the difference between d_2 and d'_2; a better approximation for the interpolated term T is given by the following:

$$T = a + n\,d_1 + \frac{n}{2}(n-1)\,d_2 + \frac{n}{6}(n-1)(n-2)\,d_3 \ldots\ldots (27)$$

Eq. 27 can be used for interpolation for cases in which v is less than 5.09. For example, let sinh 4.75 be required when the values of sinh for 4.70, 4.80, 4.90, and 5.00 are known. The calculation is performed as follows:

Magnitude	4.70		4.80		4.90		5.00
sinh =	54.969		60.751		67.141		74.203
d_1 =		5.782		6.390		7.062	
d_2 =			0.608		0.672		
d_3 =				0.064			

$$\sinh 4.75 = 54.969 + \frac{5.782}{2} - \frac{0.608}{8} + \frac{0.064}{16} = 57.788,$$ which is the correct value. Interpolation by proportional parts gives a value 54.969 + 2.891 = 57.860, which is 0.128% in error.

When Eq. 26 is applied to determine sinh 28.49 from the tabulated values of sinh 28, sinh 29, sinh 30, and sinh 31, it is found that d_2 is larger than d_1, and d_3 is larger than d_2; thus it is evident that interpolation is not possible. Calculating values of sinh for 28.4, 28.5, 28.6, and 28.7, Eq. 27 gives a value of $11{,}804.6 \times 10^8$ for sinh 28.49, which is correct to within 0.075%. In general it can be stated that interpolation can be made by proportional parts when d_2 approaches zero and that Eq. 27 will give precise results when each difference is about one tenth of the preceeding difference. When the differences approach equality, quantities cannot be interpolated without decreasing the interval.

Summary.—This discussion is offered to demonstrate that it is not necessary to have tables for the purpose of calculating large hypes. The author's method is excellent when one has access to the Hayashi tables.[8] Otherwise the procedure in this discussion will give results sufficiently precise. The writer is of the opinion that Eq. 27 is a much more workable test for interpolation than Eq. 6 and that hyperbolic functions are not indispensable. The most accurate method of computing cubic equations is that proposed by W. G. Horner, which can be used for the solution of quartic equations after the resolvent cubic is obtained. With the theorem of J. C. Sturm used to obtain the nature of the roots, the solution by the Horner method is an elegant application of mathematics. The solution of cubic and quartic equations by radicals, which involve the use of circular and hyperbolic functions, is more expedient than the Horner method and is sufficiently precise. However, when they find that the solution of a problem by a direct method involves hyperbolic functions, many engineers will use one of the longer methods, which can be worked by simple arithmetic, until they find that hyperbolic functions can be computed readily and precisely. The writer, therefore, believes that the paper is timely and important.

K. L. Cooper.[11]—For the solution of cubic and quartic equations, the author has found the need for hyperbolic functions of arguments larger than six radians. From a survey and analysis of available reference tables of natural functions he has concluded that interpolation by proportional parts may lead to excessive error for large arguments, and, therefore, that either more comprehensive tables or a better method must be sought.

More comprehensive tables are available.[12] The tables prepared by L. J. Comrie make it possible to obtain the values for the author's examples by proportional parts, although this is not necessarily either the best or the shortest method.

His examples will serve to illustrate the use of these tables.

Example 1.—To compute sinh 28.49, from the Comrie tables find $e^{28.4} = 2.157562 \times 10^{12}$ and $e^{28.5} = 2.384475 \times 10^{12}$. Hence, sinh 28.49 = $\frac{1}{2} \times 2.3618 \times 10^{12}$ or 1.1809×10^{12}.

Example 2.—To compute arcosh $(713 \times 10^4) = y$, we have $2 \cosh y = e^y = 1{,}426 \times 10^4$. Then, $e^{16.5} = 1.465072 \times 10^7$ and $e^{16.4} = 1.325652 \times 10^7$. Hence $y = 16.472$.

In the explanation to his tables, Mr. Comrie says[13]

> "* * * there are several ways in which values for non-tabular arguments may be obtained. Of these the best is probably that in which x is split into two parts a (a tabular argument) and b, so that $e^x = e^{a+b} = e^a \times e^b$."

This is the method to which the author draws attention as being somewhat less laborious and much more accurate than the use of proportional parts. It is not necessary, however, that a should be large and b should be small, as the author suggests, but only that b should fall into such tables as give sufficient decimal places. Thus, when the argument x is even, it will often be possible to reduce the author's two tabular references to one, and, when it is odd, it will be possible to reduce them to one entry for two adjacent values. For the inverse operation the given value is split into two equal parts by taking the square root. This procedure is illustrated by the same examples:

Example 3.—If $a = 14.25$ and $b = 14.24$ (both listed in Comrie's tables), then sinh 28.49 = $\frac{1}{2} \times 1.544 \times 1.529 \times 10^{12} = 1.180 \times 10^{12}$.

Example 4.—If $a = b = (2 \times 713 \times 10^4)^{\frac{1}{2}} = 37.8 \times 10^2$ from tables $e^{8.23} = 3{,}752$ and $e^{8.24} = 3{,}790$. Hence $y = 2 \times 8.237 = 16.47$.

It should not be overlooked that repeated fission will make it possible to obtain the values of the hyperbolic functions of large arguments from tables that are restricted to small arguments.

Example 5.—Sinh 28.49 = $\frac{1}{2}\,(e^6)^4 \times e^{2.25} \times e^{2.24}$ or $(e^{4.07})^7$.

Although the author has emphasized the use of natural functions, his problem may be solved without them. If the required accuracy is 1%, the following simple approximations are satisfactory for values of x from 0 to 3:

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[12] "Chambers's Six-Figure Mathematical Tables," by L. J. Comrie, W. & R. Chambers, Ltd., London, England, Vol. II, 1949.

[13] *Ibid.*, p. xvi.

$$\frac{\sinh x}{x} = 1 + \frac{0.947\,x^2}{6.65 - x} \qquad (28a)$$

$$\cosh x = 1 + \frac{2.23\,x^2}{5.2 - x} \qquad (28b)$$

These expressions combined will also give tanh x to the same degree of accuracy. At $x = 3$ the error involved in taking $\sinh x = \cosh x = \frac{1}{2} e^x$ is 0.25%, and in taking $\tanh x = 1$ the error is 0.5%. For values of x larger than 3, logarithms can be used to evaluate $\frac{1}{2} e^x$.

Example 6.—To compute $x = \sinh 28.49 = \frac{1}{2} e^{28.49}$ we have $\ln 2\,x = 28.49$. From an elementary table of logarithms $\ln 10^{12} = 27.6310$ and $\ln 2.361 = 0.8590$. Therefore, $x = \frac{1}{2} \times 2.361 \times 10^{12}$ or 1.180×10^{12}. The same solution will be obtained using common logarithms: $\log 2\,x = 28.49 \times 0.4343 = 12.3732$. Therefore $x = \frac{1}{2} \times 2.361 \times 10^{12}$.

For the inverse operation logarithms can be used for any values because

$$\operatorname{arsinh} x = \ln (x + \sqrt{x^2 + 1}) \qquad (29a)$$

$$\operatorname{arcosh} x = \ln (x \pm \sqrt{x^2 - 1}) \qquad (29b)$$

For $x = 10$, the errors involved in the approximations $\operatorname{arsinh} x = \operatorname{arcosh} x = \ln 2\,x$ are about 0.25%.

Example 7.—$y = \operatorname{arcosh} (713 \times 10^4) = \ln (2 \times 713 \times 10^4) = 2.3026 (\log 1{,}426 \times 10^4) = 16.47$. This value may be either positive or negative—a point not brought out in the paper.

A slide rule and an elementary table of logarithms suffice to solve the author's problem. With the additional aid of the writer's two simple approximations, any values of the hyperbolic functions or the inverse hyperbolic functions can be obtained to an accuracy better than 1%.

F. T. LLEWELLYN,[14] M. ASCE.—In appraising the efficiency of any mathematical formula it is helpful to consider its two phases, namely, solution and computation. The solution phase deals with algebraic forms of expression, whereas the computation phase comprises the finding of numerical values therefrom. It will be found, then, that the most "elegant" solution may not, and very often does not, lead to the most efficient method of computation.

Furthermore, in the field at issue, the author affirms that the practical need is not for additional research by mathematicians, for they have provided ample apparatus to choose from. Rather, the desideratum is to select and to standardize the fewest possible lines of procedure, so that they embrace the entire field and are arranged so that they can be readily applied without repeated reference to the theorems of advanced mathematics. This was the approach attempted in the paper, and upon this approach the writer's closure will be based.

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[14] Baton Rouge, La.

The $y^{f(n)}$ type of series, to which the correction $(1 + n)$ pertains, is useful in those rare cases in which $y < 0.0001$, which is the smallest radian usually tabulated. The writer recently encountered two problems (one in industry and one in research) wherein such meticulosity was desirable. In those problems the series converged so rapidly that the third term (where $n = 2$) would have affected only the 19th decimal and, therefore, was disregarded. Thus, provided $y < 0.0001$, four partial expressions of this type are correct to at least 18 places, as follows:—

$$\cosh (y) \doteq 1 + \frac{y^2}{2} \qquad \text{(30a)}$$

$$\cos (y) \doteq 1 - \frac{y^2}{2} \qquad \text{(30b)}$$

$$\sinh (y) \doteq y + \frac{y^3}{6} \qquad \text{(30c)}$$

$$\sin (y) \doteq y - \frac{y^3}{6} \qquad \text{(30d)}$$

The complete expressions may be found in the Hayashi[15] tables.

In the case of larger radians, the several series converge more slowly, and many additional terms may be required until the labor of computation becomes prohibitive. For this reason, rather than the one suggested in the paper, convergent series of the $y^{f(n)}$ type are not suited to general use.

Another brief note, that may clarify Table 1, is the fact that all the values of R_E are expressible also in terms of the square of the pertinent interval. Thus, Cols. 1, 4, and 6 could read as follows:

Group	Interval v	Maximum error R_E (%)
1	0.0001	12.5 v^2
2	0.001	12.6 v^2
3	0.01	12.7 v^2
4	0.1	12.9 v^2
5	1	13.1 v^2
6	5	41.0 v^2
7	10	809.0 v^2

In the matter of terminology Mr. Sherman objects to the writer's use of the words fission and radian. Apparently Mr. Sherman (but not Mr. Cooper) accepts the abbreviations "hype" and "trig." The following explanation will show that the terms were not chosen idly.

The word "fission" was adopted as being more terse than the term "trigonometric equation" (which Mr. Sherman seems to favor), or than the term "addition theorem" (as used by certain other writers). "Fission" was preferred to "splitting," because it implies also a re-combination; it is shorter, and it dignifies the title of the paper.

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[15] "Fünfstellige Tafeln der Kreis, und Hyperbelfunktionen," by K. Hayashi, Walter de Gruyter and Co., Berlin, Germany, 1921, pp. 178-179.

The writer anticipated some adverse comment on the secondary meaning which he ventured to assign to the word "radian." He would point out, however, that many technical words are currently used in more than one sense. Thus, the word "natural" denotes Naperian as opposed to Briggs logarithms, and it also distinguishes numerical from logarithmic functions. Such words as "circular," "characteristic," and "argument" are also used in two senses. These double usages cause no confusion provided the context shows which meaning is intended. If the word "argument" had been substituted for "radian," it would have been necessary, in repeated instances, to add the phrase "in circular measure." The writer sought to avoid this needless verbiage by defining the two meanings of "radian" early in the paper.

To reciprocate, the writer would question Mr. Sherman's unqualified use of the word "interpolation" in cases in which he apparently means interpolation by PP. This is not a captious criticism because two methods of interpolation are in question. For small radians (in which $y < 6$, and the standard PP method is suitable), the corresponding hypes advance (within $13 \times 10^{-4}\%$, or less) in arithmetical progression with the adjacent entries. For large radians (in which $y > 6$, and a fission method is suitable), the corresponding hypes advance (within $6 \times 10^{-4}\%$, or less) in geometrical progression with the adjacent powers of e.

Based on his reference to "Hütte,"[10] Mr. Sherman finds two radian zones, extending from 5.09 to 9.21, within which the author's Eq. 10 would be correct to stated decimal places. This concept of overlapping zones is undoubtedly the more accurate, but, as stated in the paper, transition at ($x = 6$) to distinguish large from small radians, was a practical concession to the grouping and scope of current tables.

The writer has not had occasion to use Eq. 27. However, he agrees that, for small radians, it should give very close results, but its application must be quite laborious and its need almost nil, except perhaps in computing reference tables. By contrast, the author's two simple methods (PP when $y < 6$, and fission when $y > 6$) are sufficiently precise for the entire range of most structural problems.

In his summary, Mr. Sherman recommends the "indirect" Horner and Sturm theorems as accurate aids in computing cubic and quartic equations. But, in structural problems having cumbersome coefficients, the application of these "elegant" theorems would be much too laborious (the Sturm theorem especially) to suit the routine of busy designers. Moreover, each operation gives only one real root that may, or may not, be the root sought.

On the other hand, the "direct" methods cited in the paper (Cardan[2] for cubics, and preferably Euler[4] for quartics) give all the roots at a single operation. When these methods are used, labor is also reduced to a surprising extent by the adoption of standardized work-forms. A fuller discussion on the various methods available for handling cubics and quartics would fall outside the purview of the paper.

Mr. Cooper's comments disclose the global reach of ASCE activities. Thus, the first hint to the writer as to the existence of the new Comrie tables[13] (in an English publication) has come via the antipodes. These tables are dated 1949, the year in which this paper was submitted to the ASCE. Had they been available at the time it was written, their reference would have formed a valuable addition to Table. 3.

It is no coincidence that, although working independently, Comrie used the same terms[14] as the present author in describing the splitting operation, because the basis for that operation had long been known. The writer has not seen a copy of the Comrie tables, and therefore he is not sure as to their spacing, but Mr. Cooper's examples imply that $v = 0.01$ at $x \doteq 14$, and that $v = 0.1$ at $x \doteq 28$, both intervals being closer than the spacing ($v = 1$) for the author's group 5 in either the Smithsonian or the Hayashi sets of tables. Thus, the Comrie ratios of error would here reduce to those of groups 3 or 4 (see Table 1, Cols. 1, 4, and 6), thereby affording still closer precision in the handling of large hypes.

It is to be noted, however, that Mr. Cooper's Examples 1 and 2, which were computed from the Comrie tables, entail major PP interpolation, whose scratch work he does not transcribe. The addition of this extra work would make the total labor somewhat greater than that of the author's corresponding examples, which were computed by fission at a single step, with only one easy mental interpolation in Example 2.

The possibility of computing large hypes by "repeated fission" (see Mr. Cooper's Example 5) was realized by the writer, but this method was not presented in the paper because it would be either more laborious or else less precise than the single and simple rule suggested in the section on "fission method."

The writer's more general reaction to Mr. Sherman's Eqs. 24 to 27, and to Mr. Cooper's Eqs. 28 to 29, may be combined. These contributors together present a total of six stimulating, but occasionally abstruse, formulas for computing hypes and for determining "individual" degrees of precision. Their results check quite well with those of the paper. Some of these formulas are new to the writer and others could be cited. This approach differs essentially from that of the paper, which was to standardize only two simple methods, one for large and one for small hypes (with a handy transition at $x = 6$), and to predetermine their limits of error "by groups." The author believes that the latter procedure, while perhaps less academic, is more helpful in the kind of problems encountered by structural engineers.

Both discussers envisage a free use of logs, which the author aimed to avoid in this field, primarily on account of the labor their use entails. In high precision work there is a further drawback to the use of log logs, in that the number of correct places in their mantissas is reduced by the maximum number of digits in the characteristic of the associated logs. This loss may become significant in cases in which the values of very large hypes are sought.

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HEALTH IMPLICATIONS OF AIR POLLUTION

By J. J. Bloomfield[1]

Synopsis

Although an old problem, air pollution has been the subject of increasing research because of the realization of its health implications. Studies have linked air pollution with loss of well-being or illness and have further shown that specific topographical and meteorological conditions may result in fatal concentrations of air-borne poisons. Reduction of the pollution load is not always economically feasible with available devices, but research and restriction should aid in the solution of the problem.

Industry as well as government must take steps to clean the atmosphere. The engineer in industry will play an important role in reducing the pollution load. On the governmental side, agencies already in existence have personnel which are qualified to assume responsiblity for this problem.

Introduction

The problem of atmospheric pollution is old, but the health implications have been the object of increasing scientific and lay concern throughout the United States. The first historical concern with air pollution was recorded as early as 1257, when Eleanor, the queen of Henry III of England, was reputed to have left the town of Nottingham because she was bothered by smoke. Legal cognizance of this nuisance came in 1306, when a royal proclamation was issued prohibiting the burning of coal in London. Following these early efforts to abate the smoke nuisance, England has conducted systematic studies[2,3,4,5,6,7,8,9,10,11] on atmospheric pollution since 1914.

Note.—Published in June, 1951, as *Proceedings-Separate No. 73*. Positions and titles given are those in effect when the paper was received for publication.

[1] Asst. Chf., Div. of Industrial Hygiene, U. S. Public Health Service, Washington, D. C.

[2] "Atmospheric Pollution in Leicester, A Scientific Survey," *Technical Paper No. 1*, Dept. of Scientific and Industrial Research, Atmospheric Pollution Research, His Majesty's Stationery Office, London, England, 1945.

[3] "Heat and Power Linkage," *Fuel Efficiency Bulletin No. 51*, Ministry of Fuel and Power, His Majesty's Stationery Office, London, England, August, 1948.

[4] A Report of the Joint Conference of the Institute of Fuel and the National Smoke Abatement Society, London, England.

Contemporary research in the United States has kept pace with that abroad. During the period from 1911 to 1914 the first study of all phases of the smoke problem was undertaken by the Mellon Institute of Pittsburgh, Pa.[12] Studies in Salt Lake City, Utah, and Grafton, West Va., followed in 1919. From 1926 to 1929, Baltimore, Md., Washington, D.C., Pittsburgh, New York, N.Y., Boston, Mass., Cleveland, Ohio, Columbus, Ohio, and other cities were the subject of investigations. Then, from 1931 through 1933, the United States Public Health Service (USPHS) undertook a survey of fourteen of the largest cities in the United States. The famous smoke-control program of St. Louis, Mo., in 1939, typified the smoke-abatement measures inaugurated in a number of other industrial cities.

Despite this early concern with air pollution, however, the full realization of the health implications inherent in the problem were not yet fully apparent. Understanding of the nature of air pollution has antedated this realization by less than a score of years. Only since about 1930 has it been possible to put the problem under a microscope to analyze its component parts. It is known that air pollution embodies more than the smoke aspect, which was recognized first, and consideration must also be given to the dusts, fumes, vapors, mists, and other contaminants that are emitted from industrial plants. The latter make up, by far, the more complicated phase of the problem because they do not lend themselves to a uniform solution.

The control of smoke is well understood. Smoke-abatement measures are based generally on the use of only nonvolatile or low volatile fuels or on the use of specially constructed furnaces that assure complete combustion. These smoke-abatement measures do not solve the entire polution problem, however. To rid the atmosphere of the contaminants spewed forth by countless industrial operations calls for a number of varied control methods. The spelters of zinc plants alone emit such particulate matter as zinc, lead, cadmium, sulfur, and chlorine, as well as gaseous substances like sulfur dioxide, carbon monoxide, and carbon dioxide. Considering the vast number of industries using raw materials and intermediate products, which produce great numbers of contaminants, as well as the millions of homes and transportation vehicles that contribute to air pollution, realization of the magnitude of the problem cannot be escaped.

Unless action is taken, an intensification of this problem can be expected. As new industries are developed, it is imperative that proper precautions be taken to control the resulting pollutants.

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[5] *Proceedings*, National Smoke Abatement Society, Chandos House, Buckingham Gate, London, England.

[6] "The Investigation of Atmospheric Pollution—A Report on Observations in the 5 Years Ended 31st March, 1944," His Majesty's Stationery Office, London, England, 1949.

[7] "The Weathering of Natural Building Stones," by R. J. Schaffer, *Building Research Special Report No. 18*, Dept. of Scientific and Industrial Research, His Majesty's Stationery Office, London, England, 1932.

[8] "The Smoke Problem of Great Cities," by Sir Napier Shaw and John Switzer Owens, Constable & Co., Ltd., London, England, 1925.

[9] "Smoke, a Study of Town Air," by Julius B. Cohen and Arthur G. Ruston, Edward Arnold & Co., London, England, 1925.

[10] "Smoke and Smoke Prevention," by Margaret Fishenden, *Encyclopaedia Britannica*, Vol. 20, 14th Ed., 1929, p. 839.

[11] "Dust," by S. Cyril Blacktin, Chapman & Hall, Ltd., London, England, 1934.

[12] *Bulletin Nos. 1–9*, Smoke Investigations by Mellon Institute of Industrial Research and School of Specific Industries, Univ. of Pittsburgh, Pittsburgh, Pa.

Anticipated increases in population also present difficulty, since people, as well as contaminants and certain meteorologic conditions, are factors of the air pollution problem. The congestion of population and concentration of industries are reflected in voluminous literature describing the economic and nuisance effects of air pollution. In addition to these considerations has come the awareness (and whether or not it is justified in every instance remains to be proved) that air pollution is affecting the people's sense of well-being. Air pollution has passed the nuisance stage and has more than economic and esthetic implications.

EFFECTS OF AIR POLLUTION

Much speculation has been voiced over the effect of air pollution on the course of tuberculosis, its deleterious effect in shutting out the germicidal rays of the sun, and other theories; but the case does not rest on conjecture, speculation, and theory. The first major evidence of the health implication of air pollution came in 1930, when the Meuse Valley in Belgium was blanketed by a heavy smog and sixty persons lost their lives. Prolonged stable weather conditions had bottled up the contaminants in the valley for about four days, and the concentration of chemical fumes increased until it reached fatal proportions.

In October, 1948, similar meteorologic conditions were encountered in Donora, Pa., where almost 6,000 persons were affected, twenty of them fatally. In both instances, the affected communities, located in bowl-shaped valleys where contaminants could be easily confined, experienced a meteorological phenomenon known as an inversion. At such times, the temperature of the air at gound level is less than that of the air above, whereas, normally, the reverse is true. This stable condition minimizes the vertical mixing of the air that normally takes place, and the absence of strong wind currents keeps horizontal dispersion at a minimum. In Donora these conditions lasted four and one half days. Had they lasted only a few hours or a day or so, the results, of course, would have been different. Short periods of smog are not uncommon and present no danger of wholesale sickness or death. It is the prolonged smog that may have a disastrous aftermath.

Although the term "smog" originally meant a combination of smoke and fog, its connotation now includes all kinds of fumes, gases, and other emissions, in addition to particulate matter. The duration of a smog is dependent on meteorological conditions. Ordinarily, a wind or cleansing rain will come to sweep away the contaminants before they reach dangerous concentrations, but since weather can be "a bad actor," common sense dictates preparedness for any eventuality.

Aside from these dramatic, acute episodes, periodically recurring smogs with disagreeable effects occur in many areas. In Los Angeles, Calif., for instance, about ten to twenty times each year smogs cause residents to suffer severe lachrymation as well as irritation of the nasal passages. The Los Angeles situation affords one of the best examples of the distinction between smoke and industrial emissions in the etiology of air pollution. The City of

Los Angeles burns oil and gas and, therefore, has no smoke problem as such; yet it has a genuine air pollution problem.

More serious effects of contaminants on health may be illustrated by findings of investigations of plant-working environment, which can be more closely controlled than that of the general atmosphere. Lung disease has been shown to occur among employees exposed to beryllium compounds.[13, 14, 15] In some areas, it has been alleged that people living near beryllium plants have been affected. Investigations have also emphasized the occurrence of pneumonia in manganese plant areas.

To understand the health implications of air pollution, the Donora incident should be studied in more detail.[16] This was investigated for one year by the Industrial Hygiene Division of the USPHS. The salient points in the biological data are the incidence and severity of illness during the smog, the main clinical manifestations, and the significant relationships between the affection and other factors.

Forty-three percent of the population was affected, ranging from slight to extremely severe affection.[17] There were twenty fatalities. The affection was essentially an irritation of the respiratory tract and other exposed mucous membranes. Particularly noteworthy was the direct relationship of both incidence and severity with increasing age. This observation has much meaning because of increasing interest in the science of geriatrics and concern with any influences to which the elderly may be particularly susceptible. In searching for indications of pre-existing ailments it was considered highly significant that the fatalities had histories of previous cardiorespiratory disease.

THE PROBLEM OF AIR POLLUTION

Although these findings present a substantial body of data, they cast a long shadow of unanswered questions. The mass of accumulated data has succeeded in emphasizing just how few conclusions can be drawn. The acute health effects of air pollution have been verified, but no light has been shed on the long-range effects. The effects of acute exposure to concentrated contaminants under unusual meteorologic conditions are known, but the manner in which people's health is affected by continuous exposure to the usual concentrations under normal weather conditions year in and year out must be learned.

In addition to determining the chronic effects on healthy people, a special study must be made of the influence of air pollution on cardiacs, asthmatics, and people with other respiratory diseases. As yet, it is unknown whether air pollution only aggravates or also causes the onset of respiratory diseases.

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[13] "The Toxicology of Beryllium," by F. Hyslop, E. D. Palmes, W. C. Alford, A. R. Monaco, and L. T. Fairhall, *Bulletin No. 181*, National Inst. of Health, U. S. Govt. Printing Office, Washington, D. C., 1943.

[14] "Acute Pneumonitis and Pulmonary Granulomatosis of Beryllium Workers," by Willard Machle, Emil Beyer, and Frederick Gregorius, *Occupational Medicine*, Vol. 5, 1948, pp. 671–683.

[15] "Delayed Chemical Pneumonitis Occurring in Workers Exposed to Beryllium Compounds," by Harriet L. Hardy and Irving R. Tabershaw, *Journal of Industrial Hygiene and Toxicology*, Baltimore, Md., September, 1946, pp. 197–211.

[16] "Air Pollution in Donora, Pa., Epidemiology of Unusual Smog Episode of October, 1948," *Public Health Bulletin No. 306*, U. S. Public Health Service, Washington, D. C., 1949.

[17] *Ibid.*, p. 23.

The effect of damage to the respiratory system on the cardiovascular system must also be evaluated. The possible special effects of air pollution on children and on elderly people must also be ascertained. Does the loss of sunlight lessen a child's resistance to disease? What is the effect on the aged? Science's conquest of communicable diseases has enabled people to live to more advanced years, and, since their life span has been increased, the effect of air pollution on this group must be considered to ensure that it is not a contributing factor to shortening their lives.

As one step toward answering all these questions, basic research is requisite to determine the concentration of substances necessary to produce either acute or chronic effects. It also must be learned whether the air contaminants from industry—gases, fumes, and particulate matter—always act singly on human beings, act in combination, or act on each other to produce an agent which is more poisonous than any of the single substances or than the total action in combination.

Values known as maximum allowable concentrations have been established for industrial environments where normal persons (excluding children and the aged) are exposed to in-plant contamination for 8 hours each working day. Comparable information is not available on contaminants in the outside atmosphere affecting individuals of all ages and in all physiological and pathological conditions over an uninterupted period, day and night.

Authorities who know the limitations must exercise care in encouraging air pollution ordinances. The smoke-abatement regulations in force in many cities, such as New York, are a step in the right direction, but legal standards for the control of other emissions cannot be developed until a more thorough study of the problem has been made. However, industry should take steps to correct their aspect of the situation, which, obviously, cannot be done overnight. Therefore, a pattern of action combining patience with vigilance, and, of course, a continuous program of research is indicated.

At the same time the engineer has the challenging task of finding new and more effective control methods. Although workable in most instances, available methods of control sometimes are either inadequate to perform the job of trapping contaminants, or they are too costly. To illustrate the economic impracticality encountered at times, it is conceivable that a small plant with a capital investment of $50,000 would be required, by an over-stringent law, to spend more than its total assets to reduce atmospheric pollution with techniques in current use. Obviously, under such circumstances, a company would be forced to close its doors.

In the absence of suitable controls, plants may be required to move outside the city limits. This solution, of course, leaves much to be desired for general application. It might be the answer in limited instances if stricter and better city planning were encouraged, and if industrial zones were established in areas where the factor of air pollution has been given serious consideration. Until rigid zoning laws are enacted, a company moving to an isolated location cannot assume that it has complete freedom to pollute the atmosphere and that it, therefore, need not install proper control measures. All too frequently it has been demonstrated that domestic housing is built in the vicinity of a plant. Then, regardless of the fact that the dwellings were the last to move to that site, the company is forced to take corrective steps by the pressure of public opinion. The wiser course of action would be for the company to install proper safeguards at the outset. In a true exercise of foresight in selecting a site for new operations, industry should give due consideration to the meteorologic conditions of a location as well as to transportation costs, availability of raw materials and personnel, and other factors.

Remedial Steps

By using available scientific techniques and apparatus, industry, housing, transportation, and other sources of contamination can reduce their air pollution load, despite the gaps in existing knowledge in this field. Air contamination can be readily evaluated by a combination of air sampling with specifically designed scientific equipment and subsequent laboratory analyses. This procedure is followed on a quantitative scale, enabling determination as to the concentrations of contaminants that may be present. The Donora study employed these techniques, and they will also be used in other surveys. As a result of the increasing interest in air pollution, new scientific methods and instruments are constantly being evolved. For instance, a "recorder" has been developed commercially for the continuous determination of sulfur compounds in the atmosphere. The electron microscope has facilitated the identification and classification of particulate matter. New recording instruments will determine the halogens and the chlorinated hydrocarbons in the atmosphere. In the laboratory, special equipment has been set up to indicate the reduction of ultraviolet rays and total sunlight when minute amounts of various contaminants are present in the atmosphere. Exposure chambers are being used to determine the effects of exposure to various concentrations of contaminants on human beings.

On the control side, in addition to smoke-control ordinances, emphasis must be placed on the need to control industrial emissions that do not lend themselves to that type of regulation. Here, too, various control techniques may be used effectively. The principle of dilution, using high smokestacks (some of which tower over 500 ft) may be adopted; or the answer may lie in conversion—as, for example, the complete combustion of gases to combat objectionable odors—or in collection. The latter technique utilizes a variety of principles and apparatus, such as ultrasonics, electrostatic precipitators, steam precipitators, scrubbers, mechanical filters, centrifugal separators, and gravity chambers. This variety of available measures ensures that, by and large, industrial emissions can be controlled, although better tools may have to be developed in certain instances.

To illustrate this point, the zinc plant in Donora (one of the chief contributors to the pollution load in that area) has already gone forward in the control of the operations that have been mainly responsible for contaminating the atmosphere. Furthermore, in order to be alerted to any impending adverse weather conditions in time to take further precautions, the company installed a teletype machine to receive daily reports of the United States Weather Bureau. All these actions will help to ensure that there is no repetition of the Donora crisis of 1948.

COURSES OF ACTION

There is a compelling urgency for further developmental and research activity because events have proved that air pollution is not a theory that can be shelved for a later, less busy day.

Since the intensive year-long investigation at Donora, the USPHS has been assisting the states on a consultative basis. Air pollution studies should be extended to give greater aid to the states with their problems. This activity should be in addition to, and of a different nature than, current smoke-abatement efforts, which must be continued.

In increasing numbers, industrial hygienists and sanitary engineers in state and local official agencies are concerning themselves with air pollution. Such specialists are the best qualified to assume this responsibility because control of atmospheric contaminants is only one step removed from their traditional function of controlling the working environment. By virtue of their position in health departments, they are also able to bring all the state's health resources to bear on this problem.

While official agencies are engaged in these studies, industry must work on similar projects simultaneously. There is an obligation on all industry to conduct diligent research into the status of its own problem. Each industry should study thoroughly its particular operations to determine what processes emit pollutants into the air as well as the nature and quantity of the contaminants. On this groundwork of data, industry can then base its research efforts to control the degree of pollution.

It must be realized that the operators of many plants began on such projects years ago, and they are to be highly commended. Others, however, are hiding behind the excuse that air pollution is either too vague or too overwhelming a problem.

SUMMARY

Just as the people were shocked out of their apathy by the sad pollution of the waterways when that pollution reached such alarming proportions, so, too, they must realize that the time has come when industries can no longer spew contaminants into the air as a catch-all dilutant. One tragic example has already shown what may happen when weather conditions become unusual. Additional research may also discover insidious effects of everyday exposure to contaminants which may exact a toll on the vitality and well-being of the population. Although the extent of the problem is not known, the danger can be recognized.

The responsibility for the control of air pollution rests in large part with engineers, and the public will lean heavily on their efforts. No other problem confronting the engineering profession demands more ingenuity or the taxing of all resources to cope with than this responsibility for the health of the nation.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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TRANSACTIONS

Paper No. 2485

INTERNATIONAL CARTOGRAPHY

By Robert H. Randall,[1] M. ASCE

With Discussion by Mr. Milton O. Schmidt

Synopsis

Modern transportation and communication facilities have made it imperative to secure cartographic cooperation on a much wider basis than has been necessary in the past. Attempts to secure such cooperation among the Pan American nations and among the other nations of the world are described.

The purposes, methods of operation, and accomplishments of several organizations engaged in international cartographic work are given, as well as details of the United Nations participation in the planning.

Introduction

The term "cartography" represents the science of preparing all types of maps and charts and is a term which includes every operation from original surveys to final printing of map copies. The types of maps and charts included are: (1) Topographic maps; (2) geologic maps, soil maps, vegetation maps, cadastral maps, hydrologic maps, hydrographic charts and aeronautical charts (all prepared on a topographic map base); (3) office-compiled maps showing the location, extent, and character of physical, economic, and social phenomena.

In the United States the term "surveying and mapping" has been used in the past to encompass the broad fields of activity covered by the foregoing definition. Difficulty has been encountered, however, in attempting to translate that expression into foreign languages and have it express all that it is intended to mean. For example, in the Spanish language there are no words that truly express the meaning of the term "surveying and mapping" as it is used in the United States. The term "cartography," on the other hand, is easily translated into that language, carrying the full meaning of the broad field of surveying and mapping. Also, it is the term which is generally accepted in international circles and which is being recognized in the United States as interest in world affairs increases.

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During and since World War II the people in the United States have become more and more internationally minded. Their interest in map and chart coverage has grown from a primary concern with the area within their own borders to a world-wide interest. Their responsibilities in cooperating with other countries have required them to be more deeply concerned than ever before with the mapping of foreign areas. Now that modern transportation and communication facilities have brought all people on the planet within gun-shot of one another, reducing the world in a real sense to a small community, the need is for increased geographic knowledge. Basic maps and charts can supply this knowledge so that neighbors may be able to know each other and their environment and institutions in order to work together for the common good.

Surveying and Mapping in the Americas

Pan American Institute of Geography and History.—In 1940 government officials and scientific organizations of the United States realized that closer cooperation and better understanding was needed among the nations of this hemisphere in accomplishing their surveying and mapping programs.[2] Accordingly, a group of such officials and representatives of scientific societies drew up a description of the kind of organization which they believed could accomplish the desired results. After this had been done, a study was made to determine whether or not such an organization was in existence. The study disclosed that the Pan American Institute of Geography and History was equipped by its charter and by-laws to sponsor a program of cartographic cooperation. The Institute is an intergovernmental organization (with headquarters in Mexico), which regularly holds general assemblies in which representatives of the member American republics meet to discuss common problems. At its Third General Assembly, held in Lima, Peru, in March-April, 1941, the United States delegate presented a resolution proposing the establishment of a commission on cartography. This resolution was adopted, thereby creating the Institute's present Commission on Cartography.

In general the aims of the Commission on Cartography are as follows:

a. To promulgate standards for the various classes of surveys and maps needed and produced by the various nations;

b. To exchange information between the nations on current mapping activities;

c. To exchange ideas and information on technical methods currently employed and those in the experimental or developmental stages;

d. To advance education and training in the science of map-making;

e. To exchange professors and instructors and to exchange technical personnel between mapping agencies in the various nations for in-service training.

The Commission has held five meetings, called "Consultations." The first of these meetings was held in Washington, D.C., in 1943, the second in Rio de Janeiro, Brazil, in 1944, the third in Caracas, Venezuela, in 1946, the fourth in Buenos Aires, Argentina, in 1948, and the last in Santiago, Chile, in

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[2] "Cartographers Consider International as Well as National Mapping Needs," *Civil Engineering*, Vol. 19, November, 1949, pp. 30–31.

1950. The following technical committees and subcommittees have been created to activate the work of the Commission: Geodesy (with a subcommittee on gravity and geomagnetism and a subcommittee on seismology); Topographic Maps and Aerophotogrammetry; Aeronautical Charts; Hydrography (with a subcommittee on tides and a subcommittee on fluvial hydrography); Special Maps; and Urban Surveys.

Some of the accomplishments of the Commission are: (1) The adoption of uniform standards for geodetic operations; (2) standard specifications for topographic maps; (3) standard specifications for the production of the 1:1,000,000 scale aeronautical charts; (4) standardization of methods for geomagnetic observations and tidal work; (5) production of a glossary of standard cartographic terms; (6) publication of a manual on geodetic control surveys; (7) production, in cooperation with United States governmental agencies, of narrative and training films on cartographic subjects, such as "Introduction to Cartographic Activities in the United States," "Geodesy," "Reconnaissance Mapping by Trimetrogon Photography," and "Topographic Mapping by Photogrammetric Methods." These films, besides being circulated in the Americas, have attracted attention in other parts of the world, and copies have been made for a number of foreign nations.

Through the efforts of the Commission on Cartography, considerable progress has been made in the promotion of inter-American friendship, in the advancement of techniques, and in an accelerated program of map and chart production.

Inter-American Geodetic Survey.—The Inter-American Geodetic Survey was originally sponsored by and is actively supported by the Commission. This survey, operated by the Caribbean Command of the United States Department of National Defense, is engaged in a program of establishing a network of geodetic control and precise leveling linking the nations of Central and South America with the United States and Canada. The United States government has entered into bilateral cooperative agreements with practically all of the Latin American nations. By virtue of these agreements, the personnel and facilities of the participating governments are used jointly to promote the program. It is planned that, upon completion of the triangulation and the leveling, a comprehensive program of topographic mapping and aeronautical chart revision and maintenance will be undertaken.

INTERNATIONAL CARTOGRAPHY

General Aspects.—With respect to international cartographic activities, there are existing international organizations concerned with a specific aspect of cartography, such as the International Hydrographic Bureau (IHB) and the International Civil Aviation Organization (ICAO). Both of these organizations are governmental in character. For some time, IHB has been engaged in establishing throughout the world the same kind of program in the field of hydrography that the Commission on Cartography has been conducting in the total field of surveying and mapping in the Americas. This agency has effected a continuing exchange of information about new charting activities, and considerable progress has been made in standardizing hydrographic chart construction and symbols. The ICAO has been successful in creating a firm desire by most nations to prepare aeronautical charts of their territories along standard lines and in reaching agreement for the standardization of the various types of aeronautical charts required by civil aviation. Other organizations of considerable international importance but of professional rather than governmental character are the International Union of Geodesy and Geophysics (IUGG) and the International Geographic Union (IGU), which for some time have promoted close cooperation among geodesists and among geographers throughout the world.

The IGU was instrumental in establishing the Central Bureau of the International Millionth Map of the World. The Bureau is responsible for coordinating the production of this world series of maps at the scale of 1:1,000,000.

United Nations Cartography.—Late in 1945, a group of cartographers, broadly representative of the cartographic agencies and interests of the United States, met on several occasions to consider some of the problems relating to the cartographic services which might be required by the United Nations (UN) and its specialized agencies. It was recognized that, with the development of the work of the various UN organs, commissions, and specialized agencies, requirements for map services for current operations would become of increasing importance. It was the opinion of this group that attention should be given to the development and coordination of cartographic services in the UN in order that effective results might be obtained without duplication of effort and at minimum expense. As a result of the recommendations made by this group of cartographers, the United States delegation submitted a proposal on January 22, 1947, suggesting that the Economic and Social Council examine the problem of coordinating the cartographic services of the UN and its specialized agencies.

Shortly thereafter, the Pan American Institute of Geography and History and its Commission on Cartography, in a letter to the Secretary General of the UN, endorsed the United States proposal and suggested also that consideration be given to the matter of stimulating the national cartographic progress of the respective member states of the UN. The Institute further suggested that the Secretary General call together a group of expert consultants, selected on the basis of individual professional competence, to consult with him and representatives of the specialized agencies about integrating cartographic activities within the UN and about stimulating and coordinating international cooperation among the member nations. About the same time the French delegation proposed that steps be taken to insure better international cooperation in the matter of hydrography and oceanography. This action, in turn, was followed by a communication from the IGU, suggesting the expediting of the survey of the world and the unifying of the cartographic methods and processes.

In January, 1948, the delegation from Brazil presented to the Economic and Social Council of the UN a draft resolution to implement the foregoing suggestions. At the February 19, 1948, session the Council adopted the Brazilian draft. The essential part of the resolution is as follows:[3]

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[3] "Modern Cartography," United Nations Dept. of Social Affairs, Lake Success, N.Y., 1949.

> "The Economic and Social Council * * * recommends:
>
> 1. That the Member Governments stimulate the accurate survey and mapping of their national territories;
>
> 2. That the Secretary-General take appropriate action, within the limits of budgetary availabilities;
>
> (a) to further such efforts by promoting the exchange of technical information and other means, including the preparation of a study on modern cartographic methods and development of uniform international standards;
>
> (b) to coordinate the plans and programmes of the United Nations and specialized agencies in the field of cartography, taking into account the work of the various governmental and non-governmental organizations and to report on the subject to a subsequent session of the Council;
>
> (c) to develop close co-operation with cartographic services of interested Member Governments."

During the discussion of the resolution, it was emphasized that one of the first steps in the implementation of the resolution should be the calling together of a group of experts to draw up specific recommendations for the guidance of the Secretary General. Subsequently, at the Paris General Assembly of the UN, the Secretary General was authorized to invite five international experts to meet with representatives of the specialized agencies and interested international organizations. Accordingly, the Secretary General extended invitations to five individuals: R. Ll. Brown of Great Britain, Christovam Leite de Castro of Brazil, R. H. Randall of the United States, Ir. W. Schermerhorn of the Netherlands, and Robert Verlaine of Belgium. The experts met as a committee at Lake Success, N.Y., from March 21, 1949, to April 1, 1949, and at the first session the United States representative was elected chairman of the group. The meetings were attended by a number of delegates of specialized agencies and interested international organizations, acting in an advisory capacity. As a result of its deliberations the committee submitted to the Secretary General a report on its findings, including a number of recommendations.

Report on International Cartography.—It may be well to repeat here a part of the summary of the report of the committee of experts:

> "This report undertakes to answer two questions. The first is, what should the UN do to stimulate and assist its Member Nations to procure for themselves the cartographic information they need, for their own purposes and as component parts of the United Nations; and what should be the procedure to do this. The second is, what sort of cartographic service does the United Nations need to carry out such a programme, for its own operations and in its relations with the specialized agencies; and what are the steps necessary to provide this service, and to assure its proper co-ordination with related services in those agencies."[3]

Specific recommendations were presented by the committee to promote cartographic cooperation among the UN nations. One of the proposals called for annual meetings under the auspices of the UN to review the progress of world mapping and to allow representatives of interested agencies to report on matters of mutual concern. To achieve cooperation on local matters, regional meetings were proposed at which governments having common interests in particular areas could meet and discuss cartographic problems. Official representation of the UN would be secured by establishing a cartographic office in the Secretariat. This office would coordinate all mapping activities of the UN and be a source of information and advice on cartographic problems as well as a distribution center for maps. To assist the cartographic office it was proposed to establish a panel of consultants to give expert advice in their field.

United Nations Economic and Social Council Resolution.—The report of the committee of experts and its recommendations were accepted by the Secretary General and subsequently submitted to the Economic and Social Council. At a meeting of the Council held in Geneva, Switzerland, on July 27, 1949, a resolution, submitted by Brazil, Chile, India, France, Peru, the United Kingdom, and Venezuela was adopted.[3] This resolution acknowledged the urgent need for cartographic cooperation among the nations of the world and encouraged continued efforts to stimulate accurate surveying and mapping. In addition it called for speedy action on the recommendations of the committee of experts and on the publication of periodical summaries of cartographical progress and information.

The general opinion is that the Council's resolution is a good one and that the program it outlines will prove beneficial to all the member states of the UN.

SUMMARY

Joint planning and cooperation among countries will eliminate unnecessary duplication of effort and secure speedy selection of common standards and symbols. This standardization is important since modern transportation and communication facilities have brought the nations of the world intimately in contact.

Substantial progress is being made in the field of international cartography. The kind of information which basic map coverage can supply will constitute a useful tool and an increasingly recognized force for bringing about peaceful cooperation among men and among nations.

DISCUSSION

Milton O. Schmidt,[4] **A. M. ASCE.**—It is heartening to read this summary of progress being made in mapping conferences by the peace-loving nations of the world for common action in cartographic enterprises. Specific accomplishments to date, such as the Inter-American Geodetic Survey, illustrate what can be done and give tangible evidence of the merits of such cooperative ventures.

Acceptance of the word "cartography" to include all the operations leading to the production of maps makes for effective communication between nations. This paper offers an admirable vehicle for introducing the term as well as introducing world cartographic problems to the American civil engineer. Advancement of the common defense of the Americas will stem from intensive implementation of the programs outlined by the author.

However commendable the action thus far, the individual American engineer has yet to utilize to the fullest degree the manifold possibilities of adequate and up-to-date maps founded on good topographic bases for the study and design of engineering projects. The maximum potential afforded by international cartography will not be realized until the member countries themselves push map progress relentlessly and when each individual engineer places no compromise upon the need to expedite national mapping. When each individual fully supports his nation's mapping programs, the family of nations will follow suit. International cooperation in an area of engineering endeavor and world planning will then be accomplished.

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TRANSACTIONS

Paper No. 2486

STRUCTURAL DAMPING IN SUSPENSION BRIDGES

By the Late Friedrich Bleich,[1] M. ASCE,
and L. W. Teller,[2] M. ASCE

With Discussion by Messrs. R. K. Bernhard, George S. Vincent, F. B. Farquharson, Arne Selberg, and the Late Friedrich Bleich and L. W. Teller.

Synopsis

The collapse of the Tacoma Narrows Bridge in Washington in 1940 focused attention on the importance of a better knowledge of the capacity of suspension bridges to dissipate energy imparted by wind action. This paper reports part of the investigational work undertaken by the Advisory Board on the Investigation of Suspension Bridges. It comprises: (1) A theoretical study of damping capacity of suspension bridges resulting from internal friction and from various sources of dry or Coulomb friction in the structure; (2) an account of an extensive laboratory study of frictional damping in structural members; and (3) a correlation of the theory with experimental data. The paper suggests that a substantial amount of additional damping can be provided without resort to special damping devices if consideration is given in design to the potential sources of friction available in the floor system.

PART I. THEORETICAL STUDY OF THE DAMPING CAPACITY OF SUSPENSION BRIDGES

Experiments on the behavior of suspension bridges under the action of wind, which have been conducted for several years, disclose that the capacity of a vibrating bridge structure to dissipate the energy fed into it by pulsating wind forces, in general, has an important bearing on the dynamic stability of the structure.

It is questionable whether information on structural damping in suspension bridges could be obtained by field tests on existing long-span bridges because of

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the prohibitive costs, even if suitable methods could be devised for vibrating the formidable mass of such a bridge. Moreover, such field tests can only yield data on the ovër-all damping capacity of the structure, a capacity which may vary to a large extent from bridge to bridge; they cannot provide the basic information on the actual sources of the energy-dissipating capacity of the structure, or on the real nature of the damping phenomena in suspension bridges. Nevertheless, such detailed information is necessary in studying ways and means to insure the required damping capacity in any given bridge design.

It must be realized also that laboratory tests on full models cannot furnish real information on the damping capacity of the prototype because it is impossible, as a practical matter, to duplicate the structure of a bridge to a reduced scale in such detail as to simulate, reliably, the condition of friction existing in the prototype.

However, a theoretical approach is possible for solving the rather simple problem of damping in a bridge structure if basic experimental data are available as to the logarithmic decrement of internal damping in rolled, riveted, and welded steel structures, and as to the amount of friction that can be safely assumed as acting between the sliding parts of the structure. The following theoretical study, in its essentials, is a condensation of a report submitted to the Advisory Board on the Investigation of Suspension Bridges (see "Acknowledgment").

DAMPING IN GENERAL

"Damping" is the dissipation of the energy that is imparted to a vibrating structure by exciting forces, whereby one part of the external energy is transformed into molecular energy and another part is dissipated to surrounding objects or atmosphere. When damping can be localized at certain points or regions of the structure, damping may be thought of as being caused by forces acting in those points or regions and opposing the motion. These forces are referred to as damping forces.

"Damping capacity" of a structure may be defined as the ratio of the energy ΔW dissipated on one cycle of oscillation to the maximum amount of energy W accumulated in the structure in that cycle. Therefore, the damping capacity ψ, being a nondimensional quantity, is expressed as

$$\psi = \frac{\Delta W}{W} \qquad (1)$$

Suspension bridges are built up of several structural elements, such as cables, stiffening trusses, and towers. Designating the energy losses per cycle of the various elements by $\Delta W_1, \Delta W_2, \cdots$, respectively, the damping capacity of the entire system, according to the foregoing definition, is determined by

$$\psi = \frac{\Delta W_1 + \Delta W_2 + \cdots}{W} \qquad (2a)$$

in which W is the energy of oscillation of the entire system. Eq. 2*a* remains unchanged in writing

$$\psi = \frac{\Delta W_1\, W_1}{W_1\, W} + \frac{\Delta W_2\, W_2}{W_2\, W} + \cdots \qquad (2b)$$

in which W_1, W_2, $\cdots$ denote the energies of oscillation of the respective elements. Since $\psi_1 = \frac{W\Delta_1}{W_1}$, $\psi_2 = \frac{\Delta W_2}{W_2}$, etc., represent the individual damping capacities of the elements and $r_1 = \frac{W_1}{W}$, $r_2 = \frac{W_2}{W}$, etc., the relative energy storage capacities of these elements, the over-all damping capacity ψ of the structure, is expressed by

$$\psi = r_1\psi_1 + r_2\psi_2 + \cdots \qquad (3)$$

This linear relationship is based on the tacit assumption that ψ_1, ψ_2, $\cdots$ are small as compared with unity so that the frequency of the damped vibration of the system can be considered as being identical with the frequency of the undamped oscillation. Eq. 3 is the fundamental relation for the computation of the damping capacity of a compound structure from the damping capacities of its elements.

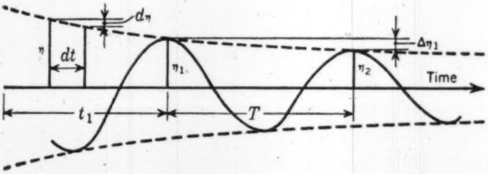


Fig. 1

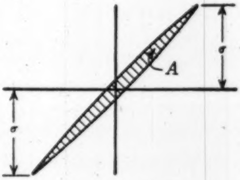


Fig. 2

Consider the amplitude-time diagram of a damped periodic motion as shown in Fig. 1. Let W_1 be the potential energy at the instant t_1 and W_2, the energy at the instant $t_1 + T$, in which T is the period of the oscillation. Assuming small motion and the validity of Hooke's law, the maximum amount of the potential energy of the structure in motion is proportional to the square of the amplitude η, so that $W = k\,\eta^2$. The difference between the potential energy W_1 at the time t_1 and the energy W_2 at the time $t_1 + T$, one cycle later, gives the loss of energy (dissipated energy) or the total damping of the structure for one cycle. Hence the damping capacity ψ may be expressed by $\psi = \frac{W_1 - W_2}{W_1}$ $= \frac{k\,\eta^2_1 - k\,\eta^2_2}{k\,\eta^2_1} = 1 - \frac{\eta^2_2}{\eta^2_1}$; or

$$\psi = 1 - \frac{(\eta_1 - \Delta\eta_1)^2}{\eta^2_1} = 2\,\frac{\Delta\eta_1}{\eta_1} \qquad (4)$$

—neglecting $\frac{\Delta\eta^2_1}{\eta^2_1}$ as a small quantity of the second order. To obtain the equation of the curve, indicated by dotted lines in Fig. 1 (which determines the decay of the amplitudes), consider an infinitely small part of the curve between t and $t + dt$. Assuming ψ to be constant and applying Eq. 4 to the infinitely small interval dt, the differential equation is

$$-\frac{d\eta}{\eta} = \frac{\psi}{2}\frac{dt}{T} \qquad (5a)$$

Integration furnishes

$$\log_e \eta = -\frac{\psi}{2}\frac{t}{T} + C \qquad (5b)$$

from which

$$\eta = e^{C-\psi t/(2T)} = \eta_0 \, e^{-\psi t/(2T)} \qquad (6)$$

is derived readily. In Eq. 6, η_0 is the amplitude at the start of vibration at the time $t = 0$. Considering two consecutive maxima η_r and η_{r+1} of the amplitude-time curve, having the distance $t = T$ and applying Eq. 6, the significant relation—

$$\frac{\eta_{r+1}}{\eta_r} = \frac{e^{-\psi(t_r+T)/(2T)}}{e^{-\psi t_r/(2T)}} = e^{-\psi/2} \qquad (7)$$

—is obtained. Eq. 7 expresses the rate of decay of the amplitudes. The ratio between consecutive maxima is constant; the amplitudes decrease in a geometric series. The quantity $\delta = \psi/2$ is dimensionless and is known as the logarithmic decrement of damping because

$$\log \frac{\eta_{r+1}}{\eta_r} = -\delta \qquad (8)$$

The relation $\delta = \psi/2$ is to be considered an approximation and applies only when δ is small compared with unity. In deriving the relation $\delta = \psi/2$, the damping capacity ψ was assumed to be a constant. In general, in steel structures ψ varies with the decaying amplitude of the vibration, and the actual curve of decay departs markedly from an exponential curve. Nevertheless, the relation $\delta = \psi/2$ also holds in cases of varying ψ since this magnitude, without appreciable error, can be considered invariable during one cycle.

In suspension bridges the damping has its origin chiefly in the following sources:

1. The imperfect elesticity of the material of the structure—that is, elastic hysteresis of the structural material. (In suspension bridges of the conventional type vibrations change the stresses in the cables only insignificantly; damping from this source is negligible, therefore, and has not been considered in this paper.)
2. Plastic yielding and friction due to small relative displacements in the riveted joints of the structure.
3. Internal friction in the concrete floor slab of the bridges.
4. Friction in the main expansion joints of the floor structure and in the secondary joints, or at the pedestals (where the stringers and other parts slide on their supports at the floor beams); also, frictional resistance at the cable saddles and at suspender connections.
5. Friction at the end bearings of the trusses or in the installed friction devices.
6. Resistance of the air.

The damping effect caused by the damping influences, sources 1, 2, and 3, is referred to as "internal damping." Sources 4 and 5 represent typical "Coulomb damping" or "dry friction" and, finally, resistance of the air causes "atmospheric damping." In this paper structural damping only, generating from sources 1 to 5, will be discussed.

INTERNAL DAMPING

"Internal damping" is the dissipation of energy as a result of friction between the molecules of the material or between the faying surfaces of structural parts riveted or bolted together. The stress-strain curve for a specimen subjected to a cycle of loading, within the range of the proportional limit, is not coïncident with that for unloading, and neither curve is an exact straight line, as Hooke's law postulates. In fact, a hysteresis loop occurs (Fig. 2), and the area A of the loop indicates the amount of energy dissipated during a completely reversed cycle. The departure from Hooke's law increases with increasing limiting stress σ, which means that the ratio of the amount of dissipated energy to the total elastic energy supplied to the specimen increases when the limiting stress σ involved in the vibration rises.

The energy dissipated by the elastic hysteresis of the steel is obviously small because of the slight departure of the stress-strain curve from a straight line, at least within the range of the small dynamic stresses which may occur in a suspension bridge and which do not exceed a few thousand pounds per square inch. The logarithmic decrement δ of steel specimens as a result of the elastic hysteresis of the material was found to be very small. For dynamic stresses between 1,000 lb per sq in.[2] and 3,000 lb per in.[2], δ varies only slightly and may be assumed to have an average value of 0.004.

The dissipation of energy is markedly increased in the riveted members of bridge structures because of the plastic slip in the riveted connections. Detailed information on the magnitude, and variation with stress, of the logarithmic decrement as observed on a model truss will be presented in Part II. The various tests showed that, in so far as small dynamic stresses between 1,000 lb per sq in. and 3,000 lb per sq in. are concerned, the logarithmic decrement δ may reach an average value about twice or three times the value of δ as observed on rolled sections.

Since elastic hysteresis in a wide range is nearly independent of the velocity of the oscillations, the internal damping capacity ψ can be considered as being independent of the frequency of the vibrating structures.

COULOMB DAMPING

It may be concluded from the previous section on "hysteresis damping" that the internal damping capacity of steel bridges is comparatively small. Observations on existing steel bridges of various types indicate damping capacities of substantially higher order than those related to internal damping. The conclusion must be drawn that a considerable part of the damping effect in a bridge structure is due to energy dissipation by internal or external dry friction.

Relatively simple theoretical studies actually reveal that Coulomb friction may play an important role in the over-all damping capacity of suspension bridges. The fact must be stressed that the effect of Coulomb friction on the damping capacity can be substantiated fairly well by theoretical considerations. The theory can be based on the experimental data as to the amount of the coefficient of dry friction, data which may be obtained in each individual case by relatively simple tests as described in Part II.

The effect of Coulomb friction on an oscillating structure is different from that of internal damping. Whereas the internal damping capacity slowly increases with rising amplitude, the frictional damping capacity (as will be shown subsequently) remains constant or decreases when the amplitude increases. In most cases, Coulomb damping reaches its maximum value at the beginning of a forced or self-excited vibration. This is a very significant fact which makes Coulomb friction an important factor in the over-all damping capacity of a bridge structure, particularly in the light of the fact that the rate of dissipation of energy by Coulomb damping often exceeds substantially the rate of energy dissipation by internal damping.

Coulomb Damping in an Oscillating Simple Span

It is convenient to discuss the theory of frictional damping in a simple span to obtain the basic concept for the analysis in the case of suspension bridges. Application of the developed method of approach to suspension bridges will prove to be a matter of somewhat more analysis. Two fundamental problems will be investigated hereinafter: (1) Damping capacity caused by friction in the end bearings; and (2) damping capacity due to friction between the stringers and the floor beams—the stringers being assumed to slide on the top of the floor beams.

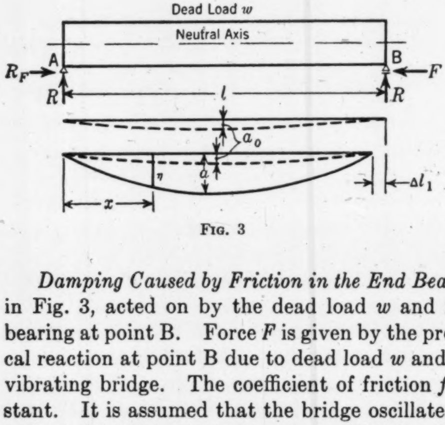


Fig. 3

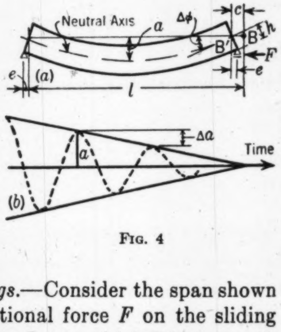


Fig. 4

Damping Caused by Friction in the End Bearings.—Consider the span shown in Fig. 3, acted on by the dead load w and frictional force F on the sliding bearing at point B. Force F is given by the product $R\,f$, in which R is the vertical reaction at point B due to dead load w and the varying inertia forces of the vibrating bridge. The coefficient of friction f is assumed to be a known constant. It is assumed that the bridge oscillates in the first symmetric mode as indicated in Fig. 3. At the start of the forced or excited vibration, when the amplitude is small, the trusses behave like two-hinged arch ribs producing a horizontal thrust H. In the first phase, H increases with increasing amplitude until it reaches the value of F at the amplitude a_o, at which the frictional resistance is overcome. When a becomes greater than a_o horizontal motion starts at point B until the truss reaches its extreme amplitude a. The path Δl_1 of point B, being a function of a, determines the energy $\Delta W_1 = F\,\Delta l_1$ dissipated in the first quarter of the cycle of vibration.

Force F varies during one cycle because the reaction R is the algebraic sum of R_w (reaction due to w) and R_I (reaction due to the inertia forces). It is easily seen, however, that R increases, in one half the cycle, to $R_w + R_I$ and decreases in the other half to $R_w - R_I$. The average value of R is equal to R_w. Hence, F may be considered a constant, $F = R_w f$.

The symbol ΔW is defined as the total energy dissipated in a complete cycle of vibration. The total path Δl_1 of the end point B of the truss during the first quarter of the cycle consists of two parts. The first part (referring to Fig. 4(a)) is the displacement $\overline{\mathrm{BB}'} = c$ of the end cross section of the bridge as a result of the curvature of the neutral axis. The second part is caused by the elongation $2e$ of the bottom chord. The total path Δl_1, therefore, is given by $\Delta l_1 = 2\,e - c$, assuming $2\,e > c$. When the girder swings back into its original position point B moves in the opposite direction. Again the displacement is $\Delta l_2 = 2\,e - c$, the small diminution of the amplitude caused by the damping being neglected. Similar reasoning shows that the path Δl during the third quarter and the fourth quarter of the cycle is $\Delta l_3 = \Delta l_4 = 2\,e + c$. Thus, the work done by the force F during one complete cycle is determined by

$$\Delta W = 2\,F\,(2\,e - c + 2\,e + c) = 8\,F\,e \quad \text{(9a)}$$

from which it can be assumed that F already has its full value at the beginning of each quarter of the cycle.

In the foregoing section it was assumed that c is smaller than $2\,e$ if $c > 2\,e$ (and it can be), the displacement Δl in the first and second quarters of the cycle is defined by $c - 2\,e$, and by $c + 2\,e$ in each of the two other quarters of the cycle; ΔW is now controlled by c and becomes

$$\Delta W = 2\,F\,(c - 2\,e + c + 2\,e) = 4\,F\,c \quad \text{(9b)}$$

The subsequent paragraphs will show that Eqs. 9 correspond to two different types of damping, referred to as the first and second types of Coulomb damping.

First Type of Coulomb Damping.—This type controls (for instance) the symmetric modes of a vibrating beam. The displacement $2\,e$ is given by

$$2\,e = 2\,h\,\Delta\phi \quad \text{(10)}$$

in which h is the distance of the neutral axis from the sliding surface of the bearing; and $\Delta\phi$ is the slope of the center line at cross section B (see Fig. 4(a)). The displacement form of the first symmetric mode of a simple beam is given by

$$\eta = a \sin \frac{\pi x}{l} \quad \text{(11)}$$

Therefore,

$$\Delta\phi = \frac{\pi}{l}\,a \quad \text{(12a)}$$

and

$$e = \pi\,\frac{h}{l}\,a \quad \text{(12b)}$$

Substituting Eq. 12*b* in Eq. 9*a*,

$$\Delta W = 8\,\pi \frac{h}{l} F\,a \qquad (13)$$

The maximum value of the total potential energy stored in the vibrating beam in its extreme position is

$$W = \frac{E\,I}{2} \int_o^l (\eta'')^2\,dx = \frac{\pi^4\,E\,I}{4\,l^3}\,a^2 \qquad (14)$$

in which I is the moment of inertia of the beam. Hence, the damping capacity ψ is determined by

$$\psi = \frac{\Delta W}{W} = \frac{32\,h\,l^2}{\pi^3\,E\,I}\frac{F}{a} \qquad (15)$$

Eq. 15 indicates that ψ decreases when the amplitude a of the vibration increases. The damping capacity reaches its maximum value at the instant when a becomes greater than a_o; that is, at the limiting amplitude, when friction in the bearing is overcome. In other words, Coulomb friction of the first type has its maximum damping effect at the starting phase of a forced vibration.

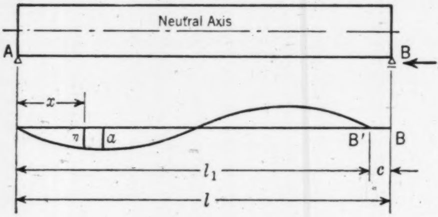


FIG. 5

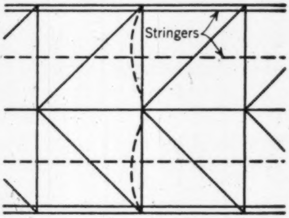


FIG. 6

Since Eq. 4 has general validity,

$$\psi = \frac{\Delta W}{W} = 2\,\frac{\Delta a}{a} \qquad (16)$$

in which Δa is the difference of the amplitudes of two consecutive cycles. Equating Eqs. 15 and 16,

$$\Delta a = \frac{16\,h\,l^2}{\pi^3\,E\,I}\,F \qquad (17)$$

Eq. 17 shows that Δa is constant, which means that the diagram of decay of the amplitude is a straight line, as shown in Fig. 4(*b*).

Second Type of Coulomb Damping.—The asymmetric modes of vibration are controlled by the second type of damping. Consider a beam AB (Fig. 5) oscillating in the first asymmetric mode. The elongation $2\,e$ of the bottom chord is zero and the work done by the friction force F depends only on the displacement c caused by the curvature of the center line. Assuming $\eta = a \sin \frac{2\pi x}{l}$, the difference c between the span l and the length l_1 of chord AB′ is

$$c = l - l_1 = \pi^2 \frac{a^2}{l} \qquad (18)$$

the energy dissipated in the sliding bearing, in one cycle of vibration, therefore, is

$$\Delta W = 4\,F\,c = 4\,\pi^2 \frac{F}{l}\,a^2 \qquad (19a)$$

The potential energy accumulated in the beam amounts to

$$W = \frac{4\,\pi^4\,E\,I}{l^3}\,a^2 \qquad (19b)$$

Combining Eqs. 19*a* and 19*b* furnishes the damping capacity—namely,

$$\psi = \frac{\Delta W}{W} = \frac{4\,\pi^2\,F\,a^2}{l}\,\frac{l^3}{4\,\pi^4\,E\,I\,a^2} = \frac{l^2\,F}{\pi^2\,E\,I} \qquad (20)$$

In Eq. 20, ψ is independent of the amplitude and may be considered a constant for any given design and mode.

Internal damping of small vibrations is characterized by a constant logarithmic decrement. The amplitude-decay diagram, practically, is an exponential curve, a type of damping frequently referred to as viscous damping. Therefore, friction damping of the second type, as described by Eq. 20, is the viscous type of damping, and the logarithmic decrement determining the decay of the amplitude is given by

$$\delta = \frac{1}{2}\frac{\Delta W}{W} = \frac{l^2\,F}{2\,\pi^2\,E\,I} \qquad (21)$$

The value of ψ as given by Eq. 20 is small compared with the value of ψ according to Eq. 15. The effect of Coulomb damping due to friction at the end bearings, in case of asymmetric vibration, may be considered negligible.

DAMPING DUE TO FRICTION BETWEEN STRINGERS AND FLOOR BEAMS

Assume that the stringers of a long single-span bridge are sliding on stringer pedestals on top of the floor beams. When the bridge deflects, a relative displacement of stringer and floor beams takes place as a consequence of the difference in length between the undistorted stringers and the distorted top chords of the trusses. In calculating the total amount of the energy dissipated by friction, the effect of expansion joints in the bridge floor must be taken into account. Furthermore, the relative displacement of stringer and floor beam is greatly affected by the lateral deflection of the floor beams caused by the horizontal friction forces. This deflection obviously reduces or completely offsets the relative displacements of stringers and floor beams. In computing the frictional forces, only those stringers that are located at points of the floor beams which cannot deflect laterally if acted on by friction forces may be considered as contributing to the energy dissipation. In Fig. 6 the solid lines indicate such stringers, whereas the stringers shown by the dashed lines may cause distortion of the top chord of the floor beams, comparable in magnitude with the relative displacement between stringer and floor beam.

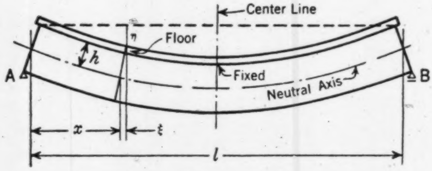


FIG. 7

The procedure for deriving the damping capacity of the bridge resulting from friction between floor and floor beams will be demonstrated under the assumption that the bridges vibrate in the first symmetric mode. The floor first may be considered tied to the trusses at midspan only, as indicated in Fig. 7. The relative displacement ξ at the distance x from the left support is

$$\xi = h\,\frac{d\eta}{dx} \qquad (22a)$$

in which η is the displacement form of the first symmetric mode; that is, $\eta = a \sin \pi\, x/l$ when the bridge vibrates with the amplitude a. Therefore,

$$\xi = \frac{\pi\, h}{l}\, a \cos \frac{\pi\, x}{l} \qquad (22b)$$

The graph in Fig. 8 computed from Eq. 22*b* and referred to as the $\bar{\xi}$-diagram shows the relative displacement ξ of stringer and floor beams in any reference point x when the bridge vibrates with the amplitude $a = 1$.

This diagram, however, does not represent the true relative displacements $\bar{\xi}$ because Eq. 22*b* has been derived under the assumption that the stringers move freely relative to the floor beams in each half span. At the expansion joints (designated E in Fig. 8) each stringer section is tied at its right end to the floor beam. Accordingly, the displacement $\bar{\xi}_f$ of the tied right-hand end of the section must be subtracted from $\bar{\xi}$. The reduced displacement, therefore, is $\bar{\xi}' = \bar{\xi} - \bar{\xi}_f$, and is represented by the ordinates of the shaded areas in Fig. 8.

With w_f denoting the weight per unit length of the floor structure and concrete slab pertaining to those stringers (which are assumed to slide on the floor beams) the friction ΔF per unit length is

$$\Delta F = w_f\, f \qquad (23)$$

Since $\Delta F\, x\, \bar{\xi}'\, a$ is the work done per unit length of bridge by the friction forces ΔF during one quarter cycle of vibration the total energy dissipated during a complete cycle is

$$\Delta W = 4\,\Delta F\, a \int_0^l \bar{\xi}'\, dx \qquad (24)$$

The integral extends over the total length of the bridge; and $\int_0^l \bar{\xi}'\, dx$ is represented by the sum of the shaded areas.

The maximum value of the total strain energy accumulated in the vibrating trusses in the case under consideration is given by Eq. 14, when I is the moment of inertia of all trusses. Finally the damping capacity ψ of the bridge structure is

$$\psi = \frac{\Delta W}{W} = \frac{16\, S\, l^3\, \Delta F}{\pi^4\, E\, I\, a} \qquad (25)$$

in which

$$S = \int_0^l \bar{\xi}'\, dx \qquad (26)$$

is the area of all shaded parts in Fig. 8.

Eq. 25 indicates that ψ is inversely proportional to a; and the damping caused by friction between stringers and floor beams is of the first type. A similar equation will be found for all other symmetric and asymmetric modes.

It is easily observed from an inspection of Fig. 8 that reduction of the distance between the expansion joints, for example, to one half of the distance shown in Fig. 8, would diminish the magnitude S in Eq. 25 to about one half, and, accordingly, would reduce the logarithmic decrement δ in the same ratio. On the other hand, the damping capacity could be increased considerably if the distance of the expansion seats could be adequately increased.

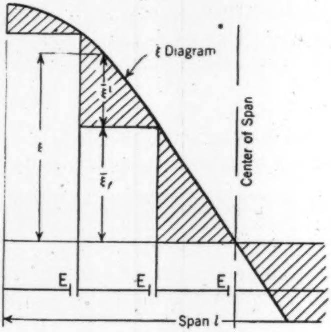


FIG. 8

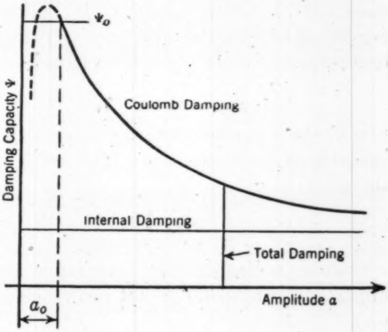


FIG. 9

The purpose of the preceding remark is to direct attention to the fact that, if careful consideration is given to the design of the floor structure in order to utilize a potential source of friction, a substantial amount of additional damping can be provided in suspension bridges without resorting to special structural devices. Most of the foregoing problems show frictional damping of the first type. The total damping capacity ψ—that is, the sum of internal and frictional damping—can be expressed in the form:

$$\psi = C_1 + \frac{C_2}{a} \qquad (27)$$

in which C_1 and C_2 are constants, when small vibrations are assumed. Plotting ψ against the amplitude a leads to the hyperbolic diagram as shown in Fig. 9.

The limiting value ψ_o corresponds to the amplitude a_o which is defined by that amplitude at which static friction is overcome. Since ψ_o defines the maximum amount of the damping capacity of the structure at the beginning of an excited vibration, a brief discussion of the determination of ψ_o in the somewhat involved case of friction in the bridge floor may be in order.

Eq. 25 has been derived under the tacit assumption that the amplitude a had reached a value where relative displacement of stringers and floor beams occurs at all points of the span. Actually, such is not the case at the beginning of a forced or excited oscillation. The longitudinal shear stresses between the floor structure and the floor beams vary along the span, from which it follows that, at small amplitudes, sliding already may have started in a part of the span while in the remaining part static friction may still prevent any movement. To obtain the relationship between the logarithmic decrement δ and the amplitude a during the state of motion under discussion it would be necessary to calculate, step by step, the consecutive values which δ assumes with increasing amplitude a, until the maximum value δ_o is reached, when friction is overcome in all parts of the span.

A rough estimate of the magnitude of δ_o at the instant when damping already has set in along the entire span can be obtained as follows:

Presume that the longitudinal shear stresses in all parts of the span have the same value, assumed equal to the average value of the actual stresses. The amplitude a at which sliding starts simultaneously at all floor beams follows immediately from the simple relation:

$$\Delta F = s\,a_o \qquad (28)$$

s being the constant shear stress per unit length of span which corresponds with the amplitude a equal to unity. The value of a at which damping sets in, therefore, is

$$a_o = \frac{\Delta F}{s} \qquad (29)$$

Since, by supposition, damping is present at all points of the span, Eq. 25 applies and furnishes (upon introducing a_o from Eq. 29, the maximum value ψ_o—

$$\psi_o = \frac{16\,S\,l^3\,s}{\pi^4\,E\,I} \qquad (30)$$

The foregoing step-by-step development of friction in the floor structure suggests a form of the damping diagram between zero amplitude and amplitude a_o as indicated by the dashed part of the curve in Fig. 9.

Corroboration of Theory by Tests.—The careful tests made on a model truss, discussed in Part II, afford a convenient opportunity to compare the prediction of the previously developed theory with the outcome of the experiments. Interpretation of these tests in the light of the theory is represented in Part III.

PART II. DAMPING TESTS ON SIMPLE BEAMS AND TRUSSES

The experimental study of the damping characteristics of structural members reported in this paper was made at the Research Laboratories of the

United States Bureau of Public Roads in Arlington County, Virginia, at the request of the Advisory Board on the Investigation of Suspension Bridges.

The initially authorized program had for its object a determination of the relative damping to be expected in solid members and in trussed structural members, information which has an important bearing on the testing of full models of suspension bridges for aerodynamic action in an air stream. As frequently happens, however, the results obtained in the work initially contemplated led to some expansion of the program so that other useful information on structural damping might be obtained while the experimental facilities were available. The work (first authorized in 1943 and expanded in 1945) has been the subject of three progress reports to the Advisory Board which, at its May, 1948, meeting, authorized the use of the data in the present paper in order to make the information available to the profession.

SCOPE OF THE PROGRAM

Four main subjects defined for investigation were as follows:

1. The damping characteristics of a trussed structural member in comparison with those of solid members of equal stiffness, when vibrated in the first symmetric mode as a simply supported beam. (The tests were planned to indicate the damping over a considerable range of dynamic stress.)

2. In the case of a bolted truss, the effect of bolt type, bolt tension, and of the procedure used in developing a given degree of bolt tension.

3. The effects of a variation in dead load on the damping characteristics of the bolted truss.

4. The effect of dry or Coulomb friction, externally applied, on the damping characteristics of the bolted truss. (The Coulomb friction was developed in three different ways: By mechanical brakes, by sliding bearings at the ends of panels of a simulated floor system, and by sliding end bearings as a replacement for ball-bearing wheels.)

To control the testing procedures properly, and for other reasons, certain collateral investigations were required. Among these were:

5. The establishment of relations between load, deflection, and strain in the specimens.

6. A determination of the ultimate strength and character of failure of the chord splice in tension, and the chord section in compression, as used in the bolted truss.

7. A determination of the maximum tightening torque for the bolt-nut combinations used in the truss.

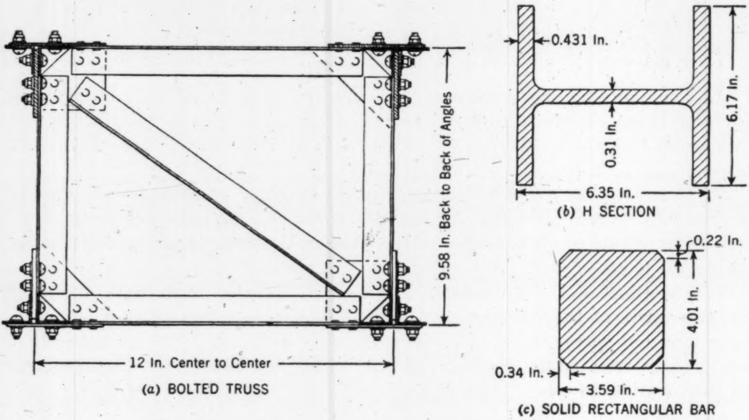
8. Segregation of the air damping effect when a simulated floor system was applied to the truss.

DESCRIPTION OF THE TESTS

The Specimens—Three types of specimens were used in the tests: (*a*) A trussed member with bolted joints, (*b*) a rolled H-section, and (*c*) a solid bar of rectangular cross section. The truss (as it will be called) was made up of angles 1 in. by 1 in. by 0.065 in. in section, cold-formed from a low alloy steel of relatively high strength. The H-section and the rectangular bar were hot-rolled, of low carbon steel. The three specimens were designed to have equal moments of inertia, although, as received, the moment of inertia of the truss was slightly greater and that of the **H**-section, somewhat smaller than the moment of inertia of the rectangular bar. The principal dimensions of the cross section of each specimen as received are shown in Fig. 10.

For testing, each specimen was placed horizontally as a simply supported beam having a rocker bearing at one end and a wheel bearing, rolling on balls, at the opposite end. The truss and the **H**-section were compared on a span of 36.83 ft. The truss and the rectangular bar were compared on a span of 28.33 ft, the truss being shortened by the removal of a section, for these tests.

Certain pertinent information as to the weight, stiffness, and other physical characteristics of each specimen are given in Table 1.



(*a*) BOLTED TRUSS (*b*) H SECTION (*c*) SOLID RECTANGULAR BAR

Fig. 10.—Cross Section Dimensions of Specimens

In order that the weight of the truss would equal that of the **H**-section or that of the rectangular bar as required, dead weight was added to the truss at the lower panel points. The dead-load weights bolted to the truss at the lower panel points may be seen in Fig. 11 which shows also the general appearance of the shortened truss as mounted on the 28.33-ft span.

Test Methods and Instrumentation.—The general test procedure has been to excite the specimen (mounted as a simple beam) to a given degree of oscillation in the first symmetric mode in the vertical plane and, having developed oscillations of the desired amplitude, to allow these to die out naturally through damping caused by (1) internal friction, (2) end bearing friction, and (3) air friction. Since end bearing friction may be assumed to be constant for specimens of equal weight and since air friction could at best exert only a very small damping force under the conditions of these tests, it was believed that any important differences found in the damping of the different specimens would be the result of differences in the damping forces within the specimens.

The rate of energy absorption or damping can be determined from the amplitude decay through the use of a quantity called the logarithmic decrement δ, introduced in Part I. This quantity may be defined as the natural logarithm of the ratio of the amplitudes of any two successive oscillations. For convenience, in these tests single amplitudes (that is, movements to one side of the mean position) have been used for decrement determination.

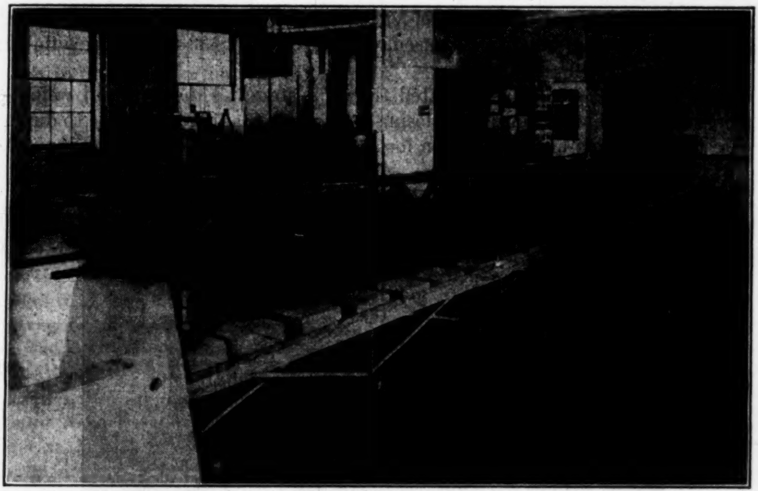


FIG. 11.—BOLTED TRUSS; SPAN, 28.33 FT AND WEIGHTS 51.75 LB EACH

TABLE 1.—COMPARISON OF PHYSICAL PROPERTIES OF SPECIMENS AS TESTED

Description	SPAN LENGTH 36.83 FT		SPAN LENGTH 28.33 FT	
	Truss	H-section	Truss	Rectangular bar
Moment of Inertia (Inches^4):				
From cross section	20.3	16.9	20.3	19.0
From load deflection	20.7	16.6	20.4	18.9
Weight per Foot (Pounds):				
Specimen proper	7.26	24.00	7.59	48.26
Attached dead-load weight	21.13	0.16	41.96	0.36
Total dead weight	28.39	24.16	49.55	48.62
Maximum deflection from dead weight (inches)	2.05[a]	2.05	1.24[b]	1.27
Natural period (second)	0.39	0.41	0.31	0.32

[a] Permanent set of 1.09 in. developed during testing. Total deflection at end of test, 3.14 in. [b] Permanent set of 0.71 in. developed during testing. Total deflection at end of test, 1.95 in.

The specimen under test was made to oscillate in the vertical plane by the simultaneous release of a number of symmetrically arranged cylindrical suspended weights of 19.5 lb, 34.5 lb, or 51.75 lb each, depending on the test condition for which the specimen was used. To distinguish them from dead-load weights that were attached to the specimens for various purposes, these suspended weights have been termed "live-load" weights. They were supported by steel hooks extending downward from the specimen at 17-in. intervals, a distance determined by the panel length of the truss. On the 36.83-ft span, twenty-six live-load weights were used, whereas on the 28.33-ft span the number was twenty.

By means of a solenoid-actuated trigger system, all the live-load weights were released simultaneously by closing one switch, making it possible to duplicate the excitation of the specimen for as many times as the program required.

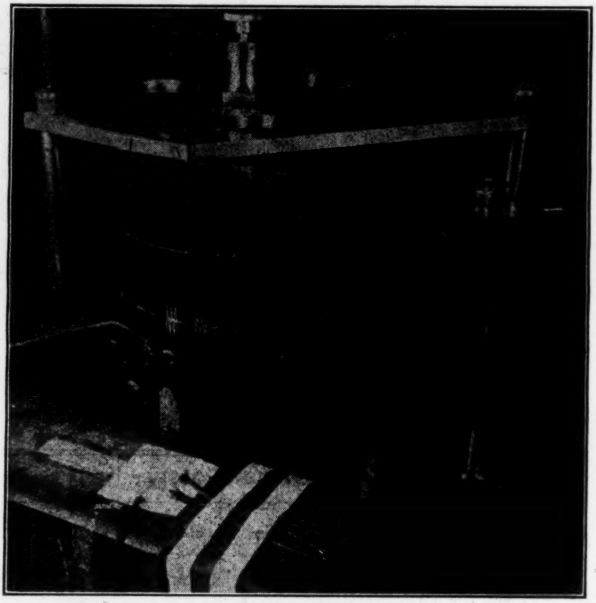


FIG. 12.—AMPLITUDE RECORD, SOLENOID-ACTUATED STYLUS AND STRAIN GAGE INSTALLATION-BAR

In most of the tests the amplitude of the oscillation was recorded at mid-span by a sharply pointed steel stylus attached to the specimen in a position such that it could be brought into light contact with a smoked paper wrapped around a slowly revolving drum. The stylus, under automatic control, periodically traced on the smoked paper complete cycles of the vertical motion of the oscillating specimen. Usually these traces were made at the beginning of the oscillation and during the first second of each 10-sec interval thereafter. A time trace with 1-sec intervals was placed on the record by another electrically-actuated stylus controlled by a calibrated pendulum. The details of this deflection recording equipment are shown in Fig. 12.

In some of the later tests with the truss where the damping rate was high, the intermittent recording of amplitude decay could not be used and a different method for recording was adopted. The method employed a magnetic displacement pickup with a galvanometer type oscillograph for recording, which made possible continuous recordings of oscillation decay with no physical contact between the test specimen and the recording medium.

Electrical resistance type strain gages were placed on each specimen at midspan to provide information regarding maximum stresses for the various conditions of test. One of the gages on the upper surface of the rectangular bar may be seen in Fig. 12. The relations between strain and load and between deflection and load were used for a number of purposes during the course of the program.

Comparison of the Internal Damping Characteristics of Trussed and Solid Steel Members.—As originally fabricated the joints in the truss, except for the lateral bracing in the plane of the upper chords, were drawn up with No. 10–24 commercial machine screws used as bolts. These passed through clearance holes in the gusset plates and truss member and were fitted with washers and hexagonal nuts of commercial grade. This was the type of joint fastening used in the initial tests.

By a number of tests with a torque-indicating wrench, it was found that this screw and nut combination would withstand a tightening torque of 40 in.-lb or more before failure occurred. On the basis of this information it was decided to test the truss at three degrees of bolt tension, measured by tightening torques of 35 in.-lb, 25 in.-lb, and 15 in.-lb, respectively. The net section of a No. 10–24 machine screw is approximately one half that of a rivet of the same over-all diameter.

The truss bolted up with the No. 10–24 machine screws tightened with a torque of 35 in.-lb was compared with the **H**-section on the 36.83-ft span, the excitation being from the release of the 19.5-lb and the 34.5-lb live-load weights. The shortened truss was compared with the rectangular bar on the 28.33-ft span, the excitation being from the release of 19.5-lb, 34.5-lb, and 51.75-lb live-load weights. Under the various test conditions initial double amplitudes at midspan varied from about 0.7 in. to about 4.1 in. and the maximum stress in the extreme fiber at midspan ranged from about 6,350 lb per sq in. to about 23,450 lb per sq in.

The measurement of both strain and deflection at midspan made it possible to relate midspan deflection to extreme fiber stress for each specimen and from this to relate instantaneous logarithmic decrement values to the maximum fiber stress developed during the particular oscillations to which they apply. Graphs showing this variation of logarithmic decrement with dynamic stress were particularly useful for the comparisons desired in this investigation.

Fig. 13(*a*) shows decrement-stress relations for the truss and for the **H**-section tested under identical conditions on the 36.83-ft span. The initial amplitudes cause the highest stresses and appear toward the right-hand side of the graph. When excited with the 34.5-lb live-load weights, the truss showed an initial decrement value of 0.0516 as compared with a value for the **H**-section of 0.0070. As the amplitudes decrease the dynamic stresses and the decrement values also decrease to a value of the general order of 0.0042 for the truss and 0.0036 for the solid section.

Fig. 13(*b*) shows a similar comparison between the shortened truss and the rectangular bar tested under identical conditions on the 28.33-ft span, three different live-load weight values being used to excite the oscillations. Even

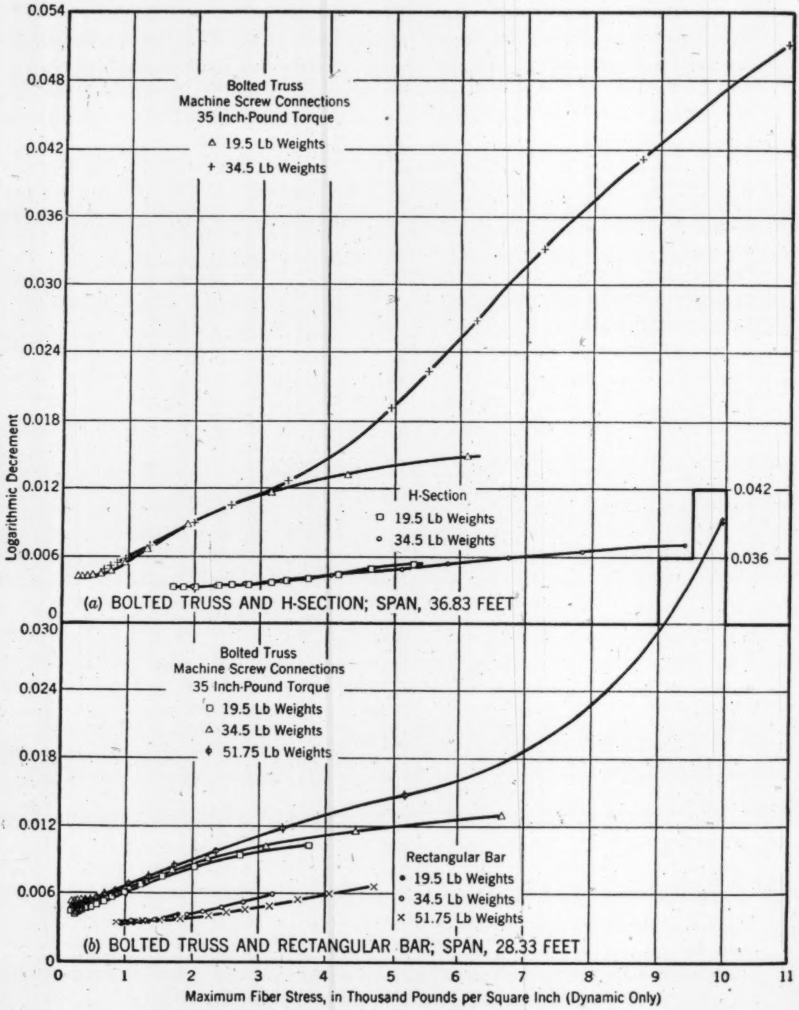
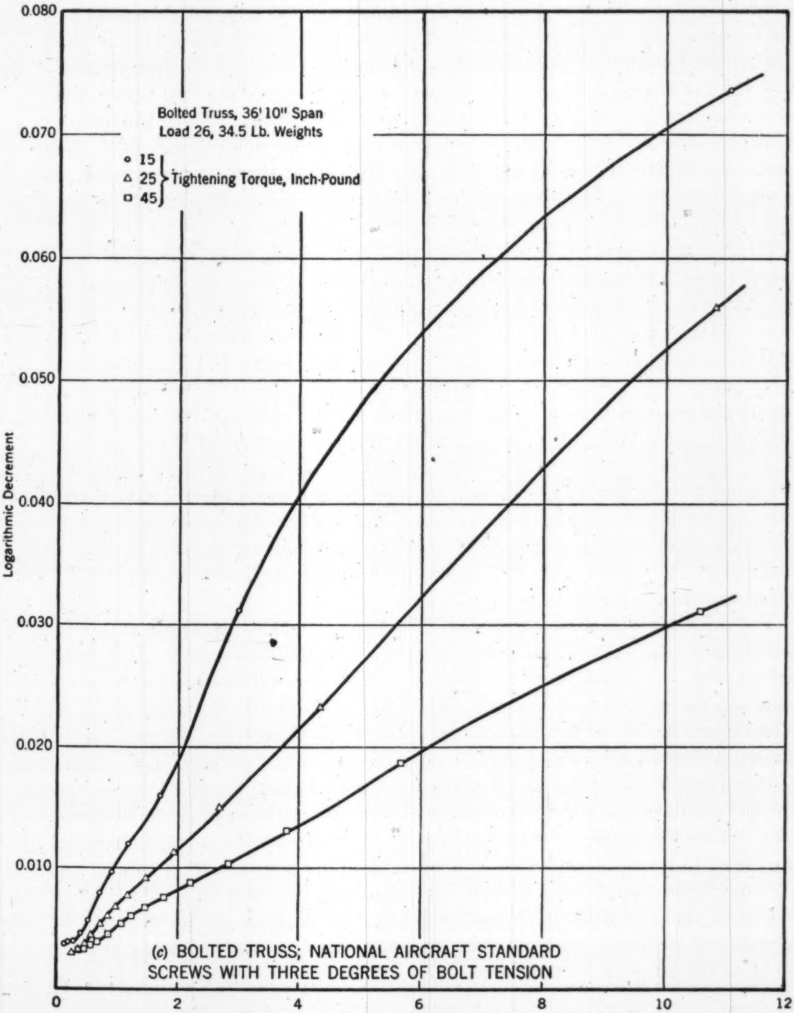


FIG. 13.—VARIATION OF LOGARITHMIC

with the 51.75-lb weights the maximum dynamic stress in the bar was only about 4,700 lb per sq in. and at this stress level the decrement value for the truss was approximately twice that for the bar. In this comparison as in the previous one the maximum damping of the truss is found near the beginning of the test where the stresses and amplitudes are greatest. It is under these conditions



(c) BOLTED TRUSS; NATIONAL AIRCRAFT STANDARD SCREWS WITH THREE DEGREES OF BOLT TENSION

Decrement δ with Fiber Stress

that the likelihood of motion in the joints, with accompanying dry friction, is greatest.

Each point on these and other graphs of the same type represents an average of five to seven tests and may be considered quite well established since the variation for a given set of test conditions was generally quite small.

In these initial tests (which involved the truss fastened with the No. 10–24 machine screws) a study was made of the damping of the truss as influenced by bolt tension. As might be expected, it was found that the more tightly the joint is bolted up the less will be the damping, particularly at the higher stresses. Since the net section of the No. 10–24 machine screw was about half that of the hole through which it passes, the question arose as to what might be the damping characteristics of the trussed specimen if the bolts more nearly filled the holes. Examination of some of the joints showed that there had been relative motion of the bolted members and that in some cases the threads along the body of the bolts had been considerably deformed by it.

Further tests were made with bolts that had a solid body where the bolt passed through the plates and which fitted closely in the hole through these plates. These bolts were of high strength steel manufactured to close tolerances, with well formed threads (32 per in.) and were available with solid (unthreaded) bodies of various lengths. They are designated National Aircraft Standard (NAS) screws. The nuts used with the NAS screws were a commercial product known as the "elastic stop nut," each nut being provided with an integral fiber washer which tends to prevent the nut from working loose. Tests with a torque wrench showed that, as used in these tests, the torque required to turn the nuts (in the new condition) on the threads of the bolt was of the order of 1 in.-lb to 2 in.-lb. Tests of the NAS screw-elastic stop nut combination showed a failure of the bolt in tension at a tightening torque of about 90 in.-lb as compared with about 45 in.-lb for the commercial machine screws used in the initial tests.

For the tests of the truss with the NAS screws, tightening torque values of 15 in.-lb, 25 in.-lb, and 45 in.-lb were selected. The first two gave a direct comparison with the data from tests with the machine screws, whereas the 45 in.-lb value was selected because tests indicated that it caused a tensile stress of about 60,000 lb per sq in. in the bolt body, a value which seemed to be a reasonable upper limit for structural bolting.

Having reamed the 1,700 holes in the bolted connections of the truss and replaced the machine screws with the NAS screws, tests were made with the truss on the 36.83-ft span with the 34.5-lb live-load weights, using tightening torques of 15 lb, 25 lb, and 45 lb. Typical data are shown in Fig. 13(*c*).

From these comparisons of bolt type and bolt tightness it would appear that in so far as self-damping characteristics of the truss are concerned the tightness with which the connections were bolted is of more importance than is the fit of the bolts in the bolt holes.

Effects of Variation in Dead Load on the Damping Characteristics of the Truss.—One of the questions raised in the discussion of the damping tests by the Advisory Board had to do with the possible effects of changes in dead load on the damping characteristics of the bolted truss. It was particularly desired to learn what might be the effect of an increase in dead load. To obtain a relatively wide spread in dead-load values without developing an excessive combined live and dead-load stress in the chord members the dead load of the truss was first reduced from its normal value of 1,059.6 lb to 843.5 lb by decreasing the dead-load weights attached at the lower chord panel points. After tests were made with this dead-load value, using both the 19.5-lb and 34.5-lb live-load weights, weights were added to the truss to bring the dead load up to 1,278.2 lb, and the tests were repeated.

Amplitude decay data for each of the four test conditions are shown in Fig. 14. The tests indicate that increasing the dead load of the truss by 51.5% caused no significant change in its damping characteristics.

Damping of the Truss Increased by Dry Friction Externally Applied.—The behavior of the truss in the early tests led to a discussion of the probable behavior of a truss equipped with a deck having sliding bearings to support the deck sections. As a result the program was extended to include some study of the effects of augmenting the dry friction within the truss with dry friction developed without, by (*a*) mechanical brakes, (*b*) sliding bearings at the ends of sections of a simulated floor system, and (*c*) sliding end bearings at the truss support.

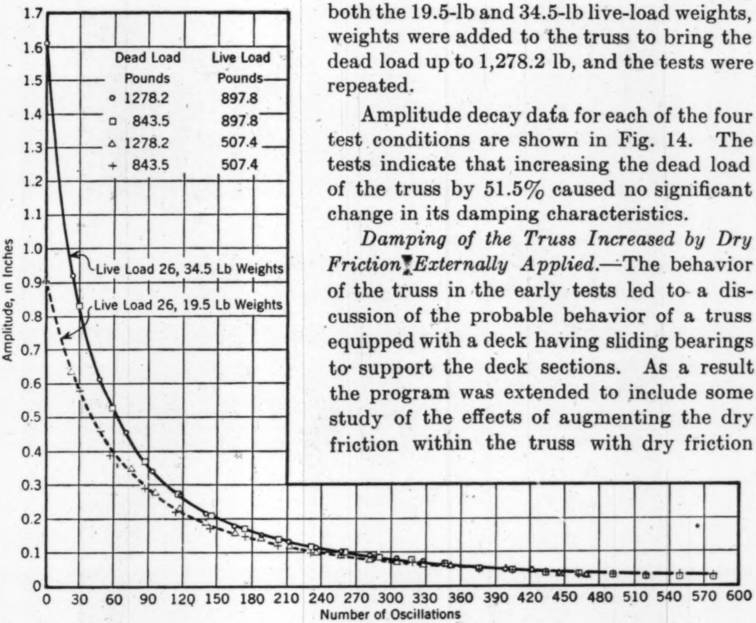


FIG. 14.—EFFECT OF DEAD-LOAD VARIATIONS ON AMPLITUDE DECAY; BOLTED TRUSS: SPAN, 36.83 FT, AND NATIONAL AIRCRAFT STANDARD (NAS) SCREWS WITH A TIGHTENING TORQUE OF 45 IN.-LB

A mechanical brake was developed which made it possible to apply dry or Coulomb friction in controlled amounts at each end of the oscillating truss. The design for the brake is quite simple and may be readily understood by reference to Fig. 15.

As the truss oscillated in the vertical plane the end frame went through a rotational motion causing a horizontal sliding motion at the contact between the friction surfaces. With the three weights used on the platform of the brake, vertical reactions of 70 lb, 140 lb, and 210 lb were developed on the braking surfaces. One of these brakes was installed at each end of the truss and both were operative in all tests.

Amplitude decay curves obtained with the mechanical brakes in operation are shown in Fig. 16(a) in comparison with that for the truss with no external dry friction. These data were obtained with the bronze lower friction surface in contact with cold-rolled steel upper friction surfaces of the three types used in the tests, and for each three pressure intensities.

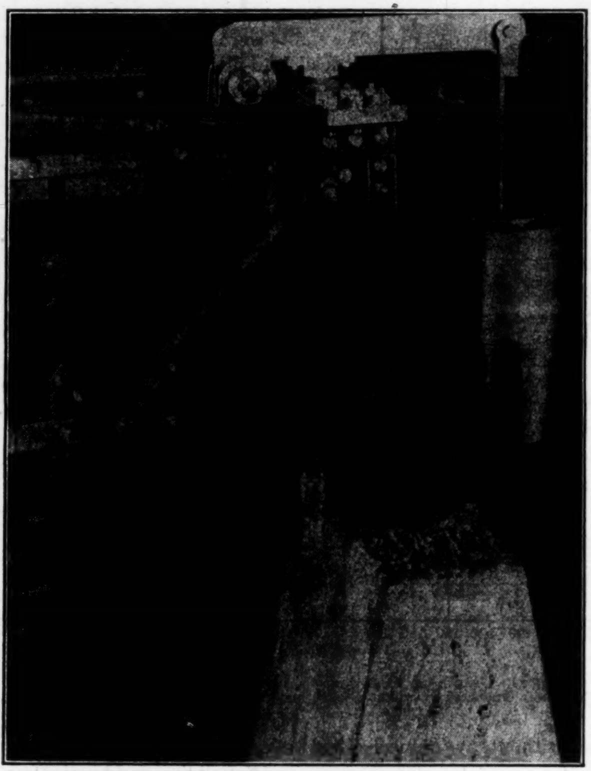
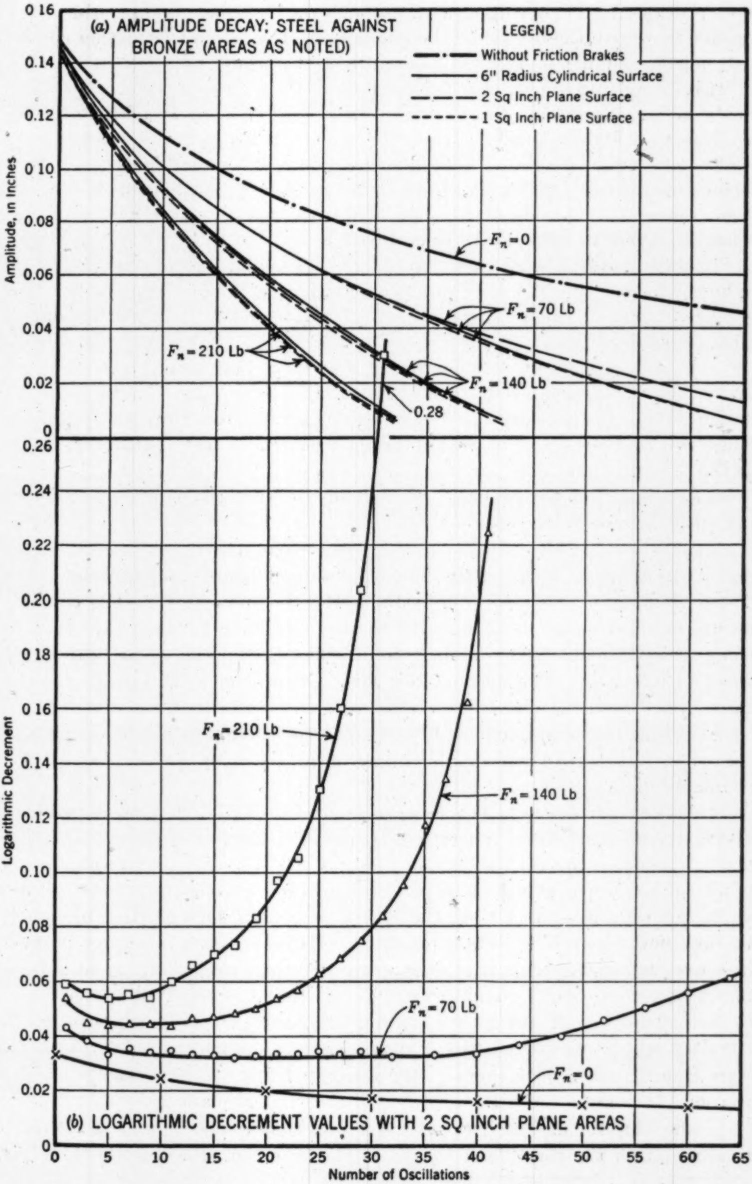


FIG. 15.—FRICTION BRAKE, SHOWING THE CYLINDRICAL SURFACE ON THE BRAKE LEVER IN CONTACT WITH THE PLANE SURFACE ON THE TRUSS

It is apparent from Fig. 16(a) that, where bronze and steel plates were used as opposing friction surfaces, the braking action was smooth and consistent, the damping increased progressively with the force (F_n) applied normally to the surfaces, and the changes in the area of the friction surfaces had little effect. In other words the laws of dry friction, or Coulomb friction, controlled.

In contrast, it was found that, where the opposing friction surfaces were of steel, even though one was much harder than the other, abrasion and scoring



(a) AMPLITUDE DECAY; STEEL AGAINST BRONZE (AREAS AS NOTED)

(b) LOGARITHMIC DECREMENT VALUES WITH 2 SQ INCH PLANE AREAS

Fig. 16.—Effect of Various Degrees of Dry Friction on Amplitude Decay and Logarithmic Decrement (Bolted Truss: Span, 36.83 Ft; Load, Twenty-six 34.5-Lb Weights; National Aircraft Standard (NAS) Screws with Tightening Torque of 45 In.-Lb; Brakes Steel Against Bronze)

were prevalent. The result is that, although the damping action at times was greater than with the bronze-steel combinations, the action was neither consistent nor dependable, and quite unsuited for test purposes.

It is interesting to note that the behavior of these friction surfaces is similar to the action observed in some bridge bearing-plate friction tests[3] reported by the Bureau of Public Roads in 1936. It was concluded from these earlier tests that combinations of like or unlike ferrous metals gave the highest, and that ferrous metals in contact with bronzes gave the lowest, coefficients of friction. Also, where the opposing plates were of ferrous metals, either like or unlike, seizure and scoring almost always occurred.

Thus the evidence indicates that where sliding plates are to be used in an unlubricated condition to develop friction for damping, more dependable action will be obtained if the opposing plates are of bronze and steel rather than of two ferrous metals.

It is evident from an examination of Fig. 16(*a*) that, with the mechanical brakes acting, the amplitudes decrease very rapidly to values of the order of 0.01 in. or less, as measured at the deflection gage position near the end of the truss, and that the shape of the amplitude decay curve tends to become linear as the braking force is increased. This is as would be expected since the Coulomb damping from the braking system tends to become the dominant component as this force is increased.

Fig. 16(*b*) shows logarithmic decrement values computed from the amplitude decay curves of Fig. 16(*a*) for the 2-sq in. friction plate area, and for four values of the braking force as measured by the normal component F_n. The complex nature of the over-all damping with the brakes acting made it expedient to compute decrement values directly from successive measured amplitudes at frequent intervals. This accounts for the slight scatter of the points for the three upper curves of Fig. 16(*b*).

Following the completion of the tests with the friction brakes at the ends of the truss, tests were made in which external dry friction was developed in a simulated deck system.

This deck was designed as a series of steel plates $7\frac{1}{4}$ in. wide by $\frac{3}{16}$ in. thick arranged end to end the full length of the truss. Except for one section 34 in. long at midspan the deck sections were each 8.5 ft long. At alternate panel points of the upper chord small plates of cold-rolled steel were bolted transversely on the truss and on these the deck sections were supported. At one end of each individual deck section (except the 34-in. section) two short vertical legs were fastened. This pair of legs was attached to one of the transverse plates by knuckle bolts, making a connection that was resistant to horizontal or vertical movement, but not to rotation. At each of the other support points, the deck section was fitted with a similar pair of short legs except that these were tipped with bronze shoes at the lower end to provide horizontal sliding friction bearings as the truss oscillated in the vertical plane.

To develop the desired deck load of approximately 500 lb, and at the same time maintain the total dead weight in the specimen, the dead-load weight attached at the lower chord panel points was reduced from 768.9 lb to 274.3 lb, a difference of 494.6 lb. Ballast weights attached to the deck sections at the support points were used to bring the weight of simulated deck up to an amount approximately equal to that removed from the lower chord panel points. The steel deck, supports, and ballast weighed 497.7 lb, making the total dead-load weight 1,062.6 lb as compared with 1,059.6 lb before the deck was added.

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[3] "Determination of Coefficients of Friction of Sliding Bearings for Bridges," by George W. Davis, *Public Roads*, December, 1936, p. 223.

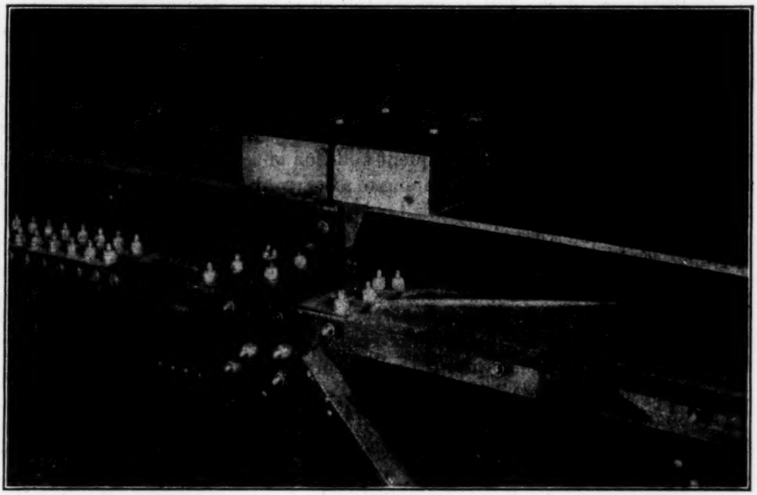

FIG. 17.—FIXED AND SLIDING BEARINGS AT THE ENDS OF STEEL PLATES OF SIMULATED DECK

In Fig. 17 are shown the details of the fixed-end connection of one deck section, the sliding bearing of the adjoining section, and the manner in which the ballast weight was applied to the plates of the deck over the support points. It has been recognized from the beginning that there has been a certain amount of air damping present in the over-all damping for which logarithmic decrement values have been determined. In view of the generally low decrement values it has been thought that the air damping probably was not an important element in the tests of the truss without the deck. With the addition of a relatively large horizontal area such as was represented by the simulated deck there was reason to suspect that the amount of air damping might become an important component of the over-all damping and some experiments were made to determine how important this element might be.

A series of deck plates of $\frac{1}{4}$-in. plywood were attached to the truss in its original form. Each plywood plate was attached to the truss only at the midlength of the plate and as the plates were separated slightly, no stiffening effect was introduced. The total area of the plywood sections was the same as that of the steel deck. The total weight of the plywood was 16.75 lb.

Fig. 18 shows the relation between the logarithmic decrement and the maximum dynamic fiber stress in the chord members for the three deck conditions (no deck, plywood plates, and the steel deck with friction shoes acting). This graph is of the same form as Fig. 13, with the high stress values measured at the beginning of the test shown at the right-hand side of the plot. The decrement values were computed from measured values of successive amplitudes in the manner described in connection with the friction brake tests. This probably accounts for the slight scatter of the points on the curve for the steel deck condition in Fig. 18.

It is apparent that the additional air resistance created by the application of the plywood plates, and the consequent addition of some 24 sq ft to the frontal area, does increase the over-all damping to a degree that is measurable, although the effect is relatively small. This tends to support the assumption that the air resistance of the truss alone was not an important component of the over-all damping measured for that condition.

With the steel deck sections added, and with the Coulomb friction at the sliding supports in operation, the increase in total damping was found to be relatively large, however, being quite similar to the effect observed with the brakes acting at the ends of the truss and corresponding approximately to the damping observed with a braking force $F_n = 70$ lb.

As might be expected, the frequency of the truss oscillations changed slightly after the steel deck sections were installed, the effect being to raise the apparent

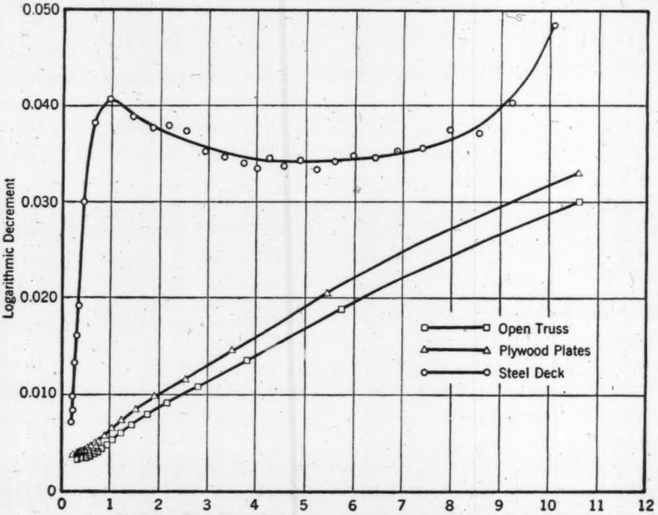

FIG. 18.—VARIATION OF THE LOGARITHMIC DECREMENT δ WITH FIBER STRESS FOR THREE DECK CONDITIONS (BOLTED TRUSS: SPAN, 36.83 FT; LOAD, TWENTY-SIX 34.5-LB WEIGHTS; NATIONAL AIRCRAFT STANDARD (NAS) SCREWS WITH A TIGHTENING TORQUE OF 45 IN.-LB)

moment of inertia about 3%. Also, it was found that the change in frequency was not constant but increased slightly as the amplitudes became smaller.

The third method used for developing external Coulomb friction included in these tests was to substitute sliding shoes for the ball-bearing wheels that supported one end of the truss in most of the tests. The shoes were mounted on ball bearings carried by the same cross shaft that normally was used for the wheel bearings so that no additional rotational resistance was added to the system by the change. The lower surface of each shoe was a smooth bronze plate 3 in. long by 0.6 in. wide and the opposing surface was of cold-rolled steel.

Comparative amplitude decay data are shown in Fig. 19, which are the curves for the truss on the wheel end bearings, for the truss on the sliding end bearings, and for the truss on the wheel bearings with the friction brakes in operation under a normal force, F_n of 210 lb per brake. It is apparent that the sliding end bearings developed a very strong damping force as the truss oscillated—stronger even than that with the two friction brakes operating. It will be recalled that the truss weighed about 1,050 lb so that the force exerted vertically on the end bearings was about 525 lb as compared with a total vertical force of 420 lb on the surfaces of the brakes.

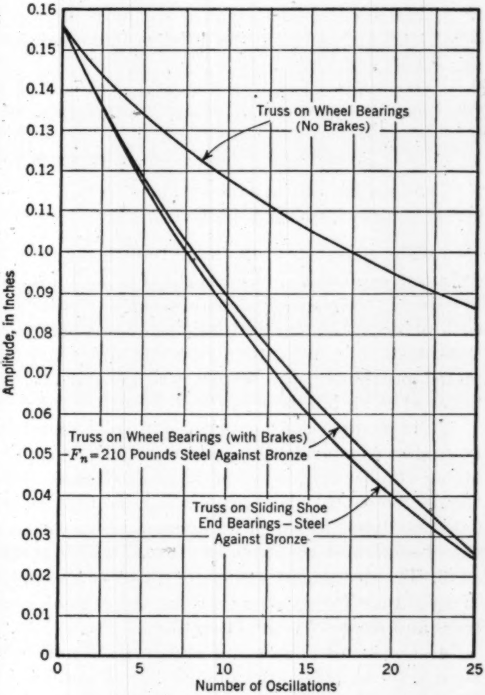

FIG. 19.—COMPARATIVE EFFECT OF BRAKE FRICTION AND SLIDING END BEARING FRICTION ON AMPLITUDE DECAY

For a better understanding of the forces that were being developed in the tests, in which external dry friction was being applied to the truss, it seemed desirable to obtain data on the frictional action of the bronze on steel surface combination, in the unlubricated condition, when the moving part was sliding with a horizontal reciprocating motion.

Accordingly, an apparatus was designed and constructed in which the force normal to the friction surfaces was developed with the same lever arrangement used in the mechanical brake on the truss; and the amplitude and frequency of horizontal motion of the friction surfaces were approximately the same as obtained when these same surfaces were used in the brakes during the testing of the truss. By means of a strain gage and oscillograph, the entire cycle of the force required to overcome resistance to sliding at the plane of the bronze-steel friction surfaces was recorded.

From tests with this device, using the same bronze and cold-rolled steel surfaces that were used in the friction brakes, it was found that:

a. The horizontal force necessary to cause the friction surfaces to slide with the reciprocating motion as described, varied with time, essentially in the form of a "square" wave;

b. The average coefficient of sliding friction for the plane surfaces of cold-rolled steel on bronze, in the unlubricated condition, as determined by six tests, was 0.141; and

c. Once sliding began, the resistance to motion (or the coefficient of friction) did not vary with the velocity of the sliding motion.

SUMMARY OF INDICATIONS FROM THE DAMPING TESTS

The analysis of the data obtained from the structural damping tests, made in the manner described, indicates that:

1. The character of the amplitude decay for the two solid sections tests was definitely exponential whereas that for the bolted truss was more complex, being apparently the result of a combination of the imperfect elasticity of the material with plastic yielding and friction due to small displacements in the joints. In these tests, of course, there was end bearing friction and air friction, but, as stated previously, the former was present as a constant and the latter was believed to be relatively unimportant.
2. In comparisons between the solid members and the bolted truss, the truss showed considerably greater capacity for self damping, particularly at the higher amplitudes (or higher dynamic stresses). With the joints tightly bolted, the decrement values ranged from about 0.006 at 1,000 lb per sq in. to about 0.030 at 10,000 lb per sq in. in comparison with a range of 0.0036 to 0.0070 for the H-section at the same limits of dynamic stress.
3. The decrement values for all three specimens increased with an increase in amplitude (or dynamic stress), the rate of increase being greatest with the truss and least with the H-section.
4. For small amplitudes and small dynamic stresses the decrement values were of the general order of 0.004 for all specimens. The decrement values for the solid members did not exceed 0.007 in any of the tests. For the truss, however, values as high as 0.073 were found at high initial amplitudes (and high dynamic stresses) in combination with low bolt tension in the bolted connections.
5. The tightness of the bolted connections had a pronounced effect on the damping of the truss, particularly at the larger amplitudes (or dynamic stresses). The tighter the bolts, the lower were the decrement values.
6. The damping action of the joints in the bolted truss, apparently, was influenced more by the pressure of the surfaces held in contact by the bolts than it was by the fit of the bolts in the holes.

7. A variation in the dead load of the truss of the order of 50% caused no measurable change in its damping characteristics.

8. The external application of dry or Coulomb friction greatly increased the damping of the bolted truss.

9. Where mechanical brakes that employed metal plates rubbing together without lubrication were used to develop frictional forces for damping purposes, more dependable action was obtained with opposing surfaces of bronze and steel than with two ferrous metals in contact.

10. The damping obtained from the rubbing of steel on bronze without lubrication follows the laws of Coulomb friction; that is, it varies directly with the total normal pressure on the friction surfaces, it is independent of their area, and it is nearly independent of velocity. The effective coefficient for the condition of this test was about 0.14.

11. The application of the plywood plates to the truss, in lieu of the steel deck panels of equal frontal area, increased the damping to a degree that could be measured, although the effect of the added air resistance on damping was relatively small.

12. The increase in the damping of the bolted truss, obtained by the application of the simulated deck sections, was quite similar in character to that obtained with mechanical brakes in operation at the ends of the truss.

13. The substitution of sliding end bearings of bronze on cold-rolled steel for the wheel bearings used at one end of the truss developed a strong damping force.

PART III. CORROBORATION OF THE THEORY OF FRICTIONAL DAMPING BY THE EXPERIMENTS

Damping Effect of Mechanical Brakes at the Ends of the Truss

The vibrating system, damped by the friction forces F which act at top chord level at both ends of the truss is shown in Fig. 20. Applying the same reasoning as in the case of damping due to friction in the bearings at one end of a single span, discussed in Part I, leads to the derivation of Eq. 14 with h denoting the distance from the sliding surfaces of the brakes to the neutral axis of the model truss. The friction coefficient f was found, by special tests, to be $f = 0.141$ from which the braking force F (for $F_n = 210$ lb normal pressure) is $F = 210 \times 0.141 = 29.6$ lb.

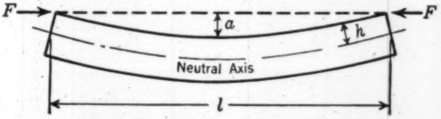

FIG. 20

Introducing this value of F into Eq. 14, with $E = 29{,}600{,}000$ lb per sq in.; $l = 442$ in.; $h = 5.35$ in.; and $I = 20.3$ in.^{4}, gives

$$\psi = \frac{0.0542}{a} \qquad (31a)$$

The logarithmic decrement, therefore, is

$$\delta = \frac{0.0271}{a} \qquad (31b)$$

The theoretical damping hyperbola is shown in Fig. 21, in which δ is plotted

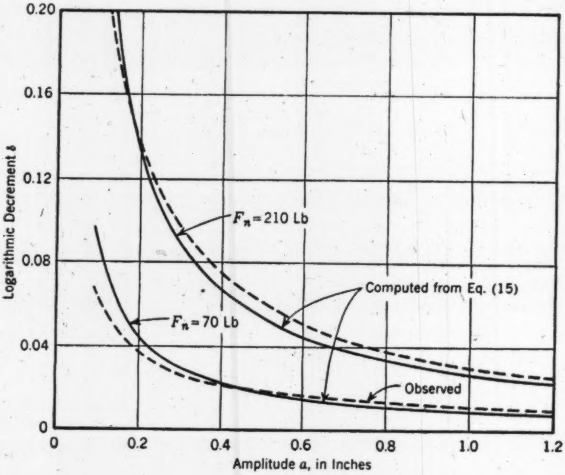

Fig. 21

against the amplitude a. A similar graph is given for $F_n = 70$ lb which follows the equation:

$$\delta = \frac{0.0090}{a} \qquad (31c)$$

From Fig. 16 the two dashed curves in Fig. 21 were developed, showing also the observed values of δ as a function of the amplitude a. These curves represent the effect of pure frictional damping, the effect of internal damping having been eliminated with the aid of the curves for $F_n = 0$ in Fig. 16.

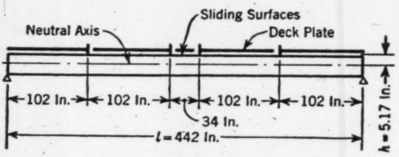

Fig. 22

Fig. 21 indicates good qualitative agreement between theory and experiment. The empirical curves deviate but slightly from a hyperbola. For values of a between 0.4 in. and 1.2 in. the observed values of δ are somewhat greater than the theoretical values, indicating that at least in that range the frictional resistance presumably was 10% to 15% greater than $f = 0.141$ which value underlies the foregoing computation.

EFFECT OF FRICTION IN THE SIMULATED DECK SYSTEM

A detailed description of the articulated deck system was given in Part II. Fig. 22 shows the arrangement of the deck plate and the location of the expansion joints. The total weight of the deck structure was 498 lb, or 498/442 = 1.13 lb per in. It follows, therefore, that the friction force $\Delta F = 0.141 \times 1.13 = 0.16$ lb per in. Following the procedure as outlined in Part I, the ξ-diagram was computed by Eq. 22*b* as $\xi = 0.0368 \cos \pi x/l$ and shown in Fig. 23(*a*). The shaded area S, for the entire truss, is 3.88 in.; and, ψ, according to Eq. 25, is

$$\frac{16\, S\, l^3\, \Delta F}{\pi^4\, E\, I\, a} = \frac{16 \times 3.88 \times 442^3 \times 0.16}{97.4 \times 29.6 \times 10^6 \times 20.3}\,\frac{1}{a} = \frac{0.0147}{a}.$$

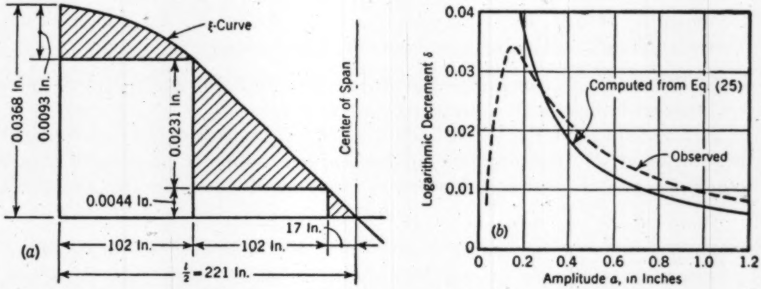

FIG. 23.—EFFECT OF FRICTION IN THE SIMULATED DECK SYSTEM

The curve $\delta = \frac{0.00735}{a}$ and, for comparison, the observed relationship between δ and a, are shown in Fig. 23(*b*). The latter curve was derived from observed diagrams after eliminating the effect of internal damping. The damping effect as observed, between $a = 0.4$ and 1.2, is distinctly greater than that predicted by the simple formula, Eq. 25. Its shape, showing a peak at $a = 0.15$ in., indicates that the considerations in Part I relating to the beginning of frictional damping in the case of an excited motion (see Fig. 9) are correct in principle.

ACKNOWLEDGMENT

The collapse of the Tacoma Narrows Bridge on November 7, 1940, created a widespread demand for a comprehensive investigation of the dynamic oscillations of suspension bridges. As a result, in 1942, the United States Public Roads Administration sponsored the organization of a group of competent engineers, representing various interested organizations, to explore the problem. This group, whose function it is to assume general direction and supervision of the investigational work undertaken, is known as the Advisory Board on the Investigation of Suspension Bridges. The co-author, Friedrich Bleich, served as consultant to this group from May, 1943, to June, 1947. Subsequent to his death, on February 17, 1950, the work of interpreting the original notes, reconciling editorial details, and answering questions raised in discussion concerning Part I of this paper has been kindly assumed by his son, Hans H. Bleich, M. ASCE.

DISCUSSION

R. K. Bernhard,[4] M. ASCE.—In their paper the authors determined experimentally the logarithmic damping decrement of steel structures by means of displacement amplitude-decay curves. They obtained a maximum value of 0.291 for the logarithmic damping decrement (Fig. 16, curve for $f_n =$ 210 lb) in a bolted truss with a span of 36.83 ft.

It might be of interest to compare this figure with values obtained by another method based on response curves produced by means of mechanical oscillators.[5] These experiments yielded values for the logarithmic damping decrement up to 0.31, hence maximum values that are only 6.5% higher than those mentioned. This slight increase might be explained as follows:

1. The response curve experiments were carried out under actual field condition on bridges with substantially larger spans.
2. The values for the logarithmic damping decrement obtained from field tests included the damping due to the bearings, supports, foundations, surrounding soil, and other such factors.

Furthermore, the statement of the authors that friction in riveted connections—as compared for example with quasi-frictionless welded connections—increases the friction damping substantially agrees with experiments of the writer.[6]

Finally, similarly to the findings of the authors, periodic field tests with oscillators showed that the tightness of the rivets has a rather significant influence on the damping characteristics. Since riveted connections of newly installed structures are normally in a tighter condition than older, worn-out structures, a decrease in the logarithmic damping decrement has been observed with aging (retrogression period). However, a slight increase in damping during the first years (training period) of riveted bridges, probably due to a working-in procedure, could also be established.[6]

It is indeed gratifying that the laboratory experiments, which could be carried out more rigorously than the writer's field tests, did lead to similar conclusions. The authors are to be congratulated for their valuable contribution in this much neglected field of mechanical vibrations.

George S. Vincent,[7] M. ASCE.—A compact, well-coordinated presentation of theory and experimental data on the problem of damping in suspension bridges, a problem distinctly different from that of damping in other types of structures, is given in this paper. The mathematical treatment is surprisingly simple and direct, and the report of tests is packed with facts, the significance of which is well-accented in the section titled "Summary of Indication from the

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[4] Prof., Eng. Mechanics, Rutgers Univ., New Brunswick, N. J.

[5] "Dynamic Test by Means of Induced Vibrations," by Rudolf K. Bernhard, *Proceedings*, A.S.T.M., Vol. 37, Part II, 1937, p. 634.

[6] "Artificial Vibrations—A New Method of Research," by Rudolf K. Bernhard, *Civil Engineering*, Vol. 7, 1937, pp. 286–287.

[7] Principal Highway Bridge Engr., U. S. Bureau of Public Roads, Structural Research Laboratory, Univ. of Washington, Seattle, Wash.

Damping Tests." This summary and the correlation presented in Part III of the paper leave little to be added by way of interpretation of results of the study.

The data given on atmospheric damping should not be given an interpretation beyond that intended by the authors. The test was made simply to determine how much of the air resistance on the 7¾-in. plate might distort the measurements of the damping effect of the carefully calibrated friction induced by a simulated deck action. The air resistance measured was applied to an abnormally narrow deck and, of course, neglects the atmospheric damping on the truss members. With these and other limitations in mind, it is interesting to analyze the plotted atmospheric decrement as revealed by the difference between the two lower curves of Fig. 18.

It is noted that δ_a, the atmospheric decrement, approaches zero at zero amplitude and increases roughly in proportion to the amplitude. By correlating the curves of Figs. 18, 13(*c*), and 14, the approximate equation for the logarithmic decrement in terms of the amplitude, η, is:

$$\delta_a = 0.001 + 0.0024\,\eta \qquad (32)$$

The second term is dominant except for small amplitudes. This seems characteristic of atmospheric damping, indicating that the principal resistance is a force varying as the square of velocity. The maximum value of the velocity of the vibration is $\omega\,\eta$, in which ω is the circular frequency of the vibration. The force will then be $C_1\,\omega^2\,\eta^2$, and in one cycle it will perform work equal to:

$$\Delta W = C_2\,\omega^2\,\eta^3 \qquad (33)$$

The energy of vibration, equal to the maximum value of the kinetic energy, is:

$$W = \frac{m\,v^2}{2} = \frac{m\,\omega^2\,\eta^2}{2} \qquad (34)$$

Hence,

$$\psi = \frac{\Delta W}{W} = \frac{C_2\,\omega^2\,\eta^3}{m\,\omega^2\,\eta^2/2} = \frac{C\,\eta}{m} \qquad (35)$$

If the air resistance were viscous, varying as the first power of the velocity, then Eq. 33 would be:

$$\Delta W = C_2\,\omega\,\eta^2 \qquad (36)$$

and Eq. 35 would be:

$$\psi = \frac{C}{m\,\omega} \qquad (37)$$

Thus Eqs. 37 and 35 represent, respectively, the first and second terms of Eq. 32. Atmospheric damping seems to be predominantly the result of the inertial force of the wind with a small constituent of viscous damping.

The fact that m is in the denominator of both Eqs. 35 and 37 is important. It means, for example, that if a bridge of the same shape is doubled in weight (without altering the frequency of vibration), δ_a will be reduced by half. A model test will indicate the atmospheric damping of a prototype only if the total weight, or weight per panel, is scaled to the cube of the linear scale. In these tests the dead load on the truss model was selected simply to correlate with tests on the H-beam. It, of course, was not intended as an aerodynamic model of any prototype. Its weight is roughly three times too great for a typical truss bridge at a scale of 1 to 30, and seven times too great if interpreted at a scale of 1 to 50. Its stiffness also exceeds that of an aerodynamic model.

Fig. 24 shows the result of tests on a model of the proposed Tacoma Narrows Bridge with slotted deck for the purpose of isolating the atmospheric damping in vertical motion. Since this was an aerodynamic model of proper scale, it will represent the atmospheric damping of the prototype if the abscissa scale is replaced by corresponding prototype amplitudes. It shows the effect of small viscous damping combined with that of a much greater damping arising from the dynamic pressure of the wind.

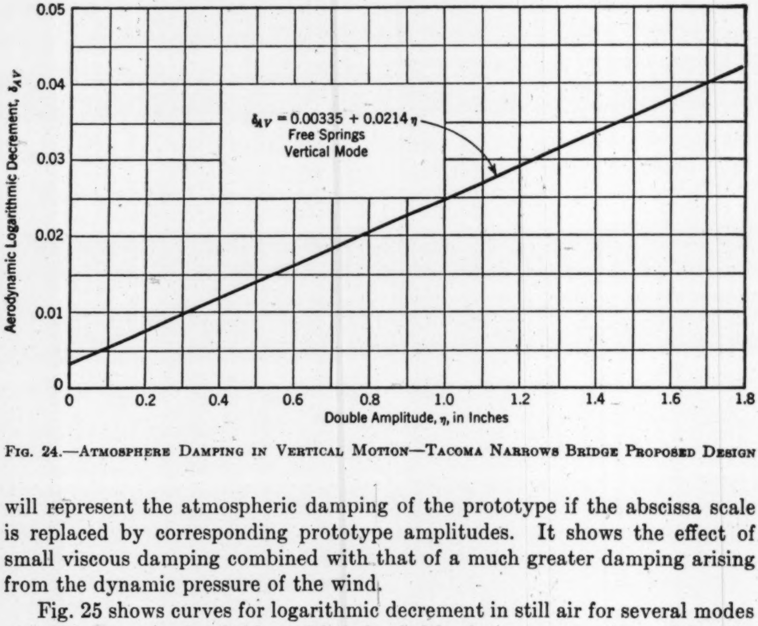

FIG. 24.—ATMOSPHERE DAMPING IN VERTICAL MOTION—TACOMA NARROWS BRIDGE PROPOSED DESIGN

Fig. 25 shows curves for logarithmic decrement in still air for several modes of a 163-ft suspension bridge with unloaded backstays supporting a 3-ft wide plank walkway. Most of these curves show the typical form revealed by theory, as illustrated by Fig. 9, and by tests, as illustrated by the upper curve of Fig. 18. The curve in Fig. 25(*b*) illustrates strong Coulomb friction damping associated with the pronounced longitudinal motion of the suspended structure in the 1-node vertical mode. Possibly finer measurements would reveal a downward hook of the curve at low amplitude, but the increasing friction damping stifled the oscillation too rapidly to permit measurement at lower amplitude.

It was found that one of the flat plate section models tested in the laboratory represented quite well a ⅓ scale model of the deck of the Twin Creeks Bridge (Washington), and from the test data the atmospheric damping of the bridge was calculated for the 2-node vertical mode, shown in Fig. 25(*a*). The dashed curve represents the true structural damping, and the interval between the curves represents the atmospheric damping. Again, the characteristic variation is observed.

The damping of a light timber-decked foot bridge is, of course, no indication of the damping of suspension bridges in general. The tests were made at the same time on a 270-ft single lane highway bridge with plank floor, which showed much higher damping, this being attributed to the articulation of the joints of a relatively deep timber stiffening truss. Arne Selberg of Trondheim, Norway, has reported tests made on a number of girder-stiffened suspension bridges in Norway[8] that had either concrete deck or timber flooring. The decrements generally lie between 0.05 and 0.15.

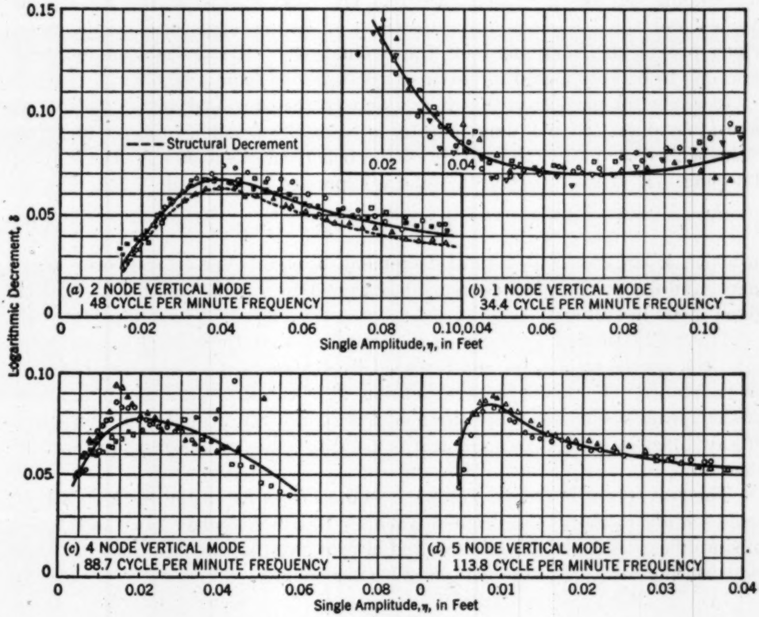

FIG. 25.—LOGARITHMIC DECREMENT IN STILL AIR FOR TWIN CREEKS BRIDGE

It is the writer's opinion that the reliable determination of the structural damping of suspension bridges will require at least a few tests in which suspension bridges of considerable size are mechanically oscillated in different modes in order to measure the rate of decay of their vibration. The theoretical analysis and laboratory tests presented in this paper and the more detailed treatment of damping by Mr. Bleich[9] should make it possible to expand the indications from a few bridges to secure a reliable estimate of the damping of suspension bridges of various types.

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[8] "Dampening Effect in Suspension Bridges," by Arne Selberg. *Publications*, International Assn. for Bridge and Structural Eng., Vol. 10, 1950, p. 183.

[9] "The Mathematical Theory of Vibration in Suspension Bridges," by Friedrich Bleich, C. B. McCullough, Richard Rosecrans, and George S. Vincent, U. S. Bureau of Public Roads, Govt. Printing Office, Washington D. C., 1950, Chapter VI.

Tests referred to in this discussion were made as a part of the cooperative research on the aerodynamic behavior of suspension bridges by the Bureau of Public Roads and the University of Washington at Seattle.

F. B. Farquharson,[10] M. ASCE.—This paper performs a very useful function in bringing before the profession the fundamentals of the mechanism of damping on this type of structure. Of equal value are the theoretical studies and the laboratory investigations on simple structural components, which provide a background for the evaluation of the probable amount of damping to be anticipated in various structural combinations.

It will be the purpose of this discussion to place on record certain additional laboratory data that will demonstrate, in connection with two different types of suspended structure, the nature of the beneficial effect of increase in total damping. In these tests the damping is expressed in terms of

$$\delta_{wo} = \delta_s + \delta_a \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (38)$$

in which δ_s equals the structural decrement, δ_a equals the atmospheric decrement, and δ_{wo} is the total decrement obtained by oscillating the model in a given mode in still air.

The evaluation of δ_s of the model requires that the model be removed from the spring suspension and replaced by streamlined weights of negligible air resistance. In this type of suspension all of the structural damping arises in the spring suspension since the model is rigid. Since neither δ_s nor δ_a are of any direct interest at this point, it is usual to make comparisons in terms of their sum, δ_{wo}.

The two sets of curves in Fig. 26 are taken from extensive tests on a 1/50 scale section model of the original Tacoma Narrows Bridge. In the interest of simplification the data are presented in prototype terms with the wind scale representing velocity in the field in miles per hour.

In Fig. 26(*a*) the restricted vertical mode has been investigated under conditions in which δ_{wo} was varied in eight steps between 0.1077 and 0.0278. It is evident that alteration in damping has a negligible effect on the critical velocity but that the effect on maximum amplitude is very marked. An increase in logarithmic decrement from $\delta_{wo} = 0.0278$ to $\delta_{wo} = 0.1077$—a 3.88 fold increase—resulted in a 3.4 fold decrease in amplitude.

In Fig. 26(*b*) torsional response curves are presented under 7 conditions of damping in which δ_{wo} varied between 0.0109 and 0.1066. Here it is clear that the critical velocity in these catastrophic response curves is highly sensitive to any change in level of damping. A 9.8 fold increase in the value of δ_{wo} corresponded to a 2.5 fold increase in critical velocity.

The data shown in Fig. 26(*b*) are taken from a very complete study of the effect of damping on the torsional mode for this section and has been somewhat simplified for this discussion through the elimination of another appearance of the same torsional mode (that appears at the same frequency). On this type of section the first appearance of the torsional mode (that is restricted or noncatastrophic) is completely suppressed by values of logarithmic decrement in excess of $\delta_{wo} = 0.039$. Below $\delta_{wo} = 0.035$ it becomes difficult to separate the catastrophic manifestation from the restricted, and the dashed portion of the curves B, C, and D represent a simplifying rationalization yielding a fair approximation of the catastrophic response.

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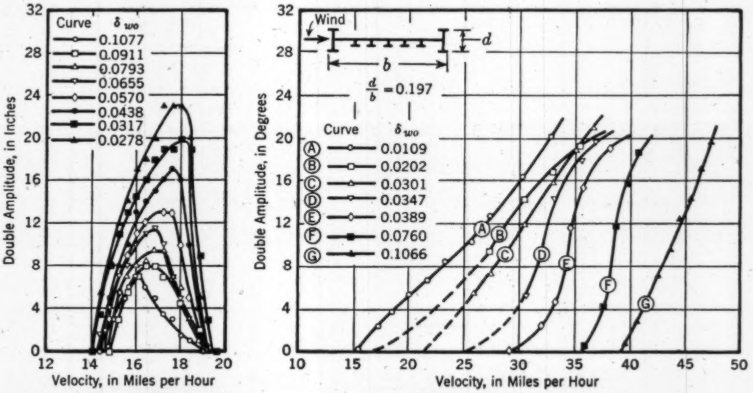
[10] Prof. Civ. Eng.; Director, Eng. Experiment Station, Univ. of Washington, Seattle, Wash.

In summary, it may be stated with respect to a girder-stiffened section that increase in damping is effective on restricted modes only in decreasing the amplitude, although in the catastrophic modes an increase in damping is effective in elevating the critical velocity of the response.



(a) VERTICAL MODE, $N = 18.1$ CYCLES PER MINUTE

(b) TORSIONAL MODE, $N = 16.4$ CYCLES PER MINUTE

FIG. 26

An additional example is shown in Fig. 27 and is taken from tests on the new Tacoma bridge in which stiffening was accomplished by means of a truss.[11] The principal improvement arising with this section, as compared with the original girder-stiffened section, was in the complete elimination of the vertical modes and the alteration of the catastrophic torsional response to a restricted form.

This restricted torsional response reacted to change in damping in exactly the same manner as did the vertical modes shown in Fig. 26(*a*). In Fig. 27 the logarithmic decrement was varied between values of $\delta_{wo} = 0.0152$ and $\delta_{wo} = 0.0365$, a 2.4 fold increase. This increase in damping resulted in 11.4 fold reduction in amplitude, and increasing the damping to $\delta_{wo} = 0.05$ resulted in the complete elimination of all motion.

It may be stated, in general, that such increases in Coulomb damping as may be realized by the author's method are chiefly significant in controlling or eliminating a restricted type of oscillation. This fact is well illustrated in Fig. 27 in which the restricted torsional response was completely controlled by a relatively moderate increase in damping. On the other hand, Fig. 26(*b*) is typical of the behavior of a section subject to a catastrophic response in which no reasonable increase in damping can be expected to increase the critical velocity to a tolerable level.

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[11] "Unusual Design Problems—Second Tacoma Narrows Bridge," by Charles E. Andrew. *Transactions*, ASCE, Vol. 114, 1949, p. 955.

This paper serves to emphasize again the need for a determination of the structural damping existing on certain representative suspension bridges. Until such information is available, it will be impossible to fully analyze the behavior of these structures in wind, and reliable prediction regarding proposed bridges will be difficult.

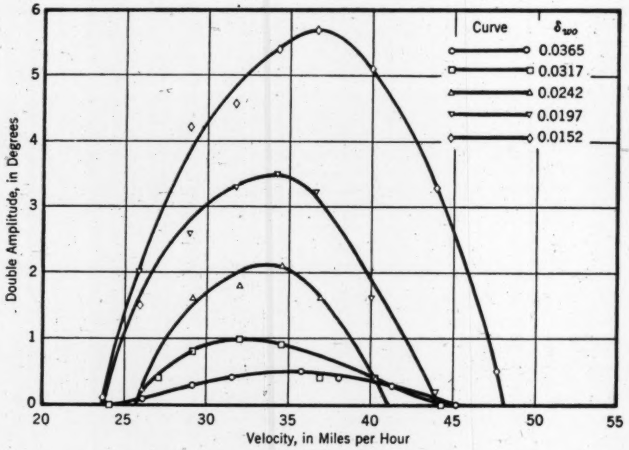

FIG. 27.—VARIATION OF AMPLITUDE IN THE TORSIONAL MODE; $N = 14.6$ CYCLES PER MINUTE AND ANGLE OF ATTACK = +10 DEGREES

The tests from which the data in this discussion were taken were made in the Structural Research Laboratory at the University of Washington at the request of the Washington Toll Bridge Authority and the Advisory Board on the Investigation of Suspension Bridges under a cooperative agreement with the Bureau of Public Roads.

ARNE SELBERG[12].—Since the writer is concerned with the same problem, this paper was of great interest to him. However the tests he conducted were on full-sized suspension bridges with spans varying from 230 to 525 ft.[13] The results of these tests are in agreement with those of the paper. For vertical oscillation with 1 node the observed values of the decrement δ varied in value from 0.07 to 0.16 for bridges with rolled steel beam stiffening girders and continous reinforced concrete slab. For bridges with timber flooring, the value of δ was between 0.12 and 0.22. Vertical oscillation with 2 nodes had δ values varying from 0.04 to 0.08 and 0.07 to 0.185 for the same bridges.

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[12] Prof., Institutt for Statikk, Norges Tekniske Høgskole, Trondheim, Norway.

[13] "Dampening Effect in Suspension Bridges," by Arne Selberg, *Publications*, International Assn. for Bridge and Structural Eng., Vol. 10, Zurich, Switzerland, 1950, p. 183.

The different values in δ are the result of frictional forces in action in bearings, suspenders, and other components, as a result of the longitudinal movements in the bridge for 1-node oscillation.

For torsional oscillation, the values of δ were between 0.185 and 0.05; for oscillation with 1 node, between 0.2 and 0.3; and for 2-nodes vibration, from 0.06 to 0.12 and 0.15 to 0.18. The tests show that timber flooring gives a high value of δ because of the internal frictional forces in the flooring. With continuous concrete flooring the frictional forces are of lesser importance, and, for some types of oscillations, dampening will be mainly the result of internal or hysteresis dampening.

As reinforced concrete is an important part of the suspended span in many suspension bridges, the writer is of the opinion that tests on the internal dampening in concrete slabs are of great interest and should be made in order to complete the investigation in the paper. However, tests with reinforced concrete slabs must also include torsional oscillations.

THE LATE FRIEDRICH BLEICH,[14] **M. ASCE, and L. W. TELLER,**[15] **M. ASCE.**—The discussions that have been presented are all contributions to the subject of the paper. The general approval of the paper by these discussers is particularly gratifying because of the intimate knowledge each has of the subject of structural damping.

The comparisons offered by Mr. Bernhard between his own experience in damping tests of full-size structures and certain observations reported in the paper indicate a good correlation.

Mr. Vincent is quite right in cautioning against attempts to extend the interpretation of the data on atmospheric damping, as given in the paper, beyond that intended. His analysis of the atmospheric component of the over-all damping observed for the truss is an interesting addition. The data obtained in the experiments with the 163-ft suspension bridge are interesting. The need for data on the damping characteristics of suspension bridges obtained from tests on full-size structures is quite apparent as Mr. Vincent points out. The conduct of such tests involves practical problems of some magnitude, however, and means for solving these problems satisfactorily are not yet available.

The laboratory data and the discussion of them offered by Mr. Farquharson are both interesting and enlightening. Damping tests with full-size suspension bridges, when they are made, will afford an opportunity for many useful comparisons with the damping data on full models obtained in the comprehensive studies conducted at the University of Washington.

The comparisons offered by Mr. Selberg as the result of his tests on a number of suspension bridges of moderate span in Norway are of interest. It is true, as Mr. Selberg points out, that tests of the internal damping characteristics of reinforced concrete slabs would have made a useful addition to the test program described in the paper.

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[14] Formerly with Frankland and Lienhard, Cons. Engrs., New York, N. Y. (Mr. Bleich died on February 17, 1950.)

[15] Prin. Highway Engr., U. S. Bureau of Public Roads, Washington, D. C.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 2487

MODEL AND PROTOTYPE STUDIES OF SAND TRAPS

By Ralph L. Parshall,[1] A. M. ASCE

With Discussion by Messrs. T. Blench, and Ralph L. Parshall

Synopsis

The deposition of water-borne gravel, sand, and silt has long been recognized as one of the most troublesome problems incident to the operation and maintenance of many of the irrigation and power canals of the west. Inordinate expenditures of time, labor, and money are made annually on this account. The reduction in the carrying capacity of a canal used for the delivery of water either for irrigation or for power means direct financial loss. In some cases periodic cleaning of the channels has resulted in such an accumulation that the spoil banks are approaching a limiting condition.

This problem has been investigated, primarily, at the Bellvue Hydraulic Laboratory at Fort Collins, Colo., by the Division of Irrigation and Water Conservation, Soil Conservation Service, United States Department of Agriculture, in cooperation with the Colorado Agricultural Experiment Station. From the investigations of various plans two practical methods have emerged to protect channels from bed-load deposits—the vortex tube and the riffle deflector, vortex-tube sand traps. They are capable of catching the bed load as it is moved along by the flowing water.

The Vortex Tube

The main feature of the vortex-tube sand trap, as shown in Fig. 1, is a tube with an opening along the top side, laid in the bottom of the channel at an angle of about 45° to the axis of flow. The elevation of the top edge or lip is the same as the bottom or grade of the channel. As the water flows over the opening, a pronounced whirling or spiraled vortical motion is established within the tube; this motion extends throughout the tube length. This

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Note.—Published in May, 1951, as *Proceedings-Separate No. 67*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Collaborator, SCS, U. S. Dept. of Agri. and Colorado Agri. & Mech. College, Experiment Station, Fort Collins, Colo.

whirling action catches the bed load as it passes over the lip of the opening and carries the sediment to the outlet at the downstream end of the tube, where it is discharged into a suitable sluiceway for disposal.

Laboratory tests, as well as field tests, indicate that the optimum action of the vortex tube occurs when the water passing over the lip is moving at or near the critical velocity, that is, when the velocity head is equal to one half the depth of water at the lip of the tube. In the laboratory various shapes, sizes, and lengths of tubes were studied. For a particular setting of a 4-in. tube, with a water velocity of approximately 2½ ft per sec over the lip and with 10% to 15% of the total flow wasted through the tube outlet, it was found that the rate of rotation near the outlet was about 200 rpm. For this condition sand, heavy gravel, and stones as large as hens' eggs were readily ejected (see Fig. 1). Further investigations with a 4-in. tube of uniform diameter produced a maximum rotation of the spiraled flow of 300 rpm., which created sufficient energy to move a cobblestone weighing 7½ lb along the tube at a uniform rate of about ½ ft per sec. The distance moved was about 7 ft, and the mean velocity of flow in the channel over the lip of the trap was 6.6 ft per sec. The axis of the tube was at a 30° angle to the axis of the channel and had a slope toward the outlet of 2 in. in 4 ft. Discharge of the tube was 3% of the total flow. When the 4-in. tube was set at an angle of 45° to the channel with a velocity of flow of 6.9 ft per sec, the maximum rate of rotation was

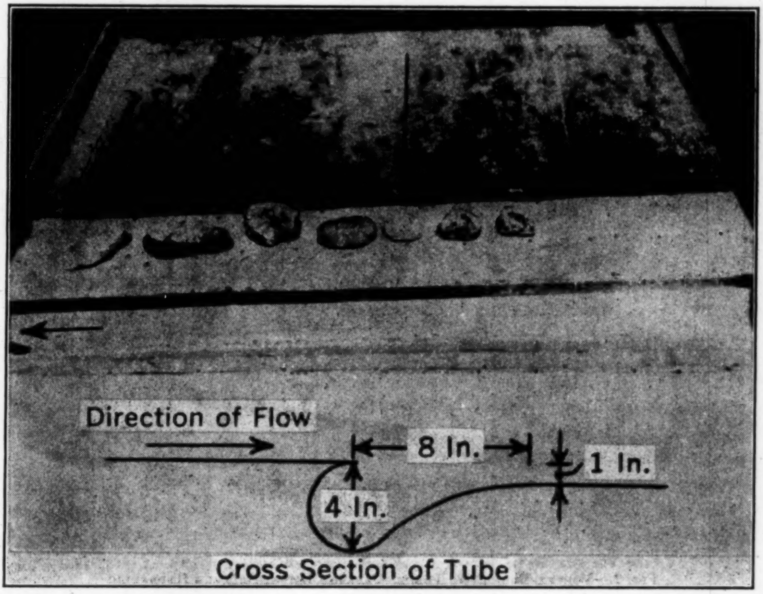

FIG. 1.—FOUR-INCH TUBE SET 90° TO THE AXIS OF THE CHANNEL

500 rpm, and enough energy was developed to transport cobblestones weighing from 3 lb to 4 lb. Laboratory tests on a vortex tube indicated that the efficiency in diverting the bed load would be approximately 90%. No particular success was attained in small-scale model studies of the vortex tube. Therefore, the laboratory work was confined largely to tube sizes ranging from 4 in. to 12 in. in diameter, which were assumed to be full-scale dimensions.

The vortex-tube sand trap has been tried in the field, and in some cases it has been a failure, whereas in other instances it has proved successful. Two conditions have been found to be awry in installations that have been ineffective: First, the velocity in the canal was too low, and, second, the tube was set below grade in the channel. In the Jackson Ditch at the Bellvue Laboratory (shown in Fig. 2) a vortex tube has been in successful operation since April, 1935. The structure is of concrete, is 8 ft wide, and has vertical walls and a channel rising slightly from the upstream end of the floor to the lip of the tube and sloping downward from the tube to the downstream end of the floor. The tube itself is cast of concrete and has a diameter at the outlet end of 8 in. and a back end diameter of 6 in. The tube is at an angle of 45° to the axis of the channel. The capacity of the Jackson Ditch is approximately 60 cu ft per sec, and during periods of moderate to low river stage the diversion in the canal is 11 cu ft per sec. It appears that the tube is especially active at this minimum flow. No tests have been made on this particular

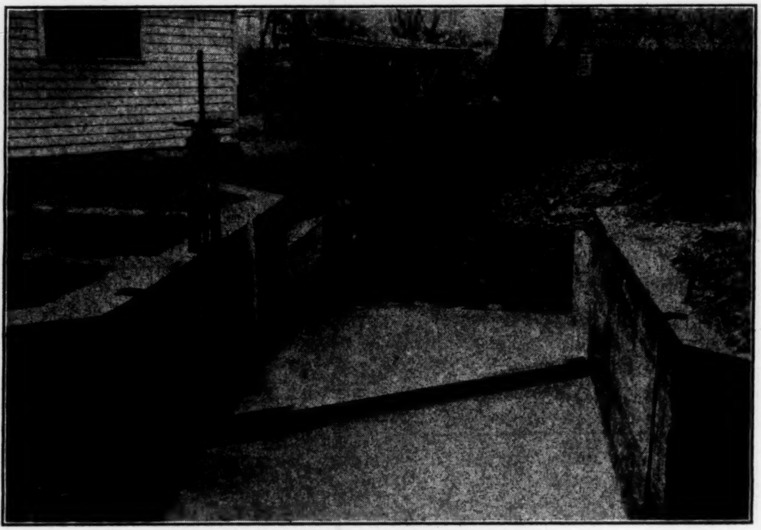

FIG. 2.—VORTEX-TUBE SAND TRAP IN JACKSON DITCH, BELLVUE HYDRAULIC LABORATORY, BELLVUE, COLO.

installation, but evidence of the deposit accumulated at the end of the outlet indicates definitely that this sand trap is efficient.

THE RIFFLE-VANE DEFLECTOR

The riffle-vane deflector sand trap consists of a series of curved sheet metal vanes. The plan view approximates a quadrant of a circle, and the side view is triangular in outline with the top point curved toward the downstream side, as shown in Fig. 3. The sheet metal vanes, 10 in. high and spaced about 10 in. apart, are attached in a vertical position to the smooth, level floor of a channel, and on some installations the floor is sloped laterally. Experiments were made with deflectors of various sizes, shapes, and spacing.

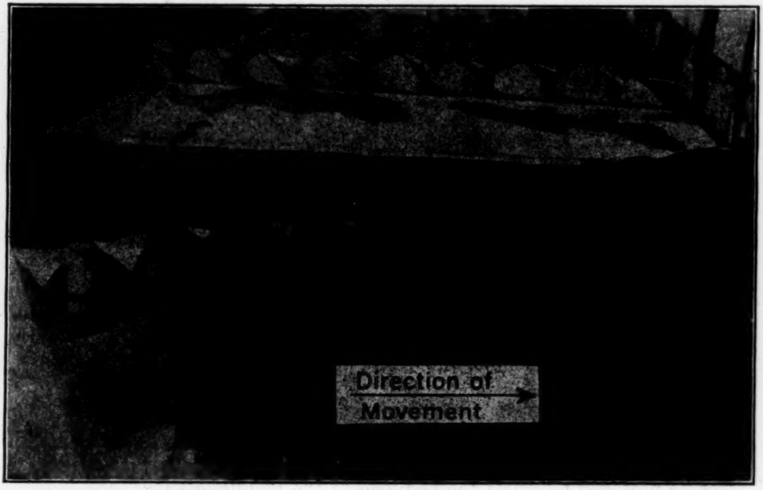

FIG. 3.—EXPERIMENTAL RIFFLE VANES SET AT ANGLES OF 90° AND 135° WITH THE AXIS OF AN 8-FT LABORATORY CHANNEL

For some settings the action in moving the bed load was remarkably good. When the riffles were set in line normal to the axis of the canal, the bed load was moved laterally at 90° toward the outlet in a well-defined, limited area immediately downstream from the line of riffles. The curvature of the riffle produced the effect which moved the load laterally, and the flow of water over the riffles resulted in a condition that maintained the bed load in a ridge downstream from the riffles. The action of the riffles was apparently independent of the depth of flow. Under maximum conditions of channel velocity (approximately 4 ft per sec) the energy developed was sufficient to move large cobblestones, exceeding sizes which would pass through a 4-in. square opening.

Field tests have been made of this type of sand trap, but the device was not too successful. One of the disadvantages is that considerable debris lodges on the riffles, thus destroying the efficiency of the trap. One installation was made in the Wannamaker Ditch near Golden, Colo., where a series of six sets of riffles were placed in a channel 6 ft wide. The riffles were 6 in. high. Observations made on this installation indicated that the sand removal under optimum conditions was about 15 lb per sec. If the vanes are kept free of lodged material, such as roots, grass, tree branches, and other fibrous matter, this type of sand trap should be highly effective. The riffle-vane deflector sand trap is best suited for moderate channel velocities of 2 to 3 ft per sec. It is not intended for wide channels.

THE RIFFLE-DEFLECTOR VORTEX TUBE

The most promising and practical sand trap developed is the riffle-deflector, vortex-tube type. In general, this device consists of a series of curved riffle deflectors on the bed of the channel. The bed load is moved laterally to the end of the riffles, where it is taken off through small vortex tubes discharging into a common compartment, as shown in Fig. 4. This compartment is provided with an outlet that sluices the total trap load back to the river, downstream from the diversion dam, or it is disposed of in basins as waste material. The riffles are parabolic in plan, have a vertical upstream face height of about 1½ ft, and a top surface sloping downward to a feather edge in a distance of about 5 ft downstream, as measured parallel to the axis of the channel. A cross section of a single riffle is a right triangle. Since the riffles are curved identically in plan, they are fitted progressively, one against the other, in a downstream direction. The number of riffles depends on the nature of the bed load. For coarse sand and gravel fewer riffles would be required, whereas for fine sand the number of riffles would be greater—in no case probably exceeding a set of ten or twelve riffles.

This type of trap can be readily adapted to either narrow or wide channels and to flows ranging from less than 10 cu ft per sec to more than 2,000 cu ft per sec. For canals 20 ft to 50 ft wide it is suggested that two series of riffles be provided to crowd the bed load to the middle or axis of the channel. Short vortex tubes at the ends of the riffles carry the load into a narrow compartment built along the center line of the channel, where an outlet pipe conducts the trapped sand and water to the point of disposal. For channels ranging in width from 50 to 100 ft a double installation with four series of riffles is recommended. The outlets from the twin installation join in a common pipe placed below the bed of the canal, leading under the canal bank. Laboratory tests on models of the riffle-deflector, vortex-tube trap indicate that it is capable of capturing 90% or more of a weighed bed-load sample introduced in the approach channel upstream from the riffles.

A few field installations of this type of sand trap have been made, and all have been successful in catching bed loads, not only of relatively coarse material but also of fine sand. As an experimental installation of such a trap, a timber structure capable of handling very fine sand was built in the Sheep Creek Ditch near Torrington, Wyo.

A reinforced concrete structure, placed in a supply canal for the Colorado Fuel and Iron Co. (Pueblo, Colo.), has a channel 40 ft wide with a capacity of 250 cu ft per sec. It consists of two sets of riffles to move the bed load toward the center of the channel, where short vortex tubes pick it up and discharge it into a narrow longitudinal compartment. A 24-in. concrete pipe serves as an outlet from the downstream end of the compartment to the river. No tests have been made on this particular structure, but, after five years of continuous use of the trap, observations along the canal downstream from the heading indicate that there is very little, if any, river sand found in the channel. It is assumed, therefore, that the efficiency of this installation is reasonably high.

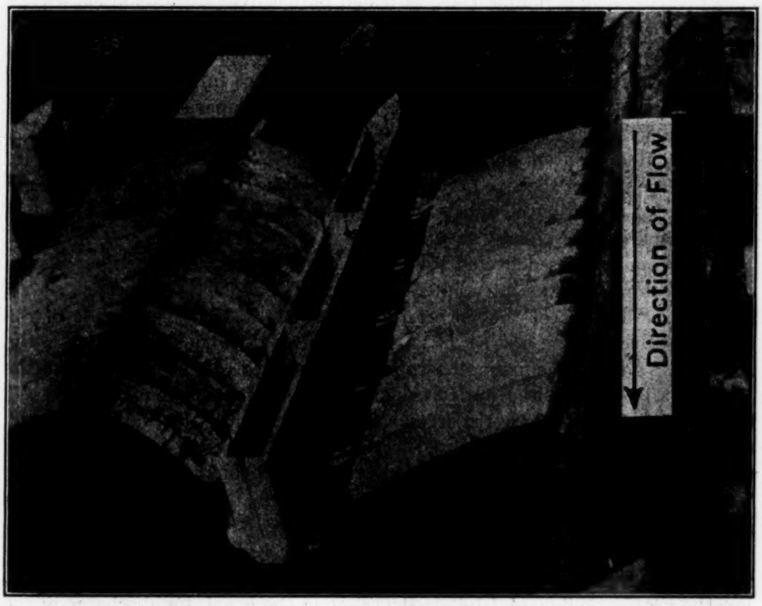

Fig. 4.—A Riffle-Deflector, Vortex-Tube Sand Trap

Another problem requiring solution was the desanding of the Consolidated Irrigation District Canal that diverts from the King's River near Fresno, Calif. This canal has a bottom width of 90 ft at the heading and a capacity of more than 2,000 cu ft per sec. Excessive cleaning operations resulted in great piles of sand at places along the canal, and in some instances the disposal piles encroached on vineyard lands that had to be purchased at $1,000 per acre. Dragline cleaning was necessary at various points every season.

Because of the apparent success of the riffle-deflector, vortex-tube type of sand trap—especially the installation in the supply canal to the Colorado Fuel and Iron steel works—the Consolidated Irrigation District arranged for the building of a model at the Bellvue Hydraulic Laboratory, where investigations and studies would be made of this type of trap.

MODEL STUDIES OF THE PROPOSED RIFFLE–DEFLECTOR, VORTEX–TUBE SAND TRAP

This model was constructed of common ¾-in. surfaced lumber. It was 41 ft long and occupied the full width of the 14-ft laboratory channel. The model consisted of a double trap, of which both halves were identical except for the type of collection chambers. In one case a straight side-wall, sediment-storage compartment was used. The compartment was 4 ft wide inside and 62.5 ft long with a streamlined upstream end to reduce turbulent flow over the riffles and the tubes. As an alternate design for comparison, the other half of the model was provided with six vertical 48-in. concrete pipe collectors spaced 10 ft on centers and connected at the base by 24-in. concrete pipes. A streamline splitter was fixed to the front side of the first vertical pipe. These two types of collection chambers were studied independently as to efficiency and were found to be equally satisfactory.

The sediment was deflected toward the central collection chamber by the riffle deflectors and was intercepted by a series of six vortex tubes, as shown in Fig. 4. The lips of the vortex tubes were 1.5 ft below the grade of the channel. Because the first set of vortex tubes caught about two thirds of the total bed load and the first and second sets combined caught about four fifths of the total bed load, the outlet from each type of collection chamber was located at the point where the second set of vortex tubes discharged into the collection chamber. This required that the sediment from the vortex tubes farther downstream would have to move upstream in the collector, but the largest quantity of sediment would move a minimum distance in the collection chamber.

To gage the discharge through the model, a 40-ft measuring flume, constructed to scaled dimensions, was placed downstream from the last riffles. For the prototype flow of 2,100 cu ft per sec in the canal, the discharge of the 40-ft model flume was 6.64 cu ft per sec, and the discharge of the sediment-removal system was 0.32 cu ft per sec. The canal discharge was regulated by means of flashboards, and the sediment discharge was regulated by slide valves at the outlet ends of the vortex tubes. The approach channel was 90 ft wide for a distance of 185 ft with a scaled grade equal to that of the actual canal and side slopes of one to one.

Model testing, especially of sand-trapping devices, has never been very satisfactory because of difficulty in selecting the media used to simulate sand of various kinds and degrees of fineness. In these studies materials used included coarse and fine sand; silica, such as standard sand for cement briquette testing; pulverized soft and anthracite coals; pulverized hard pressed brick; soy beans; wheat; ground corn; and treated sawdust. Although it was not tried, it is believed that colored plastic material in suitable form would be found adaptable as a medium for model studies. For the Consolidated Irrigation District sand trap model actual sand from the canal, ordinary river sand obtained at the laboratory, and pulverized coal and brick were tried in the tests. Coal was found satisfactory for observing the action of the riffles and vortex tubes as well as the movement within the chambers toward the outlets. For the most part the discharge was for full capacity of the canal—about 2,100 cu ft per sec. The rate of flow in the channel was determined by measurement through the 40-ft model flume, and the sediment discharge was determined by a 3-in. flume. The measured discharge over a 4-ft rectangular weir and the discharge through the 40-ft model flume showed agreement within about 2%.

The tests on the model were not extensive. Trial runs were made to permit visual observation of the general action of the riffles, of the tubes, and of the movement within the collecting chambers. Alterations in settings were made to prove the general over-all efficiency of the model.

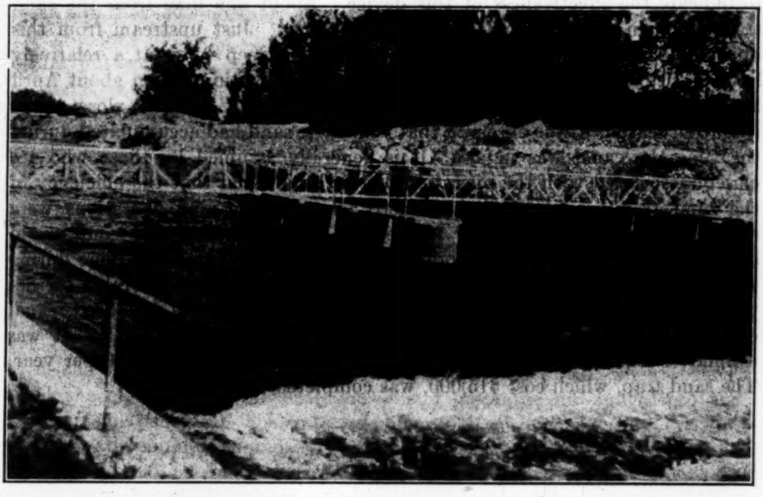

FIG. 5.—FLOW OF 1,100 CU FT PER SEC OVER THE WEIR, LOOKING UPSTREAM IN THE CANAL OF THE CONSOLIDATED IRRIGATION DISTRICT

Officials of the Irrigation District inspected the model in operation, and certain changes and alterations in design were discussed. In building the prototype, the primary change made to reduce costs was to eliminate the 40-ft measuring flume and replace it with a low-crest weir across the canal immediately downstream from the sand trap structure. The 48-in. vertical concrete-pipe collecting wells were selected in lieu of the straight-walled compartment. Because of the relatively small percentage of bed load captured by the two downstream wells, it was decided to have four wells instead of six wells, as set up in the model.

For a flow of 1,788 cu ft per sec over the control weir of the prototype, the mean velocity over the riffles was found by current meter observation to be 3.91 ft per sec. For this canal discharge, the mean velocity of the model was computed to be approximately 3 ft per sec, making a difference of about 1 ft per sec. The canal section upstream from the sand trap, as shown in Fig. 5. is straight, very uniform in width, and has a smooth cobble bed. The approaching flow, as indicated by the water surface, is smooth. Sand deposits on the bed of this section of the channel may have been present. If so, their presence was not indicated by boils at the water surface. Water was diverted uniformly into the canal through eight 10-ft radial gates. The two outside gates were closed. For low flow (1,000 cu ft per sec or less) the mean velocities over the east, central, and west sections of the sand trap were definitely unbalanced. They were greater in the central section, as could be expected.

The efficiency of this large sand trap could be tested on a full scale in the field at considerable cost in time and money. Nevertheless, detailed observations of the canal at various distances downstream from the sand trap appear to indicate the effectiveness of this device. About one mile from the sand trap the main canal is divided into two branches. Just upstream from this bifurcation the canal is of considerable width with deep water at a relatively low velocity. Before the beginning of the 1949 irrigation season about April 1, this section of the canal was cleansed of sand deposit. At the close of the irrigation season, it was observed that very little sand had accumulated during the time the canal was in use. It is generally assumed by those who are closely associated with the operation of this large canal that the efficiency of the sand trap exceeds 75%. Formerly, dragline cleaning operations were necessary each year, especially for certain stretches where the sand would deposit to a depth of 4 ft or more per season. No cleaning was done during 1949. Sand in transit in the channels will move to places favorable for depositing and later will be dredged out. Once the system is stabilized, canal cleaning should be necessary only at three- to five-year intervals. It was estimated that the previous annual cleaning operations cost $4,000 per year. The sand trap, which cost $15,000, was completed December, 1947.

DISCUSSION

T. Blench,[2] M. ASCE.—The first bed-sediment ejector in Pakistan (previously India) was devised in the early 1930's for a flow of approximately 7,000 cu ft per sec by E. S. Crump of the Punjab Irrigation Service. The circumstances are interesting. The Upper Jhelum Canal was completed about 1903, with the main object of carrying feeder water some 100 miles from the Jhelum River to the Chenab River to assist the Lower Chenab Canal. The first 50 miles were excavated in the tow of erodible foothills running along the Jhelum. In the upstream portion of these 50 miles several torrents existed of such size that level-crossings were arranged. It transpired later that, when these crossings operated during spates, coarse sediment entered the canal. Nothing untoward happened until about 1930, when water levels in the plains area of the canal downstream of the first 50 miles started to rise to such an extent that canal supply had to be curtailed in the interests of bank safety. The cause, shown by hydraulic surveys, was that the slugs of coarse sediment injected by torrents, and unnoticed in the upstream reaches (in which slope remained slightly in excess of regime because of excavation in relatively inerodible clay) had at last approached the regime reach of the plains and caused an enhancement of bed factor. Luckily, the tail end of the first 50 miles possessed several siphons, well below canal bed level, to take moderate-sized cross-drainages. So, Mr. Crump selected such a site and built what was in effect a tunnel across the canal with its roof at canal bed level. He provided holes in the roof for bed sand to drop into, blocked one end of the tunnel, and provided the other with a gate which, when opened, caused the ejector to work continuously and discharge its sludge into the cross-drainage beneath. The gate was operated whenever a slug arrived, provided water could be spared. The results were so good that several siphons were adapted in the same way. Within a few months the canal was back to normal. Since that time practically every new system has been provided with ejectors near headworks so 10,000-cu-ft-per-sec ejectors may be taken as established practice.

Such ejectors can be 100% efficient for coarse material such as gravel. For fine sand their efficiency is poor, and two or three have to be built in series to have much effect. The inefficiency for fine materials arises from the ease with which these materials go into suspension. There has been little model work as the operation of the prototypes is understood; model work has been concentrated on excluders.

Ralph L. Parshall,[3] A. M. ASCE.—The subject of the theory and design of devices to capture the bed load moving in a channel has been one of long standing. Sand traps are much like other hydraulic devices in that no one design has proved to be of universal application. Many different schemes have been applied successfully to meet particular conditions as to the nature of the bed load, width of channel, and the velocity of flow and disposal. It is the usual assumption that sand traps are intended for the removal of the bed load and not for practical application for catching a substantial amount of the suspended material. To cope with finely divided particles in suspension it is necessary to reduce the channel velocity to such a low limit that no movement is induced in the heavier bed-load material.

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The discussion by Mr. Blench is an interesting contribution to the subject of disposal of bed-load material carried in canals. The scheme developed by Mr. Crump of the Punjab Irrigation Service in India is a very practical means of protecting channels from bed-load deposit. However, the outlet conditions often dictate the efficient operation of a sand-trapping device, and for the Upper Jhelum Canal there is evidently a sufficient head for sluicing out the deposit in the cross tunnel. It is the usual experience in installations near the diversion that the head available for sluicing is quite often limited, and for this reason the sand trap must be located further downstream from the heading. When the location is at a considerable distance from the headworks, the upper section of the channel may become fouled by bed-load deposit resulting in a marked reduction in the carry capacity of the canal.

A riffle deflector vortex-tube sand trap of the latest design is now being constructed in the Beardsley Canal, Kern County Land Company, Bakersfield, Calif. The width of channel is 35 ft; the depth of water, 6 ft at a velocity of about 3 ft per sec; and has a capacity of 700 cu ft per sec. The bed load is medium size sand in variable amounts depending on the stage of the river and the extent of the canal diversion. The outlet from the sand trap consists of a 30-in. concrete pipe about 100 ft long. The head available for sluicing is ample.

This trap consists of nine concrete riffles placed on each side of a reinforced concrete compartment located on the axis of the channel. The end of the riffles are 3 ft from the face of the compartment wall. There are five short vortex tubes feeding into the compartment from each side at 10-ft intervals. These tubes are of the same design and dimension as used in the Consolidated Irrigation District Canal and described in the paper. A partition wall runs from the check structure in the canal, 50 ft downstream from the end of the compartment, separating the channel into two equal parts. This arrangement provides for a more favorable flow over the riffles for low to moderate discharges in the canal. For the full capacity of discharge in the canal both sets of riffles go into operation.

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Paper No. 2488

NATIONAL GEODESY—STATUS AND PLANNING

BY LEO OTIS COLBERT,[1] M. ASCE

WITH DISCUSSION BY MESSRS. BENJAMIN EVERETT BEAVIN, MILTON O. SCHMIDT, AND LEO OTIS COLBERT.

SYNOPSIS

From its earliest beginning the United States Coast and Geodetic Survey (USCGS) has been engaged in establishing a federal geodetic framework of horizontal and vertical control as a prerequisite to national mapping and as a base for land surveys and engineering projects. This paper describes the development of the geodetic network in the United States, the present status of control surveys, the planning of future geodetic surveys, and the part that engineers can play in helping to preserve the many thousands of monumented stations and bench marks that have been established.

INTRODUCTION

The activities of the Coast and Geodetic Survey of the United States Department of Commerce date back to the early part of the nineteenth century when maritime nations generally began to recognize the wisdom of systematically surveying and charting their coastal waters.

In those days the United States was concentrated along the Atlantic coastal plain. Fisheries and shipbuilding were important industries, and water-borne commerce was the medium for the movement and exchange of products between the various states and with foreign countries. The promotion of that commerce was the prime reason for the passage of a resolution by Congress, in 1807, directing the President of the United States to order a survey of the coast and the outlying islands and fishing banks. President Thomas Jefferson asked the advice of the American Philosophical Society, the foremost scientific body of the day, as to how the survey should be undertaken. Although the task assigned was a practical one, he intended that the survey should follow scientific principles that would stand the test of time. Various plans were discussed. The plan submitted by Ferdinand Hassler was adopted,

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and he was named the first director of the survey operations. As a scientist, and as one of the originators of the Geodetic Survey of Switzerland, Mr. Hassler knew the importance of building on a permanent scientific foundation, no matter how urgent the need for quick results. It is to his lasting credit that he did not allow himself to be stampeded into haphazard decisions. Had he planned to meet only the limited needs of his time, his work would have had to be done over at great expense in later years. From the very beginning, Mr. Hassler insisted that the surveys of harbors and coastal areas of the United States should be controlled by a backbone of geodetic measurements. His work had the foundation of accuracy adequate to satisfy future needs.

As the United States grew, the work of the Coast and Geodetic Survey was extended to the Pacific and Gulf coasts. In 1871, Congress authorized a geodetic connection to be made between the Atlantic and Pacific coasts, and charged the Coast Survey with the duty of providing the states with geographic positions and bench marks for the control of their topographic and geologic mapping. Since that time, it has been actively engaged in extending triangulation and precise levels in the interior of the United States for the purpose of establishing a federal framework on which all land surveys and engineering projects could be based.

Geodetic control surveys are necessary in connection with regional development programs so that the various aspects of topography may be correctly mapped and adequate studies of physical characteristics may be made in relation to the area as a whole. Without such control, gaps or overlaps occur between contiguous local surveys, and errors of considerable magnitude may result.

In the national mapping program, close cooperation exists between the Coast and Geodetic Survey and the Geological Survey of the United States Department of the Interior. The work of the former in providing the fundamental framework of horizontal and vertical control for the United States is planned to correlate with a comprehensive program, and priority is given when needed to areas where topographic surveys are proposed.

Horizontal Control

To span the continent from coast to coast and from north to south, widely spaced arcs of triangulation were first established—giving a preliminary framework. Intermediate arcs, spaced from 40 miles to 60 miles apart, provided control for boundary determinations and met the general needs of the various states. The next logical step has been followed, in the decade from 1940 to 1950, by filling in between arcs with area triangulation, which has been adopted to obtain uniform coverage in the area and to secure consistency in the results. Positions determined by such area triangulation are located accurately with reference to points in all directions, and consequently they tie into the previously observed arcs. The observations of area triangulation are given a final simultaneous adjustment within each 40-mile to 60-mile square.

To insure adequate agreement among the component parts of the triangulation, basic criteria are adopted for the observations. In first-order work the average triangle closure must not exceed 1 sec; and the final length, when carried throughout the chain of triangles from one measured base to another, must agree on an average of about 1 part in 75,000.

At least one triangulation station is being established in each 7½-min topographic quadrangle. Stations at 4-mile intervals are being provided along the main highways in agricultural areas and closer spacing is used in metropolitan areas. To coordinate local urban surveys, at least one federal base is measured in metropolitan areas of 100,000 population or more.

In July, 1949, there were more than 110,000 miles of arcs of first-order triangulation and second-order triangulation in the United States, with monumented stations established at an average of about 10 miles along the various arcs. Geographic positions (that is, latitude, longitude, and azimuth) have been determined for about 150,000 stations consisting of monumented points and prominent objects, such as water tanks, church spires, cupolas, chimneys, etc. To facilitate the use of the national network a near-by azimuth mark is established at each monumented station to furnish the local engineer and surveyor with directional control for local surveys without the necessity of establishing a true meridian line.

Vertical Control

Another phase of geodetic operations is the precise leveling for the determination of elevations of bench marks above mean sea level. In the distribution of the vertical control of the United States, the Coast and Geodetic Survey followed a program starting with widely spaced lines 100 miles apart, supplemented by lines spaced 50 miles apart. The present program provides for area leveling in which lines spaced not more than 6 miles apart are connected to the wider spaced lines in a consistent network of levels. Bench marks are set at 1-mile intervals along each line. For convenience in running and in the subsequent use of the bench marks, the lines of leveling follow the routes of highways and railroad systems. In July, 1949, there were more than 360,000 miles of first-order leveling and second-order leveling in the United States, and upward of 275,000 established bench marks.

Programming Future Geodetic Operations

During the past several years geodetic control has been established in some of the larger river basins and valleys—the most prominent of which are the Columbia River Basin and the Missouri River Valley—in anticipation of multiple water-use projects to be undertaken in these areas, namely, reclamation, navigation, and flood control. These projects must consider a river system as an integrated whole and require planning on an extensive area basis. Comprehensive horizontal and vertical control surveys should precede the planning stage to coordinate detailed maps of the river basins, dam sites, and reservoir locations. Local surveys based on geodetic control enable conclusive studies to be made of alternate sites with a view to selecting the most economical and efficient ones. In the final work, local construction surveys will be based on the geodetic control monuments.

Close contact is maintained with federal, state, county, and municipal organizations that use control surveys. An effort is made to plan field work to meet their most urgent needs. At the present time the demand is three times that which can be provided with the funds available. Under such conditions, only projects of the highest priority can be undertaken. This is by no means a satisfactory arrangement from the standpoint of the agencies whose programs may have to be postponed.

Geodetic Surveys Outside Continental United States

The extension of geodetic control surveys into the heart of Alaska is one of the formidable tasks that face the Survey engineers. In isolated regions, frequently with temperatures below zero, many difficult problems are encountered, not the least of which is the problem of logistics. Careful advance planning is required to insure that necessary supplies and replacements are on hand when needed.

Alaska is now tied to the continental United States by continuous arcs of triangulation that extend from Puget Sound through southeast Alaska and along the Alaska Highway through Canada and Alaska to Cape Prince of Wales and the Little Diomede Island in Bering Strait. There is an extenisve spur along the Alaskan Peninsula, which continues through the Aleutian Islands to Adak, Kiska, and Attu.

Triangulation surveys are now in progress in western Alaska and along the Arctic coast. At about 5-mile intervals along this northern coast, stations between first-order accuracy and second-order accuracy will be established. Trigonometric leveling is being carried on concurrently, and elevations and positions are being determined for the numerous peaks in the extensive mountain ranges of northern Alaska. In the summer of 1949, four parties were engaged on these surveys. Transportation for these parties was furnished by wheel and float planes, helicopters, amphibious tractors, and various types of floating equipment.

The ultimate goal of the geodetic work in Alaska, apart from furnishing control for the survey of the water areas, is the establishment of the network in the interior. Only when this network is complete will it be possible to provide large-scale mapping and to explore fully the commercial, industrial, and strategic possibilities of Alaska.

In the Hawaiian Islands, the Coast and Geodetic Survey and the Territorial Survey Office are cooperating on a geodetic project of significance—the reconstitution and extension of horizontal control. The program contemplates the reestablishment of marks and monuments destroyed or uprooted during World War II in the haste of large-scale defensive preparation. The new work will provide a more comprehensive scheme with closer spacing of monuments for use in mapping and for land boundary determination.

Office Adjustments of Geodetic Control

Field surveys are but one phase of national geodesy. If the instruments were perfect, if there were no refraction or bending of light rays, if there were no personal equations, and if exact pointings and observations were always made, there would be no discrepancies in the triangulation or the leveling observations. The surveys would have no errors of closure, the triangulation lengths and directions would agree whenever they met, and the level lines would check between loops.

Of course, these conditions do not exist. Small discrepancies develop which by themselves may be tolerated, but which in the aggregate must be eliminated by mathematical treatment so that all parts of the triangulation and leveling are in harmony. About seventy mathematicians in the Washington (D. C.) and New York (N. Y.) offices are engaged in making these computations and adjustments, some of which require the solution of as many as three hundred simultaneous equations.

To meet the impact of modern requirements on this work, together with the complexities that result from the expanded geodetic program, a study of punch-card methods of computation was made. After developing particular techniques for processing the field data, some high-speed calculating machines were installed. After a 2-year trial, these were found to be particularly applicable to the adjustment of extensive triangulation networks. These machines save many man-hours and relieve the personnel of much of the strain characteristic of this work.

Final results of the observations of the triangulation and traverse surveys of the Coast and Geodetic Survey are computed on two different systems of coordinates—geographic and rectangular. Geographic coordinates are expressed in terms of latitudes and longitudes. Such cooridnates have the advantage that they constitute a universal system, and any point on the earth is directly related to any other point.

For surveys of limited extent, the system of "State Plane Coordinates," devised by the Survey, has a distinct advantage in that it gives a method of connecting local surveys to the national network of control by the usual methods of plane surveying, with which all engineers and suveyors are familiar.

To encourage wider use of the national network and to facilitate procurement of information, two series of state maps are published, one for triangulation data and the other for leveling data. The scales are large enough (about 1:650,000) to show graphically each station in the existing scheme of first-order triangulation and traverse and of second-order triangulation and traverse and to indicate the locations of supplemental and intersectional triangulation stations. All lines of levels are shown. These maps are available on request to the Washington office.

Cooperation of Engineers

The cooperation of engineers is solicited to help preserve the many thousands of triangulation monuments and bench marks that have been established throughout the United States. These survey monuments, which are usually bronze disks, set in concrete or bedrock, are the visible results of the field surveys. They are useful only in so far as they can be recovered. For the first few years after their establishment they can usually be found quite readily. After the lapse of a few years, however, features of the surrounding terrain may have changed, making it difficult to find the monuments from the original descriptions. Engineers and surveyors who visit or use these monuments, and who find that their descriptions need modification, will do a public service by notifying the Coast and Geodetic Survey of present conditions, revising the descriptions as necessary.

New construction and improvements (particularly on highways), which sometimes mean disturbance of survey monuments, also contribute to this problem. Notification should be made by the local engineer as soon as it becomes known that a survey mark must be moved, or will be destroyed, when construction actually gets under way. In most cases, a replacement station can be established by the engineer or surveyor, and instructions will be forwarded on the methods to be followed in order that the requisite accuracy may be retained.

Conclusion

It is in the best public interest to have all surveys—no matter how localized—tied in with the national network of geodetic control. The extent to which this ideal is reached will depend largely on the availability of control points, and on their use. Triangulation stations and bench marks must be brought within the reach of the local surveyor and engineer.

Although substantial progress has been made in providing basic control data for the United States, the job is far from complete. Accurate control surveys must be provided at an accelerated rate, so that the program of national geodesy will be able to keep abreast of the expanding engineering requirements of commerce and industry.

DISCUSSION

Benjamin Everett Beavin,[2] M. ASCE.—In his paper, the author has been very generous in his praise of the first director of the survey, Ferdinand Hassler. Having had the opportunity to retrace some of the earliest first-order triangulation, the writer can subscribe heartily to those sentiments. However, the succeeding directors, among whom the author looms large should not be forgotten because of the steady, devoted manner in which they upheld the original standards of accuracy, improving upon them as scientific progress permitted.

The horizontal and vertical control surveys of the United States have always been insufficient to fill national needs, not because of shortcomings of the United States Coast and Geodetic Survey (USCGS) but because of the lack of appropriations adequate to perform the work and to replace the stations lost for one reason or another.

In many localities engineers and surveyors have not taken full advantage of the control available. If appropriations are to become adequate, the primary impetus must come from a sustained demand for such information from those who may use it to best advantage, namely, the engineers and surveyors.

Milton O. Schmidt,[3] A. M. ASCE.—Severe abbreviation characterizes this otherwise useful presentation of the progress of geodetic surveying in the United States and its possessions. What, then, can one discuss about a simple informative circular scarcely six pages in length, that provides an emasculated resumé of the activities of the Division of Geodesy of the USCGS and requests the indulgence of civil engineers in order that federal survey monuments be preserved? Some civil engineers do not know the meaning of geodesy, nor will their interest be engaged to learn something about it in the space of this article. Some engineers are not aware of the difference between the USCGS and the United States Geological Survey and are unable to distinguish between their survey markers. This ought to be quite disturbing to both agencies. The term "national geodesy" connotes to the reader the problems attending the study and choice of a spheroid of reference for the country's geodetic surveys. Many engineers, like devotees of the late Wilbur Glen Voliva, are hardly cognizant of the complexity of the earth's true shape and its impact upon national and international cartographic projects.

The author has not provided an adequate picture of existing basic control data; and estimates regarding the time, funds, and personnel needed to complete the job are completely lacking. Mention could have been made of the opportunities for graduate civil engineers in the field service of the USCGS and of such topics as studies on the longevity and stability of bench marks, astronomic position determination programs, innovations in instrumental equipment, the several sea level datums since the time of the first tidal observations, the conditions under which traverse is favored for first-order horizontal control, and the relative amounts of traverse and triangulation and of first- and second-order work of both types executed by the Survey.

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The intrinsic advantages of a uniform horizontal datum could have been enunciated again, for they still are not well understood by some engineers associated with large areal and extensive route surveys. Increasing density of horizontal control will favor greater use of state-wide plane coordinates and aid in securing better appreciation of the many thousands of survey markers placed by the federal surveying and mapping agencies.

A paper attempting to treat this topic within the restrictions of a few pages must deal with generalities and be trimmed of the meat and muscle that make for inspiring study by the interested reader.

Leo Otis Colbert,[4] M. ASCE.—The scientific foundation on which the USCGS was organized is properly emphasized by Mr. Beavin. Such a foundation is the result of the foresight of Ferdinand Hassler, the first director, who adopted at the inception rigid criteria to insure precise survey procedure. The basic principles established by Mr. Hassler have been followed throughout the history of the Survey. Geodetic stations have been established and located to a thousandth of a second in latitude and longitude. Mr. Beavin was successful in recovering the subsurface mark for Kent Island North Base, originally established in 1841, after all surface indications of the mark had disappeared. This was accomplished by establishing a temporary survey point in the general area of the original survey station and by determining the state plane coordinates of the temporary point by traverse from near-by control stations. The distance and direction from the temporary point to the original station was then computed and used in general measurements to recover the early survey point. These methods have been used quite frequently to recover early survey markers reported to have been lost. The invariability of positions of the federal triangulation stations and the fact that they may be recovered any time in the future by methods similar to those mentioned is a convincing argument for the utility of control surveys.

Mr. Schmidt mentions the brevity of the paper. The purpose of this paper was to give a general statement that would be informative and interesting to civil engineers in the hope that they would realize the advantages in relating their local surveys to federal control. Further elaboration on the subject is not within the scope of the paper and could be handled more appropriately by subsequent papers.

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[4] Rear Admiral (Retired); formerly Director, U. S. Coast and Geodetic Survey, Washington, D. C.

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Paper No. 2489

ACCEPTABLE STANDARDS FOR NATURAL WATERS USED FOR BATHING

By Charles R. Cox,[1] A. M. ASCE

Synopsis

Increasing interest in the public health aspects of bathing water quality has led to a study of standards suitable for the control of these waters. Quantitative bacteriological standards are difficult to apply without supplementary sanitary surveys and the exercise of seasoned administrative judgment.

Methods of evaluating pollution have not been developed sufficiently to permit assuming absolute standards. The connection between pollution of bathing waters and illness of people using these waters has not been thoroughly investigated.

Examples of codes based on quantitative standards are given as well as suggestions as to the proper application of these codes.

Introduction

Increasing interest in aquatic sports is leading to the construction of a large number of swimming pools and to the development of many natural bathing beaches. At the same time the currently active pollution abatement programs are focusing attention on the prevention of stream pollution in order to protect sources of water supply, shellfish growing areas, and natural bathing facilities as well as other natural water resources. These developments in turn have drawn attention to quality standards for natural waters. It is pertinent, therefore, to appraise the advantages and disadvantages of quantitative standards for bathing water quality to determine what, if any, quantitative guides or values may be developed.

Adopting Standards

There is considerable confusion regarding quantitative standards, governmental requirements, codes, and objectives; and this confusion extends to the

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[1] Chf., Water Supply Section, New York State Dept. of Health, Albany, N. Y.

appraisal of natural waters used for bathing. George C. Whipple[2] has presented a clear-cut distinction between governmental requirements and professional standards, procedures, or specifications. According to Mr. Whipple, governmental requirements must represent the reasonable exercise of police power in a way that will be supported by court decisions as representing the necessary and legitimate use of such power to prevent injury. By contrast, professional standards, procedures, or specifications are established by agreement among professional groups or societies. If established by governmental agencies, these standards should represent guides to performance rather than rigid requirements established by law or by official codes. Therefore, professional standards or specifications may be higher or more rigid than governmental requirements inasmuch as their use is voluntary and may be modified or adapted to suit specific local needs.

This distinction made by Mr. Whipple may be illustrated by reference to standards for the quality of potable waters because of their similarity to standards for bathing waters. The Drinking Water Standards of the United States Public Health Service (USPHS) apply specifically to potable waters used in interstate traffic, although these standards have been accepted as being applicable to public water supplies in general.[3] The bacteriological aspects of standards are reasonably liberal and are subject to administrative flexibility, as contrasted to those parts of the standards applying to certain toxic substances held to affect the intrinsic quality of potable waters. The standards are applied through the administrative review of all factors surrounding a given water supply, including the reliability of operation and other intangible factors. By contrast, those operating water treatment plants have accepted the total absence of coliform organisms in all portions of the treated water examined as a professional guide or criterion to the quality of a plant effluent. Any surviving organisms are held to be specific indicators of ineffective treatment, even though their presence does not presuppose that the water is inherently unsafe for potable purposes. Therefore, in one instance the governmental requirements apply to the intangible "safety" of a potable water supply, whereas in the other instance rigid professional standards are used as a guide to measure treatment plant performance.

EVALUATION OF DATA

Bacteriological Data.—The characteristics and significance of bacteriological data available to those supervising bathing beaches should be considered as important aspects of any quantitative standards. Such data do not disclose the presence or absence of pathogenic organisms, and hence they are not a measure of any intrinsic health menace to those using the waters for bathing purposes. It is evident, therefore, that they furnish only an approximate quantitative measure of bacterial pollution of a miscellaneous nature, which must be appraised through the exercise of technical and administrative judgment. In fact, such bacteriological data do not distinguish between sewage pollution of public health significance and pollution by miscellaneous surface drainage from cultivated fields and other such sources.

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[2] "Governmental Requirements and Professional Standards," by George C. Whipple, *Journal*, A.W.W.A., Vol. 12, 1924, p. 61.

[3] "Drinking Water Standards—1946," *ibid.*, Vol. 38, 1946, p. 361.

These data are influenced also by the sampling error inherent in appraising any large volume of water by the examination of a very small portion represented by any one sample or even by a series of such samples. Furthermore, the samples represent conditions existing only at the time of collection, which may not apply specifically to conditions prevailing during other bathing periods. Although the results of the examination of a series of samples are statistically more significant than those for a single sample, some one single sample may be much more significant in indicating a serious degree of pollution that might prevail at some one time, and that might be of greater over-all public health significance than averages, medians, or some other statistical grouping of a series of results. Finally, fluctuations in the degree and extent of bacterial pollution are of basic significance, and yet it does not seem to be possible to establish any statistical limits for such fluctuations.

Bacteriologists have attempted to improve this situation by the development of appropriate identifying tests for the fecal group of coliform organisms which theoretically would be more specific for sewage pollution. They also have considered the use of a test for sewage streptococci which appear to survive in polluted waters for shorter periods than coliform organisms and therefore should disclose more recent sewage pollution. Without discussing the bacteriological aspects of this subject, it is sufficient to note that these investigations have not developed any tests for the degree of bacterial pollution of bathing waters more specific or of greater public health significance than the standard plate count and test for coliform organisms.

Accordingly, it may be concluded that bacterial standards for natural waters used for bathing cannot be placed on a precise quantitative basis and assumed to disclose the intrinsic quality of the water or to indicate certain public health safety for the bathers. Therefore, currently available bacteriological procedures should continue to serve as a general guide, the results forming part of the information pertaining to any given bathing beach, subject to coordinated administrative review.

Public Health Data.—There is a dearth of information concerning the public health significance of the bacterial pollution of bathing waters. Charles-Edward A. Winslow[4] has reported on conditions prevailing at beaches in New Haven, Conn., as of 1926–1927, and he has presented bacteriological data indicating the degree of pollution of these beaches which, according to epidemiological evidence, were held to be responsible for sixty-one cases of typhoid fever.

In general, the degree of pollution of the New Haven bathing waters at that time varied from low concentration of coliform organisms to 27,000 coliforms per 100 ml of water, with concentrations of 1,000 per 100 ml to about 6,000 per 100 ml predominating. These data definitely disclosed the need for sewage treatment to protect the beaches.

In spite of the lack of precise information, there is general agreement among sanitary engineers that natural bathing waters must be appraised as possible vectors of water-borne intestinal diseases incidental to sewage pollution of such waters that may be swallowed by the bathers. Swimming pools, in contrast, may serve as possible vectors of infections of the skin and mucous membrane—that is, for the spread of infection from bather to bather.

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[4] "Bacterial Pollution of Bathing Beach Waters in New Haven Harbor," by C-E. A. Winslow and David Moxon, *American Journal of Hygiene*, Vol. 8, No. 3, p. 299.

The absence of subsequent epidemiological evidence—such as that gathered by Mr. Winslow relating the morbidity of bathers using specific beaches to the degree of bacterial pollution of the beach waters—has led to a tri-state survey of Lake Michigan waters by the USPHS and the Boards of Health of Wisconsin, Illinois, Indiana, and the city of Chicago, Ill. A preliminary report on sanitary surveys and examination of samples of water has been prepared by the USPHS,[5] and an unpublished report of the epidemiological studies was presented at the 1949 conference of the American Public Health Association (APHA) by A. H. Stevenson, A. M. ASCE, and T. D. Woolsey. Unfortunately for the purposes of the tri-state study, the degree of bacterial pollution of the bathing beaches studied was quite moderate, and there appears to be no evidence of infections associated with the use of these beaches. Studies of beaches on the Ohio River testify that the degree of bacterial pollution is considerably greater.

Sanitary Survey.—It is evident from foregoing considerations that a given bathing beach must be appraised in the light of all available information, including any epidemiological evidence, analytical data, and more especially the result of properly organized sanitary surveys of the beach and surrounding waters. Such surveys should be designed to disclose the location, magnitude, and public health significance of sources of bacterial pollution and the relative influence of factors that govern the passage of such pollution toward a given bathing beach. Consideration should be given to the location of sewer outlets, the volume of tributary sewage, the dilution factors, the influence of tidal currents and meteorological conditions, the effects of rainfall and the resulting discharge of surface drainage into the bathing waters, and the effects of the bathers themselves on the degree of bacterial pollution.

Unless such thorough sanitary surveys are made, it will be impossible, for instance, for the administrator to distinguish between pollution by coliform organisms incidental to surface drainage from cultivated areas and more significant sewage pollution, or to distinguish between residual pollution from some distant sewer outlet and the same resulting degree of pollution by a smaller but near-by sewer outlet, which may be intrinsically more important.

Obviously, the sanitary survey must be made at the time the samples are collected so that the analytical data may be properly correlated with other observations. It should be emphasized again, however, that no statistical grouping of the results of the examination of a series of samples collected at various points and at different periods of time will permit the accurate correlation of the analytical data and specific observations made at some single point at a bathing beach. Conversely, the results of the examination of a single sample collected at a given point can have only limited significance applying to the time and place of sampling. It should be emphasized, therefore, that the analytical data must be considered as a part of the available information disclosed by the sanitary survey, to be subject to administrative and technical scrutiny, and that these data have no intrinsic or precise significance. The exercise of administrative judgment, therefore, cannot be replaced by the mere checking of quantitative analytical data against rigid values used as specific governmental standards or code requirements.

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[5] "Tri-State Survey of Lake Michigan Waters," Preliminary Report, U.S. Public Health Service, Environmental Health Center, Washington, D.C., 1948.

In spite of this situation there have been repeated attempts to develop quantitative standards for the permissible degree of bacterial pollution of bathing waters. These efforts may have a sound basis when applied to artificial swimming pools where water treatment is practiced and where the permissible degree of remaining bacterial pollution is a measure of the effectiveness of the treatment process rather than a measure of safe or unsafe pool waters. In the past certain standards used for beaches, adopted from swimming pool practice, have been found so rigid as to be unworkable. Public health administrators, therefore, have been faced with the necessity of appraising bathing waters in the light of current practice, where large numbers of bathers are known to be utilizing beaches which are subject to appreciable bacterial pollution without any known effect on their health. It is not surprising, therefore, that the APHA Joint Committee on Bathing Places (hereinafter referred to as "the Committee") and the Conference of State Sanitary Engineers have concluded that it is not considered practicable or desirable to recommend any absolute standard of safety for the waters of outdoor bathing places on any of the technical or bacteriological bases reviewed in the 1948 report of the Committee. The Committee also concluded that the arbitrary wholesale condemnation of bathing beaches representing large capital investments is unwarranted without definite epidemiological evidence.

The pertinent question is whether, in the face of this situation, uniformity of practice can be encouraged by the development of guiding principles which are sufficiently flexible to permit the exercise of administrative judgment but, at the same time, are sufficiently definite to serve as useful guides.

In this connection attention is drawn to a resolution as to the significance of the coliform index as a measure of the pollution of bathing waters issued by the Great Lakes and Upper Mississippi River Boards of Public Health Engineers in 1937. The quantitative guide developed by the Michigan Stream Control Commission[6] and tentatively adopted at that time by these two boards was based on the so-called Phelps Index or "Indicated Number"[7] rather than the "most probable number." This index is based on the number of B. Coli present in a 100 cc sample.

It was resolved that in determinations of the actual extent and intensity of sewage pollution of the Great Lakes and tributary waters to be used in passing on the fitness of areas for recreational bathing purposes, the following interpretation of the significance of the B. Coli index should be tentatively adopted as a guide in connection with interstate problems:

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[6] "Coastline Pollution Surveys of Michigan, June, 1933," unpublished report of Michigan Stream Control Commission.

[7] "Standard Methods for the Examination of Water and Sewage," Am. Public Health Assn., New York, N.Y., 9th Ed., 1946, p. 205.

Index	Interpretation
10 to 100	Indicative of good water; normal for inland lakes and the Great Lakes; free of sewage pollution.
100 to 500	Normal for inland streams; free of detrimental sewage pollution; might be attributed to land wash.
1,000	Suspicious; generally indicates mild pollution in natural waters but dangerous in proximity to fresh sewage pollution.
10,000	Definite evidence of fresh sewage pollution; menace to health.
100,000	Heavy sewage pollution; definitely dangerous.

A review of available information on the subject of bacteriological guides discloses that sanitary codes applying to swimming pools and bathing beaches seldom contain any binding requirements as to the coliform density in bathing beach waters. There is a very definite trend, however, throughout the United States to adopt a median index of 2,400 coliform per 100 ml of water as an administrative guide. This policy has been followed in New York State where the Sanitary Code stipulates[8] rather detailed requirements concerning the operation of artificial swimming pools, including the allowable degree of coliform density in chlorinated pool waters. The state code gives the local health officer and the State Commissioner of Health general administrative power to close a public bathing beach if, in their opinion, the use of the beach would menace the health of the bathers. No quantitative requirements are embodied in the code, however. In view of the fact that many very important bathing beaches in the metropolitan area of New York City come under the jurisdiction of a number of separate health departments, it has been found advisable to develop a joint policy as to the administrative classification of bathing beach waters in this area. This agreement has been subscribed to by all but one health department concerned. Briefly, this policy is predicated on the basic importance of the sanitary survey of each beach, supplemented by the interpretation of the analytical data in the light of such a survey. Beaches are classified in the joint policy agreement as "approved" (Class A) and "disapproved" (Class B), with distinctions as follows:

Class A. Approved beach waters are identified by:

(*a*) Satisfactory epidemiological data pertaining to bathers utilizing the beach or adjacent beaches;

(*b*) Satisfactory sanitary survey;

(*c*) An average content of coliform organisms not in excess of 2,400 per 100 ml of water for any given series of ten or more samples from a beach.

Criteria (*a*) and (*b*) may sometimes justify the use of the lower average of 240 per 100 ml of water in criterion (*c*) for any given series of ten or more samples collected from a beach.

Class B. Disapproved beach waters are identified by:

(*a*) Epidemiological evidence and experience which disclose the prevalence of infectious diseases incidental to the use of a bathing beach under consideration;

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[8] "Sanitary Code of New York State," New York State Dept. of Health, Albany, N.Y., Chapter VI Regulation 12.

(*b*) A sanitary survey that discloses sewage material on a beach or in waters immediately adjacent to the beach;

(*c*) An average content of coliform organisms in excess of 2,400 per 100 ml of water for any given series of ten or more samples.

As in the case of Class A criteria (*a*) and (*b*) may sometimes justify the use of the average content of coliform organisms in excess of 240 per 100 ml of water for any given series of ten or more samples from a beach.

The joint policy agreement states that

> "Conditions are expected to prevail which represent borderline cases between approved beach waters and disapproved beach waters. Administrative judgment must be exercised in appraising the uncertain factors and in assigning such borderline beaches to one of the two classes noted above. In any case, administrative judgment must be used in the periodic review of sanitary conditions prevailing at each bathing area and reclassification should be practiced when necessary, leading to the revocation of a permit for the operation of an unsafe bathing beach or to the issuance of public notice that certain beach waters are unsafe for bathing."

The New York State Water Pollution Control Board (hereinafter referred to as "the Board") has developed basic principles for the classification of natural waters including bathing beaches. Their problem was not to appraise an individual beach but rather to develop guiding principles for the required degree of treatment of sewage or industrial wastes to protect natural waters, including bathing waters. The effects of currents and meteorological factors cannot be anticipated in a precise manner. Therefore, the degree of sewage or waste treatment could not be selected so that they just met any given quantitative guide for the degree of pollution of the receiving water that may reach bathing beaches. The classification of natural waters by the Board, therefore, was predicated on comprehensive field studies of each stream or body of water.

SUMMARY

It is evident that knowledge and technical procedures have not been developed sufficiently to permit the development of precise quantitative values to measure the intrinsic quality of bathing waters, and, therefore, it is not possible to select a definite value that would distinguish between bathing beaches which are safe and those which are unsafe. Under these circumstances, precise bacteriological standards cannot be developed, and all available information must be considered in the light of properly organized and properly conducted sanitary surveys of each beach. These surveys would include the results of the bacteriological examination of samples of water. Under these circumstances general administrative guides may be fruitful in encouraging uniformity of practice as long as the guides are properly utilized without being considered as precise governmental requirements. It is hoped that the study of selected beaches on the Ohio River and of the morbidity among the bathers utilizing these beaches will furnish information needed for a more definite delineation of these problems, and also that more precise analytical methods may be developed to disclose the bacterial quality of bathing waters in a more definite manner than is now possible.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 2490

DESIGN OF IRRIGATION SYSTEMS

By W. H. Nalder,[1] M. ASCE

With Discussion by Messrs. Ahmed Shukry, and W. H. Nalder.

Synopsis

An account of the planning and design considerations entering into the development of irrigation features of multiple-purpose projects of the Bureau of Reclamation (USBR), United States Department of the Interior is presented in this paper. Reviewed are the major factors affecting irrigation development and their attendant engineering implications, the elements of an irrigation system, and the influence of economics on design. Also the principal features of three reclamation multiple-purpose projects are discussed. The paper concludes with a brief discussion of irrigation's future importance in the over-all social and economic aspects of water and land resource development.

Introduction

This subject is so broad and so indefinite in its limits as to include almost any irrigation system, from a simple structure diverting water from a stream into a canal and laterals (to provide irrigation of low-lying land along the stream) to a group of structures for river control and for utilization of the water resources of an entire river system. There is very little need to describe a simple irrigation project. Although such a widespread integrated system as the Missouri River Basin Project contains many individual units that are in themselves fairly complete, simple projects, most of the uncomplicated locations are utilized and the cheap developments have been constructed. In many places, water rights have been established and water demands exist that already exceed the readily available water supply. The irrigation engineer, therefore, must seek new sources of water such as the storage and the utilization of flood flows, the diversion from sources of lower demand, and the re-use of water from return flows.

Before he can apply his talents to the design of the works that comprise the irrigation features of a project, the irrigation engineer is required to coordinate a wide variety of demands for water or for its control. Even after agreement is

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reached between various interests as to the diversion of the water, the manner in which the requirements are to be satisfied (in a structure which serves several purposes, or in multiple structures or units of a project operated according to a compromise plan) requires the services of a complex organization for planning, construction, operation, and administration of the system.

By its very nature, irrigation development is destined to become more complex as it progresses: Development of an area through irrigation fosters the development of the natural resources, land, mineral, and forest; settlement is encouraged, including the building of farms, cities, industries; the population increases, which requires food, and domestic water, power, and transportation facilities; demands for protection against floods and for conservation of resources develop; the people seek recreational facilities; and manifold influences are exerted on economic and political activities. All these factors are related to, and either are reconciled with, or are in conflict with, each other by the common denominator, the available water supply. The interrelated development necessarily depends for sustenance on the irrigation that gave birth to or nurtured the development and its demands for water cannot exceed the supply if it is to thrive.

Factors Affecting Irrigation Development

A few factors affecting irrigation development, and their engineering implications and effect on irrigation design, greatly simplified, are: Power, flood control, navigation, industrial and domestic demand, recreation, fish and wildlife, and political circumstances.

Power.—Development of water power, whether incidental to irrigation or of primary concern, imposes requirements for the utilization of head, equalization of flow, storage of water with attendant inundation of river valley lands, and use of water that in many instances conflicts with its use for irrigation. Power revenue, however, is the saving factor of many multiple-purpose projects. This factor makes the project economically feasible and helps to repay the cost that could not be borne entirely by the direct fruits of irrigation. In this way, power supports irrigation, which directly or indirectly sustains the demand for power. The two mutually supporting factors may thus become the base of the entire development.

Flood Control.—Encroachment on flood plains by agriculture, cities, industries, and service facilities such as utilities, roads, and railroads creates demand for protection against floods which requires storage of flood flows beyond channel capacity. Releases for flood-control purposes usually are not coincident with the demands of irrigation. The flood-control benefits of a project can be evaluated, and (since it is usual for the cost of flood control on reclamation projects to be borne by appropriation from nonreimbursable funds) charges allocated to irrigation development may thereby be reduced and the feasibility of the project enhanced.

Navigation.—In a section of a river system where irrigation and consumptive use of water are involved jointly with navigation, the maintenance of a fixed-depth channel for navigation imposes demands for the release of water that conflict with other uses. Navigation likewise imposes limitations on the height of dams since the cost of providing navigation facilities over a high dam would usually make the project prohibitively expensive.

Industrial and Domestic Demand.—Domestic use of water usually takes precedence over all other uses. Water supply for communities served by irrigation is not generally so great as to constitute a serious depletion of the water available for irrigation, although certain industrial uses threaten (by demand for large quantities of water or by contamination of the water) to curtail irrigation development. For example, further transmountain diversion of Colorado River water for use in irrigating the eastern slope is opposed by certain western slope users who anticipate demand for large quantities of water for the development of the oil shale resources of western Colorado.

Recreation.—Irrigation reservoirs create potential resort and recreation areas which in turn present demands for the maintenance of a level of water or of a continuity of flow which is inimical to the most effective irrigation. There is a warranted and growing demand for the development of these recreational facilities. In some instances, national park areas and primitive areas would be the logical sites for irrigation reservoirs or dams, but they usually cannot be encroached upon for such purposes.

Fish and Wildlife.—The drainage or flooding of swampland that supports wild fowl, the construction of high dams that affect the movement of migratory fish, the changing of flow characteristics of a stream, and the cultural development of an area which may be associated with irrigation development but which affects the wildlife of a region—all may be stoutly resisted as being destructive in character and as usurping the rights of the commonwealth and of posterity.

Political Circumstances.—Not the least of the factors affecting irrigation development are conflicting jurisdictional claims of districts, states, and nations when the waters of a stream must be divided between them. Development for a certain purpose, in conflict with or to the exclusion of others, may also be dictated by legislative bodies or by courts; or the impetus of a particular development may be provided by political programs.

Planning the Multiple-Purpose Project

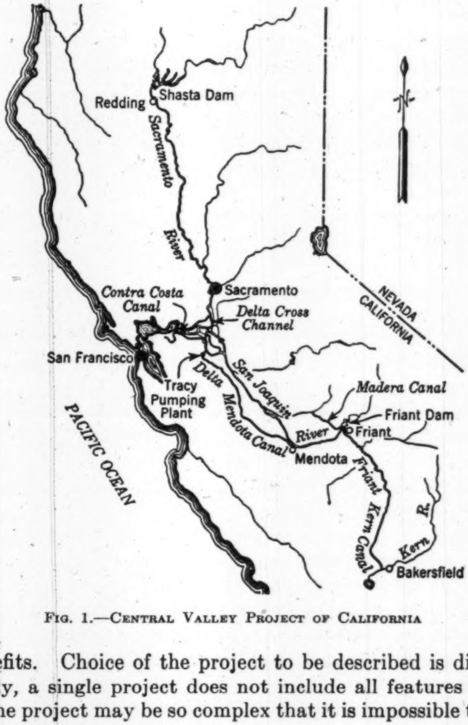
A plan is evolved which is usually the best compromise that can be worked out of the various factors and interests that are affected; and it passes through a series of steps designed to bring the project to completion. The complexity of the plan, the legislative authority required, the magnitude of the financial obligation, and the involvement of the interests of the whole community in any extensive irrigation scheme have caused irrigation development to be considered and generally accepted as a proper governmental function. The agency of the United States Government charged with this function is the USBR, and its counterpart exists in nearly every nation where irrigation is an important feature.

Although procedures vary considerably with circumstances, irrigation development may include the following steps. Community interest in irrigation for a certain area comes to the attention of the USBR. Then advance studies are made relating to its feasibility and to how best it can be fitted into a basin-wide plan embracing other adjacent or related areas and involving the best over-all use of the available water supply. This plan will be likely to include storage reservoirs with incidental power development, flood control, and other benefits.

As authorization and funds are made available, more detailed surveys and studies are made—including location and investigation of reservoir and dam sites, road and railroad relocation, canal location, irrigable land surveys, and preliminary designs and estimates—and the project plan is developed which is coordinated with federal and state agencies where their interests are involved. Construction work is programmed, construction contracts are let, and construction is completed as circumstances warrant and as funds become available.

IRRIGATION FEATURES OF MULTIPLE-PURPOSE PROJECTS

The plan of a project, evolution of the plan, and features of design of an irrigation system may be indicated by presenting a detailed description of a particular project, emphasizing the irrigation features but presenting at the same time the related features of power development, flood control, and other incidental benefits. Choice of the project to be described is difficult in itself because, usually, a single project does not include all features that might be presented; or the project may be so complex that it is impossible to describe in a short presentation.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San Francisco
Tracy Pumping Plant
Delta Mendota Canal
San Joaquin River
Madera Canal
Friant Dam
Friant
Mendota
Friant Kern Canal
Kern R.
Bakersfield
PACIFIC OCEAN

FIG. 1.—CENTRAL VALLEY PROJECT OF CALIFORNIA

As an example in the latter category, one should mention the Central Valley Project of California (Fig. 1). The initial plan of this project, well advanced in construction (1950), includes: Two high concrete dams—the Shasta Dam on the Sacramento River to provide the main source of water and the Friant Dam on the San Joaquin River; irrigation of a large area in the upper San Joaquin River Basin; transfer of water from the Sacramento River to the San Joaquin River to satisfy established claims for water below the newly irrigated regions; power development; flood control; salinity control of the delta region at the confluence of the two rivers; provision for the care of migratory fish; extensive relocation of railroad and highways; and a number of other features that could not be omitted from an adequate description of the project.

This initial plan has been expanded to a plan for the complete and comprehensive development of the water resources of the Central Valley which contemplates the construction of numerous additional dams, power plants, and canal systems.

COLUMBIA BASIN PROJECT

The most extensive compact irrigation project under construction by USBR is the Columbia Basin Project in the State of Washington. Fig. 2 shows a 426-mile canal system for carrying irrigation water to about one million acres—an area larger than the State of Rhode Island. The first major feature of the project to be constructed was the Grand Coulee Dam on the Columbia River, containing more than 10,000,000 cu yd of concrete, the largest in the world in point of volume. It is a straight gravity dam 550 ft high and 4,173 ft long.

The power development at Grand Coulee Dam has already become the world's largest capacity plant with eighteen of the largest generators in existence in operation or under contract for purchase and installation. Each unit is rated at 108,000 kva, but the units installed have been operating at about 15% overload capacity. The power plant is divided into two sections, one on each side of the spillway. At the inception of the project it was considered that the principal demand for power from the Grand Coulee Plant for many years would be to pump irrigation water from Lake Roosevelt to the Coulee Reservoir at the head of the irrigation system. During and since World War II, the industrial development of the northwest has expanded at a much more rapid rate than the power supply, and the power demand has been so heavy that construction and installation of the generating equipment have not been able to keep pace with the demand. This is an excellent example of the incidental benefits of an irrigation project becoming, for a while at least, the principal benefits. In this instance power development will repay the major cost of the project.

When Grand Coulee Dam was constructed, provision was made at the left abutment for the beginning of the canal system by building a gravity wing wall equipped with twelve, 13-ft by 20-ft hydraulically-operated gates to function as an intake structure to serve twelve pumps each capable of lifting 1,600 cu ft per sec up the 270-ft to 365-ft difference in elevation between Lake Roosevelt and the feeder canal leading to the Upper Coulee Reservoir. The pumps are driven by twelve 65,000-hp motors, two pumps serving as reserve units. The maximum demand can be supplied by ten pumps. Power to drive each set of two motors is derived from one of the 108,000-kva generators of the Grand

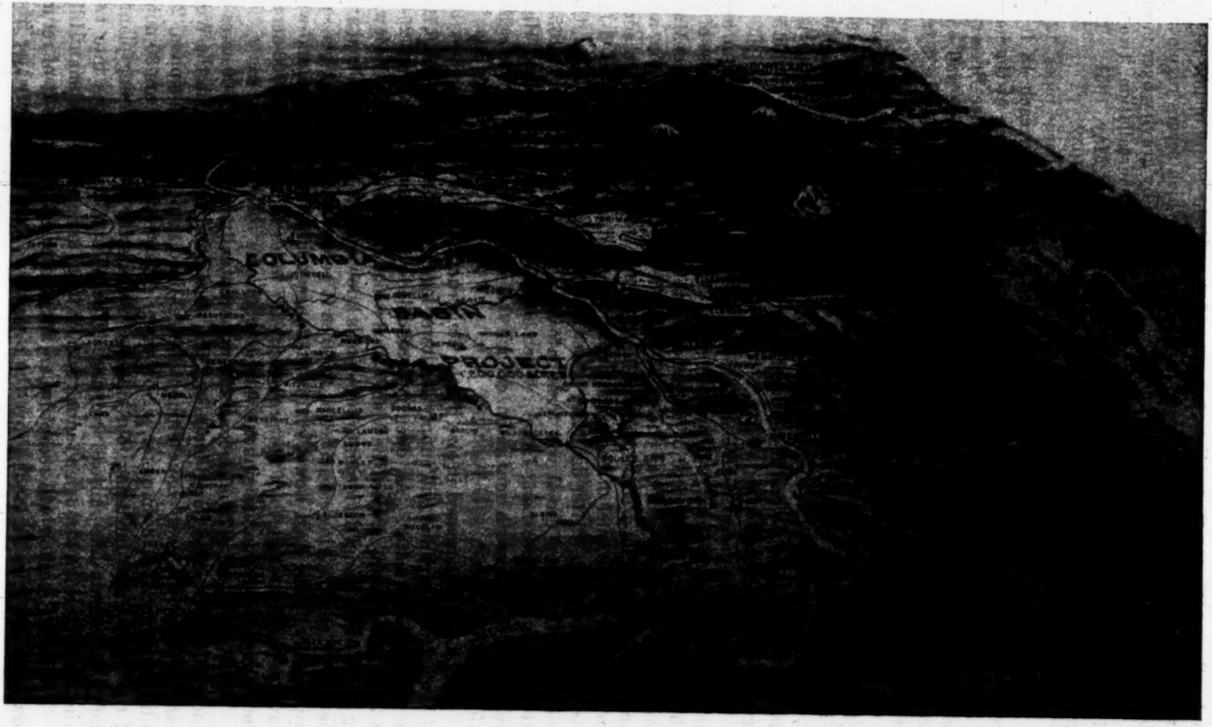
COLUMBIA
BASIN
PROJECT

FIG. 2

Coulee Power Plant. Water is to be pumped during the hours of low power demand into the Grand Coulee Equalizing Reservoir which serves regulating storage for the irrigation system, from which water flows by gravity over most of the irrigated area. Six pumps are planned for initial installation, two of which are currently (1950) being installed for operation by May, 1951. The remaining four units are to be in operation by the spring of 1952.

Emanating from the pumping plant, are twelve steel discharge pipes, 12 ft in diameter, threaded through twelve inclined rock tunnels, each 18 ft in diameter. These discharge pipes terminate in a siphon structure at the head of the feeder canal. The feeder canal leading to the Upper Coulee Reservoir has a design capacity of 16,000 cu ft per sec. It is 1.6 miles long and open concrete lined except for 2,000 ft of double-barreled concrete conduit, 25 ft in diameter, traversing a slide area.

The equalizing reservoir utilizes the Grand Coulee, which was the channel of the Columbia River thousands of years ago. The reservoir with a capacity of 1,200,000 acre-ft is formed by construction of an earth dam at each end of the 26-mile long coulee. The North Coulee Dam is rather small in comparison with its companion, the South Coulee Dam, which is 2 miles long and 115 ft high at the maximum section.

The headworks of the Main Canal are located at the South Coulee Dam, and they house a battery of six 12-ft by 18-ft radial gates to control the release of as much as 13,200 cu ft per sec of irrigation water. The first leg of the Main Canal is 11 miles long and has one 23.25-ft diameter, 1,000-ft long siphon, and one equivalent size horse shoe, concrete-lined tunnel, 10,000 ft long. An additional siphon and tunnel of the same capacity and dimensions is planned for future construction. After diverting water into the East High Line Canal, the first leg of the Main Canal will terminate by cascading water over a 165-ft natural rock drop into Long Lake. When completed, the 110-mile East High Line Canal will have a maximum capacity of 3,070 cu ft per sec to serve 215,000 acres of land.

Long Lake is increased in size by the construction of the 100-ft-high earthfill Long Lake Dam and will supply the lower leg of the Main Canal with 9,700 cu ft per sec of water. An open type of headworks, containing three 25-ft by 25-ft radial gates, services the Main Canal which is 6.6 miles long. The lower leg of the Main Canal divides into two canals known as the East Low Canal and the West Canal, having capacities of 4,500 cu ft per sec and 5,100 cu ft per sec, respectively. The East Low Canal is the longest canal in the system and will traverse the entire length of the project for 137 miles to irrigate 252,000 acres. One tunnel and numerous siphons will be required.

The West Canal is 78 miles long and will eventually irrigate 281,000 acres of land, with gravity and pump laterals. The most notable structure in the West Canal will be the Soap Lake Siphon, a steel-lined, concrete pipe siphon, 2.5 miles long and 25 ft in diameter. Now under construction (1950) are 30 miles of the West Canal including four siphons, a wasteway, two pumping plants, and a three-main-line railroad crossing. The West Canal divides into two canals after passing through a tunnel, 9,150 ft long, under the Frenchman Hills.

Potholes Reservoir will recapture an anticipated 862,000 acre-ft of return flow from the irrigated lands above the reservoir. Augmented by water from the wasteways of the East Low and West canals, storage in the reservoir will be retained behind the earth-fill, O'Sullivan Dam, which is 3.5 miles long. At some future time it is planned to use the reservoir to service the Potholes West Canal, 22 miles long, with 235 cu ft per sec, to irrigate 13,600 acres, as well as the Potholes East Canal, 60 miles long, with 3,900 cu ft per sec to irrigate 254,000 acres of land.

Several pumping plants served by the West Canal will lift water to irrigate an additional 13,000 acres of excellent quality land.

Republican River Basin

Development of the Republican River Basin, in Colorado, Kansas, and Nebraska, is a part of the integrated development of the Missouri River Basin Project, many units and features of which are now (1950) under construction by the USBR and the Corps of Engineers, United States Department of the Army. The Republican River Valley is typical of many reclamation projects in the Great Plains area and serves as a good example of how the objectives of flood control and irrigation may be achieved by coordinated development. Unlike the single large compact development characterized by the Columbia Basin Project lands, the Republican River Basin irrigable lands are scattered along the flood plains of the rivers for more than 300 miles and comprise sixteen separate irrigation units in three states. The operation of these units must be closely coordinated and timed to produce the maximum possible utilization of available water supply and at the same time to assure protection from damaging floods.

The Republican River rises in northeastern Colorado about 100 miles east of Denver. It flows through the northwestern corner of Kansas and then into Nebraska where its course follows an easterly direction along the southern border of that state. From Nebraska the river turns southeast, emptying into the Kansas River at Junction City, Kans. The drainage basin of the Republican River is approximately 425 miles long and is about 125 miles wide at the widest place. The basin has an area of 26,500 sq miles and the total drop in elevation is 3,000 ft.

Irrigation in the basin in the past has consisted principally of private developments serving about 20,000 acres. Ultimate development anticipates irrigation of about 165,000 acres of new land and provision for an additional supply of water to the lands now under irrigation.

A map of the Republican River Basin is shown in Fig. 3. The works to be constructed in the basin to serve the sixteen separate irrigation units, according to present plans, are fourteen storage dams, eight diversion dams, and about 300 miles of new canal; and several existing diversion dams and numerous stretches of existing canals are to be rehabilitated. Because of the runoff characteristics of the entire area, the control of floods is an important and controlling factor in designing the storage dams and in providing for passing high flows at the diversion structures.

The maximum flood of record on the Republican River occurred in 1935 when a peak flow of about 265,000 cu ft per sec was reached, and loss of human life and extensive property damage resulted. Hydrologists estimate that this was of the order of a 1,000-year flood for the river. The impact of such a flood may be better appreciated in light of the fact that the average annual discharge is only 740 cu ft per sec. The dams being constructed on the Republican River and its tributaries are provided with spillways that might be considered disproportionately large without full consideration of the flood potentialities of the basin. The storage reservoirs, which will have a combined total capacity of 1,786,000 acre-ft, will have more than 1,000,000 acre-ft of this capacity allocated to flood storage, leaving only about one third of the total capacity available for irrigation storage. Table 1 shows the storage dams required for the development, and their respective capacities.

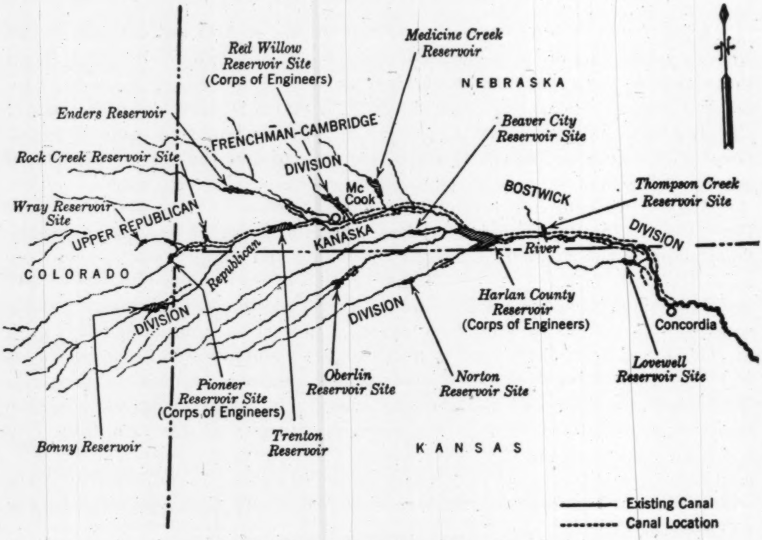
Red Willow Reservoir Site (Corps of Engineers)
Medicine Creek Reservoir
NEBRASKA
Enders Reservoir
FRENCHMAN-CAMBRIDGE DIVISION
Beaver City Reservoir Site
Rock Creek Reservoir Site
Mc Cook
BOSTWICK DIVISION
Thompson Creek Reservoir Site
Wray Reservoir Site
UPPER REPUBLICAN DIVISION
KANASKA DIVISION
Republican River
COLORADO
Harlan County Reservoir (Corps of Engineers)
Concordia
Oberlin Reservoir Site
Norton Reservoir Site
Lovewell Reservoir Site
Pioneer Reservoir Site (Corps of Engineers)
Trenton Reservoir
Bonny Reservoir
KANSAS
Existing Canal
Canal Location

FIG. 3.—MAP OF REPUBLICAN RIVER BASIN

Somewhat typical of the storage dams in the Republican River Basin is the Medicine Creek Dam located on that creek some 10 miles above its confluence with the Republican River.

This dam, of the zoned earth-fill type, has a maximum height of about 115 ft above the stream bed, and the 30-ft base-width cutoff trench excavated 60 ft upstream from the axis, extends about 50 ft below the stream bed to a shale and chalk formation. The dam is approximately 5,600 ft long at the crest, which has a width of 30 ft. The total volume of all classes of material in the embankment is 2,676,000 cu yd.

The outlet works along the right abutment consist of a vertical shaft inlet, a horseshoe conduit (8 ft in diameter) from there to the gate chamber, and an identical conduit from the gate chamber to the control house. An hydraulically-operated slide gate, 3.25 ft square, is installed in the gate chamber from which a steel outlet pipe (44 in. in diameter) installed in the conduit leads to another identical slide gate in the control house. Discharge is into a concrete stilling pool. The spillway designed for a maximum discharge of 139,000 cu ft per sec is an open channel through the left abutment. In downstream order it consists of an uncontrolled crest, a channel, and a stilling basin—all of concrete. About 32,000 cu yd of concrete is placed in the outlet and spillway structures. The storage dams completed or under construction in 1950 are shown as items 4, 5, 6, 8, and 12, in Table 1.

Operation of the storage and diversion facilities will require the development of a master plan for the coordinated control of the entire flow of the river and all its tributaries. Because of the erratic runoff characteristics of the entire area, it will be necessary to make adjustments in storage from one reservoir to another to insure flood protection at all points and, at the same time, to leave adequate water storage for late season irrigation demands. On a river development of this magnitude, the practice of irrigation over a period of years is certain to bring about substantial amounts of return flow. By proper handling of the various storage facilities, much of this return flow may be re-used to advantage.

TABLE 1.—STORAGE DAMS REQUIRED FOR THE DEVELOPMENT OF THE REPUBLICAN RIVER IN COLORADO, KANSAS, AND NEBRASKA

Line No.	Reservoirs	Approximate height, in feet	Crest length, in feet	Total embankment, in cubic yards	ACRE-FEET		
					Irrigation	Flood control	Ultimate
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	Wray	....	....	....	6,000	1,000	7,000
2	Pioneer (USCE)[a]	....	....	....	28,000	73,000	101,000
3	Rock Creek	....	....	....	10,500	....	10,500
4	Bonny[b]	130	9,300	9,207,000	43,000	132,000	175,000
5	Trenton[b]	100	8,600	7,800,000	122,820	133,760	256,580
6	Enders[b]	100	2,600	1,895,000	44,000	30,000	74,000
7	Red Willow (USCE)[a]	....	....	....	18,400	22,000	40,400
8	Medicine Creek[b]	115	5,600	2,700,000	40,000	53,000	93,000
9	Beaver City	....	....	....	21,000	20,000	41,000
10	Oberlin	....	....	....	18,000	20,000	38,000
11	Norton	....	....	....	19,000	20,000	39,000
12	Harlan County (USCE)[a,c]	100	11,000	10,714,000	350,000	500,000	850,000
13	Thompson Creek	....	....	....	13,800	10,000	23,800
14	Lovewell	....	....	....	25,000	12,000	37,000
15	Totals	....	....	....	759,520	1,026,760	1,786,280

[a] Corps of Engineers, United States Department of the Army. [b] Earth-fill dam. [c] Earth and concrete dam.

ELEMENTS OF AN IRRIGATION SYSTEM

From the foregoing examples, and from consideration of the function of an irrigation system, the structural elements of a system may be visualized. The diversity of forms and the complexity of arrangements make impossible the complete listing of all elements and their design considerations in a short paper. Generalized elements may be considered in terms of: Storage reservoirs, diversion and headworks, conveyance structures, regulating structures, protection structures, and drainage systems.

Storage Reservoirs.—Formed by dams of all types, storage reservoirs usually serve as the source of supply, sometimes to supplement normal low-water flow. These dams have extensive control structures such as spillways and outlet works.

Diversion and Headworks.—Diversion and headworks may take the form of a diversion dam of any type, with headworks or a pumping plant if the source of water is a flowing stream. Control structures at the headworks are sluice and gate arrangements with auxiliaries as required, such as stop-log provision, trashracks, and fish protection.

Conveyance Structures.—Canal lines from the large main canal at the headworks to the small branch laterals and ditches which serve individual farms, with special structures inserted in the lines as required, present many types of engineering problems and all possible combinations of forms. Structures of steel or concrete and (in some instances) wood, include siphons, flumes, conduits, culverts, chutes, drops, and transitions. Tunnels are frequently required. Canals may be unlined or lined (to prevent seepage or erosion) with a variety of materials including concrete, clay or bentonite, asphalt, and rock. Special conveyance methods such as pipe distribution systems or sprinkler systems may be used. Very extensive underground pipe systems are being built by the USBR for the first time in California.

Regulating Structures.—Checks, orifice structures, Parshall flumes, turnouts, vane and venturi meters, and weir structures of many types may be used to control and measure the flow as desired.

Protection Structures.—Culverts, drain inlets, overchutes, spillways, and wasteways are incorporated in the system as required to prevent interference of natural drainage with the irrigation system or to remove surplus water from a section of the canal line without damage to the system.

Drainage Systems.—Removal of excess water from the surface or from the subsoils may be as important to the irrigation system as the conveyance and the distribution of water to the land. Surface drains or subsoil drains may be open ditches or tile drains; or water may be drained by pumping.

INFLUENCE OF ECONOMICS ON DESIGN

The trend of irrigation development is toward larger systems, requiring larger headworks and larger main canal lines and structures than were feasible only a relatively short time ago. Use of heavy mechanical equipment that can handle large quantities of materials cheaply has been the principal contributing factor to this trend. Mechanized canal excavation and trimming, lining, and finishing equipment permit large canals to be constructed quickly and at low unit cost, and in turn require large systems and long sections of canal to justify their use economically.

Heavy earth-moving machinery has been effective in completely reversing the trend of the unit cost for moving earth as compared with the unit cost for other construction work so that large earth structures such as canal embankments and earth dams can be constructed at lower costs than their counterparts using other materials. Knowledge of soil mechanics, utilizing the earth materials at hand to the best advantage through improved design and construction procedures, and recognition of the possibilities and limitations of earth structures for hydraulic purposes have contributed to the safe construction of large, economical earth structures, particularly of high earth dams. In the case of dams there is the further consideration that dam sites must be utilized which are not suitable for supporting concrete dams but which are, or can be made, suitable for an earth dam with its wide base. Most of the dams that are a part of irrigation systems, except storage dams in mountainous country, are earth dams and the proportion of earth dams will probably increase.

With the existing heavy demand on water, it is important that full utilization be made of the water allocated to irrigation and that loss through seepage from canals be kept at a minimum. On the other hand, it may be prohibitively expensive to line all canals by conventional methods. Considerable research and experimentation have been devoted toward the development of a low-cost canal lining. In this category is included a subgrade-guided slip form for placing concrete. This is used on smaller ditches to place concrete lining about 1.5 in. thick without using rails for supporting the slip form.

One of the most promising low-cost canal linings developed by the USBR is the buried asphalt membrane. The canal is excavated about 18 in. outside the finish lines and the membrane is applied by spraying hot asphalt on the excavated section using 1.5 gal per sq yd which produces a membrane nearly 0.25 in. thick. A satisfactory membrane can be formed with semiblown asphalt although catalytically-blown asphalt is preferred. The membrane is then covered with an 18-in. thickness of earth and gravel to prevent erosion of the protective cover. Trial installations indicate that buried asphalt membrane can be applied at a fraction of the cost of concrete lining.

The great cost of developing large irrigation systems generally indicates that the water users cannot repay the entire cost of the project from primary irrigation benefits alone. Multiple-purpose projects which provide power revenue and nonreimbursable benefits such as flood control, recreation, and fish and wildlife, then become the only feasible basis for developing the water, land, and the other natural resources of a region.

THE FUTURE OF IRRIGATION

Irrigation development in the United States, as well as in more densely populated countries, will undoubtedly be relied on as the chief replacement of marginal lands removed from cultivation and as the source of supply of food for millions who reside in the semiarid regions. With virtually no new land to be opened for development except that which may be made fruitful through irrigation, the 20,000,000 additional acres that might be irrigated with the complete utilization of all the remaining water resources of the western half of the United States constitutes the last frontier that might be developed against the dwindling ratio of cultivated acres to population. Irrigation engineering, therefore, may become of greater importance in the foreseeable future than in the past, but it will not be irrigation engineering by itself. The design of irrigation systems will be linked with the design of facilities to utilize all the water available in the most effective manner for the greatest benefit of all, in larger, more extensive, and more complex multiple-purpose projects or basin developments than have yet been constructed.

DISCUSSION

Ahmed Shukry,[2] A. M. ASCE.—In the limited extent of the paper, the author has covered intelligently a subject of indefinite limits. However, the expression (under the heading "Influence of Economics on Design") "full utilization of the water allocated to irrigation" needs some broad explanation, especially when related to the future of irrigation in general. The estimated field water duties for the crops on the irrigated land directly affect the efficiency, cost, and design of any irrigation project. The capacities of storage reservoirs, the sizes of diversion and headworks, conveyance structures, regulating structures, protection structures, and drainage systems all depend primarily on these duties. The future development of profitable irrigation schemes will depend largely on the extensive scientific and experimental studies of the relations between the yields of the different crops and water supplied. At least, this may be attempted in densely populated countries that have large marginal cultivable areas but possess limited sources of irrigation water. The latter case may be represented by Egypt.

In Egypt the area cultivated in 1950 was nearly 6 million feddans (1 feddan = 1.038 acres) and the population was over 19 million. The country is mainly dependent on agriculture, the agricultural products representing approximately 90% of the national income. The population is rapidly increasing, and since 1900 it has nearly doubled. The additional area that can be cultivated was estimated in 1946[3] as 2.0 million feddans. However, studies of the nature of the soil in the deserts east and west of the Nile Delta show that the total cultivable area may reach 10 million feddans, especially when power becomes cheap and the low regions of these deserts are irrigated by pumping. Because of the rapidly increasing population, new land must be continuously reclaimed for agriculture, to avoid the possible starvation that would otherwise ensue in a few years. The area of land under cultivation per capita (in feddans) has fallen from 0.48 in 1907 to less than 0.30 in 1951.

The main water source in Egypt is the River Nile. The water required for Egypt and the Sudan is stored during the high stage of the river in three reservoirs, namely, Aswan, Sennar, and Gabel Awlia (Fig. 4). These reservoirs are in the river channel itself. The majority of the flood water, being heavily charged with silt, is wasted into the sea.

To enable the future extension of cultivation, a huge storage scheme for utilizing all the Nile sources is under discussion,[4,5] and its two major provisions can be summarized as follows:

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[2] Asst. Prof. of Irrigation, Faculty of Eng., Farouk First Univ., Alexandria, Egypt.

[3] "The Future Conservation of the Nile," by H. E. Hurst, R. P. Black, and Y. M. Simaika, *Paper No. 51*, Ministry of Public Works, Govt. Press, Cairo, Egypt, 1946.

[4] "Report of the Consultant Committee on Storage Projects," Ministry of Public Works, Govt. Press Cairo, Egypt, 1948.

[5] "Future Control of the Nile Sources," by Ali Bey Fathy, Faculty of Engineering, Farouk First Univ., Alexandria, Egypt, 1949.

1. Annual storage, by which the water requirements for a single year will be stored in the river channel itself, in the existing reservoirs, and in a fourth future reservoir at Merowe.

2. Over-year storage, by which the water will be stored in the watershed areas of the river, namely, Lake Victoria, Lake Albert, and Tana Basin. This water will be used to fill the annual reservoirs in low flood-years.

In spite of this huge program for controlling all the Nile sources and for reducing the losses in the Sudd region, the stored water will be just sufficient to irrigate 7.1 million feddans in Egypt and 2.0 million feddans in the dry regions of Sudan.

Different proposals are being studied for providing irrigation water to the remaining 2.9 million feddans in Egypt or to any other land that may be reclaimed in the distant future. The summary of four of these proposals is as follows:

a. Drawing the irrigation water by pumping from wells extending through the pervious substratum;

b. Re-use of the irrigation water by irrigating from drains;

c. Reducing the available conveyance losses of irrigation water; and

d. Economy in the use of water for the irrigated lands.

A final opinion on these proposals has not yet been given. However, it is believed that the economical use of irrigation water, based on long scientific and experimental studies, might be the last frontier against the dwindling ratio of cultivated area to population.

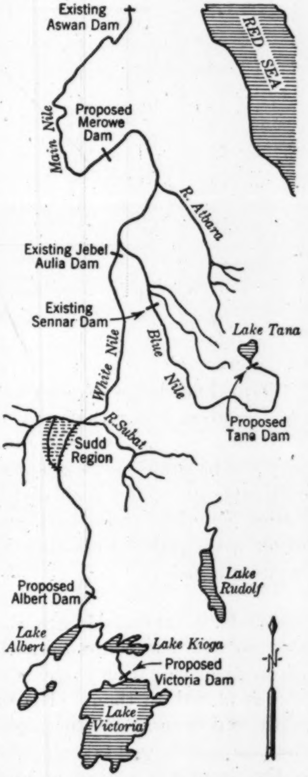
Existing Aswan Dam
RED SEA
Proposed Merowe Dam
Main Nile
R. Atbara
Existing Jebel Aulia Dam
Existing Sennar Dam
Lake Tana
Blue Nile
White Nile
Proposed Tana Dam
R. Sobat
Sudd Region
Lake Rudolf
Proposed Albert Dam
Lake Albert
Lake Kioga
Proposed Victoria Dam
Lake Victoria
N

FIG. 4.—EXISTING AND PROPOSED DAMS IN THE UPPER NILE BASIN IN 1951

The economical use of irrigation water can be explained briefly as follows: The published results of the experiments carried out on the various factors affecting the relation between the crop yield and irrigation water[6,7] show, for the majority of crops, a general trend as represented schematically in Fig. 5. Under constant conditions of the other affecting parameters, the yield increases

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[6] "Irrigation Practice and Engineering," by B. A. Etcheverry and S. T. Harding, McGraw-Hill Book Co., Inc., New York, N. Y., Vol. I, 1933.

[7] "Irrigation Principles and Practices," by Orson W. Israelsen, John Wiley & Sons, Inc., New York, N. Y., 1950.

in an approximately linear relationship with the amount of irrigation water up to a certain point (A). From point A to the point of maximum yield B the curve has a diminishing slope, and near point B it is generally so flat that big amounts of water can be saved with little reduction of the maximum yield of the crop.

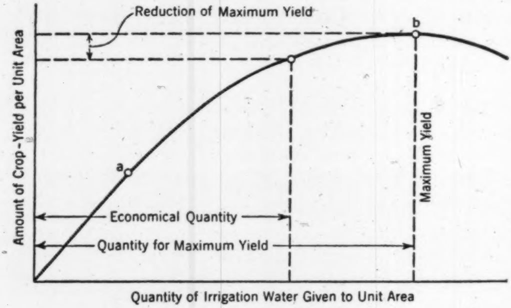

FIG. 5.—GENERAL RELATIONSHIP BETWEEN CROP YIELD AND THE QUANTITY OF IRRIGATION WATER

True knowledge of such relations may, therefore, produce economies in the irrigation water that may be given to some crops beyond their points of maximum yield. Furthermore, the available irrigation water in a district may not be sufficient to produce the maximum yields of the crops in the whole cultivable area. In such districts it may be more economical to use water duties less than those that produce the maximum yields and cultivate the whole area, rather than cultivating part of the area under the condition of maximum yield.

W. H. NALDER,[8] M. ASCE.—The discussion is a valuable contribution to this paper. The points raised by Mr. Shukry on the need for comprehensive planning of irrigation systems are cogent and deserving of the emphasis that he has placed on them. His suggestion that the paper should have devoted additional coverage to the problems of "full utilization of the water allocated to irrigation" is well taken. However, because of the limited space, this aspect of the paper was necessarily abridged.

Mr. Shukry's summary account of the proposed extension of irrigation developments in Egypt are of considerable interest and will no doubt receive widespread attention by all members of the profession who are engaged in furthering their countries' water resources and conservation developments.

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[8] Chf. Designing Engr., Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2491

SURVEYING AND MAPPING REQUIREMENTS FOR MODERN CITY PLANNING

By CHARLES A. BLESSING,[1] M. ASCE

WITH DISCUSSION BY MESSRS. HAROLD M. LEWIS, LEON T. ELIEL, AND CHARLES A. BLESSING

SYNOPSIS

The city planning profession is giving increased attention to the improvement of techniques and to the refinement of standards and criteria. Since surveys and accurate maps are the prerequisites of intelligent planning, it follows that, as the process itself is refined and perfected, so must the basic tools be refined. Planners must be alert to exploit new technical developments which will facilitate the preparation of these basic tools. Two major contributions in the field of urban surveying and mapping have been the development and perfection, largely since World War I, of aerial photography and photogrammetry and the development by the American Society of Civil Engineers of universally recommended procedures for the making of engineering surveys of cities.

Although prepared as long ago as 1934, ASCE Manual No. 10[2] continues as the best available statement on surveying and mapping for modern city planning. The emphasis in this handbook is on the making of city surveys rather than on the specific application of such surveys to problems of city planning. Manual No. 10 might well be supplemented by a basic manual covering the techniques and applications of aerial photography and photogrammetry to the economical and time-saving solutions of many city planning problems. A further supplement might be a definitive statement of surveying and mapping requirements for modern city planning from the point of view of the city planner who is the user of the survey rather than from that of the civil engineer who makes the survey.

NOTE.—Published in February, 1951, as *Proceedings-Separate No. 60.* Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Director, Planning Div., Chicago Plan Comm., Chicago, Ill.

[2] "Technical Procedures for City Surveys," *Manual of Engineering Practice No. 10*, ASCE, New York, N. Y., 1934.

PURPOSE OF PAPER

This paper is intended to stimulate thought and promote further study which might lead to such a definitive statement, with proper emphasis placed on the opportunity for a much broader application of aerial photography and photogrammetry in the solution of city planning problems.

Surveying and mapping requirements for modern city planning must be derived from an examination of standard planning techniques. Although considerable progress has been made in the development of city planning procedures since the 1920's, it must be recognized that city planners today continue to work under the handicap of an infant science. The different planners or offices use varied procedures and different classifications in surveys and studies, and all too often follow no rational standards that have been, or can be, adequately documented and justified.

Because of the increasing demands placed on a planning program to provide a master plan or official map as a specific guide for the expenditure of public funds for capital improvements, the planner has had to proceed from the general to the specific. Decisions of great importance in connection with vast redevelopment programs, superhighway programs, park, and school building programs have been thrust into the hands of the planner. Increased responsibility must be matched by increased performance. Clearly the time has come when city plans must be documented with fuller reasons for each step in the procedure leading to specific recommendations, whether for land use or for services pertaining to land use.

Webster's Dictionary defines science as " * * * accumulated and accepted knowledge which has been systematized and formulated with reference to the discovery of general truths or the operation of general laws * * * ." In their daily work and through the professional organizations, planners are formulating their ideas on city planning and reducing their principles to language. They have begun to experiment in framing a body of doctrine as a guide for more precise techniques for city planning. Careful study of the effective work being done by planning agencies, individual planners, and research groups leads to the conclusion that gradually, almost imperceptibly but surely, modern city planning is developing as a science.

Since each city is different from any other city and conditions of growth are variable and uncertain, no formalized techniques or outline of procedures can be more than a general guide. It is important, therefore, that procedures that can be reduced to a statement of principle should be formulated and that proved techniques should be established as standard practice for the encouragement of sounder planning throughout the United States. It should be emphasized that planning is more than the carrying out of specified techniques. Oliver Wendell Holmes' warning—"If a doctor hasn't got plenty of good common sense, the more science he has the worse for his patient"—applies with equal force to the city planner.

THE DEVELOPMENT OF STANDARD PLANNING PROCEDURES

Since surveying and mapping requirements are determined largely by the procedures and the scope of the city planning program, the logical approach to the mapping problem is to outline those procedures for the broad categories of planning studies which taken together constitute standard planning practice. The determination of the type of map or engineering survey required in each study is suggested by the broad classification of data required. For the major metropolitan regions of the United States or for a specific smaller city or town, it is usually possible to specify a desirable scale or range of scales commonly accepted for each type of map, the degree of accuracy required, the terminology and categories of classification of data, the detail required on the maps, and the choice of method for preparing the needed surveys and maps, based on cost, accuracy, time required in preparation, and range of use. Obviously, no two cities, even if approximately the same size, will require exactly the same kind of surveys and maps, and cities of widely varying size may require a still wider variation in maps and surveys

Essential Planning Studies

The specific mapping requirements for the different phases of city planning can be classified under twelve broad headings:

1. Geography, Topography, Geology, and Natural Resources.—For this basic study, the standard topographic survey map, as specified in Manual No. 10,[2] provides most of the necessary information. In addition to describing physiographic features, this section provides information on the underground geologic structures of the urban region and on the climatic conditions. Problems of water supply and drainage can appropriately be presented here or under the heading of public utilities.

Few large urban regions in the United States have adequate topographic maps. In a pioneer study, the New York (N. Y.) Regional Survey,[3] with the aid of the United States Department of the Interior, secured an enlargement of the United States Geological Survey map, which was made in 1923, and amplified this map by data from limited field surveys and aerial maps. The report stated:

> "The cost of making an accurate map showing all physical features based on field surveys would have been greater than the cost that has been incurred in making the whole Regional Survey and Plan–a map showing accurately the physical characteristics of the region, including its artificial features of railroad, highway, and buildings, its natural forests, swamps and water areas, and its contours to distinguish its hills and valleys is still an unfulfilled want."

Enlargements of the Geological Survey quadrangles on a scale of 3 in. = 1 mile are available in sheets, each covering 22 sq miles. The plan is reproduced on a scale of 1 in. = 1 mile. The contour interval is 20 ft. In contrast with this early regional map, the Pittsburgh, Pa., regional maps use a 2-ft contour interval and the Cleveland, Ohio, Regional Geodetic Survey provides 2-ft contour intervals on a scale of 1 in. = 200 ft.

The topographic map specified in Manual No. 10 is on a scale of 1 in. = 200 ft, with a 2-ft contour interval, and with a minimum of 1 ft for flat areas.

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[3] "Population, Land Values and Government," Regional Survey of New York and Its Environs, New York, N. Y., Vol. II, 1929, p. 219.

The topographic map shows ground contours; drainage features, such as lakes, rivers, marshes, creeks, ponds, wooded areas; and cultural features, such as buildings, pavements, property lines for streets and alleys, railroads, and bridges. Accuracy is indicated by the requirement that horizontal positions of 90% of all well-defined cultural and drainage features on the map are within from 0.01 in. to 0.02 in. of their true positions, and that the indicated elevations of 90% of the points are within one half of the contour interval of the correct elevations.

2. *Land-Use Studies.*—Real property surveys have been made in more than 250 cities in the United States since 1930. These surveys, largely standardized, generally followed the recommendations in the "Technique for a Real Property Survey" which was prepared by federal agencies in 1935.[4]

The base map is similar to the standard city survey topographic map, with the exception of the contour lines which have been removed and the individual lot lines which have been added. The monumental undertaking of the Chicago (Ill.) Land-Use Survey in 1939 was based on maps covering 238 sq miles (available as work sheets on a scale of 250 ft to the in. on 27-in. by 34-in. sheets) and also on nine single-line maps on the scale of 3 in. = 1 mile (bound in a 9-in. by 12-in. volume). The scale of 250 ft to the in. has proved entirely satisfactory for study purposes. Experience with these maps for 10 years has indicated a high degree of reliability as to both land-use information and base-map features. A scale of 1 in. = 200 ft is usually recommended. The Chicago land-use maps (scale 250 ft to the in.) are conveniently related to the wall maps with scales of 1 in. = 1,000 ft and 1 in. = 2,000 ft. The following maps were reproduced in the final report of the Chicago Land-Use Survey[5]: Base maps; land coverage; dwelling ratio (area per dwelling unit by blocks); present zoning; block data (eight factors on land use in block); economic area; problem area; transition area; median volume ratio; exempt areas tax; areas of nonconforming use; and areas of less restricted use.

In addition, other maps of the land-use survey include land use, growth trend, improved value, predominant use, number of persons per acre, land value as a percentage of total value, median area of use, and percentage of all structures in poor condition.

The continuously revised Sanborn Atlas maps,[6] with some sheets on scales of 1 in. = 50 ft and 1 in. = 100 ft provide a basic supplement to the 1939 Chicago Land-Use Survey. Current data on the Sanborn Atlas maps, together with the 1949 aerial survey covering 1,300 sq miles of the Chicago region on the scale of 1 in. = 400 ft, provide satisfactory working information on land use.

The property map, on a scale of 1 in. = 50 ft and showing lot lines and divisions of each parcel of property as described in Manual No. 10, provides substantially the same information as the Sanborn Atlas maps. Mention should be made of the general problem of standardizing terminology and categories of land use for city planning purposes. Just as it is important that city base maps meet specified standards so it is essential that land-use survey terminology be standardized.

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[4] "Technique for a Real Property Survey," U. S. Govt. Printing Office, Washington, D. C., 1935.

[5] "Residential Chicago," Chicago Land-Use Survey, Chicago Plan Comm., Chicago, Ill., Vol. 1, 1942.

[6] "Description and Utilization of the Sanborn Map," Sanborn Map Co., Pelham, N. Y.

The refinement of city planning and land-use survey techniques started in 1930 with the inauguration of the publication of a series of city planning studies at Harvard University in Cambridge, Mass. These studies differed from the then existing standard references on city planning in that they represented summaries of methodical case studies of various phases of planning in representative American cities. Since 1930, the Harvard series has produced useful working guides on such subjects as building height, bulk and form, urban land uses, transition zoning, neighborhoods of small homes, design of residential areas, model planning laws, building lines and reservations for future streets, airports, parkways, land values, and aerial photography.

Harland Bartholomew, M. ASCE, made a fundamental contribution to the techniques of city planning and zoning in his book[7] which outlined a method of estimating the total area required for each particular urban use for any given future population of between 5,000 persons and 300,000 persons. This study not only provided information in one place on land used for 22 typical American cities of less than 300,000 population but, more important, by following uniform field land-use survey techniques and by using standard and-use categories, provided a beginning toward the development of more precise tools for land-use analysis. In 1930 Mr. Bartholomew emphasized the need for some standard method of land-use analysis in order that the efforts of individual cities and city planners could be coordinated. The comparison of land uses in different cities is still difficult because different planners use slightly different classifications of land use, develop their procedures for survey and analysis as they go along, and, as often as not, work under the handicap of a lack of sound technical standards. Zoning as a legal instrument to guide the development of cities can be no sounder than the basic land-use studies that precede it.

The Bartholomew report was the forerunner of a number of significant contributions toward the same end. A milestone in the standardization of land-use terminology was the decimal system of land-use classification developed by Bryant Hall for Los Angeles, Calif. His logical basic classification included ten major categories, each being further subdivided in an orderly manner. The following standard categories are included: (1) Vacant unused land; (2) open use; (3) farming; (4) residence, one to three families; (5) residence, multiple; (6) commercial land; (7) industry; (8) utility; (9) institutions; and (10) recreation. Each of these ten major groupings was again divided into ten breakdowns.

The planning profession has been working under the handicap of a lack of uniform procedures, each planner using his own methods on work which might have been facilitated by the adoption of standard methods. Because this condition is so general, it is significant to note that a start is being made toward the improvement of procedures. Particularly significant in the direction of standardizing terminology and land-use survey procedures is the original contribution made in 1941 by Edward B. Wilkens[8] who suggested that land-use categories may be determined on the basis of the following: (1) The actual physical or inherent characteristics of the land itself; (2) the detriment caused to adjacent land use; (3) the type of structure erected; (4) the capacity of the use to crowd the land; (5) movement patterns of individuals or groups; (6) the services required or fostered by the activity pursued; (7) the service rendered to the community; (8) the taxable values produced; (9) the public or private ownership of, or access to, the land; (10) the nonflexible nature of public utilities and land-ownership patterns; and (11) the effects of pressure groups. Further work was done in this general direction by Mr. Wilkens under the sponsorship of the Public Administration Service.[9]

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[7] "Urban Land Uses," by Harland Bartholomew, Harvard Univ. Press, Cambridge, Mass., 1932.

3. Population: Density, Distribution, and Characteristics.—Population data for large cities are generally recorded for census tracts and community areas on skelton base maps (wall maps) on scales of 1 in. = 1,000 ft or 1 in. = 2,000 ft. Among the items of information normally mapped are the following data: Density; racial or nationality groupings; incidence of births, deaths, and certain diseases; crime; delinquency; and economic status. The community survey project of the University of Chicago has in its files, the recorded data by census tracts, communities, and subcommunities over a period of three decades. These published findings are an outstanding example of the contribution that has been made to city planning by educational institutions.

Spot maps have been used frequently to show total population, school age population, population changes, and racial and nationality groupings.

4. The Street System.—The basic map required in the planning of the street system is the topographic map already described under "2. Land-Use Studies." The recommended scale for this map is 1 in. = 200 ft. Acceptable scales for city planning are 1 in. = 300 ft and 1 in. = 400 ft. A smaller scale is usually required for regional plans. A scale of 1 in. = 2,000 ft was used for the New York Regional Plan studies and the Philadelphia (Pa.) regional studies. This same scale is being used for major thoroughfare studies in the City of Chicago. The Cleveland (Ohio) Regional Geodetic Survey, on the other hand, used the scale of 1 in. = 200 ft as a basis for regional highway and expressway planning, which conforms to the general specifications in Manual No. 10. This survey provides 2-ft contour intervals and shows all buildings including residences, but does not show garages. Aerial mosaics are particularly useful in the study of future street and highway rights of way. Survey maps provide information on traffic flow, origin and destination, accidents, street jogs and dead ends, street widths, street grades, railroad grade crossings and grade separations, location of traffic signals, and parking facilities.

Preliminary highway and expressway locations and design studies are often prepared at a scale of 1 in. = 50 ft.

5. Traffic, Parking, and Transit.—Studies of a general nature in large cities like Chicago can be shown on maps at a scale as small as 1 in. = 2,000 ft. More detailed studies are made at 1 in. = 200 ft, with intersection studies showing signals, signs, safety zones, and so forth, on the scale of 1 in. = 20 ft. Data commonly shown on traffic-study maps include vehicular and pedestrian traffic counts in congested districts, location of accidents, street obstructions, and one-way streets.

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[8] "The Planning Approach to Categories of Land Use," by Edward B. Wilkens, *Journal*, Am. Inst. of Planners, July-September, 1941, p. 20.

[9] "Mapping for Planning," by Edward B. Wilkens, *Publication No. 101*, Public Administration Service, Chicago, Ill., 1948.

Parking survey maps that show street parking facilities and off-street parking facilities vary in scale depending on the extent of the area being studied. Maps are prepared to show potential off-street parking sites; vacant lands, both public and private; obsolete buildings; etc. Transit survey maps show present transit routes, proposed extensions, service areas, time zones, fare zones, and flow maps. Large-scale maps of central business districts facilitate the detailed study of transit problems.

6. Transportation.—In all large cities and particularly in railroad centers, such as Chicago, the location of railroad routes and facilities is a factor of basic importance in all city planning studies. For typical railroad studies, maps are suggested as follows:

a. Map of general routes of entry and location of terminals (scale, 1 in. = 3,000 ft).

b. Map of general route studies in relation to neighborhood and community planning (scale, 1 in. = 1,000 ft).

c. Composite map of main tracks, joint tracks, sidings, industrial spurs, freight yards, coach storage, engine houses, team tracks, freight houses, grade crossings, grade separations, etc. (scale, 1 in. = 250 ft or 1 in. = 200 ft).

d. Detail map of tracks, buildings, rights of way, section lines, and boundaries of other railroad properties adjacent to rights of way. In this category are the standard right-of-way and track maps prepared by the railroads in connection with property valuation by the Bureau of Valuation of the Interstate Commerce Commission. A variety of details appear on such maps (scale, 1 in. = 100 ft).

e. Special project maps may be required for detailed studies of passenger and freight terminals, grade separations, and industrial development. (Ordinarily a scale of 1 in. = 100 ft is sufficient but when numerous details are required, a scale of 1 in. = 50 ft may be found necessary.)

f. Maps that show densities of freight and passenger traffic are usually on a distorted scale, the amount of lateral distortion being dependent on the volume of business and the area covered.

The planning of both water-terminal and air-terminal facilities should be carefully coordinated with highway, railroad, and transit plans. A convenient scale for master plan studies for transportation in a large city is 1 in. = 2,000 ft.

7. Public Utilities: Water, Sewerage, Gas, and Electricity.—The topographic map on the scale of 1 in. = 200 ft is a useful map for master plan studies of public utilities. Regional studies on scales of 1 in. = 2,000 ft and 1 in. = 3,000 ft include topography, land uses, highway system, and population distribution. Aerial photographs are useful in indicating the built-up areas of the urban region and the general character of development. ASCE Manual

No. 14,[10] published in 1937, recommends detailed atlas sheets on the scale of 1 in. = 20 ft and special intersection maps on the scale of 1 in. = 10 ft in highly developed and congested business districts. The master plan map for a single utility may be plotted on the scale of 1 in. = 200 ft. This map shows the size and location of all elements in the system, together with proposed extensions.

The planning of public utilites should be coordinated with zoning and the land-use plan to insure adequate service in all sections of the city.

8. Housing and Redevelopment.—The technique for conducting housing surveys has been standardized largely under the direction of federal agencies. Real property surveys oriented in large part to the study of housing problems have been made in more than 250 cities in the United States. Many of these surveys have followed the "Technique for a Real Property Survey."[4]

A basic housing survey such as that recorded in Chicago in 1942[5] includes information on a block basis. This information is included in the report on the scale of 1½ in. = 1 mile. Work maps on the scale of 1 in. = 250 ft are on file at the Chicago Plan Commission.

Maps are included for the following: Median rental, median year built, value of single-family structures, race, condition, median duration of occupancy, conversion and rooms, housing conditions, sanitary facilities, overcrowding, vacancies, owner occupancy, single-family structures, and control (number of structures and dwelling units). Although the commercial and industrial phases of the Chicago Land-Use Survey were not completed, the following schedule of information would indicate the content of such a survey. Block maps reproduced on a scale of 1½ in. = 250 ft would include rent profile, median annual store rental, availability status of vacancies, type of vacancies, amount of vacant business floor area, movement of business, median year built, average value of business structures, condition, working population, median duration of occupancy, hotels and rooming houses, ratio of owner occupancy, control (number of structures and business units), chain stores, transfer corners, class of business, percentage of retail establishments, and percentage of manufacturing establishments.

The redevelopment studies being conducted (in 1950) by the Chicago Plan Commission involve the use of maps at several scales: First, city-wide maps on a scale of 1 in. = 2,000 ft; second, land-use maps on a scale of 1 in. = 250 ft; third, generalized study maps on the scale of 1 in. = 500 ft. The South Central Redevelopment Area plan is drawn on a scale of 1 in. = 250 ft. Special highway studies were made using maps on a scale of 1 in. = 100 ft. Both the Sanborn Atlas maps and the 1949 Chicago Aerial Survey map have been indispensable in showing trends in development since 1940 when the land survey was made.

9. Recreation.—General city-wide maps required for a recreation survey of a large city can be on scales of 1 in. = 800 ft, 1 in. = 1,000 ft, or 1 in. = 2,000 ft depending on the size of the city. Survey maps should include the following information: Topography, existing and proposed land use, vacant area, zoning districts, street traffic flow, transit routes, spot map of pedestrian fatalities, community and neighborhood boundaries, types of planning areas, population growth, population density, school population, existing school and recreation area locations, areas not served by local playgrounds and parks, community parks and large city parks, neighborhood recreation area deficiency (in acres), community recreation area deficiency, large parks (existing and proposed), and private recreation facilities.

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[10] "Location of Underground Utilities," *Manual of Engineering Practice No. 14*, ASCE, New York, N. Y., 1937.

In the smaller cities, the base map might vary in scale from 1 in. = 200 ft to 1 in. = 600 ft. In a large city, like Chicago, square mile maps on scales of 1 in. = 200 ft or 1 in. = 250 ft would be needed in studying specific sites.

10. Subdivision Control.—ASCE Manual No. 16[11] specifies the following maps: Preliminary plat on a scale not exceeding 1 in. = 100 ft; and vicinity map on a scale not exceeding 1 in. = 400 ft that shows all existing subdivisions, street and tract lines of acreage parcels, and proposed street connections between the subdivisions that are being submitted and the adjoining areas.

The preliminary plat shows (among other information) contours at 5-ft intervals or less; location, widths, and names of all existing or platted streets within or adjacent to the tract; existing permanent buildings; railroad rights of way; section lines; political subdivisions or corporation lines; school district boundaries; and existing sewers, water mains, culverts, and other underground structures within and adjacent to the tract.

The final plat is drawn on a scale not exceeding 1 in. = 100 ft, and shows the boundary lines with accurate distances and bearings, the exact location and width of all existing or recorded streets that intersect the boundary of the tract, true bearings and distances to the nearest established street lines or official monuments, the exact location of the subdivision in reference to the county real estate records, and the exact layout of the subdivision.

11. Public Buildings.—City-wide maps of all public buildings, including schools, fire and police stations, libraries, and public administration buildings may be on a scale of 1 in. = 1,000 ft. More detailed site planning studies require a larger scale. Convenient scales are 1 in. = 50 ft, 1 in. = 60 ft, and 1 in. = 100 ft.

Specific site planning factors, such as topography, influence choice of scale and details to be included on site plans for public buildings.

12. Capital Improvement Program.—Improvements may be presented by maps showing all major projects by type. A scale of 1 in. = 1,000 ft is convenient for such a city-wide map. A great variety of maps might be used in connection with the presentation of the capital improvement program.

PHOTOGRAMMETRIC METHODS FOR CITY SURVEYS[12]

During the period from 1928 to 1948, aerial photography and photogrammetry have emerged as widely accepted supplementary surveying and mapping procedures. Although no photogrammetric method can be considered the answer to all mapping problems, or a complete substitute for standard ground-surveying techniques, photogrammetry now must be recognized as a basic and useful tool to supplement other standard methods. In many cases, a combination of photogrammetry and ground surveys produces the best result. In addition to developing and using photogrammetry in the preparation of contour maps, city planners particularly should become more familiar with the many uses of aerial photography and mosaic maps, since they are now only generally acquainted with the various categories of usefulness of these new aerial tools. It may be convenient to summarize the various applications of photogrammetry and aerial photography to city planning problems, in:

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[11] "Land Subdivision," *Manual of Engineering Practice No. 16*, ASCE, New York, N. Y., 1939.

[12] "Economics of Photogrammetry," by Leon T. Eliel, *Transactions*, ASCE, Vol. 111, 1946, p. 567.

(1) New subdivision layouts;
(2) Replotting of old and new streets;
(3) Location of recreation sites;
(4) Redevelopment studies;
(5) Selection of sites for hospitals, laboratories, and public buildings;
(6) Industrial land use and material storage;
(7) Traffic flow studies;
(8) Transportation studies;
(9) Accident studies;
(10) Conferences, printing, and reports;
(11) The planning of sewerage and disposal plants;
(12) Flood-control structures; and
(13) Drainage structures.

From the foregoing list it will be seen that there are few, if any, phases of modern city planning that cannot benefit from use of maps produced by the photogrammetric process.

The principal methods employed in the preparation of topographic maps by photogrammetry were effectively outlined in detail by Leon T. Eliel in 1945:[13]

> "Today in the United States it is probable that nearly all topographic mapping is being done by photogrammetric methods. The U. S. Geological Survey is the only organization among many now engaged upon the federal mapping program that still utilizes the plane-table method to any extent. This organization operates what is probably the largest multiplex unit in the world, so that even within the Geological Survey a large percentage of the total work is being done by photogrammetric methods. Other federal organizations and private organizations under contract to the government use the photogrammetric methods almost exclusively.
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> "* * * Photogrammetry has finally arrived at the age of full maturity. Although the methods will unquestionably continue to improve from year to year, they are already thoroughly practicable, and afford a tool which will find continuing and increased acceptance at the hands of thoughtful engineers."

In discussing photogrammetry, engineers should not lose sight of the many applications of aerial photography. A great many more miles are studied usefully from aerial photographs and mosaics than from contour maps.

USE OF AERIAL TOPOGRAPHIC MAPS BY OPERATING AGENCIES

The experience of the operating agencies of the city government in New Rochelle, N. Y.,[14] with aerial topographic maps, indicates a wide range of uses.

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[13] "Economics of Photogrammetry," by Leon T. Eliel, *Transactions*, ASCE, Vol. 111, 1946, p. 576.

[14] "How New Rochelle, N. Y. Uses Aerial Topographic Maps," by Thomas B. Ray, *American City*, August, 1949, p. 86.

These maps show 5-ft contours on the scale of 1 in. = 200 ft, and were used in the design of sanitary sewer and storm drain facilities at a great saving in time and man power.

The maps were used also in snow removal and street sanding operations and in revising routes of garbage collection equipment. The Bureau of Parks, Docks, and Harbors used the maps for studies of waterfront improvements and in the selection of future recreation areas.

Inspectors of the Department of Public Health used the maps as an aid in making sanitary surveys and studies of the extent of sewage pollution in bathing waters, streams, and reservoirs. Other operating agencies that have used these maps include the Police Bureau, the Fire Bureau, the Building Inspector's Office, and the Board of Education. The Bureau of Engineering uses the aerial topographic map continuously in working with private real estate developers on subdivision planning.

The experience in New Rochelle indicates that it would be difficult to conceive of a more widely used municipal investment than an aerial photographic map.

SUMMARY

In suggesting that a more general agreement as to mapping and surveying requirements for modern city planning will facilitate the work of a typical planning program, this paper is not intended to recommend a rigid mold into which a planning office must fit its particular mapping requirements. It is suggested, however, that reference to some standard practice may assist any planning office in establishing more precisely its own mapping specifications. It is hoped that discussion may test the general validity of the idea that some framework of mapping procedures would be helpful in any planning office. Precisely what that framework should be will, it is hoped, be more clearly stated in any future revision of ASCE Manual No. 10.

DISCUSSION

HAROLD M. LEWIS,[15] M. ASCE.—The kinds of maps in which city planners are vitally interested are discussed in this paper, which makes a strong plea for greater standardization. The writer believes the problem would be clearer if one grouped such maps under the following two headings: (1) base maps, for which the city engineer and surveyor is primarily responsible; and (2) maps presenting basic planning studies, for which the city planner is responsible and which can generally be presented with base maps as a foundation.

For many years city planners have been handicapped by the lack of adequate base maps. Often they have had to make their own based upon whatever existing maps were available. Many communities are still without an accurate street map, and in many places the only topography available is that shown on United States Geological Survey (USGS) maps made many years ago by plane-table methods. This type of mapping can and should be standardized. The second group of maps—that is, those showing basic planning studies—will vary widely, both in scale and material shown, with the size and character of the community.

There is a border line between map needs for city planning and map needs for the city engineer and various city departments. The city planner is primarily interested in the location and general character of improvements, existing and proposed. The city engineer and department staffs need enough information for the design and construction of those improvements. The former, therefore, wants a true scale, whereas the latter need true dimensions.

On this basis (in a typical case) the city planner might well settle for a base map at a scale of 1 in. equals 400 ft with 5-ft contours, while the city engineer would want a scale of 1 in. = 200 ft or 100 ft, and 2-ft contours. The city planner could make effective use of an air mosaic and a few oblique air views, but the city engineer would want these plus a photogrammetric map. Both should work together in developing a suitable mapping program and in developing the city plan.

The New York Regional Survey and Plan, referred to by Mr. Blessing, presents one extreme, and it was necessary to use the only region-wide maps available, that is, the USGS maps, which were enlarged from 1 to 62,500 to 1 to 24,000 (1 in. = 2,000 ft), at which scale the plan was drawn. The printed plan was reduced back to the scale of 1 to 62,500 (approximately 1 in. = 1 mile). Although more accurate and complete base maps would have been of tremendous help in this work, no such map as has recently been undertaken jointly by the City of Cincinnati and Hamilton County, in Ohio, would have been justified. In the latter case 414 sq miles to be mapped by aerial survey methods were estimated to cost $1,120,000.[16] At this same unit cost per square mile, it would cost today nearly $15,000,000 to map the 5,500 sq miles in the

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[15] Cons. Engr. and City Planner, New York, N. Y.

[16] "Mapping Cincinnati," by H. H. Kranz, *Civil Engineering*, Vol. 20, February, 1950, p. 37.

New York region, whereas the total cost of the New York Regional Survey and Plan, including its ten volumes of reports, was about $1,300,000. Thus the statement quoted by Mr. Blessing, from one of the Regional Plan reports, that an accurate map would have cost more than the survey and plan proves to be a very conservative one.

Three general types of base maps are needed as follows:

1. Detailed base map for comprehensive studies and conferences;
2. Simplified base map at smaller scale for publication; and
3. Outline map for statistical presentation and studies of single elements of the plan.

The detailed base map should show both the visible features, such as drainage, culture, topography, and wooded areas, and invisible features, such as mapped streets versus those in existence, property lines, and the use of land and structures. The use of land and structures is particularly essential in zoning.

Mr. Blessing states (under the heading, "Purpose of Paper") "* * * modern city planning is developing as a science." There is already such a science, with its techniques and principles well established, but these facts should not be permitted to put planning in a strait jacket. The city plan should be custom-made and not factory-made. Planning is actually more than a science. James Ford has described it as "* * * a science and an art * * *."[17] Thomas Adams defined it as "* * * a science, an art, and a movement of policy * * *."[18]

Planners have been accused of stating their principles in jargon, but are now attempting to simplify them in everyday language. If planners can get adequate base maps and simplified reports utilizing standardized basic maps at scales suited to the municipality, city planning will become better understood and will be more readily accepted.

LEON T. ELIEL[19].—The Committee on City Surveys of the Surveying and Mapping Division of the ASCE is in the process of revising Manual of Engineering Practice No. 10 ("Technical Procedure for City Surveys"). A considerable section pertaining to aerial photography and photogrammetry will be added.

In the paper a wide range of scales is mentioned ranging from 1 in. = 40 ft to 1 in. = 3,000 ft, a spread of 75-fold. Such a galaxy of scales and the attempt to assign a suitable scale for each different class of activity suggest that some basic consideration be given to the elements that go into the choice of a suitable scale for a particular map. Possibly the scale of the map is selected in order to permit measurements to be determined with a certain degree of accuracy. It is ordinarily accepted that a map can be scaled to about 0.02 in. Thus, if the map in question needs to be measured to an accuracy of 1 ft, at once there is an indication of the scale that is required (0.02 in. = 1 ft) or, to state it in the more conventional way, 1 in. = 50 ft.

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[17] "Slums and Housing," by James Ford, Harvard Univ. Press, Cambridge, Mass., Vol. I, 1936, p. 490.

[18] "Outline of Town and City Planning," by Thomas Adams, Russell Sage Foundation, New York, N. Y., 1935, p. 21.

[19] Vice Pres., Fairchild Aerial Surveys, Inc., Los Angeles, Calif.

On the other hand, if the maximum scaling accuracy required of the map is 10 ft, then a suitable scale can be arrived at by starting with the expression 0.02 in. = 10 ft. Multiplying each side of the equation by 50 results in a scale of 1 in. = 500 ft.

Oftentimes accuracy is not the criteria for establishing a scale at all, but rather the scale is determined in order to provide ample layout space. If, for example, a map or plan is to be drawn on which each tread of a stairway needs to be shown $\frac{1}{8}$ in. wide, then the scale must be $\frac{1}{8}$ in. = 1 ft (the width of the tread), or 1 in. = 8 ft. Possibly the smallest feature that must be taken into consideration on a map is a 10-ft alleyway that must show as a double line. In such a case, perhaps the double line should be $\frac{1}{10}$ in. across, resulting in a scale of $\frac{1}{10}$ in. = 10 ft, or 1 in. = 100 ft.

Again, it may be that the governing criteria in the scale of a map is the need for room to contain a great deal of descriptive matter. In this case the space available for these printed blocks will be the determining factor in choosing the map scale. Possibly the smallest feature that it is desirable to show on a map will be one city block, which is 400 ft on a side. It may be considered that, if this city block is shown $\frac{1}{4}$ in. square, it will give an adequate representation for the map user. In this case the starting expression is $\frac{1}{4}$ in. = 400 ft, which gives a map scale of 1 in. = 1,600 ft.

Another factor that frequently will govern the choice of scale will be the desire to supply maps that conform to the size of existing filing facilities. Thus, possibly, a map of a city that is 5 miles across is desired all on one sheet, unfolded, that can fit into a 40-in. file. The required scale would, of course, be 8 in. = 1 mile.

In specifying a scale, consideration should always be given to the necessity for a particular scale. Ordinarily a certain scale connotes a certain measuring accuracy, but if this accuracy is not needed it costs a lot of money, and the scale can be achieved just as well by making the map initially at a smaller scale and enlarging by either photographic or graphic processes. A large-scale map costs money because it means that extra drafting is involved, unless the large scale is achieved by enlarging a smaller map. A large-scale map also means that from the beginning the process by which the map is derived will usually be carried on to a precision beyond that actually needed. So, in considering map scale, it is always well to analyze whether a certain scale is needed for accuracy or for space.

Photogrammetric methods are playing such an important role in the preparation of contour maps that it should be kept clearly in mind that (unlike the plane-table survey in which scale is usually the criterion of cost) the cost in photogrammetric mapping is usually more closely associated with contour interval. According to the different systems of photogrammetric mapping, the flying altitude may range anywhere from 250 to 1,250 times the contour interval. For this reason, the cost of a 2-ft contour is apt to be about 3 times the cost of a 4-ft contour. The photogrammetrist will fly twice as high for the 4-ft contour, and, therefore, he will have only one fourth the number of pictures and one fourth the amount of ground control. It is well, therefore, to evaluate carefully whether a 4-ft contour would suffice when the first reaction is that a 2-ft contour is desirable. If the 4-ft contour will serve the purpose 90% as well, it obviously is good economics to buy it, because it will cost perhaps one third as much. With the remaining two thirds of the money a lot of work can be undertaken on the ground to bolster up occasional spots where there is real necessity for something better than the 4-ft contour.

In summary, it is a great extravagance to be a perfectionist as far as map requirements are concerned. A map made to the highest specifications at a large scale and a small contour interval is, of course, what everybody would like to have in approaching a project. This desire, however, must be tempered by all of the practical considerations that have been discussed.

Many cities fail to have adequate maps because of the information that has been available in trade periodicals about the cost. Great cities like Cleveland and Cincinnati, Ohio, Tacoma, Wash., and San Diego, Calif., do a comprehensive job. Many smaller cities with limited budgets get a fine job of mapping for themselves by utilizing photography only, leaving the first-order triangulation and levels, and the contour maps and lithographic reproductions for their bigger brothers. For about one tenth the cost per square mile, a city can purchase excellent photomaps made to an accurate scale and showing every feature of the city in proper relationship. Probably these photomaps will solve a large percentage of the problems that arise in the course of day-by-day city work almost as well as the more complex maps. Many small cities have been delighted to find out how much useful information they can get for prices such as $100 and $200 per sq mile, and sometimes even less.

This paper has been very thought-provoking as far as the writer is concerned, and he is grateful for this healthy stimulus to our city map thinking.

CHARLES A. BLESSING,[20] M. ASCE.—The purpose of this paper was confined to a description of the various types of base maps required in modern city planning studies. No attempt was made to describe the details involved in the preparation of finished maps showing survey data or proposed plans. It is to be greatly desired that a professional planning organization such as the American Institute of Planners, prepare a fully detailed manual of procedure for all phases of map making used by city planners in their own work. Such a manual would suggest such items as standardized symbols, patterns of hatching, and simplified titles. The standardization of survey maps as well as of base maps should accompany the general refinement of techniques in the planning field. Agreement on mapping techniques would do much to encourage sounder planning procedures in many countries that today depend on American engineers and planners for technical assistance. Such standardization would provide the means of a fuller exchange of knowledge and information on city planning practices throughout the world.

Just as the planning process increasingly involves the improvement of techniques of measurement (measurement of industrial nuisances, of standards for a healthful environment, of standards for light and air) so should it include techniques for simpler and more effective graphic presentation of ideas, simpler and more readible reports, and simpler maps as tools of analysis.

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[20] Director of Planning, Chicago Plan Commission, Chicago, Ill.

Mr. Lewis clearly contrasts the two distinct phases of work divided between the city planner and the engineer. The planner, interested in location and general character of improvements, needs a true scale whereas the engineer, interested in design and construction, needs true dimensions.

In his discussion Mr. Eliel has, with sound logic, urged that more attention be given to elements that go into the decision on the choice of a suitable scale for a particular map. He then lists the degree of accuracy required, the needs for ample layout space, the need for space to contain a great deal of written descriptive matter, and the need for a convenient sheet size that, unfolded, can fit into a standard map file. With this clarification of the reasons for determination of various scales the writer is in complete agreement.

Mr. Eliel's discussion of cost of photogrammetric mapping as affected by scale and contour interval will be helpful to the planner in his choice of maps for a particular project. His summary statement that it is a great extravagance to be a perfectionist as far as map requirements are concerned is heartily agreed with by the writer.

The writer's recent experience in planning work in a large city has convinced him that commission members prefer for conference use a simple effective map prepared with generous use of colored patterns to indicate types of land use, colored scotch tape strips to indicate transportation routes, and "single idea" presentation maps made possible by use of transparent acetate overlays.

The average plan commissioner or city official would prefer that the usual complicated study maps, filled with the detail that delights the professional practitioner, be left in the file drawer in the back room. He would prefer for public hearings, commission meetings, printed reports or exhibits, that the planner submit maps with clear-cut eye appeal, simplicity, and the abundant use of color that the popular magazines have taught the public to demand.

The planner's finest vehicle for selling his plans is a set of coordinated graphic materials, such as photographs, perspective drawings, models, charts and diagrams, and most important of all—a series of clear, simple, and attractively presented maps.

The writer is appreciative of the thoughtful discussion of this paper. He believes that an opportunity lies ahead for the city planner and the civil engineer for continued improvement both of base maps and completed survey and comprehensive plan maps.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2492

THE NEW TOWNS PROGRAM IN GREAT BRITAIN

BY T. C. COOTE[1]

WITH DISCUSSION BY MESSRS. W. STRICKLER, AND T. C. COOTE

SYNOPSIS

Beginning with the early history of the "Garden City" movement, the paper mentions the several steps in the implementation of the "new towns" program. The writer analyzes the types of new towns undertaken in England, Wales, and Scotland, and the reasons for them. The site investigation required before a proposed new city is authorized is described.

After development corporations have been established and after staff and offices have been obtained, the first step is to accumulate all necessary survey material relative to the site. This is followed by the preparation of a master plan which shows their broad intentions for the planning of the area. The surveys and the plan, together with an explanatory report, and a program for construction and estimates are submitted to the Minister of Town and Country Planning for his approval. Detailed layout plans for smaller areas within the framework then follow. The importance of teamwork in design and construction is emphasized. After mentioning the limited progress so far made, a brief review of some of the engineering problems is presented under headings of sewerage, water supplies, gas, electricity, streets, bicycle tracks, traffic circulation, and railway facilities.

EARLY DAYS

On June 10, 1899, Ebenezer Howard founded the Garden City Association in London (later named the Town and Country Planning Association), and public interest first began to be awakened in his garden city remedy for the evils of metropolitan overcrowding and the depopulation of rural areas. This led to the foundation of the first "garden city" at Letchworth in Hertfordshire, in 1903, and later to the establishment of Welwyn Garden City, in 1920

NOTE.—Published in March, 1951, as *Proceedings-Separate No. 62*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Senior Regional Planning Officer, Ministry of Town and Country Planning, London, England.

(see Fig. 1). In 1921 the reports of the Unhealthy Areas Committee (Ministry of Health) recommended the development of self-contained garden cities with state assistance in their early stages. Since early pioneering days other advocates, such as the late Sir Raymond Unwin, for the Greater London Regional Planning Committee (report of 1929 to 1933), A. Trystan Edwards with his "Hundred New Towns Association," in 1933, Thomas Adams, F. J. Osborn, Sir Patrick Abercrombie, and many others have urged forward the "new towns" idea. In 1932, with the passing of the Town and Country Planning Act, statutory provision was made for the development of garden cities under section 35 of that act.

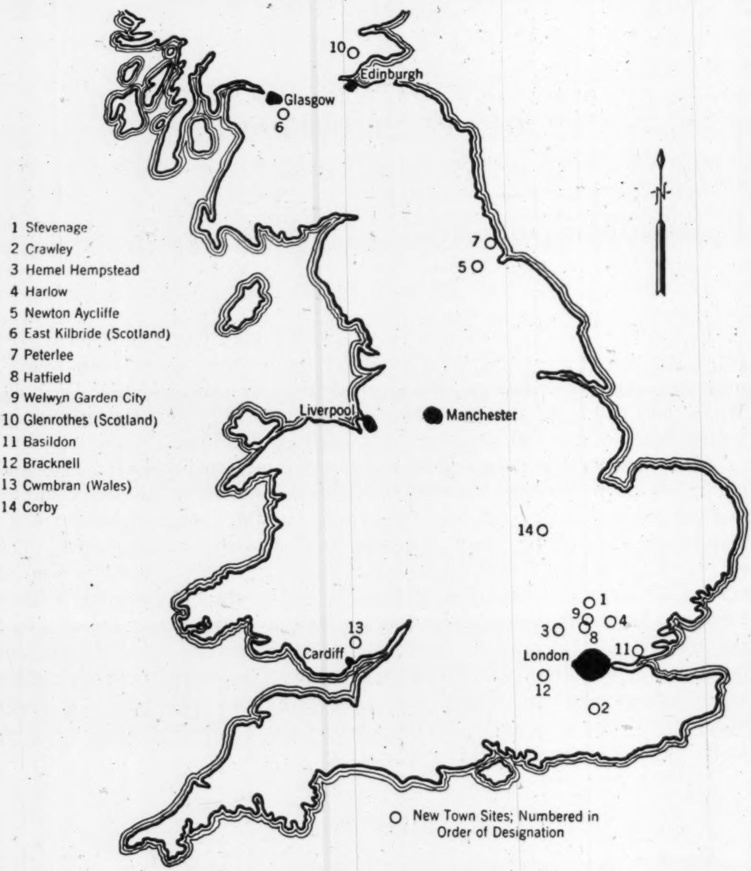
1 Stevenage
2 Crawley
3 Hemel Hempstead
4 Harlow
5 Newton Aycliffe
6 East Kilbride (Scotland)
7 Peterlee
8 Hatfield
9 Welwyn Garden City
10 Glenrothes (Scotland)
11 Basildon
12 Bracknell
13 Cwmbran (Wales)
14 Corby
10
Edinburgh
Glasgow
6
N
7
5
Liverpool
Manchester
14
1
9
3
4
8
13
London
11
Cardiff
12
2
○ New Town Sites; Numbered in Order of Designation

FIG. 1.—NEW TOWNS IN GREAT BRITAIN, AS OF JULY, 1949

The rapid growth of large cities in the years between World War I and World War II, and the further depopulation of rural areas, led the government to set up one royal commission and two committees—the Barlow,[2] the Scott,[3] and the Uthwatt[4]—whose findings drew attention to the paramount importance of remedying these evils. The Barlow Commission specifically recommended[5] the establishment

> "* * * of garden cities, satellite towns and trading estates as a useful contribution * * * towards the solution of the problem of relieving overcrowded and congested urban areas * * *."

In the County of London Plan[6] and the Greater London Plan[7] Sir Patrick recommended the immediate establishment of eight or ten new satellite towns to accommodate the population overflow from London.

The New Towns

Following the publication of the Greater London Plan, a small group of technical officers of the Ministry of Town and Country Planning was formed in September, 1945, to prepare (as a matter of urgency) a master plan for the first of the new towns at Stevenage. As no New Towns Act then existed the minister's powers were limited to those contained in section 35 of the Town and Country Planning Act (1932) mentioned previously.

At about the same time (October, 1945) the then Minister of Town and Country Planning, the Right Hon. Lewis Silkin, and the then Secretary of State for Scotland, the late Right Hon. Joseph Westwood, set up a New Towns Committee under the chairmanship of Lord Reith of Stonehaven to advise them on

> "* * * general questions of the establishment, development, organisation and administration that will arise in the promotion of New Towns in furtherance of a policy of planned decentralisation from congested areas: and in accordance therewith to suggest guiding principles on which such towns should be established and developed as self-contained and balanced communities for work and living."

The reports of this committee served as a basis for the New Towns Act which was passed through the Houses of Parliament without substantial party opposition and received royal assent in October, 1946. The act gave power for the designation of areas for new towns and the setting up of "development corporations" to undertake the work of their design, construction, and management.

The act was followed immediately by the designation of an area for the new town at Stevenage and very shortly thereafter by three others at Crawley,

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[2] "Distribution of the Industrial Population," *Report Cmd. 6153*, Royal Commission on the Distribution of the Industrial Population, His Majesty's Stationery Office, London, 1940.

[3] "Land Utilization in Rural Areas," *Report Cmd. 6378*, Committee on Land Utilization in Rural Areas, Ministry of Works and Planning, His Majesty's Stationery Office, London, 1942.

[4] "Compensation and Betterment," *Final Report Cmd. 6386*, Expert Committee on Compensation and Betterment, Ministry of Works and Planning, His Majesty's Stationery Office, London, 1946.

[5] *Ibid.*, paragraph 291.

[6] "County of London Plan, 1943," by J. H. Forshaw and Sir Patrick Abercrombie, report prepared for the London County Council, London, England.

[7] "Greater London Plan, 1944," by Sir Patrick Abercrombie, report prepared for the Standing Conference on London (England) Regional Planning.

Hemel Hempstead, and Harlow. Objections by some residents in the areas concerned led to litigation which prevented the confirmation of three of the orders for several months.

TYPES OF NEW TOWNS

Of the four new towns (Nos. 1, 2, 3, and 4, Fig. 1), Stevenage and Harlow were located exactly as proposed by Sir Patrick in the Greater London Plan, whereas Hemel Hempstead and Crawley took the place of two other towns proposed by him on near-by sites. Pressure from housing authorities in the central areas, pending progress on the construction of the new towns beyond London's Green Belt, made necessary a considerable, although limited, amount of housing in the Green Belt itself. Further new towns have consequently been designated at Welwyn Garden City (No. 9) and Hatfield (No. 8) in the north, at Basildon (No. 11) on the east, and at Bracknell (No. 12) in the west. These eight London new towns, when fully developed, will provide for an overflow population of 270,000 persons. The total overflow for the London area, however, is of the order of 1,250,000 persons and a natural increase will further affect this total.

The London region, however, is not the only part of Great Britain to which the New Towns Act has been applied. At Peterlee (No. 7), near Easington, County Durham, a new town has been designated toward the eastern edge of the coalfield, where the newest pits are situated, beside the North Sea. Near-by, inland, are a number of older pits with shorter periods of life, and each one with its old mining village clustered around the pit head. Housing conditions in these villages, by modern standards, are deplorable. Older houses are being demolished, and it has now been decided to concentrate replacements in the centrally situated new town called Peterlee where it will be possible to provide better shops, social facilities, schools, motion picture theaters, and so forth, and some employment for those women who wish to go out to work.

Investigations into the needs of the South Wales development area have also led to the designation of a new town at Cwmbran to the north of Newport in Monmouthshire (No. 13, Fig. 1). This site is just off the coalfield, and houses are needed for those persons, at present, or likely to be employed in the new factories erected there by the Board of Trade as part of the government's policy for the relief of unemployment in the South Wales Development Area.

Another type of new town is that designated at Newton Aycliffe (No. 5) in County Durham. During World War II a large royal ordnance factory was in operation at this site, and after the war this was converted into an industrial estate under the sponsorship of the Board of Trade. The plastic industry has also constructed a new factory on near-by land. Since this estate draws its labor from a radius as far away as twenty miles, it was decided to build a small new town immediately adjoining, yet separated from it, to house some of the workers and their families close to their source of employment.

Large cities, such as Liverpool and the Manchester conurbation, urgently require sites for the decentralization of population and work places, late Minister of Town and Country Planning suggested to the planning authorities that two new towns might be needed in north Lancashire, but no decisions were made at the time. A further site to the south of Manchester is also being investigated.

In Scotland two new towns have been designated, at East Kilbride in Lanarkshire, to the south of Glasgow, and at Glenrothes in east Fifeshire. East Kilbride lies over the 700-ft escarpment about seven miles from the center of Glasgow and will be required to accommodate the overflow of population and industry from that city and from northwest Lanarkshire. The government has also agreed that the new Mechanical Engineering Research Station of the Department of Scientific and Industrial Research should go to East Kilbride. This should employ about 10% of the future industrial population. Glenrothes lies about five miles north of Kirkcaldy and will be needed to house about a half of the anticipated increase of miners in the east Fife coalfield. The site was selected because it was the only area of adequate size sufficiently close to the new pits which was free from subsidence, did not involve good agricultural land, and was capable of being provided with main drainage and water supply.

Other new towns have been proposed for Scotland at Houston-Bishopton, Cumbernauld, and Lugton in the Clyde Valley and at Newton Grange, Prestonpans, Cumnock, and Lochgelly.

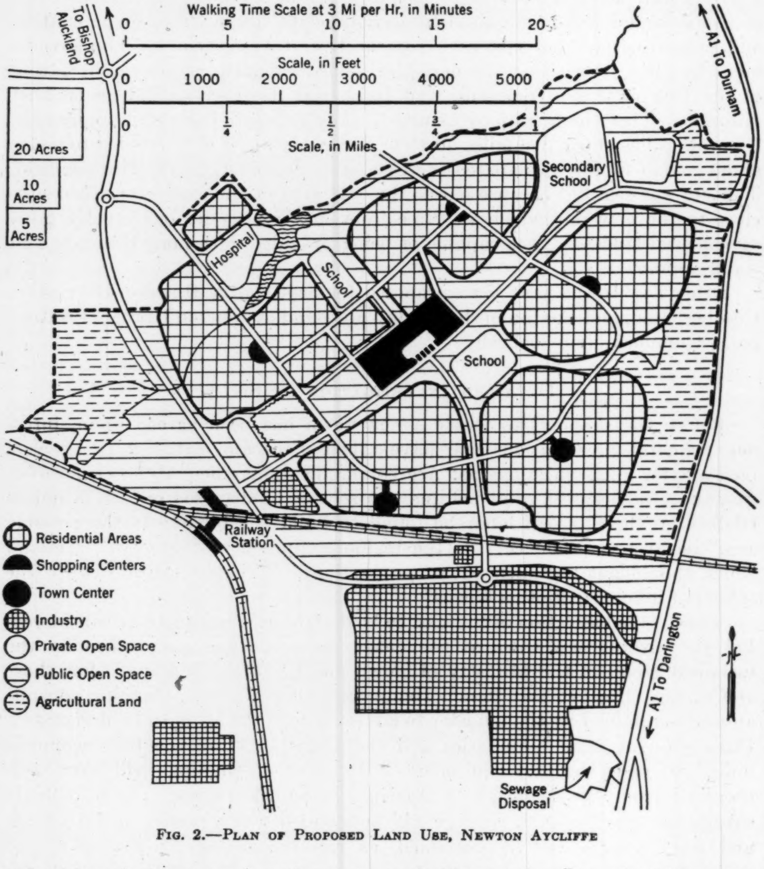
Detailed Investigation of Sites

Before the proposal to locate a new town has been agreed upon, interdepartmental consultations will have taken place and satisfactory answers received for such questions as: Are communications adequate? Can water, gas, and electricity be provided and can sewage be disposed of? Will industrialists go there? Can labor be found to build the town? Is the possible area free from subsidence? Is the region composed of high-class farmland which should not be taken out of production? Will its location disturb the sphere of influence of other towns?

The detailed selection of the area to be designated remains to be considered. For the earliest new town at Stevenage the work was undertaken by the technical group of the Ministry of Town and Country Planning. For those at Crawley, Harlow, and Hemel Hempstead eminent town planning consultants were retained by the minister to advise him on the areas to be designated. Plans showing farm boundaries and the farms that are the best economic units and have the best equipment in the way of buildings will have been obtained from the Ministry of Agriculture and Fisheries. Land liable to subsidence, flooding, and seepage will be avoided where possible. Fog, frost, and wind records will be examined, as will the geology, topography, and drainage areas. Steep slopes and rock outcrops will not be available for building. These and other factors will be taken into account when the consultant's preliminary plan is prepared. The plan will show that it is possible that a town of the requisite size could be built within the boundary to be designated and would satisfy the conditions and standards laid down. Approximately 900 acres per 10,000 population will be required.

PREPARATION OF MASTER PLAN

The first job of the development corporation is to produce a survey and an outline master plan. The survey will include: Topography, air photograph mosaics, geology and soil tests, existing land use, age and condition of the buildings (including those of architectural or historic merit), water resources and mains, drainage areas and sewers, gasholders and mains, electricity stations and mains, post office cables and exchanges, roads and footpaths, bus routes, traffic flow, rail facilities, waterways, airports, farm units, and existing planting.


Walking Time Scale at 3 Mi per Hr, in Minutes
0
5
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3/4
1
Scale, in Miles
To Bishop Auckland
A1 To Durham
20 Acres
10 Acres
5 Acres
Secondary School
Hospital
School
School
Railway Station
Residential Areas
Shopping Centers
Town Center
Industry
Private Open Space
Public Open Space
Agricultural Land
A1 To Darlington
Sewage Disposal

FIG. 2.—PLAN OF PROPOSED LAND USE, NEWTON AYCLIFFE

Only when the survey is complete can the master plan be designed. It will show the broad uses to which the land is to be put, namely: Residential, industrial, principal roads, civic, commercial, and the main shopping center. It will also show the locations of the local shopping centers, schools and their playing fields, major open spaces, railway land, and other open land (Figs. 2, 3, and 4).

The surveys and master plan will be submitted to the minister. They will be accompanied by an explanatory report which will include a broad statement of provisions (under separate heads), estimates of progress for the first three years, together with financial estimates and the labor requirements.

PREPARATION OF DETAILED PLANS

Not until the survey and master plan have been completed and the broad framework of the new town determined, will it be possible to proceed with the detailed design of the separate parts, such as the housing areas and shops lying within the framework.

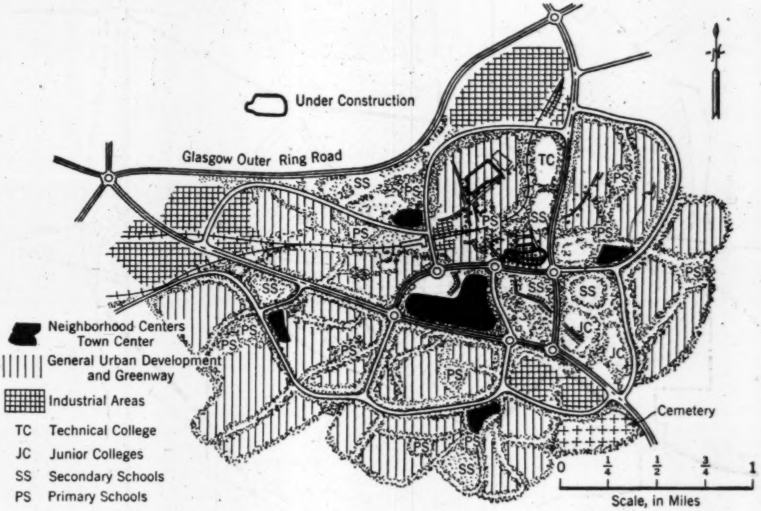
Under Construction
Glasgow Outer Ring Road
TC
SS
PS
JC
Cemetery
0
1/4
1/2
3/4
1
Scale, in Miles
Neighborhood Centers
Town Center
General Urban Development and Greenway
Industrial Areas
TC Technical College
JC Junior Colleges
SS Secondary Schools
PS Primary Schools

FIG. 3.—OUTLINE PLAN OF A NEW TOWN AT EAST KILBRIDE

Teamwork between the chief technical officers is an essential condition. As an example of such teamwork, one development corporation may be cited in which the chief architect or planning consultant is called on to produce numerous variations of possible layouts. These layouts are in turn submitted to joint meetings with the chief engineer whose requirements concerning the economics of the proposed services must be satisfied. The chief estates officer will state whether he can rent the properties in their proposed locations, and the chief legal officer will advise on the legal aspects. When plans seem reasonably economic the entire project is sent to the quantity surveyors to estimate the cost, and rentals are calculated. If these are unsatisfactory the entire process is repeated again until the best result is achieved. All functions are under the controlling hand of the general manager to keep a proper balance.

STATUS OF CONSTRUCTION SCHEDULE

When this paper was originally written (in July, 1949), little actual construction work had been undertaken in the new towns. This situation arose partly through the necessity for investigating the nature of the sites thoroughly before beginning work, partly through the slow build-up of design staffs, and largely through the national limitation on capital expenditure. A token allocation of 300 men for civil engineering work in the London "new towns" enabled a start to be made in 1948 on the provision of the first roads and

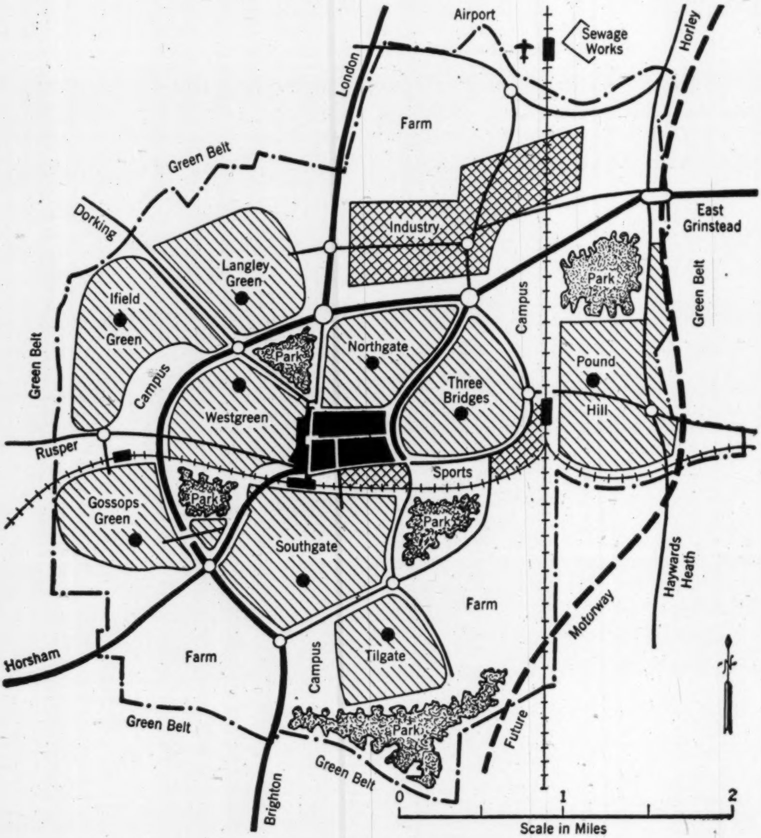
Airport
Sewage Works
Horley
London
Farm
Green Belt
Dorking
Industry
East Grinstead
Langley Green
Ifield Green
Park
Campus
Green Belt
Northgate
Three Bridges
Pound Hill
Westgreen
Rusper
Sports
Gossops Green
Park
Park
Southgate
Haywards Heath
Farm
Motorway
Horsham
Farm
Tilgate
Campus
Future
Green Belt
Park
Green Belt
Brighton
0
1
2
Scale in Miles

FIG. 4.—THE NEW TOWN OF CRAWLEY

services. Independently of this development, the inclusion of the new towns housing program in the national program (sponsored by the Ministry of Health) permitted a small number of houses to be erected by the development corporation. By November, 1950, however, all the older corporations had begun to make rapid strides forward. Actual construction work (see Table 1) was then being approved at an ever increasing rate. As more and more building and civil engineering labor can be brought to bear on the site, the speed of building will increase.

TABLE 1.—SCHEDULE OF DWELLING CONSTRUCTION, NEW TOWN PROGRAMS IN GREAT BRITAIN

NEW TOWN (EXCLUDING SCOTLAND)		ACTUAL STATUS ON OCTOBER 31, 1950			ESTIMATED STATUS ON DECEMBER 31, 1950		Plans for other buildings, etc. — approved November, 1950
Name (1)	Fig. 1 (2)	Approved (3)	Completed (4)	Incomplete[a] (5)	Complete (6)	Incomplete[a] (7)	(8)
Basildon	11	267	....	....	....	180	One factory[b]
Bracknell[c]	12	....	....	....	....	50	First housing site approved November
Corby[c]	14	....	....	....	....	....	
Crawley	2	726	69	436	240	450	Seven factories[b,d,e]
Cwmbran[c]	13	....	....	....	....	....	Corporation offices
Harlow	4	1,473	102	468	140	660	One factory; labor camp[f,g,h]
Hatfield	8	155	....	41	....	100	Eight workshops; temporary youth center
Hemel Hempstead	3	653	102	535	200	520	Labor camp; two factories[b]; fourteen shops
Newton Aycliffe[i]	5	467	131	124	210	270	
Peterlee	7	352	....	120	....	250	
Stevenage	1	393	24	241	130	600	Water supplies; two labor hostels; six shops[j]
Welwyn Garden City	9	279	16	123	60	230	Two factories; two factory extensions; eleven shops
Totals		4,066	444	2,088	980	3,310	

[a] Number of dwellings under construction. [b] Part of industrial area site works under construction. [c] In all the New Towns except Bracknell, Corby, and Cwmbran. Civil engineering works to open sites for development are in progress (January ,1951). [d] Part of main sewage works under construction. [e] Roads and sewers for two neighborhoods. [f] Part of industrial area layout approved. [g] Layout of two neighborhoods approved. [h] Trunk sewer from Harlow to Rye Meads (approx. five miles). [i] About half the main roads in town under construction. [j] Roads and sewers for one neighborhood.

SEWERAGE

By the very nature of their site requirements, most of the new towns are reasonably separated from other townships. In the London area the construction of regional sewage disposal works is likely to cause delay in the rapid construction of some of the new towns. Hemel Hempstead and part of Hatfield lie within the drainage area of the Colne Valley Regional Sewerage Board, established before the war, and work on the disposal works and trunk mains is well in hand. In the case of Stevenage, Harlow, and Welwyn Garden City, there were no such project plans and it has been proposed that these towns and other towns in the natural drainage area shall combine to form the Middle Lee Regional Sewerage Board. Crawley, Bracknell, and Basildon (near London), Aycliffe and Peterlee (in County Durham), and East Kilbride (in Scotland) will have their own sewage disposal works. Cwmbran in Wales and Glenrothes in Scotland will be in regional projects.

Separate systems for sanitary sewage and storm water will generally be installed. Consulting engineers have been appointed for most of the new towns under construction both for sewerage and water supplies.

WATER SUPPLIES

For the London new towns north of the Thames River water supplies will be obtained from deep wells drilled in the chalk. At Crawley the construction of an impounding reservoir to hold 1,200,000,000 gallons, formed by an earth dam, was begun in August, 1950. The cost of this project is estimated at more than a million pounds, and it will serve both the new town and surrounding areas.

GAS AND ELECTRICITY

The nationalization of gas and electricity in Great Britain and the formation of a gas council and of the British Electricity Authority have greatly simplified the negotiations for the supply of gas and electricity to the new towns. New towns were often in two or more statutory areas, and unification has meant that supplies can now come from one source with a consequent reduction in cost and with unified charges over the area.

DISTRICT HEATING

In the new towns it had been hoped to make considerable use of district heating and at Newton Aycliffe (Fig. 5) the work of design had proceeded to the final cost estimating stage. These estimates showed that an average weekly rental of about 9 shillings 4 pence would have been incurred all year, and it was felt that this cost would be too high. In consequence, district heating for the entire town was abandoned. A suitable boiler that could be spared was found at the royal ordnance factory, and it was proposed to use it as a source for district heating. The boiler would have generated about 100,000 lb of steam per hr at a pressure of 100 lb per sq in. The steam would have been fed through underground mains running in ducts to a ring main in the town. From this closed circuit branches would have been taken to five or more substations where the steam would heat up the water, the condensate being returned by pumps to the boiler house. The hot water would be circulated by pumps to the various houses and other buildings at a maximum temperature of 225° F flow and 175° F return through a two-pipe system designed to be reversible. The object of the reversibility was that during winter the water flowing in one direction would have entered both the heating radiators and hot water calorifier in each house; but in summer, when the flow was reversed, the circuit through the radiators would have been closed by a special nonreturn valve, leaving only the hot water supply calorifier in use.

At East Kilbride (see Fig. 3) groups of from 200 to 500 dwellings were linked to some heavier load such as a school and provided with hot water and heating. Experience with the first groups in operation indicates a cost of about 9 shillings per week. Similar plans have been put into operation for about 800 houses at Glenrothes (location 1, Fig. 1).

It may be found, therefore, that a modified form of district heating will be possible where the density of development can be kept fairly high in the residential areas and these can be linked to heavier load users.

ROADS AND STREETS AND TRAFFIC CIRCULATION

Basic traffic safety considerations in all the new towns have led to a design pattern in which main and local through traffic by-passes the living or working areas. These areas, in turn, are connected to the through ways at a strictly limited number of access points, so placed as to provide the most convenient and shortest possible routes along the line of "drag." The heaviest traveled routes will be "classified" according to the Ministry of Transport standards and rank for grant from the Road Fund, both for construction and maintenance. It is unlikely that the traffic generated inside any of the new towns will call for more than a single lane of a width of 22 ft or 30 ft, and this will be adequate for bus traffic; there will be no occasion for parking on these routes. Internal neighborhood streets will have roadways usually 16 ft or 20 ft wide where they act as collectors and less than that where they are purely "culs-de-sacs" or one-way lanes around greens.

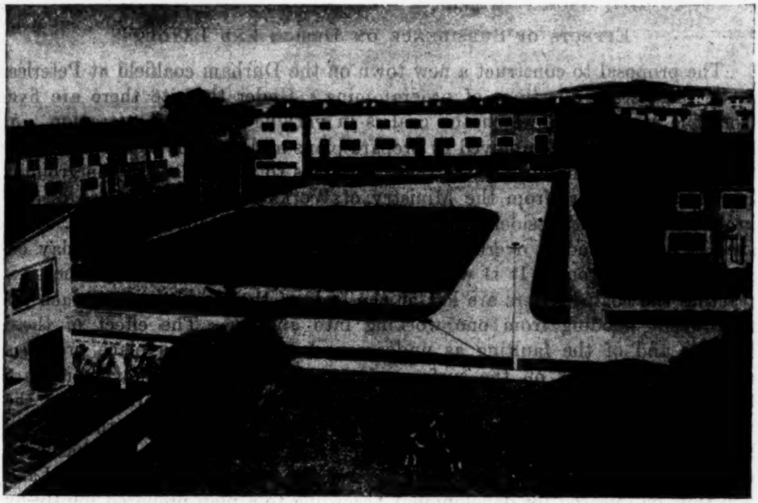

FIG. 5.—THE NEW TOWN OF NEWTON AYCLIFFE

Investigations are proceeding to determine whether it is desirable to construct separate lanes to accommodate cyclists, of whom there will be large numbers, going to and from work. It is likely that short lengths will be necessary at the factory approaches leading from the residential areas.

Research has led to changed ideas in design specifications for highways, and lighter forms of construction will be used than formerly. Soil surveys have been made and the most suitable locations for the roads determined. Stabilized base courses will be employed where suitable soils exist as at Stevenage. Construction of the first prestressed concrete road was completed at Crawley in the summer of 1950.

RAILWAYS

In selecting the sites for the new towns, care has been taken to see that existing railways are not overloaded. As the new towns are to be self-contained, commuting will not be encouraged. New stations usually will be necessary, however, and these will be combined with postal sorting offices. Additional railway sidings will be required to service the town, and some of the factories may want rail facilities.

EFFECTS OF SUBSIDENCE ON DESIGN AND LAYOUT

The proposal to construct a new town on the Durham coalfield at Peterlee raises interesting questions of programming. Under the site there are five workable seams from which the coal is in varying stages of extraction. A mining geologist was employed to prepare plans and sections of the workings, and a model was made to relate these workings to the surface of the ground. Structural engineers from the Ministry of Works have also reported on the probable effects of subsidence on the design of foundations and mains. The area is faulted and is overlain by post glacial deposits of boulder clay on magnesian limestone. It is worked from three different pits, and when extracting the coal, barriers are left at the ends of the workings from each pit to prevent flooding from one working into another. The effect of these barriers and of the faulting as underground working takes place produces complex movements on the surface of the ground. These movements are being studied carefully by releveling the surface of the ground above the workings at regular intervals of time. As far as possible construction will be undertaken on sites under which the greater part of the coal has been extracted and after time has been allowed for subsidence to occur. Land will thus be released for development according to a plan prepared jointly by the architect-planner and the National Coal Board officers.

THE NEW TOWN MASTER PLANS

Descriptive data applying to the new towns projects in Great Britain are presented in summarized form in Table 2. Unfortunately it is impossible to include illustrations of more than three of the master plans in a brief paper such as this. The three plans selected for illustration are as follows: (1) A London overflow town (Crawley); (2) a town that is dependent on an existing industrial estate (Newton Aycliffe); and (3) a Scottish new town (East Kilbride).

(1) *Crawley.*—The London overflow town of Crawley (see Fig. 4) consists of the two scattered townships of Crawley and Three Bridges. Old Crawley

High Street is now by-passed by a new length of the Brighton Road (route A 23). This will pass between the neighborhoods until relieved by a new motorway to be built on the east. The principal industrial area to the north will be accessible from either road. The central shopping area will be located off the existing High Street, exactly in the center of the town. There will be nine neighborhoods with populations varying between 4,300 and 8,000 people. The town is on the main railway from London to Brighton with a branch to Horsham. The land north of this branch line is very flat, and some pumping of sewage will be necessary.

(2) *Newton Aycliffe.*—The smallest of the new towns is Newton Aycliffe (Fig. 5). It has no marked physical features, but is bounded on the east by the Great North Road and on the south by the Railway and Trading Estate. The absence of features enabled designers to produce a plan based on sociological studies of human needs and areas of activity, and for this purpose a professional sociologist was employed who worked with the architect-planner. The minor road system will link the living areas by a rail underpass to the industrial area. Through traffic will pass the new town on the west and on the east. The central area will be close to all houses. The town will be divided into five wards, each of about 500 houses, subdivided again into precincts, which are grouped around village greens without through traffic.

TABLE 2.—BASIC DATA, NEW TOWNS OF GREAT BRITAIN

NEW TOWN		Date of Designation Order	DESIGNATED AREA, IN ACRES		DENSITY OF BUILT-UP AREA		POPULATION		Open Space[c]
Name (1)	(Fig. 1) (2)	(3)	Total (4)	Built-Up (5)	Over-all[a] (6)	Housing[b] (7)	Existing (8)	Ultimate (9)	(10)
Stevenage	1	Nov. 11, 1946	6,100	4,600	13.0	34.2	6,400	60,000	26.2
Crawley	2	Jan. 10, 1947	5,920	4,000	12.5	27.4[d]	8,000	50,000	22.0
Hemel Hempstead	3	Feb. 4, 1947	5,910	3,945	12.0	31.4	22,100	60,000	23.6
Harlow	4	March 25, 1947	6,320	4,255	14.1	38.0	4,300	60,000	23.3
Newton Aycliffe	5	April 19, 1947	867	743	13.4	31.6	60	10,000	19.5
East Kilbride	6	May 6, 1947	10,250	2,700	16.6[e]	45[f]	3,100	45,000[g]	10.0
Peterlee	7	March 10, 1948	2,350	....	....	....	100	30,000	....
Hatfield	8	May 20, 1948	2,340	1,438	17.4	34.8	10,000	25,000	13.4
Welwyn Garden City	9	May 20, 1948	4,230	2,712	14.2	28.2	18,200	36,500	13.7
Glenrothes	10	June 30, 1948	5,730	1,854	16.2	45	500	30,000	12.5
Basildon	11	Jan. 4, 1949	7,834	....	....	....	25,000	50,000	....
Bracknell	12	June 17, 1949	1,850	....	....	....	5,000	25,000	....
Cwmbran	13	Nov. 4, 1949	3,160	....	....	....	12,000	35,000	....
Corby	14	April 1, 1950	2,500	....	....	....	14,000	40,000	....

[a] Over-all density; persons per acre. [b] Average net density (approximate); persons per acre. [c] Open space of all kinds, in acres per thousand of ultimate population. [d] From 27.4 to 36 persons per acre. [e] From 16.6 to 18.5 over-all density of built-up area. [f] From 45 to 50 persons per acre. [g] From 45,000 to 50,000 ultimate population.

(3) *East Kilbride.*—The first Scottish new town was East Kilbride (Fig. 3). It will have four main neighborhoods subdivided into four or six smaller areas of about 500 houses. Two neighborhoods are large enough to support a primary school between them. Careful landscaping development is proposed

for valleys that are being retained as features. A tree nursery site has been acquired. Three industrial areas are proposed within easy walking or cycling distance of the houses; two of which can have rail facilities. A new central commercial area is planned, the existing village becoming a neighborhood center.

ACKNOWLEDGMENT

The writer is indebted to the Grenfell Baines Group of architects and town planning consultants, to Anthony Minoprio for making available several of the illustrations used in this paper, and to Oscar Faber and Partners for information related to "District Heating."

DISCUSSION

W. STRICKLER[8].—Pres. Thomas Jefferson called large cities the cancer growth of mankind. Should large cities be dissolved? Would such a step be possible under the conditions of the modern way of life—a product of industrialization? The British answer to these questions is lucidly represented in the paper, provoking thoughts beyond the scope of the problem analyzed in it.

Cities, with very few exceptions, have always grown naturally, according to their inherent needs. In Europe the "planned city" was a rare flower planted by order of an absolute monarch. For two hundred years, no planned cities were built in Europe until their revival as British garden cities, first proposed by Ebenezer Howard,[9] who borrowed his ideology from Jean Jacques Rousseau, the advocate of "back to nature." The direct contact between man and mother earth is the ancient basis not only of work and life but also of housing.

The writer does not know a single case in which Mr. Howard's basic idea found full realization. Mr. Howard's demand was for a unified city in which the citizen, rooted in the earth, lives and works. The modern garden cities, even those visualized by Mr. Howard were never completely self sufficient but were rather "sleep-cities," in which the breadwinner lived and rested for a short time after hard work in the city and tiresome commuting. The demand for harmony between work, housing, and recreation, which was fulfilled in pre-industrial towns, cannot be satisfied in the modern city.

Great Britain did not suffer from a housing shortage as much as the other European countries. Nevertheless, its planners have been anxious to "unload" its large cities in the next 20 years by building 14 new towns: 12 in England and Wales plus 2 in Scotland. The construction of eight new towns, around London, within a 30-mile circle is being planned, based on city development laws promulgated in 1946 and 1948. The planning agencies in London believe that during the coming 20 years, 1,000,000 persons will have to be resettled to unload the great cities.

When studying postwar city planning in Great Britain, the history of city development must be kept in mind in order to recognize the planning developed in an ideological rather than in a technical manner. Great Britain, foremost in industrialization at the beginning, suffered the most from the evils of industrialization. This explains why the extremely conservative English people finally promulgated laws causing a revolution in the development of their cities.

At a time when cities are "mushrooming" at an accelerated rate all over the world, Great Britain decides to move in the opposite direction, away from the large cities, to build small, lovely towns rooted in the country reflecting the character of the people living in them.

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[8] Prof., Technical Univ., Karlsruhe, Germany.

[9] "Garden City of To-morrow," by Ebenezer Howard, Karlsruhe, Germany, November 6, 1951.

The aim of the modern British city builder is not the building of a city designed with compass and rule but the promotion and fostering of the organic growth of single parts of a great city that should support each other and together form a complete living unit.

This principle applies particularly to the planning of Greater London. Because of its unique position as the largest industrial and business center, the largest port, and at the same time the metropolis of the British Empire, the City of London grew at a rate out of proportion to the remainder of the British Island. It proved to be a loss for its citizens who first expected to profit from living in such a large city. The recreation facilities dwindled with increased building activity and daily commuting routes grew longer and longer. Transportation became slower and more dangerous, and the supply system grew more intricate. The great city became the object of modern crises to a greater extent than any other contemporary community.

Forceful means cannot relieve this situation. To cure the accumulated evils is a long hard process for which a clear understanding of the inherent causes of these evils is the first prerequisite. In London, the planned area was subdivided into four concentric rings:

1. The inside circle, containing the business zone and the port, with no housing to remain after complete reconstruction.
2. The suburban ring, containing mainly the existing housing. This ring would be left untouched as far as possible.
3. The "green ring" containing parks and agricultural land. The existing tendency of suburban housing to spread out into this ring would be stopped at any cost. The green ring will be sustained in its present state, except for throughways that must cross it.
4. The outside ring—the area of new planning. Here the discussion centered on the question of whether a uniformly thin housing or a concentration of houses at a few points is preferable. The decision was for the second alternative. These new developments would be either enlargements of existing small towns or "new cities."

Planning and construction of these new cities is the most interesting and most talked about part of the reconstruction program. The city dweller would be able to form a relationship with the soil, form small units of human society, and, with his neighbors, work, as far as feasible in the plants of his own community. It means a return to the ideal of the ancient artisan—working and living under the same roof. The relation to the city proper will be quite loose (although excellent communication is planned) with the relationship more a spiritual connection than anything else. This ideal town reflects itself in many planned details: There will be no multiple-housing skyscrapers, but instead (in accordance with British tradition), two-floor, one-family houses built in small units, each surrounded by gardens rather than in monotonous endless rows, as found in some labor suburbs. Housing for 5,000 to 6,000 persons forms a "unit," and the layout of streets in such a unit prevents their use for through traffic. A family living in its house, away from the city traffic, facing its garden and its neighbors, is the final aim of the planning engineer.

The inner circle of London will retain its importance since it contains the residence of the King and the seat of the government and parliament. The city will remain the metropolis of culture and education, and the most important business and administration center and harbor of Great Britain.

An enormous amount of work is still necessary to achieve this aim. Streets and roads, already obsolete in Great Britain, must be reconstructed to fit Greater London; airports and railroads must be modernized or rebuilt; many buildings must be reconstructed; and parks and green areas must be planted to make the inner circle ring healthful and beautiful.

It is an astonishing fact that the preliminary work for this enormous program of reconstruction started during the most critical period of World War II. Delay of the planning for many years led to a thorough analysis of the problem. The design of new cities required extensive studies in different directions—agricultural soil analysis, subsidence due to mining, connections with existing road nets, water supply, gas, electricity, sewerage, and industrial development possibilities, to cite only the main problems.

The writer admires Great Britain for the fact that a radical change in its government did not impede the continuous development of its city planning and admires it for its clear and bold laws, providing, at least in theory, an excellent basis for reconstruction.

On the other hand, the writer must state that the real construction progress should follow the legislative action, and that planning phases lag far behind. Under the austerity program, construction was started only at a small part of the ring of new cities planned around London. The slow construction pace helps to eliminate errors that could be expected under a quicker tempo, and it gives plenty of time to try out new construction methods, but that does not help people looking for housing now. In addition, the "ring of green" (ring 3) is being constantly endangered by unplanned building. It may be that the main responsibility for the slow development rests with the corporation in charge of putting the program into effect—a corporation that is overorganized and overloaded with personnel. A heavy deterrent to new work is the "red tape," requiring that even the smallest development must be approved by the Ministry of Town and Country Planning. An efficient, expert regional planning organization, having the right and ability to make its own decisions, would be a better solution in many cases.

The idea of real cooperation between all concerned, as emphasized in British city planning, is excellent. This cooperation is not practiced only by governmental, economic, and scientific organizations, but is promoted among ordinary citizens. The idea of having a public relations man at each planning council in addition to the engineer, architect, geologist, and lawyer is very good. The public relations man does not simply contact newspapers and radio, but endeavors to give information regarding planning, progress of construction, and difficulties encountered in planning and construction to every person interested. It is this spirit of mutual help that makes the "Greater London Plan" and the "New Town Program" a cooperative program worthy of the British people.

T. C. COOTE[10].—Mr. Strickler implies that Ebenezer Howard's idea[9] of a self-contained town cannot be satisfied in the modern city. It is undoubtly true of the British new towns that it will be impossible entirely to prevent commuting, and this will be especially true of the London new towns. However, the main aim will be to see initially that the typical wage earner is employed in the new town and also lives there. A grown-up daughter of his, however, might decide to teach somewhere else in the county and travel to work daily from her home in the new town. In the case of Corby, Aycliffe, and Peterlee, it is thought that there should be relatively little commuting. Another point made by Mr. Strickler is that the new town will contain mainly two-floor, one-family houses, in small units, surrounded by gardens. This is largely true because there is an almost universal demand for the family house with a garden. The new town architects, however, are aware that this would result in a very monotonous appearance, and they are, in fact, breaking the horizontal line by building a number of apartment blocks, varying in height from ten floors at Harlow to three floors in Crawley. It is thought that the demand for apartments is not likely to be much more than 5% of the total accommodation to be provided. There will also be three-story blocks with shops on the street floor.

The lack of speed in construction has also been mentioned, with a suggestion that even the smallest detail of development is approved by the planning ministry. It is undoubtedly true that the speed of construction has lagged far behind what was originally thought possible, but this must be attributed first to the limitation imposed by the national capital investment program, and now to the need for rapid re-armament, which is affecting the availability of labor in the new towns. For some time, the ministry has urged upon the corporations the need to submit proposals for development of rather large pieces of land at one time, and at the same time, to avoid frequent reference to the ministry. This would have the effect also, of giving the corporations more autonomy.

Mr. Strickler has also referred to what he calls the small size of the new towns. It is interesting to note now that there is a general tendency to increase their size somewhat. Both Basildon and Harlow will now take 80,000 inhabitants each, instead of the 60,000 originally intended, and Crawley, too, may be larger than originally planned.

The discussion by Mr. Strickler is appreciated.

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[10] Senior Regional Planning Officer, Ministry of Town and Country Planning, London, England.

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LATERALLY LOADED PLANE STRUCTURES AND STRUCTURES CURVED IN SPACE

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WITH DISCUSSION BY MESSRS. UKU MÜLLERSDORF, MAURICE BARRON, AND FRANK BARON AND JAMES P. MICHALOS

SYNOPSIS

An analytical procedure is presented for determining the effects of loads normal to the plane of such structures as arches, bents, and balcony girders. This procedure is called the "shear and torsion analogy." In addition, a procedure is presented for determining moments and shears in structures curved or segmental in space and continuous between two supports. These structures may be subjected to loads in any direction and to moments about any axis. The procedure is general, and consists of an extension, into space, of the shear and torsion analogy and of the column analogy.

INTRODUCTION

Many curved and segmental structures lie in a plane, are loaded normal to their plane, and are continuous between two supports. Arches, balcony girders, and bents are examples of structures that may be loaded in this way. Analytical studies of structures thus loaded have been conducted in the United States by Milo S. Ketchum,[3] Hon. M. ASCE, I. Oesterblom,[4] M. ASCE, Robert B. B. Moorman,[5,6] M. ASCE, and Mervin B. Hogan.[7,8] Similar studies have been

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[1] Prof., Civ. Eng., Technological Inst., Northwestern Univ., Evanston, Ill.

[2] Associate Prof., Dept. of Civ. Eng., Iowa State College, Ames, Iowa.

[3] "The Design of Walls, Bins, and Grain Elevators," by Milo S. Ketchum, McGraw-Hill Book Co., Inc., New York, N. Y., 3d Ed., 1929.

[4] "Bending and Torsion in Horizontally Curved Beams," by I. Oesterblom, *Proceedings*, A. C. I., Vol. 28, 1932.

[5] "A Semi-Graphical Method of Analysis for Horizontally Curved Beams," by Robert B. B. Moorman, *Bulletin No. 29*, Eng. Experiment Station, Univ. of Missouri, Columbia, Mo., 1938.

[6] "Influence Lines for Horizontally Curved Fixed-End Beams of Circular-Arc Plan," by Robert B. B. Moorman, *Bulletin No. 35*, Eng. Experiment Station, Univ. of Missouri, Columbia, Mo., 1947.

[7] "Circular Beams Loaded Normal to the Plane of Curvature," by Mervin B. Hogan, *Transactions*, ASME, Vol. 60, 1938, p. A-81.

[8] "Circular Beams Loaded Normal to the Plane of Curvature—2," by Mervin B. Hogan, *ibid.*, Vol. 66, 1944, p. A-51.

conducted abroad by A. H. Gibson and E. G. Ritchie;[9] A. J. S. Pippard, M. ASCE, and F. L. Barrow;[10,11] Georg Unold;[12] and others.

An analytical procedure is presented for determining moments and shears in a plane structure continuous between two supports. The structure considered is subjected to loads normal to its plane and moments about axes in its plane. It may have any shape in plan and any variation in cross section along its length. The procedure is called the shear and torsion analogy. The expressions are somewhat similar in form to those of the column analogy,[13] the results in both cases being interpreted in terms of a pressure line concept.

Some structures are continuous between two supports but do not lie in a plane. These structures may be curved or segmental in space and may be subjected to loads or deformations in any direction. Many odd-shaped structures frequently used in urban freeway construction are examples of such structures. Pipe lines are additional examples. Some analytical studies of pipe lines have been conducted by William Hovgaard,[14,15] A. M. Wahl,[16] S. W. Spielvogel,[17] M. ASCE, and S. Kameros.[18] This paper presents an analytical procedure for determining moments and shears in a structure curved or segmental in space and continuous between two supports. The structure may be subjected to loads in any direction and moments about any axis. In addition, it may have any shape in space and any variation in cross section along its length. The procedure is general, and for structures lying in a plane it reduces to the shear and torsion analogy or the column analogy.

THE SHEAR AND TORSION ANALOGY

In the analysis of statically indeterminate structures, the following requirements must be satisfied:

a. Statics.—For any structure or any part of a structure the sum of the forces in any direction must equal zero, and the sum of the forces times their respective distances to any axis must equal zero.

b. Geometry.—Angle changes, slopes, and displacements must be consistent with the requirements of continuity. For any closed circuit the sum of the angle changes about any axis must equal zero. In addition, if the angle changes are small, the sum of the angle changes times their respective distances to any axis must equal zero.

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[9] "A Study of the Circular-Arc Bow-Girder," by A. H. Gibson and E. G. Ritchie, Constable and Co., London, 1914.

[10] "The Analysis of Engineering Structures," by A. J. S. Pippard and J. F. Baker, Longmans, Green and Co., New York, N. Y., 1936.

[11] "The Stress Analysis of Bow Girders," by A. J. S. Pippard and F. L. Barrow, H. M. Stationery Office, London, 1926.

[12] "Der Kreistrager," by Georg Unold, *Forschungsarbeiten No. 255*, Verein Deutscher Ingenieure, Berlin, 1922.

[13] "The Column Analogy," by Hardy Cross, *Bulletin No. 215*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., 1930.

[14] "Stresses in Three-Dimensional Pipe Bends," by William Hovgaard, *Transactions*, ASME, Vol. 57, 1935, FSP-57-12, p. 401.

[15] "Further Studies of Three-Dimensional Pipe Bends," by William Hovgaard, *ibid.*, Vol. 59, 1937, FSP-59-13, p. 647.

[16] "Stresses and Reactions in Expansion Pipe Bends," by A. M. Wahl, *ibid.*, Vol. 49–50, 1928, FSP-50-49, p. 241.

[17] "Piping Stress Calculations Simplified," by S. W. Spielvogel, McGraw-Hill Book Co., Inc., New York, N. Y., 1943.

[18] "Application of the Elastic-Point Theory to Piping Stress Calculations," by S. W. Spielvogel and S. Kameros, *Transactions*, ASME, Vol. 57, 1935, FSP-57-10, p. 165.

Geometry must be related to statics through the known or assumed properties of materials. No uncertainties are involved in the requirements of statics and geometry. Some uncertainties, however, may exist in the relationship between statics and geometry due to the characteristics of a material. This paper does not deal with uncertainties in the properties of materials. It is assumed that the properties necessary for computing the relationships between statics and geometry are known. Furthermore, the effects of shear deformations and of changes in length of an element of a structure are considered negligible. If desired, adjustments in the succeeding analyses may be made for these effects.

In general, two classes of loading may be considered for curved structures lying in a plane and continuous between two supports. These loadings are shown in Fig. 1(*a*) and are as follows:

(A) Loads applied in the plane of the structure and moments about axes normal to this plane. This class of loading is considered in the column analogy.

(B) Loads applied normal to the plane of the structure and moments about axes in this plane. This class of loading is particularly considered in the present section of the paper.

For each class of loading, the forces and moments acting on a cross section of the structure are shown in Fig. 1(*b*) and are represented as vectors. The angle changes, rotations, and displacements at a section of the structure are shown in Fig. 1(*c*) and are also represented as vectors. The sign convention used in this paper is illustrated in the same figure. Forces and moments are positive on a front face when they are in the direction of an axis. They are positive on a back face when they are against the direction of an axis. The same sign convention is used for rotations and displacements. The angle changes between adjacent cross sections of an element are positive when they are in the direction of an axis. The statical and geometrical relationships are referred to rectangular axes.

Structures loaded as in class (A) or as in class (B) may be analyzed by a procedure consisting of two parts as follows:

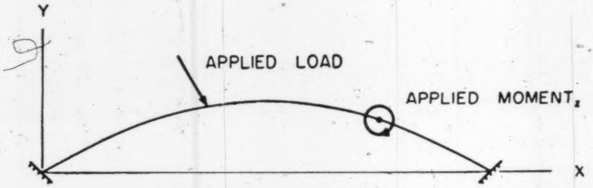
1. Assume any statically possible distribution of moments along the structure. Check to see if errors exist in the geometry of the deflected structure.

2. If such errors exist, add (or subtract) a distribution of moments that corrects these errors. This distribution, however, must not introduce errors in statics.

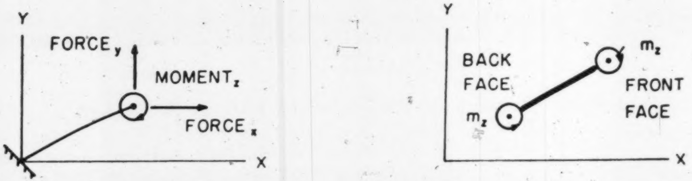
Such a procedure automatically satisfies the requirements of statics. The following discussion is restricted to the analysis of laterally loaded plane structures. The final moments and shear at any section of the structure are obtained as follows:

LOADS AS IN (A)

THIS CLASS OF LOADING IS CONSIDERED IN THE COLUMN ANALOGY.

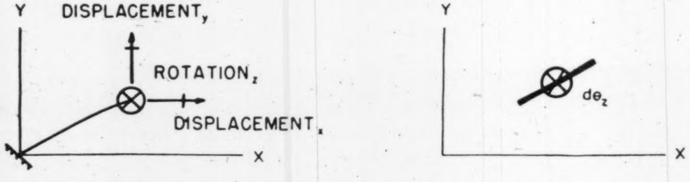

Y
APPLIED LOAD
APPLIED MOMENT$_z$
X

(a) LOADS IN THE PLANE OF THE STRUCTURE AND MOMENTS ABOUT AXES NORMAL TO THIS PLANE.


Y
FORCE$_y$
MOMENT$_z$
FORCE$_x$
X
Y
BACK FACE
m_z
m_z
FRONT FACE
X

(b) FORCES AND A MOMENT ACTING ON A CROSS SECTION OF THE STRUCTURE

MOMENTS, m_z, ON ADJACENT CROSS SECTIONS OF AN ELEMENT


Y
DISPLACEMENT$_y$
ROTATION$_z$
DISPLACEMENT$_x$
X
Y
de_z
X

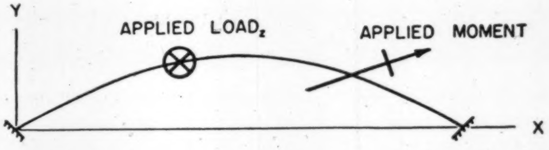
(c) DISPLACEMENTS AND A ROTATION AT A CROSS SECTION OF THE STRUCTURE

ANGLE CHANGE, de_z, ABOUT AN AXIS NORMAL TO THE PLANE OF THE STRUCTURE

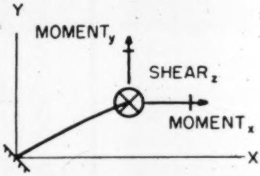
FIG. 1.—STATICAL AND GEOMETRICAL RELATIONSHIPS

LOADS AS IN (B)

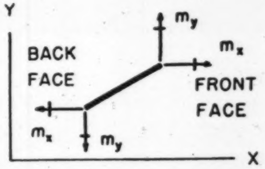
THIS CLASS OF LOADING IS PARTICULARLY CONSIDERED IN THE FIRST SECTION OF THE PAPER.


Y
APPLIED LOAD$_z$
APPLIED MOMENT
X

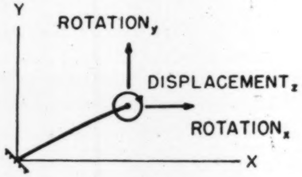
(a) LOADS NORMAL TO THE PLANE OF THE STRUCTURE AND MOMENTS ABOUT AXES IN THIS PLANE.


Y
MOMENT$_y$
SHEAR$_z$
MOMENT$_x$
X

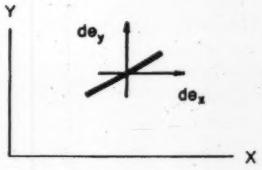
(b) MOMENTS AND A FORCE ACTING ON A CROSS SECTION OF THE STRUCTURE.


Y
m$_y$
BACK FACE
m$_x$
FRONT FACE
m$_x$
m$_y$
X

MOMENTS, m$_x$ AND m$_y$, ON ADJACENT CROSS SECTIONS OF AN ELEMENT.


Y
ROTATION$_y$
DISPLACEMENT$_z$
ROTATION$_x$
X

(c) ROTATIONS AND A DISPLACEMENT AT A CROSS SECTION OF THE STRUCTURE.



ANGLE CHANGES, de$_x$ AND de$_y$, ABOUT AXES IN THE PLANE OF THE STRUCTURE.

Moment—

$$m_x = m_{xo} - m_{xi} \qquad (1a)$$

moment—

$$m_y = m_{yo} - m_{yi} \qquad (1b)$$

and shear—

$$v_z = v_{zo} - v_{zi} \qquad (1c)$$

The subscripts *o* and *i* are associated with the assumed distributions and the correction distributions, respectively.

Now consider the distributions of correction moments and shears. Fig. 2(*a*) shows the correction shears and moments at a support and at a section of the structure. The relationships between these forces and moments are summarized in the figure. The same structure is shown in Fig. 2(*b*) but an imaginary arm is attached to an end of the structure. This arm is introduced for convenience only. It is considered rigid and extends to a position that will be referred to as a centroid. The position of this centroid and the properties of the structure with respect to this centroid are defined subsequently. Fig. 2(*b*) also shows the correction moments and shear applied at the centroid. By statics the correction moments and shear at a section of the structure are as follows:

Moment—

$$m_{xi} = m_{xic} + v_{zic}\, y_c \qquad (2a)$$

moment—

$$m_{yi} = m_{yic} - v_{zic}\, x_c \qquad (2b)$$

and shear—

$$v_{zi} = v_{zic} \qquad (2c)$$

The subscript *c* is associated with the correction moments and shear at the centroid and with distances measured from the axes at the centroid. An inspection of Eqs. 2*a*, 2*b*, and 2*c* shows that the distributions of correction moments and shears are planar.

The requirements of geometry are now restated. The sum of the angle changes about any axis in the plane of the structure must equal zero; thus:

$$\sum d\theta_x = \sum d\theta_{xo} - \sum d\theta_{xi} = 0 \qquad (3a)$$

and

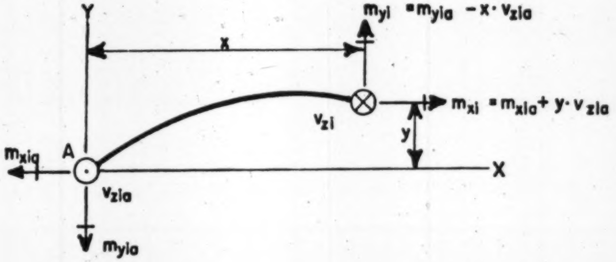
$$\sum d\theta_y = \sum d\theta_{yo} - \sum d\theta_{yi} = 0 \qquad (3b)$$

The sum of the angle changes, times their respective distances to any axis normal to the plane, must equal zero:

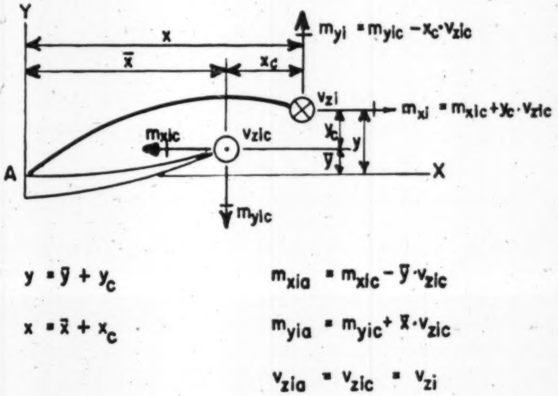
$$\left(\sum y\, d\theta_x - \sum x\, d\theta_y\right) = \left(\sum y\, d\theta_{xo} - \sum x\, d\theta_{yo}\right) - \left(\sum y\, d\theta_{xi} - \sum x\, d\theta_{yi}\right) = 0 \qquad (3c)$$

Thus, the angle changes as a result of the correction moments must balance those caused by the assumed moments.

Fig. 3(*a*) shows the angle changes associated with assumed or correction distributions of moments. These angle changes are represented as vectors. In this case each cross section has a principal axis in the plane of the structure.


Y
$m_{yi} = m_{yia} - x \cdot v_{zia}$
x
$m_{xi} = m_{xia} + y \cdot v_{zia}$
v_{zi}
y
m_{xia}
A
X
v_{zia}
m_{yia}

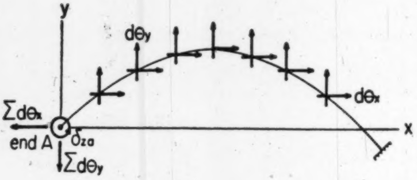
(a) CORRECTION MOMENTS AND SHEARS IN THE STRUCTURE DUE TO MOMENTS AND SHEAR AT END A.


Y
$m_{yi} = m_{yic} - x_c \cdot v_{zic}$
x
$\bar{x}$
x_c
v_{zi}
$m_{xi} = m_{xic} + y_c \cdot v_{zic}$
m_{xic}
v_{zic}
y_c
y
$\bar{y}$
A
X
m_{yic}
$y = \bar{y} + y_c$
$x = \bar{x} + x_c$
$m_{xia} = m_{xic} - \bar{y} \cdot v_{zic}$
$m_{yia} = m_{yic} + \bar{x} \cdot v_{zic}$
$v_{zia} = v_{zic} = v_{zi}$

(b) CORRECTION MOMENTS AND SHEARS IN THE STRUCTURE DUE TO MOMENTS AND SHEAR AT THE CENTROID

FIG. 2.—CORRECTION MOMENTS AND SHEARS IN A LATERALLY LOADED PLANE STRUCTURE

The general case of an element in space and oblique to the three orthogonal axes is considered subsequently. The rotations and displacement at an end A of the structure due to the assumed moments and the correction moments

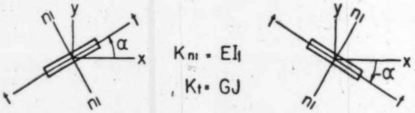


$$\sum d\theta_x = \sum\left(m_x \frac{ds}{K_{xx}} + m_y \frac{ds}{K_{xy}}\right) = \sum(m_x\, da_{xx} + m_y\, da_{xy})$$

$$\sum d\theta_y = \sum\left(m_y \frac{ds}{K_{yy}} + m_x \frac{ds}{K_{yx}}\right) = \sum(m_y\, da_{yy} + m_x\, da_{yx})$$

$$\delta_{za} = \sum y\left(m_x \frac{ds}{K_{xx}} + m_y \frac{ds}{K_{xy}}\right) - \sum x\left(m_y \frac{ds}{K_{yy}} + m_x \frac{ds}{K_{yx}}\right)$$
$$= \sum y(m_x\, da_{xx} + m_y\, da_{xy}) - \sum x(m_y\, da_{yy} + m_x\, da_{yx})$$

(a) Rotations and Displacement at End A



Positive Slope of Segment Negative Slope of Segment

$$K_{xx} = \frac{K_{n1}K_t}{K_{n1}\cos^2\alpha + K_t\sin^2\alpha} \qquad K_{yy} = \frac{K_{n1}K_t}{K_{n1}\sin^2\alpha + K_t\cos^2\alpha}$$

$$K_{xy} = \frac{K_{n1}K_t}{(K_{n1}-K_t)\sin\alpha\cos\alpha}$$

(b) Summary of Stiffness Factors

FIG. 3.—GEOMETRICAL RELATIONSHIPS FOR LATERALLY LOADED PLANE STRUCTURES

are summarized in Fig. 3(*a*). Since the angle changes due to the correction moments must balance those due to the assumed moments, the following equations of geometry may be written:

For rotations about the *x*-axis—

$$m_{xia}\sum da_{xx} + m_{yia}\sum da_{xy} + v_{zia}\sum(y\,da_{xx} - x\,da_{xy}) = \sum(m_{xo}\,da_{xx} + m_{yo}\,da_{xy}) \quad (4a)$$

for rotations about the *y*-axis—

$$m_{xia}\sum da_{yx} + m_{yia}\sum da_{yy} + v_{zia}\sum(y\,da_{yx} - x\,da_{yy}) = \sum(m_{yo}\,da_{yy} + m_{xo}\,da_{yx}) \quad (4b)$$

and, for displacements along the *z*-axis,

$$m_{xia}\sum(y\,da_{xx} - x\,da_{yx}) + m_{yia}\sum(y\,da_{xy} - x\,da_{yy}) + v_{zia}\left[\sum y(y\,da_{xx} - x\,da_{xy}) - \sum x(y\,da_{yx} - x\,da_{yy})\right] = \sum y(m_{xo}\,da_{xx} + m_{yo}\,da_{xy}) - \sum x(m_{yo}\,da_{yy} + m_{xo}\,da_{yx}) \quad (4c)$$

Moments about one axis contribute to the angle changes about both axes. In Eqs. 4*a*, 4*b*, and 4*c*,

$$\left.\begin{aligned} da_{xx} &= \frac{ds}{K_{xx}} \\ da_{yy} &= \frac{ds}{K_{yy}} \\ da_{xy} &= \frac{ds}{K_{xy}} \\ da_{yx} &= \frac{ds}{K_{yx}} \end{aligned}\right\} \tag{5}$$

The terms da_{xx} and da_{xy} represent angle changes about an x-axis due to unit moments about the x-axis and the y-axis, respectively. Similarly, da_{yy} and da_{yx} represent angle changes about a y-axis due to unit moments about the y-axis and x-axis, respectively. The unit moments are at the ends of an element which is ds in length and is oblique to the rectangular axes. For an element one unit in length, K_{xx} and K_{xy} represent moments about the x-axis and the y-axis, respectively. Each moment produces a unit angle change about the x-axis. In the same way, K_{yy} and K_{yx} represent moments about the y-axis and the x-axis, respectively. These moments produce unit angle changes about the y-axis. It may be shown that $K_{yx} = K_{xy}$ and $da_{yx} = da_{xy}$. Thus, K_{xx}, K_{yy}, and K_{xy} are measures of the stiffness of the element and for convenience are called stiffness factors. Algebraic expressions for these factors are given in Fig. 3(*b*) in terms of the normal and tangential stiffnesses of the element.

Now inspect Eqs. 4*a*, 4*b*, and 4*c* and observe that the unknowns in these equations are the correction moments and shear at end A of the structure. From the relationships shown in Fig. 2, these equations may be written in terms of the correction moments and shear at the centroid. If the distances $\bar{x}$ and $\bar{y}$ to the centroidal axes are now selected such that—

$$\sum(y_c\, da_{xx} - x_c\, da_{xy}) = 0 \tag{6a}$$

and

$$\sum(y_c\, da_{yx} - x_c\, da_{yy}) = 0 \tag{6b}$$

—then, Eqs. 4*a*, 4*b*, and 4*c* may be written as follows:

$$m_{xic} \sum da_{xx} + m_{yic} \sum da_{xy} = \sum(m_{xo}\, da_{xx} + m_{yo}\, da_{xy}) \tag{7a}$$

$$m_{xic} \sum da_{yx} + m_{yic} \sum da_{yy} = \sum(m_{yo}\, da_{yy} + m_{xo}\, da_{yx}) \tag{7b}$$

and

$$\begin{aligned} &v_{xic}\left[\sum y_c\,(y_c\, da_{xx} - x_c\, da_{xy}) - \sum x_c\,(y_c\, da_{yx} - x_c\, da_{yy})\right] \\ &\qquad = \sum y_c\,(m_{xo}\, da_{xx} + m_{yo}\, da_{xy}) - \sum x_c\,(m_{yo}\, da_{yy} + m_{xo}\, da_{yx}) \end{aligned} \tag{7c}$$

Thus, the position of the centroid is so defined that if a shear, v_{xic}, is applied at this centroid it does not contribute to rotations at end A. If the correction moments and shear at the centroid are known, the correction moments and shear at any section of the structure may be obtained by statics. In the following, the form of the expressions for these corrections is obtained by the concept that permits the interchange of computations of geometry with those of statics.

This concept is based on the definitions of a small angle and of a moment. By definition of a small angle, a displacement is equal to the product of an angle change and the normal distance to the line along which the displacement occurs. A moment about an axis is defined as the product of a force and the normal distance to the axis. Thus, computations of geometry involving small angles are interchangeable with those of statics. This is formally stated as follows:

(*a*) An angle change or rotation about an axis may be treated as a force along the axis of rotation.

(*b*) A displacement along an axis may be treated as a moment about the axis of displacement. (This generalized statement was made by Hardy Cross, Hon. M. ASCE, at Yale University, New Haven, Conn.)

Consequently, in computing the geometry of a deflected structure, the rotations and displacements at a cross section may be treated as analogous shears and moments. This treatment may readily be visualized by reinspecting Fig. 1.

From the foregoing interchange, the requirements of geometry for a closed circuit lying in a plane may be restated as analogous requirements of statics. The analogous requirements are stated algebraically:

$$\sum dp_x = \sum dp_{xo} - \sum dp_{xi} = 0 \quad \text{(8a)}$$

$$\sum dp_y = \sum dp_{yo} - \sum dp_{yi} = 0 \quad \text{(8b)}$$

and

$$\left(\sum y\, dp_x - \sum x\, dp_y\right) = \left(\sum y\, dp_{xo} - \sum x\, dp_{yo}\right) - \left(\sum y\, dp_{xi} - \sum x\, dp_{yi}\right) = 0 \quad \text{(8c)}$$

Eqs. 8*a*, 8*b*, and 8*c* are equations of statics and are analogous to the previous equations of geometry (Eqs. 3*a*, 3*b*, and 3*c*). Note that the analogous forces associated with the correction moments must balance those associated with the assumed moments.

It is now convenient to consider an imaginary strut that has the same shape in plan as the real structure. This strut is very short and is subjected to shear forces lying in the top plane of the strut. The forces are analogous to the corresponding angle changes associated with the assumed distributions of moments. The angle changes $m_{xo}\, da_{xx}$ and $m_{yo}\, da_{xy}$ are the analogous forces dp_{xo} in the x-direction. The angle changes $m_{yo}\, da_{yy}$ and $m_{xo}\, da_{yx}$ are the analogous forces dp_{yo} in the y-direction. Since a shear force is equal to a stress times a corresponding differential area, the assumed moments may be interpreted as analogous shear stresses in the top plane of the strut and the terms da_{xx}, da_{yy}, and da_{xy} may be interpreted as the corresponding differential areas. Consequently, the imaginary strut may be considered to have directional properties. The forces in the top plane of the strut are balanced by forces on cross sections parallel to this plane. Therefore, the forces on such cross sections are analogous to the angle changes associated with the correction moments. The forces dp_{xi} are analogous to the angle changes $m_{xi}\, da_{xx}$ and $m_{yi}\, da_{xy}$. The forces dp_{yi}

are analogous to the angle changes $m_{yi}\,da_{yy}$ and $m_{xi}\,da_{yx}$. Thus, the shear stresses on cross sections parallel to the top plane of the strut may be interpreted as correction moments in the real structure.

It was previously shown that the distributions of correction moments are planar; but if an imaginary strut is loaded as in the preceding case and if the shear stresses across a section of the strut are computed by the following expressions—

$$s_x = \frac{P'_x}{A'_x} + \frac{M_z}{J_z}\,y_c \qquad (9a)$$

and

$$s_y = \frac{P'_y}{A'_y} - \frac{M_z}{J_z}\,x_c \qquad (9b)$$

—the distributions of these stresses are also planar. In each expression, the first term represents an average stress due to a direct shear and the second term represents a stress due to a torque about an axis normal to the top plane of the strut, J_z being the torsional moment of inertia about the z-axis.

As a result of the preceding discussion, it is now possible to state that the correction moments in the real structure can be computed in the same way as shear stresses in an analogous strut:

$$m_{xi} = \frac{P'_x}{A'_x} + \frac{M_z}{J_z}\,y_c \qquad (10a)$$

and

$$m_{yi} = \frac{P'_y}{A'_y} - \frac{M_z}{J_z}\,x_c \qquad (10b)$$

It could also be shown that the correction shears in the real structure can be computed by means of the expression:

$$v_{zi} = \frac{M_z}{J_z} \qquad (10c)$$

The terms of Eqs. 10*a*, 10*b*, and 10*c* must be so defined that the correction moments adjust for the errors in geometry due to the assumed distributions of moments. These terms were obtained by means of the equations of geometry, Eqs. 7*a*, 7*b*, and 7*c*, and the relationships of statics shown in Fig. 2.

All terms involved in the analytical procedure are summarized in Fig. 4(*a*). Inspect Fig. 4(*a*) and interpret the expressions in the summary. For convenience first consider a structure that has an axis of symmetry parallel to the y-axis. For such a structure it may be shown that A_{xy} is equal to zero and that $P'_x = P_x$ and $P'_y = P_y$. The terms P_x and P_y represent the relative rotations of one end of the structure with respect to the other. These rotations are due to the assumed distributions of moments and are treated as total shear forces in the top plane of the strut. For an unsymmetrical structure corrections must be subtracted from P_x and P_y to obtain P'_x and P'_y, respectively.

Now inspect the term M_z. In the real structure, M_z represents a displacement along an axis normal to the plane of the structure. This displacement occurs at the defined centroid and is associated with the angle changes due to the

$$v_z = v_{zo} - v_{zi} \qquad v_{zi} = \frac{M_z}{J_z}$$

$$m_x = m_{xo} - m_{xi} \qquad m_{xi} = \frac{P'_x}{A'_x} + \frac{M_z}{J_z} y_c$$

$$m_y = m_{yo} - m_{yi} \qquad m_{yi} = \frac{P'_y}{A'_y} - \frac{M_z}{J_z} x_c$$

$$A_x = \sum da_{xx}$$

$$A_y = \sum da_{yy}$$

$$J_z = I_x + I_y - 2 I_{xy} \qquad A_{xy} = \sum da_{xy}$$

$$I_x = \sum y^2 da_{xx} - \bar{y}^2 A_x \qquad A'_x = A_x - \frac{A_{xy}A_{xy}}{A_y} \qquad A_{xy} = A_{yx}$$

$$I_y = \sum x^2 da_{yy} - \bar{x}^2 A_y \qquad A'_y = A_y - \frac{A_{xy}A_{xy}}{A_x}$$

$$I_{xy} = \sum xy\, da_{xy} - \bar{x}\bar{y} A_{xy}$$

$$K_m = EI,$$

$$da_{xx} = \frac{ds}{K_{xx}} \qquad K_{xx} = \frac{K_m K_t}{K_m \cos^2\alpha + K_t \sin^2\alpha} \qquad K_t = GJ$$

$$da_{yy} = \frac{ds}{K_{yy}} \qquad K_{yy} = \frac{K_m K_t}{K_m \sin^2\alpha + K_t \cos^2\alpha}$$

$$da_{xy} = \frac{ds}{K_{xy}} \qquad K_{xy} = \frac{K_m K_t}{(K_m - K_t)\sin\alpha \cos\alpha}$$

$$P'_x = P_x - \frac{A_{xy}}{A_y} P_y \qquad P_x = \sum m_{xo} da_{xx} + \sum m_{yo} da_{xy} \qquad M_z = M_{zx} - M_{zy}$$

$$M_{zx} = \sum y(m_{xo} da_{xx} + m_{yo} da_{xy}) - \bar{y} P_x$$

$$P'_y = P_y - \frac{A_{xy}}{A_x} P_x \qquad P_y = \sum m_{yo} da_{yy} + \sum m_{xo} da_{xy}$$

$$M_{zy} = \sum x(m_{yo} da_{yy} + m_{xo} da_{xy}) - \bar{x} P_y$$

$$x_c = x - \bar{x} \qquad \bar{x} = -\frac{C'_y}{A'_y} \qquad C'_y = C_y - \frac{A_{xy}}{A_x} C_x \qquad C_y = \sum y\, da_{xy} - \sum x\, da_{yy}$$

$$y_c = y - \bar{y} \qquad \bar{y} = \frac{C'_x}{A'_x} \qquad C'_x = C_x - \frac{A_{xy}}{A_y} C_y \qquad C_x = \sum y\, da_{xx} - \sum x\, da_{xy}$$

(a) GENERAL RELATIONSHIPS

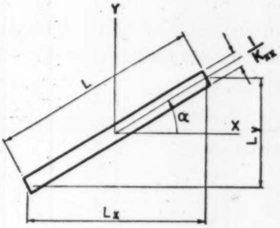
Fig. 4.—Summary for Structures Lying in a

ELASTIC AREA $= \frac{L}{K_{xx}} = da_{xx}$

$i_x = \frac{1}{12} da_{xx} L_y^2$

$i_y = 0$

$i_{xy} = 0$

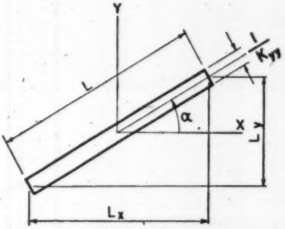

Y
L
$\frac{1}{K_{xx}}$
α
X
L_y
L_x

ELASTIC AREA $= \frac{L}{K_{yy}} = da_{yy}$

$i_x = 0$

$i_y = \frac{1}{12} da_{yy} L_x^2$

$i_{xy} = 0$

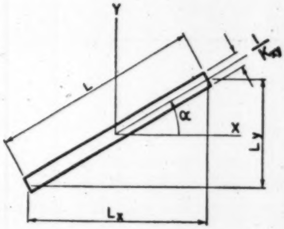

Y
L
$\frac{1}{K_{yy}}$
α
X
L_y
L_x

ELASTIC AREA $= \frac{L}{K_{xy}} = da_{xy}$

$i_x = 0$

$i_y = 0$

$i_{xy} = \frac{1}{12} da_{xy} L_x L_y$

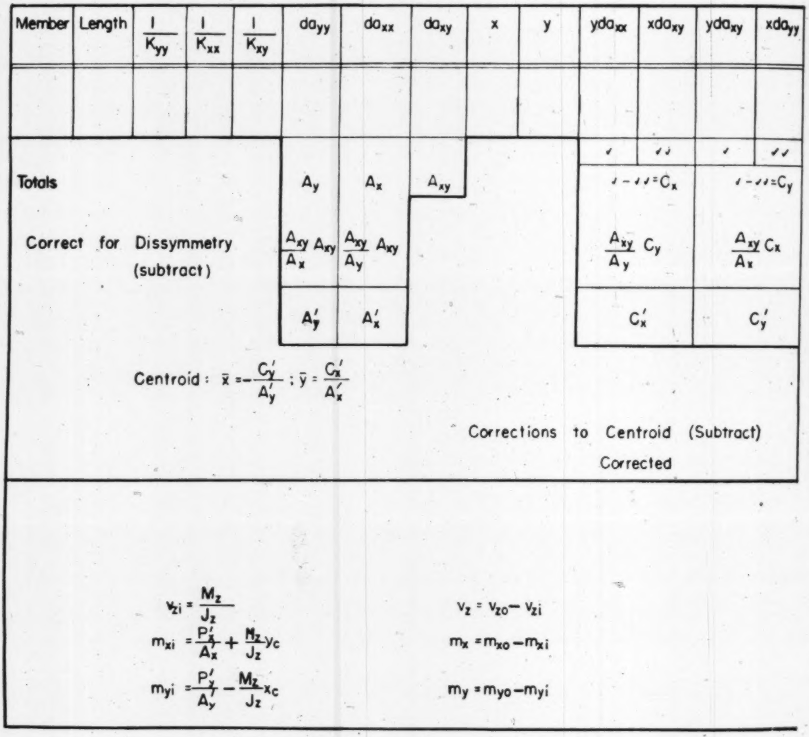

Y
L
$\frac{1}{K_{xy}}$
α
X
L_y
L_x

NOTE: K_{xy} IS POSITIVE IF THE SEGMENT SLOPES ╱, AND NEGATIVE IF IT SLOPES ╲ THEREFORE, I_{xy} FOR A SYMMETRICAL STRUCTURE IS NOT EQUAL TO ZERO.

(b) CONVENIENT RELATIONSHIPS OF FINITE ELASTIC AREAS

PLANE AND LOADED NORMAL TO THEIR PLANE

assumed distributions of moments. In the analogous strut, M_z represents a moment of the analogous forces lying in the top plane of the strut. This moment is about an axis normal to the plane of the strut and located at the centroid of the strut. This moment may be obtained by first computing the moment about an arbitrary axis and then subtracting a correction as indicated in Fig. 4(a).



Member	Length	$\frac{1}{K_{yy}}$	$\frac{1}{K_{xx}}$	$\frac{1}{K_{xy}}$	da_{yy}	da_{xx}	da_{xy}	x	y	yda_{xx}	xda_{xy}	yda_{xy}	xda_{yy}
										✓	✓✓	✓	✓✓
Totals					A_y	A_x	A_{xy}			✓ − ✓✓ = C_x		✓ − ✓✓ = C_y	
Correct for Dissymmetry (subtract)					$\frac{A_{xy}}{A_x} A_{xy}$	$\frac{A_{xy}}{A_y} A_{xy}$				$\frac{A_{xy}}{A_y} C_y$		$\frac{A_{xy}}{A_x} C_x$	
					A'_y	A'_x				C'_x		C'_y	

Centroid: $\bar{x} = -\frac{C'_y}{A'_y}$; $\bar{y} = \frac{C'_x}{A'_x}$

Corrections to Centroid (Subtract)

Corrected

$$v_{zi} = \frac{M_z}{J_z} \qquad v_z = v_{zo} - v_{zi}$$

$$m_{xi} = \frac{P'_x}{A'_x} + \frac{M_z}{J_z} y_c \qquad m_x = m_{xo} - m_{xi}$$

$$m_{yi} = \frac{P'_y}{A'_y} - \frac{M_z}{J_z} x_c \qquad m_y = m_{yo} - m_{yi}$$

FIG. 5.—TABULAR FORM FOR ANALYSIS OF

Next, inspect the terms A'_x and A'_y. Again, for convenience, consider the case of a symmetrical structure. For such a structure A_{xy} is equal to zero; therefore, $A'_x = A_x$ and $A'_y = A_y$. The terms A_x and A_y represent the relative rotations of one end of the structure with respect to the other due to moments of one unit about an x-axis and a y-axis, respectively. In the analogous strut, A_x and A_y represent total areas. If desired, A_x and A_y may be interpreted as measures of stiffness to shear deformations along the x-axis and the y-axis, respectively. For an unsymmetrical structure corrections must be subtracted from A_x and A_y to obtain A'_x and A'_y, respectively.

Finally, consider the term J_z. This term is written as follows:

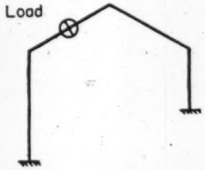
$$J_z = I_x + I_y - 2\,I_{xy} \qquad (11)$$

and represents a displacement along a z-axis at the centroid. This displacement is due to a unit shear along the same axis. Since a unit shear at the centroid does not contribute to rotation of one end of the structure with respect to the other, the displacement at end A is equal to that at the centroid. In Eq. 11 $I_x - I_{xy}$ represents a contribution to the total displacement at end A and is due to moments about x-axes resulting from the unit shear at the centroid. The contribution due to moments about y-axes is represented by $I_y - I_{xy}$. In the analogous strut, the expressions for I_x, I_y, and I_{xy} resemble the familiar transfer equations for moments and products of inertia. The term J_z is similar to the expression for a polar moment of inertia except that I_{xy} is included. If desired, J_z may be interpreted as a measure of the stiffness of the strut to a torsional deformation about a z-axis at the centroid.

$i_x+y^2da_{xx}$	$i_y+x^2da_{yy}$	$i_{xy}+xyda_{xy}$	$m_{xo}da_{xx}$	$m_{yo}da_{xy}$	$m_{yo}da_{yy}$	$m_{xo}da_{xy}$	$y_1m_{xo}da_{xx}$	$y_1m_{yo}da_{xy}$	$x_1m_{yo}da_{yy}$	$x_1m_{xo}da_{xy}$
✓	✓	✓	✓	✓✓	✓	✓✓	✓	✓✓	✓	✓✓
			✓ + ✓✓ = P_x		✓ + ✓✓ = P_y		✓ + ✓✓		✓ + ✓✓	
			$\frac{A_{xy}}{A_y}P_y$		$\frac{A_{xy}}{A_x}P_x$					
			P'_x		P'_y					
$\bar{y}^2A_x$	$\bar{x}^2A_y$	$\bar{x}\bar{y}A_{xy}$					$\bar{y}P_x$		$\bar{x}P_y$	
I_x	I_y	I_{xy}					M_{zx}		M_{zy}	

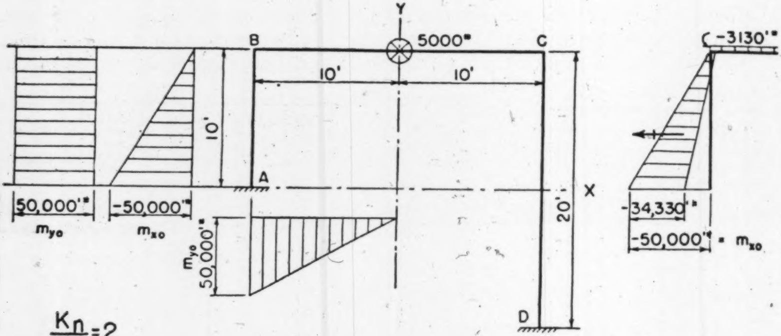
$J_z = I_x + I_y - 2\,I_{xy}$

$M_z = M_{zx} - M_{zy}$



Plane Structures Subjected to Lateral Loads

THE MOMENT CURVES BELOW ARE CONSISTENT WITH ASSUMING THE STRUCTURE CANTILEVERED FROM A


Y
B
5000'"
C
10'
10'
10'
A
X
20'
D
50,000'"
m_{yo}
-50,000'"
m_{xo}
m_{yo} 50,000'"
-3130'"
-34,330'"
-50,000'" = m_{xo}

$\frac{K_n}{K_t}=2$

LOADED STRUCTURE.

Member	Length	$\frac{K_n}{K_{yy}}$	$\frac{K_n}{K_{xx}}$	da_{yy}	da_{xx}	x	y
A B	10.0	2.0	1.0	20.0	10.0	-10	5
B C	20.0	1.0	2.0	20.0	40.0	0	10
C D	20.0	2.0	1.0	40.0	20.0	10	0
Totals				$A_y=80.0$	$A_x=70.0$		

Centroid: $\bar{x}=\frac{C_y}{A_y}=\frac{200}{80}=2.5$ ft. ; $\bar{y}=\frac{C_x}{A_x}=\frac{450}{70}=6.43$ ft.

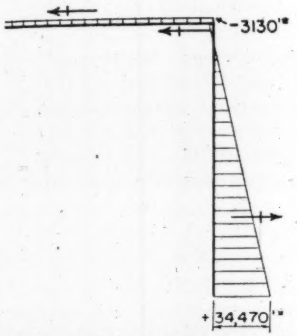
$$v_{zi}=\frac{M_z}{J_z}=\frac{1556.0\times10^4}{8273}=1880 \text{ lb.}$$

At A:

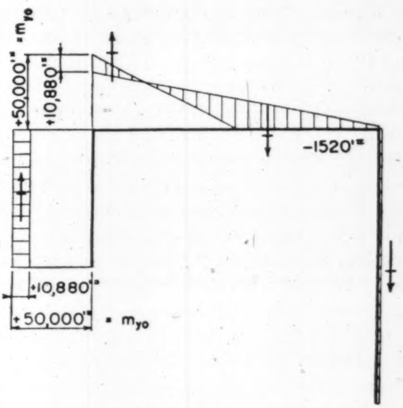
$$m_{xi}=\frac{P_x}{A_x}+\frac{M_z}{J_z}y_c=\frac{-25\times10^4}{70}+1880(-6.43)=-3750-12{,}100=-15{,}670 \text{ ft-lb.}$$

$$m_{yi}=\frac{P_y}{A_y}-\frac{M_z}{J_z}x_c=\frac{125.0\times10^4}{80}-1880(-12.50)=15{,}620+23{,}500=39{,}120 \text{ ft-lb.}$$

Fig. 6.—Unsymmetrical Ben



DISTRIBUTION OF m_x



DISTRIBUTION OF m_y

$x\,da_{yy}$	$y\,da_{xx}$	$i_x + y^2 da_{xx}$	$i_y + x^2 da_{yy}$	$m_{xo} da_{xx}$ $\times 10^4$	$m_{yo} da_{yy}$ $\times 10^4$	$y_1 m_{xo} da_{xx}$ $\times 10^4$	$x_1 m_{yo} da_{yy}$ $\times 10^4$
-200	50	333	2000	-25.0	100.0	-83.2	-1000
0	400	4000	667	0	25.0	0	-166
400	0	667	4000	0	0	0	0
$C_y = -200$	$C_x = 450$	5000	6667	$P_x = -25.0$	$P_y = 125.0$	-83.2	-1166
Corrections to Centroid		$\bar{y}^2 A_x = 2894$	$\bar{x}^2 A_y = 500$			$\bar{y} P_x = -161$	$\bar{x} P_y = 312$
Corrected		$I_x = 2106$	$I_y = 6167$			$M_{zx} = 77.8$	$M_{zy} = -1478$

$I_x + I_y = J_z = 8273$

$M_{zx} - M_{zy} = M_z = 1556.0$

THE MOMENT CURVES ABOVE ARE OBTAINED BY CORRECTING THE DISTRIBUTIONS OF MOMENTS CONSISTENT WITH ASSUMING THE STRUCTURE CANTILEVERED FROM A.

$v_z = v_{zo} - v_{zi} = 5000 - 1880 = 3120$ lb.

$m_x = m_{xo} - m_{xi} = -50{,}000 - (-15{,}670) = -34{,}330$ ft-lb.

$m_y = m_{yo} - m_{yi} = 50{,}000 - 39{,}120 = 10{,}880$ ft-lb.

with a Concentrated Load

For aid in making computations, a tabular form is presented in Fig. 5, which indicates a convenient sequence for computing the properties of the structure and the analogous forces. The final shear and moments at any section of the structure may be computed as shown in the lower part of the figure. The values of x, y and x_1, y_1 may be taken from any convenient pair of axes. If the structure is symmetrical to one of these axes, it is necessary to compute the properties of only one half the structure and to double the sums. In the analytical procedure x and y were defined as distances to differential elements. If segments of finite length are considered, x and y represent distances to the centroids of the elastic areas da_{xx}, da_{yy}, and da_{xy}. For a segment of constant cross section, the centroids of the elastic areas coincide with the centroid of the segment. However, in computing I_x, I_y, and I_{xy}, it is necessary to add corrections i_x, i_y, and i_{xy} to the values of $y^2 \, da_{xx}$, $x^2 \, da_{yy}$, and $x \, y \, da_{xy}$, respectively. These corrections are moments and products of inertia of the elastic areas about their own centroidal axes. The resulting expressions such as $i_x + y^2 \, da_{xx}$ are analogous to the familiar transfer equations for moments and products of inertia. Fig. 4(*b*) summarizes convenient relationships for computing values of i_x, i_y, and i_{xy} of a segment oblique to the x-axis and the y-axis.

It may be shown that for a finite segment x_1 and y_1 are distances to the centroids of the corresponding angle changes along the segment and not distances to the centroids of the elastic areas.

Numerical Examples.—Fig. 6 shows a rectangular bent of constant cross section with a lateral load of 5,000 lb. The ratio of bending to torsional stiffness is equal to two. For a constant cross section it is not necessary to know the absolute values of K_n and K_t. For a given $\frac{K_n}{K_t}$-ratio, values of K_n/K_{yy} and K_n/K_{xx} may be computed. For convenience such values may be plotted for several ratios of K_n/K_t and for different slopes of a segment. Similarly, curves may be drawn showing values of K_n/K_{xy}. In the computations for the rectangular bent, values of K_n/K_{yy} and K_n/K_{xx} have been used instead of $1/K_{yy}$ and $1/K_{xx}$. As a result all items involving da_{yy} and da_{xx} are in terms of K_n. An inspection of the tabulation shows that in the computations for moments and shear the quantities K_n cancel out. Fig. 6 also shows the final curves of moments. These curves are drawn in two steps: First, the assumed moments, and then planar distributions of correction moments. Note how the correction moments balance the assumed moments. This balance concept lends itself to the development of a sketching technique by which good estimates of moments can be obtained. The procedure may be considered an extension of the pressure line concept as used for an arch with loads in the plane of the arch.[19,20] In addition, the concept can be used in problems involving pure torsion or axial deformations.

Fig. 7 shows an arch with a varying cross section and with a lateral load of one kip at the quarter point. The arch is divided into twenty segments whose horizontal projections measure 5 ft. The analogous moments of inertia and products of inertia about the centroidal axes of the segments are neglected.

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[19] "Continuous Frames of Reinforced Concrete," by Hardy Cross and N. D. Morgan, John Wiley & Sons, Inc., New York, N. Y., 1932, p. 289.

[20] *Ibid.*, p. 295.

In addition, the distances to the centroids of the angle changes for each segment are taken as the tabulated distances to the centroid of the segment. The values of K_n/K_t at the various sections, and the relative values of K_n are shown in the table at the bottom of Fig. 7. These values are such as might be probable in a reinforced concrete arch. Since K_n/K_t is not constant, it is now necessary to tabulate values equal to, or proportional to $1/K_{xx}$, $1/K_{yy}$, and $1/K_{xy}$. In this example, proportional values were determined by the relative values of K_n. The final curves of moments are shown in the upper right-hand corner of Fig. 7. Again, the assumed distributions of moments are balanced by planar distributions of correction moments.

A GENERAL PROCEDURE FOR STRUCTURES CURVED IN SPACE

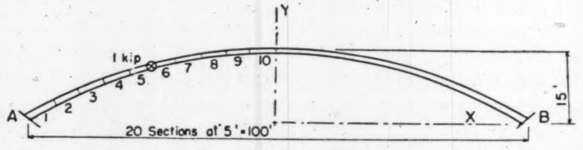
A procedure is presented, in this section, for analyzing a structure that is continuous between two supports but not lying in a plane. This structure may have any shape in space and any variation in cross section along its length. It may be subjected to loads in any direction and to moments about any axis. Three orthogonal axes are chosen for reference, but no part of the structure need lie in the planes defined by these axes. The sign convention used in this section is the same as that used in the preceding section. As before, the effects of shear deformations and changes in length of an element of the structure are considered negligible.

As in the shear and torsion analogy, an analytical procedure consisting of two steps is used. In accordance with this procedure the final moments and shears at any section of the structure are written as follows:

$$\begin{aligned} m_x &= m_{xo} - m_{xi} \qquad & v_x &= v_{xo} - v_{xi} \\ m_y &= m_{yo} - m_{yi} \qquad & v_y &= v_{yo} - v_{yi} \\ m_z &= m_{zo} - m_{zi} \qquad & v_z &= v_{zo} - v_{zi} \end{aligned} \Bigg\} \quad\ldots\ldots\ldots\ldots(12)$$

Consider the distributions of correction moments and shears. Fig. 8(*a*) shows three correction moments and three correction shears applied to the structure. These moments and shears are projected in each plane. The moments are about the *x*-axis, the *y*-axis, and the *z*-axis, and the shears are along axes which, for convenience, are called centroidal axes. Six distances are used to locate the centroidal axes. The positions of these axes and the properties of the structure with respect to these axes are defined subsequently. In each plane the projections of two moments and a shear have been grouped as in the shear and torsion analogy, and the projections of two shears and a moment as in the column analogy. The relationships between these moments and shears, and the correction moments and shears at a section of the structure, are shown in Fig. 8(*a*). Note that the resulting distributions of correction moments and shears are planar.

Fig. 8(*b*) shows rotations about the *x*-axis, the *y*-axis, and the *z*-axis, and displacements along the centroidal axes. Each axis of rotation has been chosen to correspond with that of a moment as shown in Fig. 8(*a*) and each axis of displacement corresponds with that of a shear. The expressions for the rota-



Section	ΔS	$\frac{1}{K_{yy}}$	$\frac{1}{K_{xx}}$	$\frac{1}{K_{xy}}$ (−)	da_{yy}	da_{xx}	da_{xy} (−)	x (−)	y	yda_{xx}	xda_{xy}	y^2da_{xx}	x^2da_{yy}
1	5.75	0.49	0.22	0.09	2.58	1.28	0.54	47.5	1.46	1.8	25.7	2	5825
2	5.62	0.59	0.32	0.09	3.32	1.80	0.50	42.5	4.16	7.4	21.0	31	6000
3	5.48	0.71	0.48	0.06	3.89	2.64	0.33	37.5	6.56	17.3	12.3	113	5470
4	5.36	0.78	0.58	0.05	4.18	3.09	0.24	32.5	8.66	26.8	7.9	231	4415
5	5.25	0.82	0.64	0.03	4.31	3.34	0.18	27.5	10.46	34.9	4.8	365	3264
6.	5.21	0.86	0.74	0.02	4.47	3.83	0.11	22.5	11.96	45.8	2.4	548	2263
7	5.11	0.92	0.87	0.01	4.69	4.45	0.06	17.5	13.16	58.5	1.1	771	1436
8	5.08	0.95	0.93	0	4.81	4.69	0.02	12.5	14.06	65.9	0.2	927	752
9	5.03	0.98	0.97	0	4.92	4.88	0	7.5	14.66	71.5	0	1049	277
10	5.02	1.00	1.00	0	5.02	5.02	0	2.5	14.96	75.1	0	1124	31
					42.19	35.02				405	75.4	5161	29733
					x2=	x2=				x2=	x2=	x2=	x2=
					84.38	70.04				810	150.8	10322	59466
							$\bar{y}=\frac{659.2}{70.04}=9.42$ ft.			659.2			
							Corrections to Centroid					6215	0
												4107	59466

62086

$$v_{zi}=\frac{9213}{62086}=0.148 \text{ kips}$$

At A

$$m_{xi}=-\frac{75.2}{70.04}+0.148(-9.42)=-1.07-1.39=-2.46 \text{ ft.-kips} \qquad v_z=1-0.148=0.852 \text{ kips}$$

$$m_x=-11.25+2.46=-8.79 \text{ ft.-kips}$$

$$m_{yi}=\frac{217.9}{84.38}-0.148(-50)=2.59+7.4=9.99 \text{ ft.-kips} \qquad m_y=25-9.99=15.01 \text{ ft.-kips}$$

At B

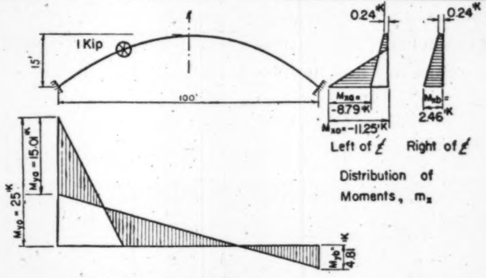
$$m_{xi}=-2.46 \text{ ft.-kips} \qquad v_z=Q-0.148=-0.148 \text{ kips}$$

$$m_x=0-(-2.46)=2.46 \text{ ft.-kips}$$

$$m_{yi}=2.59-7.4=-4.81 \text{ ft.-kips} \qquad m_y=0-(-4.81)=4.81 \text{ ft.-kips}$$

FIG. 7.—PARABOLIC ARCH OF VARYING

$\frac{K_n}{K_t}$ varies


1 Kip
15'
100'
℄
0.24'K
0.24'K
$M_{xa}=$ -8.79'K
$M_{xo}=-11.25$'K
$M_{xb}=$ 2.46'K
Left of ℄
Right of ℄
$M_{yo}-15.01$'K
$M_{yo}=25$'K
M_{yb} 4.81'K

Distribution of Moments, m_x

Distribution of Moments, m_y

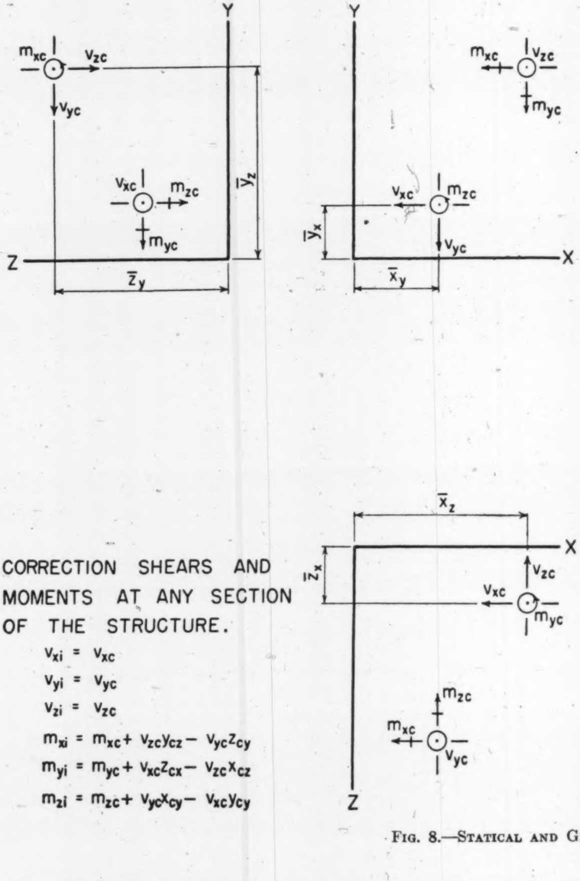
$xyda_{xy}$	m_{xo} (—)	m_{yo}	$m_{xo}da_{xx}$ (—)	$m_{yo}da_{xy}$ (—)	$m_{yo}da_{yy}$	$m_{xo}da_{xy}$	$ym_{xo}da_{xx}$ (—)	$ym_{yo}da_{xy}$ (—)	$xm_{yo}da_{yy}$ (—)	$xm_{xo}da_{xy}$ (—)
37.5	9.79	22.5	12.5	12.2	58.1	5.3	18.3	17.8	2760	251
87.5	7.09	17.5	12.8	8.7	58.1	3.5	53.0	36.0	2470	149
81.0	4.69	12.5	12.4	4.1	48.6	1.5	81.2	27.0	1824	58
68.6	2.59	7.5	8.0	1.8	31.3	0.6	69.3	15.9	1019	21
50.1	0.69	2.5	2.3	0.4	10.8	0.1	24.1	4.6	297	3
28.6	0	0	0	0	0	0	0	0	0	0
14.1	0	0	0	0	0	0	0	0	0	0
3.4	0	0	0	0	0	0	0	0	0	0
0.5	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
371.3 x2= 742.6			−48.0	−27.2	206.9	11.0	−245.9	−101.3	−8370	−482
			−75.2		217.9		−347.2		−8852	
0							−707.9		0	
743							361		−8852	

9213

Section	1	2	3	4	5	6	7	8	9	10
$\frac{K_n}{K_t}$	0.60	0.67	0.78	0.83	0.86	0.91	0.94	0.97	0.99	1.00
Rel. K_n	1.76	1.47	1.31	1.23	1.18	1.13	1.08	1.05	1.02	1.00

Cross Section with Unsymmetrical Load

tions and displacements along the centroidal axes are shown in Fig. 8(*b*). These rotations and displacements are the result of assumed moments and correction moments. The expressions are similar to those of Fig. 3 except that three dimensions are now involved, and additional stiffness factors are introduced. The stiffness factors for the general case of an element in space and oblique to the three orthogonal axes are summarized in Fig. 9.


Y
m_{xc}
v_{zc}
v_{yc}
v_{xc}
m_{zc}
m_{yc}
$\bar{y}_z$
Z
$\bar{z}_y$
Y
m_{xc}
v_{zc}
m_{yc}
v_{xc}
m_{zc}
$\bar{y}_x$
v_{yc}
X
$\bar{x}_y$
$\bar{x}_z$
X
$\bar{z}_x$
v_{zc}
v_{xc}
m_{yc}
m_{zc}
m_{xc}
v_{yc}
Z

CORRECTION SHEARS AND MOMENTS AT ANY SECTION OF THE STRUCTURE.

$v_{xi} = v_{xc}$

$v_{yi} = v_{yc}$

$v_{zi} = v_{zc}$

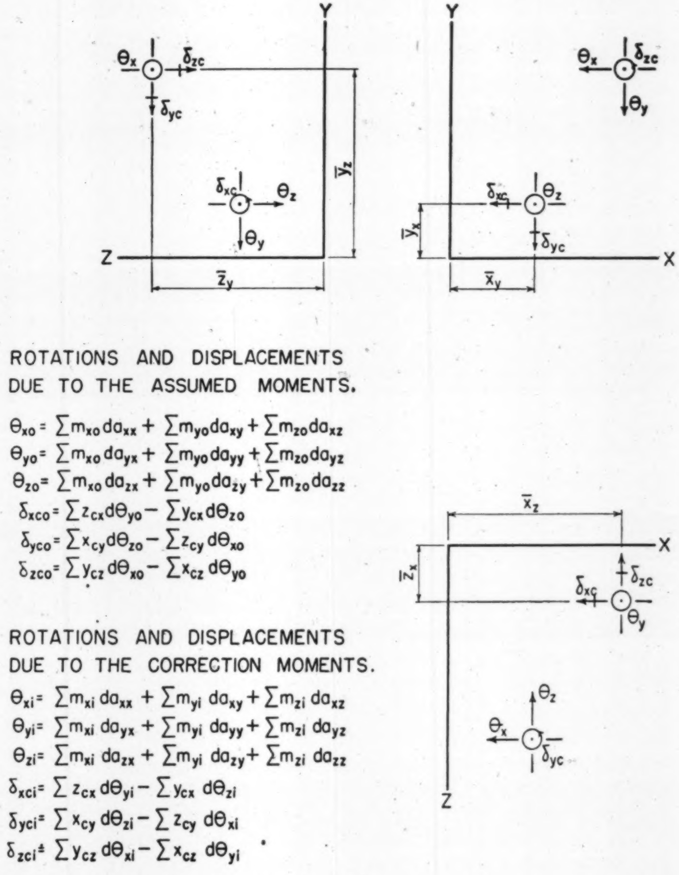
$m_{xi} = m_{xc} + v_{zc}y_{cz} - v_{yc}z_{cy}$

$m_{yi} = m_{yc} + v_{xc}z_{cx} - v_{zc}x_{cz}$

$m_{zi} = m_{zc} + v_{yc}x_{cy} - v_{xc}y_{cy}$

FIG. 8.—STATICAL AND GEOMETRICAL

It follows from the requirements of geometry that the angle changes due to the correction moments must balance those caused by the assumed moments. From the relationships shown in Fig. 8, six equations of geometry may be written. Each equation would involve the correction moments and the correction shears associated with the centroidal axes. It is convenient, however, to define the centroidal axes in such a way that a correction shear associated with one of these axes does not contribute to total rotations about axes normal to this shear. For example, a shear v_{xc} of one unit contributes no rotation about a y-axis or a z-axis if the distances $\bar{z}_x$ and $\bar{y}_x$ are selected such that


Y
θ_x
δ_{zc}
δ_{yc}
δ_{xc}
θ_z
θ_y
$\bar{y}_z$
Z
$\bar{z}_y$
Y
θ_x
δ_{zc}
θ_y
δ_{xc}
θ_z
δ_{yc}
$\bar{y}_x$
X
$\bar{x}_y$
$\bar{x}_z$
X
$\bar{z}_x$
δ_{zc}
δ_{xc}
θ_y
θ_z
θ_x
δ_{yc}
Z

ROTATIONS AND DISPLACEMENTS DUE TO THE ASSUMED MOMENTS.

$$\theta_{xo}=\sum m_{xo}\,da_{xx}+\sum m_{yo}\,da_{xy}+\sum m_{zo}\,da_{xz}$$
$$\theta_{yo}=\sum m_{xo}\,da_{yx}+\sum m_{yo}\,da_{yy}+\sum m_{zo}\,da_{yz}$$
$$\theta_{zo}=\sum m_{xo}\,da_{zx}+\sum m_{yo}\,da_{zy}+\sum m_{zo}\,da_{zz}$$
$$\delta_{xco}=\sum z_{cx}\,d\theta_{yo}-\sum y_{cx}\,d\theta_{zo}$$
$$\delta_{yco}=\sum x_{cy}\,d\theta_{zo}-\sum z_{cy}\,d\theta_{xo}$$
$$\delta_{zco}=\sum y_{cz}\,d\theta_{xo}-\sum x_{cz}\,d\theta_{yo}$$

ROTATIONS AND DISPLACEMENTS DUE TO THE CORRECTION MOMENTS.

$$\theta_{xi}=\sum m_{xi}\,da_{xx}+\sum m_{yi}\,da_{xy}+\sum m_{zi}\,da_{xz}$$
$$\theta_{yi}=\sum m_{xi}\,da_{yx}+\sum m_{yi}\,da_{yy}+\sum m_{zi}\,da_{yz}$$
$$\theta_{zi}=\sum m_{xi}\,da_{zx}+\sum m_{yi}\,da_{zy}+\sum m_{zi}\,da_{zz}$$
$$\delta_{xci}=\sum z_{cx}\,d\theta_{yi}-\sum y_{cx}\,d\theta_{zi}$$
$$\delta_{yci}=\sum x_{cy}\,d\theta_{zi}-\sum z_{cy}\,d\theta_{xi}$$
$$\delta_{zci}=\sum y_{cz}\,d\theta_{xi}-\sum x_{cz}\,d\theta_{yi}$$

RELATIONSHIPS FOR THE GENERAL PROCEDURE

$$\sum d\theta_{yi}=\sum z_{cx}\,da_{yy}-\sum y_{cx}\,da_{yz}=0\ldots\ldots\ldots\ldots(13a)$$

and

$$\sum d\theta_{zi}=\sum z_{cx}\,da_{zy}-\sum y_{cx}\,da_{zz}=0\ldots\ldots\ldots\ldots(13b)$$

in which

$$z_{cx} = z - \bar{z}_x \qquad (14a)$$

and

$$y_{cx} = y - \bar{y}_x \qquad (14b)$$

The expressions for $\bar{z}_x$ and $\bar{y}_x$ may be obtained by Eqs. 13 and 14. The expressions for $\bar{x}_y$ and $\bar{z}_y$ associated with v_{yc}, and for $\bar{y}_z$ and $\bar{x}_z$ associated with v_{zc}, can be obtained from similar equations.

The six equations of geometry may now be written as follows:

$$\left.\begin{aligned}
\sum d\theta_x &= A_x\, m_{xc} + A_{xy}\, m_{yc} + A_{xz}\, m_{zc} + f_{xx}\, v_{xc} - P_x = 0\\
\sum d\theta_y &= A_{yx}\, m_{xc} + A_y\, m_{yc} + A_{yz}\, m_{zc} + f_{yy}\, v_{yc} - P_y = 0\\
\sum d\theta_z &= A_{zx}\, m_{xc} + A_{zy}\, m_{yc} + A_z\, m_{zc} + f_{zz}\, v_{zc} - P_z = 0\\
\delta_x &= t_{xx}\, m_{xc} + J_x\, v_{xc} + J_{xy}\, v_{yc} + J_{xz}\, v_{zc} - M_x = 0\\
\delta_y &= t_{yy}\, m_{yc} + J_{yx}\, v_{xc} + J_y\, v_{yc} + J_{yz}\, v_{zc} - M_y = 0\\
\delta_z &= t_{zz}\, m_{zc} + J_{zx}\, v_{xc} + J_{zy}\, v_{yc} + J_z\, v_{zc} - M_z = 0
\end{aligned}\right\} \qquad (15)$$

in which the coefficients of the correction moments and shears are summarized in Figs. 10 and 11. These coefficients may be interpreted as total rotations and displacements due to unit moments and shears associated with the centroidal axes. For example, A_{xy} represents a rotation about an x-axis due to a moment m_{yc} equal to one unit; the term f_{xx} represents a rotation about an x-axis due to a shear v_{xc} equal to one unit; and the term t_{xx} represents a displacement along the centroidal axis associated with v_{xc} and is due to a moment m_{xc} equal to one unit. It can be shown that $t_{xx} = f_{xx}$, $t_{yy} = f_{yy}$, and $t_{zz} = f_{zz}$. The term J_{xy} represents a displacement along the centroidal axis associated with v_{xc}. This displacement is caused by a shear v_{yc} equal to one unit. It can be shown that $A_{yx} = A_{xy}$ and $J_{yx} = J_{xy}$.

In the equations of geometry (Eqs. 15), the P-terms and M-terms represent rotations and displacements resulting from the assumed distributions of moments. For example, P_x represents the total rotation about an x-axis. The term M_x represents a displacement along the centroidal axis in the x-direction. These terms may also represent unit rotations and displacements applied to the structure as in the Müller-Breslau procedure for obtaining influence lines. In addition, the M-terms may represent displacements that would occur in a pipe line if it were free to expand when subjected to temperature changes.

The correction moments and the correction shear along the centroidal axes may now be obtained by solving the equations of geometry. Having developed the expressions for these correction moments and shears, the designer can obtain the correction moments and shears at any section of the structure by the equations of statics summarized in Fig. 8(a). The expressions for these shears and moments are written in the following form:

$$\left.\begin{aligned}
v_{xi} &= \frac{\mathfrak{M}'_x}{\mathfrak{J}'_x}\\
v_{yi} &= \frac{\mathfrak{M}'_y}{\mathfrak{J}'_y}\\
v_{zi} &= \frac{\mathfrak{M}'_z}{\mathfrak{J}'_z}
\end{aligned}\right\} \qquad (16)$$

$$K_{xy} = \frac{K_{n1}K_{n2}K_t}{K_{n1}K_{n2}C_1 + K_{n1}K_t(C_2 + aC_3) - K_{n2}K_t(C_2 + bC_3)}$$

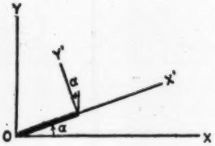
$$K_{yy} = \frac{K_{n1}K_{n2}K_t}{K_{n1}K_{n2}C_4 + K_{n1}K_t(C_5 + C_6) - K_{n2}K_t(C_5 - C_7)}$$

$$K_{yz} = \frac{K_{n1}K_{n2}K_t}{K_{n1}K_{n2}C_8 - K_{n1}K_t(C_9 + C_{10}) + K_{n2}K_t(C_9 - C_{11})}$$

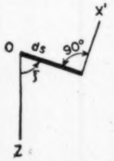
$$K_{xx} = \frac{K_{n1}K_{n2}K_t}{K_{n1}K_{n2}C_{12} - K_{n1}K_t(C_5 - C_{13}) + K_{n2}K_t(C_5 + C_{14})}$$

$$K_{zx} = \frac{K_{n1}K_{n2}K_t}{K_{n1}K_{n2}C_{15} - K_{n1}K_t(C_{16} - C_{17}) - K_{n2}K_t(C_{17} + C_{18})}$$

$$K_{zz} = \frac{K_{n1}K_{n2}K_t}{K_{n1}K_{n2}C_{19} + K_{n1}K_tC_{20} + K_{n2}K_tC_{21}}$$



PROJECTION OF ELEMENT ds in XY PLANE



TRUE VIEW OF ELEMENT ds.

$C_1 = \sin\alpha\cos\alpha\sin^2\xi$

$C_2 = (\cos^2\alpha - \sin^2\alpha)\sin\psi\cos\psi\cos\xi$

$a = \cos^2\psi - \frac{\sin^2\psi}{\cos^2\xi}$

$C_3 = \sin\alpha\cos\alpha\cos^2\xi$

$b = \frac{\cos^2\psi}{\cos^2\xi} - \sin^2\psi$

$C_4 = \sin^2\alpha\sin^2\xi$

$C_5 = 2\sin\alpha\cos\alpha\sin\psi\cos\psi\cos\xi$

$C_6 = \sin^2\alpha\cos^2\psi\cos^2\xi + \cos^2\alpha\sin^2\psi$

$C_7 = \sin^2\alpha\sin^2\psi\cos^2\xi + \cos^2\alpha\cos^2\psi$

$C_8 = \sin\alpha\sin\xi\cos\xi$

$C_9 = \cos\alpha\sin\psi\cos\psi\sin\xi$

$C_{10} = \sin\alpha\cos^2\psi\sin\xi\cos\xi$

$C_{11} = \sin\alpha\sin^2\psi\sin\xi\cos\xi$

$C_{12} = \cos^2\alpha\sin^2\xi$

$C_{13} = \cos^2\alpha\cos^2\psi\cos^2\xi + \sin^2\alpha\sin^2\psi$

$C_{14} = \cos^2\alpha\sin^2\psi\cos^2\xi + \sin^2\alpha\cos^2\psi$

$C_{15} = \cos\alpha\sin\xi\cos\xi$

$C_{16} = \cos\alpha\cos^2\psi\sin\xi\cos\xi$

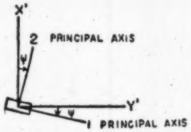
$C_{17} = \sin\alpha\sin\psi\cos\psi\sin\xi$

$C_{18} = \cos\alpha\sin^2\psi\sin\xi\cos\xi$

$C_{19} = \cos^2\xi$

$C_{20} = \cos^2\psi\sin^2\xi$

$C_{21} = \sin^2\psi\sin^2\xi$



TRUE END VIEW OF ELEMENT ds.

FOR THE SPECIAL CASE OF $\psi = 0$: A PRINCIPAL AXIS OF THE CROSS SECTION IS PARALLEL TO ONE OF THE REFERENCE PLANES.

$C_2 = 0$

$a = 1$

$b = 1/\cos^2\xi$

$C_5 = 0$

$C_6 = \sin^2\alpha\cos^2\xi$

$C_7 = \cos^2\alpha$

$C_9 = 0$

$C_{10} = \sin\alpha\sin\xi\cos\xi$

$C_{11} = 0$

$C_{13} = \cos^2\alpha\cos^2\xi$

$C_{14} = \sin^2\alpha$

$C_{16} = \cos\alpha\sin\xi\cos\xi$

$C_{17} = 0$

$C_{18} = 0$

$C_{20} = \sin^2\xi$

$C_{21} = 0$

FIG. 9.—STIFFNESS FACTORS FOR THE GENERAL CASE OF AN ELEMENT IN SPACE

Member	Length	$\frac{1}{K_{xx}}$	$\frac{1}{K_{yy}}$	$\frac{1}{K_{zz}}$	$\frac{1}{K_{xy}}$	$\frac{1}{K_{yz}}$	$\frac{1}{K_{zx}}$	da_{xx}	da_{yy}	da_{zz}	da_{xy}

	A_x		A_y		A_z		A_{xy}
Totals Corrections (subtract)	$\frac{A_{xy}^2}{A_y}$	$\frac{A_{zx}^2}{A_z}$	$\frac{A_{xy}^2}{A_x}$	$\frac{A_{yz}^2}{A_z}$	$\frac{A_{zx}^2}{A_x}$	$\frac{A_{yz}^2}{A_y}$	$\frac{A_{yz}A_{zx}}{A_z}$
	$(A_x')_y$	$(A_x')_z$	$(A_y')_x$	$(A_y')_z$	$(A_z')_x$	$(A_z')_y$	A_{xy}'

Centroids

$$\bar{z}_x = \frac{(C_y')_z}{(A_y')_z} \qquad \bar{x}_y = \frac{(C_z')_x}{(A_z')_x} \qquad \bar{y}_z = \frac{(C_x')_y}{(A_x')_y}$$

$$\bar{y}_x = -\frac{(C_z')_y}{(A_z')_y} \qquad \bar{z}_y = -\frac{(C_x')_z}{(A_x')_z} \qquad \bar{x}_z = -\frac{(C_y')_x}{(A_y')_x}$$

$z\,da_{xy}$		$x\,da_{yz}$		$y\,da_{zx}$		$(i_x)_z + y^2 da_{xx}$	$(i_y)_z + x^2 da_{yy}$	$(i_{xy})_z + xy\,da_{xy}$	$(i_y)_x + z^2 da_{yy}$	$(i_z)_x + y^2 da_{zz}$	$(i_{yz})_x + yz\,da_{yz}$	$(i_z)_y + x^2 da_{zz}$	$(i_x)_y + z^2 da_{xx}$	$(i_{zx})_y + zx\,da_{zx}$
✓		✓		✓		✓	✓	✓	✓	✓	✓	✓	✓	✓
Corrections (subtract):-														
$\bar{z}_x \cdot A_{xy}$	$\bar{z}_y \cdot A_{xy}$	$\bar{x}_y \cdot A_{yz}$	$\bar{x}_z \cdot A_{yz}$	$\bar{y}_z \cdot A_{zx}$	$\bar{y}_x \cdot A_{zx}$	$\bar{y}_z^2 A_x$	$\bar{x}_z^2 A_y$	$\bar{x}_z \bar{y}_z A_{xy}$	$\bar{z}_x^2 A_y$	$\bar{y}_x^2 A_z$	$\bar{y}_x \bar{z}_x A_{yz}$	$\bar{x}_y^2 A_z$	$\bar{z}_y^2 A_x$	$\bar{z}_y \bar{x}_y A_{zx}$
①	②	③	④	⑤	⑥	$(I_x)_z$	$(I_y)_z$	$(I_{xy})_z$	$(I_y)_x$	$(I_z)_x$	$(I_{yz})_x$	$(I_z)_y$	$(I_x)_y$	$(I_{zx})_y$

① Subtract :- ⑥	③ ②	⑤ ④	$J_z = (I_x)_z + (I_y)_z - 2(I_{xy})_z$		$J_x = (I_y)_x + (I_z)_x - 2(I_{yz})_x$		$J_y = (I_z)_y + (I_x)_y - 2(I_{zx})_y$	
			$\frac{J_{zx}^2}{J_x}$	$\frac{J_{yz}^2}{J_y}$	$\frac{J_{xy}^2}{J_y}$	$\frac{J_{zx}^2}{J_z}$	$\frac{J_{yz}^2}{J_z}$	$\frac{J_{xy}^2}{J_x}$
t_{xx}	t_{yy}	t_{zz}	$(J_z')_x$	$(J_z')_y$	$(J_x')_y$	$(J_x')_z$	$(J_y')_z$	$(J_y')_x$

$$J_x' = J_x - J_{xy}\frac{J_{xy}'}{(J_y')_z} - J_{zx}\frac{J_{zx}'}{(J_z')_y} \qquad \mathcal{A}_x = A_x - \frac{t_{xx}^2}{J_x'} \qquad \mathcal{A}_{xy} = A_{xy} + \frac{t_{xx}t_{yy}J_{xy}'}{J_x'(J_y')_z}$$

$$J_y' = J_y - J_{yz}\frac{J_{yz}'}{(J_z')_x} - J_{xy}\frac{J_{xy}'}{(J_x')_z} \qquad \mathcal{A}_y = A_y - \frac{t_{yy}^2}{J_y'} \qquad \mathcal{A}_{yz} = A_{yz} + \frac{t_{yy}t_{zz}J_{yz}'}{J_y'(J_z')_x}$$

$$J_z' = J_z - J_{zx}\frac{J_{zx}'}{(J_x')_y} - J_{yz}\frac{J_{yz}'}{(J_y')_x} \qquad \mathcal{A}_z = A_z - \frac{t_{zz}^2}{J_z'} \qquad \mathcal{A}_{zx} = A_{zx} + \frac{t_{zz}t_{xx}J_{zx}'}{J_z'(J_x')_y}$$

Fig. 10.—General Procedure for Analysi

da_{yz}	da_{zx}	x	y	z	$z\,da_{yy}$	$y\,da_{yz}$	$z\,da_{yz}$	$y\,da_{zz}$	$x\,da_{zz}$	$z\,da_{zx}$	$x\,da_{zx}$	$z\,da_{xx}$	$y\,da_{xx}$	$x\,da_{xy}$	$y\,da_{xy}$	$x\,da_{yy}$
A_{yz} $\frac{A_{zx}A_{xy}}{A_x}$	A_{zx} $\frac{A_{xy}A_{yz}}{A_y}$				$'$	$''$	$'$	$''$	$'$	$''$	$'$	$''$	$'$	$''$	$'$	$''$
					$' - '' = (C_y)_z$ $\frac{A_{yz}}{A_z}(C_z)_y$		$' - '' = (C_z)_y$ $\frac{A_{yz}}{A_y}(C_y)_z$		$' - '' = (C_z)_x$ $\frac{A_{zx}}{A_x}(C_x)_z$		$' - '' = (C_x)_z$ $\frac{A_{zx}}{A_z}(C_z)_x$		$' - '' = (C_x)_y$ $\frac{A_{xy}}{A_y}(C_y)_x$		$' - '' = (C_y)_x$ $\frac{A_{xy}}{A_x}(C_x)_y$	
A'_{yz}	A'_{zx}				$(C'_y)_z$		$(C'_z)_y$		$(C'_z)_x$		$(C'_x)_z$		$(C'_x)_y$		$(C'_y)_x$	

$$A'_x = A_x - A_{xy}\frac{A'_{xy}}{(A'_y)_z} - A_{zx}\frac{A'_{zx}}{(A'_z)_y} \qquad A'_y = A_y - A_{yz}\frac{A'_{yz}}{(A'_z)_x} - A_{xy}\frac{A'_{xy}}{(A'_x)_z} \qquad A'_z = A_z - A_{zx}\frac{A'_{zx}}{(A'_x)_y} - A_{yz}\frac{A'_{yz}}{(A'_y)_x}$$

$(i_{xy})_{xy}$ $+z^2da_{xy}$	$(i_{yz})_{xy}$ $+zx\,da_{yz}$	$(i_{zx})_{xy}$ $+yz\,da_{zx}$	$(i_z)_{xy}$ $+xy\,da_{zz}$	$(i_{yz})_{yz}$ $+x^2da_{yz}$	$(i_{zx})_{yz}$ $+xy\,da_{zx}$	$(i_{xy})_{yz}$ $+zx\,da_{xy}$	$(i_x)_{yz}$ $+yz\,da_{xx}$	$(i_{zx})_{zx}$ $+y^2da_{zx}$	$(i_{xy})_{zx}$ $+yz\,da_{xy}$	$(i_{yz})_{zx}$ $+xy\,da_{yz}$	$(i_y)_{zx}$ $+zx\,da_{yy}$
$'$ $\bar{z}_x\sum z\,da_{xy}$ $+\bar{z}_y\sum z\,da_{xy}$ $-\bar{z}_x\bar{z}_y\sum da_{xy}$	$'$ $\bar{z}_x\sum x\,da_{yz}$	$'$ $\bar{z}_y\sum y\,da_{zx}$	$'$ $\bar{x}_y\bar{y}_x\sum da_{zz}$	$'$ $\bar{x}_y\sum x\,da_{yz}$ $+\bar{x}_z\sum x\,da_{yz}$ $-\bar{x}_y\bar{x}_z\sum da_{yz}$	$'$ $\bar{x}_y\sum y\,da_{zx}$	$'$ $\bar{x}_z\sum z\,da_{xy}$	$'$ $\bar{y}_z\bar{z}_y\sum da_{xx}$	$'$ $\bar{y}_z\sum y\,da_{zx}$ $+\bar{y}_x\sum y\,da_{zx}$ $-\bar{y}_z\bar{y}_x\sum da_{zx}$	$'$ $\bar{y}_z\sum z\,da_{xy}$	$'$ $\bar{y}_x\sum x\,da_{yz}$	$'$ $\bar{z}_x\bar{x}_z\sum da_{yy}$
$(I_{xy})_{xy}$	$(I_{yz})_{xy}$	$(I_{zx})_{xy}$	$(I_z)_{xy}$	$(I_{yz})_{yz}$	$(I_{zx})_{yz}$	$(I_{xy})_{yz}$	$(I_x)_{yz}$	$(I_{zx})_{zx}$	$(I_{xy})_{zx}$	$(I_{yz})_{zx}$	$(I_y)_{zx}$
$J_{xy} = -(I_{xy})_{xy} + (I_{yz})_{xy} + (I_{zx})_{xy} - (I_z)_{xy}$ $\frac{J_{yz}J_{zx}}{J_z}$				$J_{yz} = -(I_{yz})_{yz} + (I_{zx})_{yz} + (I_{xy})_{yz} - (I_x)_{yz}$ $\frac{J_{zx}J_{xy}}{J_x}$				$J_{zx} = -(I_{zx})_{zx} + (I_{xy})_{zx} + (I_{yz})_{zx} - (I_y)_{zx}$ $\frac{J_{xy}J_{yz}}{J_y}$			
J'_{xy}				J'_{yz}				J'_{zx}			

$$\mathfrak{J}_x = J_x - \frac{t_{xx}^2}{A'_x} \qquad \mathfrak{J}_{xy} = J_{xy} + \frac{t_{xx}t_{yy}A'_{xy}}{A'_x(A'_y)_z}$$

$$\mathfrak{J}_y = J_y - \frac{t_{yy}^2}{A'_y} \qquad \mathfrak{J}_{yz} = J_{yz} + \frac{t_{yy}t_{zz}A'_{yz}}{A'_y(A'_z)_x}$$

$$\mathfrak{J}_z = J_z - \frac{t_{zz}^2}{A'_z} \qquad \mathfrak{J}_{zx} = J_{zx} + \frac{t_{zz}t_{xx}A'_{zx}}{A'_z(A'_x)_y}$$

OF STRUCTURES IN SPACE (WITH FIG. 11)

$\mathcal{A}'_{zx} = \mathcal{A}_{zx} - \dfrac{\mathcal{A}_{xy}\mathcal{A}_{yz}}{\mathcal{A}_y}$ $\qquad (\mathcal{A}'_x)_y = \mathcal{A}_x - \dfrac{\mathcal{A}_{xy}^2}{\mathcal{A}_y}$ $\qquad (\mathcal{A}'_z)_x = \mathcal{A}_z - \dfrac{\mathcal{A}_{zx}^2}{\mathcal{A}_x}$ $\qquad \mathcal{A}'_x = \mathcal{A}_x - \mathcal{A}_{xy}\dfrac{\mathcal{A}_{xy}}{(\mathcal{A}'_y)_z} - \mathcal{A}_{zx}\dfrac{\mathcal{A}'_{zx}}{(\mathcal{A}'_z)_y}$

$\mathcal{A}'_{xy} = \mathcal{A}_{xy} - \dfrac{\mathcal{A}_{yz}\mathcal{A}_{zx}}{\mathcal{A}_z}$ $\qquad (\mathcal{A}'_x)_z = \mathcal{A}_x - \dfrac{\mathcal{A}_{zx}^2}{\mathcal{A}_z}$ $\qquad (\mathcal{A}'_z)_y = \mathcal{A}_z - \dfrac{\mathcal{A}_{yz}^2}{\mathcal{A}_y}$ $\qquad \mathcal{A}'_y = \mathcal{A}_y - \mathcal{A}_{yz}\dfrac{\mathcal{A}'_{yz}}{(\mathcal{A}'_z)_x} - \mathcal{A}_{xy}\dfrac{\mathcal{A}'_{xy}}{(\mathcal{A}'_x)_z}$

$\mathcal{A}'_{yz} = \mathcal{A}_{yz} - \dfrac{\mathcal{A}_{zx}\mathcal{A}_{xy}}{\mathcal{A}_x}$ $\qquad (\mathcal{A}'_y)_x = \mathcal{A}_y - \dfrac{\mathcal{A}_{xy}^2}{\mathcal{A}_x}$ $\qquad (\mathcal{A}'_y)_z = \mathcal{A}_y - \dfrac{\mathcal{A}_{yz}^2}{\mathcal{A}_z}$ $\qquad \mathcal{A}'_z = \mathcal{A}_z - \mathcal{A}_{zx}\dfrac{\mathcal{A}'_{zx}}{(\mathcal{A}'_x)_y} - \mathcal{A}_{yz}\dfrac{\mathcal{A}'_{yz}}{(\mathcal{A}'_y)_x}$

$m_{xo}da_{xx}$	$m_{yo}da_{xy}$	$m_{zo}da_{zx}$	$m_{xo}da_{xy}$	$m_{yo}da_{yy}$	$m_{zo}da_{yz}$	$m_{xo}da_{zx}$	$m_{yo}da_{yz}$	$m_{zo}da_{zz}$	$zm_{xo}da_{xy}$	$zm_{yo}da_{yy}$	$zm_{zo}da_{yz}$	$ym_{xo}da_{zx}$	$ym_{yo}da_{yz}$
✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

✓+✓+✓ = P_x Subtract:- $P_y\dfrac{A'_{xy}}{(A'_y)_z} + P_z\dfrac{A'_{zx}}{(A'_z)_y}$	✓+✓+✓ = P_y $P_z\dfrac{A'_{yz}}{(A'_z)_x} + P_x\dfrac{A'_{xy}}{(A'_x)_z}$	✓+✓+✓ = P_z $P_x\dfrac{A'_{zx}}{(A'_x)_y} + P_y\dfrac{A'_{yz}}{(A'_y)_x}$	✓+✓+✓ $\bar{z}_x P_y$	✓+✓+✓ $\bar{y}_x P_z$
			✓	✓ ✓
P'_x	P'_y	P'_z	✓ − ✓✓ = M_x	
		Subtract:	$M_y\dfrac{J'_{xy}}{(J'_y)_z} + M_z\dfrac{J'_{zx}}{(J'_z)_y}$	
			M'_x	

$F_x = t_{xx}\dfrac{M'_x}{J'_x}$ $\qquad T_x = t_{xx}\dfrac{P'_x}{A'_x}$

$F_y = t_{yy}\dfrac{M'_y}{J'_y}$ $\qquad T_y = t_{yy}\dfrac{P'_y}{A'_y}$

$F_z = t_{zz}\dfrac{M'_z}{J'_z}$ $\qquad T_z = t_{zz}\dfrac{P'_z}{A'_z}$

$\mathcal{F}_x = F_x - F_y\dfrac{\mathcal{A}'_{xy}}{(\mathcal{A}'_y)_z} - F_z\dfrac{\mathcal{A}'_{zx}}{(\mathcal{A}'_z)_y}$ $\qquad \mathcal{T}_x = T_x - T_y\dfrac{\mathcal{J}'_{xy}}{(\mathcal{J}'_y)_z} - T_z\dfrac{\mathcal{J}'_{zx}}{(\mathcal{J}'_z)_y}$ $\qquad \mathcal{P}_x = P_x - P_y\dfrac{\mathcal{A}'_{xy}}{(\mathcal{A}'_y)_z} - P_z\dfrac{\mathcal{A}'_{zx}}{(\mathcal{A}'_z)_y}$

$\mathcal{F}_y = F_y - F_z\dfrac{\mathcal{A}'_{yz}}{(\mathcal{A}'_z)_x} - F_x\dfrac{\mathcal{A}'_{xy}}{(\mathcal{A}'_x)_z}$ $\qquad \mathcal{T}_y = T_y - T_z\dfrac{\mathcal{J}'_{yz}}{(\mathcal{J}'_z)_x} - T_x\dfrac{\mathcal{J}'_{xy}}{(\mathcal{J}'_x)_z}$ $\qquad \mathcal{P}_y = P_y - P_z\dfrac{\mathcal{A}'_{yz}}{(\mathcal{A}'_z)_x} - P_x\dfrac{\mathcal{A}'_{xy}}{(\mathcal{A}'_x)_z}$

$\mathcal{F}_z = F_z - F_x\dfrac{\mathcal{A}'_{zx}}{(\mathcal{A}'_x)_y} - F_y\dfrac{\mathcal{A}'_{yz}}{(\mathcal{A}'_y)_x}$ $\qquad \mathcal{T}_z = T_z - T_x\dfrac{\mathcal{J}'_{zx}}{(\mathcal{J}'_x)_y} - T_y\dfrac{\mathcal{J}'_{yz}}{(\mathcal{J}'_y)_x}$ $\qquad \mathcal{P}_z = P_z - P_x\dfrac{\mathcal{A}'_{zx}}{(\mathcal{A}'_x)_y} - P_y\dfrac{\mathcal{A}'_{yz}}{(\mathcal{A}'_y)_x}$

$v_{xi} = \dfrac{\mathcal{M}'_x}{\mathcal{J}'_x}$ $\qquad m_{xi} = \dfrac{\mathcal{P}'_x}{\mathcal{A}'_x} + \dfrac{\mathcal{M}'_z}{\mathcal{J}'_z}y_{cz} - \dfrac{\mathcal{M}'_y}{\mathcal{J}'_y}z_{cy}$ $\qquad v_x = v_{xo} - v_{xi}$ $\qquad m_x = m_{xo} - m_{xi}$

$v_{yi} = \dfrac{\mathcal{M}'_y}{\mathcal{J}'_y}$ $\qquad m_{yi} = \dfrac{\mathcal{P}'_y}{\mathcal{A}'_y} + \dfrac{\mathcal{M}'_x}{\mathcal{J}'_x}z_{cx} - \dfrac{\mathcal{M}'_z}{\mathcal{J}'_z}x_{cz}$ $\qquad v_y = v_{yo} - v_{yi}$ $\qquad m_y = m_{yo} - m_{yi}$

$v_{zi} = \dfrac{\mathcal{M}'_z}{\mathcal{J}'_z}$ $\qquad m_{zi} = \dfrac{\mathcal{P}'_z}{\mathcal{A}'_z} + \dfrac{\mathcal{M}'_y}{\mathcal{J}'_y}x_{cy} - \dfrac{\mathcal{M}'_x}{\mathcal{J}'_x}y_{cx}$ $\qquad v_z = v_{zo} - v_{zi}$ $\qquad m_z = m_{zo} - m_{zi}$

FIG. 11.—GENERAL PROCEDURE FOR ANALYSIS

$J'_{zx} = J_{zx} - \frac{J_{xy}J_{yz}}{J_y}$ $(J'_x)_y = J_x - \frac{J_{xy}^2}{J_y}$ $(J'_z)_x = J_z - \frac{J_{zx}^2}{J_x}$ $J'_x = J_x - J_{xy}\frac{J'_{xy}}{(J'_y)_z} - J_{zx}\frac{J'_{zx}}{(J'_z)_y}$

$J'_{xy} = J_{xy} - \frac{J_{yz}J_{zx}}{J_z}$ $(J'_x)_z = J_x - \frac{J_{zx}^2}{J_z}$ $(J'_z)_y = J_z - \frac{J_{yz}^2}{J_y}$ $J'_y = J_y - J_{yz}\frac{J'_{yz}}{(J'_z)_x} - J_{xy}\frac{J'_{xy}}{(J'_x)_z}$

$J'_{yz} = J_{yz} - \frac{J_{zx}J_{xy}}{J_x}$ $(J'_y)_x = J_y - \frac{J_{xy}^2}{J_x}$ $(J'_y)_z = J_y - \frac{J_{yz}^2}{J_z}$ $J'_z = J_z - J_{zx}\frac{J'_{zx}}{(J'_x)_y} - J_{yz}\frac{J'_{yz}}{(J'_y)_x}$

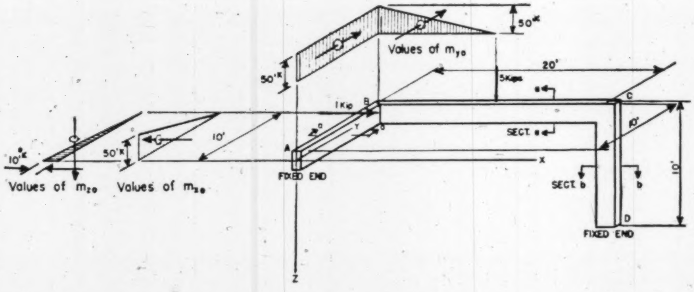
$ym_{zo}da_{zz}$	$xm_{xo}da_{zx}$	$xm_{yo}da_{yz}$	$xm_{zo}da_{zz}$	$zm_{xo}da_{xx}$	$zm_{yo}da_{xy}$	$zm_{zo}da_{zx}$	$ym_{xo}da_{xx}$	$ym_{yo}da_{xy}$	$ym_{zo}da_{zx}$	$xm_{xo}da_{xy}$	$xm_{yo}da_{yy}$	$xm_{zo}da_{yz}$
✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
	✓+✓+✓ $\bar{x}_y P_z$			✓+✓+✓ $\bar{z}_y P_x$			✓+✓+✓ $\bar{y}_z P_x$			✓+✓+✓ $\bar{x}_z P_y$		
	✓			✓✓			✓			✓✓		
	✓ - ✓✓ = M_y $M_z\frac{J'_{yz}}{(J'_z)_x} + M_x\frac{J'_{xy}}{(J'_x)_z}$						✓ - ✓✓ = M_z $M_x\frac{J'_{zx}}{(J'_x)_y} + M_y\frac{J'_{yz}}{(J'_y)_x}$					
	M'_y						M'_z					

$\mathcal{M}_x = M_x - M_y\frac{J'_{xy}}{(J'_y)_z} - M_z\frac{J'_{zx}}{(J'_z)_y}$ $\mathcal{P}'_x = \mathcal{P}_x - F_x$ $\mathcal{M}'_x = \mathcal{M}_x - T_x$

$\mathcal{M}_y = M_y - M_z\frac{J'_{yz}}{(J'_z)_x} - M_x\frac{J'_{xy}}{(J'_x)_z}$ $\mathcal{P}'_y = \mathcal{P}_y - F_y$ $\mathcal{M}'_y = \mathcal{M}_y - T_y$

$\mathcal{M}_z = M_z - M_x\frac{J'_{zx}}{(J'_x)_y} - M_y\frac{J'_{yz}}{(J'_y)_x}$ $\mathcal{P}'_z = \mathcal{P}_z - F_z$ $\mathcal{M}'_z = \mathcal{M}_z - T_z$

of STRUCTURES IN SPACE (WITH FIG. 10)



MEMBER	LENGTH	$1/K_{xx}$	$1/K_{yy}$	$1/K_{zz}$	da_{xx}	da_{yy}	da_{zz}	x	y	z	zda_{yy}	yda_{zz}	xda_{zz}	zda_{xx}	yda_{xx}	xda_{yy}	$(i_x)_z + y^2da_{xx}$	$(i_y)_z + x^2da_{yy}$
	FT.	$\times 10^9$	$\times 10^9$	$\times 10^9$	$\times 10^9$	$\times 10^9$	$\times 10^9$	FT.	FT.	FT.	$\times 10^9$	$\times 10^9$	$\times 10^9$	$\times 10^9$	$\times 10^9$	$\times 10^9$	$\times 10^7$	$\times 10^7$
AB	10	3.33	10.00	10.00	33.3	100.0	100.0	0	5	0	0	500	0	0	167	0	11.1	0
BC	20	10.00	3.33	10.00	200.0	66.6	200.0	10	10	0	0	2000	2000	0	2000	666	200.0	88.8
CD	10	10.00	3.33	10.00	100.0	33.3	100.0	20	10	5	166.5	1000	2000	500	1000	666	100.0	133.2
				TOTALS	$A_x =$ 333.3	$A_y =$ 200.0	$A_z =$ 400.0				166.5	3500	4000	500	3167	1332	311.1	222.0
									TOTALS		$(C_y)_z =$ 166.5	$(C_z)_y =$ -3500	$(C_z)_x =$ 4000	$(C_x)_z =$ -500	$(C_x)_y =$ 3167	$(C_y)_x =$ -1332		
																SUBTRACT	300.5	88.7
																	$(I_x)_z =$ 10.6	$(I_y)_z =$ 133.3
																	$J_z =$	143.9
																SUBTRACT	13.5	0.02
																	$(J'_z)_x =$ 130.4	$(J'_z)_y =$ 143.9

$\bar{x}_z = \frac{166.5}{200} = 0.833$ ft

$\bar{y}_z = -\frac{-3500}{400} = 8.75$ ft

$\bar{y}_z = \frac{3167}{333} = 9.50$ ft

$\bar{x}_z = -\frac{-1332}{200} = 6.66$ ft

$\bar{x}_y = \frac{4000}{400} = 10.00$ ft

$\bar{x}_y = -\frac{-500}{333} = 1.50$ ft

$V_x = v_{xo} - v_{xi}$

$V_{xa} = 1 - 1.328 = -0.328$ Kip

$V_{xd} = 0 - 1.328 = -1.328$ Kip

$v_{xi} = \frac{M'_x}{J_x} = \frac{32.6}{24.54} = 1.328$ Kip

$v_{yi} = \frac{M'_y}{J_y} = \frac{8.8}{215.9} = 0.04$ Kip

$v_{zi} = \frac{M'_z}{J_z} = \frac{3853}{125.6} = 3.07$ Kip

$V_y = v_{yo} - v_{yi}$

$V_{ya} = 0 - 0.04 = -0.04$ Kip

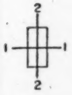
$V_{yd} = 0 - 0.04 = -0.04$ Kip

$V_z = v_{zo} - v_{zi}$

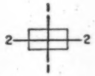
$V_{za} = 5.00 - 3.07 = 1.93$ Kip

$V_{zd} = 0 - 3.07 = -3.07$ Kip

Fig. 12.—Space Structure



SECTION a-a



SECTION b-b

$K_{n1} = 3 \times 10^8$ lb ft^2
$K_{n2} = 1 \times 10^8$ lb ft^2
$K_t = 1 \times 10^8$ lb ft^2

MEMBER AB
$K_{yy} = K_t = 1 \times 10^8$ lb ft^2
$K_{xx} = K_{n1} = 3 \times 10^8$ lb ft^2
$K_{zz} = K_{n2} = 1 \times 10^8$ lb ft^2

MEMBER BC
$K_{yy} = K_{n1} = 3 \times 10^8$ lb ft^2
$K_{xx} = K_t = 1 \times 10^8$ lb ft^2
$K_{zz} = K_{n2} = 1 \times 10^8$ lb ft^2

MEMBER CD
$K_{yy} = K_{n1} = 3 \times 10^8$ lb ft^2
$K_{xx} = K_{n2} = 1 \times 10^8$ lb ft^2
$K_{zz} = K_t = 1 \times 10^8$ lb ft^2

$(i_y)_x + z^2 da_{yy}$ $\times 10^7$	$(i_z)_x + y^2 da_{zz}$ $\times 10^7$	$(i_z)_y + x^2 da_{zz}$ $\times 10^7$	$(i_x)_y + z^2 da_{xx}$ $\times 10^7$	$(i_z)_{xy} + xy\,da_{zz}$ $\times 10^7$	$(i_x)_{yz} + yz\,da_{xx}$ $\times 10^7$	$(i_y)_{zx} + zx\,da_{yy}$ $\times 10^7$	$m_{xo} da_{xx}$ $\times 10^7$	$m_{yo} da_{yy}$ $\times 10^7$	$m_{zo} da_{zz}$ $\times 10^7$	$z_1 m_{yo} da_{yy}$ $\times 10^7$	$y_1 m_{zo} da_{zz}$ $\times 10^7$	$x_1 m_{zo} da_{zz}$ $\times 10^7$	$z_1 m_{xo} da_{xx}$ $\times 10^7$	$y_1 m_{xo} da_{xx}$ $\times 10^7$	$x_1 m_{yo} da_{yy}$ $\times 10^7$
0	33.3	0	0	0	0	0	-8,33	50.00	5	0	16.6	0	0	-27.7	0
0	200.0	266.6	0	200	0	0	0	833	0	0	0	0	0	0	27,7
11,10	100.0	400.0	33.3	200	50	33,3	0	0	0	0	0	0	0	0	0
11,10	333.3	666.6	33.3	400	50	33.3	P_x = -8.33	P_y = 58,33	P_z = 5	0	16.6	0	0	-27,7	27,7
1,39	306.4	400.0	7.5	350	47.5	11,1	SUBTRACT			48,6	43.8	50	-12,5	-79.1	388.0
$(I_y)_x$ = 9.71	$(I_z)_x$ = 26.9	$(I_z)_y$ = 266.6	$(I_x)_y$ = 25.8	$(I_z)_{xy}$ = 50	$(I_x)_{yz}$ = 2.5	$(I_y)_{zx}$ = 22.2				-48.6	-27.2	-50	12,5	51.4	-360.3
J_x = 36.6		J_y = 292.4		J_{xy} = -50	J_{yz} = -2.5	J_{zx} = -22.2				M_x = -21,4		M_y = -62,5		M_z = 411,7	
8.55	3.4	0.04	68.3	0.39	30.3	0.43	SUBTRACT			10.8 + (-64.8) = -54.0		-103.8 + 32.5 = -71.3		17.25 + 9.16 = 26.41	
$(J'_x)_y$ = 28.05	$(J'_x)_z$ = 33.2	$(J'_y)_z$ = 292.4	$(J'_y)_x$ = 224.1	J'_{xy} = -50.4	J'_{yz} = -32.8	J'_{zx} = -22.6				M'_x = 32,6		M'_y = 8.8		M'_z = 385,3	

$$J'_x = 36.6 - (-50)\frac{-50.4}{292,4} - (-22.2)\frac{-22.6}{143,9} = 24,54$$

$$J'_y = 292,4 - (-2.5)\frac{-32.8}{130,4} - (-50)\frac{-50.4}{33.2} = 215,9$$

$$J'_z = 143,9 - (-22.2)\frac{-22.6}{28,05} - (-2.5)\frac{-32.8}{224.1} = 125,6$$

$$m_{xi} = \frac{P_x}{A_x} + \frac{M_z}{J_z} y_{cz} - \frac{M_y}{J_y} z_{cy}$$

$$m_{xio} = \frac{-8,33}{3.33} + 3,07(-9,5) - 0,04(-1,5) = -31,60'\text{K}$$

$$m_{xid} = \frac{-8,33}{3,33} + 3.07(0,5) - 0,04(8,5) = -1,30'\text{K}$$

$$m_{yi} = \frac{P_y}{A_y} + \frac{M'_x}{J'_x} z_{cx} - \frac{M'_z}{J'_z} x_{cz}$$

$$m_{yio} = \frac{58,33}{2} + 1,328(-0,833) - 3,07(-6,66) = 48,50'\text{K}$$

$$m_{yid} = \frac{58,33}{2} + 1,328(9,167) - 3,07(13,34) = 0,39'\text{K}$$

$$m_{zi} = \frac{P_z}{A_z} + \frac{M'_y}{J'_y} x_{cy} - \frac{M'_x}{J'_x} y_{cx}$$

$$m_{zio} = \frac{5}{4} + 0,04(-10) - 1,328(-8,75) = 12,48'\text{K}$$

$$m_{zid} = \frac{5}{4} + 0,04(10) - 1,328(1,25) = -0,01'\text{K}$$

$$m_x = m_{xo} - m_{xi}$$

$$m_{xo} = -50 - (-31,60) = -18,40'\text{K}$$

$$m_{xd} = 0 - (-1,30) = 1,30'\text{K}$$

$$m_y = m_{yo} - m_{yi}$$

$$m_{yo} = 50 - 48,50 = 1,50'\text{K}$$

$$m_{yd} = 0 - 0,39 = -0,39'\text{K}$$

$$m_z = m_{zo} - m_{zi}$$

$$m_{zo} = 10 - 12,48 = -2,48'\text{K}$$

$$m_{zd} = 0 - (-0.01) = 0,01'\text{K}$$

WITH CONCENTRATED LOADS

$$\left.\begin{aligned} m_{xi} &= \frac{\mathcal{P}'_x}{\mathcal{A}'_x} + \frac{\mathcal{M}'_z}{\mathcal{J}'_z} y_{cz} - \frac{\mathcal{M}'_y}{\mathcal{J}'_y} z_{cy} \\ m_{yi} &= \frac{\mathcal{P}'_y}{\mathcal{A}'_y} + \frac{\mathcal{M}'_x}{\mathcal{J}'_y} z_{cx} - \frac{\mathcal{M}'_z}{\mathcal{J}'_z} x_{cz} \\ m_{zi} &= \frac{\mathcal{P}'_z}{\mathcal{A}'_z} + \frac{\mathcal{M}'_y}{\mathcal{J}'_y} x_{cy} - \frac{\mathcal{M}'_x}{\mathcal{J}'_x} y_{cx} \end{aligned}\right\} \qquad (17)$$

The form of Eqs. 16 and 17 resulted from a consideration of the concept that permits the interchange of a rotation with an analogous force, and a displacement with an analogous moment. In these expressions, the $\mathcal{P}'$-terms represent rotations resulting from the assumed distributions of moments and may be considered to represent analogous forces. The $\mathcal{M}'$-terms represent displacements along centroidal axes due to the assumed distributions of moments and may be considered to represent analogous moments. In the same manner as before, the $\mathcal{A}'$-terms and the $\mathcal{J}'$-terms represent rotations and displacements caused by moments and shears applied along the centroidal axes. The expressions for these rotations and displacements resemble expressions for areas and inertias.

Figs. 10 and 11 indicate, in tabular form, a convenient sequence for computing moments and shears in a structure curved in space and continuous between two supports. This form also serves as a summary of all terms involved in the analytical procedure. Inspection of these figures reveals that, in the special case of a structure lying in a plane, the expressions for the correction moments and shears become those of the column analogy or those of the shear and torsion analogy. Inspection also reveals that all script terms resulted from introducing the t-terms and f-terms in Eqs. 15. In numerical studies, the t-terms and f-terms are frequently negligible. In such cases, the script terms need not be computed and, consequently, Eqs. 18 and 19 may be used in computing the correction shears and moments:

$$\left.\begin{aligned} v_{xi} &= \frac{M'_x}{J'_x} \\ v_{yi} &= \frac{M'_y}{J'_y} \\ v_{zi} &= \frac{M'_z}{J'_z} \end{aligned}\right\} \qquad (18)$$

$$\left.\begin{aligned} m_{xi} &= \frac{P'_x}{A'_x} + \frac{M'_z}{J'_z} y_{cz} - \frac{M'_y}{J'_y} z_{cy} \\ m_{yi} &= \frac{P'_y}{A'_y} + \frac{M'_x}{J'_x} z_{cx} - \frac{M'_z}{J'_z} x_{cz} \\ m_{zi} &= \frac{P'_z}{A'_z} + \frac{M'_y}{J'_y} x_{cy} - \frac{M'_x}{J'_x} y_{cx} \end{aligned}\right\} \qquad (19)$$

Numerical Example.—Fig. 12 shows a structure consisting of three prismatic members, each of which is parallel to a rectangular axis. This structure is fixed at ends A and D, and is subjected to two concentrated loads. The assumed distributions of moments are for a structure cantilevered from end A.

The appropriate properties and the analogous forces and moments are tabulated as shown in Figs. 10 and 11. Since the members are parallel to the rectangular axes, many operations indicated in the latter figures need not be performed. For example, all operations involving terms written in script form may be omitted. In computing for segments of finite length the value of such a term as $(I_x)_z$, it may be necessary to add a correction $(i_x)_z$ to the value of $y^2\,da_{xx}$. This correction becomes necessary if the distance to each differential element of a finite segment is not the same as the tabulated y-distance to the centroid of the segment. Corrections such as these may be computed as moments and products of inertia of the elastic areas about their own centroidal axes. Computations for the final moments and shears at fixed ends A and D are shown at the bottom of Fig. 12. Final curves of moments may be drawn by combining the assumed distributions of moments with the planar distributions of correction moments.

If the effects of temperature changes are of interest, moments and shears in the structure may be computed in the same way as indicated in Fig. 12. The computations for the properties of the structure are exactly the same as before. The analogous moments M_x, M_y, and M_z are again equal to displacements in the x-direction, the y-direction, and the z-direction, respectively. However, these terms now represent the displacements of end D with respect to end A which would occur if the structure was free to expand. In this case, the analogous forces P_x, P_y, and P_z are equal to zero. The succeeding computations for moments and shears in the structure are identical in sequence to those indicated in Fig. 12. If the segments are not parallel to rectangular axes, the general form shown in Figs. 10 and 11 should be used.

SUMMARY

It has been shown that a shear and torsion analogy may be used in analyzing any plane structure continuous between two supports and loaded normal to its plane. The results may be interpreted in terms of a pressure line concept—that is, the balance of angle changes in a closed circuit. This balance may be conveniently checked by substituting computations of statics for those of geometry.

It has also been shown that structures curved or segmental in space and continuous between two supports may be analyzed by the methods presented under the heading, "A General Procedure for Structures Curved in Space." The shear and torsion analogy and the column analogy are special cases of this procedure.

ACKNOWLEDGMENT

The writers wish to acknowledge their indebtedness to Mr. Cross. It was he who suggested that, for structures lying in a plane and loaded normal to their plane, there is merit in drawing curves of moments along rectangular axes.

DISCUSSION

UKU MÜLLERSDORF[21].—The ingenious method of calculation described in this paper is of great interest. In the writer's opinion it constitutes a valuable contribution to the science of calculation of complicated structures, especially plane structures submitted to the action of lateral loads.

The method seems to have only one disadvantage, and that is, it implies many purely mechanical calculating operations. As a consequence there is the risk that the designer may, so to say, lose contact with the structure and its mode of action. This may result in possible errors of calculation going undiscovered until a relatively late stage.

As previously stated, the great merit of this method appears to be in connection with relatively complicated structural problems such as arches with variable rigidity. In simpler cases some other method might give a better conspectus and in many cases shorten the calculation. In this connection the method of successive approximations may be suggested to be used in accordance with the same principles that are applied when calculating the action of loads in the principal plane of the structure. The method is particularly suitable if the loads are stationary. In case of moving loads, or if influence line diagrams are to be drawn, the method suggested by the authors will probably be more appropriate. In the latter case other methods, as for instance the method of primary moments suggested by A. Efsen,[22] may also be contemplated. Originally presented as a method of solving plane problems, with certain amplifications it may also be used for the type of loads investigated by the authors. The writer has used this method in several instances, once even in the investigation of an arch structure in which the influence of the deck structure also was taken into consideration. The constants necessary for the use of this method are first calculated and thereafter used wherever loads occur. This method, as well as the method of successive approximations, may of course be applied to continuous systems.

Concerning arches, it is to be noted that by applying loads (especially if the structure is subjected to lateral loads) a combined action is established between the arch and the deck structure. As the latter generally has a great lateral rigidity, a considerable part of the load is transferred by it. However, in this case the arch may be considered as the fundamental system, and it would be very appropriate, in cases in which great accuracy is required, to calculate this system by the method suggested in the paper. In any special case, however, the lateral deformations must not give rise to perceptible additional moments, as theoretically such a system is variable in reference to lateral as well as to vertical loads. If a deformation occurs perpendicular to the principal plane of the structure, additional moments are caused even by loads in the plane of the system, just as a possible oblique position of the columns between the deck structure and the arch may give rise to additional lateral loads.

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[21] Structural Engr., Stockholm, Sweden.

[22] "Die Methode der primären Momente," by A. Efsen, Copenhagen, Denmark, 1931.

Maurice Barron,[23] M. ASCE.—Under the heading "A General Procedure for Structures Curved in Space," the authors outline a broad application of their procedure which includes "* * * any shape in space and any variation in cross section along its length * * *" and "* * * subjected to loads in any direction and to moments about any axis." This is a very broad application which covers all space structures.

The skewed rigid-frame bridge and the skewed arch are space structures and they are presumably included in the category of space structures which the authors state can be analyzed by their proposed method. It is proper, then, to compare the author's proposed method with the large amount of published material available for solid barrel skewed structures.

The writer initially calls attention to the theory, equations, and method of analysis proposed in 1924[24] by J. Charles Rathbun, M. ASCE. The counterpart of each stress and each redundant in Eq. 12, and the elastic equations, Eq. 15, may be found by such a comparison. Therefore, all the research,[25] test data,[26,27] samples of design, and the limitations, which have been collected over 30 years may be used to properly evaluate the proposed methods.

One important observation is immediately apparent. The authors assume three axes in space which are mutually perpendicular (as does Mr. Rathbun). Elastic equations (the same as those proposed by Mr. Rathbun) are then written and solved simultaneously to determine the redundant reactions. Although this procedure is theoretically correct, it has been shown[28,29] that the elastic system about each axis may be treated independently. Furthermore, the elastic system about the X-axis is a primary one, and the elastic systems about the other two axes are of such a lower order that it is impossible to solve the equations simultaneously with any degree of confidence.[30] The difficulty encountered due to small differences may not hold for the idealized problem which the authors present, but the writer has found[31] that the more idealized the problem, the greater is the difference in order between the primary elastic system and the secondary system. For idealized problems most of the effects of the secondary system on the primary system vanish completely. The initiated analyst will recognize this important fact as a principle instead of parts of equations which become insignificant (or vanish entirely) in the solution of several simultaneous equations.

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[23] Farkas & Barron, Cons. Engrs., New York, N. Y.

[24] "Analysis of the Stresses in the Ring of a Concrete Skew Arch," by J. Charles Rathbun, *Transactions*, ASCE, Vol. LXXXVII, 1924, p. 611.

[25] "Effect of Skew Angle on Rigid-Frame Reactions," by Walter C. Boyer, *Transactions*, ASCE, Vol. 116, 1951, p. 1265.

[26] "An Experimental Study of the Reactions of a Two-Span Skewed Rigid Frame Bridge," by Gordon B. Fisher, thesis presented to The Johns Hopkins University, at Baltimore, Md., in partial fulfilment of the requirements for a degree in 1948.

[27] "The Use of Models in the Solution of Indeterminate Structures," by George E. Beggs, *Journal of the Franklin Institute*, March, 1927, p. 375.

[28] "Reinforced Concrete Skewed Rigid-Frame and Arch Bridges," by Maurice Barron, *Transactions*, ASCE, Vol. 116, 1951, p. 999.

[29] "The Rigid-Frame Bridge," by Arthur G. Hayden and Maurice Barron, John Wiley & Sons, Inc., New York, N. Y., 1950.

[30] Discussion by Leo Sos of "Reinforced Concrete Skewed Rigid-Frame and Arch Bridges," by Maurice Barron, *Transactions*, ASCE, Vol. 116, 1951, p. 1042.

[31] Discussion by Maurice Barron of "Effect of Skew Angle on Rigid-Frame Reactions," by Walter C. Boyer, *ibid.*, p. 1278.

FRANK BARON[32] and JAMES P. MICHALOS,[33] ASSOCIATE MEMBERS, ASCE.—The writers appreciate the constructive discussions of the paper. Mr. Müllersdorf has raised several questions of concern to designers. He seems to be disturbed that the procedure advocated in the paper apparently has the disadvantage of implying many purely mechanical computations. This disadvantage does not exist; and, contrary to Mr. Müllerdorf's belief, the procedure permits the designer to retain the picture of structural action. The procedure is flexible and meets the requirements of the designer as well as those of the analyst. It lends itself to the conduct of either informal qualitative studies of structural behavior or of more formal quantitative studies. For this reason, particular attention has been given to the possible interpretations of the various terms appearing in the formal statement of the analytical procedure.

In addition, the results are interpreted in terms of a pressure line concept. A procedure of sketching, similar to that used for an arch with loads in the plane of the arch,[19,20] may be developed for drawing approximate diagrams of moments about the x, y, and z axes of laterally loaded plane structures and structures curved in space. Such a procedure has been indicated in the discussions of Figs. 6, 7, and 12. It consists of drawing the curves of moments in two installments. First, draw a statically possible distribution of moments about the x, y, and z axes of the structure. Such a distribution may be shown in three separate diagrams, each diagram being associated with an axis. Then draw a planar distribution of correction moments and check the requirements of geometry. A planar distribution of correction moments is represented by three straight lines, one line in each of the above diagrams. The requirements of geometry are satisfied if the angle changes associated with the moments are balanced. Additional illustrations of moments drawn in this way may be inspected elsewhere.[34]

As noted by Mr. Müllersdorf, the procedure can be used in obtaining constants required in methods of successive approximations for analyzing continuous structures. The procedure can also be used in studies of arch structures in which the influence of the deck structure is to be taken into account. For such cases, the additional requirements of geometry must be considered. In certain cases the procedure can be used in obtaining the additional moments caused by changes in the dimensions of a structure. The additional moments can be obtained by a numerical method of successive approximations provided the computations converge and are not prohibitive in number.

The writers are not in agreement with Mr. Barron's interpretation of the general procedure for structures curved in space nor with his conclusions concerning the sensitivity of computations for such structures as indicated by previous studies of skewed rigid-frame bridges or skewed arches.

Mr. Barron calls attention to the theory, equations, and methods of analysis proposed in 1924[24] by Mr. Rathbun for the analysis of skewed rigid frames and arches. Although a reasonable facsimile of each stress and each redundant in Eq. 12 and Eq. 15 may be found in corresponding equations of Mr. Rathbun's paper, the resemblance is purely coincidental and ceases at that stage. The facsimile occurs since in each paper six equations of statics and six equations of geometry are written. Mr. Barron's remarks concerning the resemblance are subject to possible misinterpretations. The writers did not set up six simultaneous equations and solve them solely to obtain the redundants in a skewed frame or arch.

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[32] Prof., Dept. of Civ. Eng., Technological Inst., Northwestern Univ., Evanston, Ill.

[33] Associate Prof., Dept. of Civ. Eng., Iowa State College, Ames, Iowa.

[34] "Effects of Lateral Loads on Arches," by James P. Michalos, *Journal*, A.C.I., Vol. 22, January, 1951, p. 377.

It is repeated that the general procedure of the paper can be used for determining moments and shears in a structure curved or segmental in space and continuous between two supports. The structure may have any shape in space, any variation in cross section along its length, and it may be subjected to loads in any direction and moments about any axis. The procedure is general, and for structures lying in a plane it reduces to the Hardy Cross column analogy or to the shear and torsion analogy of the paper.

If objections to the assumptions and limitations listed by Mr. Rathbun for the analysis of skewed frames are removed and if the relationships between moments and angle changes for a skewed element are considered acceptable, the skewed rigid-frame and the skewed arch can be classified as special examples of "structures curved in space." In that case, the concepts and tabular forms of the writers' general procedure are applicable to the analysis of such structures.

As previously stated, the writers have not concerned themselves with uncertainties in the relationships between statics and geometry caused by the characteristics of a material. It was assumed that the properties necessary for computing the relationships between statics and geometry were known. For the use of the general procedure in the analysis of skewed rigid frames or arches, it is then desirable to summarize the acceptable relationships between moments and angle changes of skewed elements in space.

Mr. Barron's further remarks leave the impression that several problems of algebra remain in obtaining solutions to the six simultaneous equations of geometry. It is emphasized that the philosophy and the various concepts introduced in the paper are independent of the precise analytical procedure. The concept of the pressure line and the concept that permits the interchange of computations of geometry with those of statics were employed by the authors in stating, restating, and obtaining an algebraic solution to the six complicated equations of geometry. It is noted, however, that the computer's task has been simplified to a purely arithmetical task. In addition, the designer's task has been simplified. For his needs as well as those of the computer, each term of the analytical procedure has been interpreted and restated. However, by means of the concepts introduced in the paper, the designer should be enabled to visualize the important aspects of structural action, to inspect the important variables, and, if desired, to obtain quick and reasonable estimates of quantities involved without the necessity of conducting lengthy arithmetical computations.

The problem of algebra concerning the sensitivity of computations caused by small differences in the coefficients of the six simultaneous equations does not exist in the sense nor to the degree indicated by Mr. Barron. It is unfortunate that Mr. Barron's conclusions concerning the sensitivity of such computations have been extrapolated from experience with the analysis of skewed frames or arches. In connection with this point the following is emphasized:

1. The precise analysis is purely arithmetical. In the analysis any convenient set of rectangular axes can be chosen and any statically possible distribution of moments can first be assumed. It should be obvious that the better the guess, the smaller the correction.
2. The numerical example in Fig. 12 is not at all an idealized problem. It is a bent in space, such as may be necessary for supporting urban elevated freeway construction. It is observed that bending about all axes is important.
3. The skewed frame or arch is, because of its relatively big width of deck, the special problem. Consequently, greatly unbalanced stiffness ratios exist. The effect of such unbalanced ratios on the $d\,a$ values and on succeeding computations can be seen by inspection of the tabular form. The inspection would indicate which operations could be neglected without materially affecting the results. This only emphasizes the versatility of this method of analysis.

In conclusion, the analytical procedures can be formal or informal. They can be numerical or pictorial and can be exact or approximate, depending on the needs of the person conducting the analysis.

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TRANSACTIONS

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TRUSS DEFLECTIONS BY THE COORDINATE METHOD

By Kuang-Han Chu,[1] J. M. ASCE

With Discussion by Messrs. L. C. Maugh, and Kuang-Han Chu

Synopsis

The method presented here is an algebraic solution of the Williot-Mohr diagram. By suitable arrangement in tabular forms, and by adopting a set of simple sign conventions, the work is minimized in such a manner as to make this method preferable to the graphical method.

Introduction

There are many procedures for finding truss deflections. The virtual work method (the method of virtual velocity or dummy unit loading) may be used to find the movement of a single point in any direction; but to find the deflections at many points by this method is rather tedious. The elastic weight method (which has many variations and different names, such as the "influence line" method, the "bar chain" method, and the method of "angle changes") may be used to find the vertical components of deflections of the joints on a continuous chain of bars in a truss. Although the method can be modified to determine the horizontal components of movements by finding the influence lines due to a horizontal unit load, separate calculations must be made for the vertical and horizontal components. To find the true movements of all the joints of a truss, the elastic weight method may require as much time as the virtual work method. The Williot-Mohr method is the only simple direct procedure which determines the true (absolute) movements of all the truss joints. However, the method is a graphical one and its accuracy is limited. Sometimes one may have difficulty in selecting a proper scale and in orientating the diagram. The method presented here, which the writer has named the "coordinate method," is an algebraic solution of the Williot-Mohr diagram. Its application is simple and rapid. Vertical and horizontal components of deflections can be determined simultaneously for all the truss joints; and it can be carried to any degree of accuracy. The letter symbols used in this paper are defined where they first appear, in the text or by illustration.

Note.—Published in January, 1951, as *Proceedings-Separate No. 54.* Positions and titles given are those in effect when the paper or discussion was received for publication.

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PART I. PRINCIPLES OF THE METHOD AND WORKING PROCEDURE

ALGEBRAIC SOLUTION OF THE WILLIOT DIAGRAM

Let δ be the change of length of any member, caused by stress, temperature change, error, or deliberate change in manufacture. If the change is caused by stress,

$$\delta = \frac{S\,L}{A\,E} \qquad (1)$$

in which S is the total stress in a member; L is the length of the member; A is the cross-sectional area; and E is the modulus of elasticity of its material. The symbol δ should have a plus sign for elongation and a minus sign for shortening. For any member PQ the deformation δ is designated as δ_{PQ}.

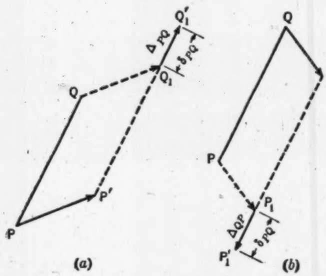


FIG. 1

Let Δ be the displacement of one end of a member, caused by the axial change in length δ of the member in its displaced position. For any member PQ, such a displacement is designated as Δ_{PQ} if the displaced position P′ of end P is known (see Fig. 1) and as Δ_{QP} if the displaced position Q′ of end Q is known. In Fig. 1, $P'Q_1$ and P_1Q' are parallel and equal to PQ; the quantity $Q_1Q'_1$ represents Δ_{PQ}; and $P_1P'_1$ represents Δ_{QP}. The deformation δ_{PQ} shown in Fig. 1 is an elongation. It should be noted that $\Delta_{PQ} \neq \Delta_{QP}$ but that $\Delta_{PQ} = -\Delta_{QP}$. However, $\delta_{PQ} = \delta_{QP}$. In other words, Δ is a vector and δ is a scalar quantity.

The displacement Δ for a member is determined by the following conditions:

(*a*) Its magnitude is numerically equal to the deformation δ of the member;

(*b*) Its slope is the same as the original slope of the member; and

(*c*) Its direction depends not only on the sign of δ but also on the relative position between the end whose displaced position is known and the end whose displaced position is to be determined.

The displacement Δ can also be defined by its x-component and y-component. The magnitude of each component is determined by conditions (*a*) and (*b*); and the sign of each component, by conditions (*b*) and (*c*). The x-component and y-component of any displacement Δ_{PQ} are designated as $(\Delta_{PQ})_x$ and $(\Delta_{PQ})_y$.

The following rules are suggested for the determination of the signs of Δ-components:

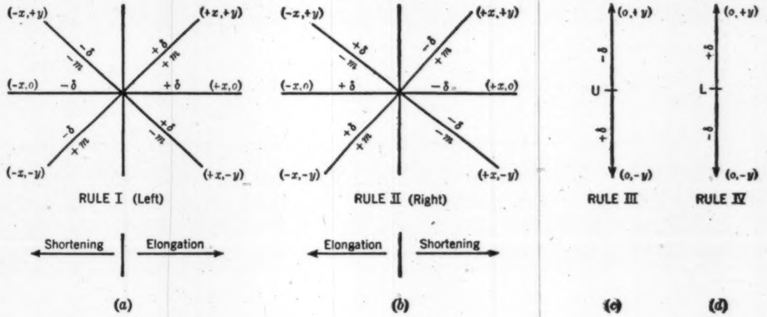
Rule I.—When the displaced position of the left end of a member is known, the components of Δ should bear signs depending on the sign of δ and the slope of the member as shown in Fig. 2(*a*), in which $+\delta$ means that the δ is positive (elongation) and $+m$ means that the slope is positive.

Rule II.—When the displaced position of the right end of a member is known, the Δ-components should bear signs depending on the sign of the deformation δ and the slope m of the member as shown in Fig. 2(*b*).

Rule III.—When the displaced position of the upper end of a vertical member is known, the y-component of Δ should bear a sign depending on the sign of the deformation δ of the member as shown in Fig. 2(*c*).

Rule IV.—When the displaced position of the lower end of a vertical member is known, the y-component of Δ should bear a sign depending on the sign of the corresponding deformation δ as shown in Fig. 2(*d*).

Generally, in a given truss, the new position of any joint can be determined, if, by means of a pair of bars, this joint is connected with two other joints whose displaced positions have been determined. (The new position of the joint is located at the intersection of the arcs drawn with the known new positions of the other two joints as centers and the new lengths of the connecting bars as radii.) Assume a bar to be fixed in direction and one of its ends to be fixed in position. The displaced position of this end is its original position. The displaced position of the other end is known when the deformation of the bar is known. Thus there are two points of known displaced positions to begin with.



(*a*) (*b*) (*c*) (*d*)

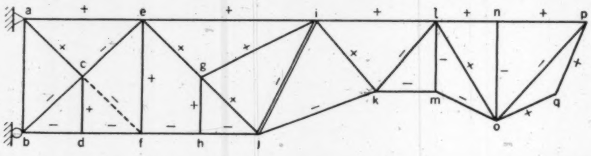
FIG. 2

Based on the rules given previously, the signs of Δ-components for all the members in a given truss can be determined easily. Fig. 3 illustrates all the possible cases. Although actually ab, Fig. 3, is the member of fixed direction and point a is the fixed point, it is assumed that member ij is the fixed member and point j, the fixed point. As point j is the left end of member ij, Rule I (with $-\delta_{ij}$ and a positive slope m of member ij) confirms that the x-component and the y-component of Δ_{ji} are both negative.

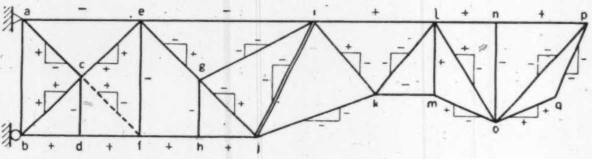
Consider first the members on the right side of member ij. With the displaced positions of joints i and j known, the displaced position of joint k can be determined as it is connected to joints i and j by the members ik and jk. Since points i and j are left ends of members ik and jk, respectively, Rule I is used for determining the signs of Δ-components. The same process is repeated for other joints and members.

For all the members on the right side of member ij except verticals and the member pq, the signs of Δ-components are governed by Rule I. The signs of y-components of Δ_{lm} and Δ_{on} are given by Rules III and IV, respectively, as ends l and o, determined by previous steps, are the upper and lower ends of member lm and member on, respectively. The signs of the components of Δ_{pq} are given by Rule II as the known end p is the right end.

Similarly, the signs of Δ-components for members on the left side of member ij can be determined. In this case Rule II applies to all members except verticals. Also, with the displaced positions of joints e and f known, the displaced position of joint c or joint d cannot be determined as neither is connected to points e and f by two members. Hence, a false member cf is added. The change of length of member cf is given by the relative movement of points c and f. The relative movement can be determined by virtual work. In using that method the only members that need to be considered are those forming the quadrilateral efdc. After the false member is inserted, the analyst proceeds as usual.



(a) SIGNS OF δ



(b) SIGNS OF Δ COMPONENTS

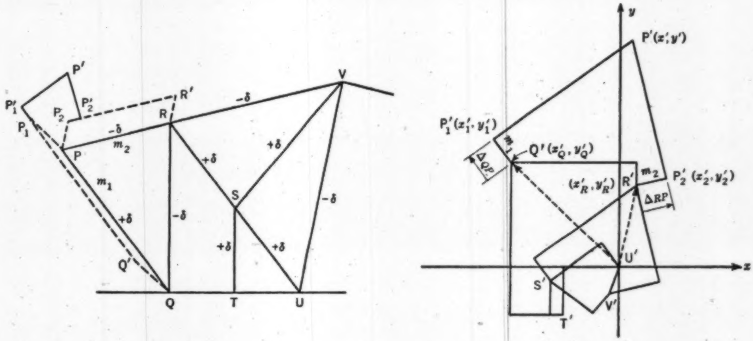
Fig. 3

There are seldom such exceptional members as pq and cf. In ordinary trusses, for all members except verticals, on the right side of the member assumed to be fixed in direction if it is inclined or vertical, or on the right side of the assumed fixed point if the assumed fixed member is horizontal, the signs of Δ-components are governed by Rule I, and those on the left side, by Rule II.

Let the joints on a given truss be A, B, C, etc., and let their displaced positions (corresponding points on the Williot diagram) be A′, B′, C′, etc., respectively. Fig. 4(*a*) shows a part of the truss; and Fig. 4(*b*), a part of the Williot diagram. Let joint U be assumed as fixed in position. The origin of the coordinate axes for the Williot diagram is taken at U′ which corresponds to point U, Fig. 4(*a*). For any point P, the coordinates are designated as x'_P and y'_P. These coordinates represent the components of the displacement from P to P′.

Let member UV be assumed as fixed in direction. Since one of its ends, U, is fixed in position, $x'_V = (\Delta_{UV})_x$ and $y'_V = (\Delta_{UV})_y$. With the displaced positions of points U and V (U′ and V′) known, the displaced positions of other joints can be located successively.

Consider a general case: A joint P is connected to joints Q and R by the bars PQ and PR (see Fig. 4). It is required to find the displaced position P′ of point P with the displaced positions (Q′ and R′) of Q and R known. Assume that joint P is disconnected. Consider the movement of end P of member QP. First joint P moves to point P_1; the displacement $\overrightarrow{PP_1}$ is equal to the displacement $\overrightarrow{QQ'}$ ($\overrightarrow{U'Q'}$ in the Williot diagram). Then point P_1 moves to point P'_1; the displacement $\overrightarrow{P_1P'_1}$ ($\overrightarrow{Q'P'_1}$ in Fig. 4(*b*)) is equal to Δ_{QP} which is parallel to member PQ. Then joint P'_1 moves along an arc with the new position of Q (point Q′) as center and the new length of PQ (line $Q'P'_1$) as radius. In a similar manner, end P of member RP moves first to point P_2, then to point P'_2, and then along an arc with point R′ as center and line $R'P'_2$ as radius.



(*a*) (*b*)

FIG. 4

The meeting of the two arcs determines the displaced position (P′) of joint P. Since the deformation of a member is very small as compared with its length, the arc of rotation is replaced by the tangent which is perpendicular to the radius vector parallel to the direction of the member.

Let the coordinates of P'_1 and P'_2 be x'_1, y'_1 and x'_2, y'_2, respectively. Then for joint P'_1,

$$x'_1 = x'_Q + (\Delta_{QP})_x \qquad (2a)$$

$$y'_1 = y'_Q + (\Delta_{QP})_y \qquad (2b)$$

and, for joint P'_2,

$$x'_2 = x'_R + (\Delta_{RP})_x \qquad (3a)$$

$$y'_2 = y'_R + (\Delta_{RP})_y \qquad (3b)$$

Eqs. 2 and 3 are to be applied to general cases and all terms in the formulas should have proper signs. Let the slopes of members PQ and PR be m_1 and m_2, respectively (slopes according to usual sign convention). The equations of the perpendiculars are

$$y' - y'_1 = -\frac{1}{m_1}(x' - x'_1) \qquad (4a)$$

and

$$y' - y'_2 = -\frac{1}{m_2}(x' - x'_2) \qquad (4b)$$

Solve Eqs. 4 simultaneously. The coordinates of P′ (that is, x', y', or x'_P, y'_P), representing the components of the displacement of P, are

$$x' = \frac{(y'_1 - y'_2) + \left(\frac{x'_1}{m_1} - \frac{x'_2}{m_2}\right)}{\frac{1}{m_1} - \frac{1}{m_2}} \ \left(\text{if } \frac{1}{m_1} = \infty, \text{ then } x' = x'_1\right) \qquad (5a)$$

and

$$y' = \frac{(x'_1 - x'_2) + (m_1 y'_1 - m_2 y'_2)}{m_1 - m_2} \ (\text{if } m_1 = \infty, \text{ then } y' = y'_1) \qquad (5b)$$

Eqs. 5 are applicable to all possible cases. The special cases $\frac{1}{m_1} = \infty$ and $m_1 = \infty$ apply to horizontal and vertical members, respectively.

For the practical solution of the problem, the procedure is as follows:

1. Compute and record on each member in the truss diagram the values of δ (+ for elongation and − for shortening).
2. Mark the member that is actually (or assumed to be) fixed in direction and mark one of its ends that is actually (or assumed to be) fixed in position. Write the sign conventions for Δ-components.
3. Draw another truss diagram and record on each member the following:
   (*a*) The slope m of the member and its reciprocal value, $1/m$ (plus or minus according to the usual convention.)
   (*b*) Vertical and horizontal components of Δ, with signs according to step 2.
4. Write the formulas to be used if necessary.
5. Applying Eqs. 2, 3, and 5, compile a table of the form described in Part II as Table 1.

ALGEBRAIC SOLUTION OF THE MOHR DIAGRAM AND ACTUAL DISPLACEMENTS OF TRUSS JOINTS

Five general principles for the solution of the Mohr diagram are as follows:

Principle 1.—Let F be an actual fixed point and let its corresponding points on the Williot and Mohr diagrams be F′ and F″, respectively. Then F′ and F″ are a common point.

Let any truss point P and its corresponding points on the Williot and Mohr diagrams be P′ and P″, respectively. Let the coordinates of P′ and P″ with respect to F′ or F″ be x'_{FP}, y'_{FP} and x''_{FP}, y''_{FP}, respectively. If the Williot diagram is started from an assumed fixed point U instead of the actual fixed point F and the coordinates of P′ and P″ with respect to U′ are x'_P, y'_P and x''_P, y''_P, respectively, then the following relations can be established:

$$x'_{FP} = x'_P - x'_F;\ y'_{FP} = y'_P - y'_F \qquad (6a)$$

and

$$x''_P = x''_{FP} + x''_F;\ y''_P = y''_{FP} + y''_F \qquad (6b)$$

In Eqs. 6, $x'_F = x''_F$ and $y'_F = y''_F$. Also, if the Williot diagram starts from the actual fixed point F, then $x'_F = x''_F = y'_F = y''_F = 0$.

Principle 2.—Let point M be constrained to move in a fixed direction. The direction of motion may be given, such as the case of a roller; or it may be determined by known conditions, such as the case of the crown hinge of a three-hinged arch. Let M′ and M″ be its corresponding points on the Williot and Mohr diagrams, respectively. Then point M″ can be located when the positions of points M′ and F′ are known.

Consider first the case in which the direction of motion of point M is given. According to the principle of the Mohr diagram, the actual movement of point M is represented by the displacement from M″ to M′ (that is, $\overrightarrow{M''M'}$) which is along the known direction. The displacement $\overrightarrow{M''M'}$ is obtained by the vectorial relation $\overrightarrow{M''M'} = \overrightarrow{F'M'} - \overrightarrow{F'M''}$ in which $\overrightarrow{F'M'}$ is the displacement of point M with respect to point F as given by the Williot diagram and $\overrightarrow{F'M''}$ is the rotational displacement of point M about point F.

Hence point M″ should lie on the line through point M′ and parallel to the known direction and it should lie on the line (arc) through point F′ and perpendicular to line FM. Let the slope of the line of known direction of motion be m_o and the slope of the line normal to that direction be m'_o, then

$$m'_o = -\frac{1}{m_o} \qquad (7)$$

The coordinates of point M″ can be found by applying Eqs. 5 with the coordinates of point M′ as x'_1, y'_1, the coordinates of point F′ as x'_2, y'_2, the slope m'_o as m_1, and the slope of line FM as m_2.

Consider next the case in which the direction of motion is determined by known conditions. For example, a three-hinged arch is shown in Fig. 5. The usual graphical method of finding the movement of the crown hinge is as follows: Beginning at a common point a′,e′ (because points a and e are given fixed points), two Williot diagrams are drawn for two halves of the arch—one for the left half, assuming that line ai remains vertical, and the other for the right half, assuming that line eg remains vertical. The positions of points c and d are given as points c′ and d′, respectively, in the Williot diagram. However, c and d represent a common point; hence c must lie on the line (arc) through c′ and perpendicular to ac and d on the line (arc) through d′ and perpendicular to ed. These lines intersect at the common point c‴,d‴ and $\overrightarrow{a'c'''}$ or $\overrightarrow{e'd'''}$ gives the true movement of the common point c,d. Point c″

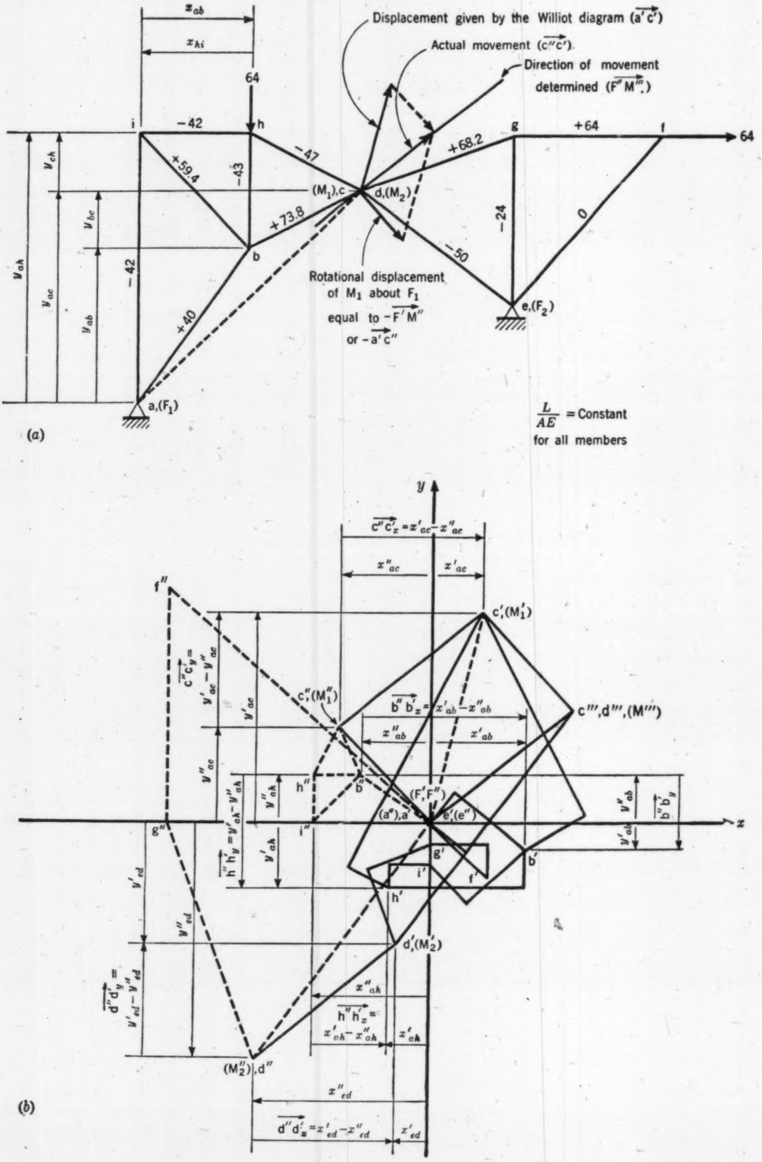

x_{ab}
x_{hi}
64
i
−42
h
−43
−47
+59.4
y_{ch}
y_{bc}
y_{ah}
y_{ac}
y_{ab}
(M$_1$),c
d,(M$_2$)
+73.8
b
−42
+40
a,(F$_1$)
Displacement given by the Williot diagram ($\overrightarrow{a'c'}$)
Actual movement ($\overrightarrow{c''c'}$)
Direction of movement determined ($\overrightarrow{F''M'''}$)
+68.2
g
+64
f
64
−24
0
−50
e,(F$_2$)
Rotational displacement of M$_1$ about F$_1$ equal to $-\overrightarrow{F'M''}$ or $-\overrightarrow{a'c''}$
$\frac{L}{AE}$ = Constant for all members
(a)
y
$\overrightarrow{c''c'_x} = x'_{ac} - x''_{ac}$
x''_{ac}
x'_{ac}
f''
c',(M$_1'$)
$\overrightarrow{c''c'_y} = y'_{ac} - y''_{ac}$
y'_{ac}
c'',(M$_1''$)
$\overrightarrow{b''b'_x} = x'_{ab} - x''_{ab}$
x''_{ab}
x'_{ab}
c''',d''',(M''')
y''_{ac}
h''
b''
(F',F'')
y''_{ah}
$\overrightarrow{h''h'_y} = y'_{ah} - y''_{ah}$
(a''),a'
e',(e'')
y''_{ab}
$\overrightarrow{b''b'_y}$
x
g''
i''
y'_{ah}
g'
y'_{ab}
b'
i'
f'
h'
y'_{ed}
y''_{ed}
d',(M$_2'$)
$\overrightarrow{d''d'_y} = y'_{ed} - y''_{ed}$
x''_{ah}
$\overrightarrow{h''h'_x} = x'_{ah} - x''_{ah}$
x'_{ah}
(M$_2''$),d''
x''_{ed}
$\overrightarrow{d''d'_x} = x'_{ed} - x''_{ed}$
x'_{ed}
(b)

FIG. 5

on the Mohr diagram (corresponding to point c′ on the Williot diagram) must lie on a line through a′ and perpendicular to ac, and on a line through c′ parallel to the known direction of motion as given by $\overrightarrow{a'c'''}$. Similarly, point d″ (corresponding to point d′) can be located.

The algebraic solution of the foregoing problem can be worked out according to the same principle as the graphical method. For instance, with the coordinates of points c′ and d′ known and the slopes of lines ac and cd given, the coordinates of the common point c‴,d‴ can be found by applying Eqs. 5. Referring to the common origin a′,e′ as F′ and the common point c‴,d‴ as M‴, then $\overrightarrow{F'M'''}$ represents the true movement of the crown hinge. The slope of the normal to the displacement $\overrightarrow{F'M'''}$ can be determined by its components $\overrightarrow{F'M'''_x}$ and $\overrightarrow{F'M'''_y}$ (signs of components referring to F′ as origin), as follows:

$$m'_o = -\frac{\overrightarrow{F'M'''_x}}{\overrightarrow{F'M'''_y}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Following the same procedure as established for the previous case with a point of given direction of motion, the coordinates of points c″ and d″ can be determined. For instance, the coordinates of point c″ can be found by applying Eqs. 5 with the coordinates of points c′ and a′ as x'_1, y'_1 and x'_2, y'_2, respectively, and the slope m'_0 as m_1 and the slope of line ac as m_2.

Principle 3.—The coordinates x''_{FP}, y''_{FP} of any point P″ on the Mohr diagram with respect to point F″ (coinciding with point F′) can be found when the coordinates x''_{FM}, y''_{FM} are known. The coordinates x''_{FM} and y''_{FM} represent, respectively, $\overrightarrow{F'M''_x}$ and $\overrightarrow{F'M''_y}$, the x-component and the y-component of the displacement $\overrightarrow{F'M''}$. Let the coordinates of any point P in the truss diagram with respect to the vertical and horizontal axes through point F be x_{FP} and y_{FP}. For point M, the coordinates are x_{FM} and y_{FM}.

Calculate the "angle of rotation" θ by the following formula:

$$\theta = \frac{\overrightarrow{F'M''_y}}{x_{FM}} = \frac{y''_{FM}}{x_{FM}} = -\frac{\overrightarrow{F'M''_x}}{y_{FM}} = -\frac{x''_{FM}}{y_{FM}} \quad \ldots\ldots\ldots\ldots (9)$$

(In Eq. 9, angle θ is considered as positive when the rotation is counterclockwise.) Consider a radius vector FM with positive slope (that is, x_{FM} and y_{FM} are both positive) rotating about its left end F counterclockwisely through an angle θ. The displacement of the right end as represented by $\overrightarrow{F'M''}$ (an arc of rotation considered to be perpendicular to the radius vector) will be upward (positive $\overrightarrow{F'M''_y}$) and toward the left (negative $\overrightarrow{F'M''_x}$). Hence the sign for the term $\overrightarrow{F'M''_y}/x_{FM}$ is positive and the sign for the term $\overrightarrow{F'M''_x}/y_{FM}$ is negative.

The coordinates of truss points with respect to F can be written directly by consulting the truss diagram. However, sometimes it may be convenient to calculate these coordinates successively from point to point. If Q and P are two consecutive points, then

$$x_{FP} = x_{FQ} + x_{QP};\; y_{FP} = y_{FQ} + y_{QP} \quad \ldots\ldots\ldots\ldots\ldots (10)$$

in which x_{QP} and y_{QP} are projections of the length of the member QP. They have algebraic signs with respect to the vertical and horizontal axes through Q.

The rotational displacement for any point about F is equal to the radius vector from F to that point times the angle of rotation. Thus, for any point P,

$$x''_{FP} = \overrightarrow{\text{F}'\text{P}''_x} = -\,\theta\, y_{FP} \quad \text{(11a)}$$

and

$$y''_{FP} = \overrightarrow{\text{F}'\text{P}''_y} = +\,\theta\, x_{FP} \quad \text{(11b)}$$

Eqs. 11 are based on the same principle as Eq. 9.

Knowing the values x''_{FP} and y''_{FP}, the coordinates x''_P y''_P of the point P″ with respect to the assumed fixed point U′ can be determined by Eq. 6*b*.

Principle 4.—The actual movement of a point is given by the vector from the corresponding point on the Mohr diagram to the corresponding point on the Williot diagram. It can also be obtained by subtracting the rotational displacement from the displacement determined by the Williot diagram with respect to point F. Hence, for any point P, the components of the actual movement are $\overrightarrow{\text{P}''\text{P}'_x}$ and $\overrightarrow{\text{P}''\text{P}'_y}$ as given by the following formulas:

$$\overrightarrow{\text{P}''\text{P}'_x} = x'_P - x''_P;\ \overrightarrow{\text{P}''\text{P}'_y} = y'_P - y''_P \quad \text{(12a)}$$

or

$$\overrightarrow{\text{P}''\text{P}'_x} = x'_{FP} - x''_{FP};\ \overrightarrow{\text{P}''\text{P}'_y} = y'_{FP} - y''_{FP} \quad \text{(12b)}$$

Principle 5.—In certain special cases, the algebraic solution of the Mohr correction diagram can be much simplified. Let L_i and U_i be, respectively, the lower and upper panel points of an ordinary truss, in which $i = 0, 1, 2, 3, \ldots, n$. If L_o at the left end is a fixed point and L_n at the right end is movable in horizontal direction only, then (see Fig. 6)

$$\theta' = \frac{y'_{Ln} - y'_{Lo}}{\overline{L_o\, L_n}} \quad \text{(13)}$$

If, in addition to the foregoing conditions, the lower chord is horizontal and the upper chord U_hU_i in the panel *h-i* (panel length = $\overline{\text{L}_h\text{L}_i}$ and $h = i - 1$) has a slope equal to m_{ci} (the end post also being considered as an upper chord), then (see Fig. 6)

$$x''_{Li} = x'_{Lo} \quad \text{(14a)}$$

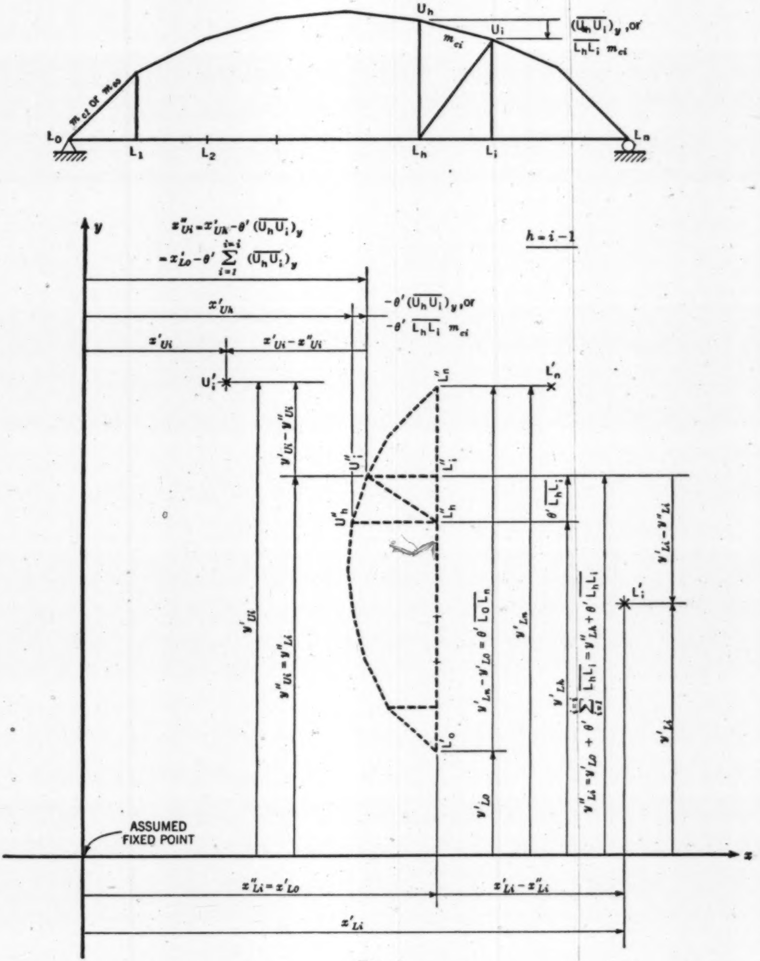
$$x''_{Ui} = x'_{Lo} - \theta' \sum_{i=1}^{i=i} (\overline{\text{U}_h\text{U}_i})_y = x'_{Lo} - \theta' \sum_{i=1}^{i=i} \overline{\text{L}_h\text{L}_i}\, m_{ci} =$$
$$x''_{Lh} - \theta' (\overline{\text{U}_h\text{U}_i})_y \quad \text{(14b)}$$

$$y''_{Li} = y'_{Lo} + \theta' \sum_{i=1}^{i=i} \overline{\text{L}_h\text{L}_i} = y''_{Lh} + \theta' \overline{\text{L}_h\text{L}_i} \quad \text{(14c)}$$

and

$$y''_{Ui} = y''_{Li} \quad \text{(14d)}$$

in which $(\overline{\text{U}_h\text{U}_i})_y$ is the *y*-projection of the length $\overline{\text{U}_h\text{U}_i}$ bearing algebraic signs with respect to the vertical and horizontal axes through U_h. If, in addition to



U_h
U_i
$(\overline{U_hU_i})_y$, or
$\overline{L_hL_i}\; m_{ci}$
m_{ci}
m_{ci} or m_{co}
L_o
L_1
L_2
L_h
L_i
L_n
y
$x''_{Ui}=x'_{Uh}-\theta'(\overline{U_hU_i})_y$
$=x'_{Lo}-\theta'\sum_{i=1}^{i=i}(\overline{U_hU_i})_y$
$h=i-1$
x'_{Uh}
$-\theta'(\overline{U_hU_i})_y$, or
$-\theta'\;\overline{L_hL_i}\; m_{ci}$
x'_{Ui}
$x'_{Ui}-x''_{Ui}$
U'_i
L''_n
L'_n
$y'_{Ui}-y''_{Ui}$
U''_i
L''_i
U''_h
L''_h
$\theta'\;\overline{L_hL_i}$
$y'_{Li}-y''_{Li}$
L'_i
y'_{Ui}
$y''_{Ui}=y''_{Li}$
$y'_{Ln}-y'_{Lo}=\theta'\;\overline{L_oL_n}$
y'_{Ln}
y'_{Lh}
$y''_{Li}=y'_{Lo}+\theta'\sum_{i=1}^{i=i}\overline{L_hL_i}=y''_{Lh}+\theta'\;\overline{L_hL_i}$
y'_{Li}
L'_o
y'_{Lo}
ASSUMED FIXED POINT
x
$x''_{Li}=x'_{Lo}$
$x'_{Li}-x''_{Li}$
x'_{Li}

FIG. 6

the foregoing conditions, the truss is of equal panel length, then instead of Eqs. 13, 14*a*, and 14*b*, respectively, the following equations may be used:

$$\theta''=\frac{y'_{Ln}-y'_{Lo}}{n}\qquad\qquad(15a)$$

$$x''_{Ui}=x'_{Li}-\theta''\sum_{i=1}^{i=i}m_{ci}=x''_{Lh}-m_{ci}\,\theta''\qquad\qquad(15b)$$

and

$$y''_{Li} = x'_{Lo} + i\,\theta'' \qquad \text{(15c)}$$

If, in addition to the foregoing conditions, the truss has a horizontal upper chord, then, instead of Eq. 15*b*,

$$x''_{Ui} = x'_{Lo} - m_{eo}\,\theta'' \qquad \text{(16)}$$

in which m_{eo} is the slope of the end post at point L_o. The actual movements for all the aforementioned cases are (corresponding to Eqs. 12)

$$\left.\begin{aligned} x\text{-movement of } L_i &= (\overrightarrow{L''_iL'_i})_x = x'_{Li} - x''_{Li} \\ y\text{-movement of } L_i &= (\overrightarrow{L''_iL'_i})_y = y'_{Li} - y''_{Li} \end{aligned}\right\} \qquad \text{(17a)}$$

and

$$\left.\begin{aligned} x\text{-movement of } U_i &= (\overrightarrow{U''_iU'_i})_x = x'_{Ui} - x''_{Ui} \\ y\text{-movement of } U_i &= (\overrightarrow{U''_iU'_i})_y = y'_{Ui} - y''_{Ui} \end{aligned}\right\} \qquad \text{(17b)}$$

PART II. ILLUSTRATIONS AND TABULAR FORMS

To illustrate the principles stated in Part I, an example and several tabular forms are presented herein. The general method of solution will be illustrated first and some special cases will be discussed subsequently.

An Example for the General Method of Solution

Consider the three-hinged arch shown in Fig. 5(*a*). It is required to find the deflections of the joints under the loading shown in the figure by the coordinate method.

Step 1. Deformations and Dimensions.—The δ-components, the load, and the dimensions of the truss are shown in Fig. 7. For simplicity, the $\frac{L}{A\,E}$-values are assumed to be equal to a constant K for all members. Hence, the δ-components are coefficients of K and are equal to the stress components.

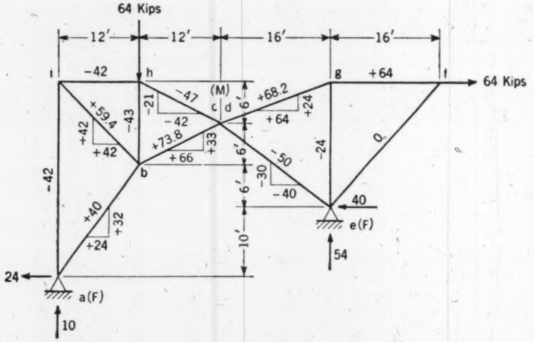


Fig. 7

Step 2. Sign Conventions.—The two halves of the arch on the left and the right sides of the crown joint are considered separately. In these two halves, a and e are considered as the respective fixed points and ai and eg as the respective bars fixed in direction (see Fig. 8). Note the different sign conventions for Δ-components used for the left and the right sides of a member assumed as fixed in direction. (In this example, there is no such exception as the member pq in Fig. 3.)

FIG. 8

Step 3. Slope and Displacement.—In Fig. 9, observe the change from the signs of δ-components to the signs of Δ-components.

Step 4. Formulas.—Eqs. 3 and 5 are used for the algebraic solution of the Williot diagram. For example, referring to Eq. 2*a*, $x'_{1,P} = x'_Q + (\Delta_{QP})_x$, and similarly for Eqs. 2*b*, 3, and 5.

Step 5.—The algebraic solution of the Williot diagram is demonstrated in Table 1.

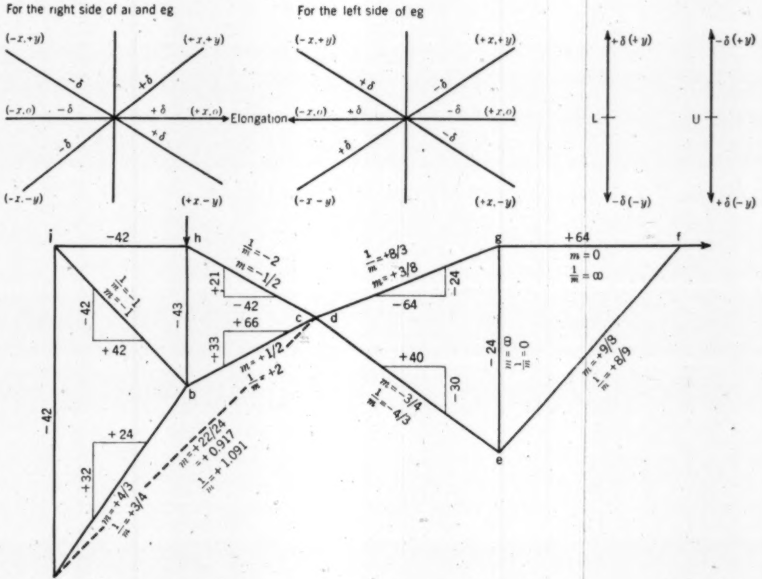

For the right side of ai and eg
(-x,+y)
(+x,+y)
-δ
+δ
(-x,o)
-δ
+δ
(+x,o)
+δ
-δ
(-x,-y)
(+x,-y)
Elongation
For the left side of eg
(-x,+y)
(+x,+y)
+δ
-δ
(-x,o)
+δ
-δ
(+x,o)
+δ
-δ
(-x -y)
(+x,-y)
+δ (+y)
L
-δ (-y)
-δ (+y)
U
+δ (-y)
i
-42
h
g
+64
f
$\frac{1}{m}=-2$
$m=-1/2$
$\frac{1}{m}=+8/3$
$m=+3/8$
$m=0$
$\frac{1}{m}=\infty$
$\frac{1}{m}=-1$
$m=-1$
-42
-43
+21
-42
-24
-64
c
d
+66
+42
+33
+40
-24
$m=\infty$
$\frac{1}{m}=0$
$m=+1/2$
$\frac{1}{m}=+2$
$m=+9/8$
$\frac{1}{m}=+8/9$
b
-30
$m=-3/4$
$\frac{1}{m}=-4/3$
-42
+24
e
$m=+22/24$
$=+0.917$
$\frac{1}{m}=+1.091$
+32
$m=+4/3$
$\frac{1}{m}=+3/4$
a

FIG. 9

The actual fixed points and the points that move in directions fixed by known conditions are designated F and M in subscripts, respectively. As point a is fixed in position, the coordinates of a′, item 1, are 0 as given in Cols. 4 and 8. Since line ai is assumed to be fixed in direction, the coordinates of i′, item 2, are the *x*-component and the *y*-component of Δ_{ai}. These components are also entered in Cols. 4 and 8, respectively.

With the coordinates of a′ and i′ known, point b′ can be located since b is connected to points a and i by the bars ab and ib, respectively. Referring to Col. 4, the x-value of i′ plus the x-component of Δ_{ib} in item 3 gives x'_1 in item 4. (This value is designated x'_1 in applying Eqs. 5. Col. 4, Table 1, is headed x without any subscript because the column is not restricted to values of x'_1. Other notations like x'_2, m_1, m_2, y'_1, and y'_2, used in this section have meanings similar to x'_1.) This value (item 4, Col. 4), multiplied by $1/m_1$ for line ib in Col. 5, gives x'_1/m_1 in Col. 6. The x-value of point a′, plus the x-component of

TABLE 1.—COORDINATES OF THE POINTS ON THE WILLIOT DIAGRAM

Item	Joint[a] (1)	Member (2)	x (3)	x (4)	$\frac{1}{m}$ (5)	$\frac{x}{m}$ (6)	y (7)	y (8)	m (9)	$m\,y$ (10)
1	a'_F			[0]				[0]		
2	i′			[0]				[−42]		
3		ib		+42				−42		
4				+42	−1	−42		−84	−1	+84
5		ab		+24	+3/4	+18		+32	+4/3	+42.67
6				+18	−7/4	−60		−116	−7/3	+41.33
7						−116[b]				+18.00[c]
8	b′			[+100.57] ×	−7/4 =	−176		[−25.43] ×	−7/3 =	+59.33
9		bh		0				−43.00		
10				+100.57	0			−68.43	∞	
11		ih		−42.00	∞			0		
12	h′			[−42.00][d]				[−68.43][e]		
13		hc		−42				+21		
14				−84	−2	+168.00		−47.43	−1/2	+23.72
15	b′		+100.57				−25.43			
16		bc	+66				+33			
17			+166.57	+166.57	+2	+333.14	+7.57	+7.57	+1/2	+3.79
18				−250.57	−4	−165.14		−55.00	−1	+19.93
19						−55.00[b]				−250.57[c]
20	c'_M			[+55.04] ×	−4 =	−220.14		[+230.64] ×	−1 =	−230.64
21	e'_F			[0]				[0]		
22	g′			[0]				[−24]		
23		gd		−64				−24		
24				−64	+8/3	−170.67		−48	+3/8	−18.0
25		ed		+40	−4/3	−53.33		−30	−3/4	+22.5
26				−104	+4	−117.34		−18	+9/8	−40.5
27						−18.00[b]				−104.0[c]
28	d'_M			[−33.84] ×	+4 =	−135.34		[−128.44] ×	+9/8 =	−144.5
29	e′			0				0		
30		ef		0	+8/9			0	+9/8	0
31	g′		0				−24			
32		gf	+64				0			
33			+64	+64	∞		−24	−24	0	0
34				−64				+24	+9/8	0
35										−64[c]
36	f′			[+64]				[−56.89] ×	+9/8 =	−64

[a] Subscript F denotes fixed joints and subscript M denotes points that move in directions fixed by known conditions. [b] From Col. 8. [c] From Col. 4. [d] Since $1/m_2 = \infty$, $x' = x'_2$ and this item is the same as the item above it. [e] Since $m_1 = \infty$, $y' = y'_1$ and this item is the same as the second item above it.

Δ_{ab} gives x'_2 in item 5. The process of addition is omitted since the x-value of point a′ is zero. The x'_2-value in item 5 is then multiplied by $1/m_2$ for line ab in item 5 to produce x'_2/m_2 in item 5, Col. 6. The values $x'_1 - x'_2$, $\frac{1}{m_1} - \frac{1}{m_2}$, and $\frac{x'_1}{m_1} - \frac{x'_2}{m_2}$ are recorded as item 6 in Cols. 4, 5, and 6, respectively. The same procedure is used to obtain $y'_1 - y'_2$, $m_1 - m_2$, and $m_1 y'_1 - m_2 y'_2$ in item 6, Cols. 8, 9, and 10, respectively. The value in item 6, Col. 4, is then carried to item 7, Col. 10, and it, plus the value of $m\,y'$ in item 6, Col. 10, gives the value entered in item 8, Col. 10. This value is $(x'_1 - x'_2) + (m_1y'_1 - m_2y'_2)$ and it is divided by $m_1 - m_2$, the value in item 6, Col. 9, to produce the y-value for point b′ in line 8, Col. 8. Similarly, the x-value for point b′ is determined and entered in item 8, Col. 4.

With the coordinates x and y of points b′ and i′ known, point h′ can be located. The x-value for point b′, plus the x-component of Δ_{bh} in item 9, Col. 4, gives x'_1 in item 10, Col. 4. The values in Cols. 5 and 9, item 10, need no explanation. The value x'_2 in item 11, Col. 4, is known to be the x-component of Δ_{ih}. However, it is to be noted that the ratio $1/m_2$ for the member ih is ∞. Therefore, x for point h′ in item 12, Col. 4, is the same as x'_2 in item 11. Similarly, the y-value of point h′ is determined.

With x and y of points h′ and b′ known, point c′ can be located. Again, referring to Col. 4, the x'_1-value in item 14 is obtained by the addition of the x-value of point h′ in item 12 and the x-component of Δ_{hc} in item 13. The x'_2-value in item 17, Col. 3, is obtained by the addition of the x'_1-value of point b′ (from item 8, Col. 4) in item 15, Col. 3, and the x-component of Δ_{bc} in item 16, Col. 3. This step is shown in Col. 3 instead of Col. 4 to avoid confusing this step with the next. Then the x'_2-value in item 17, Col. 3, is carried to item 17, Col. 4, and it is subtracted from x'_1 in item 14, Col. 4, to give $x'_1 - x'_2$ in item 18, Col. 4. Col. 7 for y serves the same purpose as Col. 3 for x. Follow the usual procedure to obtain the x-value and the y-value of point c′.

Starting anew from point e′, locate points g′, d′, and f′ in the same manner as indicated previously. In Col. 4, note that, although the value in item 36 is equal to x'_2 in item 33 for m_2 is ∞, the x'_2-value in item 33 is subtracted from x'_1 in item 30 to obtain the value $x'_1 - x'_2$ in item 34. The value $x'_1 - x'_2$ is necessary for computing y' in item 36, Col. 8. However, in the case of a joint such as h, which is located by one vertical bar and one horizontal bar, neither $x'_1 - x'_2$ nor $y'_1 - y'_2$ is necessary (see items 10 to 12).

Step 6. Algebraic Solution of the Mohr Diagram.—The computations for the rotational correction of the Mohr diagram are carried out in Table 2, which is based on Principles 1, 2, and 3 (under the heading, "Algebraic Solution of the Mohr Diagram and Actual Displacements of Truss Joints") in Part I. Its form is almost the same as that of Table 1, although Cols. 3 and 7, being unnecessary in this example, have been omitted. The coordinates of point c‴ in Cols. 4 and 8, item 5, are determined according to Principle 2 stated in Part I. Item 6 can be omitted for this example because the origin of the Williot diagram is an actual fixed point. However, when it is not actually fixed, the x-component and the y-component of $\overline{\text{F}'\text{M}'''}$ in Cols. 4 and 8, item 7, should be given by the differences between items 5 and 6, Col. 4, and items 5 and 6, Col. 8, respectively. The m-value in item 8, Col. 9, is $m'_o = -\dfrac{\overrightarrow{F'M'''}_x}{\overrightarrow{F'M'''}_y} = -\dfrac{\text{Col. 4, item 7}}{\text{Col. 8, item 7}}$. The ratio $1/m$ in item 8, Col. 5, is the reciprocal of item 8, Col. 9. The coordinates of point c″ in Cols. 4 and 8, item 13, and of point d″ in Cols. 4 and 8, item 20, are determined according to Principle 2 stated in Part I. Items 14 and 15, and items 21 and 22, serve the same purpose as items 6 and 7, respectively. Items 23 to 26, Table 2, are for the calculation of the angle of rotation θ. The coordinates of the point of known direction of motion, M, with respect to the fixed point F (that is, the projections of line FM) should have proper signs. For example, from the truss diagram of this example, point d_M is on the left side of the point e_F; hence, x_{ed} in item 24, Col. 4, is negative.

TABLE 2.—ALGEBRAIC SOLUTION OF THE MOHR DIAGRAM; COMPUTATION OF CORRECTIONS

Item	Joint (1)	Member (2)	x (3)(4)	$1/m$ (5)	x/m (6)	y (7)(8)	m (9)	$m\,y$ (10)
1	d′	ed	−33.84 ×	−4/3 =	+45.12	−128.44 ×	−3/4 =	+96.33
2	c′	ac	+55.04 ×	+12/11 =	+60.04	+230.64 ×	+11/12 =	+211.42
3			−88.88	−80/33	−14.92	−359.08	−5/3	−115.09
4					−359.08[a]			−88.88[b]
5	c'''_M		[+154.28]		−374.00	[+122.38]		−203.97
6	a′		0			0		
7		$a'c'''$, $\overrightarrow{F'M'''}$	[+154.28]			[+122.38]		
8		$a'c'''$ or $e'd'''$		[−0.7932][c]			[−1.2607][c]	
9	c′	$a'c'''$	+55.04	−0.7932	−43.66	+230.64	−1.2607	−290.77
10	a′	ac	0	+1.0909	0	0	+0.9167	0
11			+55.04	−1.8841	−43.66	+230.64	−2.1774	−290.77
12					+230.64[a]			+55.04[b]
13	c''_M		[−99.24] ×	−1.8841 =	+186.98	[+108.26] ×	−2.1774 =	−235.73
14	a'_F		0			0		
15		$a'c''$, $\overrightarrow{F'M''}$	[−99.24]			[+108.26]		
16	d′	$e'd'''$	−33.84	−0.7932	+26.84	−128.44	−1.2607	+161.92
17	e′	ed	0	−1.3333	0	0	−0.7500	0
18			−33.84	+0.5401	+26.84	−128.44	−0.5107	+161.92
19					−128.44[a]			−33.84[b]
20	d''_M		[−188.11] ×	+0.5401 =	−101.60	[−250.79] ×	−0.5107 =	+128.08
21	e'_F		0			0		
22		$e'd''$, $\overrightarrow{F'M''}$	[−188.11]			[−250.79]		
23	Length	ac	+24			+22		
24	$\overline{FM}$	ed	−16			+12		
25	$\theta = \dfrac{\overrightarrow{F'M''}_y}{x_{FM}} = -\dfrac{\overrightarrow{F'M''}_x}{y_{FM}}$		$\theta_1 = \dfrac{\overrightarrow{a'c''}_y}{x_{ac}} = +\dfrac{108.26}{+24} = -\dfrac{\overrightarrow{a'c''}_x}{y_{ac}} = -\dfrac{(-99.24)}{+22} = +4.5108$					
26			$\theta_2 = \dfrac{\overrightarrow{e'd''}_y}{x_{ed}} = -\dfrac{250.79}{-16} = -\dfrac{\overrightarrow{e'd''}_x}{y_{ed}} = -\dfrac{(-188.11)}{+12} = +15.675$					

[a] From Col. 8. [b] From Col. 4. [c] Based on Eq. 8.

The actual movements of the truss points are computed in Table 3, which is based on Principles 1, 3, and 4, stated in Part I. Either Eqs. 6b, 10, 11, and 12a or Eqs. 6a, 10, 11, and 12b may be used. The coordinates for a point in the given truss with respect to the actual fixed point are x_{FP} and y_{FP} recorded in

Cols. 4, Table 3. These coordinates can be written directly by consulting the truss diagram, and Cols. 2 and 3 can be omitted, although they may be used for convenience, or as a check. The projections of the length of a member should have proper signs with respect to its starting point. For example, x_{ch} starts from point c and the truss diagram shows that point h is on the left side of point c; hence, x_{ch} is negative. In Table 3(*b*) this value x_{ch}, given in item 4, Col. 3, plus the value x_{ac} in item 3, Col. 4, yields the value x_{ah} in item 4, Col. 4.

Col. 5, Table 3(*a*), is for $-\theta y_{FP}$ whereas Col. 5, Table 3(*b*), is for $+\theta x_{FP}$. For convenience, the θ-values are entered in items 1 and 6, Col. 5. These spaces are available as it is not necessary to enter the values x''_{FP} and y''_{FP} for the actual fixed points as they are always equal to zero.

TABLE 3.—COMPUTATION OF ACTUAL MOVEMENTS OF TRUSS POINTS

Column No.	Eq. No.	Table (3*a*)	Table 3(*b*)
4	10	$y_{FP} = y_{FQ} + y_{QP}$	$x_{FP} = x_{FQ} + x_{QP}$
5	11	$x''_{FP} = \overrightarrow{F'P''}_x = -\theta y_{FP}$	$y''_{FP} = \overrightarrow{F'P''}_y = +\theta x_{FP}$
6	6*b*	$x''_P = x''_{FP} + x'_F$	$y''_P = y''_{FP} + y'_F$
6 (alternate)		x'_P	y'_P
7[a]		x'_P	y'_P
7 (alternate)	6*a*	$x'_{FP} = x'_P - x'_F$	$y'_{FP} = y'_P - y'_F$
8	12*a*	$\overrightarrow{P''P'}_x = x'_P - x''_P$	$\overrightarrow{P''P'}_y = y'_P - y''_P$
8 (alternate)	12*b*	$\overrightarrow{P''P'}_x = x'_{FP} - x''_{FP}$	$\overrightarrow{P''P'}_y = y'_{FP} - y''_{FP}$

Note: The alternate headings should be taken together.

Item	Joint (Fig. 9) (1)	Member (2)	(*a*) *x*-COMPONENTS y_{QP} (3)	y_{FP} (Eq. 10) (4)	x''_{FP} (Eq. 11) (5)	x'_P $(=x'_{FP})$[a] (6)	$\overrightarrow{P''P'}_x$ (Eq. 12*b*) (8)	(*b*) *y*-COMPONENTS x_{QP} (3)	y_{FP} (Eq. 10) (4)	y''_{FP} (Eq. 11) (5)	y'_P $(=y'_{FP})$[a] (6)	$\overrightarrow{P''P'}_y$ (Eq. 12*b*) (8)
1	a_F			0	(θ_1=4.511)		0		0	(θ_1=4.511)		0
2	b	ab	+16	+16	−72.18	+100.57	+172.75	+12	+12	+54.13	−25.43	−79.56
3	c	bc	+ 6	+22	−99.24	+55.04	+154.28	+12	+24	+108.26	+230.64	+122.38
4	h	ch	+ 6	+28	−126.31	−42.00	+84.31	−12	+12	+54.13	−68.43	−122.56
5	i	hi	0	+28	−126.31	0	+126.31	−12	0	0	−42.00	−42.00
6	e_F				(θ_2=15.675)		0		0	(θ_2=15.675)		0
7	d	ed	+12	+12	−188.10	−33.84	+154.26	...	−16	−250.80	−128.44	+122.36
8	g	dg	+ 6	+18	−282.15	0	+282.15	...	0	0	−24.00	−24.00
9	f	gf	0	+18	−282.15	+64.00	+346.15	...	+16	+250.80	−56.89	−307.69

[a] Col. 7, in the standard form, is not needed in the solution of the present example as $x'_P = x'_{FP}$ and $y'_P = y'_{FP}$ and is omitted.

For Cols. 6, 7, and 8, Table 3, either the set of headings shown or the alternate heads at the top of the table should be used. The values x'_P and y'_P are copied from Table 1. For the given example, either Col. 6 or Col. 7 may be omitted. However, if the origin of the Williot diagram is not the actual fixed point, Cols. 6 and 7 are both necessary. The values in Col. 8 give the components of the actual movements of truss points. (The values given for this example are coefficients of $K = L/(A\,E)$ which is assumed to be the same for all members.)

SOME SPECIAL CASES

The foregoing example illustrates all the general principles involved in determining truss deflections by the coordinate method. The procedure can be applied to any special case. However, for some cases, the algebraic solution of the Mohr diagram can be much simplified. In certain cases (such as a symmetrical truss under symmetrical loading with the line of symmetry as the fixed member, or a truss with supporting conditions as shown in Fig. 3 using ab instead of ij as the fixed member), no Mohr diagram correction is needed and the coordinates of the points on the Williot diagram will give directly the x-component and the y-component of the true movements of the joints.

1. Consider the case of a truss which satisfies the following conditions: (a) The left end L_o (Fig. 6) is fixed; (b) the right end L_n is movable only in the horizontal direction; (c) the lower chord L_o L_n is horizontal; (d) the upper chord is also horizontal; and (e) the panels are of equal length. After obtaining the values x'_{Li}, y'_{Li}, x'_{Ui}, and y'_{Ui} by filling out a computation form similar to Table 1, the actual movements of the joints can be computed by tabular forms demonstrated by Fig. 10. The value θ'' (Fig. 10(a)) is obtained by applying Eq. 15a and the value x''_{Ui} is obtained by applying Eq. 16; that is, $x''_{Ui} = x'_{Lo} - m_{eo}\,\theta''$ (in which m_{eo} is the slope of the end post at point L_o).

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
y'_{Lo}	y'_{Ln}	$y'_{Ln}-y'_{Lo}$	$\theta''=\left(\frac{y'_{Ln}-y'_{Lo}}{n}\right)$	m_{eo}	$m_{eo}\theta''$	x'_{Lo}	$x''_{Ui}=x'_{Lo}-m_{eo}\theta''$

(a) DETERMINATION OF θ

(7)	(6)	(5)	(4)	(3)	(2)	(1) (1)′	(2)′	(3)′	(4)′	(5)′	(6)′	(7)′
x'_{Li}	$x'_{Li}-x''_{Li}$ $=(7)-x'_{Lo}$	POINT	$y'_{Li}-y''_{Li}$ $=(3)-(2)$	y'_{Li}	$y''_{Li}=y'_{Lo}+i\theta''$	$i\theta''$	$y''_{Ui}=y''_{Li}=(2)$	y'_{Ui}	$y'_{Ui}-y''_{Ui}$ $=(3)'-(2)'$	POINT	$x'_{Ui}-x''_{Ui}$ $=(7)'-x''_{Ui}$	x'_{Ui}
		L_1				θ''				U_1		
		L_2				$2\theta''$				U_2		
		• • • • •				• • • • •				• • • • •		
		L_i				$i\theta''$				U_i		
		• •				• •				• •		

(b) DETERMINATION OF JOINT MOVEMENT

FIG. 10.—FORMS FOR COMPUTING THE MOVEMENT OF JOINTS FOR ORDINARY PARALLEL CHORD TRUSSES

The actual movements of the points are computed by application of Fig. 10(b). The values in Col. 1 are equal to $i\,\theta''$. For example, the value for point L_2 is $2\,\theta''$. The values in Col. 2 are given by the values in Col. 1 plus

y'_{L_0} as shown by Eq. 15c. The value of y''_{U_i} in Col. 2′, Fig. 10, is equal to the (y''_{L_i})-value in Col. 2 as shown by Eq. 14*d*. The values x'_{L_i}, y'_{L_i}, x'_{U_i}, and y'_{U_i} in Cols. 7, 3, 7′, and 3′, respectively, are those obtained by the algebraic solution of the Williot diagram. The value of x''_{L_i} is equal to x'_{L_0} according to Eq. 14*a* and it is entered at the proper place in the heading of Col. 6. The value of x''_{U_i} which has been obtained in Fig. 10(*a*) is entered into the heading of Col. 6′. The actual displacements are given Cols. 6, 4, 6′, and 4′, Fig. 10, which are obtained by applying Eq. 17.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	POINT	$x'_{Li}-x''_{Li}$ $=(3)-(4)$	x'_{Li}	x''_{Li} $=x'_{Lo}$	$\overline{L_hL_i}$	$\theta'\overline{L_hL_i}=$ $(5)\times\theta'$	$y''_{Li}=$ $y''_{Lh}+(6)$	y'_{Li}	$y'_{Li}-y''_{Li}$ $=(8)-(7)$	POINT
0	L_o		x'_{Lo}	x'_{Lo}		$\theta'=$	$y''_{Lo}=y'_{Lo}$			L_o
1 2 • • • •	L_1 • • • • •		x'_{L1}	x'_{Lo} x'_{Lo} x'_{Lo} x'_{Lo} x'_{Lo} x'_{Lo}			y''_{L1}	y'_{L1}		L_1 • • • • •
$h=i-1$	L_h						y''_{Lh}			L_h
i	L_i					$\theta'\overline{L_hL_i}$	y''_{Li}			L_i
$j=i+1$										

(*a*) LOWER CHORD

	(1)′	(2)′	(3)′	(4)′	(5)′	(6)′	(7)′	(8)′	(9)′	(10)′
	POINT	$x'_{Ui}-x''_{Ui}$ $=(3)'-(4)'$	x'_{Ui}	$x''_{Ui}=$ $x''_{Uh}+(5)'$	$-\theta'(\overline{U_hU_i})y$ or $-\theta'\overline{L_hL_i}m_{ci}$	$(\overline{U_hU_i})y$ or m_{ci}	y''_{Ui} $=y''_{Li}$	y'_{Ui}	$y'_{Ui}-y''_{Ui}$ $=(8)'-(7)'$	POINT
0	$U_0=L_0$			$x''_{Uo}=x'_{Lo}$	$\theta'=$					
1 • • •	U_1 • • •									U_1 • • •
$h=i-1$	U_h			x''_{Uh}						U_h
i	U_i			x''_{Ui}	$-\theta'(\overline{U_hU_i})y$					U_i-

(*b*) UPPER CHORD

FIG. 11.—FORMS FOR COMPUTING THE MOVEMENT OF JOINTS FOR CURVED UPPER CHORD TRUSSES

2. If the given truss satisfies conditions (*a*), (*b*), and (*c*) but not conditions (*d*) and (*e*), then the solution can be worked out as follows:

Compute the value of θ' by Eq. 13—either directly or by adapting a table with four columns similar to the first four columns in Fig. 10(*a*). Then compute the actual movements of the joints by the tabular forms shown in Fig. 11. Fig. 11(*a*) applies to the lower chord points and Fig. 11(*b*) to the upper chord points. These tables are based on Eqs. 14 and 17. For two consecutive points L_h and L_i, the value of y''_{L_h} in item h, Col. 7, plus the value $+\ \theta'\ \overline{L_hL_i}$ in line i, Col. 6, gives the value y''_{L_i} in line i, Col. 7. Similarly, x''_{U_h} in line h, Col. 4′, adding $-\theta'(\overline{U_hU_i})_y$ in line i, Col. 5′, gives the value x''_{U_i} in line i, Col. 4′. The values in Col. 5′ can be found either by multiplying $-\theta'$ by the y-projection of the chord U_hU_i (as shown in the truss diagram and as recorded in Col. 6′), or by multiplying the value $\theta'\,\overline{L_hL_i}$ in Col. 6 by $-m_{ci}$ given in Col. 6′. The value m_{ci} is the slope of U_hU_i, and $(\overline{U_hU_i})_y$ is its y-projection, which should have the same sign as m_{ci} when point U_h is the left end of member U_hU_i. The values in Col. 7′ are the same as those in Col. 7. The coordinates of L'_i and U'_i given in Cols. 3, 8, 3′, and 8′ are those found by the algebraic solution of the Williot diagram. The components of the actual movements of truss joints are given in the Cols. 2, 9, 2′, and 9′.

CONCLUSION

The procedure described herein is an algebraic equivalent of the graphical method known as the Williot-Mohr diagram. The sign conventions suggested are easy to remember. The formulas to be used are rather simple in form; and they can be transformed into simple operations arranged in tabular forms. For a rather complicated problem, not more than three forms of tables need be used. When the designer is familiar with the operations, he can forget all the formulas.

The knowledge of truss deflections is important because it has applications in many cases such as cambering, erection by the cantilever method, stress analysis for indeterminate trusses, and the computation of secondary stresses.

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DISCUSSION

L. C. MAUGH,[2] M. ASCE.—An interesting and accurate solution for the common problem of determining the movement of the panel points of triangular trusses is presented in this paper. The method provides an algebraic procedure that corresponds to the graphical solution obtained by use of the Williot-Mohr displacement diagrams. In fact the calculations involve the determination of the coordinates of the various points in the Williot and Mohr diagrams by expressing the components parallel and perpendicular to the members in equation form. By means of such equations the successive increments of motion that any point undergoes because of the change in length of the various members can be determined. Undoubtedly many engineers will prefer such an algebraic procedure to the graphical solution, although it is doubtful if any person can fully understand the algebraic method that is presented unless he is able to use the Williot-Mohr diagrams. Mr. Chu has shown that, once the sign convention is familiar, the calculations can be made without much difficulty.

In many practical problems it is not necessary to determine both coordinates for every joint but only the vertical displacements of some points and the horizontal movement of others. When this condition exists, the writer prefers the application of the virtual work theorem in its most general form. As applied to the calculation of truss displacements this theorem can be stated as follows:

Theorem.—

> When the displacements of any two points of a truss are known the movement of a third point can be obtained by applying the virtual work principle to the simplest stable assemblage of members connecting the three points.

This generalized concept of virtual work was developed in detail in a dissertation[3] by M. Megahid in 1948. The application of the principle of virtual work to the problem that Mr. Chu has solved by analytic geometry will be illustrated by a few numerical examples.

If the vertical displacement of point b in Fig. 7 is known, then the horizontal movement of point i can be calculated by considering the virtual forces shown in Fig. 12(*a*). By the principle of virtual work the work done by the external virtual forces acting through the real displacements must be equal to the work that is obtained from the internal virtual forces acting through the actual change in length of the members. For the forces shown in Fig. 12(*a*) and the change of lengths given in Fig. 7. this equality of external and internal work gives $1 \times i_x + 2.33\ b_y = -1.0 \times -42.0 + 1.414 \times 59.4 + -1.67 \times$

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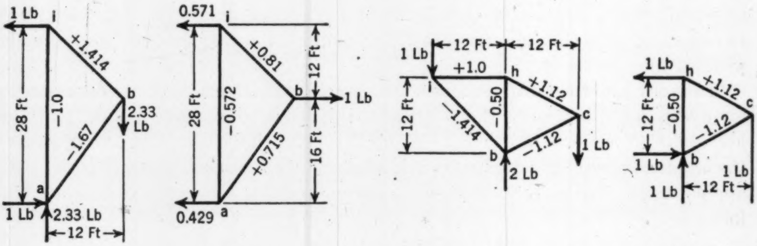
[2] Prof., Civ. Eng., Univ. of Michigan, Ann Arbor, Mich.

[3] "The Free Main System," by M. Megahid, dissertation presented to Fouad I University, Giza, Egypt, in 1948, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

$+ 40$. If the value of b_y is 79.6 downward, then the value of i_x is $i_x + 2.33 \times 79.6 = 59.3$; and $i_x = -126.4$. The minus sign indicates that the displacement is opposite in direction to the assumed force.

The horizontal displacement of point b can now be determined by means of the virtual forces in Fig. 12(*b*). Again the external work of the virtual forces in Fig. 12(*b*) acting through the actual displacements in Fig. 7 are equated to the corresponding internal work; thus: $1 \times b_x - 0.571 \times 126.4 = -0.571 \times -42.0 + 0.715 \times +40.0 + 0.810 \times +59.4$, from which $b_x = 172.8$.

The vertical displacement of point c is easily determined by means of the virtual forces shown in either Fig. 12(*c*) or Fig. 12(*d*). If the force system in Fig. 12(*c*) is used, the following equation is obtained: $1 \times C_y - 2 \times 79.6 + 1 \times 42.0 = 1.12 \times -47.0 + -1.12 \times +73.8 + -0.5 \times -43.0 + -1.414 \times +59.4 + 1.0 \times 42.0$, from which $C_y = -122.7$ (upward). If the force system in Fig. 12(*d*) is used, the equation becomes: $1 \times C_y - 1 \times 79.6 + 1 \times 172.8 - 1 \times 126.4 - 42.0 = 1.12 \times -47.0 + -1.12 \times +73.8 + -0.5 \times -43.0$, which gives $C_y = -122.7$.



(*a*) (*b*) (*c*) (*d*)

FIG. 12.

The procedure that is used in the foregoing examples has the advantage of involving only a few members of the truss and only one sign convention. It is interesting to note that for every geometrical solution there is an analogous solution by the principle of virtual work.

KUANG-HAN CHU,[4] J. M. ASCE.—The writer appreciates the comments made by Mr. Maugh and his introduction of Mr. Megahid's method. This method can be used advantageously for problems in which, as Mr. Maugh states, it is not necessary to determine both coordinates for every joint of a truss but only for the vertical displacements of some points and the horizontal movement of others. The method involves a few members of the truss and only one sign convention. However, Mr. Megahid's method becomes tedious when the displacements of many joints are needed, as it is necessary to find a new stress pattern for each displacement of each joint although only a few members are involved. The Williot and Mohr method has been proved useful for many practical problems, and it is discussed in detail by Mr. Maugh in his book[5] because "* * * it provides an extremely practical method for the determination of the actual movement of all joints." The writer's method inherits the advantages and disadvantages of the Williot and Mohr method, except that the writer's method is an algebraic one and therefore, can be carried to any desired degree of accuracy.

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[4] Structural Designer, D. B. Steinman, New York, N. Y.

After discussions had been formally closed, the writer's attention was called to a discussion submitted by Mr. Yen-chi Lu of the National Taiwan University (Formosa, China). Since it was received too late to be processed as a formal discussion, two main points brought out in this discussion that are worth noting are presented as follows:

1. Mr. Lu recommends that the magnitude and the signs of the components be determined by the following equations:

$$(\Delta_{QP})_x = \delta_{QP} \cos \phi_{QP} \qquad (18a)$$

$$(\Delta_{QP})_y = \delta_{QP} \sin \phi_{QP} \qquad (18b)$$

in which ϕ_{QP} is the angle that the bar QP makes with the x-axis through the starting point Q in conventional orientation—counterclockwise as positive (see Fig. 13). The main advantage of using Eqs. 18, as Mr. Lu states, is that the signs of Δ components are automatically determined and the four rules as outlined by the writer are unnecessary. However, in actual applications, the writer's method may be more expedient than Mr. Lu's because in using Eqs. 18 one has to watch carefully the signs of δ, $\cos \phi$, and $\sin \phi$, and then carry out the multiplication to determine the signs of Δ components. In the writer's method, the signs can be determined by merely following the rules as shown in Fig. 2. Eqs. 18, however, are easy to remember and express the mathematical meaning of the writer's rules more clearly and concisely.

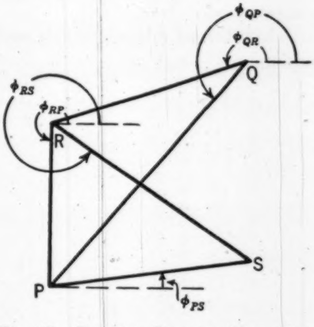

FIG. 13.—DIAGRAM SHOWING ANGLE + ϕ WITH Q, R, AND P IN TURN AS STARTING POINTS

2. In case the actual displacements of the point M of known condition of motion, $\overrightarrow{F''M'}_x$ and $\overrightarrow{F''M'}_y$, are known, Mr. Lu recommends the use of the following equations to determine the values of x''_{FM} and y''_{FM}.

$$x''_{FM} = x'_{FM} - \overrightarrow{F''M'}_x \qquad (19a)$$

and

$$y''_{FM} = y'_{FM} - \overrightarrow{F''M'}_y \qquad (19b)$$

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[5] "Statically Indeterminate Structures," by L. C. Maugh, John Wiley & Sons, Inc., New York, N. Y., 1946, p. 40.

Eqs. 19 can be derived from the writer's Eqs. 12. In the example given by the writer, $\overrightarrow{F''M'}_x = \overrightarrow{F'M'''}_x$ and $\overrightarrow{F''M'}_y = \overrightarrow{F'M'''}_y$. Therefore, the coordinates of the points c''_M and d''_M can be determined as shown in Table 4. Thus, items 8 to 22 in Table 2 can be replaced by the six items as shown in Table 4, which is a material simplification.

TABLE 4.—COMPUTATION OF COORDINATES

Item[a]	Joint	x	Computation	y
		(*a*) Point c''_M		
7	c'''_M	[+154.28]		[+122.38]
9	c'	+55.04		+230.64
14	c''_M	[−99.24]	(Item 9) − (Item 7)	[+108.26]
		(*b*) Point d''_M		
7	d'''_M	+154.28		+122.38
16	d'	−33.84		−128.44
20	d''_M	[−188.12]	(Item 16) − (Item 7)	[−250.82]

[a] Item numbers refer to Table 2.

Besides the two points presented, Mr. Lu also proposes a table form and gives equations involving trigonometric functions of ϕ based on the same principles as outlined by the writer. They are not presented here because of the following considerations:

a. Mr. Lu's equations are essentially the same as the writer's but more complicated in form; and

b. Different engineers can make different tabular forms for their own convenience.

In conclusion, the writer wishes to thank Mr. Lu for his contributions.

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TRANSACTIONS

Paper No. 2495

VALUATION AND DEPRECIATION RELATED TO INCOME TAX

BY MAURICE R. SCHARFF,[1] M. ASCE

WITH DISCUSSION BY MESSRS. GEORGE J. SOFFER, EUGENE L. GRANT, AND MAURICE R. SCHARFF

SYNOPSIS

The importance of income tax considerations in comparing the economic feasibility of engineering projects and the effect of these taxes on valuation and depreciation of existing public utility plants is emphasized in this paper.

An example is cited to show the effect of income taxes on comparative annual costs of two types of power plants. Equations are presented to express the relation between value and income tax. In effect, the income tax is equivalent to an additional fixed charge on the investment.

To illustrate the application of the methods developed, a computation of service value and accrued depreciation is developed for a hypothetical manufactured gas company.

INTRODUCTION

Previous discussions of the valuation and depreciation of public utility properties[2] have suggested logical methods of determining these amounts by computations of present worth of reasonable earning power. A review of the solutions offered has led to the recognition that it is necessary to give specific effect to income taxes in such computations. To the same extent and in somewhat the same manner this consideration enters into engineering-economic comparisons of alternative projects. Logical procedures can be developed which will give full recognition to the influence of income taxes on these economic factors.

ENGINEERING-ECONOMIC COMPARISONS

Before proceeding to the consideration of the valuation and depreciation of public utility property, the importance of income taxes in engineering-economic

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NOTE.—Published in May, 1951, as *Proceedings-Separate No. 69.* Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Cons. Engr., New York, N. Y.

[2] "Valuation and Depreciation of Public Utility Property," by M. R. Scharff, *Transactions*, ASCE, Vol. 114, 1949, pp. 907–954.

comparisons can be illustrated by an example. It is assumed that a comparison is to be made between a hydroelectric plant with a transmission line and a steam-power plant as alternative methods of providing a capacity of 100,000 kw and supplying 450,000,000 kw-hr of energy per year. The cost per kilowatt of the hydroelectric plant and transmission line is assumed to be $500, and the cost per kilowatt for the steam plant is assumed to be $200. Assume further that the rate of return to be earned after income taxes is 6% and that the capital for either project will be provided in the ratio of 50% in bonds paying 3% and 50% in capital stock. The capital stock, then, must earn 9% after income taxes in order that the average return on capital may be 6%. Assume also that (*a*) the operating expenses of the hydroelectric plant and transmission line can be taken at $4.00 per kw per year; (*b*) the fuel cost for the steam plant is $0.004 per kw-hr; and (*c*) the other production expenses of the steam plant are $6.00 per kw per year.

Then if depreciation is computed on the 6% sinking-fund basis for 80 years in the case of the hydroelectric plant and for 40 years in the case of the steam plant, the comparative annual costs exclusive of income tax will be as follows:

Type of plant	Hydro	Steam
Operating Expenses	$ 400,000	$2,400,000
Depreciation	29,000	129,000
Return	3,000,000	1,200,000
Total	$3,429,000	$3,729,000

A comparison on this basis would seem to indicate that the hydroelectric plant was more economical by $300,000 per year. However, if the resulting saving in income taxes from the customary application of straight-line depreciation rates for income tax purposes is neglected and if income taxes are determined on the basis of the foregoing returns, less bond interest only, it would theoretically be necessary to add to the annual cost $1,500,000 of income tax at an assumed 40% rate. This added sum is the amount that the owner of the hydroelectric plant would have to pay to earn a taxable income corresponding with a net income of $2,250,000 after deducting $750,000 bond interest. Similarly, the $600,000 income tax that the owner of the steam plant would have to pay in order to earn a taxable income corresponding with a net income of $900,000, after deducting $300,000 bond interest, must be added. The total cost, including income tax, on this basis would be $4,929,000 for the hydroelectric plant compared with $4,329,000 for the steam plant, indicating that, from the point of view of the rate-payer, the steam plant is more economical by a sum equal to $600,000 a year.

On the other hand, if the allowable total charge to the rate-payers were to be limited to the foregoing totals (plus income tax) and if income taxes were computed more realistically on the basis of the aforementioned sums of depreciation and return—less straight-line depreciation over 80 years and 40 years, respectively (neglecting salvage), and less bond interest—the resultant income taxes would be $1,102,700 for the hydroelectric plant and $352,700 for the steam plant. On this basis the total costs would be $4,531,700 and $4,081,700, respectively, indicating that the steam plant is more economical by $450,000 per year.

VALUATION OF PUBLIC UTILITY PROPERTY

The writer has suggested previously[3] a formula defining value as the present worth of the earning power of existing utility property at reasonable rates. This may be expressed algebraically as

$$V_e = [E\,A_s - (O_c - O_s)]\,P_e \quad \text{(1a)}$$

or

$$V_e = \frac{E\,A_s - (O_c - O_s)}{A_e} \quad \text{(1b)}$$

in which E is the cost of the most economical facilities; A_s is the annuity at reasonable rate of return (R) whose present worth for the capitalization period of the most economical facilities is 1; O_c is the operating expenses of existing facilities; O_s is the operating expenses of most economical facilities; P_e is the present worth at rate R of an annuity of 1 per year for the future capitalization period of the existing facilities $\left(\frac{1}{A_e}\right)$; and A_e is the annuity at rate R whose present worth for the future capitalization period of the existing facilities equals $1\left(\frac{1}{P_e}\right)$.

The formulation expressed by Eqs. 1 is incomplete in that it fails to give effect to the relation of income taxes to the valuation and depreciation of public utility property to which reference has been made previously. The total reasonable revenue corresponding with the economical cost of rendering the service must include income taxes necessary to earn the required return on the cost of the most economical facilities. The income taxes necessary to earn a corresponding return on the value of the existing facilities must be provided out of these reasonable revenues, in addition to operating expenses, depreciation, and return.

For any given situation in which capital structure, bond interest rate, and income tax rate are given, the income tax is equivalent, in fact, to a specific additional fixed charge on the investment. If the saving in income tax due to the use of straight-line depreciation for income tax purposes is neglected and if the income tax is based on return less bond interest, such additional fixed charge is equal—for a capitalization with $B\%$ bonds and $(1 - B)\%$ stock—to

$$t = \frac{I\,(R - B_i)}{1 - I} \quad \text{(2)}$$

In Eq. 2, I is the income tax rate expressed as a percentage of net income after bond interest; R is the rate of return expressed as a percentage of total investment; B is the ratio of bonds to total capitalization; and i is the interest rate on bonds. For example, if I is 40%, R is 6%, B is 50%, and i is 3%, the income tax

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[3] "Valuation and Depreciation of Public Utility Property," by M. R. Scharff, *Transactions*, ASCE, Vol. 114, 1949, p. 914.

TABLE 1.—SUMMARY OF COMPUTATION OF SERVICE VALUE AND ACCRUED

Account		Cost of reproduction new of existing plant[a] (thousands of dollars)	Cost of most economical facilities to furnish service[a] (thousands of dollars)	Capitalization period for most economical facilities[b] (years)	Annuity factor at 6% for period in Col. 5 plus income tax ratio	Annuity plus income tax on cost of most economical facilities[e] (thousands of dollars)
No.	Designation					
(1)	(2)	(3)	(4)	(5)	(6)	(7)
11	Land	2,500	2,500	Indefinite	0.08760	219.0
312	Structures and improvements	4,000	3,500	60	0.08947	313.1
313	Boiler plant equipment	900	1,000	35	0.09657	96.6
314	Other power equipment	300	400	50	0.09104	36.4
316	Coke ovens	2,000	1,800	35	0.09657	173.8
317	Producer gas equipment	400	400	38	0.09496	38.0
318	Water gas generating equipment	1,300	1,500	40	0.09406	141.1
321	Coal, coke, and ash handling equipment	2,200	2,200	55	0.09014	198.3
322	Gas reforming equipment	60	60	40	0.09406	5.6
323	Purification equipment	1,600	1,500	34	0.09720	145.8
324	Residual refining equipment	600	550	48	0.09150	50.3
325	Other production equipment	1,440	1,390	34	0.09720	135.1
	Total production plant	17,300	16,800			1,553.1
341	Land	500	500	Indefinite	0.08760	43.8
342	Structures and improvements	4,500	4,200	70	0.08863	372.2
	Total storage plant	5,000	4,700			416.0
361	Land	300	300	Indefinite	0.08760	26.3
362	Structures and improvements	900	800	60	0.08947	71.6
363	Mains	18,600	14,200	65	0.08899	1,263.7
364	Pumping and regulating equipment	1,000	800	35	0.09657	77.3
365	Services	6,800	6,200	48	0.09150	567.3
366.1	Meters	4,800	4,700	30	0.10025	471.2
366.2	House regulators	8	8	48	0.09150	.7
366.3	Meter installations	1,792	1,792	50	0.09104	163.1
	Total distribution plant	34,200	28,800			2,641.2
370	Land	300	300	Indefinite	0.08760	26.3
371	Structures and improvements	2,200	2,000	60	0.08947	178.9
372	Office furniture and equipment	500	500	20	0.11478	57.4
373	Transportation equipment	200	200	10	0.16347	32.7
374	Stores equipment	15	15	25	0.10583	1.6
375	Shop equipment	25	25	28	0.10219	2.6
376	Laboratory equipment	20	20	22	0.11065	2.2
377	Tools and work equipment	220	220	12	0.14688	32.3
379	Miscellaneous equipment	20	20	22	0.11065	2.2
	Total general plant	3,500	3,300			336.2
	Total plant in service	60,000	53,600			4,946.5

Estimated. [b] Assumed same as for existing facilities except for account 363, for which total capitalization land excluded this value is 0.210. [f] With land excluded this value is 0.097. [g] With land excluded this value is

by Eq. 2 equals 3% of the investment. If I is 38% and the other factors are unchanged, the income tax is 2.76% of the investment.

On this basis the value of an existing property affected by obsolescence can be expressed algebraically, either in a form corresponding to Eq. 1*a*, the method of calculation used previously,[4] which is

$$V_e = \frac{[E\,(A_e + t) - (O_c - O_e)]}{1 + P_c\, t}\, P_c \ldots\ldots\ldots\ldots\ldots (3a)$$

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[4] "Valuation and Depreciation of Public Utility Property," by M. R. Scharff, *Transactions*, ASCE, Vol. 114, 1949, p. 917.

DEPRECIATION FOR HYPOTHETICAL MANUFACTURED GAS COMPANY

Redistribution of Col. 7 (thousands of dollars) (8)	Annual operating expense savings by most economical facilities[c] (thousands of dollars) (9)	Amount available for capitalization,[d] Col. 8 − Col. 9 (thousands of dollars) (10)	Future capitalization period for existing facilities (years) (11)	Annuity factor at 6% for period in Col. 11 plus income tax ratio (12)	Service value of existing facilities Col. 10 ÷ Col. 12 (thousands of dollars) (13)	ACCRUED DEPRECIATION Col. 3 − Col. 13 (thousands of dollars) (14)	Col. 14 ÷ Col. 3 (15)
219.0	....	219.0	Indefinite	0.08760	2,500	....	....
333.6	20	313.6	45	0.09230	3,398	602	0.150
90.7	20	70.7	21	0.11260	628	272	0.302
29.1	10	19.1	39	0.09449	202	98	0.327
169.7	30	139.7	25	0.10583	1,320	680	0.340
38.0	....	38.0	26	0.10450	364	36	0.090
140.1	50	90.1	29	0.10118	890	410	0.315
196.4	2	194.4	42	0.09328	2,084	116	0.053
5.6	....	5.6	29	0.10118	55	5	0.083
137.0	30	107.0	21	0.11260	950	650	0.406
51.3	3	48.3	39	0.09449	511	89	0.148
136.1	5	131.1	29	0.10118	1,296	144	0.100
1,546.6	170	1,376.6			14,198	3,102	0.179[e]
43.8	....	43.8	Indefinite	0.08760	500	....	....
373.9	3	370.9	49	0.09126	4,064	436	0.097
417.7	3	414.7			4,564	436	0.087[f]
26.3	....	26.3	Indefinite	0.08760	300	....	....
72.5	2	70.5	43	0.09293	759	141	0.157
1,263.7	....	1,263.7	57	0.08985	14,065	4,535	0.244
79.3	12	67.3	28	0.10219	659	341	0.341
567.3	....	567.3	33	0.09787	5,796	1,004	0.148
471.2	....	471.2	17	0.12304	3,830	970	0.202
0.7	....	0.7	39	0.09449	7	1	0.125
163.1	....	163.1	31	0.09939	1,641	151	0.084
2,644.1	14	2,630.1			27,057	7,143	0.209[g]
26.3	....	26.3	Indefinite	0.08760	300	....	....
180.8	3	177.8	35	0.09657	1,841	359	0.163
57.4	....	57.4	12	0.14688	391	109	0.218
32.7	....	32.7	6	0.23096	142	58	0.290
1.6	....	1.6	16	0.12655	13	2	0.133
2.6	....	2.6	14	0.13518	19	6	0.240
2.2	....	2.2	12	0.14688	15	5	0.250
32.3	....	32.3	6	0.23096	140	80	0.364
2.2	....	2.2	12	0.14688	15	5	0.250
338.1	3	335.1			2,876	624	0.178[h]
4,946.5	190	4,756.5			48,695	11,305	0.188[i]

period of existing facilities is 80 years. [c] Col. 6 times Col. 4. [d] As service value of existing facilities. [e] With 0.211. [h] With land excluded this value is 0.195. [i] With land excluded this value is 0.200.

or, more simply, by the equivalent expression (compare with Eq. 1*b*):

$$V_e = \frac{E\,(A_s + t) - (O_e - O_s)}{A_e + t} \qquad \text{(3b)}$$

On the other hand, if the effect of straight-line depreciation on income tax is taken into account, and if L_s is defined as the service life and total capitalization period of the most economical facilities and if L_e is defined as the remaining life and future capitalization period of the existing facilities, then the ratio of income tax to value will differ in the two cases. For the most economical facilities, it will be

$$t_e = \frac{\left(A_e \frac{-1}{L_e} - B\,i\right) I}{1 - I} \quad \text{(4a)}$$

and for the existing facilities—

$$t_c = \frac{\left(A_c - \frac{1}{L_c} - B\,i\right) I}{I - 1} \quad \text{(4b)}$$

As an example for $L_e = 40$ years with other constants the same as before, then $t_e = 1.764\%$ with a 40% income tax rate or 1.622% with a 38% income tax rate. Using t_e and t_c for these expressions, the value of an existing property affected by obsolescence can be expressed in a form analogous to Eq. 1*a*:

$$V_c = \frac{[E\,(A_e + t_e) - (O_c - O_e)]}{1 + P_c\,t_c}\,P_c \quad \text{(5a)}$$

or for greater ease in computation by the equivalent expression (compare Eq. 1*b*):

$$V_c = \frac{E\,(A_e + t_e) - (O_c - O_e)}{A_e + t_c} \quad \text{(5b)}$$

As a matter of interest, the application of Eq. 3*b* (neglecting the reduction of income tax through use of straight-line depreciation for income tax purposes) to the example of the "Hypothetical Manufactured Gas Company,"[5] has been calculated using a capitalization of 50% bonds paying 3% interest and 50% stock and having an income tax rate of 38%. The result of the calculation is shown in Table 1. It will be noted that consideration of "theoretical" income tax results in a total service value of the existing facilities of $48,695,000 compared with $46,969,600 as previously cited. Furthermore the ratio of accrued depreciation to the cost of reproduction new of the existing plant is 18.8% if land is included and 20.0% if land is excluded. Ignoring income taxes, the corresponding ratio of accrued depreciation to cost of reproduction new was 21.717% if land was included and 23.104% if land was excluded.

A similar computation, giving consideration to "realistic" income tax, by the application of Eq. 5*b*, has also been made, which yielded a total service value of existing facilities equal to $48,857,000 compared to $46,969,600 computed without considering income taxes. The comparable ratio of accrued depreciation to cost of reproduction new of existing plant is 18.6% if land is included and 19.8% if land is excluded, instead of 21.717% and 23.104%, respectively, without giving effect to income taxes.

In all of the computations referred to herein, the "unadjusted compound interest" method of calculation has been used for simplicity in calculation and presentation, without taking into account the effect of probable dispersion of retirements.

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[5] "Valuation and Depreciation of Public Utility Property," by M. R. Scharff, *Transactions*, ASCE, Vol. 114, 1949, Table 1, pp. 920–921.

Property taxes and insurance are generally related in some manner to the value of the property. Nevertheless, the methods of fixing insurance values and tax assessments are of such uncertain relation to value in the sense of present worth of earning power at reasonable rates that it may well be preferable to consider these elements of cost as expenses determined independently and not related to value on the basis of service value. On the other hand, if these items were considered to be related directly to the service value that is being sought, it would be necessary to take them into account in exactly the same manner as has been suggested for income tax. The algebraic formulations previously presented would be extended by adding a term for the ratio of such insurance and property taxes to value.

The subject of this paper has received little attention in the past, and others who are interested may wish to extend the consideration of the subject by criticism and futher analysis of the problem.

DISCUSSION

George J. Soffer[6].—The basic premise of this paper is to treat the cost of income tax as a fixed annual cost. This fundamental assumption must be analyzed in all its details. The first question to be considered is how does this method of computation affect the consumer's rate for power? Evidently, when the annual cost of operating a power plant is increased, then an increase in price to the consumer must result. This is contrary to the purpose of a public utility corporation.

The second question to consider is how will the author's premise affect the basic structure of the income tax? The aim of this tax is to function at that level of income after which all operating expenses have been deducted. Now, if this tax is included in the cost of production, the whole purpose of the income tax is defeated. If this were permitted, then other industries may seek to take advantage of this method of computation.

However, the advantage of using the author's method in preliminary studies and investigations is clearly shown by his excellent example of an engineering economic comparison between a hydroelectric plant with a transmission line and a steam power plant.

Eugene L. Grant,[7] M. ASCE.—Two important points in Mr. Scharff's interesting paper might be rephrased as follows:

1. The conclusions of engineering economy studies may be altered by the introduction of income tax considerations. Generally speaking, proposed investments intended to decrease operating expenses are less attractive under income taxation than they would be without income taxes (or would seem to be if income tax considerations were disregarded).
2. Assume that existing fixed assets are appraised at a figure that will make them equally economical with the most economical new substitute assets. Appraised values under this principle will be higher if the income tax aspects of the matter are considered than if income taxes are disregarded.

The first point is recognized by many persons responsible for economy studies in industry and has been discussed in the literature of engineering economy. Although the second point would seem to follow logically from the first, few people seem to be aware of it. He believes that this paper is notable in presenting a formal analysis of this relationship between income taxes and appraisals.

The paper is entirely in a public utility setting and assumes that revenues will just equal costs regardless of what costs may be (including as costs both income taxes and capital recovery with a stipulated return). Doubtless this is the most reasonable single assumption that could be made for a regulated public utility even though it is not inevitable that high costs will result in high revenues and low costs will result in low revenues. However, it is clearly not a reasonable assumption in competitive industry.

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[6] Structural Designer, Burns and Roe, Inc., Eng. Consultants, New York, N. Y.

[7] Executive Head, Dept. of Civ. Eng., Stanford Univ., Stanford, Calif.

If, for example, an electrometallurgical company were comparing the hydro and steam plants in the illustration, the most reasonable assumption would doubtless be that revenues from the sale of metal would be the same regardless of whether hydro or steam power were used in the reduction process. With revenues unaffected by the choice, the difference in taxable income would depend solely on the difference in the expense items deductible for tax purposes. In competitive industry, one important aspect of income tax considerations in economy studies may be the difference between the pay-off period used in the economy study and the life of the proposed fixed assets that will probably be allowed in computing annual depreciation expense for tax purposes. The influence of a proposed plant investment on the excess profits tax base may also require consideration.

It should be recognized, therefore, that the validity of the analysis presented is limited to the public utility setting for which it is given, and that a somewhat different type of analysis of this problem is necessary in competitive industry.

Maurice R. Scharff,[8] M. ASCE.—The analysis in this paper is applicable, completely, only to the public utility industry, where allowable revenue is customarily limited to cost of service, including income taxes. The writer is in agreement with Mr. Grant in this respect. He believes, however, that the analysis has a limited application to competitive industry, particularly, where prices tend to be "administered" rather than to be fixed by free competition. Lewis H. Kimmel states[9] that answers to questionnaires sent to the industrial clients reflected a growing tendency to relate pricing policies to corporate income tax, 59.8% of the answers to the latest questionnaire answering "yes" to the question "Has the corporate income tax consciously influenced your pricing policies?"

The writer also believes that Mr. Grant was too generous in crediting him with the first published statement of the relationship between income taxes and appraisals. Although the disscussion might be considered applicable more directly to the first of the two points formulated in Mr. Grant's discussion rather than to the second (propounded by Alex Bauhan[10]), it at least carries the implication of the same relation of income tax to valuation.

The writer also feels that Mr. Grant's statement that "***proposed investments intended to decrease operating expenses are less attractive under income taxation than they would be without income taxes***" is an understatement of the effect of present high corporate income tax rates in the United States. They amount to confiscating a large proportion of the savings in expense that can be gained by substituting modern efficient equipment for obsolete inefficient equipment, and thus such tax rates discourage replacements and improvements in efficiency.

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[8] Cons. Engr., New York, N. Y.

[9] "Taxes and Economic Incentives," by Lewis H. Kimmel, Brookings Institution Publication, Washington, D. C., 1950, pp. 27–31.

[10] "Interconnection Services, Their Classification and Evaluation," by Alex Bauhan, *Transactions*, Am. Inst. of Elec. Engrs., Vol. 50, 1931, p. 1247.

Mr. Soffer's two questions leave out of account the fundamental nature of the public utility industry and public utility regulation as they have developed under the social and legal institutions in the United States. The increase in utility rates to consumers with increase in cost may be undesirable; but it is inevitable under a system that prohibits competition as a means of regulating utility rates and that, through intervention of the police power, restricts rates to cost of service, including return and income taxes.

Similarly, the passing on of income taxes to consumers, in rates, is a necessary consequence of the method of fixing utility rates and has no application to other industries in which prices are presumably controlled by competition.

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Paper No. 2496

WATERWAY TRAFFIC ON THE GREAT LAKES

BY JOHN R. HARDIN,[1] M. ASCE

SYNOPSIS

Since the earliest times in United States history, the Great Lakes have been a vital artery of commerce, serving an area rich in natural resources and industrial facilities. From meager beginnings as storm havens for sailing vessels, great harbors have been developed and are maintained by the Great Lakes Division of the Corps of Engineers, United States Army.

The work of this division in improving harbor and navigational facilities is recounted. Statistics are cited to show the magnitude of commerce carried by the lakes and the dependence of this shipping on constant maintenance of channels and harbors.

The increase of Great Lakes trade to world-wide scope would be made possible by construction of the St. Lawrence River Seaway facility, and the cost and significance of this project is discussed.

INTRODUCTION

Early History.—During the advance and retreat of the glacial ice sheet many centuries ago, the ancestors of our present Great Lakes were born. From a geographic, historic, and economic standpoint, the Great Lakes with their connecting channels constitute the most important body of fresh water in the world. This paper is intended to treat in general terms the development and characteristics of navigation on the Great Lakes.

For sheer size, the lakes are tremendously impressive. With their connecting channels they form a natural transportation highway having a water area of more than 95,000 sq miles and a shore line of more than 8,300 miles. From Montreal, Que., Canada, at the head of deep draft ocean navigation on the St. Lawrence River, the sailing distance to the westerly end of Lake Superior is about 1,340 miles and to the south end of Lake Michigan about 1,250 miles.

NOTE.—Published in August, 1951, as *Proceedings-Separate No. 83.* Positions and titles given are those in effect when the paper was received for publication.

[1] Military Constr. Div. of the Office, Chf. of Engrs., Washington, D. C.

Surrounding the lakes is a region rich in many natural resources. At one time the shores were covered with virgin timber. Iron ore, limestone, and coal, also have been found in abundance in the area. Some of the most productive of our agricultural lands lie close to the lake shores. With all these blessings from nature, the Great Lakes area was destined from its very discovery to become an important center of industrial and commercial development. Because of the ease of transportation on the lakes, for example, the French had established outposts as far west as the Saskatchewan Valley, while in the same period the Dutch and English were penetrating only a few miles inland from the Atlantic seacoast. The French, of course, followed the Indians who had settlements along the shores of the various lakes and had worked the copper deposits found in rocky outcroppings. Frail birch bark canoes were the first type of transportation on the lakes used by the Indians, French explorers, and missionaries. The hardy fur trappers, however, built bigger canoes, some holding as many as 9 persons and as much as 8 tons of furs.

Not satisfied with the Great Lakes waterway system as nature left it, these dynamic fur traders made the first improvements to lake navigation facilities. They built a canal and lock to eliminate the portage around St. Marys Falls between Lake Superior and Lake Huron. This canal and lock was built by the Northwest Fur Company in 1797 on the Canadian side of St. Marys River. The lock was 38 ft long, 8 ft 9 in. wide and had a lift of 9 ft. A towpath along the shore permitted oxen to pull the canoes and bateaux through the entrance channels and lock. The remains of this old lock may still be seen at Sault Ste. Marie, Ont., Canada.

In those early days when fur and trade goods were the principal cargoes, oversize canoes and flat bottom bateaux were adequate for the economic needs, and the natural navigation conditions of the lakes required little improvement. The restless march of progress was underway, however, and in 1797 pioneers at Erie, Pa., built the first sailing vessel on the lakes. About 20 years later, the first steam-powered vessel for Great Lakes use was built at Oswego, N.Y. An interesting account of navigational activities during the early days has appeared previously.[2]

FEDERAL ACTIVITIES

As the nation's commercial and industrial tempo quickened and more ships were built, both shippers and the general public demanded better harbor facilities. In consequence, the United States federal government authorized its first river and harbor program in 1824. Federal harbor improvements were authorized on the Great Lakes at Erie, in 1824; at Cleveland, Ohio, and Fairport, Ohio, in 1825; at Buffalo, N.Y., and Ashtabula, Ohio, in 1826; and at Chicago, Ill., in 1833. A few years later, in 1841, Congress authorized a $15,000 appropriation for the charting of 9,500 sq miles of the Great Lakes system. The survey was for the purpose of furnishing charts to lake navigators and determining further improvements necessary to maintain the prosperity of the rapidly growing lake commerce. Under the direction of the Corps of

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[2] "Great Lakes Transportation," by M. C. Tyler, *Transactions*, ASCE, Vol. 105, 1940, p. 167.

Engineers, the Lake Survey has been a continuing work and has to its credit numerous contributions to the science of hydrographic surveying. Among other things, the Lake Survey devised and perfected the "wire sweep" which today is the accepted method of locating underwater obstructions. The Survey designed and built the first metal tower for use in triangulation survey work, the first electric recording current-meter, and it was the first United States governmental agency to print navigational charts in color. Since its inception, the Lake Survey unit of the Corps of Engineers has been the recognized leader in the American effort to facilitate navigation on the Great Lakes.

In the early days, when the federal government first became concerned with improving inland harbors, inter-lake traffic was restricted by physical obstacles between Lakes Erie, Huron, and Michigan. Navigation between Lake Erie and Lake Ontario was blocked by the falls of the Niagara River, and communication with Lake Superior was impracticable because of the falls of St. Marys River. The original Northwest Fur Company lock had been destroyed in 1814. Until 1855, the movement of freight to and from Lake Superior was by boat to Sault Ste. Marie, at which point cargoes were unloaded, portaged across a mile-long tramway, and reloaded on boats. When the State of Michigan completed a 10-ft draft canal around the rapids in 1855, Lake Superior was again opened to the small boats then in use. A few years earlier, traffic had been opened between Lake Erie and Lake Ontario when Canadian private interests built a small canal in the general vicinity of the present Welland Canal.

Vessels Used in Great Lakes Traffic.—As the Great Lakes region developed agriculturally and industrially, lake traffic increased steadily, with a corresponding increase in the number and size of vessels. Generally speaking, the federal government in the ensuing years developed harbors and connecting channels to keep pace with prevalent lake transportation needs. Until 1883, there were more sailing ships than steamers on the lakes. Obviously, the harbors and connecting channels required by light-draft sailing vessels and small side-wheel steamers were quite different from those required for larger vessels. For example, in the days when three-masted schooners plied the lakes, carrying manufactured and package freight to the outposts of the lake region and returning with cargoes of lumber and grain, there was a vital need for many harbors of refuge, to which the light-draft vessels could run for protection when severe storms and squalls made the lakes hazardous. By 1900, the vast majority of the lake schooners had disappeared. If technological progress had not already doomed the colorful sailing ships, their ultimate fate was sealed in 1892 when iron ore from the Mesabi Range was first shipped to lower lake ports. To handle this heavy bulk cargo economically, larger ships with deeper drafts were a necessity. Bigger ships, of course, required deeper channels and better harbors. The demand for these improvements generally followed the flow of commerce on the lakes.

When the ore trade started on the lakes, steamers as well as sailing ships were poorly adapted for ore carrying. They had small capacity and were difficult to load and unload. To Capt. Alexander McDougall goes the credit of designing the highly specialized ship commonly known as the bulk freighter. These ships have the pilot house moved forward and the engines aft, leaving nothing amidships to interfere with the main cargo space. Both the ships and the shore installations for loading and unloading are fully coordinated, each having been designed to fit the other. Hatch openings on most ships have the same spacing as the loading chutes on the ore wharves so as to facilitate loading without moving the ship. Unloading equipment has been designed so that as little time as possible need be spent in port.

More than $\frac{2}{3}$ of the ships on the lakes are over 450 ft long, and some are about 600 ft long, with a beam of 60 ft and a draft of 32 ft. The larger boats carry about 17,000 tons of iron ore at maximum draft. A trend toward even larger boats for the ore trade may be seen in the launching of the *S. S. Wilfred Sykes*, which is 678 ft long, 70 ft wide, and can carry 20,000 tons of ore at 24-ft 6-in. draft. With a loaded speed of 16.5 miles per hr, the *Sykes* will be able to make 44 round trips during the navigation season, as compared with 34 round trips for the average ship in the ore fleet.

Commerce on the Lakes

Discounting the lumber trade that went into a decline about 1910, since the turn of the 20th century at least 80% of the commerce on the Great Lakes has been in four commodities—grain, iron ore, limestone, and coal. Therefore, development of navigation facilities has been influenced to an appreciable extent by the requirements of vessels hauling these cargoes.

In 1948, when lake traffic was the heaviest on record up to that time, iron ore, coal, limestone, and grain composed 86% of the commerce on the Great Lakes. Of a total of 217,548,101 net tons of traffic, iron ore alone constituted 91,791,144 net tons. Most of this ore was mined in the Lake Superior regions, and moved through the Soo Canal Locks to destinations on the southern end of Lake Michigan and to ports on Lake Erie.

Coal accounted for 66,924,551 tons of the lakes' traffic in 1948 and to a large extent reversed the flow taken by iron ore. Most of the coal was shipped from Lake Erie ports, with Lake Superior and Lake Michigan ports as its destination.

Stone, which constituted the third largest tonnage at 22,349,287 net tons, was shipped principally from Lake Huron and upper Lake Michigan ports. While most of the stone shipments were generally southbound to lower Lake Michigan and Lake Erie destinations, a substantial amount of stone also was shipped into Lake Superior.

Grain comprised 6,496,769 tons of lake commerce in 1948. The major portion of this tonnage was shipped from Lake Superior ports, although sizable shipments of corn were made from Chicago and Milwaukee, Wis., on Lake Michigan. More then half the grain carried on the lakes in 1948 was destined for shipment to Boston, Mass.

Of course, in addition to these four major items there are many commodities carried on the lakes. It may be interesting to note that cargoes of timber and pulpwood at one time formed the original bulk freight traffic on the lakes and then practically disappeared, but these shipments are returning. However, the forestry products now are imported mainly from Canada and Scandinavian countries. Package freight, once a thriving form of traffic, appears to be reviving to some extent. All these changes have been reflected in the growth, and sometimes the decline, of harbors on the lakes. To meet the needs of one period in history ports have sprung up, some have changed radically, and others have dropped into obscurity. Two good examples of how harbor growth is affected by changes in commerce are the cases of Grand Marais, Mich., and Holland, Mich.

In the late 1890's Grand Marais was a bustling, prosperous lumber shipping port. In one year the harbor handled 160,000 tons of cargo, mainly forest products. When the virgin forests were stripped, however, the town went into a decline, and in 1947, the only shipments at the harbor were 222 tons of a local catch of fresh fish. Holland also got its start back in the heyday of the lumber trade. In the 1890's Holland harbor in one year handled 307,000 tons of cargo, of which 104,000 tons was lumber. When the lumber trade fell off, Holland harbor had a few bad years but soon kept moderately busy with package freight business, despite competition of rail and highway transportation. The harbor has again been teeming with activity in coal, gasoline, cement, and stone shipments, and in 1947 Holland harbor handled more than 240,000 tons of commercial traffic. Grand Marais still serves a useful purpose as a base for Lake Superior fishermen and as a harbor of refuge for recreational craft and other lake vessels. Therefore, it is desirable that the federal government continue to maintain it for use. Holland harbor, of course, has justified its continued existence as a commercial harbor, eligible for federal maintenance.

NAVIGATIONAL WORKS

Federal navigation works on the Great Lakes, as far as the Corps of Engineers is concerned, have been primarily of two categories—development of harbors and improvement of connecting channels between the lakes. The question of why the federal government assumed the responsibility for river and harbor work is sometimes asked. The answer is quite simple, in that such improvements are costly and the cost usually far exceeds the benefits ordinarily derived by any local community in which a harbor may be located. Take, for example, the harbor at Ashtabula. At that port, millions of tons of coal are shipped and iron ore received but none or very little of it is used locally, most of it being shipped to and from the Ohio-Pittsburgh (Pa.) area. The benefits derived from harbors are national in scope, and Congress decided at a very early date that the national government is responsible for making improvements to harbors and connecting channels. The Corps of Engineers, under authority from Congress, provides and maintains entrance channels, main harbor channels, harbor areas, anchorage areas, and protective breakwaters and jetties required for the needs of general commerce and navigation.

Development of Harbors.—The first improvements undertaken by the United States government on the Great Lakes were mainly the construction of channel entrance works. Because the Great Lakes have few natural harbors, the mouths of the larger rivers entering the lakes and, to a lesser extent, the lee sides of islands and points of land afforded protection and anchorages for the first commercial vessels to ply these waters. The early settlements usually were made at or near these important river mouths on natural lines of communication with the hinterland. Therefore, early harbor improvements consisted largely of building jetties to improve navigation over the bars which normally form at the mouths of streams that enter currentless bodies of water. These works were first built by local governmental agencies, or in some cases by private interests, but since 1825 the major commercial harbors have been improved by the federal government. About 80 active commercial lake harbors, representing an aggregate expenditure for construction and maintenance of about $350,000,000 over a period of about 125 years, have been constructed or improved and are being maintained by the federal government.

Harbor works on the Great Lakes are generally similar in character to those of artificial or partly artificial harbors on seacoasts. They are, however, in some cases lighter in section since they are subject to less severe wave attack. Furthermore, the absence of the marine borer, known as the teredo, and a former abundance of timber along Great Lakes shores has led to a much wider use of timber crib and timber pile construction than is usual in salt water. The fact that the Great Lakes are, to all intents and purposes, tideless bodies of water has also simplified harbor design and construction. Federal works at the harbors usually consist of construction and maintenance of breakwaters and protective piers, and the dredging and maintenance of main navigation channels. While a generalization about harbors on the Great Lakes is difficult because of widely different conditions encountered, it is safe to say that harbor entrances usually are built 500 to 700 ft wide. As a general rule of thumb, it also may be said that the minimum distance from the entrance gap to the nearest dock line is at least 3 to 3.5 boat lengths.

Depths of harbors vary within wide limits, depending on the primary purpose for which they were constructed and their present importance. Harbors in which large ore docks are located, for example, are at least 26 ft deep. In the Chicago area there are five harbors, and each of them is of a different depth, ranging from 18-ft to 28-ft drafts. All harbors are slightly deeper at the entrance to allow for pitch of the ship in open water as it enters the protected area. In developing channels within harbor areas, the Corps of Engineers is guided by the criteria that such channels must be in use by general navigation. In other words, the Corps of Engineers is not authorized to develop private wharves or slips. Before recommending any navigation improvement, the Corps of Engineers, acting only upon directives of Congress, makes a thorough study of the proposal, including the anticipated annual cost of the improvement as compared with anticipated annual benefits. As a general rule, only those improvements in which the benefits may be expected to equal or exceed the charges and in which the benefits tend to accrue to the public at large are recommended for favorable action.

Among the largest of the major improvement projects currently in progress (1951) in the Great Lakes Division of the Corps of Engineers are the harbors at Cleveland, Ohio, and Buffalo. The work at Cleveland consists of improvement of the Cuyahoga and Old rivers by dredging to a depth of 23 ft and the replacement or reconstruction of 7 railroad bridges. The estimated cost of this work is about $19,000,000. At Buffalo the project consists of deepening the north entrance of the harbor to 25 and 26 ft and deepening the Buffalo River and Ship Canal to 22 and 23 ft. This project is estimated to cost approximately $12,000,000.

In these and other harbor projects, local interests are cooperating in making the improvement possible. The amount of local cooperation required varies widely, but the element of local cooperation must always be present. Local interests participate in port development by contributing funds when warranted, furnishing necessary lands, easements, and rights of way, providing adequate terminal facilties, and maintaining berthing areas and approaches to docks. On the other hand, such "local cooperation" is not required in the improvement of channels outside harbor areas. It would clearly be unfair to ask the City of Sault Ste. Marie, for example, to pay part of the costs of the locks at St. Marys Falls, or to ask the little town of Marine City, Mich., to pay for channel dredging in the St. Clair River.

Maintenance of Navigation Channels.—Improvement of navigation channels, particularly those connecting the various lakes, has been a vast undertaking that was started around 1850 and is still continuing. Of special interest from an engineering point of view has been the construction of the Soo Canal Locks and rock excavation and general dredging in the St. Marys and Detroit rivers. The execution of the work in such a manner as not to disturb the water levels in Lake Superior, Lake Huron, and Lake St. Clair has been a delicate and very interesting problem.

Lakes Superior and Huron are connected by the improved St. Marys River, 63 miles long. Because Lake Superior is 22 ft higher than Lake Huron, the Soo Locks and compensation works are necessary. Connection between Lakes Michigan and Huron is by way of the Straits of Mackinac, which have required little improvement to provide suitable depths for navigation. From the lower end of Lake Huron the improved connection with Lake Erie, about 80 miles long with a drop of 8 ft, consists of the St. Clair River, Lake St. Clair, and the Detroit River. In dredging and maintaining these navigation channels, it is common to make channels for loaded ships a minimum of 25 ft deep and channels for empty vessels at least 21 ft deep. At average lake levels, these depths provide for 23-ft draft for loaded oreboats sailing from Lake Superior to Lake Erie, Lake Huron, and Lake Michigan ports, and a somewhat shallower draft for the same boats returning loaded with coal and other products that do not weigh as much per cubic foot as iron ore.

Maintaining these channels to authorized depths is a very exacting and serious matter and a continuous activity of the Corps of Engineers. Because the navigation season on the upper lakes is only about 8 months per year, shippers load their vessels as heavily as possible, consistent with safety and marine regulations. Therefore, a 1-ft or even a 6-in. difference in channel depths is a matter of grave concern from the standpoint of safety as well as of economics. Complicating the problem is the fact that mean annual lake levels fluctuate from year to year—sometimes as much as several feet. The importance of these interconnecting channels to the economic well-being of North America may be judged from the fact that more tonnage passes through the St. Marys Falls Canal Locks than through any other canal in the world—and the further fact that the Port of Duluth-Superior handles a greater tonnage than any other port in North America, excepting the Port of New York.

The greatest amount of annual traffic on the Great Lakes during the period from 1935 to 1950 was 217,548,101 tons in 1948. The total cost of maintenance and operation of navigation channels and structures for the same year was $5,584,906 or about 2.5 cents per ton of commerce. As previously mentioned, the total federal expenditures to date (1950) for navigation works, including operation and maintenance of both harbors and channels, but excluding expenditures for lighthouses, channel markers, and similar aids to navigation, is about $350,000,000. If this entire expense were to be accounted for in one year, it would amount to only $1.60 per ton of freight carried in 1948. The total cost related to the total commerce carried on the lakes since 1825 amounts to only a few cents per ton.

Throughout the years, the savings in freight costs made possible by the water route of the Great Lakes have been many times more than the federal expenditure for improvements and maintenance. Because these benefits have extended to the entire nation and ultimately have resulted in lower prices to the American consumer, no toll or fee has ever been charged for the use of the federally improved navigation facilities which are wholly within the United States. This policy of toll-free waterways is in keeping with the earliest national declaration of policy concerning the use of the lakes and other inland watercourses.[3]

> "The navigable waters leading into the Mississippi and St. Lawrence, and the carrying places between the same, shall be common highways, and forever free as well to the inhabitants of the said territory as to the citizens of the United States and those of any other state that may be admitted into the confederacy without any tax, impost, or duty thereon."

Congress also has repeatedly stated that its policy is to maintain the inland waterways as toll-free highways for commerce. This toll-free policy has contributed materially to the industrial development of the Great Lakes area and to the prosperity of the peoples of the United States and Canada.

The Continuing Need for Lake Commerce

While the facts speak for themselves concerning the value of Great Lakes navigation, questions sometimes arise concerning the long-range aspect. Actually, it is unthinkable that the United States government would fail to provide adequate transportation facilities for the iron and steel industry in the lakes region. Too much of the economic life blood is tied up with the production of iron and steel at competitive prices to permit the collapse of lake navigation for want of adequate facilities. Much is said and written about the future of iron ore sources for this industry. Most authorities say available sources of high-grade ore in the lakes region are limited; others deny it, claiming that it is only required to explore and develop additional deposits in the Mesabi Range or develop methods of using different types of ores. However, the long-term view would indicate that when the time comes, suitable channels for that traffic should be ready, if existing mills in the Great Lakes area are to continue to function.

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[3] "The Ordinance of 1787," The Northwest Territorial Government, The Confederate Congress, July 13, 1787, Article IV.

The St. Lawrence Seaway Project.—Therefore, the key to the long-term future of the Great Lakes commerce is the St. Lawrence River Project. The St. Lawrence Seaway would provide a direct gateway for vessels between the Great Lakes and the sea. Engineering studies including the extensive planning for the St. Lawrence River portion indicate that the problem of providing channels and locks is of considerable magnitude, but traffic estimates made by the United States Commerce Department show that the proposed expenditures for the navigation phases of the St. Lawrence Seaway are economically sound.

From the head of the Great Lakes at Duluth to Montreal, the Great Lakes-St. Lawrence Seaway Project proposes a controlling channel depth of 27 ft with a depth of 30 ft over lock sills. Canadian engineers have exercised great foresightedness in building the Welland Canal with locks of 30-ft depth over the sills, and with 25-ft depths in the canal. These depths in the canal can be increased, as required by future traffic, to provide a 30-ft depth between Lakes Erie and Ontario. Such foresight greatly simplifies the problem of building the seaway. The cost of completing a 27-ft channel project for the St. Lawrence River is estimated at $818,063,000, based on December, 1950, price levels. A seaway with a 27-ft channel can carry a great amount of traffic, as witnessed by activities on the Great Lakes where such depths are in use. The trend in American shipping, however, is to deeper and faster ships for reasons of economy; so studies have been made also of the possibility of constructing the St. Lawrence Seaway with 30-ft and 35-ft channels. The maritime commission estimates that a 30-ft channel could readily accommodate 80% to 85% of the present United States ocean-going merchant fleet with little or no restriction. The cost of completing a 30-ft channel, based on December, 1950, price levels, is estimated at $1,078,815,000, and the cost of a 35-ft project is estimated at $1,840,889,000.

Had the St. Lawrence Seaway been constructed prior to World War II, there is little doubt that combat ships, up to and including the destroyer and cruiser classes, and merchant ships of the Liberty type could have been constructed in the relative security of the Great Lakes area. At Great Lakes ports ships could have been loaded with supplies, munitions, and war armament produced in factories in the lake region, and they could have then proceeded in convoy through the St. Lawrence River and by the Great Circle route to Europe, greatly reducing the submarine hazard.

There can be no question concerning the long-range value of the St. Lawrence project. Some shippers on the Great Lakes, however, have raised the question as to whether opening of the seaway would make present navigation facilities obsolete. Of course, some channels, harbors, wharves, and slips would have to be enlarged to accommodate larger vessels when such are built and enter the Great Lakes from the ocean. This can be accomplished gradually, for it will require several years to complete the seaway after the project is approved and funds are made available. The question of the practicability of using present lake vessels for Labrador ore transportation also has been raised, and has been given careful consideration by the Lake Carriers Association. It is their conclusion that, with some minor changes, the larger ships now in use on the lakes for ore transportation are well adapted for the Labrador run.

Conclusion

As near as can be determined from traffic statistics, commerce on the Great Lakes has a bright future. The guiding policy will continue to advocate a step-by-step development of federal navigation facilities in keeping with shipping needs as evidenced. In all cases, benefits will have to be definitely demonstrated on favorable economic ratios; in this way unwise expansion can be avoided and expenditures reduced to annual amounts within a reasonable budget and in keeping with the other financial burdens on the United States.

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Paper No. 2497

RESINOUS ION EXCHANGERS IN WATER TREATMENT

By William W. Aultman,[1] M. ASCE

Synopsis

Resinous exchangers are rapidly assuming a place of ever greater importance in the field of water conditioning for municipal and industrial purposes. Whereas the more familiar greensands and synthetic zeolites were useful only for removal of hardness and other metallic ions from water, the newer cation exchangers and anion exchangers of the resinous type provide a means of securing any degree of mineral removal up to complete demineralization of a water. The paper describes the application of resinous cation exchangers to water softening, boiler water treatment, process water treatment, and demineralization of brackish water, and discusses costs of several types of treatment.

Introduction

In 1850 J. T. Way,[2] consulting chemist to the Agricultural Society of England, discovered that some soils had the ability to retain the "base," or positive radical, of solutions passing through them while the "acid," or negative radical, passed through unaltered. This appears to be the first recorded case of ion exchange.[3] Mr. Way's discovery was an example of base exchange (cation exchange), as he called it. Since that time many other cation exchangers have been found, and materials having anion exchange (acid exchange) properties have been discovered and developed.

This paper will show how the developments in the field of ion exchange may influence the present concepts of water treatment.

The Meaning of Ion Exchange

Ion exchange may be defined as a reversible interchange of ions between a liquid phase and a solid body. This exchange does not involve any radical

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[1] Water Purification Engr., Metropolitan Water Purification Dist. of Southern Calif., La Verne, Calif.

[2] "The Capacity of Soils to Absorb Manure," by J. T. Way, *Journal*, Royal Agri. Soc., Vol. 11, 1850, p. 313.

[3] "Ion Exchange," by H. C. Walton, *Journal of Chemical Education*, Vol. 23, 1946, p. 454.

change in the solid structure.[4] The ion exchanger, or solid body, must have ions of its own to exchange for others, and it must also have a highly porous structure or large exposed surface to allow these ions to move in and out freely. These characteristics were found to be present in the zeolite minerals, a group defined as

> "Any of a family of hydrous silicates, * * * so called because many species intumesce before the blowpipe * * * In composition they are analogous to feldspars, their chief metals being aluminum, sodium, potassium, and calcium."[5]

Because of cation exchange phenomena were identified for so many years with the zeolites; the term "zeolite" has been used by many persons to include all ion exchange materials. Today cation exchange softening of water is generally referred to as zeolite softening. This is not strictly correct and, with the advent of the organic exchangers, an effort is being made to identify such softening reaction as cation exchange softening. Similarly when anion exchangers are being used such identification is also necessary.

TABLE 1.—CLASSIFICATION OF ION EXCHANGERS

CATION EXCHANGERS		ANION EXCHANGERS	
Inorganic	Organic	Inorganic	Organic
Natural zeolite (greensand) Processed greensand Synthetic zeolite	Natural (the lignites) Sulfonated coal (carbonaceous "zeolite") Synthetic resins Phenolic base Polystyrene base	Metallic oxides	Synthetic resin

The two classes of ion exchangers, the cation exchangers and the anion exchangers, include both inorganic and organic materials.[6] A general classification of ion exchangers is given in Table 1. Both inorganic and organic anion exchangers have been produced, but only the organic anion exchangers are widely used because the inorganic materials do not have good operating characteristics for deionization application.

In general all the cation exchangers may be regenerated with common salt (sodium chloride) and thus be used to remove hardness forming calcium and magnesium from solutions. The organic cation exchangers may also be regenerated with dilute acid solutions and in the resulting hydrogen condition they will remove all metallic cations (sodium, potassium, calcium, magnesium, iron, copper, and manganese) from solution. The anion exchangers, used to remove the acid ions (sulfates, chlorides, and nitrates) from solution, may be regenerated with either soda ash (sodium carbonate), caustic soda, or ammonia.

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[4] "Ion Exchange Between Solids and Solutions," by Harold Frederic Walton, *Journal*, The Franklin Inst., Vol. 232, 1941, p. 306.

[5] "Webster's New International Dictionary," G. & C. Merriam Co., Springfield, Mass., 2d Ed., 1942.

[6] "High-Capacity Cation Exchangers," by F. K. Lindsay, *Journal*, A.W.W.A., Vol. 42, January, 1950, p. 75.

DEVELOPMENT OF ION EXCHANGE PROCESSES

The Cation Exchangers.—The lime and lime-soda ash methods of water softening can be dated before 1850[7] and these methods have widespread application in industrial and municipal water treatment plants throughout the world. With the introduction of the zeolites early in the 20th century, cation exchange softening was initiated.

It was around 1900 that the first industrial water softening plant using natural greensand as the cation exchanger was put into service in Europe. The first municipal zeolite plant was placed in operation in the United States in 1923, at Wyomissing, Pa.[8] Since that time higher capacity cation exchangers have been developed including the processed greensands, synthetic siliceous zeolites, carbonaceous exchangers, and the resinous exchangers. The relative exchange capacities of these various materials are given in Table 2.

TABLE 2.—RELATIVE EXCHANGE CAPACITY OF CATION EXCHANGERS

Cation exchanger (1)	Nominal exchange capacity (grains per cubic foot) (2)	REGENERATIVE SALT DOSAGE	
		Volumetric[a] (3)	Effective[b] (4)
Greensand	2,800	1.26	0.45
Processed greensand	5,500	2.47	0.45
Synthetic siliceous zeolite	11,000	4.95	0.45
Carbonaceous	7,000	3.15	0.45
Resin, phenolic	15,000	6.75	0.45
Resin, polystyrene	28,000	12.60	0.45
Resin, polystyrene	20,000	5.00	0.25

[a] Pounds of salt per cubic foot of exchanger. [b] Pounds of salt per kilograin of hardness removed.

Since they were first introduced commercially, the cation exchangers have held a major place in both industrial and municipal water treatment. At first they were used in water softening only for calcium and magnesium removal. Then it was found that iron and manganese could be removed from water by some of the inorganic exchangers, with manganous sulfate and potassium permanganate used as regenerants, or by the organic exchangers with acid as a regenerant. Many industrial installations used cation exchangers for boiler water treatment, because the water thus produced had zero hardness and did not tend to form a hard scale. However, silica is not removed from the water by this treatment; in fact it is often increased by passing through the siliceous zeolites, and the silica can cause trouble in high pressure boilers.

Over 700 municipalities have installed cation exchange materials in their domestic treatment plants. After thorough investigation the Metropolitan Water District of Southern California (hereinafter called "the District") determined that the lime-zeolite method of treatment was the most economical to use in the 100 mgd water softening plant placed in service in 1941. This plant, doubled in capacity in 1949,[9,10] still utilizes the cation exchangers for softening and is the largest zeolite process softening plant in the world with a total of 56,700 cu ft of cation exchanger installed.[11,12]

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[7] "Water Supply and Treatment," by C. P. Hoover, National Lime Assn., Washington, D. C.

[8] "Review of Performance of Municipal Zeolite Plants," by H. M. Olson, Ohio Salt Company, Wadsworth, Ohio.

[9] "Metropolitan Water District Doubles Capacity of La Verne Softening and Filtration Plant," by R. B. Diemer, *Civil Engineering*, Vol. 19, 1949, p. 669.

[10] "World's Largest Water Softening Plant," by William W. Aultman, *Water and Sewage Works*, Vol. 96, 1949, p. 456.

Resinous Exchangers.—The progress that has been mentioned in the application of the cation exchangers is based primarily on the use of the siliceous or carbonaceous type of exchanger. The advent of the resinous type exchangers introduces an entire new field of use and efficiency and requires a complete reevaluation of the treatment processes.

The development of the resinous exchangers was first reported by the English scientists B. A. Adams and E. L. Holmes in 1935.[13] Four years later the commercial production of these resins was started in the United States. Since that time the development of cation and anion resinous exchangers has been phenomenal. It is believed that the potentialities of these resins are just beginning to be realized.

In addition to the use of the ion exchangers in the water treatment field, the new resinous exchangers are being used extensively in the chemical process industries with an evergrowing field of application. For example, the inorganic ash may be eliminated from sugar juice by passing the juice through a cation-anion treatment plant, thereby materially increasing the yield of crystallizable sugar and reducing evaporator scale and operating difficulties. Small concentrations of the metallic ions, such as copper, nickel, aluminum, etc., may be recovered from solution in more concentrated form by use of an organic exchanger with a regenerant of the desired properties. The resinous exchangers have been used for: (*a*) The treatment of milk for curd modification; (*b*) the treatment of whey for the recovery of lactose; (*c*) the purification of formalin; (*d*) the refining of hydrocarbons; (*e*) the purification of polyalcohols; (*f*) the separation of inorganic cations; and other uses too numerous to mention.[14] This paper, however, will deal only with their use in water treatment.

APPLICATION OF RESINOUS EXCHANGERS IN WATER SOFTENING

The chemical treatment of water, other than for sterilization or coagulation, is usually undertaken for three general purposes: (1) Softening of water for domestic and industrial use; (2) boiler water conditioning; and (3) process water preparation. Under the two latter main headings may fall a number of subheadings depending on the quality of the raw water, the boiler water pressure encountered, and the requirements of the process industry. What is the place of the new resinous exchangers in these three phases of water treatment?

The high-capacity polystyrene resinous exchangers have three major characteristics that cause them to be given special consideration when determining the type of exchanger to use for water softening.

First, their high capacity permits a smaller volumetric installation, with corresponding smaller equipment cost, for a softening capacity equal to any of the other exchangers.

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[11] "Use of Organic Cation Exchangers for Municipal Water Softening," by W. W. Aultman and A. E. Bowers, *Proceedings*, 9th Annual Water Conference, Engrs.' Soc. of Western Pennsylvania, 1948, p. 1.

[12] "Cation Exchangers for Municipal Water Softening," by Lee Streicher and A. E. Bowers, *Journal*, A.W.W.A., Vol. 42, January, 1950, p. 81.

[13] "Absorptive Properties of Synthetic Resins," by B. A. Adams and E. Leighton Holmes, *Journal of the Society of Chemical Industry*, Vol. 54, 1935, p. 1T.

[14] "Ion Exchange," by Robert Kienin, *Industrial & Engineering Chemistry*, Vol. 42, 1950, p. 65.

Second, if the water being softened is clear, its quality in other respects has, almost without exception, no deleterious effect on the nonphenolic exchangers. They are not affected by a pH within the range of 1 to 14, or a temperature of more than 200° F, and the chemical composition of the water does not tend to break down the exchanger. This is not true of the siliceous synthetic zeolites that should be operated within a pH range of 6.0 to 8.3, at temperatures not exceeding 140° F, and will be dissolved slowly by waters low in silica.

Third, the operating efficiency of the polystyrene resin is appreciably higher than any other type of exchanger in commercial use.

In actual operation of the District plant every attempt is made to operate the exchangers as efficiently and economically as possible. For the synthetic siliceous zeolites an average exchange capacity of 8,700 grains per cu ft is obtained at a regeneration salt requirement of 0.37 lb of salt per kilograin of hardness removed. The polystyrene resins, on the other hand, have a capacity of 22,000 grains per cu ft when regenerated at the rate of 0.24 lb of salt per kilograin of hardness removed. Thus, in comparison with the synthetic siliceous zeolite, the polystyrene resins show 2.5 times the exchange capacity with only $\frac{2}{3}$ the salt consumption per kilograin of hardness removed.

What does this mean in dollars and cents? For example, the Colorado River water, as received at the District softening plant, had an average hardness in 1950 of 321 ppm or 18.8 grains per gal. Salt costs the District $4.46 per ton delivered (1950). To soften 1,000,000 gal of this water to zero hardness with zeolite requires 6,956 lb of salt (costing $15.51) although it requires only 4,512 lb of salt (costing $10.06) when a polystyrene resin is used. The resulting net saving is $5.45 per million gallons. If the salt cost $9.00 a ton, or more, as it does at most places in the United States, the saving would exceed $11.00 per million gallons. This is a saving in salt alone and does not include savings in rinse water quantities or lower equipment cost.

The saving in salt, however, is offset to some extent by the increased cost of the resin. The relative cost of zeolite and polystyrene resinous exchanger is approximately proportional to the exchange capacity of the two materials, the cost of the resin being about 2.5 times that of the zeolite. But the useful life of the two exchangers is not equal. Under identical operating conditions, using Colorado River water, the life of zeolite is found to average about 2,000,000 per cu ft of exchanger whereas the life of the polystyrene resin is in excess of 5,000,000 gal per cu ft of exchanger. With all factors considered, an appreciable saving is shown by the use of the polystyrene exchanger, rather than zeolite.

For those cities located near the ocean another possibility of reducing the cost of treatment is available by using the polystyrene resin with ocean water as the regenerant. Although this is not a new scheme (there are several zeolite plants now using sea water for regeneration), it is the first time that relatively high-capacity exchange has been available and is obtainable with a small amount of sea water circulation. A new plant at Ventura, Calif., uses sea water for regeneration. The exchange capacity of the resin at various dosages of sea water for regeneration is shown in Fig. 1. It is evident that exchange

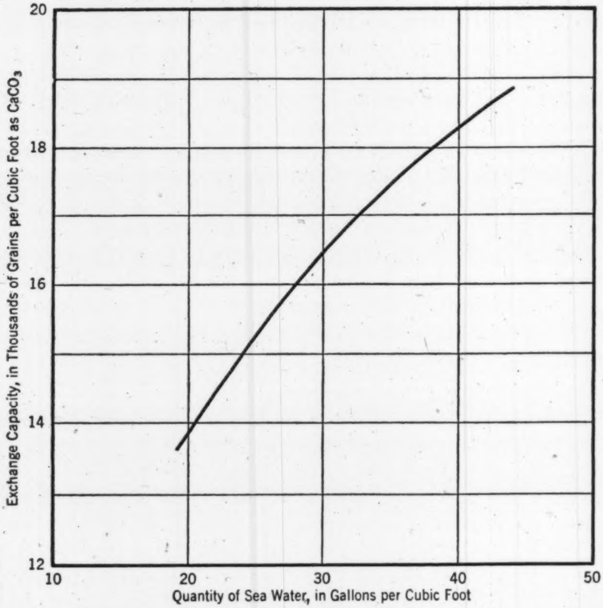

FIG. 1.—EFFECT OF SEA WATER REGENERATION ON THE OPERATING EXCHANGE CAPACITY OF A POLYSTYRENE RESIN.

capacities in excess of those obtained from any inorganic exchanger are possible with this polystyrene resin under these conditions.

RESINOUS EXCHANGERS IN BOILER WATER TREATMENT

The factors pertaining to the use of the resinous exchangers for municipal water softening are likewise true for water softening in connection with boiler water conditioning. However, there are additional factors in boiler water treatment, particularly in high-pressure boilers, that must be considered.

Boilers require a water low in silica, but the true zeolites, either natural or synthetic, are siliceous materials. Water passing through these zeolites tends to pick up silica, the degree of pickup depending upon the silica content, temperature, pH, and other characteristics of the water. This silica pickup is so great at times that municipal plants for treatment of domestic water have added silica to the raw water to protect the zeolite.[15] Carbonaceous exchangers have been extensively used for boiler water treatment to avoid the introduction of silica from siliceous materials. This, likewise, is an advantage of the resinous exchangers. With the highly basic resinous anion exchangers available, silica may be removed from water, and this is an important forward step in the processing of boiler waters.

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[15] "Operating Characteristics of Synthetic Siliceous Zeolite," by L. Streicher, H. E. Pearson, and A. E. Bowers, *Journal*, A.W.W.A., Vol. 39, 1947, p. 1133.

The ability of the resinous cation exchangers to withstand temperatures in excess of 200° F makes them suitable for use in plants in which hot water is processed for boiler feed purposes in order to conserve heat.

At times it may be desirable to partly or wholly demineralize the water used in a boiler. This can be accomplished with the resinous cation and anion exchangers whose operation will be described subsequently.

USE OF RESINOUS EXCHANGERS FOR PROCESS WATER TREATMENT

Many process industries require a very pure or a very uniform quality of water. When low salinity water has been required in the past, distillation units have usually been employed. With the advent of the organic cation and anion exchangers a water equal in quality to distilled water can be produced, generally at a lower cost.

In the demineralization process, water is passed through an organic cation exchanger that has been regenerated with dilute acid, that is, a unit operating on the hydrogen cycle. In this unit all of the cations in the water (calcium, magnesium, sodium, potassium, and dissolved iron and manganese) are absorbed on the exchange material, and hydrogen ions are released from the resinous exchangers. The water is then passed through an anion exchanger unit where the negative ions in the water (chlorides, sulfates, and nitrates) are absorbed on the exchanger and the carbonate or hydroxyl ions are released, depending upon whether soda ash or caustic soda is used for regenerating. The resultant water consists essentially of H_2O and CO_2. The carbon dioxide (CO_2) may be removed by aeration, leaving a pure water. Recently developed resins, operating in suitably designed equipment have produced demineralized water having a specific electrical resistance of between 10,000,000 and 20,000,000 ohms per cu cm. This compares with good quality distilled water with a resistance of 500,000 to 1,000,000 ohms per cu cm. The chemical purity of the water is proportional to its electrical resistance, 1 ppm of $CaCO_3$ salinity in water representing a specific resistance of about 460,000 ohms.

The cost of demineralization is directly proportional to the salinity of the water being treated. The cost of distilled water, however, is affected very little by the quality of water being used, except for the production of scale within the distillation unit. At the chemical prices prevailing in the California area the cost of demineralizing a water containing 400 ppm total dissolved salines is about $0.30 per 1,000 gal. The cost of distilling such water is from five to ten or more times this cost, depending upon the type of distillation equipment being used and its size. As the salinity of the water being treated increases, the cost of demineralization increases proportionately, until a point is reached where demineralization by means of the resinous cation and anion exchangers becomes greater than the cost by distillation methods.

From the foregoing discussion it is evident that a water of almost any quality desired may be produced by the resinous cation exchangers operating on either the sodium or the hydrogen cycle coupled with the resinous anion exchangers. This gives the sanitary or chemical engineer an excellent tool to use in solving some of the intricate problems of water quality presented by many of the process industries.

DEMINERALIZATION OF BRACKISH AND SEA WATERS

With the publicity given the advances in the science of demineralization many laymen seem to think that it is a simple and inexpensive matter to produce a potable water from brackish or salt waters. Where cost is not a consideration it must be admitted that the demineralization of sea water is a possibility. A cost of $5.00 per pint for drinking water produced from sea water by special ion exchangers aboard life rafts during World War II was inconsequential when it meant saving men's lives. However, the cost of demineralizing water for municipal or industrial use must compare favorably with the cost of local sources of supply to be given serious consideration.

Considerable research on the partial demineralization of the brackish waters of Palestine[16] developed a new, cheaper method of operation for large-scale treatment of waters containing from 1,000 ppm to 2,500 ppm total dissolved salines, enabling the production of a potable water of 300 to 500 ppm total salines. This is accomplished by using counterflow of water and regenerant solutions with lime as the anion exchange regenerant. Lime regeneration has been tried on pilot plant scale only, but it appears to operate satisfactorily. With this method of treatment it appears possible to produce a water suitable for domestic use at a cost that is within the realm of economic feasibility, if the natural water contains no more than about 2,500 ppm total dissolved salines.[17]

Unfortunately, sea water contains from 32,000 to 36,000 ppm total salines, and to produce a potable water from this source with the established methods of demineralization would cost about $25.00 per 1,000 gal for chemicals alone (1950). Vapor compression distillation is much cheaper than demineralization of sea water, but with the equipment available water so produced would still cost over $1.25 per 1,000 gal when all factors are taken into consideration.[18,19] Perhaps more efficient means of producing potable water from sea water may evolve if Congress authorizes appropriations for research on this problem, but available scientific knowledge does not indicate that the cost of such reclaimed water can ever approach that of locally developed water supplies within the United States.

SUMMARY

The development of the new resinous cation and anion exchangers may influence the present concepts of water treatment in the following respects:

a. Waters of almost any desired quality may be produced for municipal and industrial use with the economic feasibility of treatment depending upon the quality of the raw water being treated.

b. The quality of the water being treated has little or no effect on some of the resinous exchangers, a factor of considerable value in water conditioning for boilers and the process industries.

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[16] "Ion Exchange Process for Brackish Waters," *Chemical and Engineering News*, Vol. 27, 1949, p. 3306.

[17] Private communication from Ionics, Inc., Cambridge, Mass., January 11, 1950.

[18] "Fresh Water from Salt," by W. W. Aultman, *Engineering and Science Monthly*, Vol. XII, 1949, p. 3.

[19] "Desalting Sea Water for Domestic Use," by William W. Aultman, *Journal*, A.W.W.A., Vol. 42, August, 1950, p. 786.

c. Because of their high salt efficiency, the polystyrene resins operating on the sodium cycle will warrant serious consideration in any studies to determine the best method of softening any specific water.

d. Silica may be removed from water with comparative facility and at a reasonable cost.

The development of the resinous ion exchangers is in its infancy, and all engineers in any way concerned with water treatment when using these materials should keep abreast of the progress made in the perfection of operating techniques and procedures.

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Paper No. 2498

PRESSURES IN A SHALLOW RECTANGULAR BIN

BY RAYMOND L. MOORE,[1] A. M. ASCE,
AND J. R. SHAW[2]

SYNOPSIS

In the design of shallow rectangular bins to support granular materials, some estimate must be made of the distribution and magnitude of the pressures involved. If the bin is used to transport granular materials that are subjected to impact or vibration during movement, the final pressures will be different from the initial pressures. The tests described were undertaken to obtain some information regarding the magnitude and possible variations in such pressures. Strain and deflection measurements were made on the sides and bottom of a small bin for loads of wheat, gravel, and cement under both static and dynamic conditions; and direct comparisons were made with corresponding determinations for static water loadings that provided known pressures.

INTRODUCTION

The investigation described in this paper was undertaken to obtain information on (1) the bulging pressures produced by granular materials in shallow containers and (2) the increases in pressure likely to occur if these materials are compacted by impact or vibration. The problem arose in connection with the design of light-weight railway freight cars but would seem to be of equal interest in the design of other types of containers used to transport granular materials.

In the design of railway freight cars the usual practice is to compute lateral bulging pressures by means of the Rankine formula,[3] assuming a full load of wheat with an angle of repose of 25°. On this basis the bulging pressure is computed to be 41% of that which would be obtained if the wheat were re-

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[3] Report of the Committee on Car Construction (Div. 5, Mechanical) through its Subcommittee on Fundamentals of Car Design, *Proceedings*, Am. Ry. Assn., 1925, p. 332.

placed by a fluid of the same weight. The distribution of pressure is assumed to be hydrostatic, varying uniformly from zero pressure at the top to a maximum at the bottom. Although it scarcely seems reasonable to believe that such a loading adequately covers all cases of interest in the design of railway cars, there appears to be no basis on which to judge the merits or shortcomings of this simple design rule. The assumption of full fluid pressure offers a possible solution, but in view of past experience such a procedure seems unduly conservative.

This paper describes an attempt to evaluate experimentally the over-all pressure effects produced on the sides and bottom of a small rectangular bin by loads of water, wheat, common river gravel, and cement. The granular samples were selected to represent widely different classifications of material and were used in both loose and vibrated conditions. The basic assumption underlying the investigation was that a direct comparison of the stresses and deflections produced in the bin by a water loading (providing known pressures) and of the corresponding values produced by the granular materials would provide an approximate basis for evaluating relative pressure effects.

A secondary but nevertheless interesting feature of the tests was that they afforded an opportunity to study the structural behavior of the bin itself under a known water-pressure loading. Where possible, comparisons have been shown between measured and computed stresses and deflections to illustrate the basic types of action involved.

Description of the Test Bin

The container for the tests was a small, open-topped rectangular bin, constructed of aluminum alloy sheet and shapes. Fig. 1, showing the test bin bolted to the vibration table, gives the essential structural details. The sides and bottom were fabricated from aluminum alloy sheets by welding, and all stiffening members were attached by riveting. The capacity of the bin was 30 cu ft (inside dimensions, 30 in. by 36 in. by 48 in.).

Procedure

The first step in the investigation was to obtain a measure of the maximum stresses and deflections produced in the principal elements of the bin by water loading. Subsequently, the bin was filled with wheat, gravel, and cement, and all or parts of this stress and deflection survey were repeated.

The results of two different series of loading tests are reported. The first series provided a general survey of the static stresses and deflections produced by loadings of water, wheat, and gravel. The second series showed the influence of vibration and settlement of the materials on the magnitude of the principal stresses and deflections. Dry cement as well as wheat and gravel were used in these tests.

Fig. 1 shows the nature of the vibration tests. The table was designed originally for tests of loaded gasoline tanks and consisted of a bed plate (supported by springs at the corners) to which was attached a rotating shaft carrying an unbalanced flywheel. The rotation of the shaft at about 1,800 rpm produced vibration amplitudes of 1/64 in. to 1/32 in. under the bin loads. This action was sufficient to cause an appreciable settling of the materials. The vibration of the bin was interrupted at intervals to permit the measurement of strains and deflections, and a test was stopped when it was evident that no further increases in these determinations would be obtained. Although the total time of vibration was several hours in some cases, the maximum stresses and deflections were obtained after a relatively short interval. Total settlements of as much as 2 in. to 3 in. were obtained from a level starting position in some of the tests. The quantity of material in the bin, rather than the depth of contents, was considered to be the significant factor in these tests, however, and no additions to the original "full" load were made.

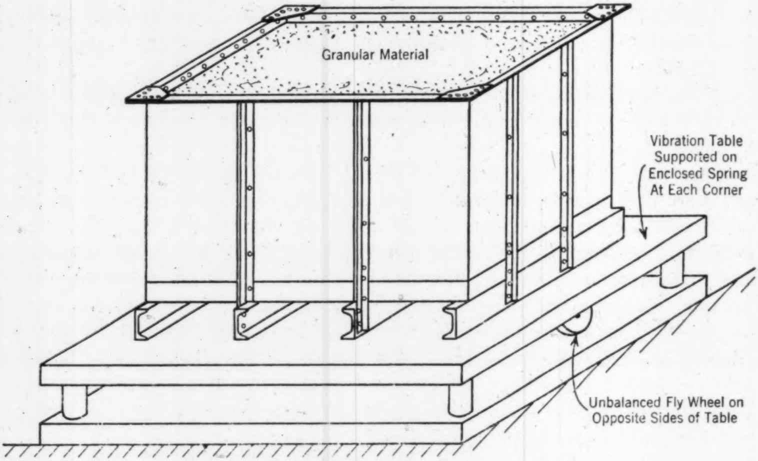

Granular Material
Vibration Table Supported on Enclosed Spring At Each Corner
Unbalanced Fly Wheel on Opposite Sides of Table

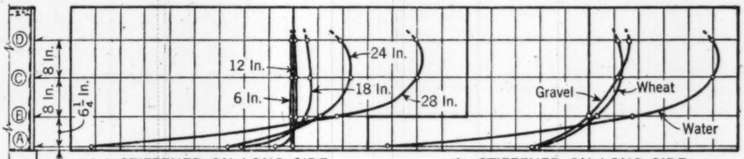
Fig. 1.—Apparatus Used in Vibration Tests

The measurements on the bin included three types of determination: (*a*) Lateral deflections of the side panels, vertical stiffeners, and top rail; (*b*) bending stresses in the side and bottom panels and in the vertical stiffeners and top rail; and (*c*) vertical deflections of the bottom.

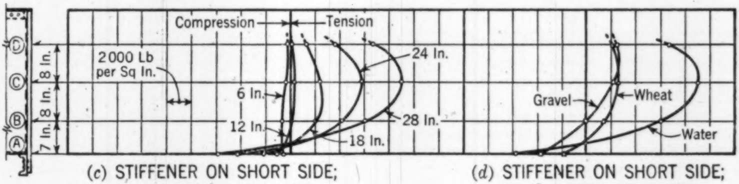
Lateral deflections were measured by means of a dial indicator and suitable reference bar or by means of mirrored scales attached to the member in question with a fine reference wire stretched between its ends. The vertical deflections of the bottom, relative to the floor, were measured with a dial indicator.

Strains were determined by means of SR-4 bonded wire, resistance-strain gages, and an SR-4 strain indicator. The corresponding stresses were computed, using a modulus of elasticity of 10,000,000 lb per sq in. for the sheet and 10,500,000 lb per sq in. for the stiffeners and top rail. Distinctions in moduli of elasticity were necessary because all parts of the bin were not made of the same aluminum alloy. A value of Poisson's ratio equal to 1/3 was used where strains at a point were measured in two directions, 90° apart, and where it was necessary to consider biaxial stress relationships.

Test loads were applied by increments up to a maximum depth of fill of 28 in., in most cases, or to within 2 in. of the top of the bin. Although care was taken not to compact the material in the initial filling, it was evident, from a comparison of some of the stresses and deflections obtained for the same kind of loadings in the two series of tests, that quite different lateral pressures were obtained in a few cases. For each increment of loading the top surface was leveled off, and the corners completely filled before any measurements were taken.

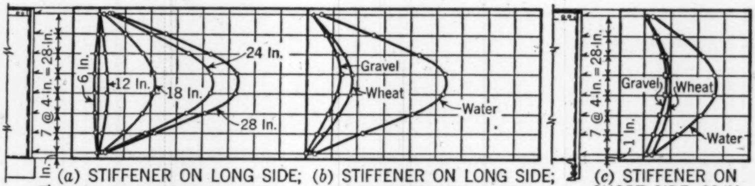


(*a*) STIFFENER ON LONG SIDE; VARIOUS DEPTHS OF WATER (*b*) STIFFENER ON LONG SIDE; 28-IN. DEPTH OF WATER

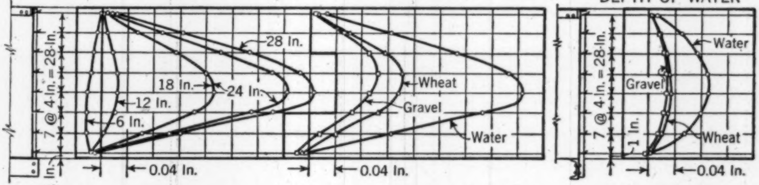


(*c*) STIFFENER ON SHORT SIDE; VARIOUS DEPTHS OF WATER (*d*) STIFFENER ON SHORT SIDE; 28-IN. DEPTH OF WATER

FIG. 2.—STRESSES MEASURED IN THE STEM OF THE VERTICAL STIFFENERS



(*a*) STIFFENER ON LONG SIDE; VARIOUS DEPTHS OF WATER (*b*) STIFFENER ON LONG SIDE; 28-IN. DEPTH OF WATER (*c*) STIFFENER ON SHORT SIDE; 28-IN. DEPTH OF WATER



(*d*) SHEET ON LONG SIDE BETWEEN STIFFENERS; VARIOUS DEPTHS OF WATER (*e*) SHEET ON LONG SIDE BETWEEN STIFFENERS; 28-IN. DEPTH OF WATER (*f*) SHEET ON SHORT SIDE BETWEEN STIFFENERS; 28-IN. DEPTH OF WATER

FIG. 3.—LATERAL DEFLECTIONS MEASURED IN THE VERTICAL STIFFENERS AND SIDE SHEETS

RESULTS AND DISCUSSION

Analysis of Lateral Pressures.—The data presented in Figs. 2, 3, 4, and 5 and in Tables 1 and 2 indicate the general behavior of the bin in the static tests and emphasize the fundamental difference between the lateral pressures produced by water and those produced by granular materials. (For the location of sections in Table 2, see Fig. 5(*a*).) The maximum static stresses and deflections, measured in the sides under a 28-in. depth of water, exceeded by a wide margin those produced by the same depth of wheat and gravel. Differences

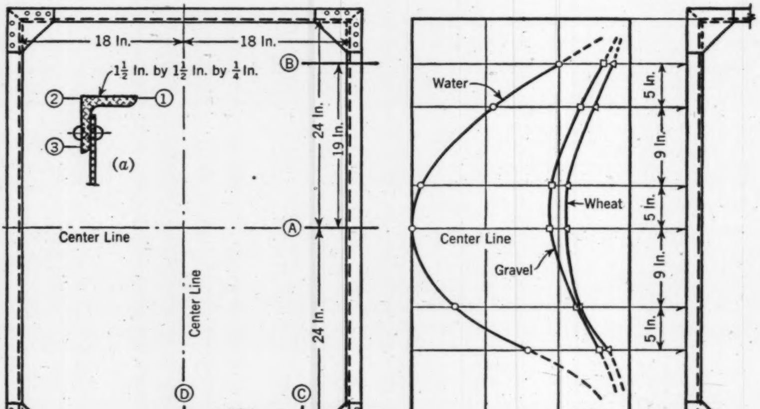
TABLE 1.—STRESSES IN POUNDS PER SQUARE INCH IN TOP RAIL FOR 28-INCH DEPTH OF WATER

Section (see Fig. 4)	POINT 1. FIG. 4(*a*)		POINTS[a] 2 AND 3	
	Measured	Computed	Measured	Computed
(1)	(2)	(3)	(4)	(5)
A	+3,200	+4,500	−2,400	−2,000
B	−1,700	−1,800	+900	+800
C	−2,100	−2,700	+1,200	+1,200
D		+200		−100

[a] Average of stresses observed at points 2 and 3, Fig. 4(*a*).

TABLE 2.—MEASURED STRESSES IN POUNDS PER SQUARE INCH AT CENTER OF BOTTOM FOR 28-INCH DEPTH OF LOADING

Material	PARALLEL TO x − x AXIS		PARALLEL TO y − y AXIS	
	Top side	Under side	Top side	Under side
(1)	(2)	(3)	(4)	(5)
Water	−10,100	+12,100	−3,000	+4,400
Wheat	−9,500	+10,800	−2,300	+4,000
Gravel	−9,500	+10,400	−2,800	+3,500


18 In.
18 In.
$1\frac{1}{2}$ In. by $1\frac{1}{2}$ In. by $\frac{1}{4}$ In.
②
①
③
(*a*)
Ⓑ
24 In.
19 In.
Ⓐ
Center Line
Center Line
24 In.
Ⓓ
Ⓒ
13 In.
Water
Wheat
Center Line
Gravel
5 In.
9 In.
5 In.
9 In.
5 In.
0.02 In.

(*b*) PLAN VIEW OF TOP SHOWING LOCATION OF STRAIN-GAGE SECTIONS Ⓐ TO Ⓓ ON TOP RAIL

(*c*) LATERAL DEFLECTIONS OF TOP RAIL OF LONG SIDE UNDER 28-IN. DEPTH

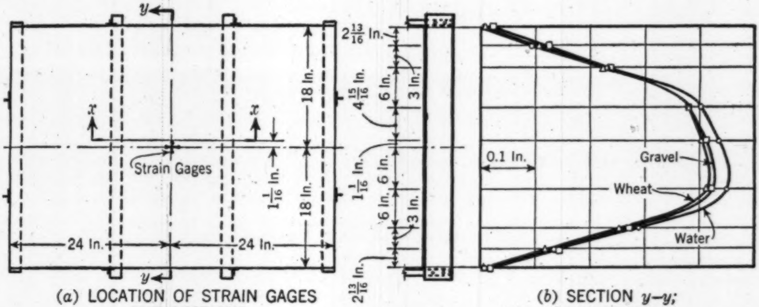
FIG. 4.—STRESSES AND DEFLECTIONS IN TOP RAIL

between the lateral pressures exerted by various granular materials are emphasized by the fact that the stresses and deflections under a 28-in. depth of wheat exceeded in many cases those obtained for a 28-in. depth of gravel weighing almost twice as much.

Table 3 lists the maximum stresses and deflections measured in the first series of tests and gives the corresponding fluid-pressure factors, determined by the following relation:

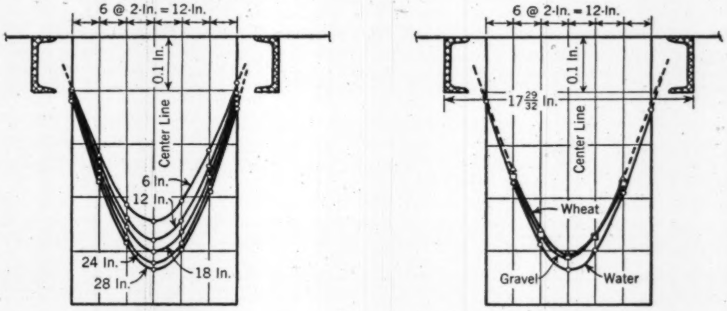
$$F = \frac{s_g}{s_w} \times \frac{\gamma_w}{\gamma_g} \qquad (1)$$

in which F is the fluid-pressure factor; s_g is the stress (or deflection) produced by granular material; s_w is the stress (or deflection) produced by water; γ_w is the weight of water; and γ_g is the weight of granular material. The fluid-pressure factor, introduced for comparative purposes, is the ratio of the pressure produced by the granular material to the pressure produced by the same depth of a liquid of equal weight (see Table 4). It will be noted that the factors


y
x
x
Strain Gages
18 In.
18 In.
$1\frac{1}{16}$ In.
24 In.
24 In.
y
$2\frac{13}{16}$ In.
$4\frac{15}{16}$ In.
6 In.
3 In.
$1\frac{1}{16}$ In.
6 In.
6 In.
3 In.
$2\frac{13}{16}$ In.
0.1 In.
Gravel
Wheat
Water

(*a*) LOCATION OF STRAIN GAGES AND SECTIONS

(*b*) SECTION y–y; 28-IN. DEPTH



(*c*) SECTION x–x; VARIOUS DEPTHS OF WATER

(*d*) SECTION x–x; 28-IN. DEPTH

FIG. 5.—MEASURED VERTICAL DEFLECTIONS IN BOTTOM OF TEST BIN

TABLE 3.—SUMMARY OF MEASURED DATA AND COMPUTED FLUID-PRESSURE FACTORS

Loads[a]	Test Series I Before Vibration				Test Series II Before Vibration				Test Series II After Vibration			
	Stress[b] (lb per sq in.)	Factor[c] F	Deflection (in.)	Factor[c] F	Stress[b] (lb per sq in.)	Factor[c] F	Deflection (in.)	Factor[c] F	Stress[b] (lb per sq in.)	Factor[c] F	Deflection (in.)	Factor[c] F
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
(a) VERTICAL STIFFENER ON LONG SIDE; STRESSES ¾ INCH FROM THE BOTTOM; AND DEFLECTIONS 17 INCHES FROM THE BOTTOM												
Water	−16,400	1.00	0.212	1.00	−14,800	1.00	0.197	1.00	−14,800	1.00	0.197	1.00
Wheat	− 4,400	0.33	0.069	0.40	− 4,900	0.39	0.067	0.40	− 6,300	0.50	0.108	0.64
Gravel	− 4,600	0.17	0.053	0.16	− 8,800	0.42	0.128	0.46	−10,600	0.51	0.175	0.63
Cement	....	....	....	....	− 6,500	0.33	0.098	0.38	− 9,700	0.50	0.176	0.68
(b) VERTICAL STIFFENER ON SHORT SIDE; STRESSES 15 INCHES FROM THE BOTTOM; AND DEFLECTIONS 17 INCHES FROM THE BOTTOM												
Water	+ 8,900	1.00	0.108	1.00	....	....	....	....	....	....	....	....
Wheat	+ 2,600	0.36	0.039	0.44	....	....	....	....	....	....	....	....
Gravel	+ 2,200	0.15	0.033	0.19	....	....	....	....	....	....	....	....
(c) SIDE SHEET AT THE MIDDLE PANEL ON THE LONG SIDE; DEFLECTIONS 13 INCHES OR 17 INCHES FROM THE BOTTOM												
Water	....	....	0.324	1.00	....	....	0.324	1.00	....	....	0.324	1.00
Wheat	....	....	0.141	0.53	....	....	0.163	0.59	....	....	0.228	0.82
Gravel	....	....	0.103	0.20	....	....	0.250	0.55	....	....	0.301	0.66
Cement	....	....	....	....	....	....	0.215	0.50	....	....	0.310	0.72
(d) SIDE SHEET AT THE MIDDLE PANEL ON THE SHORT SIDE; DEFLECTIONS 13 INCHES FROM THE BOTTOM												
Water	....	....	0.090	1.00	....	....	....	....	....	....	....	....
Wheat	....	....	0.033	0.45	....	....	....	....	....	....	....	....
Gravel	....	....	0.028	0.19	....	....	....	....	....	....	....	....
(e) TOP RAIL AT THE CENTER OF THE LONG SIDE												
Water	+ 3,200	1.00	0.060	1.00	....	....	....	....	....	....	....	....
Wheat	+ 1,350	0.52	0.018	0.37	....	....	....	....	....	....	....	....
Gravel	+ 1,600	0.31	0.022	0.23	....	....	....	....	....	....	....	....
(f) TOP RAIL ALONG THE SHORT SIDE, 13 INCHES FROM THE CENTER												
Water	− 2,100	1.00	....	....	....	....	....	....	....	....	....	....
Wheat	− 600	0.35	....	....	....	....	....	....	....	....	....	....
Gravel	− 600	0.18	....	....	....	....	....	....	....	....	....	....
(g) BOTTOM SHEET AT THE CENTER OF THE MIDDLE PANEL												
Water	+12,100	1.00	0.434	1.00	+11,500	1.00	....	....	+11,500	1.00	....	....
Wheat	+10,800	1.09	0.405	1.14	+11,600	1.18	....	....	+11,500	1.17	....	....
Gravel	+10,400	0.54	0.409	0.58	+11,000	0.68	....	....	+13,800	0.85	....	....
Cement	....	....	....	....	+13,100	0.86	....	....	+14,100	0.92	....	....

[a] For total weight of loads at different depths refer to Table 4. [b] Maximum stress in pounds per square inch. [c] Fluid-pressure factor; see Eq. 1.

given for wheat, based on measurements on the vertical stiffeners and on the side sheets of the bin (Tables 3(*a*), 3(*b*), 3(*c*), and 3(*d*), Cols. 3 and 5) ranged from 0.33 to 0.53. The mean of this observed range compares quite favorably with the factor of 41% for wheat computed in accordance with the usual freight car design specifications. The corresponding observed factors for the gravel loading were appreciably lower, ranging from 0.15 to 20.0.

TABLE 4.—TOTAL WEIGHT OF LOADS IN POUNDS, CORRESPONDING TO STRESSES AND DEFLECTIONS IN TABLE 3

Test series (Table 3)	Depth, in inches	MATERIAL			
		Water	Wheat	Gravel	Cement
(1)	(2)	(3)	(4)	(5)	(6)
I	28	1,750	1,430	2,830	....
II	26 to 29	1,780	1,520	2,500	2,350

Table 3 also lists the stresses and deflections and corresponding fluid-pressure factors obtained in the second series of tests. It appears from a comparison of results that some rather marked differences in lateral pressure were obtained under conditions of loading that were supposed to be about the same. The first time the bin was filled with gravel, for example, the maximum measured stress in the stiffener on the long side was 4,600 lb per sq in. Under the second loading, in which the total weight of material was about 10% less than in the first tests, the stress at the same point was 8,800 lb per sq. in. The measured lateral deflections for the stiffener on the long side indicated approximately the same wide difference in pressures. For the wheat loadings in the two series of tests, however, the stresses and deflections were in close agreement. These results emphasize the complex nature of the problem considered and the difficulty of obtaining a very exact solution.

The vibration tests demonstrated that, when a bin loaded with granular materials was vibrated, the lateral pressure increased. The maximum stresses measured in the vertical stiffener on the long side indicated fluid-pressure factors after vibration of 0.50 to 0.51 for all three granular materials (Col. 11, Table 3), whereas the corresponding range of factors before vibration was 0.17 to 0.42 (Cols. 3 and 7, Table 3). Factors based on the maximum measured deflections of the vertical stiffener and the side panel midway between stiffeners on the long side ranged from 0.63 to 0.82 (Col. 13, Table 3) after vibration, as compared to a range of 0.16 to 0.59 (Cols. 5 and 9, Table 3) before vibration.

Fig. 6 shows the range of fluid-pressure factors indicated for the sides of the bin for all loadings summarized in Table 3. Corresponding computed factors, determined by the Rankine formula, are included for comparison. The Rankine formula is

$$P = \left(\frac{W H^2}{2}\right)\left(\frac{1 - \text{Sin}\,\phi}{1 + \text{Sin}\,\phi}\right) \dots\dots\dots\dots\dots\dots (2)$$

in which P is the total lateral pressure per horizontal foot, in pounds; H is the depth of material in feet; and ϕ is the angle of repose in degrees. A value of ϕ equal to 25° was assumed for wheat in accordance with the procedure usually followed in railway car design. Values of ϕ equal to 30° and 35° were selected for gravel and cement, respectively, as being within the range of values commonly quoted for these materials.

The diagrams in Fig. 6 indicate several significant trends regarding lateral bulging pressures. It will be noted that the range of observed fluid-pressure factors for the loose loadings (before vibration) were generally above those computed by the Rankine formula. Close agreement between pressures produced by different lots or loadings of the same kind of materials should not be expected. Gravel and cement are apparently more susceptible to such variations than wheat, which was generally the most "fluid" in its behavior. Vibration of the test bin resulted in definite increases in bulging pressures over those observed in the initial static loading. It is significant that the mean fluid-pressure factor indicated for wheat after vibration was about 50% greater than the computed value. Moreover, the factors for cement and gravel were almost equal to that for wheat. On the latter basis, cement and gravel may produce considerably greater bulging pressures than wheat, a possibility which apparently is not recognized at present in railway car specifications. The findings of these tests, therefore, indicate that a lateral fluid-pressure factor of $F = 0.6$ should be assumed for all the granular materials considered, where it is evident that impact or vibration may be encountered.

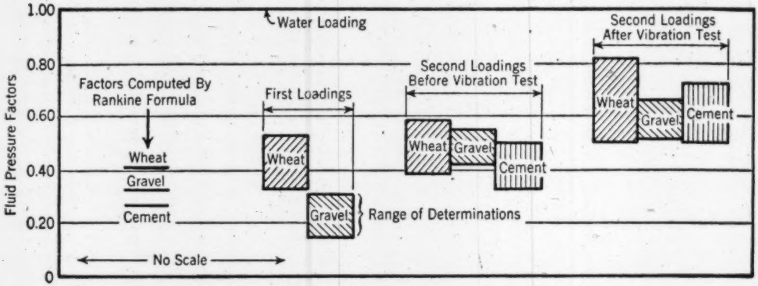


FIG. 6.—RANGE OF OBSERVED FLUID-PRESSURE FACTORS

The measurements of stress (shown in Table 2) and deflection (shown in Fig. 5) on the central panel of the bottom reflect a behavior quite different from that which would normally be assumed in design. Nevertheless, the action was probably quite typical of that to be found in thin-walled, flat-sided containers under small normal pressures. The central panel of the bottom had an initial upward arc of almost 5/16 in. Under the first 6-in. increment of load this bow "snapped out," and the panel acquired a sag almost as great as the initial upward bow. In such a position the bottom was subjected to membrane action, involving direct stresses as well as bending, much sooner than if the downward deflections had increased gradually from a flat starting position.

As will be noted in Tables 2 and 3, the differences between the measured stresses and deflections for the bottom under the water and the granular-material loadings were not nearly as marked as those observed for the sides of the bin. Consequently, the behavior of the bottom did not provide as sensitive a basis for estimating fluid-pressure factors. Although the factors shown for wheat were slightly greater than unity, this result is interpreted to mean that the pressure on the bottom was close to the full fluid-pressure value. Vibration of the wheat resulted in no significant change in pressure on the bottom. Cement and gravel, which are generally considered to be less fluid than wheat under static conditions, showed an increase in fluid-pressure factor under vibration, the final values ranging from 0.85 to 0.92. As far as the practical design of the bottom of shallow containers under these types of loading is concerned, it is believed that a value of $F = 1$ should be assumed; that is, the bottom should be designed to carry the full weight of the material supported.

Analysis of Structural Behavior.—The attempt made to analyze the measured stresses and deflections at a few locations in the interest of design procedures has given some insight into the actual structural behavior of the bin. The vertical stiffeners exhibited the highest stresses, and for that reason they were perhaps the elements most susceptible to a study of this kind. Three factors are involved in any computation of stiffener stresses: (1) The amount of side sheet effective with the stiffeners in bending; (2) the conditions of restraint at the ends of the stiffeners; and (3) the total load carried by the stiffeners, assuming a hydrostatic distribution of pressure over their depth.

Effective widths of side sheet depend on the type of loading, the length of the stiffener, and the number and effectiveness of the connections between the stiffener and the sheet. These widths were evaluated experimentally by comparing the average measured stresses in the stiffener flanges with those measured in the outstanding stem. It was assumed that the point of zero

TABLE 5.—ANALYSIS OF LATERAL BENDING STRESSES IN VERTICAL STIFFENERS UNDER 28-INCH WATER LOADING

Strain-gage section number (1)	Distance from bottom of bin, in inches (2)	AVERAGE MEASURED STRESSES,[a] POUNDS PER SQUARE INCH		Effective width, in inches (5)	Indicated bending moment, pounds-inches (6)	Computed bending moment, pounds-inches (7)	CORRESPONDING COMPUTED STRESSES, POUNDS PER SQUARE INCH	
		On stem (3)	On flange (4)				On stem (8)	On flange (9)
				(a) STIFFENER ON LONG SIDE				
1	¾	−16,400	....	0	−358	−358	−16,400	+5,900
2	7	+ 3,500	+100	3.9	+ 99	+ 96	+ 3,400	+ 140
3	15	+10,000	+100	2.2	+272	+286	+10,500	0
4	23	+ 9,900	+ 50	2.2	+270	+181	+ 6,600	0
				(b) STIFFENER ON SHORT SIDE				
1	0	− 5,800	....	0	−135	−135	−5,900	+1,170
2	7	+ 6,000	+150	3.9	+170	+158	+5,600	+ 230
3	15	+ 8,900	+250	3.9	+254	+240	+8,500	+ 360
4	23	+ 6,700	....	3.9	+190	+164	+5,800	+ 240

[a] The location of strain measurements on the stem is shown in Fig. 2. Measurements were made at the edges of the flanges at the same sections.

bending stress and the location of the neutral axis in bending at a given section could be determined by linear extrapolation from these measured stresses. On this basis the effective widths were estimated to range from zero at the bottom of the stiffener to a maximum value of 3.9 in. Values for all sections are listed in Table 5.

The end restraint at the bottom of the stiffeners was estimated from the shape of the bending stress-distribution diagrams shown in Fig. 2. Since the stresses in the outstanding stem were not particularly sensitive to the effective width of sheet for a given value of bending moment, it was assumed that the point of zero moment coincided with the point at which the stress-distribution diagrams for the water loading passed through zero stress. These locations corresponded to degrees of end fixity of about 80% for the stiffener on the long side (where the outstanding stem was riveted to the channel base support) and of 40% for the stiffener on the short side, where only the flange was riveted to the base member. The moment at the top of the stiffeners was assumed to be zero in both cases.

The total lateral pressure carried by the stiffeners was determined as that necessary to produce a bending moment at the bottom (Section A, Fig. 2) consistent with that indicated by the measured stresses on the stem of the stiffener at that section. For stiffeners on both the long and short sides of the bin the width of loaded strip necessary to provide this moment was roughly only about one half that which would have been used if the total lateral load on each side had been distributed between the ends and the intermediate stiffeners.

Table 5 compares the stresses measured in the stiffeners and those determined by computation for the conditions of effective width, end fixity, and loaded width indicated previously. It appears from the generally good agreement shown that a fairly accurate evaluation of these design factors was obtained.

It should be pointed out that the maximum measured lateral deflection of the stiffener on the short side of the bin was within a few percent of that computed on the basis of the same assumptions made in the computation of stresses. The deflection computed for the stiffener on the long side, however, was only about one half of that measured. The latter result does not indicate as marked an inconsistency as might first appear, however, since the deflections and stresses were not measured on the same stiffener. Apparently there was considerable difference in the loads carried by adjacent stiffeners on the long side investigated.

As shown in Table 1, the stresses measured in the top rail were in fair agreement with those computed, assuming that the top acts as a continuous rectangular frame of uniform moment of inertia. The question of effective width of sheet was again involved, but, in view of the location of the point of attachment of the sheet to the vertical leg of the top angle, it seemed reasonable to neglect this factor. The maximum lateral deflection computed for the top rail on the long side was also in good agreement with that measured. The lateral load on each rail was assumed to be uniformly distributed along the length and equal to one third of the total pressure on the side below it.

In computing maximum stress and deflection in the bottom, assumptions as to end conditions were made. Fig. 7 demonstrates the assumptions for the center of the middle panel. The comparison of measured and computed stresses and deflections shown for the bottom of the bin in Fig. 8 illustrates the marked influence that initial out-of-flatness may have on the behavior of sheets under small normal pressures. The initial upward bow of the central panel, previously referred to, greatly altered the action of the sheet from that which would normally be assumed. Nevertheless, the observed relations between loads, deflections, and stresses paralleled quite closely those computed, assuming that the central bottom panel acts as a flat plate bent to a cylindrical surface.[4] If the sheet had been flat in the beginning, it appears that a fair agreement between measured and computed stresses and deflections would have been obtained.

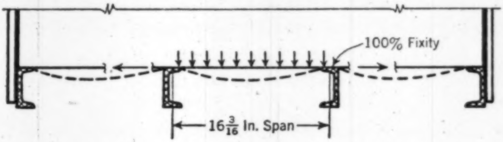
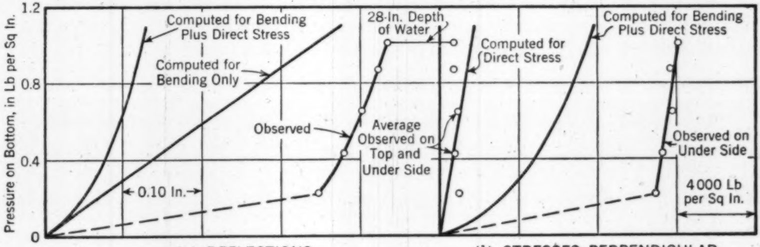


FIG. 7.—SECTION THROUGH BOTTOM OF BIN



(*a*) DEFLECTIONS

(*b*) STRESSES PERPENDICULAR TO CHANNEL SUPPORTS

FIG. 8.—STRESSES AND DEFLECTIONS AT THE CENTER OF THE BOTTOM

SUMMARY AND CONCLUSIONS

Although this investigation of bin pressures was undertaken in the interest of railway freight car design, the results are believed to be equally applicable to the design of other types of shallow containers for granular materials. The following observations regarding the pressures indicated in the test bin for wheat, cement, and gravel loadings, in both loose and vibrated conditions, appear most significant:

a. The lateral bulging pressures indicated by the loose granular loadings (before vibration) were generally somewhat higher than those computed by the Rankine formula, using accepted values of angle of repose;

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[4] "Strength of Materials," by S. Timoshenko, D. Van Nostrand Co., Inc., New York, N. Y., 2d Ed., Part II, 1941, p. 119.

b. The lateral pressure produced by any given granular material may vary over quite wide limits, depending on the manner in which the container is loaded and on whether the material, after being placed, is compacted by impact or vibration;

c. In the design of shallow containers where vibration is known to be a factor, it is proposed that the lateral bulging pressure be taken as 60% of that computed for a fluid having the same weight as the granular material considered;

d. The bottom of such containers should be designed to carry the full weight of the loading;

e. The measurements on the bottom of the bin indicate that the structural action of the panels between stiffeners was complicated considerably by the initial waviness of the sheet. Since this is a common characteristic of thin-walled, welded containers, it appears that attempts at refined or exact analyses of the bending action of side or bottom panels under small normal pressures are not justified;

f. The analysis of the observed behavior of the vertical stiffeners and of the top rail (which were the basic strength elements of the bin) indicate the importance of such considerations as effective widths of sheet, end restraint, and load distribution in the design of members of this type.

ACKNOWLEDGMENTS

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REGIME THEORY FOR SELF-FORMED SEDIMENT-BEARING CHANNELS

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With Discussion by Messrs. Emmett M. Laursen, Gerard H. Matthes, and Thomas Blench

Synopsis

A dynamical framework—that of regime theory—on which to base a study of the behavior of channels that have formed boundaries from their own transported material or material of like nature is given. The most important system of natural channels to which the theory can be applied is that of the rivers of alluvial plains. Rivers, in general, are not susceptible to quantitative analysis except for certain broad effects. The theory has grown from the unrivaled mass of data provided by the irrigation canal systems of northern India. These canals were built by British engineers in India at the end of the nineteenth century and were extended regularly until they formed the largest composite body of canals in the world.

It is shown that canals of the type considered provide formulas for the self-formation of width, depth, and slope. Rivers require two more formulas to give the general pattern of meandering. The developed laws fit into the framework of accepted hydraulics and generalize it. Although these laws apply to rivers, exactly as to canals, the disturbing influences in the former require judgment in the application of the formulas. Self-forming rivers cannot be described adequately in terms of the one flow formula of rigid-boundary hydraulics.

Regime-Type Rivers

Rivers of alluvial plains have the peculiarity that they build their own boundary conditions out of their transported material or out of material transported by previous rivers. In detail the action is complicated; but the over-all results, when studied in terms of a suitable time scale, show a definite

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pattern of behavior consistent with a theory of regime that will be expounded in this paper.

The term "regime" was adopted many years ago, before there was any definite theory, by irrigation engineers who dealt with canals of fairly constant discharge, excavated in alluvium and carrying a sediment load. The term, which implies both an obedience to certain laws and a considerable freedom of action, is an appropriate one. To declare that a river has acquired a regime is comparable in a general sense to stating that a territory has acquired a climate. The vagaries of the weather from day to day are great and have not yielded to exact analysis. Nevertheless, a regime of weather such as a climate is recognizable; and reasonably definite laws have been discovered, such as those causing the trade winds, those limiting rainfall, and those affecting temperature.

A regime-type river is defined as one that has formed a major part of every cross section from material that has been transported or could be transported at some stage of flow.

A river is said to be "in regime" in a reach if its mean measurable behavior during a certain time interval does not differ significantly from its mean measurable behavior during comparable times before or after the given interval.

These definitions will help to simplify the discussion of river behavior. A few examples of classification will be given:

(*a*) Suppose that a record of river gages for twenty years, corresponding to a discharge of 200,000 cu ft per sec, is plotted against time. If this "specific gage" record shows a definite trend, then the river is not in regime, but the behavior is consistent with its classification as a "regime type." If the record shows no trend, the river is practically certain to be in regime during the period for all factors provided it is of regime type as previously defined.

Regime implies a time scale. A record of a few weeks would be worthless, and to require a record of centuries would probably disqualify every channel on earth from passing the test for regime. This peculiarity has caused considerable controversy in the more exact applications of the term. Actually there is no need for controversy since the difficulty arises from a lack of sense of proportion. All measurements of physics imply a scale. A simple example is the case of measuring the velocity of flow in a channel, using a current meter. Measurements occupying a few seconds are inconsistent because of turbulence. Measurements over a period of a few minutes are satisfactory and are now used universally because they yield values practically suitable for an average velocity. Measurements over a period of many minutes would be influenced by variations of the mean and would prove useless. There is no dispute as to the practical existence and utility of a "mean velocity," although rigorous logic might prove that no such quantity existed. As a further example of the importance of a scale, in conditioning viewpoint, engineering lengths would not be measured by a scale designed to indicate molecular subdivisions.

(*b*) Suppose that observations were made monthly for several years along the logitudinal sections of a main line canal presumed to be in regime and that they always indicated practically the same result. This is remarkable behavior for a regime canal, and the chances are that the section is not of regime type. It would probably be lined with stiff clay without a sand cover. Beds of regime channels usually oscillate seasonally about equilibrium. Channels excavated in rigid material and sections that prevent deposition or scour are not of regime type. However, an entirely cohesive boundary may have become stable after erosion and have no sand deposit, in spite of sand being available in suspension, because the section is too constricted to permit deposition. This case is unusual, and, if the observer did not know that the scour had occurred, he would be justified in classing the channel as a nonregime type if he were interested in channels with sand beds. If he were interested in the history of clay-boundary channels carrying sand-free water and if he knew that scour had occurred, he would classify the channel as being of regime type. A rock channel would always be nonregime when free from deposition.

(*c*) Suppose surveys of a considerable length of river were available every year for thirty years, and suppose the mean meander length of fully developed meanders were calculated from plans every year. If the first fifteen-year mean came to 1 mile plus or minus a standard deviation of 20%, and if the second fifteen-year mean came to 1.2 miles plus or minus 15%, it would indicate that these particular calculations are consistent with regime so far as meandering is concerned. (The question whether such a test makes the best use of the statistics does not matter for the present illustration.)

(*d*) Suppose that a torrential river has a shingle bed that does not move except in high flood. In low stage the shingle is masked by very fine sand. Such a river is of regime type. It can be said to be in one regime when running low and moving the sand by saltation and in another regime when in spate (freshet flow) with the sand in suspension and the shingle rolling. To decide whether the river is in regime the best measure would be to record "specific gages" at different points. Practically, it would not matter much whether the gages were for high stage or low stage. Such a river would not be found in alluvial plains as they are generally defined.

Enough has been said to indicate the use of the term "regime" and to show that even simplified cases may introduce complications. The question now arises as to how much is known, without special regime theory, of behavior that would indicate the existence of simple laws. The following statements should meet with no objection:

(1) Osborne Reynolds observed[2,3] that, in regime-type rivers, small discharge meant relatively great depth with respect to width, and he concluded that the exaggeration in the slope of small channels was the same as the exaggeration of depth-to-width ratio. This is borne out by the experience of river model experimenters, who have all found that the bed material refuses to move without vertical exaggeration;

(2) It is a general observation that, aside from the effect of material, large rivers have relatively flat slopes and great widths compared with depth;

(3) Coarse material is associated with steep slope and greater width-to-depth ratio on an equal discharge basis;

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[2] "Uniform Flow in Alluvial Rivers and Canals," by Gerald Lacey, *Minutes of Proceedings*, Inst. of Civ. Engrs., London, England, Vol. 237, 1933–34, p. 421.

[3] "Stable Channels in Alluvium," by Gerald Lacey, *ibid.*, Vol. 229, 1929–1930, p. 259.

(4) Large rivers have large meanders; small rivers have small ones;

(5) The varieties of meander pattern occurring in large rivers find their counterpart in small rivers.

Apparently, simple laws are at work in regime-type rivers; yet there are many examples of engineers who fail to foresee retrogression below dams on such rivers and aggradation upstream from the reservoirs. Perhaps the best hint of the nature of one of the laws lies in the analogy between "smooth-boundary" flow in pipes and regime-type flow. In the former case a simple fluid forms its own boundary with its own substance so that the resistance law is expressible in terms of viscosity without any reference to the material of the pipe. In the latter case a water-sediment complex arranges itself so as to form a water-sand bed zone of considerable complexity and sometimes to build a pair of banks of fine sand, silt, and clay. Consequently, the flow formula for regime-type rivers might be expected to be a generalized case of that for rigid "smooth" boundary pipes, and this is precisely what has been found (see Fig. 1).

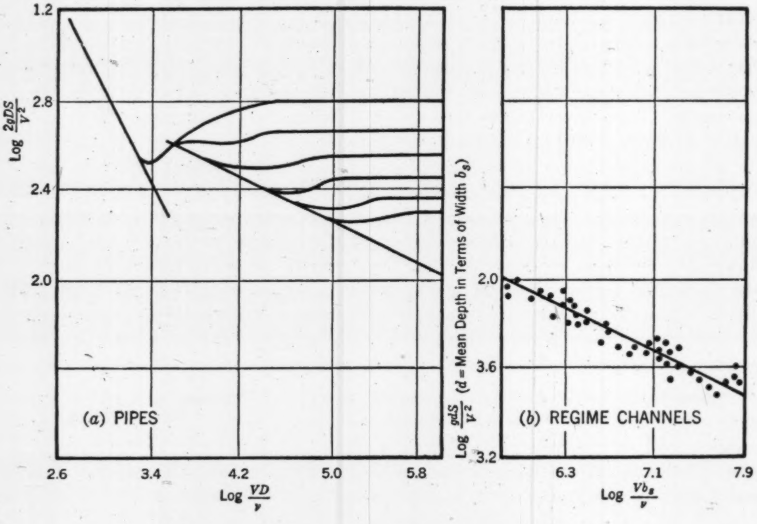

FIG. 1.—FLOW FORMULAS, GENERALIZED REGIME THEORY

GROWTH OF REGIME THEORY

The vagaries of discharge and the heterogeneous nature of banks render regime-type rivers practically useless for analysis aimed at deriving quantitative laws. This explains why regime theory originated on the great canal systems of India and Pakistan, where discharges are controlled and meandering is prevented. As these canals provide what is virtually the laboratory of regime science, a history of the growth of the theory will be presented. Special reference is given to the canal systems of Punjab Province, India, with 70,000 cu ft per sec of authorized supply and 20,000,000 acres of irrigation. The United Provinces (India) and Sind (Pakistan) provide comparable amounts. The channels of all three provinces are mostly in alluvium and derive water from regime-type rivers without storage. The American counterpart of one of the Indian canals is the All-American Canal, Arizona-California. The Punjab, being the major irrigating province, affords most information. Major canal work started in the 1880's with two canals of 7,000 cu ft per sec and 9,000 cu ft per sec capacity. They were excavated in alluvium on the basis of the Kutter formula, using arbitrary slopes and widths. They were characterized by an excessive sedimentation—worse in the one with the lesser ground slopes—that persists to this day. These difficulties caused R. G. Kennedy[4] to study the relation between velocity and depth in some channels that gave no trouble, and he produced a formula which, unfortunately, is still quoted:

$$V = c\,d^N \qquad (1)$$

in which V is the mean velocity, d is the mean depth, and c and N are variables. Mr. Kennedy produced design diagrams that give the simultaneous solutions of the Kutter equation and Eq. 1. As canals have three self-adjusting variables, he was one equation short; so engineers were still free to design erroneously but with only one degree of ignorance instead of two. In the course of time unofficial rules of thumb were produced to make up for the lack of a third equation. Mr. Kennedy was unlucky in having chosen channels of different sediment grades and side conditions; so his formula is nonrepresentative of any simple physical condition and has not been used in India since 1930.

In 1903 three more canals were completed in India with capacities of 4,500 cu ft per sec, 9,000 cu ft per sec, and 11,000 cu ft per sec, respectively. One exhibited serious sediment trouble on one branch but not on the other, because engineers did not understand regulator designs that could divide sediment suitably. Another developed very severe trouble largely because of failure to understand the effect of approach conditions on sediment distribution in the river. The third developed trouble later, in spite of initially adequate slope, because of heavy sediment entering from level crossings provided for drainage torrents. This condition has been overcome by bed ejectors. In 1914 a feeder canal with a capacity of 13,000 cu ft per sec was built to divert adjacent river water into a system with a capacity of 7,000 cu ft per sec. The feeder had adequate slope and has given little trouble, but the main line of the receiving system became scoured, and material settled in the laterals.

The result of the experiences on all these canals was a statement by E. S. Lindley,[5] A. M. ASCE, crystallizing in 1919 the principles that had been forced on engineers in their losing battle against nature:

> "When an artificial channel is used to convey silty water, both bed and banks scour or fill, changing depth, gradient and width, until a state of

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[4] "The Prevention of Silting in Irrigation Canals," by Robert Greig Kennedy, *Minutes of Proceedings*, Inst. of Civ. Engrs., London, England, Vol. CXIX, 1894-95, p. 281.

[5] "Regime Channels," by E. S. Lindley, *Minutes of Proceedings*, Punjab Eng. Congress, Lahore, India, Vol. 7, 1919.

balance is attained at which the channel is said to be in regime. These regime dimensions depend on discharge, quantity and nature of bermsilt, and rugosity of the silted section; rugosity is also affected by velocity, which determines the size of wavelets into which the silted bed is thrown."

Although details of the statement may be criticized, the point to notice at present is that the existence of three self-adjusting variables had been accepted, with nature choosing the magnitude of each to suit the discharge and sediment load. Mr. Lindley also stated the following results of observations made throughout the feeder canal (13,000 cu ft per sec) while he was in charge of it:

$$V = 0.95\, d^{0.57} \qquad \text{(2a)}$$

and

$$V = 0.57\, b^{0.355} \qquad \text{(2b)}$$

in which b = average width of the channel. He still accepted the Kutter formula, and apparently he did not suspect that the exponents indicated physically significant laws based on the values $\frac{1}{2}$ and $\frac{1}{3}$.

A system of canals from four diversion dams was completed in the latter half of the 1920's. Although the Kennedy design diagrams were still accepted officially, some engineers had obtained knowledge published soon thereafter and had used it unofficially so that these canals gave little sediment trouble.

The major theoretical step followed the United Provinces Government's placing Gerald Lacey on special duty to analyze data to find the laws of regime. The results are available[2,3,6,7] although not very easy to apply as written. One account that is applicable in practice is an unavailable Indian publication. The formulas have been accepted officially by the Central Board of Irrigation of the Government of India since 1934,[8] although they had considerable use before that date.

Since the introduction of the Lacey theory, the Irrigation Research Institutes of the provinces of India and Pakistan have subjected it to most searching field tests by choosing channel field sites and observing them at intervals of a few days for several years. The results vindicate the exponents in the Lacey equations but show that the constants depend on sediment grade; the single "silt factor" F is not adequate. In fact the insertion of field data into the equations derived by Mr. Lacey gives values of F that are found to be functionally related. In 1941 the writer and C. King showed a method of separating F into a bed factor and a side factor,[9,10,11] and these factors have been found to apply more consistently than the original silt factor F. This generalized Lacey theory is the one presented in this paper.

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[6] "A General Theory of Flow in Alluvium," by Gerald Lacey, *Journal*, Inst. of Civ. Engrs., London, England, Vol. 27, 1946, p. 16.

[7] "The Behaviour and Control of Rivers and Canals," by Sir Claude C. Inglis, *Publication No. 13*, Govt. of India Central Waterpower, Irrig. and Navigation Research, Yeravda Jail Press, Poona, India, 1949.

[8] Constitution of the Research Committee of the Central Board of Irrigation with a Complete List of Resolutions of Research Committees up to and Including the 10th Research Committee in July, 1940; Central Board of Irrig., Simla, India, 1940.

[9] "Effect of Dynamic Shape on Lacey Relations," by T. Blench and C. King, *Annual Report* (*Tech*), Central Board of Irrig., Simla, India, 1941.

[10] "Practical Design Formula for Stable Irrigation Channels," by C. King, *Annual Report* (*Tech*), Central Board of Irrig., Simla, India, 1943.

[11] "Theorie de L'Ecoulement Turbulent," by T. Blench, *La Houille Blanche*, N. S., No. 3, May, 1946, p. 163.

A canal system built in India in 1936 with a capacity of 5,000 cu ft per sec was designed entirely according to the Lacey theory and has given no appreciable trouble. A system twice as large (10,000 cu ft per sec) was designed similarly in 1947. The old canal systems have been remodeled considerably and with notable success according to Lacey' principles since the inception of the theory.

It may be said that, nowadays, armed with regime theory, the engineer finds what the canals want to do, and he does it first. With care he can adapt the theory for rivers; but without an enormous range of canal information he never would have discovered regime theory.

DATA SUPPORTING REGIME THEORY

The Lacey regime theory[2,3,6] was developed from observations on both canals and rivers. The generalized theory used in this paper rests on the analysis of canal data alone, although it is noted that the behavior of rivers is consonant with the findings from canals. The findings of the Lacey theory, as supported by canals, have been accepted in general (although modified in detail) so that his data are used implicitly. Data taken from field observations of Irrigation Research Institutes of the provinces and others are used in addition. The scope and nature of the Lacey and subsequent data are given in eight items as follows:

a. The canals from which data are accepted were assumed to be in regime. Because of the various sources of information used, some of the data accidentally refer to steady but not regime sections. It should be clear from the beginning that perfect regime does not exist in nature any more than perfect equilibrium exists, but there is no doubt about the existence of a practical regime. Observations on a practical regime channel may be likened to observations of the position of a planet describing an equilibrium path with slight perturbations. Sufficient observations, when plotted, indicate the major law of equilibrium.

b. The regime canals observed possessed berms. A berm is a step in the bank, at about the elevation of full-supply level, and it exists whether the section is excavated or has deposited its own sides with engineering assistance to ensure regularity and no meandering. Common practice is to excavate the sides below berm level to a slope of 1 on $\frac{1}{2}$, this being about the inclination of self-formed sides. Sometimes sides are excavated at slopes of 1 on 1. Maintained sides acquire a slight curvature. Maintenance consists of periodic cutting back to prevent meandering (the sides usually grow in again by deposition) and cutting back of grass edges that would tend to overhang. Usually the side material is of silty clay-loam nature.

c. The beds of the regime canals observed were covered by clean sand that moved in dune or ripple formation. Fig. 2, from a model of selective deposition flume shows the high-level bed in typical formation. Apart from the dunes the bed, in straight canal reaches, was of visibly uniform elevation across any section; cross sections plotted from soundings showed a slight dishing. There is no doubt about the distinct nature of the sides and bed nor the legitimacy of referring to the depth *d* (depth from full-supply level to practically flat bed) instead of a mean depth. There have been references to ideal elliptic sections; they find no support from observations on channels with cohesive sides.

d. Discharges in the records ranged from about 5 cu ft per sec to 10,000 cu ft per sec with most of the data between about 20 cu ft per sec and 500 cu ft per sec. Standard practice is to run laterals to full supply or close them. Laterals technically include those canals with discharges up to 500 cu ft per sec, but few exceed 150 cu ft per sec. Channels in excess of 150 cu ft per sec have short-supply periods, but the bed is then unlikely to be active, and there will seldom be appreciable suspended load. Some large systems operate only during the high river season; therefore, a large supply does not necessarily mean that the channel has appreciable periods of short supply. Generally, however, the data for channels with flows as great as 500 cu ft per sec may be expected to give least scatter due to variable discharge. The discharge used is usually the authorized full supply. Channels with capacities of 20 cu ft per sec or less have rather poor discharge accuracy if measured by current meter, but discharge measurement will be accurate if the channels are fitted with meter flumes.

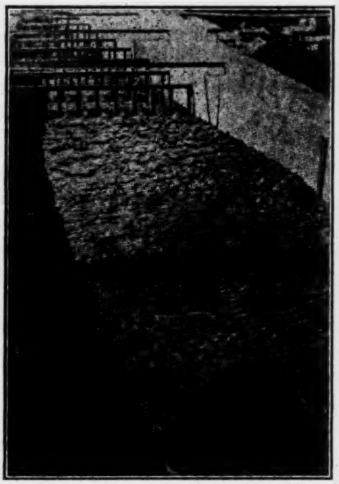
FIG. 2.—TYPICAL BED DUNES AND RIPPLES

e. The temperature variation of water was from about 50° F to 85° F, causing a considerable variation in viscosity. Turbidity caused a small but appreciable change of viscosity. Fortunately most formulas contain viscosity raised to a small fractional power.

f. The sediment grade exposed on the bed was from about 0.1 mm to 0.5 mm, the vast preponderance of sample mean values being approximately 0.25 mm. The sediment in suspension during a year varied from extremes of zero to about 1% by weight of flow and seemed to have little effect on the movement of bed material. However, suspended sediment is important in berm formation. The movement of bed material in the saltation phase is probably about $\frac{1}{100}\%$ by weight of the water flow.

g. The important Reynolds number is the one in terms of width, and it ranges from about 10^5 to 10^8. The square of the Froude number in terms of depth ranged from about 0.5 to 1.5 times $1/g$. Width to depth ratios varied between 5 to 1 and 30 to 1.

h. The data included torrents transporting material up to approximately 4-in. size; but the generalized theory does not rely on these sizes. There is no reason for excluding coarse material merely on the grounds that it does not saltate like fine material. This peculiarity would alter the form of any relation between bed factor (defined subsequently) and size and other factors of bed material, but should not alter relations between V, d, b, slope, and bed factor. The grounds for objection are that channels with coarse material are usually those with variable discharge, and they do not present definite cohesive banks and reaches free from meandering.

If the scope and nature of the data are not appreciated, there will be timidity in extrapolating where permissible, and possibly error will be committed by extrapolating to small models where the sides mask the bed completely.

GENERALIZED REGIME THEORY

In a complete exposition it would be necessary to give an account of the Lacey theory without which the present generalized theory would not exist. However, such a course would require too lengthy a dissertation. Therefore the presentation here will refer to the steps of development, with acknowledgment to Lacey theory, and will otherwise ignore that very valuable treatment. However, it should be mentioned that (presumably because of the nature of the data available from records at the time) Mr. Lacey used a single sediment factor F, thereby effectively obtaining results for a physically possible set of channels whose relative importance of side action to bed action had been "averaged out" to some arbitrary value. This permitted him to work in terms of wetted perimeter (P) and hydraulic radius (R). Mr. Lacey sought simple exponents as indicated by his plottings within the limits of statistical significance. The exponents in the Lacey formulas are suggestive of relations that have physical interpretations consistent with the findings of rigid boundary hydraulics. As the channels are generally wide compared with depth, an analysis in terms of depth (d) and mean width (b) will yield the same exponents within the limits of statistical significance. This fact justifies replacement of P and R, as used in the Lacey formula, by b and d in the generalized theory. Mr. Lacey always used P and R.

The first equation of generalized regime theory is[2]

$$\frac{V^2}{d} = B \qquad (3)$$

in which V is mean velocity of flow, d is depth from water level to flat bed, and B is a bed factor related to the nature of the bed load and constant as long as the factors relevant to bed load are constant. The formula is accepted as exact when constant discharge, constant bed load, and regime conditions have been attained. Dividing both sides of Eq. 3 by g (the gravitational constant) shows that the Froude number in terms of depth is constant when the bed load is fixed. Obviously Eq. 3 provides the means for defining a dynamically satisfying bed factor.

The formula was derived by continuing the Kennedy line of investigation (Eq. 1) with more complete and accurate data. From channels of many sizes in one system V and d were plotted logarithmically against each other. When the formula was indicated (with an exponent close to 2) for one system, then another system with an obviously different type of sediment load was analyzed, and so on. It is noteworthy that some care must be taken in choosing channels at random within a system. Thus channels of a given discharge range must be selected from different parts of a system, for some systems have channels equipped with head regulators that take more than a fair share of bed sediment, so that the bed factor drops progressively from head to tail of the system some 150 miles away. With proper random sampling the resulting plot should give a line corresponding to an average bed factor, more or less representative of the bed load entering at the head of the main line.

The second equation of generalized regime theory is

$$\frac{V^3}{b} = s \qquad (4)$$

in which b is chosen for convenience as the width that, multiplied by d, gives the cross-sectional area of flow. Eq. 4 defines a side factor, s, that (after some dimensional adjustment explained subsequently) measures the square of the tractive force intensity over the sides. Eq. 4, like Eq. 3, may be taken as exact for perfect regime conditions and is practically applicable to real regime conditions.

The formula was derived by theoretical argument[9,10,11] in the process of finding the reason for universal agreement on the exponent $\frac{1}{2}$ in the Lacey formula:

$$P = 2.67\, Q^{\frac{1}{2}} \qquad (5a)$$

(in which Q is the discharge). The formula is equivalent to

$$\frac{P}{R\, V} = 7.11 \qquad (5b)$$

There is serious disagreement over the constant 2.67, which obviously needed to be replaced by a variable dependent on the nature of the bed sediment. The detailed derivation of the meaning of Eq. 4 requires a little more than simple hydraulics and will be found in Appendix I. The line of reasoning is as follows:

Universal acceptance of the exponent $\frac{1}{2}$ in Eq. 5a, based on data covering a very wide range (see, for example, Fig. 3), suggests that the expression $P/V\,R$ has a physical significance. Replacing P and R by b and d, the expression can be written as

$$\frac{V^2/d}{V^3/b} \qquad (6)$$

in which the numerator has physical significance. Therefore there should be physical significance in the denominator. Since V^2/d is a force per unit mass, V^3/b (which is associated with the sides) is probably in some way a measure of tractive force on the sides. Appendix I shows that $\rho^2\, \nu\, V^3/b$ measures the square of the tractive force intensity on the sides (except for a multiplying constant) provided that there is a laminar film at the sides (that is, provided that the sides are technically "smooth"). Therefore, the presumption is that the sides of most of the canals observed were "smooth" and that the Lacey formula expressed nothing more than an averaging out of the relative importance of bed to sides as measured by the ratio B/s. That is, all Lacey channels are regime channels, but not all regime channels are Lacey channels.

Logically, s would be better replaced by νs. Practically, this step is not desirable. The tractive force intensity on the sides of a regime channel may lie anywhere between that preventing the deposition of material in suspension to that which scours material already in place. A considerable range of values of s is possible without violating the condition of regime. Furthermore, viscosity varies considerably, and it is hard to assess when water is turbid. There is no practical loss in using s in design.

The third equation of generalized regime theory is

$$\frac{V^2}{g\,d\,S} = C\left(\frac{V\,b}{\nu}\right)^{\frac{1}{4}} \qquad (7)$$

in which S is channel slope measured as a fraction, C is a nondimensional constant, and $V\,b/\nu$ is the Reynolds number in terms of width.

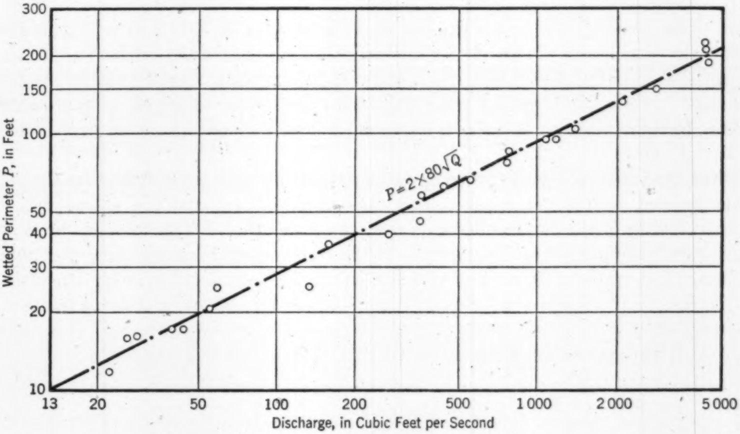

FIG. 3.—WETTED PERIMETER AS A FUNCTION OF DISCHARGE; PUNJAB DATA

Data supporting Eq. 7 are shown in Fig. 1(*b*). Plotted to the same scale, displaced to make pipe and canal data more comparable, Fig. 3(*a*) shows the data for artificially roughened pipe (D = diameter) reported by J. Nikuradse.[12] The Nikuradse lines in Fig. 1(*a*) are roughly plotted and are presented for general information only. The physical significance lies in the parallelism of the lines in the two sections of Fig. 1. Mr .King[13] made them collinear merely to emphasize his point that the flow formulas are similar for different boundary conditions.

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[12] "Strömungsgesetze in rauhen Röhren," by J. Nikuradse, *Forschungsheft Verein Deutscher Ingenieure*, Heft 361, 1933.

[13] "Stable Channels in Alluvium," by Gerald Lacey, *Minutes of Proceedings*, Inst. of Civ. Engrs., London, England, Vol. 229, 1929–1930, opposite p. 58.

It has been suggested previously in this paper that the flow formula for regime channels should be a generalized form of that for channels or pipes with smooth rigid boundary, and Eq. 7 shows such is the case. Advocates of the Prandtl-von Kármán logarithmic type of formula may dispute the applicability of the Blasius formula. The Blasius formula for pipes gives the friction coefficient

$$f = \frac{0.316}{(V\,D/\nu)^{\frac{1}{4}}} \quad \text{(8)}$$

in which D is diameter. Recent data of large conduits for Reynolds' numbers greater than 10^6, plotted on a diagram of f against Vd/ν, contradict the Prandtl-von Kármán line markedly;[14] but they form groups consistent with the correctness of the Blasius line (the locus of Eq. 8).

Eq. 7 can be transposed into "rough boundary" form by using it in combination with Eqs. 3 and 4. It then yields

$$V = \sqrt{C}\,\sqrt[4]{\frac{d}{\sqrt{\nu\,s/B}}}\,\sqrt{g\,d\,S} \quad \text{(9a)}$$

in which $\sqrt{\nu\,s/B}$ is a linear dimension that may be called an "equivalent protuberance height." This shows that there is no contradiction between Eq. 7 and the fact that bed dunes obviously produce what is usually called "roughness." In fact, the results merely indicate that there seems to be a universal flow formula, just as there is a universal velocity distribution (to proper nondimensional coordinates). This universal formula yields the special formulas when equivalent protuberance is properly expressed. As a flow formula may be derived by integrating a velocity distribution formula, the assumption of universality of the one implies universality of the other. This idea has been put forward previously[15] and suggests that the well-known Manning formula should have the exponent $\frac{3}{4}$ instead of $\frac{2}{3}$ for R. Such a change removes the defect inherent in the Manning formula, in which n is dependent on a small fractional power of Q.

A little algebraic manipulation of Eqs. 3, 4, and 7 will yield

$$g\,V\,S = \frac{B\,(\nu\,s)^{\frac{1}{4}}}{C} \quad \text{(9b)}$$

which shows that a regime channel adjusts itself so that the rate at which gravity does work per unit mass per unit time depends on the three factors specifying the water-sediment complex and does not depend on anything else—particularly the discharge.

Thus the three formulas of regime (Eqs. 3, 4, and 7) not only fit into the framework of conventional hydraulics but also generalize it.

Practical Regime Formulas.—Eqs. 3, 4, and 7 are unsuitable for practical use because the designer wishes to know values of b, d, and S to select for given values of Q, B, and s. Algebraic manipulation of the equations will give the design equations:

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[14] *Transactions*, ASCE, Vol. 113, 1948, p. 1067.

[15] "A New Theory of Turbulant Flow in Liquids of Small Viscosity," by Thomas Blench, *Journal*, Inst. of Civ. Engrs., London, England, Vol. 11, 1938–1939, p. 611.

$$b = \sqrt{\frac{B\,Q}{s}} \quad \text{.........................} (10)$$

$$d = \sqrt[3]{\frac{s\,Q}{B^2}} \quad \text{.........................} (11)$$

and

$$S = \frac{B^{5/6}\, s^{1/12}\, Q^{-1/6}}{\frac{C\,g}{\nu^{\frac{1}{4}}}} \quad \text{.....................} (12)$$

The value of $C\,g/\nu^{\frac{1}{4}}$ when ν averages 10^{-5} has been found to be 2,080.

The designer must assess the value of B that nature (restricted by the sediment exclusion and ejection actions of engineering structures) will impose on his canal system. There is no scientific formula to assist in making this assessment any more than there is a scientific formula to permit assessing a value of Manning's n or an equivalent sand-roughness size for a rigid pipe or channel. With the behavior of an existing canal known, an estimate can be made by inserting observed values of b, d, S, Q in the equations, just as a given pipe factor (n) can be found from performance and will then give an idea of what to expect from similar material. There is a rough empirical formula, adapted from an equation proposed by Mr. Lacey[16] (who was careful to emphasize its empiricism), to the effect that

$$B = 2\sqrt{D_m} \quad \text{.......................} (13)$$

in which D_m is the mean sample diameter of bed material (in millimeters) to be expected in the canal. Scientifically the formula is thoroughly unsatisfactory; practically it fits many cases quite well probably because the quantities in movement in saltation are very minute in most practical canal cases. If the canal is fed by a river with sand shoals, the estimation of D_m from the sand should be safe, for the expert engineer has many ways of preventing his canals from receiving the full share of river-bed sand.

The lowest value of B found in Indian canals is about 0.6; the highest value is about 1.25. The common value is approximately 1.0. These values refer to canals of the alluvial plains.

The value to assume for s is about midway between that likely to cause scour of berm material in place and that likely to permit deposition of new material on the berms from suspension. For glacial till material the designer can use 0.3; for silty clay-loam material, about 0.2; for material with little cohesion, 0.1. Sand sides of rivers below low tide level give values between about 0.05 and 0.10. The use of average values in design permits the maintenance engineer to make changes to canal widths within limits, and thereby produce permanent alterations to slopes. Although such alterations of slope are small, they can produce useful changes to command of flat land.

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[16] "Uniform Flow in Alluvial Rivers and Canals," by Gerald Lacey, *Minutes of Proceedings*, Inst. of Civ. Engrs., London, England, Vol. 237, 1933–34, p. 16.

Regime Formulas as Applied to Rivers.—As the regime formulas have proved the existence of definite laws relating to the self-formation of regime-type channels, they have an application to rivers of regime type. However, an allowance for disturbing effects and variable discharges must be made in terms of engineering common sense and the formulas regarded with suspicion until supported by results. Some apparently satisfactory applications have been found to be:

(1) Determination of width between incised banks;
(2) Determination of scour between bridge piers;
(3) Determination of scour downstream of piers, along groins, and at spur heads;
(4) Aggradation upstream from reservoirs;
(5) Degradation downstream from reservoirs;
(6) Estimation of dredging; and
(7) Determination of model scales.

Width Between Incised Banks.—Where a river in flood flows like a canal between incised banks and is fairly straight, the width between those banks is fairly well represented by Eq. 10 provided maximum flood discharge is used for the discharge Q. Using this Q Mr. Lacey has found[17] considerable information showing that

$$b = 2.67\, Q^{\frac{1}{2}} \qquad (14)$$

is in substantial agreement with actual conditions. Eq. 14 is also in accordance with the spans of several Indian railway bridges over large rivers that had been reduced in span during successive remodelings to suit the waterway really required. The practice of the nineteenth century was to span everything that looked like a river during flood spill. The formula is acceptable for "shingle" rivers also, because their sides are not like those of a canal. In "shingle" rivers s and B increase together, thereby making the square root of their quotient relatively unvariable. It is now considered good practice to choose the spans of bridges and barrages in Indian plains' rivers in terms of Eq. 14.

Scour Between Bridge Piers.—If the approach of a river to a bridge is fairly straight, Eq. 11 may be used to estimate the scour between the piers. It is usual to solve it in conjunction with Eq. 10 to become

$$d = \sqrt[3]{\frac{q^2}{B}} \qquad (15)$$

in which $q = Q/b$ is estimated from the peculiarities of the river. For example, two thirds of the river discharge may be assumed to go through half the spans of the bridge. This element of judgment based on experience cannot be avoided. If the flow is oblique, the depths are doubled for design.

Scour Downstream from Piers, Along Groins, and at Spur Heads.—Investigations on scour at various installations have shown that Eq. 11 fits observed behavior exceedingly well in terms of empirical coefficients. The variation with $Q^{\frac{1}{3}}$ is confirmed excellently, Q being the maximum flood discharge and d being the maximum scoured depth. Different types of structures had different coefficients.[7]

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[17] "Stable Channels in Alluvium," by Gerald Lacey, *Minutes of Proceedings*, Inst. of Civ. Engrs., London, England, Vol. 229, 1929–1930, p. 270 and p. 274.

Aggradation Upstream from Reservoirs.—The backwater that occurs upstream from a reservoir causes deposition in the valley upstream because the slope is less than necessary for regime. The river tends to keep filling its valley in an attempt to regain regime. At the same time a delta advances into the reservoir, giving ever-advancing pivot points from which the river tries to set up regime slope. The ultimate fate of all such rivers is that they will fill the reservoir and run with regime slope which can be drawn back from the site of the dam. Although the ultimate fate may not occur for centuries, the rise in specific levels at the original lake head in a couple of decades can be serious.[18] There is a good record of the behavior of the rivers of the Indus system[19] as a consequence of diversion dams that have been built on it regularly since the end of the nineteenth century. Such records confirm the fact that a barrage ultimately raises the river to be parallel to its old slope up to the next diversion dam.

Degradation Downstream from Reservoirs.—Obviously the removal of bed sediment by deposition in a reservior must result in retrogression of levels downstream. It should be allowed for in the location of points of hydraulic jump on spillway floors.

Estimation of Dredging.—In 1948 comparison of the Meyer-Peter formula[20] for sediment quantity movement with the Lacey formula led to good agreement with the observed dredging quantities of several years on two rivers in the United States. The discharge used was the arithmetic mean over the time of record. That the arithmetic mean happened to be so successful is fortuitous. Obviously, the Lacey formulas do not apply for off-regime conditions, but some mean should be expected to satisfy the formulas.

Model Scales.—It is common Indian practice[7] to design the scales of river models instead of detemining such scales by the lengthy process of trial and error. Eqs. 10, 11, and 12 would be multiplied by different coefficients to allow for the variability of such factors as discharge, heterogeneity of banks and meandering. However, if arrangements are made to give about the same bed factor in model and prototype (by using the same kind of bed material and injecting suitable quantities) and if the side irregularities are reproduced faithfully in effect, the scale ratios of the coefficients will be unity, and

$$b_r : d_r : S_r = Q^{1/2}{}_r : Q^{1/3}{}_r : Q^{-1/6}{}_r \ldots\ldots\ldots\ldots\ldots\ldots (16a)$$

in which the subscript r denotes the ratio of model to prototype dimension.

Eq. 16*a* explains the rule—

$$(\text{Depth scale}) = (\text{length scale})^{2/3} \ldots\ldots\ldots\ldots\ldots\ldots (16b)$$

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[18] "Future of Lake Mead and Elephant Butte Reservoir," by J. C. Stevens, *Transactions*, ASCE, Vol. 111, 1946, p. 1231.

[19] "Regime Levels on the Indus System, Part I," *Punjab Irrigation Branch Paper No. 16*, Class B, Lahore, India, 1944.

[20] *Proceedings*, Federal Interagency Sedimentation Conference, USBR, Washington, D. C., January, 1948, p. 114.

—which is sometimes stated to be empirical.[21] There is no objection to decreasing the width or using a different bed material except that Eq. 16 would then have to take cognizance of the possibility of B_r and s_r and other scale ratios related to the indefinite factors not being unity. As their exact values would not be known, the model would display difficulties before functioning effectively.

Meandering.—Excellent work has been done in studying meandering.[7,22] When disturbing causes are removed, meander pattern appears as something just as definitely characteristic of a channel as width or depth. The conclusion is that meander length and meander width can be scaled in exactly the same way as $\sqrt{Q}$—according to observations on models and on masses of river data. The nature of the coefficients has not been determined, and unfortunately most laboratory work has been done without the aid of regime theory.

Perhaps the best indication that meander size varies as width or as $Q^{\frac{1}{2}}$ is the fact that no modelmaker has ever found that he had to distort meanders; but modelmakers who did not distort the vertical scale have all met with failure, unless they were not modeling regime-type conditions. There are two extra regime equations to define the influence of meandering in rivers in terms of meander breadth M_b and meander length M_l:

$$M_b = C_b\, Q^{\frac{1}{2}} \qquad \text{(17a)}$$

and

$$M_l = C_l\, Q^{\frac{1}{2}} \qquad \text{(17b)}$$

In Eq. 17*a*, Q is a representative discharge (usually the maximum flood discharge). The coefficient C is a function of B and of a quantity of the nature of a very disturbed side factor, s. It is also related to the form of the river hydrograph, because the selected representative discharge has to be converted into the mean that is dynamically correct for the purpose.

CONCLUSION

Regime theory provides a dynamic framework for the study of the behavior of rivers that form themselves from their transported materials or from similar material in place. It is precisely applicable to controlled canals of similar type.

APPENDIX I.—TRACTIVE FORCE INTENSITY DEDUCED FROM LAMINAR FILM HYPOTHESIS

The first step is to find an expression for the relative thickness of the laminar film when flow is turbulent. Considering a channel in uniform turbulent flow, of depth d and of great width, the x-axis is taken in the direction of flow and the y-axis is upward at right angles to it. Then, using the conventional symbolism of hydrodynamics, the Navier-Stokes equation for the mean motion can be written as

$$\frac{D\bar{u}}{Dt} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} + X + \nu\,\nabla^2 \bar{u} - \frac{\partial}{\partial x}\overline{\mathrm{u'u'}} - \frac{\partial}{\partial y}\overline{\mathrm{u'v'}} - \frac{\partial}{\partial z}\overline{\mathrm{u'w'}} \qquad \text{(18)}$$

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[21] "Scale Models in Hydraulic Engineering," by Jack Allen, Longmans, Green Co., London, England, 1947.

[22] "A Laboratory Study of the Meandering of Alluvial Rivers," by J. F. Friedkin, U. S. Waterways Experiment Station, Vicksburg, Miss., 1945.

in which $\bar{u}$ = mean velocity along Ox; t = time; ρ = fluid mass per unit volume; ν = kinematic viscosity; X = body force per unit mass; and ∇^2 = the Laplace operator. Eq. 18 reduces to the following in terms of the special problem: For the laminar film—

$$0 \doteq X + \nu \frac{\partial^2 \bar{u}}{\partial y^2} \ldots\ldots\ldots\ldots\ldots\ldots (19a)$$

and for the fully turbulent portion of the fluid—

$$0 \doteq X - \frac{\partial}{\partial y} \overline{\mathrm{u'v'}} \ldots\ldots\ldots\ldots\ldots\ldots (19b)$$

In Eqs. 19 $X(= g\ S)$ is the same for all points.

It is assumed that, as suggested by the form of observed velocity distributions, there is some small distance δ from the boundary beyond which the turbulent component suddenly begins to become important compared with the viscous component of shear stress. That is, at distance δ from the boundary the turbulent component is unimportant, but at $k\ \delta$ (in which k is of the order of 1) it is important. This implies that $\left(\nu \frac{\partial^2 \bar{u}}{\partial y^2}\right)_\delta$ in the film is equal to $-\left(\frac{\partial}{\partial y} (\overline{\mathrm{u'v'}})\right)_{k\delta}$ just beyond the film. The first bracketed expression can then be represented by some constant number of times V/δ^2 and the second by some constant number of times V^2/d, if it is accepted that there is a universal velocity distribution law in some system of coordinates. Because the respective results depend on the relative positions of the two points in the laminar and turbulent zones, respectively, δ and d are used.

Therefore $\nu\ V/\delta^2 \ \alpha\ V^2/d$; or

$$\left(\frac{\delta}{d}\right) \alpha \left(\frac{V\ d}{\nu}\right)^{-\frac{1}{2}} \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

One way to see that a formula of the type of Eq. 20 is plausible is to use a variation of an argument that has been applied to establish a stability criterion that decides when laminar flow is likely to change to turbulent. This approach, however, will not reveal what happens after the breakdown occurs, which is the present problem. The extended argument is that the change from laminar to turbulent flow is likely to depend on the Reynolds number in terms of film thickness and on the relative thickness of film to depth; that is,

$$\frac{V\ \delta}{\nu}\left(\frac{\delta}{d}\right)^n \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

If n equals 1.0, Eq. 20 is verified.

It is to be noticed that Eq. 20 is of exactly the form suggested by L. Prandtl for the related problem of a boundary layer thickness along a flat plate and does not agree with any of the several conventional forms devised to give an idea of the magnitude of thickness of a laminar film without consideration of specific dynamical meaning.

If Eq. 20 is accepted and if it is assumed that there is a laminar film along the sides of a channel of width b, the relative laminar film thickness over the sides can be represented by $(V\,b/\nu)^{-\frac{1}{2}}$. The shear stress intensity can then be represented by

$$\tau = \frac{\mu\,V}{\delta} \propto \frac{\mu\,V}{b} \times \frac{b}{\delta} \propto \frac{\mu\,V}{b}\left(\frac{V\,b}{\nu}\right)^{\frac{1}{2}} \propto \rho\left(\frac{\nu\,V^3}{b}\right)^{\frac{1}{2}} \qquad (22)$$

Eq. 22 explains the physical significance given to the term V^3/b that is indicated by an analysis of the formulas derived from the data of regime canals.

DISCUSSION

Emmett M. Laursen,[23] A. M. ASCE.—The author will probably frankly admit that his three equations of regime (Eqs. 3, 4, and 7) are empirical, and that any attempts to rationalize them are simply efforts to integrate specific observations into a more general body of empirical law, thereby increasing the confidence with which they may be used. This comment must not be taken as a criticism since fundamentally all scientific knowledge is empirical—that is, based on observed facts; and "laws" are true if, and only if, they allow the prediction of future conditions. Whether the regime equations achieve general acceptance will depend on further measurements extending the limits of observation, rather than any rationalization. The following analysis may aid in the interpretation of the author's regime theory: First, by outlining the relations necessary for the solution of the problem, and, second, by examining the factors involved in the constants of his equations.

Only the Manning equation $\left(Q = \frac{1.49\, R^{\frac{1}{6}}}{n} A \sqrt{R\, S} \right)$ and two assigned values are needed for the solution of flow in a rigid channel. For any cross-sectional shape, the value of the hydraulic radius, R, is determined by the area, A. The material of the channel, of course, determines the value of the roughness coefficient, n. If the quantity, Q, is known, the one equation gives a relation between the two unknowns (the area, A, and the slope, S) such that the choice of one fixes the other. Usually, this choice is based on economic balancing of costs.

The freedom of choice disappears, however, when a certain load of sediment must be transported in a lined channel. Since existing transport equation is generally similar in form, the argument does not depend on which is selected. The Schoklitsch equation[24] is used merely for illustration:

$$G = \alpha \, (Q - Q_O) \, \frac{S^{\frac{3}{2}}}{\sqrt{D}} \qquad \text{(23)}$$

in which D is the diameter of the sand in mm; α is a variable coefficient; and Q_O is a critical term. Despite the introduction of a second equation, the solution of this problem is still straightforward.

If, now, a channel that is self-formed in alluvium is considered, the shape is no longer a matter of choice but is determined by the flow and sediment characteristics. A third equation is necessary to replace the former arbitrary choice of shape, and the author's second equation of regime represents an example of this desired relationship.

Although the three equations of regime, as advanced, are mathematically sufficient for the complete solution of the problem, their usefulness is open to a specific criticism. Just as the successful use of the Manning equation depends on the judgment employed in selecting the correct value of n, the successful use of the regime equations depends on the selection of the constants B, s, and C, not to mention the question that still remains as to the validity of the relationships for other than conditions in India. Unfortunately, very few engineers have the requisite experience to exercise such judgment.

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[23] Research Engr., Iowa Inst. of Hydr. Research, Univ. of Iowa, Iowa City, Iowa.

[24] "Engineering Hydraulics," ed. by Hunter Rouse, John Wiley & Sons, Inc., New York, N. Y., 1950 p. 795.

To assess the physical significance of the author's regime theory, the foregoing analysis may be carried to a parallel form. A guide to the selection of coefficients will be provided in that way. Eq. 23 may be combined with the Manning equation and rearranged to give

$$\frac{V^2}{d} = \frac{2.2\, d^{\frac{1}{3}}}{n^2}\left(\frac{G}{Q - Q_O}\frac{\sqrt{D}}{\alpha}\right)^{\frac{2}{3}} \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

so that the bed factor (B) becomes

$$B = \frac{2.2\, d^{\frac{1}{3}}}{n^2}\left(\frac{G}{Q - Q_O}\frac{\sqrt{D}}{\alpha}\right)^{\frac{2}{3}} \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

It may be expected, therefore, that the value of B will vary beyond the limits of 0.6 to 1.25 (as given under the heading "Generalized Regime Theory: Practical Regime Formulas"), if conditions are markedly different from that of the Indian plains. That is, the factors implicitly involved in the bed factor are now explicit. Unfortunately, every bed-load formula will give a relation that is slightly different, although composed of essentially the same parameters.

By rearranging terms, the third equation of regime can be put in a form parallel to the Manning equation:

$$V = (C\, g)^{\frac{1}{2}}\left(\frac{V\, b}{\nu}\right)^{\frac{1}{8}}\sqrt{dS} \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

This is equivalent to saying that the Manning evaluation of the Chezy coefficient is given by

$$\frac{1.49\, d^{\frac{1}{6}}}{n} = (C\, g)^{\frac{1}{2}}\left(\frac{V\, b}{\nu}\right)^{\frac{1}{8}} \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

The use in this equation of the generally accepted Reynolds number based on depth would seem more appropriate, since in a wide channel the depth rather than the width characterizes the flow. However, it might be pointed out that since the Reynolds number is raised to the $\frac{1}{8}$ power, by including an appropriate constant the evaluation of the Chezy coefficient would differ by only ±12% for the range of b/d from 5/1 to 30/1. Thus, $\dfrac{1.37\left(\dfrac{V\, b}{5\, \nu}\right)^{\frac{1}{8}}}{\dfrac{V\, b}{\nu}} = 1.12$; and $\dfrac{1.37\left(\dfrac{V\, b}{30\, \nu}\right)^{\frac{1}{8}}}{\left(\dfrac{V\, b}{\nu}\right)^{\frac{1}{8}}} = 0.90$. Any other factors that might affect the value of the coefficient, such as roughness or sediment transport, must be contained in the author's C term. Hence, in the application of the regime theory to areas other than India, cognizance should be taken of a possible variation in the recommended numerical values, as well as in the function itself.

By combining the first and second equations of regime, a width-depth relation may be obtained:

$$\frac{b}{d} = V \frac{B}{s} \qquad (28)$$

This relation expresses explicitly the factors determining a self-formed shape. As E. W. Lane,[25] M. ASCE, has pointed out, a high width-to-depth ratio is required if the sediment load is large or if the bank material is easily erodible. The variations in the factors B and s, as given by the author, agree with this interpretation. To question the inclusion of the velocity, V, in the relation is actually a question of why V occurs to the cube in the second equation of regime and to the square in the first. The writer's only comment would be that the first equation is a modified transport equation and the second, somehow, a critical equation that is applicable when erosion begins. The corresponding derivation of the author's Appendix is incorrect, because $\tau = \mu \frac{V_s}{\delta}$.

Therefore, Eq. 22 should read

$$\tau \propto \frac{\rho \left(\frac{\nu V^3}{b} \right)^{\frac{1}{2}}}{\frac{1}{2} \ln \frac{V b}{\nu}} \qquad (29)$$

Moreover, the assumption that $\frac{\delta}{b} \propto \left(\frac{V b}{\nu} \right)^{-\frac{1}{2}}$ is questionable for a wide channel.

The author advances three relations of the type necessary for the solution of self-formed, sediment-bearing channels. Whether the simple forms advocated will be modified or supplanted in the future will depend on further measurements, especially of channels outside the Indian plains area for which the original equations were derived. Even assuming that the form of these equations may eventually prove to be valid, a high degree of judgment will still be required for the selection of numerical values of the factors B, s, and C in the equations. However, the relations do offer at the very least a method of solution that does not depend on pure guess or assumption.

Gerard H. Matthes,[26] **Hon. M. ASCE.**—The engineering profession is indebted to the author for having sketched the growth of the regime theory and for having illustrated its application to practical problems. Although the author tactfully has refrained from expressing wonderment at the scant interest shown in this theory by American engineers, it is clear that his able paper is designed to awaken their interest in the subject. Undeniably, formulas that over a period of more than a quarter of a century have proved of considerable worth in the hands of the irrigation engineers in India are entitled to better recognition in the United States. The fact that interest in them has lagged in this country is due to certain conditions that also need recognition. The object of this discussion is to make this aspect clear.

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[25] "Stable Channels in Erodible Material," by E. W. Lane, *Transactions*, ASCE, Vol. 102, 1937, p. 123.

[26] Cons. Hydr. Engr., New York, N. Y.

Considering first R. G. Kennedy's well-known formula $V_k = 0.84d_k^{0.64}$, it is unfortunate that from the start in the United States it was called a formula for "nonsilting, nonscouring" channels. Unfortunate, also, was the fact that for a long time American engineers were not aware that the term "silt" as used by British engineers in India included all materials from the finest grain sizes to gravel, stones, and boulders. Thus, in the case of the large flows diverted from the rivers in the Punjab into irrigation canals capable of carrying 10,000 (or more) cu ft per sec, with which Mr. Kennedy was concerned, the "silt" entrained included gravel in sizes up to 1-in. and 1½-in. sizes. Although most of this coarse bed-load material was prevented from entering the canals at the headworks, a certain volume of it did enter the canals. The writer's analyses of the original Kennedy data have indicated to him that the formula is distinctly an empirical tractive force formula, aimed at preventing these coarser materials from forming deposits in the canals. The finer materials in suspension (that is, that portion which American engineers designate as silt) were thus automatically prevented from forming deposits, but the formula primarily was not aimed at these finer materials. In other regions where the bed-load material is of a different type, the Kennedy formula has required different coefficients as well as exponents.

On August 24, 1939, the writer had a conference with Sir Claude C. Inglis (M. ASCE), then Director of Central Irrigation, India, and C. M. White of Imperial College, London, when these gentlemen visited the lower Mississippi River. During the conference the writer inquired whether the Kennedy formula was still felt to be satisfactory. He was assured that it was. Inquiring further, about trouble with sedimentation when canals are operated at part capacity, he was told that since the canals in India were operated only at the full capacity for which they were designed, or "not at all," sedimentation had been small. The writer explained, in turn, that this plan of operation, however desirable, could not be followed in the arid regions of the United States where scarcity of water did not tolerate running canals full and wasting the unused surplus. Consequently, he explained, irrigation canals frequently must operate at part capacity, inviting settlement of all grades of sediment and requiring periodic cleaning out; for these reasons American irrigation canals can not be designed on the basis of a Kennedy-type relation.

The foregoing is cited as it has an important bearing on the underlying philosophy of the regime theory evolved by Gerald Lacey. That theory fundamentally has the same basic concept that actuated Mr. Kennedy; namely, it applies only to bank-full flow, whether in canals or in natural channels. The canals have shown that they are self maintaining when operated as described. Rivers at bank-full flow are known to recondition their channels, provided they flow in "noncohesive" alluvium, which may consist of silt, sand, or gravel (shingle). When rivers are reconditioning their channels, they are said to be "in regime," according to Mr. Lacey. However, bank-full, dominant, or regime flow (all synonymous terms) is only one of the many stream-flow conditions that engineers are confronted with in their day-by-day river problems. In most of their problems the regime formulas do not apply or are of no assistance. Furthermore, many alluvial rivers in the United States have become affected by contraction works, bank revetment, channel shortening, and encroachments on their widths; according to Mr. Lacey, his regime formulas are not applicable in any of these cases. This situation places many limitations on the use of regime formulas in the United States. Conceivably, the author may still find a field for their application in Canada, where alluvial rivers have suffered relatively little as yet from restraints imposed by man.

THOMAS BLENCH,[27] M. ASCE.—The fundamental points raised by Mr. Laursen are treated in detail by the writer in other publications.[28, 29] The following reply to Mr. Laursen is based on that treatment.

Human needs required that rigid-boundary hydraulics be a practical subject long before knowledge of fluid motion within rigid boundaries reached even its present inadequate form. So theoretical development was marred by "plausible assumptions" (often obscured by mathematics slightly beyond ordinary engineering standards), by a neglect of essential variables, and by a failure to test theories over a wide range of variables. The compilation of tables and charts containing "variable constants" has served to make these results serviceable in practice. As a result the science has advanced under the stigma of "empiricism." Gerald Lacey, who founded modern regime theory as applied to mobile boundaries, followed the normal routine of scientific investigation by correlating measurable variables. He used these variables to define measures of intangible variables over the entire range encountered in practice; and he sought to express correlations as simple functional relationships containing dimensional or nondimensional groups of magnitudes with definite physical meanings. The foundation laid by Mr. Lacey has served his successors so well that regime theory is now empirical only in the sense that all physics is empirical.

It is true that rigid-boundary and flume transport formulas can be used (instead of the better regime formulas) to demonstrate that the three degrees of freedom in most regime channels require three independent equations. Nevertheless, the development of a working theory from such formulas is not recommended. For example, the use of the popular Manning formula to fix one self-adjusting variable is tempting; but that formula, in spite of its implicit dimensional soundness, is merely a mathematical alternative to the Kutter formula with its dimensionally incongruous "variable constant" and its unsatisfactory background. When the formula was applied to regime canals, n had to be given values ranging from 0.018 to 0.0225 to explain the behavior of channels of the one system with discharges ranging from several thousands to a few units of cubic feet per second. (This misbehavior of the formula, with n a function of discharge, cannot be detected in ordinary rigid pipe work, where size range is small and is accompanied by genuine changes of roughness.) However, in the absence of a correct formula, the Manning formula can be used as in the past.

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[27] Associate Prof., Civ. Eng., Univ. of Alberta, Edmonton, Alta, Canada; and Con. Engr.

[28] "Hydraulics of Sediment-Bearing Canals and Rivers," by T. Blench, published privately (Evans Industries, printer), Vancouver, B. C., Canada, 1951.

[29] "Civil Engineering Reference Book," Butterworth Scientific Publications, London, England, 1951 (Chapter on Rivers and Channels).

The second degree of freedom in regime channels can be disposed of by solving Eq. 23 with Eq. 24, but the result (Eq. 25) makes the bed factor B depend on four quantities:

(1) n, which the computer must guess, really depends on the other terms. (It is a function of Q because of the empirical derivation of the formula that defines it);

(2) G, which cannot be measured, in practice;

(3) Q_o, which is a vague "critical term"; and

(4) α, which is a "variable coefficient" that cannot be assessed with assurance, and whose nature needs to be further explained.

Regime theory teaches that channels carrying the same water-sediment complex have the same values of B. Under such circumstances, the right-hand side of Eq. 25 is constant, which makes the equation appear as an "empirical" formula, in the very worst sense of that word, in which the depth is related to a mass of nondeterminable interrelated parameters which are associated somehow or other with the nature of the complex. This example shows why the use of rigid-boundary equations mixed with flume transport formulas has had little success in the design of regime channels.

With even the best transport equations in use, the extent of the current basic ignorance is obscured (not removed) by the several practically unassessable terms involved. In the case of regime equations, the ignorance is concentrated in the quantity B, and this fact is generally recognized. Actually, however, the practical estimation of values of B by an experienced engineer is not much worse than the selection of n-values by inspecting the material in a rigid boundary.

The constant C is fixed for all conditions, as defined implicity in the sentence following Eq. 12. For practical purposes s need not be evaluated to a fine degree of accuracy; even a novice can determine its value without trouble (see heading "Generalized Regime Theory: Practical Regime Formulas"). Except for the unfortunate separation of flume-transport experimenters from regime knowledge there might now be a reasonable formula linking B with the sediment factors on which it depends. Instead, a great deal of potentially useful experimentation seems to have been wasted in observing nonregime (and, therefore, unspecifiable) conditions. Unrealistic materials were used in testing formulas with little scientific background. There are noteworthy exceptions, and the prospect of correlating B with its several constituents seems bright. The prospect would improve if original data of earlier tests were made available.

The third degree of freedom would be disposed of exactly as in regime theory. An expression is found for tractive force intensity over the sides, the upper and lower limits of which are determined by the nature of the sides and by the water-sediment complex. However, the relevant theories of rigid-boundary hydraulics are related to concepts involved in the flow formula; and the two theories are still in conflict. The widely advocated logarithmic type of flow formula does not fit practical conduit data[14,30] any better than do the tests of regime channels (see Fig. 1). A revision of laminar film theory is needed so that s can be explained as attempted in Appendix I. In fact laminar films should now be explained quantitatively in terms of data observed on large conduits and regime channels instead of in terms of assumptions that lead to formulas that are inconsistent with those data.

Mr. Matthes states correctly that the object of the paper is to awaken interest; but the writer denies that he is surprised by the backwardness of the subject in the United States. It took a long time to develop interest in India where the facts were readily available for analysis. Little advance can be expected until the subject is presented as a whole in accessible form. The case is analogous to that of rigid-boundary pipe flow, where the prerequisite to a clear understanding is a plot of observed friction factors against the fullest available range of Reynolds' numbers. Such a plot demonstrates that there are three distinct transitions that yield to no obviously simple expression. Unless the whole physical picture is seen and appreciated, investigators must be expected to continue experimenting with their own little assortments of mixed data, deriving contradictory and unintelligible flow formulas and accusing each other of "selecting data." In this paper and elsewhere,[28,29] the writer has tried to make the new knowledge available and to outline its complete picture. Once the reader has understood it, and has acquired a sense of proportion, he will find that engineering ingenuity will permit him to obtain reasonable approximate solutions to most, if not all, of the problems that Mr. Matthes very reasonably believes to be insoluble. The sense of proportion is not easy to obtain by reading. For example, Fig. 4 is derived from a drawing presented[31] by E. W. Lane. It shows two lines between which the writer would expect

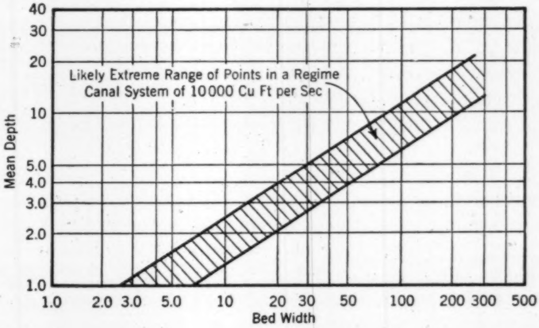

FIG. 4.

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[14] *Transactions*, ASCE, Vol. 113, 1948, p. 1067.

[30] "A Revised Manning Flow Formula," by Thomas Blench, *Technical Bulletin No. 2*, The Eng. Inst. of Canada, Ottawa, Ont., Canada, June, 1950, p. 7, Figs. 1 and 2.

[31] "Stable Channels in Erodible Material," by E. W. Lane, *Transactions*, ASCE, Vol. 102, 1937, p. 130, Fig. 2.

points, taken from a regime canal system of 10,000 cu ft per sec, to lie with a scatter that depends largely on the way in which real canal heads distribute sediment load among the individual channels. Obviously, the mass of data required to prove or disprove regime theory will be very great until enough has been learned to justify scientific selection; yet the inexperienced would probably test their theories in terms of half a dozen points or by taking points from different systems. For actual application, the writer has yet to find a report of the misbehavior of American rivers, which shows a state of affairs that was unforeseeable or even unassessable. When he visited the All-American Canal (Arizona-California), he found that he could predict the order of slope and width-to-depth ratio for well-behaved large channels from the corresponding data for small channels; and he could even estimate the order of size of the bed material. Perhaps most noteworthy is the fact that an analysis of model scales of the United States Waterways Experiment Station at Vicksburg, Miss., since 1938, shows them to follow regime theory very closely although they were determined, apparently, by prolonged trial and error.[32]

Sir Claude Inglis' quoted views of 1939 on the Kennedy formula seem to have been revised[7] by 1949. Possibly he did not feel inclined to condemn, completely, in a place where no alternative was known, a formula that had some empirical value. Furthermore, the reasons for running canals full or closed seems to have been misunderstood. Distributing channels are run full because equitable supply under conditions of limited command would be impossible any other way; but the branches from which these distributing channels are supplied are run partly full and are "headed up" at control points. Incidentaally the "heading up" results in temporary inactivity of the bed, and regime analysis would be applied only to full supply conditions when the bed is active.

Even if data for canals were collected to the limit of their availability on the North American continent, there seems to be no prospect that there would be any improvement on Indian data or extension of their range. The Indian discharge range covers the largest canals ever built; the sediment loads include examples comparable with those of the Colorado River; the bed materials include all sizes in the sand range; the Reynolds numbers (in terms of breadth) are many times greater than the maximum laboratory values and compare with those of the world's largest conduits; slopes are measured with great accuracy (the laboratory cannot measure them as accurately); and widths are self adjusting, negatively as well as positively. The most potentially fruitful line of advance in North America seems to be in applying the theory to existing canals and rivers. Its application, like that of rigid-boundary hydraulics, requires some training, experience, and familiarity with phenomena observed in the field. Unfortunately, only two colleges in North America teach the subject.

The writer acknowledges that Eq. 22 should have a sign of proportionality instead of a sign of equality. The implications of the formula then depend on the definition of laminar film thickness, leading to a lengthy discussion beyond the scope of the paper.

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[32] "Scales for Moving-Bed Models," by T. Blench, Dock and Harbour Authority, London, England March, 1951, p. 353, Fig. 1.

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TURBULENT TRANSFER MECHANISM AND SUSPENDED SEDIMENT IN CLOSED CHANNELS

By Hassan M. Ismail,[1] A. M. ASCE

With Discussion by Messrs. Emmett M. Laursen and Pin-Nam Lin; M. R. Carstens; and Hassan M. Ismail

Synopsis

Tests on a rectangular channel, 10.5 in. by 3 in. in cross section, transporting water mixed with different proportions of sand, are described in this paper. In terms of mean sedimentation diameters, two sizes of sand were used—0.10 mm and 0.16 mm. The experimental data were used to study (*a*) the effects of the presence of sand in suspension on the characteristics of the flow, and (*b*) the momentum transfer coefficient and the concentration transfer coefficient at the profile through the center line of the channel, assuming a two-dimensional flow.

The present experiments show that the universal constant of turbulent exchange, k, decreases with the increase of the suspended material. The value of k decreases to 0.20 when the average concentration is 4.3% by weight. No limit was approached for the decrease of k. The friction coefficient λ was hardly affected by the presence of the sand when the velocity of flow was higher than a certain rate. This velocity is related to the bed load; it is the velocity at which all the dunes on the bed are carried in suspension; and it increases with the total load in the flume and the size of the sand used. Below that velocity, λ exceeds that of clear water.

The sediment transfer coefficient ϵ_s was found to be equal to 1.5 times the momentum transfer coefficient, ϵ_m, for the 0.10-mm sand and to 1.3 ϵ_m for the 0.16-mm sand. Both ϵ_s and ϵ_m follow the normal parabolic form at the outer two thirds of the channel and have a constant value at the middle third.

Introduction

Notation.—The letter symbols introduced in this paper are defined where they first appear, in the text or by illustration, and are assembled alphabetically in the Appendix for convenience of reference.

The modern developments in the turbulent flow theory have created interest in the sediment suspension and the heat flow problems. It was Morrough P.

Note.—Published in February, 1951, as *Proceedings-Separate No. 56*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Civ. Engr., Ministry of Public Works, Cairo, Egypt.

O'Brien,[2] M. ASCE, who first made use of the Taylor-Schmidt findings in deriving the equation for the distribution of suspended sand in a turbulent flow. By equating the rate of upward transfer, $-\epsilon_s \frac{dC}{dy}$, of suspended sediment resulting from turbulent exchange and the rate of settling, $> w\,C$, under gravitational force, Mr. O'Brien obtained the equation:

$$\epsilon_s \frac{dC}{dy} + w\,C = 0 \qquad (1)$$

in which y is measured vertically upward; C is the concentration of suspended material; w is the settling velocity of the sediment in the still fluid; and ϵ_s is the transfer coefficient for the sediment.

Theodor von Kármán,[3] Hon. M. ASCE, gave the analogy between the transfer of mass or of heat and the transfer of momentum. He showed that the coefficient of all these kinds of transfer should have the same form, but he stated that it was subject to further discussion as to whether those coefficients have the same values.

Another step was taken by T. K. Sherwood and B. B. Woertz[4] who found that the transfer coefficient for water vapor in a turbulent gas stream was not equal to the momentum transfer coefficient ϵ_m, but that they seemed to bear a constant relation to each other. Other investigators disclosed that this approximation can be assumed also in case of heat transfer. Therefore, it may be assumed that ϵ_s can be related linearly to ϵ_m:

$$\epsilon_s = \beta\,\epsilon_m \qquad (2)$$

in which β is the coefficient of proportionality. It is known that, for turbulent flow, $\tau = \rho\,\epsilon_m\,du/dy$, in which τ is the shearing stress at any point; u is the velocity; and ρ is the density of the fluid. For two-dimensional flow, $\tau = \tau_o\,(1 - y/y_m)$; in which τ_o is the shearing stress at the boundary; and y_m is the vertical distance to the maximum velocity—that is, the plane of zero shear. Thus, $\epsilon_m = \frac{\tau_o\,(1 - y/y_m)}{\rho\,du/dy}$. The von Kármán universal velocity defect law[5] is

$$\frac{u - u_{\max}}{U_f} = \frac{1}{k} \log_e \frac{y}{y_m} \qquad (3)$$

in which $u_{\max}$ is the maximum velocity and $U_f = \sqrt{\tau_o/\rho}$. Therefore, when du/dy is evaluated from Eq. 3, the result is

$$\epsilon_m = k\,U_f\,y\left(1 - \frac{y}{y_m}\right) \qquad (4a)$$

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[2] "Review of the Theory of Turbulent Flow and Its Relation to Sediment-Transportation," by Morrough P. O'Brien, *Transactions*, Am. Geophysical Union, Vol. 14, 1933, pp. 487–491.

[3] "Some Aspects of the Turbulence Problem," by Theodor von Kármán, *Mechanical Engineering*, Vol. 57, 1935, pp. 407–412.

[4] "Mass Transfer Between Phases," by T. K. Sherwood and B. B. Woertz, *Industrial & Engineering Chemistry*, Vol. 31, 1939, p. 1034.

[5] "The Mechanics of Turbulent Flow," by Boris Bakhmeteff, Princeton Univ. Press, Princeton, N. J., 1936.

or

$$\epsilon_s = \beta\, k\, U_f\, y \left(1 - \frac{y}{y_m}\right) \quad \text{(4b)}$$

and, in its dimensionless form,

$$\frac{\epsilon_s}{\beta\, k\, U_f\, y_m} = \frac{y}{y_m}\left(1 - \frac{y}{y_m}\right) \quad \text{(5)}$$

The universal constant k is used on the left-hand side so that the right-hand side will be a function of only the position.

Eqs. 4 give zero values for both ϵ_m and ϵ_s at the center of the channel. Many investigators maintain that the transfer coefficients are not zero at the center and, therefore, that the equations are not valid at this point. Mr. von Kármán[6] explained how these relations were derived from the assumption of similar flow pattern; then he stated that in the center part of a pipe the similarity assumption cannot be correct. F. A. Brooks and W. Berggren[7] showed how L. Prandtl tried to correct that region and how the experiments of J. Nikuradse gave definite values of ϵ_m at the center of the pipes in his experiments. They showed that the two reasonable assumptions which can be made at the center are either to take ϵ_m as a constant according to the results of Messrs. Sherwood and Woertz[4] or to assume an error curve for ϵ_m.

If ϵ_s is constant over a certain region, Eq. 1 can be integrated for that region to give

$$\log_e \frac{C}{C_a} = \frac{w}{\epsilon_s}(y - a) \quad \text{(6)}$$

in which C_a is the concentration at any arbitrary reference level $y = a$.

If ϵ_s follows Eq. 4b in another region, Eq. 1 will be integrated to give

$$\log_e \frac{C}{C_a} = \frac{w}{\beta\, k\, U_f} \log_e \left(\frac{y_m - y}{y} \times \frac{a}{y_m - a}\right) \quad \text{(7a)}$$

or

$$\frac{C}{C_a} = \left(\frac{h}{h_a}\right)^z \quad \text{(7b)}$$

in which $z = \frac{w}{\beta\, k\, U_f}$ and $h = \frac{y_m}{y} - 1$.

Vito A. Vanoni,[8] M. ASCE, verified Eq. 7a and showed how well it fitted the experimental points in an open channel. He also showed that the existence of a suspended load tends to suppress or damp out the turbulence, causing a decrease in the value of k, the von Kármán universal constant of turbulent exchange (Eq. 3). The main lines of Mr. Vanoni's work were extended in the present paper to cover the study of a closed channel.

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[6] "Turbulence and Skin Friction," by Theodor von Kármán, *Journal of the Aeronautical Sciences*, Vol. 1-2, 1934, pp. 1-20.

[7] "Remarks on Turbulent Transfer Across Planes of Zero Momentum-Exchange," by F. A. Brooks and W. Berggren, *Transactions*, Am. Geophysical Union, Pt. VI, 1944, pp. 889-896.

[8] "Transportation of Suspended Sediment by Water," by Vito A. Vanoni, *Transactions*, ASCE, Vol. 111, 1946, pp. 67-133.

OBJECTIVE OF EXPERIMENTS

The two main objectives of the present experiments are as follows:

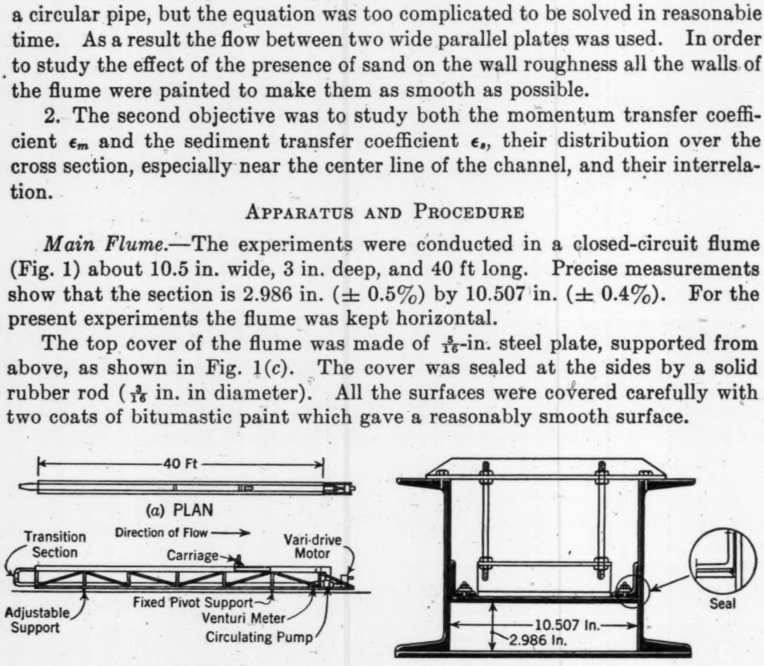
1. The first objective was to continue the study of the effects of the suspended sediment on the characteristics of the flow. A closed channel was selected for these studies instead of an open channel because of the relative ease with which steady uniform conditions can be established and accurate measurements made. An attempt was made to solve the partial differential equation of the second degree that represents the sediment distribution in the case of a circular pipe, but the equation was too complicated to be solved in reasonable time. As a result the flow between two wide parallel plates was used. In order to study the effect of the presence of sand on the wall roughness all the walls of the flume were painted to make them as smooth as possible.

2. The second objective was to study both the momentum transfer coefficient ϵ_m and the sediment transfer coefficient ϵ_s, their distribution over the cross section, especially near the center line of the channel, and their interrelation.

APPARATUS AND PROCEDURE

Main Flume.—The experiments were conducted in a closed-circuit flume (Fig. 1) about 10.5 in. wide, 3 in. deep, and 40 ft long. Precise measurements show that the section is 2.986 in. (± 0.5%) by 10.507 in. (± 0.4%). For the present experiments the flume was kept horizontal.

The top cover of the flume was made of $\frac{5}{16}$-in. steel plate, supported from above, as shown in Fig. 1(*c*). The cover was sealed at the sides by a solid rubber rod ($\frac{3}{16}$ in. in diameter). All the surfaces were covered carefully with two coats of bitumastic paint which gave a reasonably smooth surface.



(*a*) PLAN

(*b*) ELEVATION

(*c*) TYPICAL SECTION

FIG. 1.—LAYOUT AND DIMENSIONS, 10-IN. CLOSED CIRCUIT FLUME

The pitot tube and the samplers can traverse the cross section through an opening in the flume cover (Fig. 2). This working section is 28.5 ft from the entrance to the channel. On the downstream side of the working section, there were glass windows 5 ft long on each side wall and a plastic window in the cover 10.5 in. long and 6 in. wide. The water or the mixture of sand and water was circulated in the flume by a propeller pump driven by a variable speed motor.

Measuring Equipment.—A standard Prandtl pitot-static tube $\frac{1}{4}$ in. in diameter was used to measure the velocity. The differential pressure on the tube was read to 0.001 ft on a water-air manometer and converted to velocity by assuming a coefficient of unity for the tube. Two samplers were used to determine the suspended sand concentration at different points. One of them reached the upper surface and the other reached the bed (see Fig. 3). Both were made of brass tubing with an external diameter of $\frac{3}{16}$ in. Their tips were flattened so that the inside dimensions were 0.22 in. by 0.05 in. Each sample was siphoned from the flow and collected in a bottle of 1-liter volume.

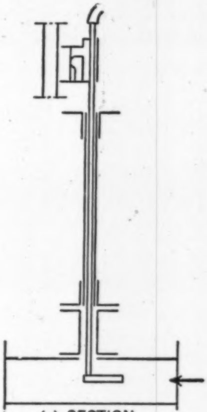
The elevation of the bottle and, hence, the head on the siphon could be adjusted to give the same velocity at the inlet to the sampler as the local stream velocity at the sampling point. The effect of the rate of sampling on the proportion of sediment removed with the sample, as well as the sampling technique, has been clearly explained elsewhere.[8,9] The sediment was removed from the sample by filtering and was dried and weighed to the nearest 0.005 g.

A general sample was collected by a special sampler at the end of the flume. Three liters were collected for each run and they were used to find an average value of concentration of suspended sand over the section. This average concentration was called $\bar{c}$.

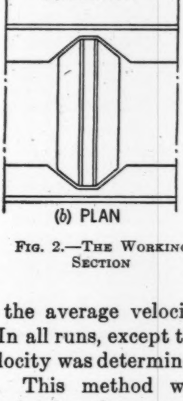
The pitot tube, or one of the two samplers, could be fixed to a carriage contrived so that it could move across and along the stream and in the vertical direction. The three coordinates of the instrument on the carriage could be read from scales that enabled the operator either to locate the position of the instrument or to set it at a predetermined position.

The flow rate and the average velocity, U, in the channel were determined from velocity measurements with the pitot tube. From complete velocity traverses in runs 1, 2, 5, 6, 7, and 71, the relation—

$$U = 0.95\,u'' \quad \text{(8)}$$



(a) SECTION



(b) PLAN

FIG. 2.—THE WORKING SECTION

—was found, in which u'' is the average velocity along the center-line profile. In all runs, except the aforementioned, the average velocity was determined from this empirical relation. This method was adopted in preference to that of using the venturi meter. The meter behaved erratically, probably because of the presence of sediment in the flow and because of the disturbances from the pump, which is close to the meter.

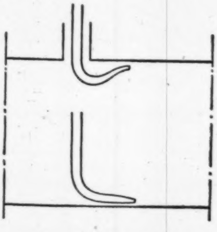


FIG. 3.—SAMPLERS

The head lost in the flume was measured by four pressure taps connected at 9.8 ft, 18.8 ft, 28.25 ft, and 33.25 ft from the inlet. Settling pots were included

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[9] "Laboratory Investigations of Suspended Sediment Samplers," *Report No. 5*, U. S. Engr. Dist. Suboffice, St. Paul, Minn., December, 1941.

in all pressure lines to trap sediment and prevent it from lodging in the lines. The static head could be read to the nearest 0.05 in. The temperature of the fluid was measured by a thermometer at the end of the flume.

It was very helpful that the same sand which was used by Mr. Vanoni[8] was still available. The sands with sedimentation diameters of 0.10 mm and 0.16 mm were used in this case. Their size distributions were checked by sieve analyses and were found to be still the same. Therefore, his measured values of the settling velocity w were used in the present work. The size distribution is given in Fig. 4.

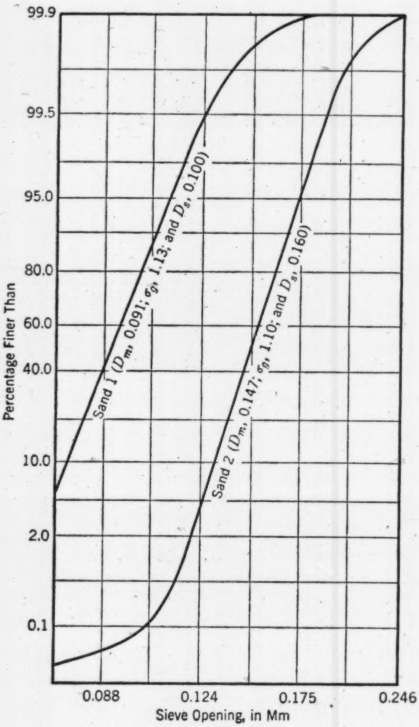

FIG. 4.—SIZE DISTRIBUTION OF SAND

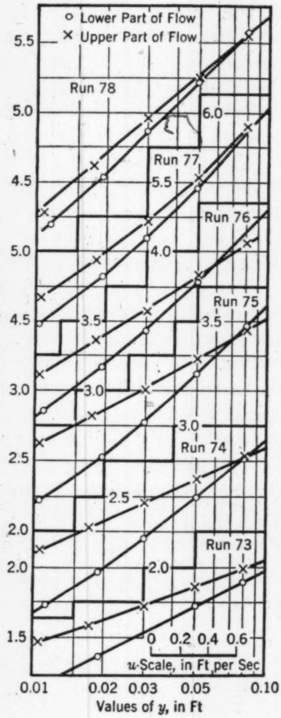

FIG. 5.—VELOCITY PROFILES

Procedure.—A complete series of experiments was made for each of the two sand sizes. Each series had two variables, the total weight of sand that existed in the flume and the velocity of the flow. Beginning with clear water, 50 g of sand were added; the weight added each time was about the same as that already in the system, following more or less the terms of a geometric series. In general, for each sand content, experiments were made with four flow velocities. They were about 1.7 ft per sec, 2.6 ft per sec, 3.5 ft per sec, and 4.6 ft per sec.

Results

A sample of the basic experimental data is given in Figs. 5, 6, and 7. Fig. 5 gives the semilogarithmic plot of the velocity profiles along the center line of the channel; Fig. 6 gives log C versus log h; and Fig. 7 gives log C versus y. The results obtained from all the experiments are collected and summarized in Table 1.

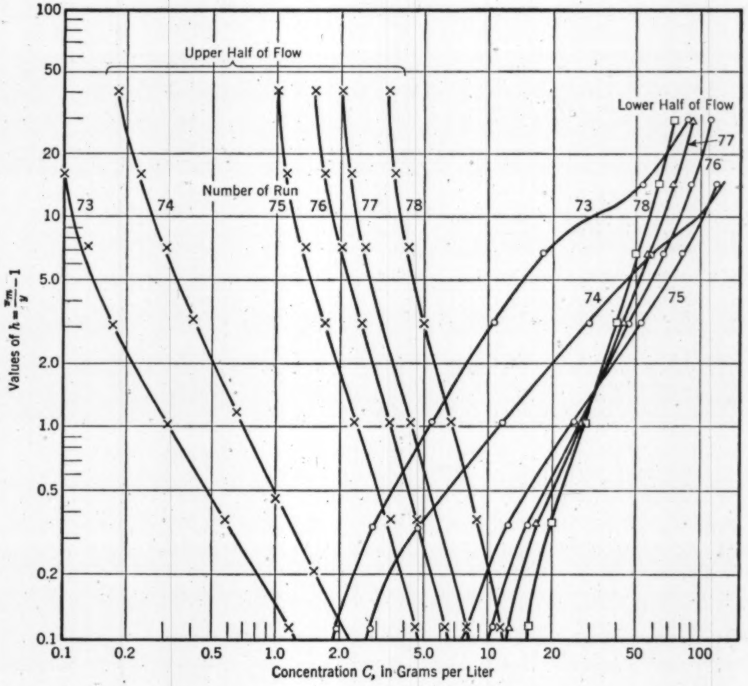

Fig. 6.—Measured Sediment Distribution at Center of Channel Plotted Against $h = y_m/y - 1$

It is to be noted that, except for the study of the central part of the channel, the flow was divided into two parts, one part covered nearly the upper half and the other part covered the lower half. Each part was treated alone as if it were a completely separate flow. This was studied and proved to be true by H. Schlichting.[10] H. A. Einstein,[11] A. M. ASCE, divided the section of open trapezoidal channels into three parts, each corresponding to one of the sides, and he assumed that the roughness of each wall would control the flow in the corresponding part.

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[10] "Experimental Investigation of the Roughness Problem," by H. Schlichting, *Proceedings*, ASCE, November, 1937, p. 16 following p. 1834.

[11] "Formulas for the Transportation of Bed Load," by H. A. Einstein, *Transactions*, ASCE, Vol. 107, 1942, p. 575.

TABLE I.—SUMMARY

Run	Mean velocity, U	Hydraulic gradient, i	Total sand load, W	Concentration, $\bar{c}$	Temperature, t	Viscosity, ν	Settling velocity, w	Friction velocity, U_f	Depth, y_m	Friction Velocity	
										Top, U_{tf}	Bottom U_{bf}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	1.78	0.00244	0	....	15.9	1.26	....	0.0883	....	....	....
2	2.45	0.00452	0	....	18.0	1.21	....	0.12	....	....	....
3	3.00	0.00587	0	....	19.3	1.17	....	0.1367	....	....	....
4	3.00	0.00687	0	....	17.9	1.30	....	0.148	....	....	....
5	3.66	0.01026	0	....	18.0	1.28	....	0.181	....	....	....
6	4.38	0.013	0	....	18.0	1.20	....	0.204	....	....	....
7	5.08	0.0172	0	....	19.5	1.16	....	0.234	....	....	....
8	1.78	0.00257	50	....	21.4	1.15	....	0.0906	....	....	....
9	2.41	0.0045	50	....	15.6	1.26	....	0.12	....	....	....
10	3.06	0.0067	50	....	21.7	1.10	....	0.146	....	....	....
11	3.70	0.0093	50	....	22.0	1.10	....	0.172	....	....	....
12	4.40	0.0128	50	....	20.7	1.11	....	0.202	....	....	....
13	5.12	0.0167	50	....	23.9	1.05	....	0.231	....	....	....
14	1.17	0.0011	100	....	21.3	1.10	....	0.0593	....	....	....
15	2.01	0.00312	100	....	25.2	1.05	....	0.0998	....	....	....
16	2.37	0.00404	100	0.015	21.0	1.11	....	0.1135	....	....	....
17	2.39	0.00413	100	....	22.8	1.08	....	0.115	....	....	....
18	3.53	0.00863	100	0.025	17.0	1.22	....	0.116	....	....	....
19	1.74	0.00248	150	....	16.0	1.26	....	0.089	....	....	....
20	2.56	0.00477	150	0.015	19.0	1.19	....	0.1233	....	....	....
21	2.60	0.00486	150	....	20.0	1.15	....	0.1246	....	....	....
22	3.58	0.00881	150	0.033	20.7	1.14	....	0.168	....	....	....
23	4.93	0.0155	150	0.057	20.8	1.14	....	0.222	....	....	....
24	1.12	0.0011	200	....	19.0	1.16	....	0.0593	....	....	....
25	1.74	0.00239	200	0.01	19.0	1.16	....	0.0873	....	....	....
26	2.40	0.0045	200	0.045	20.9	1.12	....	0.120	....	....	....
27	3.29	0.00744	200	0.07	19.0	1.16	....	0.154	....	....	....
28	4.23	0.0115	200	0.05	21.8	1.10	....	0.1915	....	....	....
29	5.28	0.0179	200	0.057	20.5	1.13	....	0.239	....	....	....
30	1.76	0.00239	290	0.02	22.5	1.07	....	0.0873	....	....	....
31	2.39	0.00422	290	0.035	18.5	1.18	....	0.116	....	....	....
32	3.26	0.0078	290	0.06	18.7	1.18	....	0.158	....	....	....
33	4.25	0.01191	290	0.065	19.3	1.17	....	0.195	....	....	....
34	1.73	0.00238	390	0.02	20.5	1.13	....	0.0872	....	....	....
35	2.65	0.00504	390	0.06	17.6	1.21	....	0.127	....	....	....
36	3.58	0.00835	390	0.08	21.6	1.10	....	0.163	....	....	....
37	4.97	0.0151	390	0.11	20.0	1.15	....	0.2196	....	....	....
38	1.73	0.00229	490	0.035	21.0	1.12	0.0267	0.0855	....	....	....
39	2.60	0.00449	490	0.08	17.5	1.22	0.025	0.12	....	....	....
40	3.60	0.0085	490	0.115	18.7	1.19	0.0255	0.165	....	....	....
41	4.89	0.0147	490	0.16	20.0	1.15	0.0262	0.216	....	....	....
42	1.08	0.00101	600	....	18.0	1.2	....	0.0567	0.13	0.0554	0.057
43	1.56	0.00193	600	0.02	18.2	1.2	0.0252	0.0785	0.124	....	....
44	1.72	0.00229	600	0.015	20.6	1.13	0.0265	0.0855	0.124	....	....
45	2.37	0.00413	600	0.06	21	1.12	0.0267	0.1148	0.124	....	....
46	2.96	0.00582	600	0.12	23.4	1.05	0.028	0.1363	0.124	....	....
47	3.59	0.0084	600	0.15	24.4	1.02	0.0285	0.163	0.124	....	....
48	4.69	0.0142	600	0.17	23.7	1.05	0.0281	0.212	0.124	....	....
49	1.18	0.00116	800	....	19.2	1.16	....	0.0608	0.122	0.0612	0.0604
50	1.83	0.0025	800	....	19.2	1.16	....	0.0893	0.124	....	....
51	1.75	0.00227	800	0.055	24.8	1.02	0.0287	0.0851	0.124	....	....
52	2.65	0.00499	800	0.165	21.2	1.14	0.0268	0.126	0.124	....	....
53	3.58	0.0086	800	0.26	21.0	1.12	0.0267	0.165	0.124	....	....
54	4.91	0.0148	800	0.34	25.5	1.01	0.0291	0.217	0.124	....	....
55	1.71	0.00224	1,200	0.175	25.8	1.01	0.0292	0.0845	0.124	....	....
56	2.58	0.00457	1,200	0.47	20.5	1.14	0.0264	0.121	0.122	0.122	0.120
57	3.59	0.00832	1,200	0.60	23.8	1.04	0.0282	0.163	0.121	0.164	0.162
58	4.71	0.0136	1,200	0.60	25.5	1.01	0.0291	0.208	0.124	....	....
59	1.18	0.0013	2,000	....	20.5	1.14	....	0.0644	0.124	....	....
60	1.42	0.00166	2,000	....	21.0	1.13	....	0.0728	0.124	....	....
61	1.71	0.00226	2,000	0.53	21.8	1.11	0.0271	0.085	0.13	0.083	0.0865
62	2.61	0.00467	2,000	1.23	21.5	1.11	0.027	0.122	0.124	....	....
63	3.55	0.0083	2,000	1.46	23.8	1.04	0.0282	0.163	0.124	....	....
64	4.67	0.0138	2,000	1.54	21.6	1.11	0.027	0.210	0.124	....	....

[a] Dunes and streaks.

[b] Not able

OF EXPERIMENTS

UNIVERSAL CONSTANT			Reynolds' number, $R \times 10^{-3}$ (16)	Friction coefficient, $\lambda \times 10^{2}$ (17)	RATIO, $w/(kU_f)$		$s = w/(\beta k U_f)$		$\frac{\epsilon_{se}}{\beta k y_m U_f}$ (22)	Bed condition[a] (23)	Run (1)
Top, k_t (13)	Bottom, k_b (14)	Average, k_{avg} (15)			Top (18)	Bottom (19)	Top (20)	Bottom (21)			
....	....	0.370	4.35	1.92	....	....	....	....	....	....	1
....	....	0.370	6.50	1.88	....	....	....	....	....	....	2
....	....	0.374	8.51	1.64	....	....	....	....	....	....	3
....	....	0.378	6.91	1.91	....	....	....	....	....	....	4
....	....	0.375	8.70	1.92	....	....	....	....	....	....	5
....	....	0.372	11.80	1.70	....	....	....	....	....	....	6
....	....	0.373	14.65	1.67	....	....	....	....	....	....	7
....	....	0.373	5.22	2.03	....	....	....	....	....	Smooth	8
0.392	0.368	0.380	5.89	1.94	....	....	....	....	....	Smooth	9
0.369	0.365	0.367	9.80	1.79	....	....	....	....	....	Smooth	10
....	....	0.360	11.86	1.70	....	....	....	....	....	Smooth	11
....	....	0.375	13.9	1.65	....	....	....	....	....	Smooth	12
0.391	0.369	0.380	18.0	1.60	....	....	....	....	....	Smooth	13
....	....	0.388	3.76	2.00	....	....	....	....	....	Streaks	14
....	....	0.362	7.16	1.94	....	....	....	....	....	Streaks	15
....	....	0.363	7.46	1.80	....	....	....	....	....	Streaks	16
....	....	0.368	7.97	1.81	....	....	....	....	....	Streaks	17
....	....	0.358	9.21	1.73	....	....	....	....	....	Smooth	18
....	....	0.376	4.25	2.04	....	....	....	....	....	Streaks	19
0.387	0.329	0.358	6.47	1.82	....	....	....	....	....	Streaks	20
0.388	0.368	0.378	7.64	1.80	....	....	....	....	....	Streaks	21
0.368	0.346	0.357	10.70	1.72	....	....	....	....	....	Smooth	22
0.37	0.358	0.364	14.75	1.60	....	....	....	....	....	Smooth	23
....	....	0.402	3.24	2.19	....	....	....	....	....	....[a]	24
....	....	0.372	5.02	1.97	....	....	....	....	....	Streaks	25
....	....	0.370	7.42	1.95	....	....	....	....	....	Streaks	26
....	....	0.353	9.50	1.72	....	....	....	....	....	Smooth	27
0.367	0.345	0.356	13.60	1.61	....	....	....	....	....	Smooth	28
....	....	0.355	16.0	1.60	....	....	....	....	....	Smooth	29
....	....	0.365	5.90	1.93	....	....	....	....	....	Streaks	30
....	....	0.357	6.68	1.85	....	....	....	....	....	Streaks	31
....	....	0.362	9.10	1.83	....	....	....	....	....	Smooth	32
0.374	0.35	0.362	12.04	1.65	....	....	....	....	....	Smooth	33
0.378	0.365	0.371	5.30	1.99	....	....	....	....	....	Streaks	34
0.382	0.350	0.366	7.02	1.80	....	....	....	....	....	Streaks	35
0.368	0.354	0.361	11.50	1.63	....	....	....	....	....	Smooth	36
0.369	0.344	0.357	14.6	1.53	....	....	....	....	....	Smooth	37
0.371	0.334	0.353	6.00	1.92	....	....	....	....	....	Streaks	38
0.355	0.330	0.343	8.30	1.66	....	....	....	....	....	Streaks	39
0.372	0.348	0.36	11.75	1.65	....	....	....	....	....	Smooth	40
....	....	0.355	16.5	1.54	....	....	....	....	....	Smooth	41
0.368	0.366	0.367	3.5	2.17	....	....	....	....	....	Dunes	42
....	....	0.360	5.05	1.98	....	....	....	....	....	....[a]	43
0.352	0.334	0.343	5.9	1.94	....	....	....	....	....	....[a]	44
0.357	0.348	0.353	8.3	1.84	0.65	0.67	....	0.467	....	Streaks	45
0.357	0.346	0.352	10.94	1.66	0.575	0.593	0.40	0.40	....	Streaks	46
0.362	0.342	0.351	13.8	1.63	0.486	0.514	0.355	0.356	....	Smooth	47
0.366	0.346	0.356	17.38	1.62	0.366	0.388	0.198	0.262	....	Smooth	48
0.375	0.348	0.362	3.95	2.08	....	....	....	....	....	Dunes	49
0.387	0.332	0.360	6.12	1.86	....	....	....	....	....	Streaks	50
0.363	0.338	0.351	6.67	1.85	0.93	0.996	....	0.75	....	Streaks	51
....	....	0.354	9.03	1.78	0.60	0.60	....	0.46	....	Streaks	52
....	....	0.365	13.6	1.68	0.444	0.444	0.325	0.352	....	Smooth	53
0.378	0.367	0.372	18.9	1.54	0.355	0.366	0.227	0.269	....	....[b]	54
0.360	0.309	0.335	6.58	1.91	0.96	1.12	0.58	0.58	....	Streaks	55
0.375	0.320	0.348	8.8	1.72	0.577	0.69	0.438	0.438	....	....	56
0.376	0.362	0.369	13.4	1.62	0.458	0.483	0.355	0.38	....	....[b]	57
0.357	0.355	0.356	18.1	1.53	0.392	0.394	0.30	0.30	....	....[b]	58
0.41	0.37	0.39	4.01	2.33	....	....	....	....	....	....[a]	59
0.356	0.356	0.356	4.88	2.06	....	....	....	....	....	....[a]	60
0.335	0.312	0.324	6.0	1.93	....	1.003	....	0.65	0.123	Streaks	61
0.347	0.323	0.335	6.66	1.71	0.638	0.685	0.437	0.51	0.195	....[b]	62
0.354	0.338	0.346	13.2	1.64	0.489	0.512	0.37	0.385	0.197	....[b]	63
0.348	0.348	0.348	16.35	1.58	0.37	0.37	0.262	0.293	0.204	....[b]	64

to see the condition of the bed.

TABLE I.—

Run	Mean velocity, U	Hydraulic gradient, i	Total sand load, W	Concentration, $\bar{c}$	Temperature, t	Viscosity, ν	Settling velocity, w	Friction velocity, U_f	Depth, y_m	FRICTION VELOCITY	
										Top, U_{tf}	Bottom, U_{bf}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
65	1.74	0.0024	3,600	0.92	24.5	1.03	0.0285	0.0875	0.128	0.086	0.0883
66	2.61	0.00476	3,600	2.90	19.0	1.18	0.0257	0.123	0.113	0.128	0.1167
67	3.48	0.00818	3,600	3.08	21.6	1.11	0.0271	0.1617	0.12	0.164	0.158
68	4.89	0.0153	3,600	3.10	20.7	1.14	0.0265	0.221	0.124	....	....
69	1.66	0.0025	6,800	1.49	20.5	1.14	0.0265	0.0893	0.13	0.0873	0.095
70	2.61	0.00471	6,800	5.40	24.0	1.04	0.0283	0.1225	0.119	0.1248	0.117
71	3.56	0.00827	6,800	5.77	23.0	1.09	0.0278	0.162	0.118	0.166	0.155
72	4.74	0.0141	6,800	6.68	25.0	1.02	0.0288	0.212	0.130	0.207	0.217
73	1.69	0.00277	13,200	2.39	21.0	1.11	0.0267	0.094	0.136	0.0895	0.096
74	2.39	0.00415	13,200	7.23	25.0	1.02	0.0288	0.115	0.115	0.119	0.111
75	2.96	0.00591	13,200	12.57	21.6	1.11	0.027	0.1373	0.112	0.144	0.126
76	3.55	0.00827	13,200	12.65	24.5	1.025	0.0285	0.1622	0.105	0.174	0.145
77	4.18	0.01115	13,200	12.28	26.2	1.00	0.0294	0.188	0.114	0.195	0.176
78	4.91	0.01487	13,200	12.38	25.4	1.01	0.029	0.218	0.12	0.221	0.211
79	1.61	0.0029	23,200	2.92	24.1	1.03	0.0284	0.0963	0.152	0.085	0.1066
80	2.64	0.00535	23,200	13.66	22.7	1.07	0.0277	0.131	0.124	....	....
81	3.62	0.00815	23,200	31.10	25.5	1.01	0.0291	0.161	0.116	0.166	0.156
82	4.93	0.015	23,200	23.0	26.5	0.99	0.0296	0.219	0.116	0.266	0.212
83	1.74	0.00222	0	....	23	1.06	....	0.0842	....	....	....
84	2.64	0.00471	0	....	26.3	1.00	....	0.1227	....	....	....
85	3.67	0.00863	0	....	24	1.03	....	0.166	....	....	....
86	4.71	0.0136	0	....	26	1.00	....	0.2084	....	....	....
87	1.71	0.00199	100	....	24.3	1.03	....	0.0797	....	....	....
88	2.60	0.00476	100	....	23	1.06	....	0.1232	....	....	....
89	3.55	0.00808	100	....	24.7	1.02	....	0.1605	....	....	....
90	4.72	0.0136	100	....	26.5	1.00	....	0.2084	....	....	....
91	1.74	0.00222	300	0.004	23.3	1.05	....	0.0842	....	....	....
92	2.60	0.00466	300	0.065	21.5	1.11	....	0.1220	....	....	....
93	3.58	0.00823	300	0.125	24.8	1.06	....	0.1622	....	....	....
94	4.60	0.01327	300	0.142	22.0	1.09	....	0.206	....	....	....
95	1.715	0.00226	700	0.034	22	1.09	0.0584	0.085	....	....	....
96	2.59	0.00489	700	0.27	21	1.13	0.0573	0.125	....	....	....
97	3.56	0.00823	700	0.35	24.7	1.02	0.0612	0.1622	....	....	....
98	4.61	0.01322	700	0.44	23.0	1.06	0.0595	0.2058	....	....	....
99	1.73	0.00235	1,500	0.215	25.5	1.01	0.0622	0.0866	....	....	....
100	2.60	0.00484	1,500	0.868	22.8	1.07	0.0592	0.1243	....	....	....
101	3.52	0.0084	1,500	1.067	21.0	1.13	0.0572	0.164	....	....	....
102	4.57	0.0131	1,500	1.05	24.6	1.02	0.0611	0.2045	....	....	....
103	1.72	0.00244	3,100	0.385	25.0	1.02	0.0616	0.0884	....	....	....
104	2.59	0.00494	3,100	1.94	23.2	1.05	0.0596	0.1257	....	....	....
105	3.55	0.0085	3,100	2.21	21.5	1.11	0.0578	0.1648	0.121	0.167	0.1628
106	4.58	0.01323	3,100	2.16	24.7	1.02	0.0612	0.206	....	....	....
107	1.64	0.0026	6,300	0.45	23.0	1.06	0.0595	0.091	0.126	0.0902	0.092
108	2.58	0.00492	6,300	3.10	27.0	0.98	0.0638	0.1253	0.124	....	....
109	3.54	0.00851	6,300	5.0	23.5	1.04	0.060	0.1649	0.124	....	....
110	4.59	0.01317	6,300	4.7	27.1	0.97	0.0638	0.205	0.119	0.209	0.202
111	1.64	0.00274	12,700	0.57	27.0	0.98	0.0638	0.0935	0.134	0.09	0.097
112	2.55	0.0054	12,700	5.1	24.5	1.02	0.0612	0.1313	0.127	0.1297	0.1332
113	3.50	0.00886	12,700	10.7	24	1.03	0.0607	0.168	0.124	....	....
114	4.60	0.01385	12,700	13.8	26.8	0.99	0.0636	0.2104	0.121	0.2126	0.2085
115	1.53	0.00318	25,500	1.3	22.5	1.088	0.0589	0.1008	0.148	0.091	0.110
116	2.58	0.00563	25,500	4.8	28	0.95	0.0648	0.1341	0.128	0.1322	0.1362
117	3.43	0.0096	25,500	13.5	26	1.00	0.0628	0.175	0.125	0.174	0.1762
118	4.47	0.0135	25,500	24.0	28	0.95	0.0648	0.208	0.124	....	....
119	1.44	0.0029	38,300	1.4	26	1.00	0.0628	0.096	0.154	0.083	0.107
120	2.56	0.0059	38,300	4.5	26.7	0.98	0.0636	0.1372	0.128	0.1343	0.1402
121	3.46	0.01005	38,300	13.0	24	1.03	0.0607	0.1794	0.126	0.1772	0.182
122	4.44	0.01425	38,300	30.0	25	1.02	0.0616	0.2135	0.125	0.212	0.215

[a] Dunes and streaks.

[b] Not able

(Continued)

UNIVERSAL CONSTANT			Reynolds' number, R × 10^{-5}	Friction coefficient, λ × 10^2	RATIO, $w/(kU_f)$		$z = w/(\beta k U_f)$		$\frac{\epsilon_{sc}}{\beta k y_m U_f}$	Bed condition[a]	Run
Top, k_t (13)	Bottom, k_b (14)	Average, k_{avg} (15)	(16)	(17)	Top (18)	Bottom (19)	Top (20)	Bottom (21)	(22)	(23)	(1)
0.341	0.313	0.327	6.55	1.98	0.97	1.03	0.562	0.65	0.224	....[a]	65
0.366	0.293	0.330	8.6	1.75	0.55	0.753	0.36	0.49	0.195	....[b]	66
0.347	0.299	0.323	12.2	1.68	0.476	0.574	0.318	0.372	0.20	....[b]	67
0.346	0.346	0.346	16.62	1.60	0.347	0.347	0.208	0.264	0.205	....[b]	68
0.386	0.30	0.343	5.67	2.26	0.786	0.93	0.482	0.593	0.207	Dunes	69
0.346	0.23	0.288	9.75	1.73	0.655	1.05	0.37	0.66	0.204	....[b]	70
0.370	0.283	0.327	12.7	1.63	0.452	0.634	0.30	0.39	0.203	....[b]	71
0.355	0.31	0.332	18.0	1.57	0.40	0.43	0.28	0.28	0.196	....[b]	72
0.343	0.260	0.301	5.9	2.42	0.87	1.07	0.56	0.62	0.233	Dunes	73
0.365	0.195	0.28	9.1	1.82	0.664	1.33	0.395	0.86	0.210	....[b]	74
0.360	0.216	0.288	10.35	1.69	0.52	0.99	0.30	0.655	0.174	....[b]	75
0.364	0.216	0.29	13.4	1.64	0.451	0.91	0.284	0.486	0.178	....[b]	76
0.346	0.256	0.301	16.2	1.60	0.436	0.653	0.265	0.40	0.188	....[b]	77
0.356	0.294	0.325	18.9	1.54	0.362	0.458	0.26	0.32	0.185	....[b]	78
0.350	0.285	0.318	6.08	2.8	0.95	0.936	0.61	0.62	0.226	....[b]	79
0.343	0.201	0.272	9.6	1.92	0.61	1.05	0.37	0.88	0.180	....[b]	80
0.326	0.221	0.274	13.9	1.56	0.538	0.843	0.263	0.56	0.181	....[b]	81
0.340	0.268	0.304	19.3	1.54	0.386	0.52	0.254	0.35	0.181	....[b]	82
....	....	0.373	6.40	1.84	....	....	....	....	....	....	83
....	....	0.367	10.2	1.70	....	....	....	....	....	....	84
....	....	0.368	13.9	1.60	....	....	....	....	....	....	85
....	....	0.37	18.4	1.53	....	....	....	....	....	....	86
0.360	0.348	0.354	6.45	1.70	....	....	....	....	....	Streaks	87
0.368	0.346	0.357	9.50	1.76	....	....	....	....	....	Streaks	88
....	....	0.356	13.6	1.60	....	....	....	....	....	Smooth	89
0.368	0.36	0.364	18.4	1.53	....	....	....	....	....	Smooth	90
0.38	0.364	0.372	6.45	1.83	....	....	....	....	....	Streaks	91
....	....	0.343	9.1	1.73	....	....	....	....	....	Streaks	92
0.366	0.349	0.357	13.2	1.61	....	....	....	....	....	Smooth	93
0.359	0.346	0.353	16.4	1.57	....	....	....	....	....	Smooth	94
0.390	0.376	0.383	6.12	1.92	....	....	....	....	....	Streaks	95
0.364	0.347	0.356	8.9	1.82	....	1.32	....	1.16	....	Streaks	96
0.363	0.328	0.346	13.6	1.62	....	1.15	....	0.85	....	....[b]	97
0.381	0.324	0.353	16.9	1.56	0.76	0.89	0.86	0.65	....	....[b]	98
0.338	0.333	0.336	6.66	1.96	....	2.14	....	1.63	0.242	Streaks	99
0.349	0.322	0.336	9.45	1.79	....	1.48	....	1.11	0.250	....[b]	100
0.362	0.337	0.350	12.2	1.69	0.96	1.03	0.83	0.80	0.226	....[b]	101
0.365	0.341	0.353	17.4	1.56	0.82	0.87	0.68	0.68	0.215	....[b]	102
0.376	0.351	0.363	6.55	2.06	....	1.98	....	1.57	0.40	Streaks	103
0.370	0.311	0.341	9.60	1.84	....	1.57	....	1.09	0.25	....[b]	104
0.372	0.30	0.336	12.4	1.69	0.93	1.18	0.72	0.94	0.223	....[b]	105
0.362	0.327	0.345	17.5	1.58	0.82	0.91	0.64	0.73	0.209	....[b]	106
0.376	0.353	0.364	6.01	2.41	....	1.83	....	1.47	0.42	Dunes	107
0.356	0.292	0.324	10.4	1.85	....	1.74	....	1.36	0.173	....[b]	108
0.348	0.265	0.307	13.2	1.70	1.04	1.37	0.31	1.07	0.236	....[b]	109
0.359	0.295	0.327	18.4	1.56	0.85	1.07	0.58	0.86	0.222	....[b]	110
0.385	0.262	0.323	6.50	2.55	....	2.51	....	1.73	0.53	Dunes	111
0.363	0.274	0.318	9.7	2.08	....	1.68	....	1.43	0.245	....[b]	112
0.368	0.21	0.289	13.2	1.80	0.98	1.7	0.67	1.21	0.23	....[b]	113
0.350	0.230	0.240	18.1	1.64	0.855	1.33	0.56	0.99	0.223	....[b]	114
0.388	0.272	0.330	5.5	3.40	....	1.97	....	1.67	0.54	Dunes	115
0.384	0.272	0.328	10.6	2.12	....	1.74	....	1.67	0.244	....[b]	116
0.380	0.215	0.297	13.3	2.04	0.95	1.65	0.78	1.28	0.208	....[b]	117
0.352	0.199	0.276	18.3	1.69	0.88	1.56	0.70	1.11	0.215	....[b]	118
0.337	0.272	0.305	5.23	3.62	....	....	....	....	0.59	....[b]	119
0.329	0.221	0.275	10.2	2.25	1.44	2.05	1.20	1.83	0.215	....[b]	120
0.374	0.187	0.280	13.1	2.10	0.92	1.78	0.82	1.37	0.193	....[b]	121
0.354	0.183	0.268	17.0	1.81	0.825	1.57	0.60	1.2	0.20	....[b]	122

to see the condition of the bed.

Like the objective of the experiments, the results can be divided into two parts. A third part will be given on the observations at the bed.

EFFECT OF SUSPENDED SEDIMENT ON THE CHARACTERISTICS OF THE FLOW

The characteristics of the flow can be represented by three quantities: (*a*) von Kármán's universal constant k; (*b*) the momentum transfer coefficient ϵ_m; and (*c*) the coefficient of friction λ.

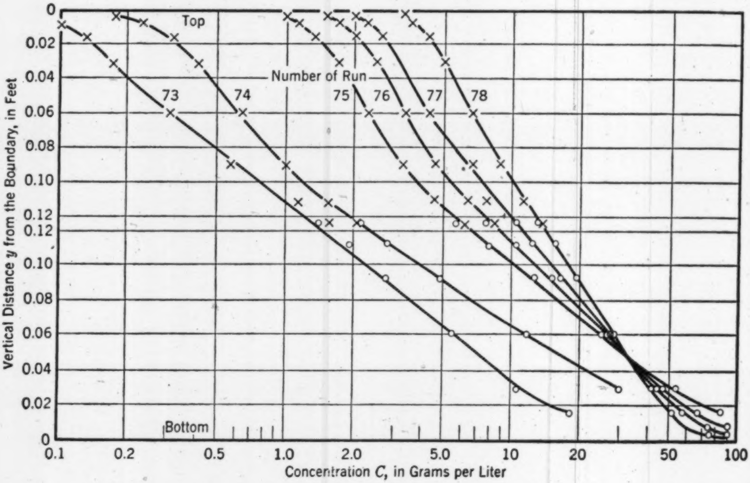

FIG. 7.—SEDIMENT CONCENTRATION PROFILES

von Kármán's Universal Constant, k.—The constant k can be calculated from the velocity defect law, Eq. 3, as follows: $\frac{u - u_{\max}}{U_f} = \frac{2.3}{k} \log \frac{y}{y_m}$; that is, the slope of the semilogarithmic velocity profile will give k/U_f. Then

$$K = 2.3\ U_f \times \text{slope of the semilogarithmic velocity profile} \ldots\ldots (9)$$

but

$$\overline{U}_f = \sqrt{\frac{\tau_o}{\rho}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

in which τ_o equals the shear at the point of intersection of the profile with the boundary. Thus, in order to find k, τ_o must be known.

By using the experimental data of M. Bazin, Garbis H. Keulegan[12] showed that the value of the shear changes along the bottom of rectangular channels. He found that the values of U_f at the center of the bed, divided by the average $\overline{U}_f$ varied from 1.00 to 1.07 for moderate roughness. He did not give any data for smooth channels. A few measurements, in the present work, were made covering the entire section—runs 1, 2, 5, 6, and 7 for clear water and run 71 for loaded water. Runs 1, 2, 5, and 6 showed that U_f was uniform along the boundary, whereas in run 7, the ratio $U_f/\overline{U}_f$ at the center was 1.04. The distribution of U_f is an interesting problem and needs more study. This distribution might have been responsible for the small variation in the values of k found for clear water in channels of different shapes. However, it was clear that the distribution of U_f could contribute to the variation in the slopes of the semilogarithmic velocity profiles, and, hence could introduce errors into the proposed method of computing k. Its effect, however, seemed to be very small with respect to the values observed here (k decreased as much as 50%). In the present work U_f was assumed to be uniform along each wall.

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[12] "Laws of Turbulent Flow in Open Channels," by Garbis H. Keulegan, *Journal of Research*, National Bureau of Standards, U. S. Dept. of Commerce, Vol. 21, 1938, pp. 707-741.

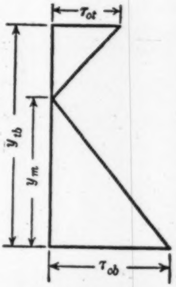

FIG. 8.—SHEAR DISTRIBUTION

To find the shear velocity at top and bottom (U_{tf} and U_{bf}), the flow was assumed to be two dimensional and, therefore, $d\tau/dy = dP/dx$, in which p is the pressure intensity and x is the horizontal distance along the axis of the channel. Assuming a uniform pressure gradient in the direction of flow, τ will be linearly related to y (Fig. 8)—that is, $\tau_{ot} = \tau_{ob}\,(y_{tb}/y_m - 1)$, in which y_{tb} is the total depth from top to bottom boundary; τ_{ot} is the intensity of shear at the top wall; and τ_{ob} is the intensity of shear at the bottom wall. (The symbol y_m is the coordinate of the point of zero shear or the point of maximum velocity.) Also, $\frac{1}{2}(\tau_{ob} + \tau_{ot}) = \tau_o$ in which τ_o is the average intensity of shear over the entire boundary. Therefore, $\tau_{ob}\, y_{tb}/y_m = 2\,\bar{\tau}_o$, and $\sqrt{\tau_{ob}} = \sqrt{\bar{\tau}_o}\,\sqrt{2\, y_m/y_{tb}}$ or $U_{fb} = \overline{U}_f\,\sqrt{2\, y_m/y_{tb}}\,\sqrt{\rho \text{ average}/\rho \text{ bottom}}$.

For a concentration of 33 g per liter, the ratio of the density for loaded water to that of clear water was 1.02 and its square root was 1.01; that is, it would cause an error of 1%, which is within the experimental error. For concentrations less than 30 g per liter the correction for the density was neglected; that is, $U_{fb} = \overline{U}_f\,\sqrt{2\, y_m/y_{tb}}$ and $U_{ft} = \overline{U}_f\,\sqrt{2\,(1 - y_m/y_{tb})}$.

These values are given in Cols. 11 and 12, Table 1. The values of k which were computed from these values and the slopes of the (u- versus-log y)-lines, are given in Cols. 13 and 14. For both sands k decreased from 0.373 to about 0.20 as the average concentration increased.

Fig. 9 shows the relation between the values of k and the total load of sand, W, in the system for two mean velocities in the channel. The points were somewhat scattered but Fig. 9 shows the definite tendency of k to decrease with the increase in the load. For the same total loads Fig. 9 shows that the 0.16-mm sand had more effect on k at the bottom than the 0.10-mm sand. This is not true because, for the same total load and for the same speed, concentration at the bottom is higher for the 0.16-mm sand than for the 0.10-mm sand; and so it can be assumed that both sands have nearly the same effect on k.

Moment Transfer Coefficient ϵ_m.—Some of the semilogarithmic plots of velocity profiles are given in Fig. 5. The experimental points follow reasonably well the logarithmic law over most of the section. This proves that ϵ_m still follows Eq. 4a for all those different concentrations. The only changes in ϵ_m will be due to changes in k.

The central region should be excluded from this rule. Messrs. Brooks and Berggren[7] showed that a very slight change in fitting the Nikuradse experiments will cause an appreciable difference in the values of ϵ_m. For example, a constant ϵ_m will give a second-degree parabola for the velocity profile, and this parabola can fit the velocities reported by Mr. Nikuradse for $R_e = 43{,}400$ within 0.6% over the middle half of the section. It is difficult to evaluate ϵ_m at this region from velocity measurements, because $\epsilon_m = \frac{\tau}{\rho\, du/dy}$ and both τ and du/dy approach zero at the center. For this reason, no attempt is made at this stage to evaluate ϵ_m from the velocity profiles. It will be evaluated indirectly by studying ϵ_s and the relation between ϵ_m and ϵ_s.

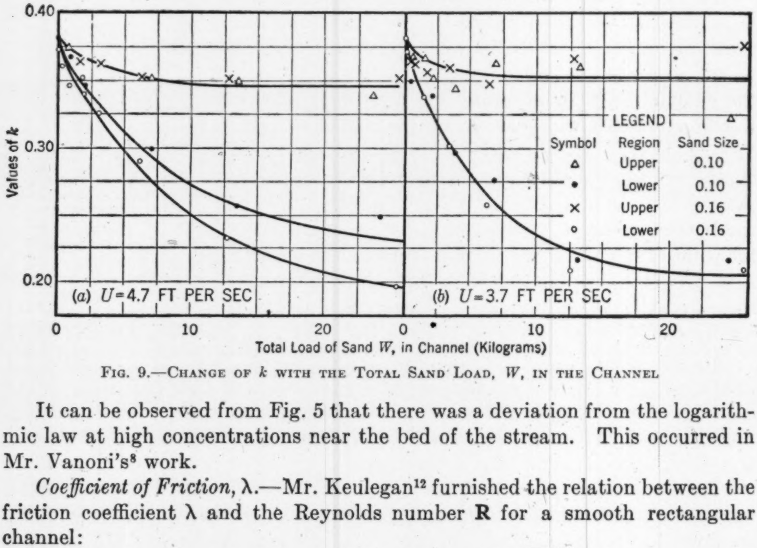

FIG. 9.—CHANGE OF k WITH THE TOTAL SAND LOAD, W, IN THE CHANNEL

It can be observed from Fig. 5 that there was a deviation from the logarithmic law at high concentrations near the bed of the stream. This occurred in Mr. Vanoni's[8] work.

Coefficient of Friction, λ.—Mr. Keulegan[12] furnished the relation between the friction coefficient λ and the Reynolds number $\mathbf{R}$ for a smooth rectangular channel:

$$\sqrt{8/\lambda} = a_s - 2.25 + \frac{2.3}{k}\log\left(\frac{\mathbf{R}\sqrt{\lambda}}{4\sqrt{8}}\right) \ldots\ldots\ldots\ldots (11)$$

In Eq. 11, $a_s = N - 2.3/k \log N$; $\mathbf{R} = 4\, r_h\, U/\nu$; and N is the characteristic value of the parameter $U_f\, y/\nu$, to which the transition between the boundary laminar film and the inner turbulent region corresponds.

Taking $N = 11.6$ according to the experiments of Mr. Nikuradse, and $k = 0.373$ according to Table 1, runs 1 to 7, Eq. 11 becomes

$$\frac{1}{\sqrt{\lambda}} = -1.32 + 2.18 \log \mathbf{R}\sqrt{\lambda} \ldots\ldots\ldots\ldots (12)$$

Fig. 10 shows such a curve with the experimental results. The curve fitted the points of the clear water quite satisfactorily. This proves that the walls

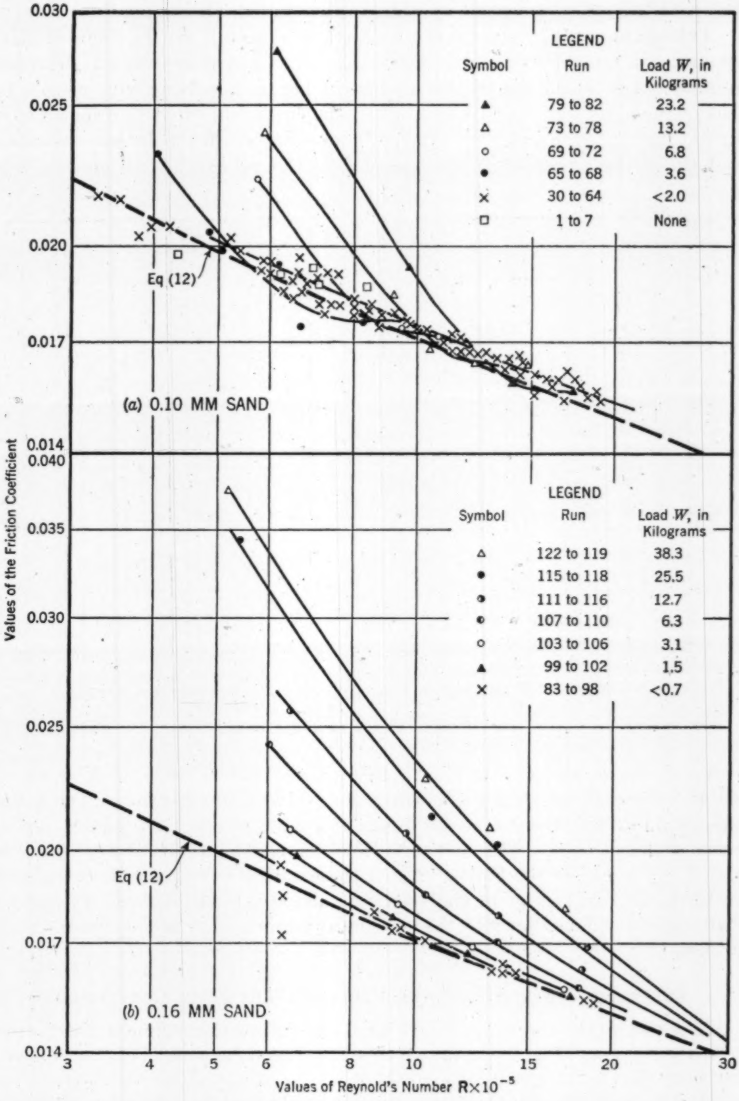

FIG. 10.—EFFECT OF SEDIMENT ON λ

are smooth. The general pattern of the results plotted in Fig. 10 agrees very well with the results collected by Mr. O'Brien and R. G. Folsom.[13]

Taking the curve λ versus **R** of Eq. 12 as a reference, it can be observed that, for each concentration of each sand, there was a certain critical Reynolds number below which the friction coefficient began increasing with respect to that of the clear water, and above which the coefficient of friction for all concentrations was practically the same. It was observed from the glass windows that the critical value of **R** corresponded to the velocities that were able to sweep all the dunes from the floor. Below these critical values of **R** an appreciable fraction of the sand was moving as a bed load in dune forms, and above these values of **R** there were no dunes and most of the sand was carried in suspension as clouds. This fact was also observed from the values of $\bar{c}$ given in Col. 5, Table 1 and from Fig. 6. Above these critical values of **R** it was difficult to increase $\bar{c}$ since most of the sand was already in suspension. It just redistributed itself, as shown by the crossing and overlapping of the concentration profiles (Fig. 6).

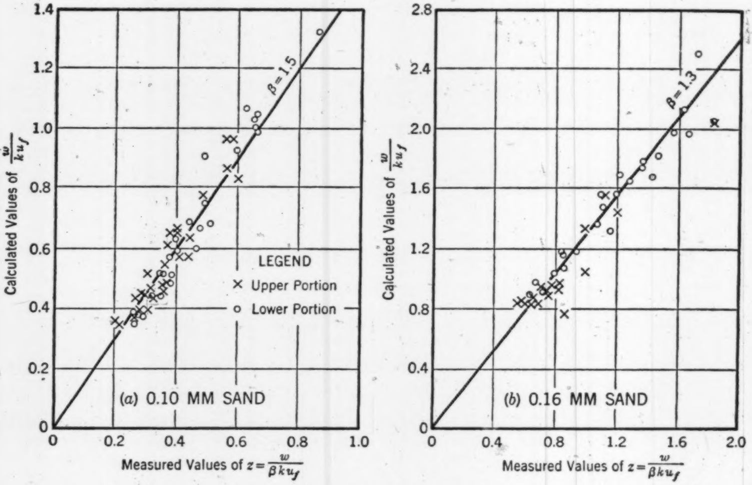

FIG. 11.—EVALUATION OF THE FACTOR β

SEDIMENT DISTRIBUTION AND SEDIMENT TRANSFER COEFFICIENT

The two sets of curves given in Figs. 6 and 7 show the sediment distribution over the section. Fig. 6 was drawn to verify Eqs. 7, whereas Fig. 7 was drawn to verify Eq. 6.

For most of the depth of the channel (excluding the central part) Fig. 6 shows that Eqs. 7 hold well. The slight curvature at the top wall will be discussed subsequently to prove that this deviation is due to sand grading. According to Eqs. 7, the slope of the lines of log C and log h must be equal to z. The values of z measured by this method are given in Cols. 20 and 21, Table 1, whereas the calculated values of $w/k\ U_f$ are given in Cols. 18 and 19. The upper and lower halves of the channel were treated separately. Fig. 11 shows the relation between the measured values of z (which should be $w/(\beta\ k\ U_f)$) and the values of $w/(k\ U_f)$. The linear relation in both curves for the two sands shows that the assumption of Eq. 4 is fairly reasonable. For the 0.10-mm sand, $\beta = 1.5$; and, for the 0.16-mm sand, $\beta = 1.3$.

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[13] "The Transportation of Sand in Pipelines," by M. P. O'Brien and R. G. Folsom, *Publications in Engineering*, Univ. of California, Berkeley, Calif., 1937, Vol. 3, No. 7, pp. 343–383.

Fig. 7 helps in studying the central part of the flume. The straight parts of the curves (which always occur in the center) indicate that the assumption of a constant ϵ_s here is quite satisfactory. The constant value of ϵ_s in the central parts of the flow (which will be called ϵ_{sc}), can be evaluated directly from Eq. 6 and from these curves. The nondimensional values $\epsilon_{sc}/(\beta\ k\ y_m\ U_f)$ are given in Col. 22, Table 1. This value is nearly constant for most of the runs, for both the sands, and its average value is about 0.22. The only runs that are far from this value are those at low speed and high concentration. In the latter runs big dunes were observed and the conditions of the flow were believed to be unsteady because of the periodical disturbance caused by the travel of dunes past the measuring point, so these readings were excluded. The straight part, and accordingly the constant ϵ_{sc}, was observed to cover at least the middle third of the channel depth in each run (Fig. 7). When ϵ_{sc} was measured and compared with the parabolic form of Eq. 4*b*, it was found that it would intersect it at the third points, as in Fig. 12. According to the present experiment, Fig. 12 would be the most reasonable approximation of the dimensionless form of ϵ_s. As previously mentioned, it could also represent the form of ϵ_m.

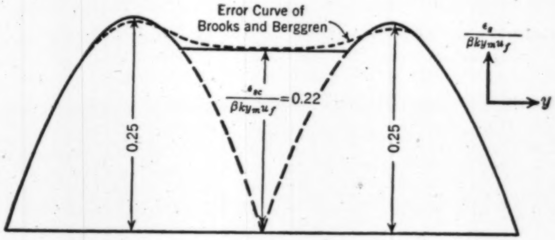
Error Curve of
Brooks and Berggren
$\frac{\epsilon_s}{\beta k y_m u_f}$
y
$\frac{\epsilon_{sc}}{\beta k y_m u_f} = 0.22$
0.25
0.25

FIG. 12.—DIMENSIONLESS FORM OF ϵ_s OR ϵ_m

BED LOAD

The bed load was observed through the glass windows. For high speeds and high concentrations it was not possible to see what was occurring. For small loads of sand and for low velocities the bed load was in the form of streaks. These streaks were not very steady and their particles were moving in steps. When the velocity was increased, all the streaks disappeared and the particles went into suspension.

When the concentrations were somewhat increased for low speed, dunes began to appear on the bed as isolated individual dunes of a crescent shape, small in size compared to the channel width. At the beginning, a single dune might appear every 5 ft or 6 ft along the bed. When the concentration was increased, the number of those dunes increased and they started joining each other to form one dune that extended across the channel. The form of the bed load is described by letters in Col. 23, Table 1.

A new phenomenon was observed for the maximum loads reached in the case of the 0.16-mm sand. During run 119, which was made with the low velocity of 1.7 ft per sec, very large dunes or waves were observed on the bed. They occurred irregularly every now and then; sometimes two might follow each other closely and sometimes more than half an hour elapsed between these waves. A wave was about 20 in. long and 0.6 in. in maximum height and would cover the full width of the flume. The dunes and waves of sand in that run were so large and unsteady that the entire flow was considered unsteady; then the results of the run were rejected.

With the same concentration but with higher velocities it seemed as if the dunes and waves were smoothed out and that a uniform layer was moving along the floor with small dunes on its surface. In run 120 this layer was about 0.25 in. thick, and the dunes were about 0.05 in. high over the layer.

DISCUSSION OF RESULTS

Effect of Suspended Load on the Flow.—The results of the studies can be appraised by a discussion of the effect of the suspended load on four characteristics of the flow: (*a*) the universal constant; (*b*) the momentum transfer coefficient; (*c*) the friction factor; and (*d*) the velocity distribution.

(*a*) The Universal Constant, *k*.—In his similarity theory of turbulence pattern Mr. von Kármán[3,6] introduced k as the factor of proportionality between the velocity derivatives and the mixing length $\frac{du/dy}{d^2u/dy^2} = l/k$, in which l is the Prandtl mixing length.

Assuming that the turbulence pattern will remain similar from point to point, k will remain constant over the cross section of the flow. The decrease in the value of k, which was observed by Mr. Vanoni, varied from run to run, but in each run k was the same for the entire profile. In the present work, k for the upper part of the section was constant; and k for the lower part was also constant but had a different value than for the upper part. This means that the change in k was not local from point to point following the different concentrations at these points, but changed from section to section and remained constant over each section. It seems, then, that the presence of a certain proportion of sediment in suspension will affect the main pattern of the turbulence in the entire region from the controlling boundary to the zero shear plane.

The slight curvature in the semilogarithmic velocity profiles observed in the case of high concentration near the bed (Fig. 5) cannot be explained with any local change in k. If k is affected by the local concentration, it should decrease as the bed is approached and this will curve the line in the opposite direction to that shown in Fig. 5. This curvature was observed to occur only near the bottom for appreciable concentrations and it increased with the increase in the load. This might be evidence that the curvature is due to some interaction between the flow and the presence of the particles at the bed. The question of the local effect of sand on k has been raised by E. R. Van Driest.[14]

By measuring the velocity profiles of the wind above a circular concrete pavement, P. A. Sheppard[15] observed an approximate variation in the values of k from 0.6 to 0.2 in the lower atmosphere. The constant of turbulent transfer decreases as the atmospheric stability increases. Mr. Sheppard declared these values are very rough; the wind profiles at the experimental site were not representative of the boundary layer because of the limited extent of the measuring surface (concrete.)

The only source of error in evaluating k can be due to a large variation in the U_f-distribution along the boundary. Run 71 showed that the existence of sand could never change the U_f-distribution to take care of even a small part of the changes in the semilogarithmic velocity profile slopes.

(*b*) Momentum Transfer Coefficient, ϵ_m.—It has been shown how ϵ_s was calculated and how Fig. 12 represents its distribution. The relation (Eq. 2) between ϵ_m and ϵ_s was verified in the part of the section where both ϵ_m and ϵ_s could be measured directly. It is very reasonable to assume that this relation will hold over the remainder of the section—that is, the central part, where ϵ_m is not known. Therefore, Fig. 12 can represent the distribution of ϵ_m as well as ϵ_s. This curve agrees very well with the curve given by Messrs. Brooks and Berggren[7] under their assumption of the error curve (see Fig. 12). The present relation is easier to handle.

(*c*) The Friction Factor, λ.—The results of the present work as given in Fig. 10 agree very well with the results reported by Messrs. O'Brien and Folsom.[13] The findings of G. W. Howard,[16] M. ASCE, when represented graphically (as recommended by Josef E. Montgomery[17]) agree with the other results. Below certain Reynolds numbers λ increases with the concentration, whereas, for greater values the concentration has no effect. Comparing the different curves of λ versus **R** given in this paper and elsewhere,[13,17] it is clear that the curves of the different concentrations approach the clear water curve in different ways which depend mainly on the size of the suspended material. Messrs. O'Brien and Folsom[18] show how the resistance of the clay slurry deviates sharply from the clear-water curve. This is similar to Fig. 10(*a*) for the 0.10-mm sand. When the size of the sand was increased to 0.16 mm (Fig. 10(*b*)) the curves approach the clear water more smoothly over a longer region.

From the careful observation of the bed in the present work, these critical velocities are related to the existence of the dunes on the bed. It is clear that for the same load and velocity the bed load will increase with the size of the suspended material. It is also clear that the chances of forming dunes increase with the increase of the bed load. This is the reason for the dependence of the critical Reynolds numbers on the size of the suspended material. The initial roughness might affect these critical numbers slightly. If a channel has very rough walls, say, in the case of Fig. 10, it can be expected that the reference line of clear water will be raised. There will still be a transition region, but the critical values might be reached at smaller values of **R** than in the case of a smooth wall.

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[14] Discussion by E. R. Van Driest of "Transportation of Suspended Sediment by Water," by Vito A. Vanoni, *Transactions*, ASCE, Vol. 111, 1946, p. 106.

[15] "The Aerodynamic Drag of the Earth's Surface and the Value of von Kármán's Constant in the Lower Atmosphere," by P. A. Sheppard, *Proceedings*, Royal Soc. of London, January, 1947, p. 208.

[16] "Transportation of Sand and Gravel in a Four-Inch Pipe," by G. W. Howard, *Transactions*, ASCE, Vol. 104, 1939, pp. 1334–1380.

[17] Discussion by Josef E. Montgomery of "Transportation of Sand and Gravel in a Four-Inch Pipe," by G. W. Howard, *ibid.*, p. 1350, Fig. 9.

[18] "The Transportation of Sand in Pipelines," by M. P. O'Brien and R. G. Folsom, *Publications in Engineering*, Univ. of California, Berkeley, Calif., 1937, Vol. 3, No. 7, p. 361, Fig. 9.

With the indicated substitutions, Eq. 11 for smooth channels becomes

$$\sqrt{\frac{8}{\lambda}} = \frac{N - 2.3}{k \log N} - 2.25 \frac{+ 2.3}{k \log \dfrac{r_h U \sqrt{\lambda/8}}{\nu}} \qquad (13)$$

in which N is represented in a Reynolds-number form for the thickness of the laminar sublayer δ, as follows: $U_f \delta/\nu = N$. It was found empirically that $N = 11.6$ for the clear fluids. For rough walls λ follows the relation:

$$\sqrt{\frac{8}{\lambda}} = -\frac{1}{k} + \left(\frac{U_w}{U_f} - \frac{2.3}{k} \log \frac{y_m}{e}\right) + \frac{2.3}{k} \log \frac{y_m}{e} \qquad (14)$$

in which e is the equivalent roughness height parameter and U_w is the wall velocity.

Two main factors will affect λ. One factor is the internal system of flow represented by k and can be called the internal factor. The other one is the boundary conditions represented by either δ and ν if the walls are smooth (or by the roughness height e if they are rough); and this can be called the boundary factor. There are some other factors, but they appear in terms of secondary order of magnitude and so can be neglected.

Varying the internal factor by varying k alone and holding the boundary factor constant, λ will change approximately as the square of k, according to Eq. 14. It must decrease in the presence of the suspended material, because of the decrease in k. As for the effect of the boundary factor, it can be seen that with a sediment-laden flow and a continuous interchange of particles between the bed and the fluid, it is quite difficult to visualize the picture at the boundary. It is conceivable that the laminar sublayer will be broken up by the continuous bombardment of the sand particles through it. This will increase λ and the bottom region will be somewhat similar to the case near a rough surface even if it looks smooth. When there are dunes at the bed, it is obvious that they greatly increase the bed roughness and λ will increase. Now when both the factors are considered, λ will decrease or increase according to the ratio between the effects of the two factors.

At low velocities, below the critical value, the dunes will increase the boundary effects and accordingly λ will increase, as seen from Fig. 10. Just after the dunes are swept off the bed, in the case of the fine particles, it seems as if the effect of k exceeds the boundary-layer effect of the sediment at the bottom and λ is less than for the clear water. This can be seen in Fig. 10(*a*). At high velocities the two effects are nearly balanced and λ is not changed.

(*d*) Velocity Distribution.—Fig. 13 shows some of the velocity profiles at the center line, for high concentrations, compared to the profiles of clear water flow, which have nearly the same values of U.

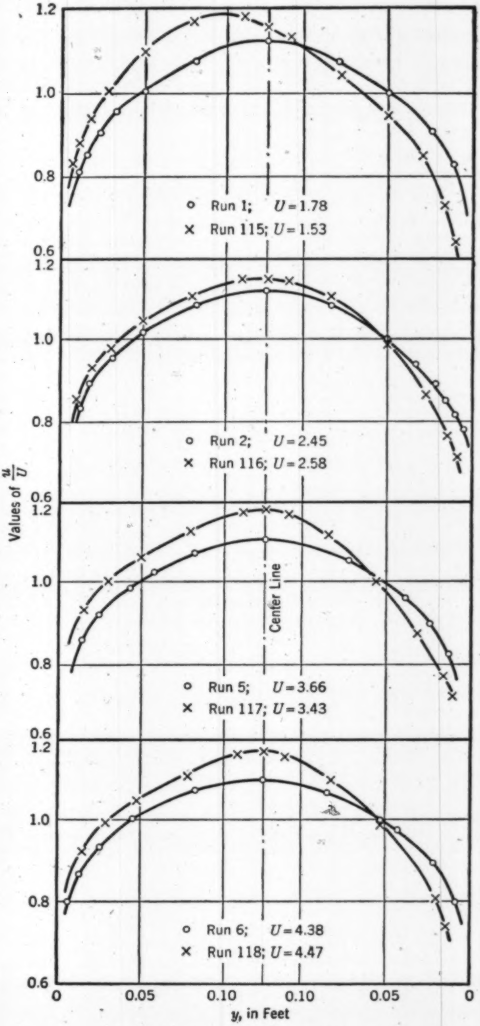


FIG. 13.—EFFECT OF SUSPENDED SAND ON VELOCITY PROFILES

Fig. 13(*a*) shows the effect of dunes on the velocity distribution. It shows how the maximum velocity and, correspondingly, the plane of zero shear were shifted upward. This indicated that the shear at the bed was increased (Fig. 8) causing λ to increase with the dunes. When the effect of the dunes was not so strong the other factors came into the picture and resulted in profiles such as Figs. 13(*b*), 13(*c*), and 13(*d*).

For high velocity it was shown in Fig. 10 that λ (or u/U_f) was the same for both clear and sediment-laden water. Fig. 13(*d*) and Col. 10, Table 1, showed that in these cases the maximum velocity always lies at the center; therefore, U_{tf} at the top must equal U_{bf} at the bottom. There is no other way to explain the big difference between the velocity profile slope at the top half and the bottom half except by the change in k. It is also of interest to compare the two sides of the velocity profile for high concentration (see Fig. 13(*d*)). The velocity near the bottom shows quite a decrease in the wall velocity U_w exactly as if the roughness of that wall had been increased. This could be explained by the effect of the sand in breaking up the laminar sublayer and decreasing its thickness in the same way as increasing its roughness.

Sediment Distribution.—The sediment concentration curves of Fig. 6 are slightly curved near the upper surface of the channel. The concentrations in this region were usually very low; and, when some of the samples were examined carefully by eye, they appeared to be finer than the sand customarily used. It is known that however carefully the sand is sieved it will always have a certain standard deviation of particle sizes, as shown in Fig. 4. Dividing the sample into different sizes, it is clear that the very fine portion will be more uniformly distributed than the coarser portions. Now, knowing that Fig. 6 is for log C and that the concentration is very low at the top, it is clear that the small proportion of the very fine uniformly distibuted particles will amount to an appreciable fraction of the sample at the top, whereas it is negligible at the bottom. This grading is then the cause of the observed curvature.

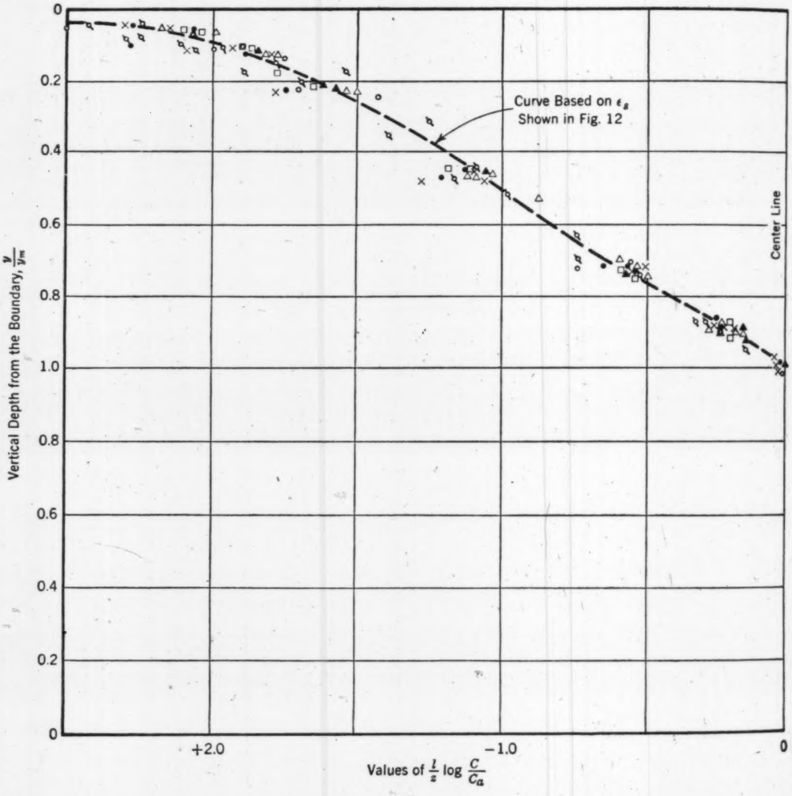
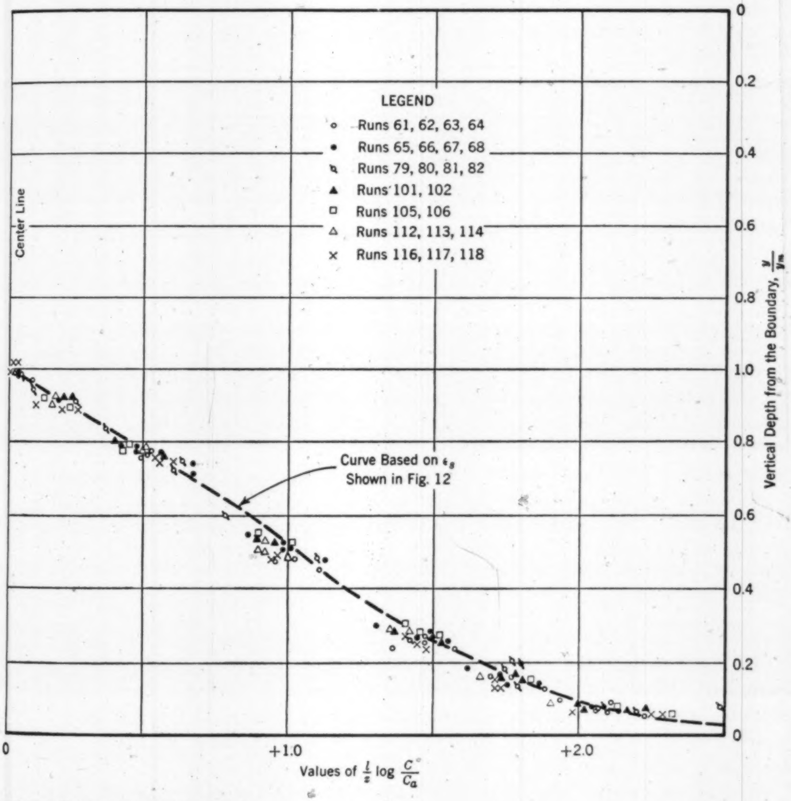


FIG. 14.—DISTRIBUTION OF THE

It was shown in the results that $\epsilon_s = 1.5\ \epsilon_m$ for one sand and $\epsilon_s = 1.3\ \epsilon_m$ for the other sand. These two observed values are not sufficient to establish the relation between β and the size of sand. More sizes and mixtures of sizes should be tested in the future. Messrs. Sherwood and Woertz[4] found the eddy diffusion in the main body of the turbulent fluid for the case of vaporization of water into a turbulent gas stream to be equal to $1.6\ \epsilon_m$. W. Corcoran[19] found that ϵ_c, the heat transfer coefficient between two parallel plates, varied between 1.5 and 1.22 ϵ_m with an average of 1.3 ϵ_m.



SEDIMENT CONCENTRATION ($a = y_m$)

Fig. 12 represents the assumed distribution of the nondimensional form of ϵ_s or ϵ_m across the central profile of the channel. Fig. 14 shows a curve drawn for the sediment concentration distribution under this assumption. It must be noticed that the measured values of k were always used for plotting the points in this curve. The points of twelve runs for the 0.10-mm sand and ten runs for the 0.16-mm sand are shown in Fig. 14. The agreement between the experimental results and the assumed theoretical curve is remarkable, which proves that the assumption in Fig. 12 is reasonably correct.

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[19] "Temperature Gradients in Turbulent Air Streams," by W. Corcoran, thesis presented to the California Inst. of Technology at Pasadena, Calif., in 1948, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

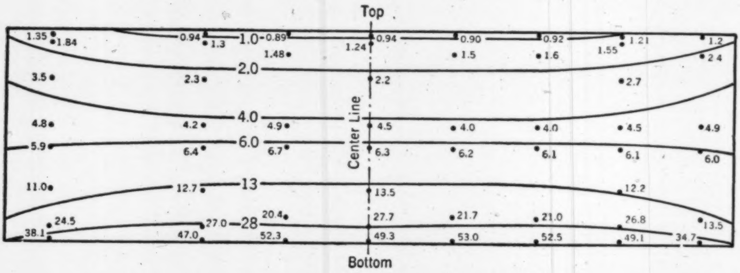

FIG. 15.—SEDIMENT CONCENTRATION DISTRIBUTION OVER THE SECTION FOR RUN NO. 71 (GRAMS PER LITER)

Fig. 15 shows the concentration distribution over the entire cross section for run 71. It is clear that in the middle half the contours run parallel to the top and bottom, showing that the assumption of a two-dimensional flow is very reasonable. Near the side walls the concentration is more uniform over a vertical section, which indicates that ϵ_s is stronger near the side walls than it is in the center. Mr. Van Driest[14] has explained the more uniform sediment concentration near the wall by saying that ϵ_s in any vertical line is more uniform near the wall. This could not be the reason because, as seen from Fig. 7, the average slope of the concentration profile was mainly controlled by the average value of ϵ_s at the profile and not by its variation.

CONCLUSIONS

1. The von Kármán constant k decreases when the load in suspension is increased. Both of the sands used in the experiments have nearly the same effect on k, which decreased to as low as 0.20 for a load of sand equal to 25 kg, in other words, when the sand content of the system was 43 g per liter. The decrease of k indicates damping of the turbulence.

2. The change in k does not follow the change in the concentration from point to point over one cross section, but it varies from section to section, maintaining a constant value over each one. This value decreases with the increase of the total quantity of suspended sand in the section.

3. The value of the momentum transfer coefficient is affected by the presence of the sand only through the changes in k. The form of its distribution curve will remain the same for sediment-laden water as for clear water. This coefficient cannot be measured directly at the center of the channel but, because it is similar to ϵ_s, one can reasonably assume that it follows Fig. 12.

4. The coefficient of friction for a stream carrying suspended sediment exceeds that for clear flow only when dunes form on the bed. If there are no dunes, the present experiments show that λ can hardly be affected by the presence of sand. This is for smooth walls and probably for walls roughened by particles smaller than, or equal to, the suspended particles.

5. The sediment transfer coefficient ϵ_s follows Fig. 12. It is approximately equal to 1.5 ϵ_m for the 0.10-mm sand and 1.3 ϵ_m for the 0.16-mm.sand.

6. Two devices—the basic equation for sediment distribution Eq. 1 and the distribution curve of ϵ_s in a vertical profile in the channel (see Fig. 12)—can define, accurately, the concentration distribution as represented in Fig. 14. The only difficulty in this procedure is that k and β cannot be predicted from the theory and they must be assumed or determined experimentally.

Acknowledgment

The research forming the basis for this paper was presented as a thesis to the California Institute of Technology at Pasadena, in 1948, in partial fulfilment of the requirements for the degree of Doctor of Philosophy. The writer wishes to express his sincere gratitude to Mr. Vanoni for his patient guidance and suggestions during the completion of this study.

APPENDIX. NOTATION

The following letter symbols, introduced in this paper, are assembled for the convenience of discussers:

a = a reference level; $a_s = N - \dfrac{2.3}{k \log N}$;

C = concentration of suspended material, expressed as mass per unit volume; C_a denotes a value of C at an arbitrary reference level;

c = concentration over a given cross section; $\bar{c}$ denotes a measured mean value of c in grams per liter;

D = diameter of sand particles:

  D_m = geometric mean sieve size;

  D_s = sedimentation diameter of sand;

e = an equivalent roughness-height parameter;

g = acceleration of gravity;

$h = \dfrac{y_m}{y} - 1$; $h_a = \dfrac{y_m}{a} - 1$;

i = hydraulic gradient slope;

k = von Kármán universal constant of turbulent exchange:

  K_b = the constant k in the bottom region of a section;

  K_t = the constant k in the top region of a section;

l = mixing length;

N = the characteristic value of the parameter $\dfrac{U_f\,\delta}{\gamma}$;

P = pressure intensity;

R = Reynolds' number = $\frac{4\,r_h\,U}{\nu}$;

r_h = hydraulic radius;

t = temperature;

U = velocity of flow; mean velocity over the cross section:

- U_b = value of U at the bottom wall;
- U_f = the Prandtl friction velocity = $\sqrt{\frac{\tau_o}{\rho}}$;
- U_t = value of U at the top wall or boundary;
- U_w = value of U at the wall of a flume or a channel;

u = velocity (see also U):

- u' = turbulent velocity;
- u'' = average velocity along the center-line profile;
- $u_{\max}$ = maximum value of u;

W = total load of sand in the flume or channel in grams;

w = settling velocity of a particle in still fluid;

x = horizontal distance along the axis of the channel;

y = vertical distance from the boundary:

- y_m = y measured from the boundary to the plane of maximum velocity, also, the coordinate of the point of zero shear;
- y_{tb} = the distance y from the top boundary to the bottom boundary;

$z = \frac{w}{\beta\,k\,U_f}$;

β = factor of proportionality;

δ = thickness of a laminar sublayer;

ϵ = transfer coefficient:

- ϵ_m = momentum transfer coefficient;
- ϵ_s = sediment transfer coefficient;
- ϵ_{sc} = value of ϵ_s at the center of the profile;

λ = friction coefficient;

ν = kinematic viscosity in centistokes;

ρ = density of fluid; mass per unit volume;

σ = standard deviation; σ_g = geometric standard deviation; and

τ = shearing stress:

- τ_o = value of τ at the point of intersection of the profile with the boundary;
- τ_{ob} = value of τ at the bottom boundary;
- τ_{ot} = value of τ at the top boundary.

DISCUSSION

Emmett M. Laursen,[20] A. M. ASCE, and Pin-Nam Lin[21].—A fundamental issue unintentionally raised by this paper is the proper interpretation of experimental evidence. If the author's interpretation of his data is accepted, no fault can be found with his conclusions. However, a different interpretation can lead to conclusions that are just the reverse. Although the outcome of this discussion is an almost complete, but perfectly straightforward, disagreement with the author's views, the essence of the argument is far more subtle in that it hinges upon certain basic concepts of the philosophy of measurement.

Experimental research may be reduced schematically to three successive steps: (*a*) The direct measurement of a few easily perceived quantities; (*b*) the evaluation from these measurements, by means of definitions and physical laws, of other quantities not susceptible to direct observation; and, finally (*c*) the deduction of further laws from the correlated measurements and evaluations. It should be evident that the significance of the quantities measured or evaluated and of the numerical values assigned must be clearly understood if the laws that are deduced are to be statements of true physical relationships.

A few quantities, such as length and time, are usually susceptible to direct observation and, therefore, to direct measurement by comparison with known standards. The numerical values assigned by the measurement are a matter of convention. Questions may arise as to the precision of the measurement, but seldom does any misunderstanding arise as to what quantity has been given a numerical value or what the meaning of the value is.

Most so-called "measurements" are not this simple since many quantities are not susceptible to direct observation and hence cannot be measured directly by comparison with a known standard. The evaluation of such a quantity is then indirect. If a basic definition is used in the evaluation, no misunderstanding is likely to arise; however, if a physical law is used, the validity of the law is a pertinent question. Thus, if the velocity of an object is evaluated by timing it over a measured distance and using the definition $V = L/T$, no more questions arise than with direct measurement, even though the procedure is inherently different. On the other hand, if a pitot-tube assembly is used to obtain the velocity at a point in the flow, a very basic question arises. Is the quantity $V = C\sqrt{2\,g\,h}$ the same as the quantity $V = L/T$? The answer, of course, is in the affirmative provided that the Bernoulli equation is a true law, properly applied. Since this equation is well established, and since its limitations are well known, the actual questions are more likely to be on the technique level—that is, whether the coefficient C is applicable to the particular boundary and flow conditions.

This brief consideration of the concepts implicitly involved in so ordinary a derived measurement as that of velocity gives a clue to the difficulty that can arise in evaluations that depend on such incompletely established laws as those relating to sediment transportation. The numerical values of the coefficients λ, κ, and β on which the author bases his conclusions are such derived measurements based upon the application of certain approximate laws and procedures. In fact, the author's unquestioning acceptance and use of the Kármán-Prandtl velocity equation and its counterpart for sediment distribution has led to conclusions that, although they seem to have fundamental significance, are in no way justified by a more careful analysis. The writers, therefore, submit herewith a detailed criticism of the author's principal points, together with an independent analysis in which each indirect evaluation is carefully examined.

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Since λ is defined by the Darcy-Weisbach formula, no other relationship can be used for the computation of its numerical values. Furthermore, any other coefficient of resistance, such as the Manning n, depends on exactly the same values of flow (Q), area (A), hydraulic radius (R), friction head (h_f), and length (L) and, therefore, should exhibit a similar pattern of variation. The significance of the quantity λ is that it is a coefficient that is required to make the Darcy-Weisbach formula an equality. The accuracy of its evaluation, however, depends on the accuracy of the measurements of the more basic quantities just listed. With regard to the measurement of these quantities, only two questions can be raised: One in connection with h_f, which is usually difficult to measure precisely; and one in connection with A and R, as their effective values may be influenced appreciably by the depth of sand in the flume. However, since it is reasonable to suppose that the dunes would act as roughness elements, the results are in qualitative agreement with past experience. Therefore, the author's conclusion in the section on "Discussion of Results" that the dunes and not the suspended sediment affect the resistance to flow would seem to be substantiated, despite the scatter of the experimental points.

In contrast, perhaps the most striking example of an unwarranted conclusion is the one—based on Fig. 9, in which κ is plotted against the total sand "load" —that κ decreases with increasing concentration and that the decrease indicates damping of the turbulence by the suspended sediment. The total sand "load," which could be more expressibly termed the total sand "charge," actually bears little relation to the concentration except as an upper limit. In the examination of this conclusion, κ has been plotted in Fig. 16 against the concentration at $h = (y_m/y) - 1 = 20$, as representative of the concentration in the section. Although the κ values are generally lower for higher concentrations, the scatter is too great to allow any convincing conclusion to be drawn as to cause and effect. Therefore, it is not only proper, but advisable, to inquire into the significance of the κ values.

The computation procedure for the determination of the κ values is the use of the Kármán-Prandtl velocity equation, according to which κ is the ratio of the shear velocity to the slope of the line through the u: log y values. If κ is also to be considered as a measure of the turbulence, the numerical values are the result of the semilogarithmic equation and the numerical values of u, y, and U_f. (It be might pointed out that von Kármán's original kappa (κ_o), in which $l/\kappa_o = \frac{du/dy}{d^2u/dy^2}$ or $\sqrt{\tau/\rho} = -\kappa_o \frac{(du/dy)^2}{d^2u/dy^2}$ is not, as implied, the same as the kappa (κ) the author has computed. They are related by $\kappa_o = \sqrt{1 - y/y_m}\,\kappa$.) Since the variation in κ could not be explained satisfactorily in this manner, the computation of the κ values was checked by replotting the velocity profiles and by distributing the shear according to the method of H. A. Einstein,[11] A. M. ASCE, but without sensibly different results. The lack of correlation may be the result of one or more unknown reasons—most plausibly a lack of precision in the original direct measurements or an inability of κ to express fully the turbulence properties of the flow. Therefore, it was concluded that another procedure involving fewer derived values was needed for the evaluation of the effect of the suspended sediment on this characteristic of the flow.

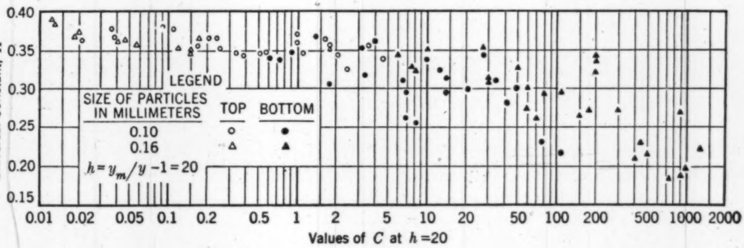

FIG. 16.—VALUES OF κ VERSUS CONCENTRATION AT $h = (y_m/y) - 1 = 20$

If the turbulence pattern is modified by the presence of sediment, such an effect should be detectible in the shape of the velocity profile. If one assumes that the velocity distribution may be written in the exponential form:

$$u = a\, y^{1/m} \qquad (15)$$

m will describe the shape of the profile. This form has the advantage that m is evaluated from the u: y values alone and not from any relationship for the shear. The excellence of the fit of the experimental points in Fig. 17 indicates the reasonableness of the assumption. For the more pertinent runs, m values are given in Table 2 together with the total charge, the mean concentration, the estimated inert charge, and the author's κ values. A definite system can be observed in the variation of m. For any total charge, the m value is smallest for the lowest concentration and greatest for the highest concentration. Since the concentration and the inert charge are inversely related, for any total charge the m value is smallest for the largest inert charge and increases as the inert charge decreases. In fact, if the velocity is sufficient to sweep most of the sediment into suspension, m approaches the value for clear water. From this it can be inferred that the suspended material has little, if any, effect on the velocity distribution. Since the head loss is also unaffected by the concentration, neither the turbulence production nor the turbulence distribution can be influenced by the presence of the suspended sediment. Instead, the major effect seems to be caused by the material on the bed, either because of roughness or because of transport as bed load. Unfortunately, the inert charge is not a sufficient criterion to permit quantitative description of this effect. However, the author's observations (Table 1, Col. 23) tend to substantiate the writers' conclusion that bed conditions alone govern the variation of *m*.

It is interesting to note that J. Nikuradse[22] found a similar variation of *m* with roughness in his classic experiments, the results of which are summarized in Fig. 18. For his roughest condition ($r_O/k = 15$) m was equal to 4.2. A comparable equivalent roughness height for the author's flume would be 0.1 in., which could easily be produced by the inert charge in the shape of dunes. In fact, the author cites dunes 0.05 in. high in run 120 and a sand wave 0.6 in. high in run 119.

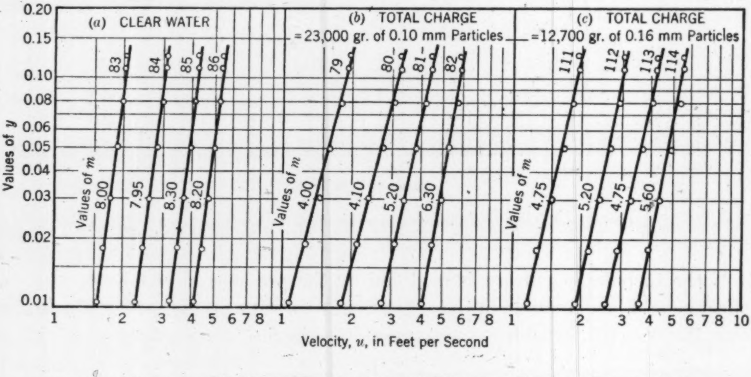

FIG. 17.—COMPARISON BETWEEN EXPONENTIAL AND MEASURED VELOCITY DISTRIBUTIONS

All the questions raised in regard to velocity distribution may be repeated in regard to sediment distribution for two reasons: (1) Basically, since both depend on turbulence; and (2) particularly, since the author's determination of sediment distribution implicitly involves the Kármán-Prandtl velocity equation. Specifically, β as determined by the author equals $\frac{w}{\kappa\, U_f}$ divided by the slope, z, of the line representing log C versus log h, and $\frac{\epsilon_{sc}}{\beta\, \kappa\, y_m\, U_f}$ equals a constant 0.22. The Kármán-Prandtl equation is implicitly used twice in the computation of β—once in the evaluation of κ and then again in the expression of the form of the sediment distribution equation. In the instance of ϵ_{sc} it is only involved in the form of the sediment-distribution equation, since $z = w/\beta\,\kappa\,U_f$. It is thus clearly apparent that sediment distribution cannot be considered independently of velocity distribution. Moreover, this examination reveals that Fig. 14 is obtained from the averaged reverse order of the foregoing computational procedure. Therefore, it is not indicative of the validity of the assumptions or conclusions, but merely of the correctness of the computations—the derived β values necessarily forcing the theoretical curve to fit the data with good approximation.

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[22] "Stromungsgestze in Rauhen Rohren," by J. Nikuradse, *Verein Deutscher Ingenieure, Forschungsheft* 361, 1933.

TABLE 2.—VALUES OF *m* FOR SELECTED RUNS

Run (1)	Total sand charge, *W* (2)	Concentration (3)	Inert sand charge (4)	*m* (bottom) (5)	Universal constant, κ (bottom) (6)
65	3,600	0.92	3,065	5.65	0.313
66	3,600	2.90	1,910	6.60	0.293
67	3,600	3.08	1,810	7.25	0.299
68	3,600	3.10	1,800	7.25	0.346
69	6,800	1.49	5,935	4.95	0.300
70	6,800	5.40	3,660	5.30	0.230
71	5,800	4.77	3,450	6.85	0.283
72	6,800	6.68	2,910	7.35	0.310
73	13,200	2.39	11,810	4.40	0.260
74	13,200	7.23	9,000	4.30	0.195
75	13,200	12.57	5.890	4.70	0.216
76	13,200	12.65	5,840	5.30	0.216
77	13,200	12.28	6,050	7.15	0.256
78	13,200	12.38	6,000	8.25	0.294
79	23,200	2.92	21,500	4.00	0.285
80	23,200	13.66	15.250	4.10	0.201
81	23,200	31.10	5,100	5.20	0.221
82	23,200	23.00	9,850	6.30	0.268
83		CLEARWATER		8.00	
84				7.95	
85				8.30	
86				8.20	
103	3,100	0.38	2,875	6.60	0.351
104	3,100	1.94	1,970	6.20	0.311
105	3,100	2.21	1,815	6.50	0.300
106	3,100	2.16	1,845	7.65	0.327
107	6,300	0.45	6,040	6.25	0.353
108	6,300	3.10	4,490	5.75	0.292
109	6,300	5.00	3.400	5.75	0.265
110	6,300	4.70	3,570	6.40	0.295
111	12,700	0.57	12,370	4.75	0.262
112	12,700	5.10	9,740	5.20	0.274
113	12,700	10.70	6,480	4,75	0.210
114	12,700	13.80	4,670	5.60	0.230
115	25,500	1.3	24,740	3.30	0.272
116	25,500	4.8	22,710	4.30	0.272
117	25,500	13.5	17,680	4.00	0.215
118	25,500	24.0	11,550	4.55	0.199
119	38,300	1.4	37,490	3.85	0.272
120	38,300	4.5	35,680	3.65	0.221
121	38,300	13.0	30,750	3.45	0.187
122	38,300	30.0	20,800	3.90	0.183

Another factor neglected by the author that should be considered in any computational procedure to determine ϵ_s or β is the variation of the fall velocity, w, with the concentration. Thus, Eq. 1 should read

$$\epsilon_s = \beta\, \epsilon_m = \frac{w_{ac}\, C}{-dC/dy} \quad \text{.....................} (16)$$

in which w_{ac} is the actual fall velocity. The actual fall velocity for a homogeneous suspension of uniform particles is given[23] in Fig. 19 as a function of the concentration and of the Reynolds number. This correction factor can be sizable (for a 0.1-mm sand at a concentration of 30 g per liter the decrease in fall velocity is about 20%) and should not be neglected. Although an analytic expression can be written for the correction factor, its inclusion makes the integration of Eq. 16 too formidable (the expression for w_{ac}/w contains fifteen terms). Nevertheless, an approximate evaluation indicates that use of the actual fall velocity would decrease the author's β values, but not sufficiently to change his conclusion qualitatively—that is, β would still be greater than unity, whereas the following considerations require that it be equal to or less than unity.

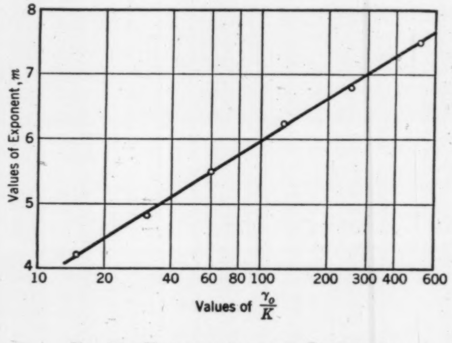

FIG. 18.—VARIATION OF m WITH ROUGHNESS (AFTER NIKURADSE)

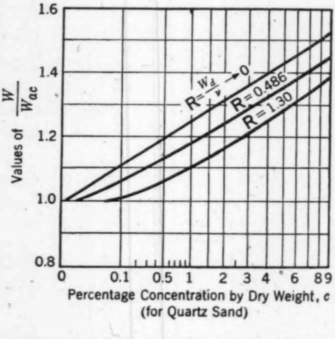

FIG. 19.—EFFECT OF CONCENTRATION ON FALL VELOCITY

Since the questions that arose in regard to κ and U_f in the velocity distribution analysis still remain, the writers have chosen to integrate the differential equation for sediment distribution on the basis of an exponential velocity distribution. Thus, the momentum-exchange coefficient becomes

$$\epsilon_m = \frac{\tau/\rho}{du/dy} = \frac{\tau_o/\rho\,(1 - y/y_m)\, y}{U_{\max}/m\,(y/y_m)^{1/m}} \quad \text{..............} (17)$$

---

[23] "Effect of Spacing and Size Distribution on the Fall Velocity of Sediment," by Pin-Nam Lin, thesis presented to the University of Iowa, at Iowa City, Iowa, in 1951, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

Inserting Eq. 17 into Eq. 16, rearranging terms, and intergrating produces

$$\ln \frac{C}{C_a} = \frac{w_{ao}\,(1 + 1/m)^2}{\beta\, U_{\max}\,\lambda}\,[I_\eta - I_\alpha] \ldots\ldots\ldots\ldots\ldots (18)$$

in which

$$I_\eta = \frac{1}{m}\int_o^\eta \frac{d\eta}{\eta^{\frac{m-1}{m}}\,(\eta - 1)}$$

and

$$\eta = y/y_m,\ \alpha = a/y_m$$

The necessary values of the *I*-function have been obtained numerically and are presented in Fig. 20. (By chance, an equivalent function has been tabulated by B. A. Bakhmeteff as a varied-flow function.[24] His values have therefore been incorporated into the figure.)

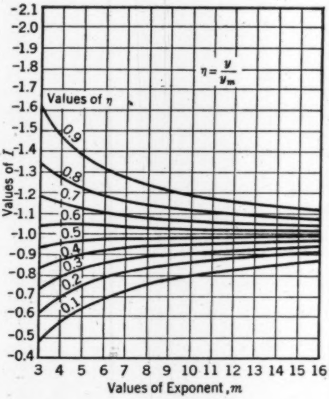

FIG. 20.—VALUES OF *I*-FUNCTION FOR SEDIMENT DISTRIBUTION

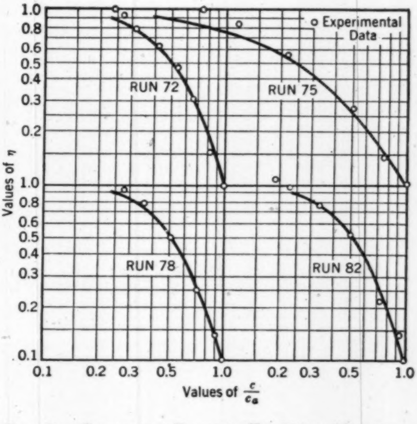

FIG. 21.—COMPARISON BETWEEN EQ. 18 AND MEASURED SEDIMENT DISTRIBUTION

Fall velocities corrected for the concentration at mid-depth and λ values from Eq. 12 were used in the theoretical curves of Fig. 21. The curves of Fig. 21 are based on the information given in Table 3 used in Eq. 18 with a value of $\beta = 1$.

Although in all runs the β values are very close to unity, the theoretical treatment has been approximate, and another consideration indicates that the true β value, as defined by the ratio of the mixing coefficients of sediment and momentum, would be less than unity. There is, of course, a sorting of the sediment in the vertical such that the mean size of the particles decreases with increasing y values. For this reason, and possibly others, the mean size of the material in suspension is smaller than the mean size of the material on the bed. Since finer material would have a smaller fall velocity, the derived β values would be less than unity if the data included this factor. The sorting effect would also explain the deviation of the theoretical curve from the experimental points as the ratio $\acute{y}/y_m$ approaches 1.

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[24] "Hydraulics of Open Channels," by B. A. Bakhmeteff, McGraw-Hill Book Co., Inc., New York, N. Y., 1932, p. 310.

TABLE 3.—DATA FOR CURVES OF FIG. 21

Run	Actual fall velocity in terms of theoretical (w_{ac}/w)	Percent concentration $C\ y_{m/s}$	Friction coefficient λ	Maximum velocity U_{max}	m
72	0.85	1.35	0.0157	5.65	7.60
75	0.81	2.80	0.0170	3.72	4.67
78	0.81	2.85	0.0152	5.86	7.25
82	0.76	5.20	0.0150	6.07	6.85

An independent analysis of β has been made, both theoretically and experimentally, by M. R. Carstens,[25] J. M. ASCE, for a quartz sphere in simple harmonic motion. As a first approximation, Fig. 22 can be used to evaluate β as a function of the frequency ω, the amplitude of the motion having a negligible role. Although a weighted mean of the turbulence frequencies would have to be employed, it is evident that, according to the analysis by Mr. Carstens, β should always be less than unity. The qualitative conclusion

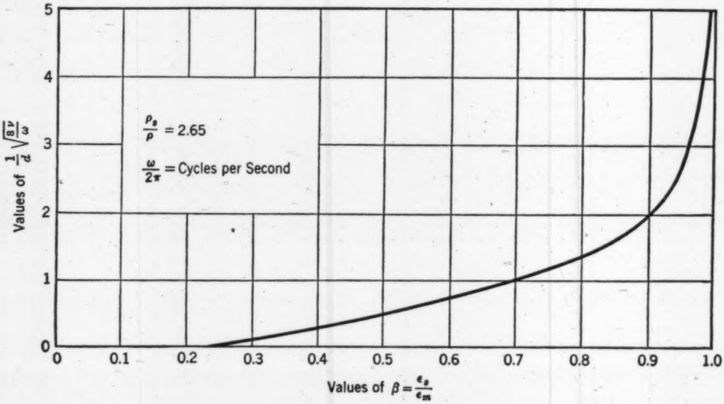

FIG. 22.—VARIATION OF β WITH FREQUENCY AND PARTICLE DIAMETER (AFTER CARSTENS)

that ϵ_s is smaller than ϵ_m is thus checked independently. In order to obtain a quantitative relation between ϵ_s and ϵ_m by either method, further data would be necessary. In the one method the turbulence frequencies, and in the other the size frequency of the suspended sediment, must be known.

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[25] "Accelerated Motion of a Sphere," by M. R. Carstens, thesis presented to the University of Iowa, Iowa City, Iowa, in 1950, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

The exponential velocity distribution has been used herein as a matter of convenience and not as an argument against the logarithmic form. By this expedient the possible errors in the head-loss measurements, which are reflected in the shear values, have been minimized. The point in question is not which should be used, for both are assuredly empirical and approximate. Admittedly, Eq. 18 is also merely an approximation even without considering the variation of β with y and of w with c. However, even if a completely satisfactory analysis can not be made, the rigorous examination of each procedural step has indicated that the experimental data do not substantiate the author's conclusions except in regard to λ. In fact, the writers' analysis leads to just the opposite conclusions:

1. That the nature of the quantity κ is not sufficiently well defined to allow reliable conclusions to be drawn from the variation of its values;
2. That the presence of suspended sediment has little or no effect on the flow; and
3. That β is equal to or less than unity.

M. R. Carstens,[26] J. M. ASCE.—The general diffusion process is characterized by the differential equation,[27]

$$N_R = -D_R \, dn/dy \qquad (19)$$

in which N_R is the net rate of transfer of the fluid characteristic, D_R is the diffusion coefficient, and n is the characteristic concentration. Since the net rate of transfer is in the direction of decreasing concentration, the sign is negative. This process is represented by the suspended sediment diffusion process in the equilibrium state, in which the net rate of transfer of sediment upward is equal to the rate of settling, that is,

$$w\,c = -\epsilon_s \, dc/dy \qquad (20)$$

The magnitude of the diffusion coefficient ϵ_s is representative of the intensity of the transfer mechanism in the y direction. The diffusion equation of fluid momentum is identical in form. The diffusion coefficient ϵ_m is representative of the fluid transfer mechanism in the y direction. Since momentum is a characteristic of the fluid, the diffusion coefficient of momentum is in reality a diffusion coefficient of the fluid. Since the suspended sand is of greater density than the water, the sand particles will be out of phase with the movement of the fluid particles and will be transported a lesser distance than the fluid particles. Consequently, the momentum diffusion coefficient ϵ_m is never smaller in magnitude than the sediment diffusion coefficient ϵ_s if the sediment is greater in density than the fluid.

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[26] Associate Prof., School of Civ. Eng., Georgia Inst. of Technology, Atlanta, Ga.

[27] "Engineering Hydraulics," ed. by Hunter Rouse, John Wiley & Sons, Inc., New York, N. Y., 1st Ed., 1950, p. 95.

Studies by the writer[25] and by M. Wagenschein[28] were performed with a spherical particle in order to determine the ratio of the particle amplitude to the fluid amplitude. There is available for the amplitude ratio an analytical solution in which the ratio is less than 1.00 if the foreign particle is more dense than the fluid particle. Conversely, if the fluid particle is more dense, then the amplitude ratio is between the limits of 1.00 and 3.00. In other words, in Eq. 2, $0 < \beta < 1.00$ if $\rho_s/\rho > 1.00$ and $1.00 < \beta < 9.00$ if $\rho_s/\rho < 1.00$. The upper limit corresponds to the case of air bubbles in water, for which ρ_s/ρ is approximately zero.

The Boussinesq relationship for the apparent shear stress in turbulent flow is

$$\bar{\tau} = \rho \, \epsilon \, \overline{du}/dy \qquad (21)$$

in which the bars denote temporal mean values. A rational derivation for the apparent shear stress can be performed by means of a momentum flux equation applied to a small fluid element. From this derivation the apparent shear stress is

$$\bar{\tau} = -\rho \, \overline{v'u'} \qquad (22)$$

in which u' and v' are the fluctuating components of the velocity in the x and y directions, respectively. Measurements[29] of the velocity fluctuations in a two-dimensional channel verify the validity of Eq. 22. Comparing the terms of Eqs. 21 and 22, it is apparent that the fluctuating component v' is the lateral transport velocity and, hence, is proportional to the diffusion coefficient ϵ. It then follows that $d\bar{u}/dy$ in Eq. 21 is proportional to u'. Since the diffusion coefficient or eddy viscosity ϵ contains this proportionality factor, the magnitude of ϵ is not equal to the diffusion coefficient of fluid or momentum. In fact, Eq. 21 is useful mainly as an analogy to the viscous shear of laminar flow and is a descriptive aid in explaining the apparent shear of turbulent flow. C. B. Millikan[30] has shown that the logarithmic velocity distribution in channels and pipes can be derived without the use of Eq. 21 and with the aid of more fundamental assumptions.

The general diffusion equation (Eq. 19) will result in a zero net rate of transfer when either the diffusion coefficient D_R is zero or the gradient of the quantity being transported, dn/dy, is zero. As the center line of a two-dimensional channel the net rate of transfer of momentum is zero because the gradient of momentum is zero. The diffusion coefficient of momentum ϵ_m is not zero at the center line of the channel. As a consequence, fluid, sediment, coloring matter, heat, and the like are transported across the center line when there is a gradient of these quantities.

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[28] "Experimentalle Untersuchung über das Mitschwingen einer Kugel in einer Schwingenden Flüssigkeit-oder Gasmasse," by Martin Wagenschein, *Annalen der Physik*, Series 4, Vol. 65, 1921, pp. 461–480.

[29] "Investigation of Turbulent Fow in a Two-Dimensional Channel," by John Laufer, *Technical Note No. 2123*, National Advisory Committee for Aeronautics, 1950.

[30] "Turbulent Flows in Channels and Circular Tubes," by C. B. Millikan, *Fifth International Congress of Applied Mechanics*, John Wiley & Sons, Inc., New York, N. Y., 1939.

Hassan M. Ismail,[31] A. M. ASCE.—The final results of any research definitely depend on the interpretation of experimental evidence. The selection of the functions and the form in which they appear in the different relations are among the most important parts of the research. A comparison between Figs. 9 and 16 will illustrate the effect of grouping the experimental data in a manner that will make them representative. In Fig. 16 the concentration at a certain value $h \left(= \frac{y_m}{y} - 1 \right) = 20$ was taken as a base for studying the variation of the values of K. The scatter was too great. When the experimental points were grouped and plotted as in Fig. 9, a definite relationship was shown. Fig. 7 shows that, for the same total quantity of sand, the curve of concentration for different velocities cross each other at different values of y. This is why the concentration at a certain depth could not be taken as a basis for comparison. Other factors discouraged the use of a big value of h, such as $h = 20$ (that is, $y = 0.006$ ft). The concentration curve at the bottom was very steep, and any small error in evaluating y would cause an appreciable error in the value of C_a. Also, using a value of h anywhere in the bottom half would not permit using results of the upper half. For these reasons the curves in the paper did not involve a reference plane, except Fig. 14 for which the reference plane was taken at the center of the profile.

Mr. Bakhmeteff[32] explained the relation between κ and κ_0 referred to by Messrs. Laursen and Lin. He used the same symbol κ for both concepts, and showed that, when applied to the final forms of velocity distribution, the Nikuradse experiments gave $\kappa = 0.4$ as compared with the so-called $\kappa_0 = 0.38$.

Eq. 15 may be a very good approximation expression for the velocity distribution; but since the velocity distribution is mainly dependent on the shear stress, m must be a function of the shear stress. The advantage of the von Kármán formula is that the shear stress appears in the formula, leaving K as a universal constant that depends only on the mechanism of the turbulence. The advantage of being able to evaluate m from the u-values and y-values alone is the same as that of introducing a new factor—say, $n = \frac{K}{u_f}$— in the velocity defect law, $u - u_{\max} = \frac{2.3}{n} \log \frac{y}{y_m}$. As a matter of fact, n will be a combined function, and there is no doubt that the use of $\frac{K}{u_f}$ is very advantageous.

The conclusion of Messrs. Laursen and Lin that the presence of suspended sediment has little or no effect on the flow was based on values of that combined function m. They state that, as the velocity increases, m approaches the value for clear water. The results given in Table 2 for runs 103 to 122 do not agree with this conclusion.

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[31] Civ. Engr., Ministry of Public Works, Cairo, Egypt.

[32] "The Mechanics of Turbulent Flow," by B. A. Bakhmeteff, Princeton Univ. Press, Princeton, N. J., 1941, pp. 67–69.

Mr. Carstens[25] as well as Messrs. Laursen and Lin[23] base their conclusion, that β must be less than unity, on research at the University of Iowa at Iowa City. The results reported by T. K. Sherwood and B. B. Woertz[4] and by W. Corcoran,[19] as well as the studies reported in the paper, prove that β can be greater than unity.

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TRANSACTIONS

Paper No. 2501

LIMIT DESIGN OF BEAMS AND FRAMES

By H. J. Greenberg[1] and W. Prager,[2] M. ASCE

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Synopsis

The paper is concerned with the limit design of statically indeterminate beams or frames under the action of given loads. The limit moments under which individual sections act as yield hinges are supposed to be known throughout the structure and the safety factor against collapse is sought. This safety factor is defined as the multiple to which the actual loads must be increased before yield hinges develop in a number sufficient to render the structure unstable. Two extremum principles are established for the safety factor. With these principles two estimates of the safety factor can be obtained, one of which is known to be too small whereas the other is known to be too large. The gap between these estimates can be reduced by a repeated application of the extremum principles.

1. Introduction

As applied to statically indeterminate beams and frames, the method of limit design[3] is based on the assumption that the relation between the bending moment M and the curvature c of an elastic-plastic beam is of the type shown in Fig. 1. The absolute value of the curvature can increase indefinitely under the constant bending moments $\pm M_o$. The shape of the transition curve AB between the fully elastic branch OA and the fully plastic branch BC of the curve is immaterial for the application of limit design.

The fact that the absolute value of the limit moment (M_o in Fig. 1) is the same regardless of the sign of the moment means that it is necessary to consider only absolute values of the bending moments. As a rule, the value of the bending moment on a member will equal the limit moment only for discrete locations along the member. At these cross sections "yield hinges" are formed,

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[3] "Theory of Limit Design," by J. A. Van den Broek, John Wiley & Sons, Inc., New York, N. Y., 1948.

which reduce the degree of indeterminacy of the structure. A sufficient number of yield hinges transforms the structure (or part of it) into a mechanism. To test whether this stage has been reached, it is sufficient to replace all yield hinges by perfect hinges and to consider the structural members which are joined by these hinges as completely rigid. If the structure modified in this manner is capable of deformation, however small, despite the assumed rigidity of its members, the actual appearance of the yield hinges under consideration would transform the structure into a mechanism and lead to the collapse of the structure. For instance, the frame shown in Fig. 2 becomes a mechanism if yield hinges develop at the seven cross sections marked A, B, $\cdots$, G in Fig. 3. If these yield hinges are replaced by perfect hinges and the members between these hinges are considered rigid, the system is capable of the type of deformation indicated by dotted lines in Fig. 3.

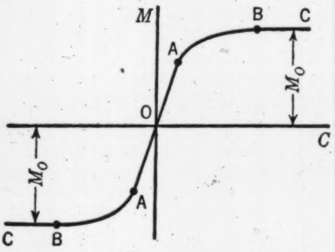

FIG. 1

The limit design problems considered in this paper are concerned with statically indeterminate beams or frames under the action of given loads. The limit moments are known throughout the structure, and the safety factor s against collapse is sought. This safety factor is defined as the multiple to which the actual loads must be increased before yield hinges develop in a number sufficient to transform the structure into a mechanism, thus precipitating collapse.

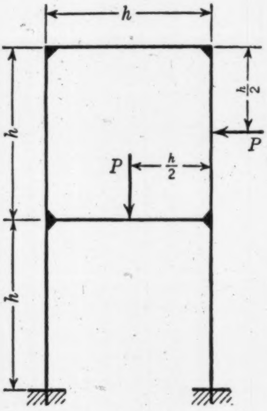

FIG. 2

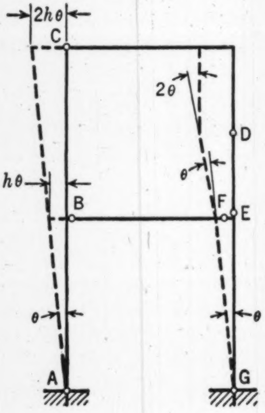

FIG. 3

At first glance, a problem of this kind would seem to require a detailed investigation of the elastic-plastic behavior of the structure under gradually increasing loads. This laborious investigation is not necessary, however, if only the safety factor against collapse is required. It is this fact which makes the method of limit design so attractive. Although in most cases the exact value of the safety factor cannot be computed directly, the following sections of this paper offer principles from which lower and upper limiting values ("bounds") for the safety factor can be obtained.

2. A Lower Bound for the Safety Factor

The principle to be established in this section furnishes the lower bound for the safety factor against collapse. Any such lower bound may be considered as an estimate of the safety factor, which can err only on the conservative side.

Consider a statically indeterminate beam or frame under the action of given loads. Any bending moment distribution that is computed from these loads, and from arbitrarily assigned values of the statically indeterminate quantities, will be said to be "statically compatible" with the given loads. Any bending moment distribution such that at no cross section does the bending moment exceed the limit moment for the cross section will be called an "admissible" distribution of bending moments.

The given loads (design loads) are not expected to cause collapse. Consider, now, the system of loads obtained by multiplying each of the given loads by the same multiplier. This multiplier will be called "statically admissible," if there exists an admissible distribution of bending moments which is statically compatible with the increased loads.

With this terminology the principle for the safety factor can be formulated as follows:

> Theorem I.—The safety factor against collapse is the largest statically admissible multiplier.

A proof of the principle stated in theorem I is given in the Appendix. As an illustration of the manner in which this principle is used, consider the indeterminate beam shown in Fig. 4(*a*). For simplicity, assume that the limit moment M_o is constant along the beam. If the statically indeterminate clamping couples X and Y are known, the bending moments of this beam are obtained by superimposing: (*a*) the bending moments which the given loads would produce if the beam were simply supported at its ends; and (*b*) the bending moments which the couples X and Y would produce in this simply supported beam. Fig. 4(*b*) shows this superposition for arbitrarily assumed values of X and Y. The polygon ABCD corresponds to the given loads and, therefore, is fixed. The line A′D′ corresponds to the assumed values of X and Y and may be moved.

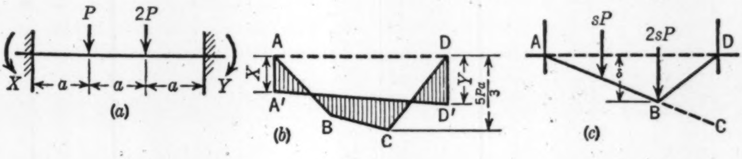

Fig. 4

For any assumed bending moment distribution which is compatible with the given loads (that is, for any assumed position of the line A′D′, Fig. 4(*b*)), it is easy to determine the largest multiplier m that can be applied to these bending moments if the distribution is still to remain admissible. Indeed, this factor m is found by dividing the limit moment by the largest absolute value of the bending moment. Obviously, m is a statically admissible multiplier in the sense defined in connection with theorem I, and hence a lower bound for the safety factor s. Moreover, the safety factor is the largest value that can be obtained for m by changing the position of the line A′D′. To increase m, one must decrease the largest bending moment. Inspection of Fig. 4(*b*) reveals that, for the given loads, the value of the bending moment at a certain cross section can be decreased only at the cost of increasing the value of the bending moment at some other cross section. The optimum result is achieved by drawing the line A′D′ so that the bending moments at the two ends and under the load $2\,P$ become equal. A simple computation shows that this smallest possible value of the largest bending moment equals $5\,P\,a/6$. The safety factor of the beam, therefore, equals

$$s = \frac{6\,M_o}{5\,P\,a} \qquad (1)$$

The fact that the bending moments at the ends and under the load $2\,P$ had to be made equal in absolute value indicates that the yield hinges in the collapse state are located at these cross sections. Of course, in simple problems such as the one just considered, there can scarcely be any doubt concerning the location of the yield hinges. In more complicated, highly indeterminate structures, however, even an experienced stress analyst will frequently fail to guess the location of all yield hinges correctly. In such cases the principles of this section and of section 3 become really useful.

3. An Upper Bound for the Safety Factor

The principle to be established in this section furnishes the upper bound for the safety factor. By definition, such an upper bound is larger than the safety factor, or at least equal to it. By the joint use of both principles, the safety factor can therefore be bounded from above and below. This fact is of great practical importance, because the precise determination of the safety factor is unnecessary when the gap between the lower and upper bounds is sufficiently small for practical purposes.

Consider a statically indeterminate beam or frame in a state of collapse. The given loads have been multiplied by the largest statically admissible multiplier and cannot be increased any further. At the same time, the structure has been transformed into a mechanism by the appearance of a sufficient number of yield hinges, and this mechanism begins to move under the collapse loads (the given loads multiplied by the largest statically admissible multiplier). This beginning collapse can be treated as a quasi-static process:

> Theorem II.—The work that the collapse loads do on the displacements of their points of application must equal the work that the limit moments in the yield hinges do on the relative rotations of the parts connected by the hinges.

If the location of the yield hinges were known, this application of the principle of virtual work would furnish the ratio of the collapse loads to the given loads—that is, the safety factor.

To illustrate the use of this principle, consider again the beam in Fig. 4(*a*), assuming it to be known that the yield hinges in the state of collapse are located at the ends and under the larger load. Fig. 4(*c*) shows the beam just after collapse has begun. If δ denotes the infinitesimal displacement of the load $2\,s\,P$, the displacement of the load $s\,P$ is $\delta/2$ and the relative rotations at the hinges (angles DAB, CBD, and BDA in Fig. 4(*c*)) are $\frac{\delta}{2\,a}$, $\frac{\delta}{2\,a} + \frac{\delta}{a}$, and $\frac{\delta}{a}$, respectively. The work of the limit moments in the yield hinges is therefore

$$W_o = 2\,M_o\left(\frac{\delta}{2\,a} + \frac{\delta}{a}\right) \qquad (2)$$

and the work of the collapse loads is

$$W_c = 2\,s\,P\,\delta + s\,P\,\frac{\delta}{2} \qquad (3)$$

Equating these expressions and solving for s yields Eq. 1 as before.

Instead of the actual yield hinges, whose location is not known *a priori*, consider now any arrangement of yield hinges which transforms the structure into a mechanism. The principle of virtual work, applied in the manner described, then furnishes the multiplier n which must be applied to the given loads if the assumed yield hinges are to become active, and if no attention is paid to the fact that the limit moment might then be exceeded at some cross sections other than the assumed hinges. Any multiplier n obtained in this manner will be called "kinematically sufficient" (to produce collapse).

With this terminology, the principle for the safety factor can be stated as follows:

> Theorem III.—The safety factor against collapse is the smallest kinematically sufficient multiplier.

A proof of this principle is given in the Appendix.

4. Example

The most practical method of combining the principles of sections 2 and 3 is as follows:

(*a*) Trying to guess the actual mode of collapse, assume an arrangement of yield hinges that transforms the structure into a mechanism and determine the corresponding kinematically sufficient multiplier n. This is an upper bound for the safety factor s—that is, $s \leqq n$.

(*b*) For the collapse mode assumed in step (*a*) determine the bending moments M. These are caused by the limit moments in the yield hinges and the given loads multiplied by the factor n found in step (*a*). In practical structures, these bending moments are statically determinate as a rule; the exceptional case is discussed in section 5. Denote by r the largest value of the ratio $|M/M_o|$ occurring anywhere in the structure. Obviously, $r \geqq 1$ because $r = 1$ at the yield hinges as assumed in step (*a*). The bending moments M/r will be admissible in the sense defined in section 2, and n/r is a statically admissible multiplier. Accordingly,

$$\frac{n}{r} \leqq s \leqq n \qquad (4)$$

In particular, if $r = 1$, the assumed mode is the actual mode of collapse and $s = n$. On the other hand, if $r > 1$ at some cross section, this usually is an indication that a yield hinge should have been assumed at this particular cross section. If the procedure is repeated with this yield hinge replacing one of the yield hinges originally assumed, closer boundaries for the safety factor are obtained, as a rule. It can be proved that this substitution can always be made in at least one manner so as to narrow the gap between the bounds for the safety factor. However, the mathematical tools needed for this proof would exceed the scope of the present paper. Moreover, the way in which this substitution must be made in order to have the desired effect is not known *a priori* but must be found by trial and error. The fact that the substitution will have this effect if made properly, thus, is only of academic interest.

As an example illustrating the foregoing procedure, consider the two-story frame shown in Fig. 2. For brevity, assume that the limit moment has the same value M_o at all cross sections. An arrangement of yield hinges which transforms this frame into a mechanism and the type of displacement made possible by these hinges are shown in Fig. 3. If the infinitesimal relative rotation at yield hinge A is denoted by θ, the relative rotations at yield hinges B, C, E, F, and G have the absolute values θ, whereas the rotation at yield hinge D has the absolute value $2\,\theta$. The signs of these relative rotations are unimportant for the application of the principle of virtual work, because the signs of the limit moments in the yield hinges are always such that the work of each limit moment on the relative rotation at the corresponding yield hinge is positive. Since the sum of the absolute values of the relative rotations at all yield hinges is $8\,\theta$, the work done by the limit moments M_o at these yield hinges is $8\,M_o\,\theta$.

Inspection of Fig. 3 shows that both horizontal beams undergo purely horizontal translations, the displacement of the upper beam being $2\,h\,\theta$ and that of the lower beam $h\,\theta$. Accordingly, the horizontal load $n\,P$ does the work $2\,h\,\theta\,n\,P$, whereas the vertical load $n\,P$ does not do any work. The principle of virtual work therefore yields

$$8\,M_o\,\theta = 2\,h\,\theta\,n\,P \qquad (5)$$

and, hence,

$$n = 4\,\frac{M_o}{P\,h} \qquad (6)$$

This value of n constitutes an upper bound for the safety factor.

Fig. 5 shows the parts of the frame under the action of the multiplied loads and the limit moments at the hinges. From Eq. 6, the multiplied load $n\,P = \frac{4\,M_o}{h}$. To simplify the lettering, M_o is taken as the unit for the couples and M_o/h as the unit for the forces. This designation implies that the length h must be treated as the unit of length. The limit moments and the exterior loads are indicated by full lines; and the forces which the parts of the frame exert on each other, by dotted lines. The sense of the limit moments follows from the sense of the relative rotations shown in Fig. 3. The forces which the parts of the frame exert on each other are obtained by applying the conditions of equilibrium to each of these parts.

Consider first the length DE. For moment equilibrium, the horizontal reaction at the lower end must have the intensity 4. Since this reaction and the exterior load 4 at hinge D satisfy the condition for horizontal equilibrium, no horizontal force is transmitted across hinge D. Horizontal equilibrium of the segment CD requires then that no horizontal force be transmitted across hinge C either, and moment equilibrium of this segment requires that the vertical forces transmitted across hinges C and D have the intensity 2. Continuing in this manner, one finds the forces indicated by dotted lines in Fig. 5. The bending moments for the members of the frame are shown in Fig. 6; the bending moments for a member are plotted perpendicularly to the axis of the member and toward that side of the member in which they produce tension. The numbers indicate the absolute values of the bending moments (in multiples of M_o).

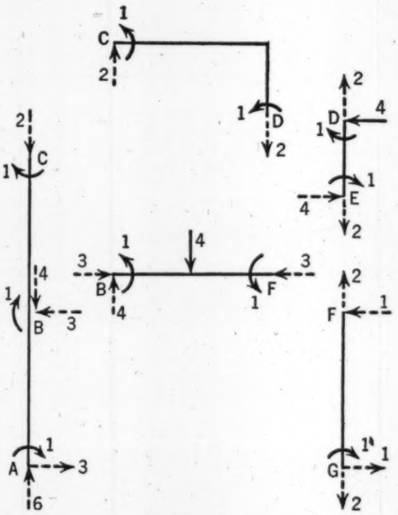

FIG. 5

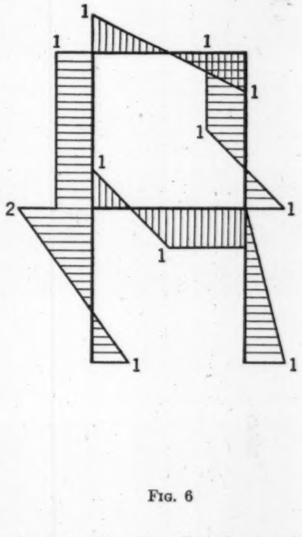

FIG. 6

Inspection of Fig. 6 shows that the absolute largest bending moment, occurring just below hinge B, has the intensity 2 M_o. The ratio r, therefore, has the value 2. With the value of n obtained, Eq. 4 gives

$$2\frac{M_o}{P\,h} \leqq s \leqq 4\frac{M_o}{P\,h} \qquad (7)$$

The bounds for the safety factor are still too far apart for practical purposes. The procedure must be repeated, therefore, with a hinge at the location of the greatest bending moment (that is, in the left-hand column, just below the first floor) replacing one of the previously assumed hinges. The decision as to which hinge is to be so replaced is somewhat facilitated by the condition that the structure must still be transformed into a mechanism by the new arrangement of hinges. Thus, it would not be admissible to replace hinge G in Fig. 3 by the new hinge.

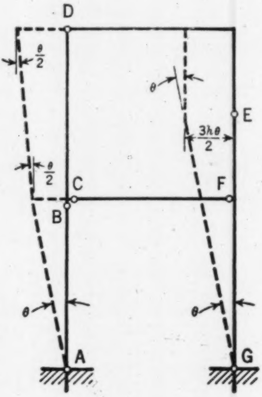

FIG. 7

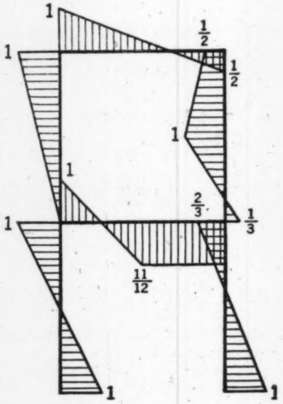

FIG. 8

Fig. 7 shows the selected arrangement of hinges and the type of displacement made possible by these hinges. With θ as defined in Fig. 7, the relative rotation at hinges A, E, F, and G has the absolute value θ; and the relative rotation at hinges B, C, and D has the absolute value $\theta/2$. The horizontal displacement of hinge E now equals $3\,\theta\,h/2$. Thus, the principle of virtual work furnishes the kinematically sufficient multiplier:

$$n = \frac{11\,M_o}{3\,P\,h} \qquad (8)$$

The value of n defined by Eq. 8 constitutes an upper bound for the safety factor. To obtain a lower bound, one must determine the bending moments corresponding to the assumed mode of collapse. Fig. 8 shows these bending moments. Since the absolute value of the bending moment nowhere exceeds the limit moment, $r = 1$, and the upper bound is also a lower bound. Hence, the safety factor against collapse equals

$$s = \frac{11\,M_o}{3\,P\,h} \qquad (9)$$

and Fig. 7 shows the actual mode of collapse.

5. Local Versus Total Collapse

In the preceding example the actual yield hinges are so arranged that the entire structure is transformed into a mechanism. In other cases, however, the actual yield hinges may transform only part of the structure into a mechanism without permitting any displacements of the remainder of the structure. In this section these two types of collapse will be called "total collapse" and "local collapse."

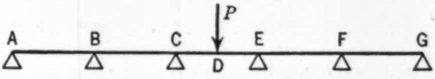

Fig. 9

For continuous beams, collapse is very often local. In fact, for a continuous beam with a single transverse load, collapse must be local. Fig. 9 illustrates this fact. For each of the segments AB, BC, $\cdots$, FG of the beam the bending moment varies linearly with the distance measured along the beam. Accordingly, the largest absolute value of the bending moment for any one of these segments can occur only at the ends of this segment. Yield hinges can thus develop only at points B, $\cdots$, F. However, yield hinges at points B and F would not contribute to the mobility of the system because of the supports there and the absence of hinges inside the adjacent spans. The only way to transform the beam into a mechanism, therefore, is to insert hinges at points C, D, and E. This arrangement of hinges, however, means local collapse.

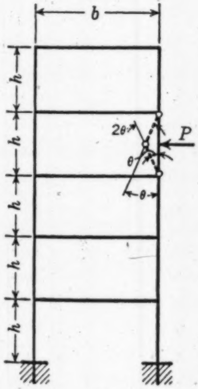

Fig. 10

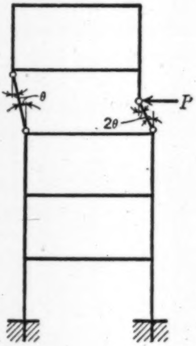

Fig. 11

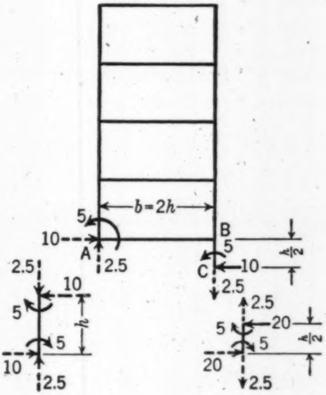

Fig. 12

For multistory frames, on the other hand, this particular type of local collapse constitutes the exception. To demonstrate this fact, consider the frame of Fig. 10. Assume that the ceiling beam and the columns of the topmost story have the limit moment M_o, the ceiling beam and the columns of the next story, the limit moment $2\,M_o$, and so forth; and, finally, the ceiling beam and the columns of the lowest story, the limit moment 5 M_o. Consider, now, a single horizontal load P applied at the center of one of the columns. The dotted lines in Fig. 10 indicate the simple type of local collapse which is typical for continuous beams. The corresponding kinematically sufficient multiplier is found to be

$$n_1 = \frac{8\,k\,M_o}{P\,h} \quad \text{(10a)}$$

if the horizontal load is applied to the kth story from the top. Fig. 11 shows a more complicated type of local collapse. The corresponding kinematically sufficient multiplier equals

$$n_2 = \frac{6\,k\,M_o}{P\,h} \quad \text{(10b)}$$

in which k has the same meaning as Eq. 10a. Since both n_1 and n_2 are the upper bounds for the safety factor and since $n_2 < n_1$, the simple type of local collapse shown in Fig. 10 cannot occur in the frame considered here.

To show that the type of local collapse indicated in Fig. 11 can actually occur, consider the case in which the horizontal load is applied to the lowest story. The kinematically sufficient multiplier n_2 (and therefore the upper bound for the safety factor) then has the value $30\,\frac{M_o}{P\,h}$. Fig. 12 shows the couples and forces acting on the parts of the frame for the assumed mode of collapse. (As in Fig. 5, M_o is used as unit for the couples and M_o/h as unit for the forces, and b is assumed to equal 2 h.) To find a lower bound for the safety factor, one must determine bending moments in the structure corresponding to these couples and forces. At first glance, this might appear to be a difficult task because of the indeterminacy of the upper part of the frame. However, all that is needed is a distribution of bending moments which is "statically compatible" with these forces and couples (as explained in section 2). Such a distribution is obtained, for instance, by letting the forces and couples, which are applied to the upper part of the frame, be carried by beam AB and column BC (Fig. 12) alone. As a result, the beams and columns of the higher stories are free from bending moments. The bending moments in the lowest story, which are obtained in this manner, are shown in Fig. 13. Nowhere do they exceed the limit moment 5 M_o of the members of this story. Thus, the assumed mode of collapse is the correct one and the safety factor equals 30 $M_o/P\,h$.

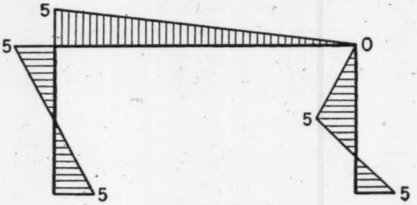

FIG. 13

ACKNOWLEDGMENT

The results presented in this paper were obtained during the course of research conducted under contract N7-onr-35806 with the Office of Naval Research, United States Department of the Navy.

APPENDIX

The characterization of the safety factor against collapse as the largest statically admissible multiplier is an immediate consequence of:

> Theorem IV.—Collapse cannot occur under the loads obtained by multiplying the given loads by a factor which is smaller than a statically admissible multiplier.

The following proof is an adaptation, to beams and frames, of a more general discussion presented by Mr. Greenberg before the Second Symposium on Plasticity at Brown University, Providence, R. I., in April, 1949.

Let m denote a statically admissible multiplier and m' a positive number which is smaller than m. According to the definition of a statically admissible multiplier, there exist bending moments which are statically compatible with the loads multiplied by m and which, at no cross section, exceed the limit moment of this cross section in absolute value. Consequently, there exists a distribution of bending moments M' which is statically compatible with the loads multiplied by m' and satisfies the continued inequality:

$$-M_o(x) < M'(x) < M_o(x) \qquad (11)$$

at any cross section x.

To prove theorem IV, suppose that, in contradiction to this theorem, collapse were to occur under the loads multiplied by m'. Then there would exist admissible bending moments $M(x)$ statically compatible with the loads multiplied by m' and reaching the limit moment at a sufficient number of cross sections to transform the structure into a mechanism. Let $x_1, x_2, \cdots, x_n$ be the cross sections at which yield hinges are active during this collapse. Then,

$$|M(x_1)| = M_o(x_1), \cdots, |M(x_n)| = M_o(x_n) \qquad (12)$$

As long as the displacements are small enough to be disregarded in establishing the equilibrium conditions for the various parts of the collapsing structure, the collapse takes place under constant forces and bending moments. Thus, no elastic deformations occur during this initial phase of the collapse and the parts into which the structure is separated by the yield hinges move as rigid bodies. At a generic instant during this initial phase, let $\theta(x_1), \cdots, \theta(x_n)$ be the relative rotations at the yield hinges. Then

$$\sum_{k=1}^{n} M(x_k)\,\theta(x_k) = W_m \qquad (13a)$$

in which the sum W_m equals the work done by the multiplied loads (multiplier m') on the displacements of their points of application since the beginning of the collapse. According to the principle of virtual work, this work can also be written as

$$W_m = \sum_{k=1}^{n} M'(x_k)\,\theta(x_k) \qquad (13b)$$

because the bending moments $M'(x)$ are statically compatible with the same loads as the bending moments $M(x)$. From the equality of Eqs. 13, it follows that

$$\sum_{k=1}^{n} [M(x_k) - M'(x_k)]\,\theta(x_k) = 0 \qquad (14)$$

Now, the bending moment $M(x_k)$ must equal the limit moment $M_o(x_k)$ in absolute value. Moreover, each term of Eq. 13*a* is positive because it represents the work of the limit moment in a yield hinge on the relative rotation at this hinge.

It is readily shown that every term of Eq. 14 must be positive, too. Indeed, if $M(x_k) = M_o(x_k)$, then $\theta(x_k) > 0$ because the contribution of the hinge x_k to the sum in Eq. 13*a* must be positive. Moreover, Eq. 11 shows that $M'(x_k) < M(x_k)$ in this case. Thus, the contribution of the hinge x_k to the sum in Eq. 14 is positive. If, on the other hand, $M(x_k) = -M_o(x_k)$, then $\theta(x_k) < 0$ and $M'(x_k) > M(x_k)$. Therefore, the contribution of hinge x_k to the sum in Eq. 14 is again positive. All terms of this sum being positive, Eq. 14 cannot be fulfilled and the assumption that collapse occurs under the loads multiplied by m' has led to a contradiction. This completes the proof of the maximum principle of section 2.

In the proof of the minimum principle of section 3 the following theorem[4] is used:

> Theorem V.—If a beam or frame is strengthened (that is, if its cross sections are changed in such a manner that the limit moment is increased for, at least, one cross section and decreased for none), the safety factor for a given system of loads cannot decrease as a result of this strengthening.

Indeed, any distribution of bending moments that is admissible for the original structure is also admissible for the strengthened structure. Thus, any multiplier that is statically admissible for the original structure is also statically admissible for the strengthened structure. Consequently, the safety factor of the original structure which is the largest statically admissible multiplier for this structure is also a statically admissible multiplier for the strengthened structure. Therefore, the safety factor of the strengthened structure that is the largest statically admissible multiplier for this structure cannot be smaller than the safety factor of the original structure.

Consider, now, the manner in which a kinematically sufficient multiplier n is obtained: Yield hinges are assumed in sufficient number to transform the structure into a mechanism, and the multiplier n is determined which must be applied to the given loads if the structure is to collapse by yielding at the assumed hinges. Of course, the assumed mode of collapse may not be the actual mode of collapse for the given structure. It is easy, however, to define a structure for which the assumed mode of collapse is the actual one. Indeed, consider the structure derived from the given structure by leaving the limit moments unchanged at the assumed yield hinges and increasing them indefinitely everywhere else. The actual collapse mode of this strengthened structure will obviously be the mode used in the determination of the kinematically sufficient multiplier n. In other words, n is the safety factor of the strengthened structure and, therefore, cannot be smaller than the safety factor of the given structure. This completes the proof of the minimum principle of section 3.

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[4] "The Principle of Limiting Stress," by S. M. Feinberg, ***Prikladnaya Matematika i Mekhanika***, Vol. **12, 1948**, pp. 63–68 (in Russian).

DISCUSSION

L. E. Grinter,[5] M. ASCE.—Using mathematical language this paper states two physical principles which engineers may prefer to consider without need of such rigorous proofs. The principles are readily established without mathematical equations. Their usefulness, as will be indicated, is limited by the need for considerable judgment in their application.

Consider any continous frame that supports a fixed load P. The frame is redundant to the (n-1)th degree. When n plastic hinges develop, collapse will begin and progress if a flat stress-strain curve is assumed. Choose by guess or by experience n locations for the successive development of n plastic hinges. This guess may be good or bad without influence on the conclusions. By simple geometry compute the angular discontinuities at the remaining n-1 plastic hinges when a unit angle change develops at one convenient hinge; also by geometry compute the corresponding movement of the load P in the direction of P. Then, for assumed collapse, the virtual work of the load, P(collapse) will equal the virtual work of M(yield) at the n hinges; and M(yield) may be constant or may vary from hinge to hinge, but it must be known. This equation of virtual work may be solved for P(collapse), the theoretical collapse load, if the n hinges are correctly located. However, since the hinge locations were merely assumed, the following applies:

> *Principle* (a).—If even one hinge is misplaced, that should have been at another point H, to preclude collapse it must be assumed that the steel near H had been locally hardened to increase M(yield). Therefore, the value of P(collapse), as computed for assumed hinge locations, is the collapse load for a frame locally hardened or strengthened at one or more points and is larger than P (collapse) for the frame with uniform properties. Therefore, the computed value of P(collapse) is an "upper bound."

The second principle is simplified even more than the first by adopting the physical viewpoint. Consider the frame as explained in (a) to be locally hardened $x\%$ at point H so that it does not collapse but is just at the point of collapse. The percentage x may be obtained from the moment diagram that is determined by statics with the aid of known yield moments at the n hinge points. Now reduce the yield point proportionately everywhere and reduce the value of P(collapse) computed in item (a) by the same proportion that will depend on the value of $x\%$ as indicated below. The frame still is on the point of collapse, but it has become a frame that has the original yield point only at point H and has a reduced yield point everywhere else. Hence the reduced value of the collapse load that is the original P(collapse) times $\left(\frac{100}{100 + x\%}\right)$ is an obvious lower bound on the collapse load. This principle may be stated as follows:

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Principle (*b*).—If the frame from item (*a*), which has been locally hardened $x\%$ at point H and by smaller percentages at one or two other points, is then reduced everywhere in yield point by the ratio $\frac{100}{100 + x\%}$, it will return to its normal yield point only at point H and will be of reduced yield point everywhere else. This frame, which is much weaker than the original frame of uniform properties, is on the point of collapse under a reduced load $P(\text{collapse}) \frac{100}{100 + x\%}$. Hence this reduced load is a lower bound for the collapse load of the frame with uniform properties.

Limitations of Usefulness.—Although a single load P has been considered, the application of the principle is equally evident for any fixed loading. The factor x is the greatest localized percentage increment in yield point needed to prevent collapse in item (*a*). No assumptions have been made as to the form or number of redundants in the frame. The main limitation upon usefulness is the need for very good engineering judgment. Unless the operator can almost guess the exact locations of the plastic hinges, the upper and lower bounds obtained are too widely separated to be very useful. For instance, it is not very helpful to know that a frame will probably collapse at 200,000 lb of loading but will not collapse at 100,000 lb of loading. An experienced engineer could study a set of plans and guess the collapse loading of a frame more closely than this without making such calculations.

The weakness of the principle seems to lie primarily in item (*b*). In item (*a*) the theoretical collapse load is completed for a frame that is of normal yield point everywhere, except at one or two local areas where it must be hardened, say, x and $y\%$. In item (*b*) however, all of the frame is softened except near one plastic hinge below the original yield level. This reduction is determined from the larger percentage of local hardening, that is, x or $y\%$. Hence, the lower bound upon the collapse loading is an extremely conservative value which exaggerates the spread between upper and lower bounds as computed by examples (*a*) and (*b*). Of course, improved guesses can be used to reach better and better agreement between upper and lower bounds until they become identical and a solution is obtained. This, however, may be a slow process for complex structures, where such a technique would be most helpful.

These comments have served to show that the two principles established mathematically by the authors may be proved quite simply by physical reasoning. However, this approach does not change the fact that their convenient use in structural design depends upon the judgment of the engineer in guessing the approximate locations of the plastic hinges.

I. K. Silverman,[6] A. M. ASCE.—A method for solving the problem of determining the load required to cause collapse in a structure of known dimensions is indicated in this paper. The reverse of this problem is designing for the dimensions of a structure that will give the greatest economy for given loads.

Most engineers deal with the latter problem. The method of analysis given by the authors furnishes answers, using an iterative process, to both problems. The engineer assumes a design, determines the safety factor (as defined by the authors), examines this safety factor in the light of the desired factor, and, if necessary, modifies his design. It will be found that there are a great many designs (relative dimensions) that will satisfy any criterion of safety factor. Further analyses as to relative economy will fix the final design.

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Except for local failure the number of yield hinges necessary to transform a structure into a mechanism is one greater than the structure's degree of statical indeterminateness; but there is no theoretical reason why a structure cannot be so proportioned that simultaneous failure occurs in all elements of the structure with the occurrence of more than the minimum number of required yield hinges. This brings to mind the "one-hoss shay" principle:[7]

> "Have you heard of the wonderful one-hoss shay
> That was built in such a logical way
> It ran for a hundred years to a day?
> And then of a sudden * * *
> It went to pieces all at once—
> All at once and nothing first,
> Just as bubbles do when they burst."

The theory of limit design contains in itself a direct method for the design of structures on the "one-hoss shay" principle. Consider the beam in Fig. 14

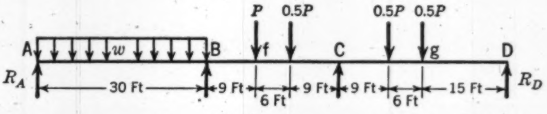

FIG. 14

carrying the fixed loads shown. The loads are expressed as a function of a single parameter P. The center span is designed to furnish three yield hinges located at B, C, and f. Assuming a constant limit moment at B, C, and f equal to M_2 the following equations may be written:

$$M_B + M_C = M_F = M_2 \dots\dots\dots\dots (15)$$

$$M_f + M_B - 9\,V_B = 0 \dots\dots\dots\dots (16)$$

$$M_B + 15\,P + 4.5\,P = M_c + 24\,V_B \dots\dots\dots\dots (17)$$

in which V_B is the shear to the right of the support B acting upwards. It follows that

$$V_B = \frac{2}{9}\,M_2 = \frac{19.5\,P}{24} \dots\dots\dots\dots (18a)$$

and

$$M_2 = 3.66\,P \dots\dots\dots\dots (18b)$$

The next step is to determine the value of the limit moment so that a hinge occurs at the point of the maximum bending, which in span AB is at the point of zero shear. Denoting the maximum bending moment in span AB as M_x,

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[7] "The Deacon's Masterpiece," by Oliver Wendell Holmes, *World's Best Loved Poems*, Harper and Bros., New York, N. Y., 1927, p. 362.

$$M_x = R_A\, x - \frac{w\, x^2}{2} \qquad (19)$$

$$X = R_A/w \qquad (20)$$

$$M_B = 30\, R_A - 450\, w = -\, M_2 \qquad (21)$$

These equations give

$$M_x = \frac{(15\, w - 0.122\, P)^2}{2\, w} \qquad (22)$$

For span CD it is assumed that the maximum moment occurs under the load nearest the support D and that consequently a yield hinge occurs there. The conditions existing in span CD are expressed analytically as follows:

$$M_g = 15\, R_D = M_3 \qquad (23)$$

$$M_c + 30\, R_D - 12\, P = 0 \qquad (24)$$

$$M_c = M_2 \qquad (25)$$

It follows that

$$M_3 = 4.17\, P \qquad (26)$$

Assume, after applying a safety factor, that P equals 36 kips and w equals 2 kips per ft. The limit moments are then $M_x = 164$ kip ft, $M_2 = 131.5$ kip ft, and $M_3 = 150$ kip ft. The section moduli for steel I-beams having a minimum yield point stress of 36,000 lb per sq in. and a shape factor of 1.15 are: $S_x = 47.6$ cu in.; $S_2 = 38.2$ cu in.; and $S_3 = 43.5$ cu in.

Jack R. Benjamin,[8] A. M. ASCE.—The method of limit design of beams and frames presented suffers from several serious shortcomings. The first is one of lack of emphasis on the basic assumptions. The method-of-collapse analysis depends primarily on two factors: The stress-strain diagram and the absence of direct tension or compression in the members. If the material assumptions are satisfactory, the second assumption is more often not satisfactory. All members of frames are subject to direct tension or compression, but the usual single-story frame has only minor tensile stresses and small-to-moderate compressive stresses. Thus, under certain conditions, collapse could be predicted for most one-story frames.

The method-of-collapse analysis does not apply to multiple-story bents primarily because of the compression members. Members under combined direct compressive stress and bending do not develop plastic hinges similar to those in a beam. In a beam under a concentrated load the region of plasticity is localized, and a sharp kink will occur at failure. In contrast, in a compression member the advent of plasticity involves a considerable portion of the member, and the zone of plasticity is not localized. The advent of plasticity sharply changes the action of the member, causing rapid changes in geometry that in turn cause more plasticity. This action is highly nonlinear. The simple rectangular, mild steel section used as a beam will develop 50% higher moment as a plastic hinge than that developed at the elastic limit. However, the same section used as a compression member may show almost no load variation between the capacity at the elastic limit and the capacity at collapse. Fig. 1 should include other curves for various values of axial load other than zero. Fig. 15 will answer part of the problem. However, the solution of the section is not the same as that of the member because of the influence of the change of geometry. The member capacity can decrease while the section capacity increases. The compression member problem is not simple. The method of the authors can apply only to members without direct compressive stress.

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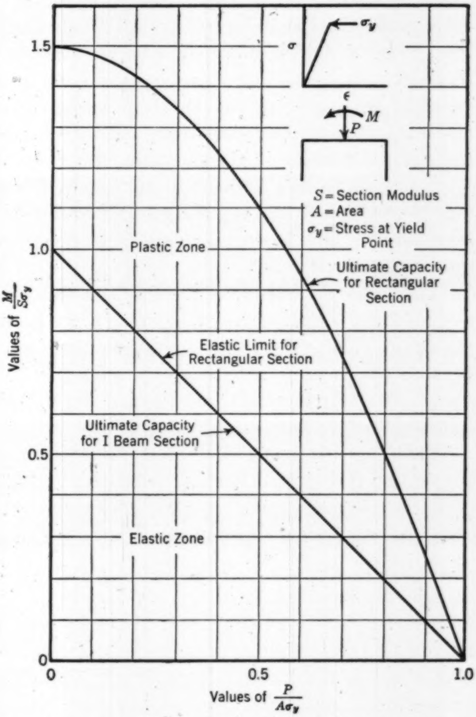

FIG. 15.—VARIATION OF MOMENT CAPACITY WITH DIRECT LOA FOR TWO MILD STEEL SECTIONS

The second serious limitation of the method concerns the mode of failure. The failure load of a single bent, as computed by the method of this paper, may be much greater than that of a structure made up of several of these bents. The structure may fail torsionally in a lower mode than the single-bent analysis would indicate. Any analysis of an actual structure for collapse must search out the lowest failure mode. Unfortunately, torsional failures must be considered. Loadings on actual structures are far from the simple loadings used, and in almost every case torsional failure is a possibility.

A further practical problem to be solved before frame collapse can be considered a design criterion is that of loadings. The loadings used in the paper do not reflect the designer's problem. The designer must consider distributed loads of three types:

1. Dead loads whose values are fairly well known and cannot change.

2. Live loads whose service impositions on the structure are estimated by intelligent guess.

3. Transient loads (such as wind and earthquake) that are normally accounted for by using higher allowable stresses, thereby recognizing the transient nature of the loads.

The first two loading types are essentially vertical, but the latter is essentially horizontal. The paper does not indicate how these three distinctly different loadings shall be treated, although this is a fundamental design problem. The writer's studies of collapse indicate that this problem must be resolved before collapse can be considered a satisfactory design criterion. It might be mentioned also that any loading that is a function of time might momentarily give rise to collapse conditions in a structure without particularly damaging it because of the short time duration of the loading.

A second loading problem is that of the factor of safety. Should all loadings be multiplied by the same factor of safety? This course does not appear reasonable. This is a particularly knotty problem yet the paper assumes a uniform factor of safety.

The methods and ideas presented in this paper are very interesting from an academic standpoint. However, their limitations and assumptions are not fully explained. These limitations and assumptions should be underlined because design of actual structures using just these principles could be highly dangerous. Furthermore, a large number of knotty practical problems must be solved before limit analysis for collapse can be of much value.

This paper is a fine beginning toward a more rational basis for design of beams and frames, but it must be emphasized that it is just a beginning.

E. P. Popov,[9] A. M. ASCE.—An outstanding addition to the methods of structural analysis was made in this paper. The method developed by the authors for establishing the upper and lower bounds for the load factor at collapse is very ingenious, and the suggested trial-and-error procedure for narrowing the bounds appears to be quite simple for many ordinary frames. The method developed is general and has already been extended to space frames.[10]

Although the problem of limit design is admirably solved from the mathematical point of view, some of the assumptions and limitations of the method are not clearly indicated, and adequate references to related work are lacking. The following remarks are intended to urge caution in the application of the method to practical problems.

The writer considers that no treatment of limit design of frames is complete without some mention of the extensive analytical and experimental work done at Cambridge University, Cambridge, England, under the direction of J. F. Baker, A. M. ASCE. For example, the virtual work technique for limit design that may appear novel to some readers was described by Mr.

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[9] Associate Prof., Civ. Eng., Univ. of California, Berkeley, Calif.

[10] "The Limit Design of Space Frames," by Jacques Heyman, *Journal of Applied Mechanics*, Vol. 18, No. 2, June, 1951, p. 157.

Baker.[11] However, the application of the virtual work principle to obtain an upper bound for the safety factor is a creditable accomplishment of the authors.

In applying the limit design method to frames, several limitations must be clearly kept in mind. First, it is assumed that all compression members possess a sufficiently low slenderness ratio so that instability does not occur. Thus, while it is true that, for example, most building columns would fulfil such a condition, the instability of compression members cannot be ruled out for all conceivable cases. Second, in the "Introduction," it is stated: "The limit moments are known throughout the structure * * *"

This presumably means that the effect of axial force and shear on the limit moment of a member is known. Actually, much experimental research is needed to determine such quantities for common members. On the other hand, in the numerical example given in the paper, complete disregard of axial forces and shears found in the frame is implied. The axial forces and shear in various members of Fig. 6 and of Fig. 8 are different. Thus, from Fig. 5 it is seen that the upper member, according to the first assumption of hinges, is under no axial load, while the member AB is compressed by a force equal to 6 M_o/h. Similarly, by applying the conditions of equilibrium to the various members of the frame solved in Fig. 8, it may be easily shown that the axial forces vary from a tensile force equal to 3 $M_o/2\ h$ to a compressive force of $M_o/3\ h$. However, the safety of the structure is invariably based upon a common limit moment M_o alone. To be sure, Mr. Baker contends[12,13] that for small axial stress the reduction in the limit moment is surprisingly small, and thus the limit design is sufficiently developed for single-story bents with light roof loads. However, the writer feels that, without further experimental verification, the method cannot be safely extended to practical cases in which large axial forces exist in the members. For example, an interaction curve for the collapse of an 8 WF 31 beam under axial loads and applied bending moments (as established in preliminary tests[14] at Lehigh University, Bethlehem, Pa.) indicates that, if there is no axial load, $M_o = 1{,}200$ in.-kips. However, if $P = 100$ kips, M_o is reduced to approximately 1,000 in.-kips. Similarly, when $P = 150$ kips, $M_o = 800$ in.-kips.

A. Hrennikoff,[15] A. M. ASCE.—Presenting concisely some of the recent developments in the theory of limit design, this paper is interesting and instructive. It deals with a subject that has been commanding much attention from engineers.

Although the rise of interest in this field is comparatively new, dating from the 1939 publication of the well-known paper[16] by J. A. Van den Broek, M. ASCE (who introduced the term "limit design"), the basic idea has been used by engineers, in application to certain types of members, for a long time. Thus, the standard design of rivets, splices, and connections of tension and compression steel members is nothing but limit design, since different parts of the members involved (such as rivets and sections of the main material and splice plates) are arbitrarily assigned capacity loads fully consistent with statics, so that no part is overstressed, while at the same time no cognizance is taken of the deformation of different parts. This established method was devised for the design of connections out of sheer necessity, since the problem was found too difficult for the elastic theory. The method proved to be safe largely because of the ample ductility of structural steel.

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[11] "The Design of Steel Frames," by J. F. Baker, *The Structural Engineer*, Vol. XXVII, 1949, p. 397.

[12] *Ibid.*, p. 408.

[13] *Ibid.*, p. 428.

[14] "Tests of Columns Under Combined Thrust and Moment," by Lynn S. Beedle, Joseph A. Ready, and Bruce G. Johnston, *Proceedings of the Society for Experimental Stress Analysis*, Vol. VIII, 1950, p. 109.

[15] Prof., Civ. Eng., Univ. of British Columbia, Vancouver, B. C., Canada.

[16] "Theory of Limit Design," by J. A. Van den Broek, *Transactions*, ASCE, Vol. 105, 1940, p. 638.

Mr. Van den Broek's theory of limit design and the authors' elaboration of it projects the same principle (assigning capacity values to the members) into the realm of complete statically indeterminate flexural structures consisting of continuous beams and rigid frames. The element of compelling necessity is lacking in this field, since continuous beams and rigid frames devoid of side sway can be analyzed conveniently by moment distribution and other methods of elastic theory, and frames subject to side sway can be analyzed by some adaptation of statics, such as the portal and cantilever methods. Nevertheless, the method of limit design offers some economy compared to the other methods and for this reason is fully justifiable. At the same time, in view of its less conservative nature, its adoption should be preceded by a searching investigation of underlying assumptions. It is gratifying, therefore, to see in this paper formal rigorous proofs of the statements establishing the upper and lower limits of the factors of safety in the structure from the viewpoint of limit design.

There are, however, some other important considerations which have not received due attention. One of these is the question of plastic hinges. In the opinion of the authors, such hinges develop freely at several sections of the structure and provide all the angle changes necessary for consistency of deformation that takes place prior to the collapse of the structure. It is tacitly assumed here that the structure cannot fail by actual physical breaking at some of the earlier formed hinges before the limiting state is reached and the structure begins to act as a mechanism. The basis for this unstated assumption apparently lies in the shape of the assumed (M-ϕ)-curve (Fig. 1), in which the limiting moment M_o continues constant indefinitely as the angle change ϕ continues to increase. However, the absolute value of ϕ that a beam can sustain before breaking depends on the length l, over which the limiting moment is acting. If this length is small, the maximum possible angle change is small too, in spite of a long horizontal part of (M-ϕ)-curve in Fig. 1. For example, a 1-ft length of an 18-in. I-beam can easily have its flanges deformed as much as 1 in. each, and can thereby undergo (without ill effects) an angle change of approximately 6°, but the same beam, 1 in. long, will probably break before bending through the same angle of 6°, because such bending would mean doubling the length of the tension flange and compressing the other flange to a length of zero. An impossibility of such a failure at the plastic hinge must be firmly established before the theory of limit design can be accepted, and the conditions that would prevent hinge failure must be established definitely.

One of the main factors that determine whether the beam should fail in this manner is the shape of the (M-ϕ)-curve beyond the elastic stage. Assume first an extreme case in which the (M-ϕ)-curve consists of two straight lines (Fig.

16), the elastic part OA and the plastic part AB, and consider an example of bending of a fixed-end beam under a gradually increasing uniform loading w (Fig. 17). As long as the elastic range is not exceeded, the moment curve is a parabola with the negative end ordinates two times greater than the positive center ordinate. When the load intensity reaches the value w_e, the end moments just reach the limit value M_o—in other words $M_o = \frac{w_e L^2}{12}$, and the moment diagram assumes the shape FDHEG. To this point all angle changes have been elastic. If, now, w exceeds w_e, plastic hinges form at the ends, preserving the same values of moments there, whereas the positive center moments gradually increase beyond $\frac{w_e L^2}{24}$ up to the limiting value $\frac{w_e L^2}{12}$, and the moment diagram approaches the limit curve FD_1KE_1 G.

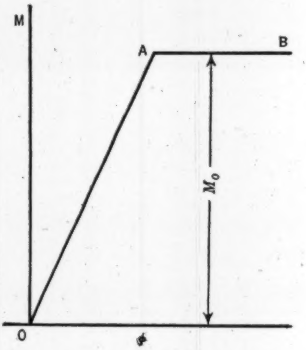

Fig. 16

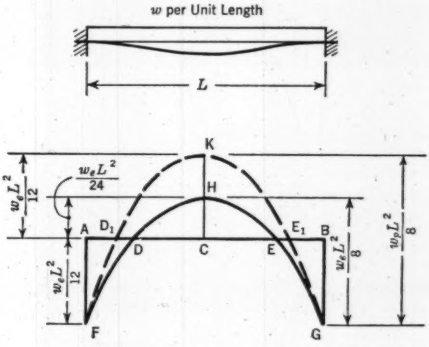

Fig. 17

It must be remembered that the total angle change on one half the length of the beam AC must be zero both in the elastic and plastic stages, as demanded by deformation of the fixed-ended beam. At the end of the elastic behavior the positive angle change on the length DC equals numerically the negative angle change on the length AD, both these angle changes being proportional to the corresponding areas under the moment curve. Toward the limiting condition corresponding to the load w_p, the still-elastic moment at the center, doubles its value compared to the value under the load w_e, and the length of the positive moment region D_1C becomes greater than DC. These circumstances combine to yield a positive angle change that is more than twice the magnitude of the one created by load w_e. However, the region of negative moments shrinks to the length AD_1, and the negative bending moments are everywhere smaller in value than under the load w_e, except at one point A, where the moment remains the same. This means that all increase in the negative angle change necessary to balance the increased positive angle change and to compensate for the loss of some negative angle change must take place at one section—that is, on an infinitesimal length at point A, Fig. 17. Since such a concentration of angle change would result in infinite strains, the beam is likely to fail at the ends before the limit condition visualized in limit design is materialized.

Assume now a more general case of the (M-ϕ)-curve in Fig. 18, and let ϕ represent from now on an angle change occurring on a unit length of the beam corresponding to the value of the bending moment M. An increment of moment dM occurs on the length of the beam dl determined by the value of the shearing force V in this manner: $dl = \frac{dM}{V}$. The angle change on this length is evidently equal to

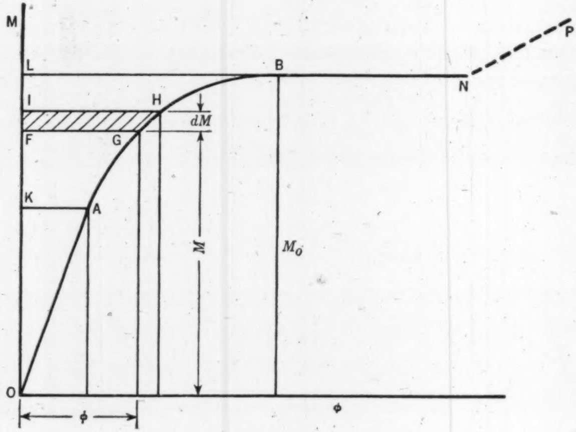

FIG. 18

$$\phi \, dl = \frac{\phi \, dM}{V} \qquad \text{(27a)}$$

In Eq. 27*a* $\phi \, dM$ represents the area of the horizontal strip FGHI, Fig. 18. By summation of these strips the total angle change between any two points on the curve, such as points A and B (assuming the shearing force V constant in the interval), is found to be

$$\phi = \int_A^B \phi \, dl = \frac{\int_A^B \phi \, dM}{V} = \frac{\text{area } \overline{\text{AKLB}}}{V} \qquad \text{(27b)}$$

The area in the numerator of Eq. 27*b* is bounded by the axis of M, the (M-ϕ)-curve, and the two abscissas, corresponding to the points on the curve between which the angle change is evaluated. It is interesting to note that, with the (M-ϕ)-curve extending horizontally without limit, no contribution to the angle change is provided by the horizontal or plastic part of the curve because advancing the point B along the curve to the right will not increase the area AKLB. Thus, contrary to intuition, the plastic part of the $(M\text{-}\phi)$-curve proves incapable of contributing to the necessary angle change at the plastic hinge and to preclude failure of the structure there while other plastic hinges are developing.

The situation is different if the $(M\text{-}\phi)$-curve exhibits a strain hardening part, indicated in Fig. 18 by a dotted line NP. When a stress condition in the beam extends into the strain hardening region, the area in the numerator of Eq. 27*b* begins to increase again, thus providing the necessary angle change at the hinge. The presence of a plastic region prior to strain hardening increases the referred-to area of the $(M\text{-}\phi)$-curve, and thus decreases the necessary rise of moment along the strain hardening part, thus assisting in the formation of the hinge.

As follows from Eq. 27*b*, the angle change occurring in a certain range of moments is inversely proportional to the shearing force.

An exact solution of the problem of bending of a typical fixed-end uniformly loaded I-beam, made of typical mild steel,[17] shows that the ends of the beam would extend into the strain hardening region even before the limit of proportionality is reached at the center.

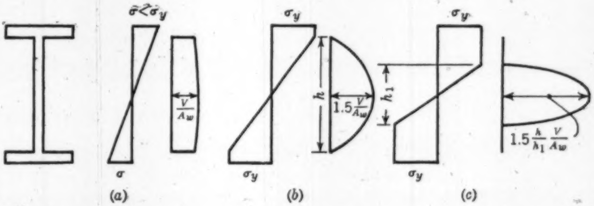

FIG. 19

In the case of I-beams and channels, there is another factor which interferes with the behavior of beams not subject to strain hardening as visualized in the theory of limit design. The I-beams are characterized as a rule by appreciable shearing stresses in the web. In the elastic range these stresses are roughly uniform and equal to $\frac{V}{A_w}$ (Fig. 19(*a*)) where A_w is the area of the web. When the flanges have just reached the yield point, the shearing stresses follow a parabola with the greatest ordinate $1.5\,\frac{V}{A_w}$ at the center. As yielding approaches the center, the shearing stresses concentrate near the center (Fig. 19(*c*)) increasing in the ratio h/h_1, compared to the condition shown in Fig. 19(*b*). These deductions follow from a consideration of the static equilibrium of an element of beam, as used in the elementary determination of shearing stresses. When the depth h_1, over which the normal stress in the web of the beam is still elastic, becomes quite small, the shear stress at the center becomes excessive. In the absence of strain hardening this condition cannot be relieved by warping of the section, and shear failure must necessarily ensue. Thus, an I-beam, with the (M-ϕ)-characteristic assumed in Fig. 1, is incapable of sustaining any shearing force at and near its horizontal plastic section, unless strain hardening comes in and redistributes the shearing stress again over the entire section of the web.

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[17] "Theory of Inelastic Bending with Reference to Limit Design," by Alexander Hrennikoff, *Transactions*, ASCE, Vol. 113, 1948, p. 235.

Thus, the major characteristic of the beam essential for formation of plastic hinges is the strain hardening rather than the plastic stage although the latter is very valuable in keeping down the hinge moments in beams subject to strain hardening. Without strain hardening the beams would fail at the values of the load below the limit load.

Most of the ductile metals are subject to strain hardening and thus may be suitable for limit design; but the moments present at plastic hinges are unknown beforehand. As the hinges form and extend, the hinge moments grow larger. The moments are greater at the earlier formed hinges than at the later ones. This complicates the application of the theory of limit design considerably. It is possible that an assumption of constant moment values is still admissible, but the limits of errors involved in it should be investigated.

The degree of strain hardening necessary to avoid an early failure becomes an important question. Most ductile metals are likely to have the necessary requisites, but some aluminum and steel alloys are probably unfit for limit design.

An exact theory of bending beyond the elastic range is needed to answer some of these questions. Some investigations have been made along these lines.[17] Further extension of this theory, allowing for the effect of shear and for the warping of the section, is now in order.

In conclusion, the writer wishes to state that the purpose of his discussion has been not to disparage or to minimize the excellent work performed by the authors, but to state the limitations of the theory and to point to some comparatively unexplored but basic phases of the problem, which must be examined and settled prior to further advances along the road taken by most of the investigators in the field of limit design.

P. S. SYMONDS[18].—The authors have rendered an invaluable service in stating and proving in rigorous terms two theorems that are basic in the plastic methods of analysis and design of beams and frames. They have filled a serious gap in the theory. Since they were concerned with basic theory, they could not elaborate on the physical significance or justification of the underlying hypotheses, nor could they deal at any length with practical applications.

"Limit moment" has also been termed the "fully plastic moment" of a given section since it represents the limiting condition in a mild steel beam when plastic zones have spread all the way across the section except for a zone of infinitesimal thickness in which the stress changes from the yield stress in tension to that in compression. As the fully plastic moment is approached, the curvature at a given section tends toward infinity. Thus, when the fully plastic moment is reached, by hypothesis a finite change of slope in an infinitesimal distance can occur. At such a section free relative rotation would take place under constant moment, that is, as if a "plastic hinge" were inserted.

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[18] Associate Prof. of Eng., Brown Univ., Providence, R. I.

These notions of fully plastic moments and plastic hinges represent idealized physical quantities since they are never strictly realized in physical structures. For example, the relative rotations at a hypothetical plastic hinge imply infinite curvatures and hence infinite strains; this means that the material actually enters the strain hardening range, and the actual moment must rise above the theoretical limit moment. More importantly, the fully plastic moments in the authors' theory and, in general, in the simple plastic methods of analysis and design, are treated as constants for a given section and material. In fact, they are always functions of local loading conditions such as the existence of transverse shear or axial force, and depend upon the rate of loading. Moreover, effects of changes of shape are neglected, and these may be important when large axial forces are present.

The basic hypotheses must be regarded as semi-empirical postulates whose utility and justification must be verified by tests on actual structures. Much of the needed experimental corroboration has been supplied by H. Maier-Leibnitz[19] in his tests on continuous beams and by J. F. Baker,[20] A. M. ASCE, in tests on small portal frames. More work is needed, especially on frames of full-scale size and standard commercial members, but the work already done is strong evidence that the plastic theory based on the hypotheses of the paper is not only simple and direct, but leads to the prediction of failure loads in reasonably close correspondence with observed loads at which the rate of increase of deflection with load rises sharply to excessive values. Mr. Baker[11] and his associates have also shown clearly the potential practical advantages of the plastic methods, in suitable problems, over conventional elastic methods.

An alternative method for the calculation of plastic failure loads may be based directly on the authors' theorem that if plastic hinges are inserted at sufficient sections of the frame to render it a kinematic mechanism, then the corresponding load is always either greater than, or equal to, the actual collapse load of the frame. It leads to especially rapid solutions for rectangular frames and would seem to make possible the determination of the plastic collapse load of a multi-story, multi-bay frame, or of a shed-type frame in a surprisingly short time.

Considering the two-story frame treated as an example by the authors, any mechanism, including the actual one, can be regarded as built of combinations of certain elementary mechanisms. In any problem there are three types of elementary mechanisms, namely—those associated with failure of single beams under their own loads, failure of individual panels or stories o frame, and "failure" (in a special sense) corresponding to rotation of a joint at which three or more members are connected.

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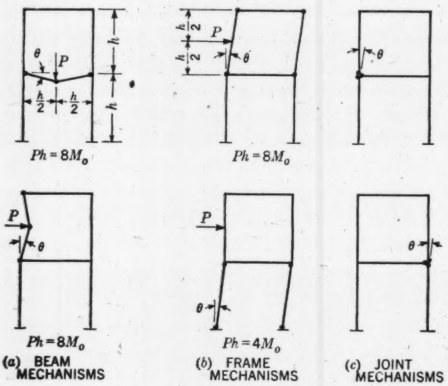
[19] "Test Results, their Interpretation and Application," by H. Maier-Leibnitz, *Preliminary Publications*, 2d Congress, International Assn. for Bridge and Structural Eng., Berlin, Germany, 1936, p. 97.

[20] "A Review of Recent Investigations into the Behaviour of Steel Frames in the Plastic Range," by John Fleetwood Baker, *Journal*, Inst. C. E., London, England, Vol. 31, 1949, p. 188.

Fig. 20 shows these three types of elementary mechanisms for the authors' frame and loading. The dots represent plastic hinges, and angle θ in each case is arbitrary but small. Joint mechanisms (Fig. 21(c)) are devices for representing arbitrary rotations and need not be true mechanisms.

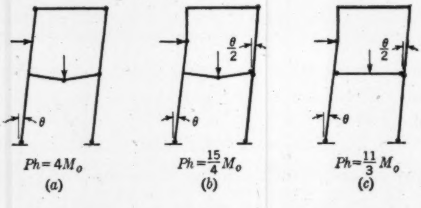
There are always just as many of these elementary mechanisms as there are independent equations of equilibrium relating the moments which must be considered in the plastic collapse analysis; thus in the authors' example there are six elementary mechanisms. These can be combined with each other just as can equations of equilibrium. The actual mechanism is either one of these elementary mechanisms or the particular combination of them that has the lowest corresponding load of any possible combination. The guide in combining them is the elimination of plastic hinges in the mechanisms being combined. In practice it is easy to see what combinations are likely to be advantageous. Furthermore, when the principle of virtual work is used, as in the paper, it is a matter of only a minute or two to compute the failure load corresponding to any combination.



(a) BEAM MECHANISMS (b) FRAME MECHANISMS (c) JOINT MECHANISMS

FIG. 20.—ELEMENTARY MECHANISMS

Fig. 21 shows a number of possibly advantageous combinations. The correct combination is that of Fig. 21(c) which can be thought of as composed of the lower of the mechanisms of Fig. 20(a) and those of Fig. 20(b), with the angle θ in each diagram chosen so as to eliminate the plastic hinge at the top left-hand joint. The test for correctness is that no other elementary mechanism can be combined with that of Fig. 21(c) to give a new mechanism with a lower collapse load than 11/3 M_o/h. With a little practice it is not difficult for the analyst to see when all possibly advantageous mechanisms have been tried. There is then no need of making any elastic calculations or of seeking a lower bound, and this is often a major practical advantage of the method. A more complete account is to be published elsewhere.[21]



(a) (b) (c)

FIG. 21.—ADVANTAGEOUS COMBINATIONS OF ELEMENTARY MECHANISMS WITH CORRESPONDING FAILURE LOADS

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[21] "The Rapid Calculation of the Plastic Collapse Load for a Framed Structure," by B. G. Neal and P. S. Symonds, *Journal*, Inst. of C. E., London, England (publication pending).

This technique (with different physical concepts) has also been used successfully in the determination of shakedown load limits of continuous frames,[22] that is, of the load limits within which, no matter in what order or how many times a set of loads are applied, plastic flow ultimately ceases. This general problem always includes, as a special case, the problem of failure under a single application of a set of loads. Its practical importance lies in the fact that when loads are repeatedly applied in certain sequences, failure of the type considered by the authors may occur at lower load values than are required for failure under a single load application. The authors' theorems apply only in this rather restricted problem.

Finally, the writer would raise a question of terminology. The term safety factor as used in this paper seems unfortunate since it differs from ordinary structural usage. The method of the paper and others available really determine the failure load of a given frame not a safety factor in the usual sense. The latter is a number chosen in advance by the designer, for instance, 2.0 or 1.75, that depends on his estimate of uncertainties of materials, fabrication methods, loading, and other quantities in a given type of structure. The structure is so designed that the given working loads, when multiplied by this factor, are just capable of producing failure. Safety factor as used in the paper has no relation to the one defined and would seem to make for some unnecessary confusion by conflicting with ordinary terminology.

H. TACHAU,[23] J. M. ASCE.—A valuable contribution to the theory of limit design has been made by the authors of this paper. Because it is clearly presented and based on principles familiar to structural engineers, the new procedure is easy to learn and easy to apply. The proposed methods make it a relatively simple matter to analyze the collapse of rigid frames with several bays and several stories. Thus, complex frames can now be subjected to limit design, whereas this was not practically feasible previously.

Certainly, this procedure is much shorter than the conventional analysis, and it would be of great benefit to the analyst if the new theory could eventually supercede elastic methods. However, there still exists a gap between the theory as presented in the paper and its application to the actual design of framed structures. Since the authors' efforts were evidently concentrated on the establishment and proofs of the new theorems, it could hardly be expected that they would cover this aspect in the short space allotted to one paper. Part of this discussion is intended to define that gap in more detail.

By basing their theory on a bending moment curvature diagram (Fig. 1), the authors have not encountered the difficulty of relating the limit moment to the stress-strain curve of the material (usually obtained from a standard tension test). The main purpose of Fig. 1 is to establish a working value for the limit moment M_o. Suppose all members of a structure have the same cross section at the critical locations where yield hinges will occur. At first it may appear that in this particular case M_o would be the same at all sections involved. On second thought, however, this may not be strictly true as there are a number of factors influencing the value of the limit moment M_o.

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[22] "Recent Progress in the Plastic Methods of Structural Analysis," by P. S. Symonds and B. G. Neal, *Journal of the Franklin Institute*, Vol. 252, 1951 (November, p. 383, and December, p. 469).

[23] Asst. Prof. of Civ. Eng. and Mechanics and Hydraulics, State Univ. of Iowa, Iowa City, Iowa.

In a general way the following variables may have to be considered in establishing bending moment curvature curves for a given material: (1) Geometric shape of the member, which involves the shape of cross section and also changes along the axis of the member; (2) distribution of bending moments along the member; and (3) presence of axial and shear stresses in addition to the bending moments.

To be more specific, consider a frame of structural steel. Apparently, at the present time (1951) limit moments are not readily available for structural sections of various sizes. However, as a crude approximation, limit moments for wide-flange beams can be based on the flexure formula, using the ultimate strength in place of the working stress. For other sections limit moments can often be approximated by assuming the entire cross section to be subjected to its ultimate strength. More exact values of M_o can be calculated by a method developed by Mr. Hrennikoff.[24] This method is applicable to prismatic members symmetrical about their neutral axes; but many steel beams are composed of relatively thin parts, and, while local crinkling of the compression flange is not usually a factor in elastic analysis, it might be of importance after yielding has begun. Likewise, lateral instability is of importance for many economical steel sections, especially wide-flange beams. The latter topic has been studied in connection with the conventional elastic analysis[25] and is incorporated in the present American Institute of Steel Construction (AISC) design specification for steel buildings. Possibly the formula for elastic instability can be modified so as to apply to limit design. An investigation of lateral instability in the plastic range has been published,[26] but the results of this research are applicable to beams of rectangular cross section only.

To obtain rigid connections between members, the joints are generally reinforced with plates, seat angles, or brackets. Thus, the bending resistance right at the joint will be higher than elsewhere, and the critical sections will occur at some distance away from the connection. Obviously this effect is important for short members only.

In addition, other less important factors may have to be considered under certain circumstances, such as lack of uniformity of the material (which may become important in cases where the bending moment varies gradually, as, for example, on the right side of the lower horizontal member in Fig. 22, or for members of variable cross section), duration of loading, rate of load application, nonsymmetrical cross sections, and the restraining actions of web stiffeners (at concentrated load points).

In reinforced concrete frames all joints are rigid by the very nature of the construction. Limit design concepts have been applied to concrete beams and columns[27] but, to the knowledge of the writer, there is no published record of application to concrete frames. Very few load-deflection curves or similar experimental data are to be found in the literature, but the deflections at the "yield" and ultimate can be calculated,[28] and in some cases the moment curvature relation will be of the type shown in Fig. 1. The main difficulty in applying the method of the paper to concrete design is the necessity of placing the reinforcing symmetrically about the bending axis. Otherwise, M_e will be different for positive and negative bending. Therefore, it would be worthwhile to try extending the new method to cases in which the positive limit moment is different from the negative limit moment.

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[24] "Theory of Inelastic Bending with Reference to Limit Design," by Alexander Hrennikoff, *Transactions*. ASCE, Vol. 113, 1948, p. 213.

[25] "Strength of Beams as Determined by Lateral Buckling," by Karl deVries, *ibid.*, Vol. 112, 1947, p. 1245.

[26] "The Lateral Instability of Yielded Mild Steel Beams of Rectangular Cross-Section," by B. G. Neal, *Philosophical Transactions of the Royal Society of London*, Series A, Vol. 242, No. 846, 1950, pp. 197–242.

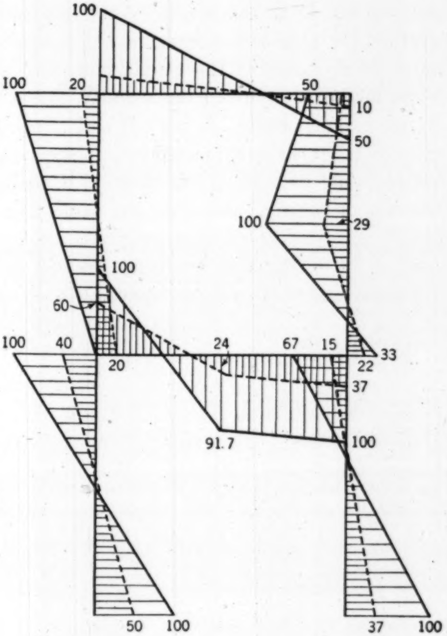

FIG. 22

The two-story frame described in Figs. 2 to 8 serves as an excellent medium for demonstrating the new method. The selection of the location of the yield hinges is not at all difficult. They will be at joints and corners and under concentrated loads. If the number of yield hinges is less than the number of redundants, the equations of statics will not be sufficient to solve the forces on all members; but, in such instances arbitrary values may be assigned to these forces, so long as they are statically compatible with the applied loads. It can probably be proved that the number of yield hinges required for over-all collapse must be at least equal to one more than the number of redundants.[29] On the other hand, the hinges could be arranged so the movements of the members are not uniquely defined. There is no obvious reason for deciding whether or not the mechanism created by the introduction of yield hinges should be such that the motion of each link is completely controlled by and depends on the motions and positions of each of the other links.

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[27] "The Design of Reinforced Concrete Structures," by Dean Peabody, Jr., John Wiley & Sons, Inc., New York, N. Y., 2d Ed., 1946.

[28] "Ultimate Strength of Reinforced Concrete Beams as Related to the Plasticity Ratio of Concrete," by Vernon P. Jensen, *Bulletin No. 345*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., Vol. 37, 1943, p. 35.

[29] "Theory of Limit Design," by J. A. Van den Broek, John Wiley & Sons, Inc., New York, N. Y., 1948, p. 32.

As the frame is subjected to gradually increasing forces the moments grow beyond the elastic limit, as shown by Fig. 22. In this figure, the bending moments based on the elastic distribution are represented by dotted lines and figures, and the distribution at collapse is shown by solid lines. The limit moments M_o are assumed as 100, and the elastic distribution has a maximum value of 60. The two arbitrary values of 60 and 100 represent a reasonable relation for a wide-flange beam. The similarity between the shape of the moment diagrams at the elastic limit and at collapse is quite striking and lends further support to the validity of the proposed method. The highest elastic moment occurs at the left end of the lower horizontal member. This same member is also subjected to high shearing forces and this is where inelastic action would first begin. Ordinarily, this place should be made stronger than the rest of the structure. At the same time, the left column is very highly stressed at the fixed end: The moment is 50, the axial compression is 198, and the shear has a value of 90 at this point. All forces are expressed in terms of 100 M_o/h. (For comparison see Table 1.) The elastic moment of 60 is reached when $P = 142.4$, but collapse occurs only when P reaches 367. So the margin of safety is roughly 150% based on the elastic limit. Of course, the amount of the increase depends on the shape and other variables mentioned previously.

TABLE 1.—LIMIT DESIGN OF BENT OF FIG. 22

Member	Elastic Limit[a]			Total Collapse[a]		
	Maximum Moment	Axial Stress	Maximum Shear	Maximum Moment	Axial Stress	Maximum Shear
Lower left vertical	50	−198	90	100	−533	200
Upper left vertical	20	−29	39	100	−150	100
Lower right vertical	37	+55	52	100	+167	167
Upper right vertical	29	+29	103	100	+150	267
Lower horizontal	60	−51	168	100	−100	383
Upper horizontal	20	−39	29	100	−100	150

[a] Tension is designated +, Compression −.

After the distribution of moments at collapse has been determined, the axial and shear forces can be found for each member. If they reduce M_o, the critical section may be increased in size, and if necessary, the previous analysis can be reviewed. Maximum moments, axial stresses, and maximum shears for each member of the bent in Fig. 3 are summarized in Table 1.

At failure, the right-hand portion of the lower horizontal in Fig. 3 is subjected to moments varying from M_o at the right end to 0.917 M_o at the center. Although the original hypothesis that yield hinges occur at discrete locations has not been strictly violated, it is doubtful if this condition could be approached. Almost half the member is in a plastic condition, and collapse might occur at a lower load than required by the mechanism of Fig. 7.

VINCENZO FRANCIOSI.[30]—The elegant method of calculation of the safety factor introduced in this paper permits the neglect of all that happens between the first appearance of the plastic phenomena and the total collapse, since it considers the structure only in this final phase. It does not consider, however, the effects of the axial force that may, in some cases, play a very important role. Owing to the presence of the axial force, in fact, both the value of the moment corresponding to the beginning of the plastic phenomenon and the value of the limit moment M_o decrease. The point of interest, as was stated in the Introduction, is not to know the relation between the bending moment and the curvature in the elastic-plastic state but the new reduced value M'_o of the limit moment.

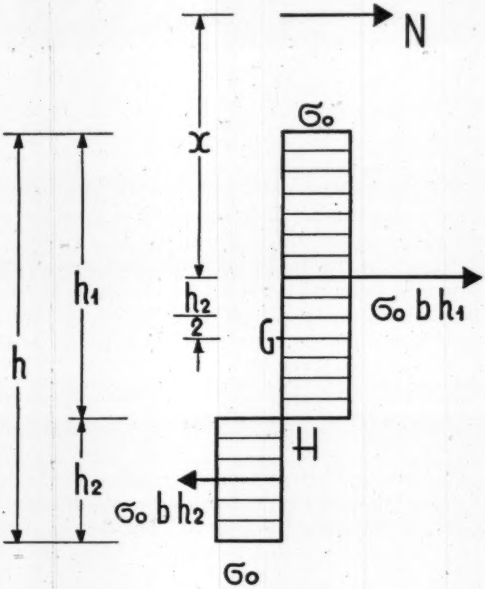

FIG. 23

For the calculation of the reduced limit moment the rectangular cross section that is the most frequently met with in the technical practice is used. The diagram of the stresses originated by an axial force N and by the limit moment reduced to M'_o is shown in Fig. 23. The conditions of equilibrium to a translation along the beam axis give

$$\left.\begin{matrix} h_1 \\ \\ h_2 \end{matrix}\right\} = \frac{h}{2}\,\frac{N_o \pm N}{N_o} \qquad (28)$$

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[30] Prof. of Construction Science, Univ. of Naples, Naples, Italy.

in which $N_o = \sigma_o b h$ (σ_o being the yield stress) equals the normal force that would determine by its sole action the yielding of the section. The condition of equilibrium to rotation gives the value

$$x = \frac{N_o}{N} \frac{h_2}{2} \qquad (29)$$

Thus

$$M'_o = N\left(x + \frac{h_2}{2}\right) = \frac{N_o h}{4} = \left(1 - \frac{N^2}{N^2_o}\right) \qquad (30)$$

Since[31]

$$\frac{N_o h}{4} = \frac{\sigma_o b h^2}{4} = M_o \qquad (31)$$

$$M'_o = M_o \left(1 - \frac{N^2}{N^2_o}\right) \qquad (32)$$

If the forces N are known, from Eq. 32 the value of the reduced limit moments M'_o is drawn, and the calculation of the safety factor goes on by means parallel to those proposed in the paper. This procedure is used, for instance, in the case of a deflected beam loaded with an axial constant force.

Generally, however (as, for instance, in the case of the frames), the axial forces N depend on the loads applied, and in the collapse phase they are linear homogeneous functions of the couples M'_{oi} applied at the yield hinges.

Consider, for instance, the structure shown in Fig. 24. In the line CD the shearing (constant) force τ is given by

$$\tau = \frac{M'_{o5} + M'_{o3} - M'_{o1}}{h} \qquad (33)$$

in which M'_{oi} is the reduced limit moment relative to the ith bar. (Such moment has one value along the whole bar, as the axial force it contains is constant, and the section is presumed constant.) For the equilibrium of the line DE the axial force in the Cols. 4 and 5 (tension forces considered positive) is

$$-N_4 = N_5 = \frac{N'_{o1} - M'_{o3} + 3\,M'_{o5}}{2\,h} \qquad (34)$$

For the equilibrium in the line AB the shearing force is

$$\tau = 2\,\frac{M'_{oi}}{h} \qquad (35)$$

For the equilibrium of the line EG the value of n is

$$n = \frac{M'_{o1} + \frac{2}{3}\,M'_{o2} + M'_{o3} + M'_{o5}}{P\,h} \qquad (36)$$

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[31] "Scienza delle Costruzioni," by A. Galli, Pellerano, Naples, Vol. 2, 1951, p. 216.

Studying finally the beam CF two reactions are obtained:

$$R_c = \frac{n\,P}{2} + \frac{2\,M'_{o3}}{h} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (37a)$$

$$R_f = \frac{n\,P}{2} - \frac{2\,M'_{o3}}{h} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (37b)$$

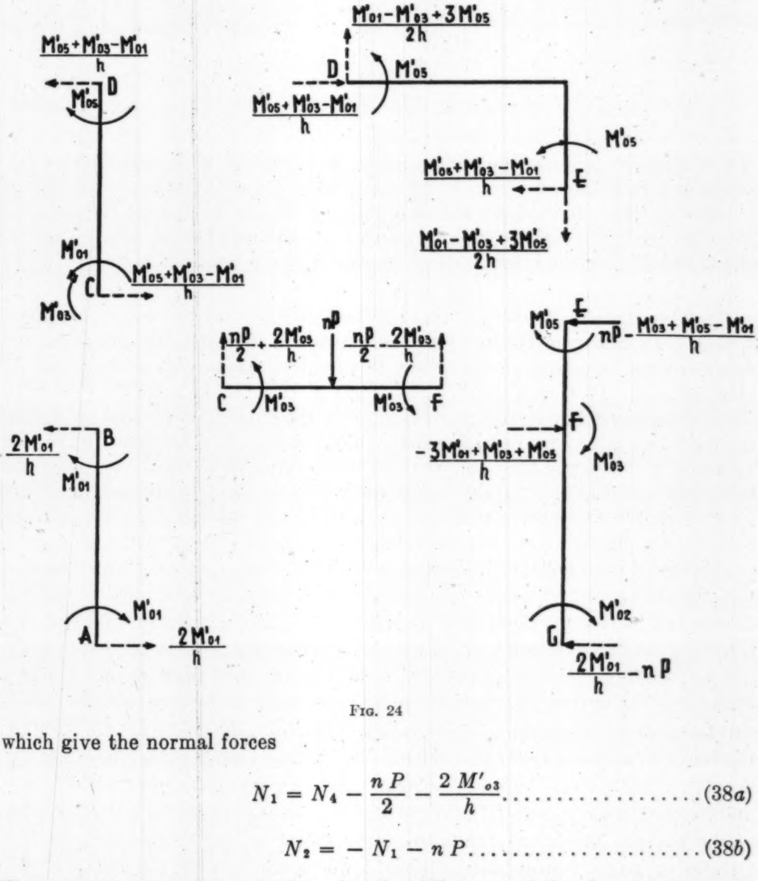
M'05+M'03-M'01 / h
D
M'05
M'01
C
M'05+M'03-M'01 / h
M'03
M'01-M'03+3M'05 / 2h
D
M'05
M'05+M'03-M'01 / h
M'05
M'05+M'03-M'01 / h
E
M'01-M'03+3M'05 / 2h
nP/2 + 2M'03/h
nP
nP/2 - 2M'03/h
C
M'03
M'03
F
E
M'05
nP - M'03+M'05-M'01 / h
F
-3M'01+M'03+M'05 / h
M'03
2M'01 / h
B
M'01
M'01
A
2M'01 / h
M'02
G
2M'01 / h - nP

Fig. 24

which give the normal forces

$$N_1 = N_4 - \frac{n\,P}{2} - \frac{2\,M'_{o3}}{h} \ldots\ldots\ldots\ldots\ldots\ldots (38a)$$

$$N_2 = -\,N_1 - n\,P \ldots\ldots\ldots\ldots\ldots\ldots\ldots (38b)$$

The force N_3, in its turn, is given by the difference of the shearing forces in lines AB and CD

$$N_3 = -\,\frac{2\,M'_{o1}}{h} + \frac{M'_{o5} + M'_{o3} - M'_{o1}}{h} \ldots\ldots\ldots\ldots (39)$$

The value of force N_6 is of no interest as no yield hinge is situated in bar 6. Eqs. 34, 38, and 39 give the values of the normal forces as linear homogeneous functions of the couples M'_{oi}. Therefore, for a generic frame,

$$N_i = \sum_1^n {}_j\, a_{ij}\, M'_{oj}\ (i = 1, 2, \cdots n) \qquad (40)$$

in which n is the number of the bars of the frame on which some yield hinges are situated.

By means of Eq. 32 the system

$$N_i = A_i + \sum_1^n {}_j\, A_{ij}\, N^2_j\ (i = 1, 2, \cdots n) \qquad (41)$$

of n quadratic nonhomogeneous equations in which the rectangular terms are absent is arrived at.

Having thus obtained the values of N_i and, therefore, those of the reduced M'_{oi}, the next step is either to ascertain whether the coefficient n is the safety factor or to define the interval in which said factor is contained.

H. J. Greenberg[32] and W. Prager,[33] M. ASCE.—Mr. Grinter's intuitive approach to the main theorems of the paper will doubtless be welcomed by many readers because it makes these theorems plausible with a minimum of mathematics. It is even possible that some readers would be willing to settle for such a plausibility argument reinforced by the authoritative statement that the theorems can be proved rigorously. Others, however, will insist on seeing an actual proof before they feel justified in applying these theorems. For the following reasons, Mr. Grinter's argument does not constitute such a proof. First of all, the assumption that a frame with $(n-1)$ redundancies will collapse only after n plastic hinges have developed is erroneous because it excludes the possibility of local collapse discussed in section 5 of the paper. The assumption that the choice of a collapse mode leads to statically determinate bending moments likewise overlooks the possibility of local collapse. Next, Mr. Grinter's way of determining the necessary hardening x from the supposedly unique bending moments associated with the assumed collapse mode effectively uses the writers' theorem I which Mr. Grinter is about to prove. Since this theorem is not yet shown to be valid at this stage, the only way of modifying the frame in such a manner that the assumed collapse mode cannot fail to be the actual one is to leave the limit moments unchanged at the chosen yield hinges and increase them indefinitely everywhere else, as is done in the last paragraph of the paper. This corresponds to $x = \infty$; therefore, the correct form of Mr. Grinter's argument furnishes the trivial lower bound zero for the collapse load. Finally, even though the assumed collapse mode becomes the actual one when the limit moments of all cross sections, excepting the chosen yield hinges, are increased indefinitely, the conclusion that the collapse load obtained from this mode is an upper bound for the collapse load of the considered frame is based on the tacit assumption that theorem V is valid. Actually, this theorem is a consequence of theorem I which Mr. Grinter has not yet established at this stage.

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[32] Associate Prof. of Math., Carnegie Inst. of Technology, Pittsburgh, Pa.

[33] Prof. of Applied Mechanics, Brown Univ., Providence R. I.

The writers agree fully with Mr. Grinter's remark that good engineering judgment is needed if the bounds obtained from the two principles are to bracket the collapse load closely. Since even the best engineering judgment may fail in the case of highly indeterminate structures of unconventional design, iterative methods providing a systematic narrowing of the gap between the bounds are needed. In this connection the work of B. G. Neal and Mr. Symonds[34,20,11] and W. Nachbar and J. Heyman[35] must be mentioned. The methods developed by these authors are based on the theorems presented in this paper and go far toward eliminating the need for engineering judgment in the determination of collapse loads and modes.

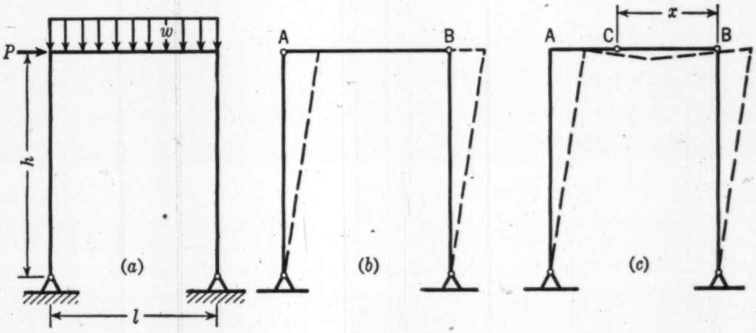

FIG. 25

Mr. Silverman's remarks reveal the fact that the term "limit design," as currently used, is misleading and should be replaced by the term "limit analysis" introduced in 1951 by W. Prager and P. G. Hodge, Jr.[36,37] Mr. Silverman's contribution to the discussion contains an example of true limit design; other examples have been presented by Mr. Heyman.[38]

As stated by Mr. Benjamin, a common safety factor applied to various types of loads is not realistic. The writers were well aware of this fact but did not think it advisable to obscure the exposition of the basic principles by the introduction of the numerous complicating conditions encountered in practice. To illustrate the application of the method to the case of concen-

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[34] "The Calculation of Collapse Loads for Framed Structures," by B. G. Neal and P. S. Symonds, *Journal*, Inst. C. E., London, England, Vol. 351, 1950.

[35] "Approximate Methods in the Limit Design of Structures," by J. Heyman and W. Nachbar, *Proceedings*, First U. S. National Congress for Applied Mechanics, Chicago, Ill. (publication pending).

[36] "Theory of Perfectly Plastic Solids," by W. Prager and P. G. Hodge, Jr., John Wiley & Sons, Inc., New York, N. Y., 1951 Chapter VII, Sec. 33.

[37] *Ibid.*, Chapter VIII, Sec. 39.

[38] "Plastic Design of Beams and Plane Frames for Minimum Material Consumption," by J. Heyman, *Quarterly of Applied Mathematics*, Vol. 8, 1951, p. 373.

trated or distributed loads which vary independently, consider the portal frame with hinged feet shown in Fig. 25(*a*). Applied to the collapse modes shown in Figs. 25(*b*) and 25(*c*), the principle of virtual work yields the relations:

$$P\,h = 2\,M_o \quad\text{.........................}(42)$$

and

$$P\,h - w\,l\,\frac{l - x}{2} = 2\,M_o\,\frac{l}{x} \quad\text{..................}(43)$$

in which the limit moment is assumed to have the same value M_o for all members of the frame. The discussion of these relations is facilitated by the use of Fig. 26 in which $\frac{P\,h}{M_o}$ and $\frac{w\,l^2}{8\,M_o}$ are used as coordinates. Eq. 42 represents the straight line DE. According to Theorem III, the value of P obtained from Eq. 42 is an upper bound of the collapse load. This means that the points to the right of the line DE in Fig. 26 cannot represent safe states of loading. According to the ratio of P and w, the hinge C may develop at any point between the corner A and the center of the span AB (Fig. 25(*c*)). Therefore, Eq. 43 must be discussed for $\frac{l}{2\,x\,l}$. For a fixed value of x between these limits, Eq. 43 represents the straight line which makes the intercepts $\frac{P\,h}{M_o} = \frac{l}{x}$ and $\frac{w\,l^2}{8\,M_o} = \frac{l^2}{4\,x\,(l - x)}$ on the coordinate axes in Fig. 26. The envelope EF of all these lines corresponding to various values of x between $l/2$ and l is readily sketched when a few lines corresponding to selected values of x have been drawn. Since all collapse modes that may occur under the considered type of loading are of the kinds shown in Figs. 25(*b*) and 25(*c*), or combinations of them, the preceding discussion is exhaustive. The safe states of loading, therefore, are represented by the points of the shaded area ODEF in Fig. 26.

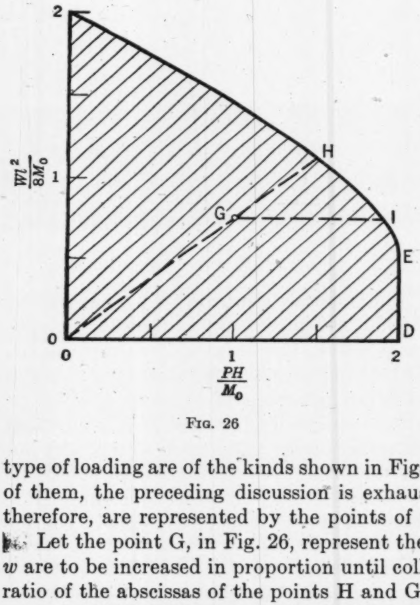

FIG. 26

Let the point G, in Fig. 26, represent the given state of loading . If P and w are to be increased in proportion until collapse occurs the safety factor is the ratio of the abscissas of the points H and G, Fig. 26. On the other hand, if w represents a dead load and only P is to be increased until collapse occurs, the safety factor is the ratio of the abscissas of the points I and G.

The difficulties created by the presence of axial forces have been stressed by several discussers. As Mr. Popov states, it is assumed that all compression members possess a sufficiently low slenderness ratio so that instability does not occur. To those feeling that this assumption deprives the limit analysis of its value, it must be emphasized that the customary elastic analysis is based on precisely the same assumption. Indeed, the deformations of the members are disregarded when the equilibrium conditions are set up. This means that the possibility of buckling is arbitrarily excluded in the determination of the redundant forces or couples. Only after the axial forces have been found in this manner is the possibility of buckling investigated. The same procedure can be followed in the case of limit analysis. In both cases, the results of the preceding analysis can be accepted only if the subsequent check shows that there is no danger of instability. Otherwise, the cross section of the critical member must be increased, and the first analysis must be repeated regardless of whether it was of the elastic or limit type.

Aside from raising the stability question, axial forces are important because they lower the limit moments. Again, the writers were perfectly aware of this fact but did not wish to complicate the exposition of their basic principles by the inclusion of secondary considerations.

When Mr. Franciosi's M_o and N_o are identified with Mr. Benjamin's 1.5 $S\,\sigma_y$ and $A\sigma_y$, respectively, Eq. 32 is seen to represent the parabola in Fig. 15. The straight line in Fig. 15 can be obtained in a similar manner for an I-beam section if the flanges are considered as indefinitely thin and the contribution of the web is neglected. Mr. Franciosi shows how this type of information can be incorporated in the statical analysis based on theorem I. An alternative kinematic analysis (not published) based on theorem III has been developed by E. T. Onat at Brown University, Providence, R. I. In this analysis the effect of axial forces are taken into account by replacing the ordinary yield hinges of limit analysis by modified hinges which allow not only a relative rotation but also an approach or separation of adjacent cross sections.

Mr. Hrennikoff's remarks are concerned with one aspect of an important limitation of the theory. Limit analysis is based on the assumption that the beam or frame under consideration can fail only when sufficient yield hinges have developed to transform it into a mechanism. As R. Hill[39] has shown, this assumption is completely justified only in the case of a rigid-plastic beam that cannot bend until the bending monent has reached the value M_o and may then bend indefinitely under the constant bending moment M_o. Of course, this concept of a rigid-plastic beam is not realistic. On the other hand, a beam with the more realistic moment-curvature diagram of Fig. 1 may cease to be serviceable before the collapse load furnished by limit analysis is reached. This may be due to excessive local strain at a yield hinge, as Mr. Hrennikoff states, or to the occurrence of large deflections made possible by the cooperation of several yield hinges, none of which bends excessively. The development of a general, simple method for estimating the deformation at the instant of impending collapse would constitute, therefore, a most significant contribution to the theory of limit analysis.

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[39] "On the State of Stress in a Plastic-Rigid Body at the Yield Point," by R. Hill, *Philosophical Magazine*, Series 7, Vol. 42, 1951, p. 868.

For the sake of brevity, the writers assumed the same absolute value M_o for the limit moments for positive and negative curvature. Actually, the methods presented in the paper are readily extended to the case discussed by Mr. Tachau in which the positive limit moment differs in absolute value from the negative limit moment. In the statical analysis based on theorem I, the sign of the bending moment at each yield hinge must then be taken into account in determining the limit moment for this hinge. In the kinematic analysis based on theorem III, the sense of the relative rotation at each yield hinge must be considered in determining the limit moment at this hinge and its work on the relative rotation.

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Paper No. 2502

EXPERIMENTAL STUDY OF WATER FLOW IN ANNULAR PIPES

By W. M. Owen[1]

Synopsis

Head loss and velocity distribution measurements were made on three annular pipes formed by centrally supporting pipe cores in outer pipes. Knowledge of flow conditions in pipes of this type is important in many engineering applications. The fluid used in the tests was water, and the range of Reynolds numbers tested was from 4,000 to 700,000. A description is given of the apparatus used and the precautions taken to assure conformity to the test conditions.

The test results are analyzed and compared with previous research in this field, and conclusions are drawn regarding flow conditions to be expected in actual applications.

Introduction

Knowledge of the head loss and velocity distribution for the turbulent flow of fluids in the annular space between two coaxial cylindrical pipes is important in various engineering problems, especially in those encountered in the oil-producing industries in which pipes of this type are in common use. In this paper such pipes will be referred to as annular pipes; the inner pipe will be called the core, and the outer pipe alone, without the core, will be referred to as a circular pipe. The investigation that is reported herein was conducted to reconcile the conflicting results reported in various investigations listed in the bibliography and to obtain reliable results, especially with respect to the head loss and velocity distribution for turbulent flow in annular pipes of substantial sizes and high rates of flow.

Notation.—The letter symbols used in this paper are defined where they first occur and are assembled in Appendix I for convenience of reference.

Note.—Published in September, 1951, as *Proceedings-Separate No. 88.* Positions and titles given are those in effect when the paper was received for publication.

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TEST CONDITIONS

Range of Tests.—One brass and two steel annular pipes were used in the investigation. The brass annular pipe consisted of an outer pipe having a nominal inner diameter (d_1) of 2 in. and an inner pipe or core having a nominal outer diameter (d_2) of $\frac{1}{2}$ in. (ratio $d_1/d_2 = 2.47$). One steel annular pipe used in the test consisted of an outer pipe having a nominal diameter of 6 in. and a nominal core diameter of 2 in. ($d_1/d_2 = 2.55$). The other steel annular pipe consisted of the same 6-in. outer pipe but had a core with an outside nominal diameter of 3 in. ($d_1/d_2 = 1.76$). Head loss and velocity determinations were made while water flowed through the pipes at Reynolds numbers varying from 4,000 to 700,000. The water entered the annular pipes from a standpipe that maintained a constant head of approximately 60 ft. Head loss measurements were also made on the outer pipes without the core so that a comparison of the head loss for annular and circular pipes could be made.

The characteristics of laminar flow in annular pipes were not investigated, mainly because of the difficulty in measuring the small values of head loss and velocity. Furthermore, theoretical expressions for the velocity and the rate of discharge for laminar flow in annular pipes have been given by H. Lamb (8)[2] and confirmed experimentally by F. C. Lea and A. Tadros (9) for annular pipes with values of the ratio d_1/d_2 of approximately the same magnitude (1.75–2.5) as those of the test pipes used in this investigation. This range includes the values of the ratio d_1/d_2 of annular pipes that are in general use. However, it was found by Messrs. Lea and Tadros that, for considerably higher values of the ratio d_1/d_2, the experimental results for laminar flow did not agree satisfactorily with those obtained from the theoretical expressions. For example, it was found that for $d_1/d_2 = 23.0$ the theoretical values of the quantity flowing were 26% less than the experimental values. This difference in results has been attributed to turbulence adjacent to the surface of the core.

DESCRIPTION OF APPARATUS

Six-Inch Steel Pipe.—A diagrammatic sketch representing the apparatus used to test the 6-in. annular pipe is shown in Fig. 1(*a*). The outer circular pipe consisted of five lengths or sections of 6-in. steel pipe, each approximately 20 ft in length. The actual inside diameter of the pipe was 6.06 in. Piezometer rings were placed between the pipe flanges at places where two sections of pipe were joined, and the piezometer rings were connected to a 4-column manometer by means of which the head loss for any one or for all of the 20-ft sections of pipe could be measured. The gage fluids used were mercury (specific gravity 13.56), carbon tetrachloride (specific gravity 1.595), and a mixture of carbon tetrachloride and gasoline (specific gravity 1.23). The gage fluid in the manometer was changed to a lighter fluid whenever the difference in levels of the gage fluid columns was so small as to cause appreciable errors in reading the manometer. Each of two galvanized steel cores (2.375-in. and 3.499-in. outisde diameter) was placed inside the 6-in. pipe and supported and centered by means of metal spiders that also served as connections for the core sections. The spiders were located downstream from the region of pressure or pitot tube connections to minimize disturbances that might affect the measured values of pressures or velocities. Both cores consisted of sections 10 ft long and both started at the upstream end of the first section of 6-in. pipe and extended to within 10 ft of the end of the last section of the pipe.

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[2] Numbers in parentheses, thus: (8), refer to corresponding items in the Bibliography (see Appendix II)

The discharge was controlled by a gate valve located at the downstream end of the last section of pipe, and discharge was measured by means of a calibrated 90° V-notched weir. The first section of pipe was connected to the laboratory standpipe as shown in Fig. 1. Straightening vanes were placed in the elbow and reducer, and the straightening section consisted of a nest of tubes. These measures were designed to eliminate spiral motion of the water and produce a uniform velocity distribution in the test pipe. A constant head of approximately 60 ft was maintained on the test pipes by means of a skimming weir located at the top of the standpipe.

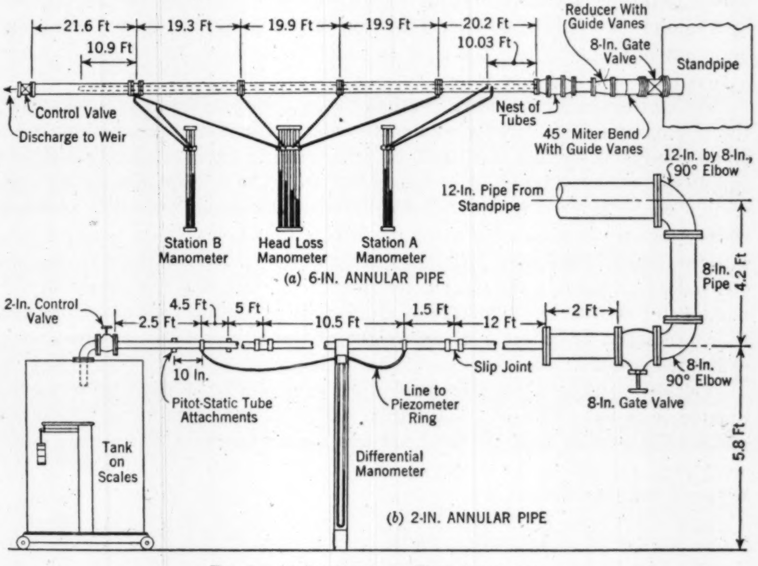
21.6 Ft
19.3 Ft
19.9 Ft
19.9 Ft
20.2 Ft
10.9 Ft
10.03 Ft
Reducer With Guide Vanes
8-In. Gate Valve
Standpipe
Control Valve
Discharge to Weir
Nest of Tubes
45° Miter Bend With Guide Vanes
12-In. by 8-In., 90° Elbow
12-In. Pipe From Standpipe
Station B Manometer
Head Loss Manometer
Station A Manometer
(a) 6-IN. ANNULAR PIPE
8-In. Pipe
4.2 Ft
2-In. Control Valve
4.5 Ft
5 Ft
2.5 Ft
10.5 Ft
1.5 Ft
12 Ft
2 Ft
10 In.
Slip Joint
Pitot-Static Tube Attachments
Line to Piezometer Ring
8-In. 90° Elbow
8-In. Gate Valve
Tank on Scales
Differential Manometer
5.8 Ft
(b) 2-IN. ANNULAR PIPE

FIG. 1.—ARRANGEMENT OF TEST APPARATUS

Pitot tube stations for making velocity traverses were located in the first and last sections of pipe. Provisions were made to take velocity traverses in the radial direction at diametrically opposite points at both pitot tube stations and the pitot tube traverses at the second station were made at 90° to those at the first station. Static pressure connections at the pitot tube stations were made by drilling several holes in the 6-in. pipe, connecting the holes to a piezometer ring, and connecting the piezometer ring to one side of the pitot tube differential manometer. The pitot tubes were kept in alinement with the axis of the pipe by rotating the tube so that a vane mounted on the tube outside the pipe was parallel to the pipe.

Two-Inch Brass Pipe.—A diagrammatic sketch representing the 2-in. annular pipe apparatus is shown in Fig. 1(*b*). The core is not shown in this sketch. The outer pipe consisted of smooth brass pipe composed of three sections, each 12 ft long with an actual inside diameter of 2.075 in. The couplings connecting the three sections of pipe were of the sleeve type that permitted the ends of the pipes to butt against one another and held the pipes in alinement. The core consisted of 6-ft sections of ½-in. diameter (0.840-in. actual outside diameter) smooth brass pipe. The core was supported and centered within the pipe by means of spiders. The core was fastened to the spiders by pins which fixed the ends of the core and tended to reduce the deflection of the core to a minimum. The maximum deflection of the core which occurred at the midpoint of the length of the core was calculated to be 3% of the distance between the core and pipe wall.

The core extended the entire length of the pipe and was designed to eliminate the placing of a spider near a manometer ring. The test apparatus consisted of an approach section of 13.5 ft of pipe, leading from an 8-in. standpipe supply line, followed by a test section of 20.0 ft. The pressure drop in the test section was measured by means of a differential manometer. The pressure connections to the pipe were made by drilling four holes, 0.1 in. in diameter, at right angles in the pipe. All burrs were carefully removed from the piezometer holes that led into the piezometer ring from which the connection to the manometer was made. The rate of discharge was obtained by measuring the time necessary to collect a given weight of water in a tank mounted on a calibrated scale.

Velocity measurements were made by means of a pitot-static tube of ⅛-in. outside diameter. The pitot-static tube was connected to the same differential manometer as that used for the head loss measurements. Three gage fluids were used so that the manometer could be read accurately throughout the range of test conditions.

Interpretation of Test Results

Head Loss.—A dimensional analysis of the flow in an annular pipe reveals that the friction factor is a function of the dimensionless ratios Vd_1/ν, d_1/d_2, ϵ_1/ϵ_2, and ϵ_2/d_1. For each pipe used in this investigation the variation of the friction factor with the dimensionless ratio Vd_1/ν can be determined from the Stanton diagram, Fig. 2. The ratio Vd_1/ν for any annular pipe is equal to a constant times the Reynolds number $(4VR/\nu)$ shown as abscissas in Fig. 2. If the diameter of the core in an annular pipe is increased and all other factors remain constant, the values of the last two of the foregoing ratios remain constant, but the values of the ratios d_1/d_2 and Vd_1/ν change. From the dimensional analysis result shown above, the Stanton diagram for the flow in annular pipe should consist of several distinct families of curves. Each family of curves would be expected to represent a different hydraulic roughness and each curve in the family, a different value of the ratio d_1/d_2.

Fig. 2 shows this expectation to be true for the pipes used in the investigation and also for the pipes tested by F. R. Lorenz (11). The dependence of the friction factor on the ratio d_1/d_2 indicates that average curves showing the relation between the friction factor and Reynolds number (such as those given by D. H. Atherton (1) and A. P. Kratz, H. J. MacIntire, and R. E. Gould (7)) are unreliable. The curves showing the relation between friction factor and Reynolds number for annular pipe given by Messrs. Kratz, MacIntire, and Gould fall below the curve for the circular pipe without a core and are not in order of the values of the ratio d_1/d_2. This was probably caused by the short initial section used in the test pipes. In general, it can be said that the curves showing the relation between the friction factor and Reynolds number lie above and approximately parallel to the curve for circular pipe of the same roughness, and their range is in the order of the values of the ratio d_1/d_2. The curve of J. Caldwell's data is reproduced in Fig. 2 to show the gradual transition from laminar to turbulent flow noted by several experimenters and also to show another experimentally confirmed fact that the transition from laminar to turbulent flow for annular pipe occurs at the same value of Reynolds number as for circular pipe when the Reynolds number is based on the mean hydraulic radius. It should be pointed out that the calculations of the friction factors in Fig. 2 are based on the mean hydraulic radius and, therefore, are only $\frac{1}{4}$ the value given when the diameter is used.

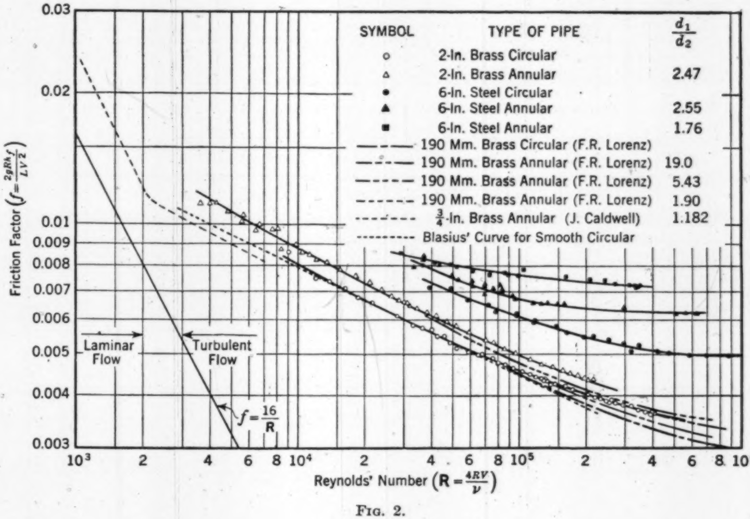

Fig. 2.

Velocity Distributions in Annular Pipes.—The results of the velocity traverses in annular pipes are shown in Figs. 3 through 6. Fig. 3 shows typical velocity distribution curves for various velocities in the three annular pipes used in this investigation. It should be noted that the velocity distribution curves for corresponding velocities in the three annular pipes are similar in form and are quite flat for the lower velocities, becoming more pointed for increasing velocity. The position of the point of maximum velocity remains constant within the limits of experimental error, for a given annular pipe. From the velocity distribution curves it is also noted that the point of maximum velocity is nearer to the core than to the pipe wall and that the velocity next to the core is always greater than that next to the pipe wall. This is in disagreement with the results of V. Mikrjukov (12) but agrees with the results of Mr. Lorenz (11).

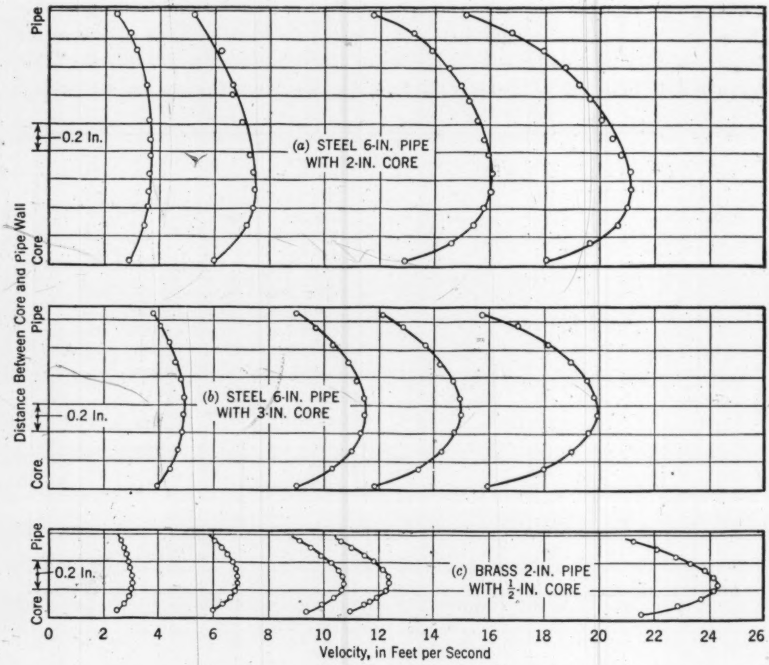

FIG. 3.—VELOCITY TRAVERSES IN ANNULAR PIPE

Fig. 4 is a graph showing the relation between the dimensionless ratios $v/V_{\max}$ and $y/r_1 - r_2$ for the 2-in. brass annular pipe for all the velocity traverses. No curves are drawn through the points shown, for the reason that the individual curves would not be distinguishable since the points for the various velocities fall in a very narrow band. However, the points adjacent to the core lie in the same order as the velocities, with one exception, thus indicating a dependence of the velocity distribution curve on the Reynolds number. This fact is in contradiction to a conclusion made by Mr. Lorenz in which he states that, in a dimensionless velocity distribution curve, such as the one shown in Fig. 4, the dependence of the velocity distribution curve on the Reynolds number vanishes. The dependence of the velocity distribution curve on Reynolds number was also noted in the two 6-in. steel annular pipes used in the investigation reported herein. The fact that the points in Fig. 4 converge in the region of the maximum velocity is an indication that the point of maximum velocity for geometrically similar annular pipes remains constant.

Fig. 5 shows dimensionless velocity distribution curves obtained by plotting the ratios $v/V_{\max}$ and $y/r_1 - r_2$ for three annular pipes used in this investigation. The curves shown in Fig. 5 are velocity distribution curves obtained by averaging the values of $v/V_{\max}$ for the various velocities in each annular pipe. The effect of the roughness on the shape of the velocity distribution curve is clearly shown by examining the velocity distribution curves for the 2-in. brass pipe with a $\frac{1}{2}$-in. brass core and the 6-in. steel pipe with the 2-in. steel core.

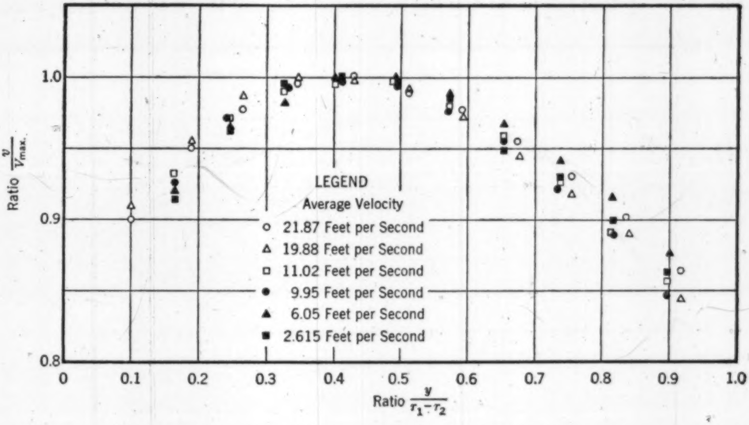

FIG. 4.—DIMENSIONLESS VELOCITY DISTRIBUTION CURVES

These annular pipes have approximately the same d_1/d_2 ratios, but they vary considerably in roughness. The velocity distribution curve for the steel annular pipe has a much more elongated form than that for the brass annular pipe. This fact is also found to be true for the flow of fluids in circular pipe. The explanation for this fact is that the greater shearing stress in the fluid in the rougher pipe requires a greater velocity gradient.

The effect of varying the ratio d_1/d_2 on the velocity distribution curves can be seen by examining the curves for the two steel annular pipes in Fig. 5. It should be realized that, although these pipes were made of the same material, their hydraulic roughnesses were different and therefore the effect of roughness is not entirely eliminated. The two curves practically coincide at all points from the core up to and including the point of maximum velocity. From the point of maximum velocity to the outer wall the curves diverge progressively, the pipe with the 3-in. core lying uppermost. The reason that these two curves were quite similar is probably a result of the fact that the values of the ratio d_1/d_2 did not vary appreciably in this investigation. Mr. Lorenz found a considerable change in the velocity distribution curve over a range of the ratio d_1/d_2 of 1.9 to 19.0. His results show a logarithmic dimensionless graph giving the relations between the ratios—

$$\frac{\text{distance from core to point of maximum velocity}}{\text{distance from pipe wall to point of maximum velocity}} \quad \text{and} \quad \frac{d_1}{d_2}$$

—for three brass annular pipes. Mr. Lorenz found that the data for his three annular pipes fell approximately along a straight line. The point representing the data for the brass annular pipe used in this investigation did not fall along this line, however. According to Mr. Lorenz, as the ratio d_1/d_2 approaches unity, the point of maximum velocity approaches the midpoint of the annular space. The fact that no shift of the point of maximum velocity was noted in this investigation might be due to the narrow range of the values of d_1/d_2 used.

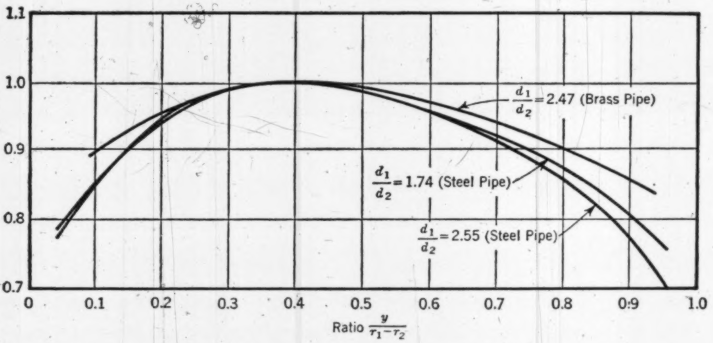

FIG. 5.—AVERAGE DIMENSIONLESS VELOCITY DISTRIBUTION CURVES

In order to find a mathematical expression for the velocity distribution curve the velocity data were plotted to logarithmic scales. This was done to determine whether or not the velocity distribution curve could be expressed as a simple exponential function, as is done in the case of pipes of circular cross section. Fig. 6 is a graph plotted to logarithmic scales showing the relation between the velocity v and distance from either the core or the wall for the three annular pipes used in this experiment. The open circles represent velocity distribution data for the region from the core to the point of maximum velocity, and the closed circles represent the data for the region from the pipe wall to the point of maximum velocity. The fact that these data for each velocity fall on two straight lines indicates that the equation for the velocity distribution curve is made up of two parts, one on each side of the maximum velocity and each having the form,

$$v = a\,y^n$$

in which a is a constant and the other symbols are as previously defined. The value of n for the core is always equal to or less than that for the pipe wall and decreases with an increase in Reynolds number, with a few exceptions. This substantiates the statement made earlier that the velocity distribution curve appears to be a function of Reynolds number.

The difference in the slopes of the steel and brass annular pipes is caused mainly by the difference in roughnesses of the surfaces of the pipes. The value of n for the two steel annular pipes was found to decrease as the ratio d_1/d_2 decreases. This statement is in agreement with the conclusions given by Mr. Lorenz.

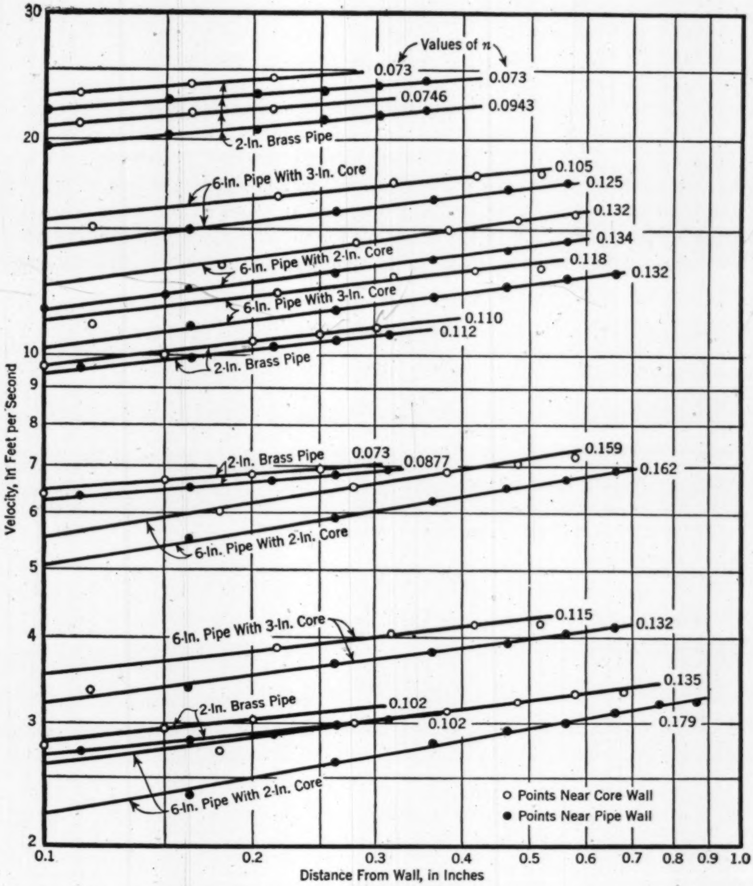

FIG. 6.—VELOCITY DISTRIBUTION CURVES NEAR CORE AND PIPE WALLS

CONCLUSIONS

From the results of this investigation the following conclusions can be drawn concerning the turbulent flow of water in the annular space between two coaxial cylindrical pipes for a range of values of d_1/d_2 from 1.76 to 2.55 and for a range of Reynolds numbers from 4,000 to 700,000.

Head Loss.—(1) The friction factor f for annular pipe is a function of Reynolds number, the roughness of the pipe, and the ratio d_1/d_2; (2) the friction factor at a given Reynolds number is greater for an annular pipe than for the outer circular pipe without the core; and (3) the friction factor, at a given Reynolds number, increases as the diameter of the core placed in the outer circular pipe increases.

Velocity Distributions.—(*a*) The point of maximum velocity of flow in an annular pipe lies approximately 0.4 of the distance between the core wall and pipe wall; (*b*) the velocity next to the core is always greater than that near the pipe wall; (*c*) the velocity distribution curve in an annular pipe is a function of the roughness of the surfaces; (*d*) the velocity distribution curve for a given annular pipe appears to be a function of Reynolds number; (*e*) the exponent n in the equation for the velocity distribution curve, $V = a\, y^n$, is less at the core wall than at the pipe wall; and (*f*) for annular pipes of the same hydraulic roughness the value of n in the equation for the velocity distribution curve is less for a smaller value of ratio d_1/d_2, therefore the velocity distribution is a function of the ratio d_1/d_2.

ACKNOWLEDGMENT

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APPENDIX I.—NOTATION

The following letter symbols have been adopted for use in this paper and in the discussion:

d_1 = inside diameter of outer pipe;

d_2 = outside diameter of inner pipe;

r_1 = inside radius of outer pipe;

r_2 = outside radius of inner pipe;

v = velocity at a given point;

V = average velocity of fluid;

$V_{\max}$ = maximum velocity of fluid;

g = acceleration of gravity;

f = friction factor in Darcy's equation, $h_f = f \frac{L}{R} \frac{V^2}{2g}$;

$\bar{R}$ = Reynolds number, $\bar{R} = \frac{4R\,V}{\nu}$;

h_f = head loss due to friction;

L = length of pipe test section;

A = cross-sectional area of annular section;
P = wetted perimeter;
R = mean hydraulic radius, $\frac{A}{P}$;
y = distance from core wall;
n = an exponent, the slope of the velocity distribution curve in Fig. 6;
ϵ_1 = measure of roughness of outer pipe;
ϵ_2 = measure of roughness of inner pipe;
μ = coefficient of viscosity; and
ν = kinematic coefficient of viscosity.

APPENDIX II.—BIBLIOGRAPHY ON FLOW IN ANNULAR PIPES

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Paper No. 2503

NATIONAL TOPOGRAPHIC MAPPING

BY W. E. WRATHER[1]

WITH DISCUSSION BY MESSRS. MILTON O. SCHMIDT, D. P. KRYNINE, AND W. E. WRATHER

SYNOPSIS

The present status of topographic mapping of the United States and its possessions leaves much to be desired. Many of the maps are inadequate, and most are in danger of becoming obsolete. Changing standards and requirements have increased the rate of obsolescence.

The United States Geological Survey (USGS) has cooperated successfully with other federal and state agencies to produce maps that were valuable in the planning and supervision of many projects. Mapping standards of the Geological Survey are discussed.

An expanded, comprehensive mapping program is proposed to expedite future work in this field and to eliminate unnecessary duplication of work.

INTRODUCTION

The United States has long been recognized as a land of unlimited opportunity. Settlers from all points of the compass and from every continent have flocked to its shores since Columbus and his contemporaries first discovered this hemisphere. The early pioneers found a land of abundant natural resources. With the lavish abundance of nature on every hand, there seemed no need for conservation, so they indulged in many wasteful practices which decimated the nation's wildlife and destroyed large areas of forest. These conditions prevailed when the United States Department of the Interior was created as the "Home Department" in 1849. One of its primary functions was to establish practical conservation for the proper use of national resources.

THE BASIS FOR PRACTICAL PLANNING

Its natural resources have provided the means for the development of the nation's industrial might and have made possible the highest standard of living in the world. Many of these resources and raw materials, which formerly

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[1] Director, U. S. Geological Survey, Washington, D. C.

seemed plentiful, cannot now fully supply national requirements, and virtually no resources are overabundant in relation to anticipated needs. The United States is still a land of opportunity, but to maintain its progress and position of leadership requires all possible tools and facilities, coupled with vigorous development and comprehensive conservation policies.

One of the primary national requirements is a sound knowledge of the terrain in every part of the country. Good topographic maps which provide such information are versatile, fundamental tools that can be of service in solving a variety of urgent problems. In the past these maps have not always been available when needed, and plans for future development should be made to provide adequate, accurate mapping coverage.

After the Civil War, the need for reconnaissance and exploratory maps of the West was partly met by the work of the Wheeler, King, Hayden, and Powell surveys.[2, 3] These expeditions produced small maps of large areas (small-scale maps) which provided general rudimentary geographic knowledge for the location of transportation routes, and otherwise met the needs of a pioneer population. However, it was not until the USGS was organized in 1879 "to classify the public lands and to examine the geologic structure" that topographic mapping was undertaken as a continuing national program.

Organized Topographic Mapping

Small-scale reconnaissance mapping was continued by the Geological Survey until about 1900 when one third of the United States had been charted. As the development of the country progressed, the trend in map requirements was toward larger scales, more detail, and greater accuracy, which led to the adoption, at about 1900, of the standard scale of 1:62,500 (approximately one inch to a mile). The quality of the maps constantly improved as better instruments were designed and new methods developed.

During the 1930's it became apparent that higher standards of accuracy were needed to insure that the national series of quadrangle maps would be suitable for all major uses, and also would be of such quality that they could be conveniently revised at any time in future years. Such standards were adopted, and as a result, modern maps produced by the Geological Survey should be adequate for an unlimited period, except for occasional revision required by changes in man-made, or so-called "cultural," features.

Considerable credit must go to the various states for much of the mapping that has been done. Local cooperation has been encouraged, and several states (especially in the East) have underwritten extensive cooperative mapping programs since this policy was established in 1885. In 1930, Connecticut, Delaware, Maryland, New Jersey, New York, Ohio, Rhode Island, West Virginia, the District of Columbia, and the Hawaiian Islands were completely represented by maps which were considered adequate at that time. Unfortunately, maps, like automobiles, do not improve with age but suffer obsolescence rapidly.

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[2] "Contributions to the History of American Geology," by George P. Merrill, U. S. Govt. Printing Office, Washington, D. C., 1906.

[3] "Service Monographs of the United States Government, No. 1, The U. S. Geological Survey," by the Institute for Government Research, D. Appleton & Co., New York, N. Y., 1918.

By 1950, topographic map coverage was complete on approximately one half of the total area of the United States. However, because of changing requirements and higher standards of accuracy, much of this mapping was of questionable value. Actually, only Massachusetts and Rhode Island were then completely covered with adequate maps, and only one fourth of the entire United States could be considered adequately charted.

Several attempts were made to inaugurate a long-range federal mapping program during the years that followed World War I. The Temple Act, which was passed by Congress in 1925, provided an initial appropriation of approximately $1,000,000 to be increased yearly until the United States was completely mapped. Although the program was thus approved in principle, funds were not appropriated to implement it. Federal funds directly appropriated for topographic mapping for 1935 were less than $200,000, or about the sum provided in 1885.

The failure to implement the Temple Act resulted in a serious lack of maps required for planning national defense against invasion when World War II started. It was necessary at that time, therefore, to produce maps for military use as rapidly as possible by utilizing all available facilities at a time when these facilities were also needed in the mapping of foreign areas closer to the immediate scenes of military action. Since trained technical personnel were not available in sufficient numbers and since really modern equipment was not available in quantity, many of the maps prepared as part of this emergency program are substandard. It has been necessary to remap some of these areas, and many more must eventually be remapped. It is inconceivable that a recurrence of this deplorable situation, with its inevitable waste of public funds, will ever be permitted. Intelligent planning on a national scale, for both civilian and military needs, requires good topographic maps, and the demonstrated need demands a long-range program to accomplish this objective.

Planned development under way at the close of 1950 or proposed by federal and state governments promises to result in construction programs of a magnitude heretofore unknown in the history of the nation. Proposed federal expenditures during the next several decades for water resource projects alone are estimated to cost more than $52,000,000,000. E. R. Needles,[4] M. ASCE, has stated that $60,000,000,000 and fifteen to twenty years would be needed to extend and improve the highway system for handling present and anticipated traffic. The improvement of airports and related facilities is another construction program that will also require billions of dollars before 1960. With current and proposed development programs of such unprecedented magnitude in prospect, it would seem that one of the first requisites for comprehensive planning—complete topographic information—should be available. Unfortunately, adequate maps of the required standards are not now available. This is a serious situation because it cannot be alleviated over night.

It is axiomatic that carefully prepared plans are a prerequisite to the success of any major project. Planning is greatly handicapped when basic information is lacking. In fact, the ultimate success or failure of a project can depend on the availability of adequate fundamental data. Accurate topographic maps provide essential basic data in many engineering activities and should be available for preliminary as well as for detailed planning.

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[4] "Enter the Road Builders," by E. R. Needles, *Fortune Magazine*, November, 1949, p. 109.

VALUE OF MAPS IN DEVELOPMENT PROJECTS

When the Tennessee Valley Authority (TVA) was first established in 1933, Arthur E. Morgan, M. ASCE, was appointed chairman and chief engineer. Mr. Morgan immediately recognized the need for complete map coverage for a basin-wide program of land and water development. An analysis of the status of mapping for the 40,000 sq miles of the Tennessee River Valley revealed, in general, that no adequate maps of the basin existed, although reconnaissance maps were available for some areas. The Corps of Engineers, United States Department of the Army, had prepared strip topography for many sections of the main river and for some of the tributaries.

Under Mr. Morgan's direction, studies were made of several mapping proposals and a program for complete topographic mapping in cooperation with the USGS was decided upon. Although mapping operations were somewhat curtailed during World War II, by 1950 about 85% of the valley had been covered by topographic maps published on the scale of 1:24,000.

These topographic quadrangles have been used constantly by the various divisions of the TVA engaged in numerous activities within the basin. Discussing the value of topographic maps before the American Congress on Surveying and Mapping, Reed A. Elliot, Chief of the Project Planning Branch of TVA, said

> "It would be practically impossible to exaggerate the value of high-grade topographic maps for the preliminary planning of water control projects such as those of the TVA system. Most of the considerations that contribute toward the final selection of the dam sites and the degree of development are, of course, directly or indirectly dependent upon the available maps."

He also said that in the case of highway relocations, tremendous savings were effected by the use of these maps. Locations were plotted on the maps from which it was usually possible to select the final route in the office.

Other important related uses of topographic maps by the TVA include malaria control, forest resource investigations, watershed protection planning, recreational development, general management of reservoir areas, and the selection of industrial sites to suit specific requirements. In addition to these uses, the topographic maps are available for the operation of existing projects and the planning of all future TVA development.

Although a sizable topographic mapping program is contemplated for the Missouri River Basin, topographic map coverage is sparse for the Pacific Northwest and for many other areas of great potential development. Adequate programs of topographic mapping should be initiated throughout the West to enable the sound and economic planning of any contemplated development projects. Undoubtedly topographic maps will be made at one time or another for the entire United States. The value of a planned mapping program is in having accurate terrain information available when needed to expedite contemplated developments.

PROGRESS IN THE MAPPING PROGRAM

The extensive mapping operations of the Geological Survey cover a broad territory throughout the forty-eight states, Alaska, Hawaii, and Puerto Rico. Map preparation is a slow and painstaking process, and the annual production is limited to approximately 500 quadrangle sheets, or to an equivalent ground area of 50,000 sq miles. It would require fifty years to complete the adequate mapping of the entire United States at this rate.

Government agencies and private individuals have condemned this slow rate of progress. A coordinated assembly and careful analysis of basic data on topography, geology, water, and soils prior to authorization of large federal projects have been strongly recommended. Given the necessary personnel and equipment, an accelerated program of topographic mapping would make coverage of the unmapped and inadequately mapped areas of the United States possible by 1970.

This proposal would involve a considerable expansion of skilled personnel and the manufacture of additional equipment for the application of photogrammetric and other modern techniques.

Mapping Standards.—The Geological Survey has adopted two publication scales for the standard topographic maps of the United States. The larger scale of 1:24,000 is used for most urban areas and sections of major economic importance, whereas areas of lesser importance are published on the scale of 1:62,500. The demands of map users for the publication of maps on the former scale have been increasing during the past several decades to meet their requirements for more detail and smaller contour intervals. The use of this larger scale results in higher mapping costs per acre but produces maps of greater utility. Under the proposed expanded program, about half of the unmapped area of the continental United States would be covered by maps published on the 1:24,000 scale. The remaining half of the unmapped area and about a third of Alaska would be covered for map publication on the scale of 1:62,500, and the remaining two thirds of Alaska on the 1:250,000 scale.

The mapping schedule would be based on priorities determined by an appraisal of the map requirements of federal and state agencies and other map users. Concurrent with the mapping of new areas, maps previously completed would be revised periodically to overcome obsolescence. After the completion of the twenty year program, maps would be revised as necessary to maintain their full usefulness.

The estimated cost of this topographic mapping averages twenty-five cents per acre. With about three fourths of the United States to be mapped, the estimated cost of a twenty-year mapping program would be between four hundred and five hundred million dollars. This is only a fraction of 1% of the total estimated cost of anticipated federal and state construction and development programs.

Valuable By-Products

Many incidental benefits would be realized through full activation of an expanded topographic mapping program. Aerial photographs would be available in broad areas a year or two before completion of the maps, and those photographs would often serve the immediate critical needs of many agencies and individuals. Copies of the original large-scale manuscript map could be supplied upon request many months in advance of publication. Other by-products of topographic mapping are the geodetic control and general geographic knowledge garnered during field surveys and editorial processing.

One of the greatest needs, commonly recognized by government and public alike, is for an authoritative center where information may be obtained on the availability of maps, aerial photographs, geodetic control and related data, and on the status of current and planned mapping operations. The Map Information Office of the Geological Survey has been expanded to provide this much-needed clearing house. This facility is available to the general public as well as to federal and state agencies.

Conclusion

The USGS, which is the principal mapping agency of the federal government, has received strong endorsement for the expansion of the national mapping program from the Conference of State Governors, the Inter-Agency River Basin Committee, the Bureau of Public Roads, the United States Army, and many other public organizations and interested groups. Critical evaluation of the nation's mapping needs and establishment of a logical, comprehensive plan of map development are steps toward this expansion. The Geological Survey is prepared to pursue vigorously whatever mapping operations may be required of it.

DISCUSSION

Milton O. Schmidt,[5] A. M. ASCE.—The brevity of this paper is in marked disparity with the importance of its subject in a time of armament for world-wide struggles and the use of military strategies geared to the availability of reliable maps and charts. Although maps play a vital role in the conservation of natural resources and are indispensable in modern warfare, the national mapping program still limps. The author might have scored the Congresses following the passage of the lamentable Temple Act for not making it a workable piece of legislation. Other abortive ventures such as the Hayden-Ickes Bill of 1937 followed in the same tradition.

It would be difficult for many engineers to understand why the plan proposed in 1951, calling for the completion of the topographic atlas of the United States in a 20-yr period at an annual cost of $25,000,000, may also prove a fiasco. Such a calamity can be prevented by telling the story of map needs to Congress in vivid language that the lawmakers can understand. Congress must be convinced that the implementation of a surveying and mapping program (that will cost no more than one aircraft carrier a year) may bring to the military establishment data vitally needed for the defense of the Great Lakes Region and other areas of the United States. Such implementation would also provide a topographic map base of incalculable value for the peacetime planning needs of city, state, and the nation.

Engineers have long berated the legislative bodies for their indifference toward supporting national mapping programs. Furthermore, the apathy of the general public has added to the disinterest of the solons. The engineering profession must strive as earnestly for legislative championing of mapping as the medical profession attacked plans for socialized medicine. The mysteries of topographic symbolization of the typical map must be translated so that a map is seen as a document of great worth. It can become a tool of vast utility and make possible more efficient planning and use of the nation's resources.

One element of a sound mapping program must be an assurance that financial support will continue. Otherwise, normal obsolescence will make the sporadic mapping program useless. Furthermore, the program should be entirely federal because the disinterest of some strategically situated state might cause that unmapped area to become the nation's Achilles' heel. A measure that affects the safety of the entire nation cannot brook the indifference of a single area and for lack of state funds hazard the security of the United States.

Under a program of the scope outlined by the author, it can be confidently expected that improvements in techniques and equipment that are expected concomitants of increased mapping activity will effect a lowering of unit costs in all cartographic operations. The magnitude of the undertaking and the pride of accomplishment engendered in the personnel cannot help but hasten more rapid and cheaper mapping processes.

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[5] Associate Prof. of Civ. Eng., Univ. of Illinois, Urbana, Ill.

Maps are invaluable to the federal agencies and through them to 150,000,000 Americans and their allies to the north and south. Toleration of cartographic programs falling below the level of a 20-yr project is economically indefensible and morally derelict in case of war on American soil. The task of convincing Congress and the state legislatures of the need for accelerated mapping devolves on the people who use maps and recognize their great potentialities. The problem has yet to be an engineering colossus. It is one for the civilian engineering groups and the military to persuade Congress logically and relentlessly to extend modest but continued financial help to an enterprise vital to the nation in peace and war.

D. P. Krynine,[6] M. ASCE.—An interesting and exceedingly important matter has been brought to the attention of the engineering profession by the author of this paper. At a time when life expectancy has risen considerably there are quite a few living and still active engineers approximately of the same age as the United States Geological Survey (USGS) who saw the adventure of the development of topographic mapping by the USGS in the United States, the continuous progress in the preparation of maps, and their increasing usefulness in the engineering projects (especially those that spread across an area, such as irrigation projects, or extend along a distance, such as highways or canals). The railroad location era is practicaly over in the United States, but the old railroad men should remember with gratitude the help received from the USGS maps in their days.

The writer wishes to emphasize the necessity of providing all topographic maps with grid systems. Engineers need maps on larger scales, referred to orthogonal axes of coordinates for convenience in measurements. A grid consists of two sets of parallel lines intersecting at right angles to form a network consisting of squares. This grid is applied to plane areas that, for mapping purposes, replace actual areas at the earth's surface. This replacement, though made according to certain rules, involves some distortion that may be considered negligible so far as the length of the lines and the angles made by them are concerned. The side of a square of the grid may be as large as 3000 yd or as small as 1000 ft. The x-coordinates increase in the west-east direction, and the y-coordinate in the south-north direction (mnemonic rule "read right up").[7]

There are several systems of grids. The United States military grid system subdivides the continental United States into seven zones labeled from A to G in the east-west direction. Thus, the State of Connecticut is located within the A-zone whereas California is located mostly within the G-zone. The world grid, as the writer understands, is practically an extension of the United States and British military grid systems. The state coordinate systems[8,9] are based on the fact that in a strip of the earth 158 statute miles wide (counting along a meridian) the difference in length between the actual or geodetic length of a line and its length on the grid constructed for that strip is negligible (less than 0.01%). The territories of the states are thus subdivided into grid areas not exceeding 158 statute miles in the south-north direction. In this connection, California has seven grid areas and Connecticut only one. The state grids are being introduced in various states by legislative enactment.

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[6] Cons. Engr., San Francisco, Calif.

[7] "Military Maps and Air Photographs: Their Use and Interpretation," by Armin K. Lobeck and Wentworth J. Tellington, McGraw-Hill Book Co., Inc., New York, N. Y., 1944.

[8] "The State Coordinate Systems," by H. C. Mitchell and L. G. Simmons, *Special Publication No. 235*, U. S. Coast and Geodetic Survey, Govt. Printing Office, Washington, D. C., 1945.

[9] "Use of State Plane Coordinates in Route Surveying," by H. C. Mitchell, U. S. Coast and Geodetic Survey, Washington, D. C., 1950 (lith.).

Besides the USGS, the United States Coast and Geodetic Survey (USCGS) is active in the preparation of the topographic maps. It establishes the primary vertical and horizontal control nets which, afterward, are subdivided into third-order and fourth-order control nets by the USGS. Topographic maps based on the United States military grids are prepared by the Corps of Engineers. On these maps (which generally are carefully prepared) highways and roads are well drawn, and their condition is recorded.

Engineers interested in shore and harbor structures have to consult "charts" prepared by the USCGS.[10] These charts cover the Atlantic, Gulf, and Pacific Coasts, and also Alaska, Puerto Rico, Hawaii, and the Philippine Islands. These charts are conveniently drawn on decimal scales.

Air photos are very useful in engineering practice, as the author of the paper also states, and in many occasions they are preferable to topographic maps. In general, however, air photos are not, and will not be, competing with topographic maps; they simply complement them. An analogy in the field of transportation is the role of railroads that are still very useful in the country's economics, but are complemented by the highway and aircraft traffic.

The writer is far from being a specialist in mapping, but he is a user of maps to a considerable extent, and this discussion should be considered as an expression of the opinion of a rank-and-file engineer who greatly appreciates the work of the USGS along these and other lines.

W. E. WRATHER[11].—The writer is naturally in full agreement with Mr. Krynine in believing that national topographic mapping is important to the efficient use of natural resources and to the logical planning of engineering projects.

Mapping of large areas on the surface of the earth is referred most conveniently to the basic geographic system in terms of latitude and longitude; so, from the beginning of organized mapping efforts many years ago, the topographic quadrangle maps that are units of the national mapping system have been framed within evenly spaced meridians and parallels. This is a good arrangement for the map maker.

Although the same geographic reference system is also satisfactory to many map users, engineers very frequently prefer rectangular coordinates for local surveying projects. To meet such requirements the two systems mentioned by Mr. Krynine were developed by the USCGS. The "Grid System for Progressive Maps in the United States," in yards, was primarily for military use. It has been superseded by the "Universal Transverse Mercator" (UTM) grid, in meters, which was adopted in 1947. The "State Coordinate Systems," in feet, were designed for the convenience of local surveyors, project engineers, and other civilian users; they are based either upon the Lambert conformal projection for long E–W zones, or upon the transverse Mercator projection for long N–S zones.

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[10] Catalog of Nautical Charts and Related Publications, U. S. Coast and Geodetic Survey, *Serial No. 665* (Nautical Charts, Tidal Current Charts, Coast Pilots, Tide Tables, Current Tables), October, 1949.

[11] Director, U. S. Geological Survey, Washington, D. C.

Mr. Krynine is in agreement with many other important map users in believing that the topographic maps should show the rectangular coordinate systems. These provide the information required to correlate local engineering surveys, using rectangular coordinates, with the details on the maps. The method used to show the grids is to print small "ticks" in the margins of the maps, so that anyone who so desires may draw the grid lines across the map he is using. The "ticks" for the state coordinate systems appear on the civil editions of practically all new quadrangle maps published by the USGS since 1942, and on reprints of many older maps. The military grid, in yards, was generally indicated on maps published from 1925 to 1941. The new military grid, in meters, is now indicated only on military editions, and "ticks" for the state grids, in feet, are shown only on the maps for civilian use.

Mr. Schmidt is quite right in his statement that a national mapping program cannot be executed properly and economically without a stable long-range plan for financing such work. The Temple Act was workable legislation, but the mapping program it authorized failed to become a reality because the funds to support it were not appropriated.

In fairness to the government agencies charged with the responsibility of proposing and defining the mapping program, and the Congress which must consider the proposals and appropriate the funds, it should be stated that since World War II significant progress has been made. Funds have increased each year and the USGS is now staffed and equipped to a point where a noticeable advance is made each year toward getting the mapping job done. The present rate of progress, although not sufficient to finish this work before 1970, is within striking distance of that goal.

The existing national emergency has had the effect of stimulating the program in some respects, but it has also retarded progress somewhat because of the necessity of utilizing considerable capacity in the preparation of maps and charts covering foreign areas. At the present time, the military continues to support strongly an expanded mapping program and, if the engineering groups throughout the United States would consolidate their interests into an equally specific indication of their needs, a mapping program of the scope and magnitude needed to provide adequate topographic map coverage for the United States and its territories would soon be launched.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 2504

RECTIFICATION OF THE PAPALOAPAN RIVER IN MEXICO

BY REYNALDO SCHEGA[1]

WITH DISCUSSION BY MESSRS. LEO M. ODOM, SAMUEL SHULITS, AND REYNALDO SCHEGA

SYNOPSIS

In order to curb the frequent inundations of the fertile valley lands along the Papaloapan River, State of Veracruz (Ver.), Mex., the Papaloapan River Commission has undertaken a project to effect a lowering of flood stages by cutting off the worst bends of this meandering river. This project is to be supplemented eventually with flood-water storage on the tributaries for the dual purpose of flood control and the generation of electric energy. The rectification program, as regards procedure, is similar to that adopted on the lower Mississippi River.[2] Only pilot cuts have been excavated, and the final enlargement of these cuts, together with the closure of the abandoned river channels, is left to the river to accomplish. Six cutoffs in the process of development will shorten the 120-mile stretch of river 34 miles. When fully developed, the lowering of flood stage is conservatively estimated to range from 1.5 ft at the head of the tidal reach, increasing upstreamward to a reduction of as much as 6 ft where the confluence of the Santo Domingo and Tonto rivers forms the Papaloapan River. A stage lowering approximating this magnitude was actually accomplished in this locality at the crest of the great flood of October, 1950, although none of the cutoffs was as yet fully developed.

THE PAPALOAPAN RIVER

General Features.—The Papaloapan is a navigable river which ranks among the largest five rivers of Mexico. It drains a 17,390 sq mile watershed (2.3% of the total area of Mexico), largely mountainous, with headwaters descending from altitudes higher than 9,000 ft. The river empties into the Gulf of Mexico near Alvarado, Ver. Precipitation on the watershed ranges from a minimum of

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[1] Chf. Executive Engr., Papaloapan Commission, Cuidad Aleman, Ver., Mexico.

[2] "Mississippi River Cutoffs," by Gerard H. Matthes, *Transactions*, ASCE, Vol. 113, 1948, p. 1.

15 in. per year in the arid easterly portion to a maximum of 100 in. per year in the mountainous areas of the interior, the gross average being 60 in. annually. The main stem which is formed by the confluence of the Tonto and Santo Domingo rivers takes a tortuous northeasterly course 120 miles in length through a wide alluvial valley discharging first into the Laguna de Alvarado and from the latter into the Gulf of Mexico. Complicating the flood problem, two important tributaries from the southeastern part of the watershed discharge into the Papaloapan River a short distance before it reaches the Laguna. These are the Rio Tesechoacan and the Rio San Juan, which drain 2,040 sq miles and 3,950 sq miles, respectively. From the west the Rio Blanco, with a drainage area of 1,060 sq miles, also empties into the lagoon.

The valley of the Papaloapan is noted for its fertile soil, important crops being sugar cane, pineapples, oranges, and bananas. Several sugar mills and refineries and several canning factories are located along the banks. The Papaloapan is navigable for light-draft river craft for 144 miles except during extreme low stages, when the crossing bars in the upper reaches afford less than 3-ft draft. Below Cosamaloapan, Ver. (mile 62) the river is deep, and towboats hauling strings of six to eight barges laden with sugar cane can navigate. Above Alvarado the channel is tidal for 44 miles to Amatlan, Ver., and tides continue to be felt as far upstream as Cosamaloapan. Beyond this point the channel becomes a succession of deep bends alternating with shallow crossing bars characteristic of meandering streams.

Low-water slopes increase progressively from 0.25 ft per mile in the tidal reach to 1.0 ft per mile at mile 115, where the river bed attains an elevation of 16.5 ft above mean sea level. The river bed at this point is composed of coarse gravel and cobblestones. The principal tributaries, the Tonto, Valle Nacional, and Santo Domingo rivers, are typical mountain streams.

Discharge Records.—At the time the project was first conceived little was known concerning the river's discharge. A gaging station had been established in 1947 at Papaloapan, Ver. (mile 115), where the river is crossed by a steel railroad bridge. The first gagings were made in May, 1947, and a continuous accurate discharge record has been kept. In the absence of any river survey, an aerial photographic mosaic, prepared by the Compania Mexicana Aerofoto at a scale of 1:20,000, has proved of great assistance in planning the location of the cutoffs. High-water marks preserved at a number of points along the river were utilized in the preparation of flood profiles.

The hydrographs of the gaging station at Papaloapan show the runoff from the watershed to have two well-defined periods, one of moderate or low flow (which begins usually in October and ends about the middle of May), and a high-water season, which extends from the middle of May to the last of September and sometimes into October. During the calendar year 1947 the total volume passing the gaging station at Papaloapan amounted to 13,196,520 acre-ft. If the runoff from the lower watershed is added, the total discharge at the river mouth for 1947 is estimated to be 24,300,000 acre-ft or 1,380 acre-ft per sq mile of drainage area. This is equivalent to an average depth of runoff of 26 in. The runoff for the years 1948 and 1949 was 13,263,000 acre-ft and 9,177,000 acre-ft, respectively, at the Papaloapan gaging station.

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IMPORTANCE OF FLOODS

The discharge capacity of the river channel below the confluence of the Santo Domingo and Tonto rivers is about 106,000 cu ft per sec. As flood peak discharges of 140,000 cu ft per sec and larger occur almost every year, overflow of marginal lands between the towns of Papaloapan and Amatlan is a common occurrence. Damage so caused is estimated at some 5 million pesos annually. Historic floods of catastrophic proportions have occurred in 1552, 1604, 1714, and 1888, causing widespread loss of life. Recent great floods of this type occurred in September, 1944, and October, 1950, and had maximum discharges exceeding 200,000 cu ft per sec or about 15 cu ft per sec per sq mile of drainage area. These caused great material damage with loss of life and created unsanitary conditions. Towns along the river were inundated, as were the vast agricultural lands extending along both sides of the river, thereby effecting complete loss of crops.

TABLE 1.—PAPALOAPAN RIVER FLOOD LEVELS, IN FEET

River mile[a] (1)	Location (2)	1944 FLOOD		Elevation of flood crest 1947 (5)	Elevation of flood crest 1950 (6)	Estimated reduction[b] (7)
		Elevation of flood crest (3)	Depth of water on river bank (4)			
0.0	Alvarado, Ver.	5.2	2.5	1.4	1.4	....
18.5	Tlacotalpan, Ver.	9.8	2.5	5.7	6.6	....
30.0	San Miguel, Ver.	11.8	2.5	6.9	8.8	....
43.5	Amatlan, Ver.	15.4	2.6	11.6	13.3	1.4
52.0	San Cristobal, Ver.	17.4	2.5	14.5	17.1	....
62.5	Cosamaloapan, Ver.	21.0	2.5	18.4	19.7	3.9
71.0	Chacaltianguis, Ver.	24.2	2.7	21.2	22.6	....
80.0	Tuxtilla, Ver.	28.2	3.3[c]	23.6	25.6	....
83.0	Novillero, Ver.	30.3	3.3[c]	25.9	26.7	3.3
86.5	Tlacojalpan, Ver.	31.4	3.3[c]	27.1	29.6	3.0
106.0	Otatitlan, Ver.	43.3	4.0[c]	37.6	36.6	4.6
115.0	Papaloapan, Ver.	50.6	4.8	43.0	44.1	6.1
....	Tuxtepec,[d] Oaxaca	73.4	7.8	57.7	....	8.2

[a] Zero point for river miles is at Port of Alvarado. [b] Reduction of 1947 flood level after 3 yr cutoff operation.
[c] Depth of water in town averaged about 5 ft. [d] Tuxtepec is on the Santo Domingo River.

Table 1 shows the elevation above mean sea level of recent flood crests at the various towns. The average depth of water on the river banks is shown for the flood of 1944, in which the maximum discharge exceeded 200,000 cu ft per sec. Within the towns, which are built on ground that is somewhat lower than the tops of banks, the depth of flooding was a foot or more greater. The town of Cosamaloapan escaped inundation in 1950 owing to levee protection. In the last column of the table are shown the highest stages registered during the flood of October, 1950, when the maximum discharge, as measured at the Papaloapan gaging station, was 220,000 cu ft per sec. Each flood season produces, as a rule, a succession of sudden stage rises at intervals of ten to twenty days followed by equally rapid subsidences. These rises decline toward the end of the flood season but recur characteristically in a minor way during the low-water season.

THE PAPALOAPAN RIVER PROJECT

Organization.—The catastrophic flood of 1944 was of such magnitude that the President of the Republic, Gen. Avila Camacho, after an inspection tour of the affected region, declared it essential to the public welfare that measures for flood protection be instituted. He placed the planning and execution of the necessary works in the hands of the National Irrigation Commission, which was equipped with an organization and technical staff capable of undertaking the work. General Camacho's successor in office, Pres. Miguel Aleman, expanded this organization into the present Ministry of Hydraulic Resources (Secretaria de Recursos Hidraulicos) headed by Adolfo Orive Alba, M. ASCE, secretary, whom he also appointed president of the Commission of the Papaloapan, created by presidential decree in April, 1947. The writer was placed in executive charge of operations.

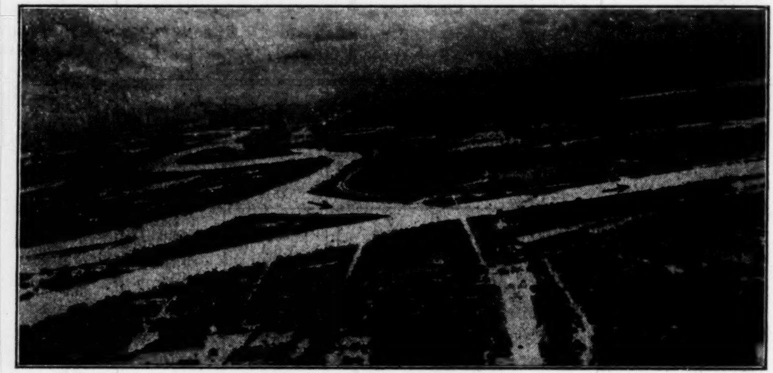

FIG. 1.—OTATITLAN CUTOFF, LOOKING WEST

The project as conceived, is comprehensive. It is aimed not only at flood control but also at the development of all the natural resources of the region, including public water supplies and sanitation, the generation and transmission of electric energy by means of hydroelectric plants at high dams on the tributaries, the building of highways and bridges, the improvement of agriculture, and the general betterment of the standard of living of the people within this region. The Papaloapan Commission operates under the general direction of the Ministry of Hydraulic Resources and has the administrative and technical direction of planning and construction of all works required for the integrated development of the watershed.

Construction of Levees.—Prior to 1950 a vast expanse of agricultural lowland has remained subject to inundation by floods, because of the total absence of levees. This has since been remedied in part by building an embankment for a main highway along the river. This embankment, although not of great height, has served as a levee protecting the best agricultural areas west of the river.

The road extends the full length of the valley, connecting the towns of Ciudad Aleman, Novillero, Cosamaloapan, San Cristobal, Amatlan, San Miguel, and Tlacotalpan. This road is visible in Fig. 1 on the far side of the river at Otatitlan Cutoff. Built on one side of the river only, the presence of this embankment did not increase flood heights to any marked extent during the great flood of October, 1950. It did, however, save the important town of Cosamaloapan from being flooded on that occasion. It is realized that a similar embankment on the east side of the river would, through confinement of flood flow, inevitably create increased flood stages, as is the case in the lower Mississippi River. Therefore, it is planned to defer further levee construction until after flood-water storage on the upper tributaries has been completed in connection with hydroelectric development. A number of dam and reservoir sites has been investigated on all the principal tributaries, including those tributaries emptying into the lower reaches of the main river, and construction is under way on a dam at Temazcal (Ver.) on the Rio Tonto. This stream was a principal contributor to the flood of October, 1950, with a discharge of 107,600 cu ft per sec. This project includes a hydroelectric development.

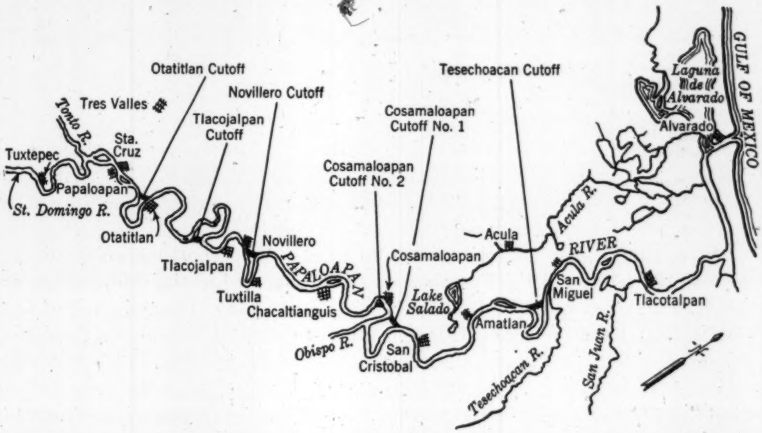
Otatitlan Cutoff
Tres Valles
Tonto R.
Sta. Cruz
Tuxtepec
Papaloapan
St. Domingo R.
Otatitlan
Tlacojalpan Cutoff
Novillero Cutoff
Tlacojalpan
Novillero
Tuxtilla
PAPALOAPAN
Chacaltianguis
Cosamaloapan Cutoff No. 2
Cosamaloapan Cutoff No. 1
Cosamaloapan
Obispo R.
San Cristobal
Lake Salado
Amatlan
Tesechoacan Cutoff
Acula
Acula R.
RIVER
San Miguel
Tesechoacan R.
San Juan R.
Tlacotalpan
Laguna de Alvarado
Alvarado
GULF OF MEXICO

FIG. 2.—RECTIFICATION OF THE PAPALOAPAN RIVER, SCALE: 1 IN. EQUALS APPROXIMATELY 13 MILES

Planning the Cutoffs.—Inasmuch as dam building is time consuming, attention was given to river rectification and levee construction as early as possible as a means of bringing some measure of protection to the towns and agricultural interests of the valley. Rectification, in the case of the Mississippi River, had demonstrated the advantage to be achieved by use of the pilot cut method, since an appreciable measure of flood-stage reduction could be realized long before the pilot cuts developed into full-sized river channels. Accordingly, on November 8, 1947, a plan for a rectification project was submitted to the Papaloapan River Commission. This plan contemplated a series of seven cutoffs spaced to eliminate excessive curvature yet without straightening the river unduly. Gerard H. Matthes, Hon. M. ASCE (former head engineer of the Mississippi River Commission), after detailed inspection and study both in the field and at the office, approved the plan, indicating certain modifications intended to increase the stability of the future channel alinement as well as to promote proper automatic enlargement of the pilot cuts through the scouring action of floods.

Referring to the map, Fig. 2, the seven cutoffs were located and named as shown in Table 2. Except Tesechoacan, which was named for the tributary by that name, the cutoffs were named for the towns nearest to them.

A total length of 38.9 miles of river bends was thus to be eliminated, constituting a net shortening of 33.75 miles since the aggregate length of the cutoff channels is 5.15 miles.

TABLE 2.—Papaloapan River Cutoffs

Name of cutoff (1)	Location by River Miles: From (2)	Location by River Miles: To (3)	Length of river cutoff, in miles (4)	Length of pilot cut, in miles (5)
Tesechoacan	30.6	37.3	6.7	0.78
Cosamaloapan No. 1	51.6	59.6	8.0	0.72
Cosamaloapan No. 2	60.6	61.8	1.2	0.40
Novillero	75.3	80.7	5.4	0.58
Tlacojalpan	85.5	94.5	9.0	0.81
Otatitlan	102.3	108.5	6.2	0.66
Tuxtepec	114.6	117.0	2.4	1.20
Total length			38.9	5.15

As originally planned, the pilot cuts were to have bottom widths of 50 meters (164 ft) except the Tesechoacan Cutoff, located in the tidal section of the river that was to be made 100 meters (328 ft) wide. The widening of the pilot cuts and their eventual enlargement to the full width and depth of the stream were to be left to the natural forces of the river. The river's flow was expected to erode a total of 13,000,000 cu yd in the cutoff channels alone. This would be increased by 2,600,000 cu yd of spoil from excavation placed on the banks of the cuts, plus an amount of river-bed erosion estimated at 1,300,000 cu yd, making a total of about 17,000,000 cu yd to be moved by the river in the course of time. This time element could not be estimated since it was dependent not only on the magnitude and frequency of floods but also on the difference of time elements for individual cutoffs.

It was estimated that, after the first three years of operation of the pilot cuts, the height of a flood equal to that of 1947 would be lowered as indicated in Table 1.

Comparison of the flood crest elevations of 1950 with those of 1944 in Table 1 reveals a cumulative increase in flood-height reduction at the upper river stations due to the cutoffs. However, at points below Cosamaloapan no comparison is practicable because of tides and flood inflow from the lower tributaries.

CONSTRUCTION OF CUTOFFS

Bids for cutoff construction were invited in the late winter of 1947–1948 during the low-water season. All bids presented had to be rejected, as none of the contractors was able to meet the time limit imposed by the beginning of high water in the latter part of May. The specifications were strict in regard to the necessity of completing all excavation before the advent of the flood season because failure to do so meant the ruin by flood flow of any uncompleted pilot cuts with small prospect of repairing the damage except at considerable expense. Moreover, the difficulties involved in getting equipment to the sites of the work meant a serious loss of time. Owing to the urgency of getting the work started a contract was let to a company operating near the town of Cosamaloapan for the excavation of the pilot cut for Corte (Cutoff) Cosamaloapan No. 1 with reduced dimensions (Fig. 3). It was felt that this plan would be advantageous by affording an opportunity to observe the functioning of this pilot cut during the high-water season, thereby utilizing it as a guide for undertaking further pilot cut excavations during the next low-water season. The contractor had at his disposition only two $2\frac{1}{2}$-yd draglines, two bulldozers, and auxiliary equipment and could not attempt to excavate a channel larger than 20 meters (65.6 ft) bottom width, with side slopes of 1:1 and bottom elevation 6.56 ft below mean sea level, in the time remaining. Work was begun April 20, 1948, and completed May 27 of the same year. The excavation equaled 207,300 cu yd.

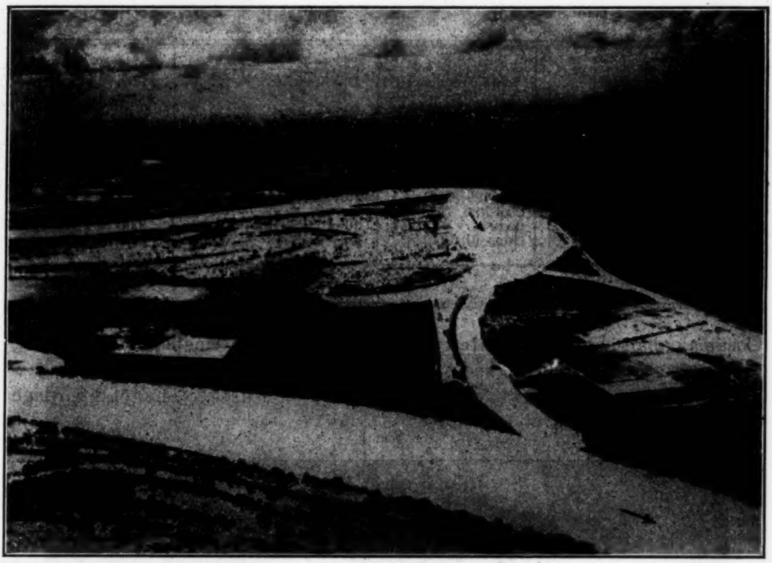

FIG. 3.—COSAMALOAPAN CUTOFFS, LOOKING WEST

As the functioning of Cosamaloapan Cut No. 1 would set up strong currents along the left bank bordering the town of Cosamaloapan, where bank caving in previous years had destroyed several buildings, the need for excavating the pilot cut for Corte Cosamaloapan No. 2 was felt to be urgent. This was a short cutoff across a sharp point intended to relieve the high-water attack on the Cosamaloapan shore. A contractor offered to make a small pilot cut, using four tractors, four scrapers, and two bulldozers. His offer was accepted with a view of making enlargement with draglines at a later date. The total yardage of this excavation was 66,900 cu yd.

TABLE 3.—PILOT CUT DATA—

Name of cutoff (1)	Date commenced (2)	Date finished (3)	Length (feet) (4)	Bottom width (feet) (5)	Side slopes (6)
Tesechoacan	February 15, 1949	June 5, 1949	4,048	165.0	1 on $\frac{1}{2}$
Cosamaloapan No. 1	April 20, 1948	May 27, 1948	3,677	65.6	1 on $\frac{2}{3}$
Cosamaloapan No. 2	May 14, 1948	June 17, 1948	2,066	65.6	1 on $\frac{2}{3}$
Cosamaloapan enlargement	May 20, 1949	July 22, 1949	2,066	42.6	1 on $\frac{1}{2}$
Novillero	April 28, 1949	June 21, 1949	3,198	98.4	1 on $\frac{1}{2}$
Tlacojalpan	February 27, 1949	June 14, 1949	4,986	82.0	1 on $\frac{1}{2}$
Otatitlan	June 10, 1949	July 18, 1949	3,608	82.0	1 on $\frac{1}{2}$

[a] Rate of exchange at the time contracts were

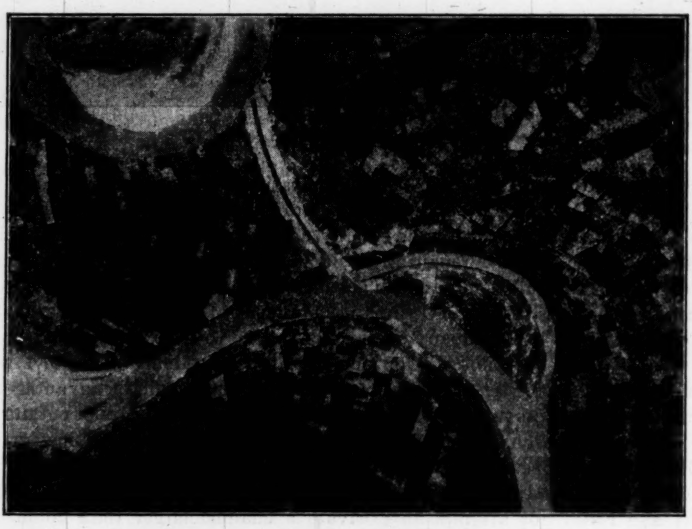

FIG. 4.—TLACOJALPAN CUTOFF WITH END PLUGS UNEXCAVATED

Soundings taken August 10, 1948, in the two Cosamaloapan pilot cuts revealed that the bottom of cutoff No. 1 had been eroded progressively in an upstream direction commencing at the downstream end for a distance of 1,150 ft. This erosion equaled a depth of 10 ft bringing the bed elevation to 16.5 ft below mean sea level. Also, the cross section of the cutoff had been widened by 5 meters (16.4 ft) in a fairly uniform manner, thus preserving the gently curving alinement on which the pilot cut had been laid out. Pilot Cut No. 2 was found to have eroded its bed somewhat irregularly to a depth of about 1.5 ft, except at the two ends where bottom erosion had caused considerable deepening. Its cross section had gained in width about 15 ft.

PAPALOAPAN RIVER CUTOFFS

Entrance elevation (feet) (7)	Outlet elevation (feet) (8)	Maximum cut (feet) (9)	Minimum cut (feet) (10)	Excavation (cubic yards) (11)	Total cost (pesos)[a] (12)	Unit cost (pesos per cu yd) (13)
−14.8	−14.8	24	16.3	555,800	1,189,802.01	2.14
−6.6	−6.6	24	16.0	206,300	788,697.00	3.82
+1.6	+1.6	17	3.6	66,800	153,215.25	2.29
−6.6	−6.6	33	14.5	77,200	148,778.40	1.93
+1.3	−2.0	27	13.5	271,600	498,329.28	1.84
+3.3	0.0	30	3.3	449,000	820,860.10	1.83
10.5	+8.9	31	21.0	303,660	557,125.68	1.84

let was 8.65 pesos = 1 United States dollar.

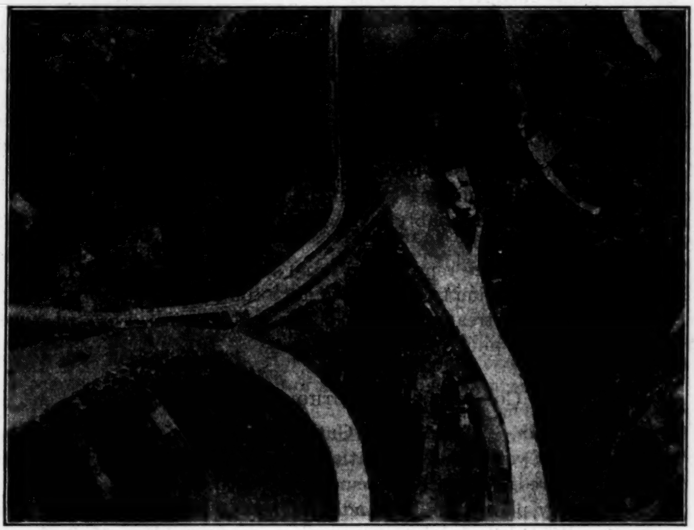

FIG. 5.—TESECHOACAN CUTOFF

Encouraged by the enlargement that had taken place in Pilot Cut No. 1 in three months' time, considering the fact that no floods of any magnitude had occurred (the largest, on July 22, had a peak discharge of 76,800 cu ft per sec), it was decided to invite bids for the excavation of the remaining pilot cuts, except for Cutoff Tuxtepec, whose construction was to be held in abeyance pending formualtion of plans for a hydroelectric development in its neighborhood. On November 20, 1948, bids were advertised calling for excavation of the quantities shown in Table 3.

The bottom elevations in the pilot cuts were planned to afford sufficient depth for boats of 2-ft draft during the low-water season. In the Tesechoacan cut the depth of excavation was purposely increased because the river, being tidal and having a mild slope, could not be depended upon to effect much erosion except during major floods. Fig. 4 shows the Tlacojalpan Cutoff during construction, with the end plugs still unexcavated.

Sequence of Pilot Cut Construction

In the execution of the project the procedure adhered to, as far as circumstances would permit, was to excavate the pilot cuts farthest downstream first and work progressively upstream. The principal advantage of this procedure was that any lowering of the river level affected by the opening of one pilot channel became at once transmitted with small loss to the site of the next pilot cut upstream. The enlargement of the upstream cut was thereby accelerated, due to steepening of the river and swifter currents. Stage lowerings, therefore, tended to be cumulative from pilot cut to pilot cut, diminished only by losses due to channel friction.

Materials eroded from individual pilot cuts often entered the abandoned channels of the cutoffs downstream and thus became permanently sidetracked and disposed of. The remainder found no lodgment in this manner but continued downstream, sometimes building bars in the river. Upon entering a pilot cut the eroded material was invariably carried through and passed on to the reaches below.

The outlined procedure represented in no sense a rigid schedule, nor did it limit the work of excavation to one pilot cut at a time. However, by having the work advance in an upstream direction, the benefits described proved material factors in accelerating pilot cut development.

Construction Methods and Cost

Corte Tesechoacan.—Tesechoacan Cutoff (Fig. 5) was by far the largest pilot cut in terms of yardage of excavation, exceeding half a million cubic yards. As no hydraulic dredge could be procured, its width had been reduced to one half that originally planned. To handle this excavation with land machines, the contractor divided the job into two sections, separated by a 49-ft plug midway, in addition to the end plugs. Utilizing two $2\frac{1}{2}$-yd, one $1\frac{1}{2}$-yd, and one $\frac{1}{2}$-yd draglines, assisted by four bulldozers, and four 13-yd trucks, the contractor started excavation simultaneously on both sections of the pilot cut, working in the dry until water level in the river was reached at El. 8.2 ft above mean sea level, a fairly high stage. He then concentrated all his equipment in the lower section and excavated it down to grade (El. $-$ 14.8 ft mean sea level) keeping down seepage water by pumping into the upper excavation with two 8-in. pumps. From this point pumps lifted the water into the river upstream. On completion of the lower section, the equipment was transferred to the upper section, and after excavating this section, including the central plug, the two end plugs were removed. As may be noted from Table 3, the bottom elevation was given no slope, being uniformly at El. − 14.8 ft mean sea level. The river being tidal at this point, it was permitted to establish its own feeble gradient. Fig. 6 is an aerial view of the cutoff in operation during the 1950 high-water period. On the near side of the river can be seen the road embankment that acted as a levee to prevent flooding of adjacent lowland areas.

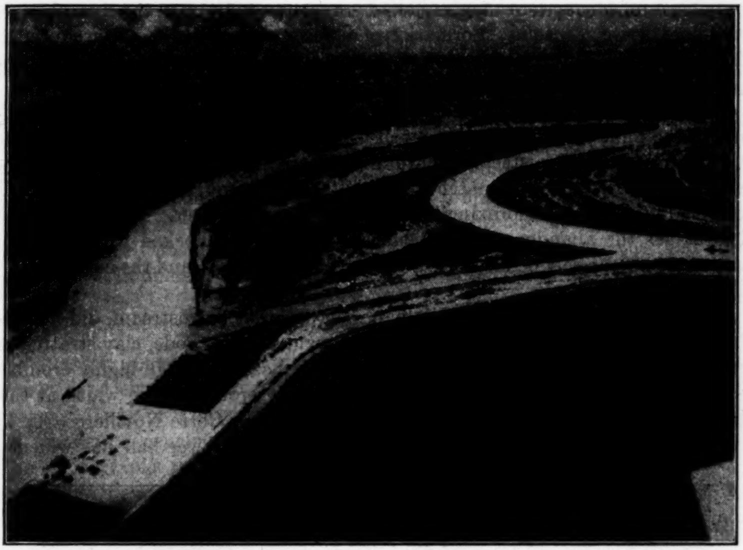

Fig. 6.—Tesechoacan Cutoff, Looking East

Other Cutoffs.—At the other cutoffs, the equipment and methods of procedure were, with some variation, essentially similar to those described for Cosamaloapan No. 1, except that in all cases final opening was effected by excavating the upsteram plug in its entirety. The dates of commencement and completion of excavation at each pilot cut, together with the cost, are listed in Table 3.

Development of the Pilot Cuts

High river stages constitute the chief agency that effects the development of pilot cuts. Unfortunately, the high-water season of 1949 was lacking in high rates of runoff. Consequently, the development of the two Cosamaloapan pilot cuts (the only ones then built), although satisfactory, had been less than expected. This situation became reversed in 1950 when, a few months after the completion of the remaining four pilot cuts, a flood of extraordinary proportions swept down the river. Its magnitude, coming during the early stages of these four pilot cuts, did not portend any good, and some damage was expected to result. On the whole, however, the results were better than expected. The only harmful effects were a few sand and gravel bars built up in the river channel at points between cutoffs. Formation of deposits of this kind had been expected. In anticipation the procurement of a light-draft cutterhead dredge for their removal had been urged.

The flood of October, 1950, effected considerable widening and also deepening of the pilot cuts. The pilot cut of Corte Otatitlan, which is located where the river's gradient is steepest, naturally suffered the greatest amount of erosion. In width it closely matches the width of the river. Its discharge capacity had increased to approximately 70% of that of the river. At ordinary and low stages the abandoned river bend appeared to be blocked with sediment deposits immediately below the head of the cutoff channel, and the cutoff carries the entire flow.

The pilot cut for Corte Tlacojalpan, next in order downstream, did not enlarge as much as at Otatitlan, owing to a tough, slow-to-erode, clay stratum at its mouth. The downstream portion of the cut passes through highly erodible sandy-silt soil and consequently has been heavily eroded. The width at this point approaches that of the river. The pilot cut for Corte Novillero (Fig. 7) has developed satisfactorily along its upper half; the lower half has widened much less because of compacted silt deposits in a former oxbow lake that the cut must cross. Erosion there will necessarily be slow.

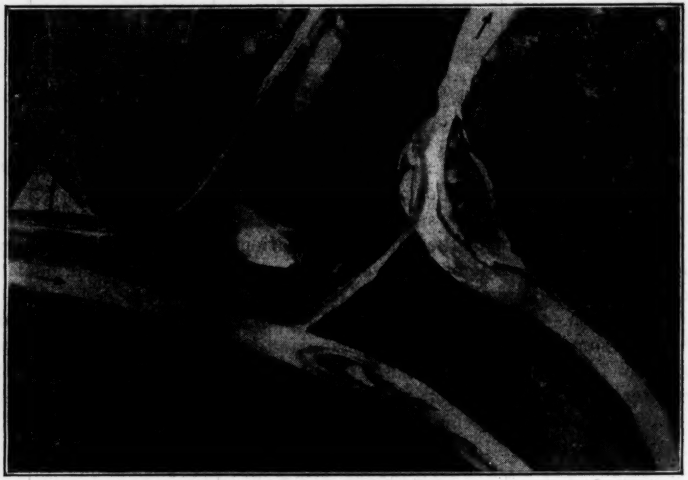

FIG. 7.—NOVILLERO CUTOFF

The two Cosamaloapan pilot cuts widened some 16 ft each. No. 1 showed this widening over two thirds of its lower length, while the small No. 2 pilot cut eroded somewhat irregularly. The pilot cut for Corte Tesechoacan, which had been expected to enlarge very slowly owing to its location in the estuarial portion of the river, showed surprisingly good enlargement, gaining more than 40 ft in width.

RESULTS

At the gaging station at Papaloapan the flood peak of October, 1950, registered 6.5 ft lower than the great flood of 1944. Since the maximum discharges of these two floods are believed to be fairly comparable, reduction in flood height of about 6 ft appears to have taken place in the reach of the river above Corte Otatitlan. This is in agreement with original estimates for three-year prospective results.

In general, the development of the cutoff project has been satisfactory as regards over-all aspects, more particularly in respect to flood-stage lowerings as described. Considering the short time that has elapsed since the pilot cuts were placed in operation, the project gives promise that it will justify its cost.

ACKNOWLEDGMENT

Successful completion of this project is the result of vigorous backing by President Aleman of Mexico. His foresighted policy established the comprehensiveness of the project. The aerial photography that was the basis for the actual construction was done by the Compania Mexicana Aerofoto. The photographs used in this paper were produced by that company. The original paper was written in Spanish and was translated into English by Frank C. Carey, M. ASCE. Subsequent revision and rearrangement was made by Mr. Matthes.

DISCUSSION

LEO M. ODOM,[3] M. ASCE.—This paper is welcome as a report on one phase of the program of the Mexican national government's Ministry of Hydraulic Resources. The slightest acquaintance with the terrain and the economic development of Mexico indicates the forward step that the creation of such a national bureau with its specific duties constitutes.

The recognition by the federal government that major flood-control problems cannot be solved by local interests alone is recent in the United States. Only in 1927, when the valley of the Mississippi River from Cairo, Ill., to the Gulf of Mexico was overwhelmed by a great flood, was the federal government forced to recognize this fact.

In carrying out its programs for river control the Mexican government can no doubt profit greatly by application of procedures that have been developed in the United States. However, each stream has to be treated individually, and general principles must be carefully separated from particular developments. The success of adaptations will depend largely on the adequacy and the interpretation of the data collected on each stream.

The program for flood control on the Papaloapan River, as outlined in the paper, consists of a levee along the left descending bank and a series of cutoffs patterned somewhat after those constructed in the Mississippi River.

The levee, if of sufficient height and strength, will probably be an unmixed blessing to those who reside on the left bank. However, the statement that this levee will not result in higher stages, although levees on both banks would do so, appears illogical. The writer has never known of a case in which a levee constructed along one bank did not result in increased flood heights. The storage capacity represented by the area protected by the levee is subtracted from the available storage, and consequently gages go up.

From the map (Fig. 2) and the description, the Papaloapan River looks surprisingly like a Mississippi River in miniature in spite of the great differences in origin and slopes. The meander pattern is quite similar in appearance. The fact that the banks are higher than the land farther back is typical of alluvial streams running in beds composed of their own alluvium.

The selection of a cutoff program as the means of lowering flood heights, as successfully demonstrated by the system used on the Mississippi River, is thus easily understood. If it operates successfully it will have advantages over a complete levee program. The comparative costs were undoubtedly considered in advance.

Considering that practice on the Mississippi River was used as a model, it is surprising that the program was commenced with such a small amount of engineering data. The Mississippi River Project was preceded by years of study and by collection and analysis of a huge amount of specific data applicable to the cutoff program. Only thus was it possible to locate the cutoffs properly and to anticipate the subsidiary work that would be required.

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[3] Cons. Engr., Baton Rouge, La.

The subject of cutoffs on the Mississippi River was the source of argument among well-known engineers for many years before these cutoffs were undertaken. The Mississippi River Commission finally went ahead and made the cutoffs, and this construction has accomplished successfully the lowering of the flood plane between Memphis, Tenn., and Old River, La. However, a victory such as this one does not necessarily mean that all the opposing arguments were wrong. Valley storage has been lost, as the opponents forecasted, and the valley dwellers below the latitude of the cutoffs would be in a worse position than before had not the discharge capacity of the Atchafalaya River been enlarged under the same program, so as to divert a larger proportion of the flow of the Mississippi. The cutoffs have resulted in caving the banks both at places at which the directive was changed and in places at which velocities were increased. An unprecedented revetment program has been required to keep pace with this occurrence, the funds required being much greater than those that the opponents thought could be obtained.

The argument that navigation would be hindered (by the lowering of the low-water plane and the deposition of material scoured out of the cuts) would have been valid if a huge dredging program had not been carried on in conjunction with the cutoffs.

In the case of the Papaloapan River, the cuts seem to have been made without the safeguards mentioned previously. The pilot cuts have already been made and the report is optimistic, but it is believed advisable to point out that the art of cutoff procedure as developed by H. B. Ferguson,[4] M. ASCE, includes a great deal more than merely making cutoffs.

The optimism expressed in the report as to the benefits already obtained from the cutoffs does not appear to the writer to be justified in the light of the information presented. This information does not indicate to the writer that the flood of 1950 was appreciably lowered. Certainly, it does not indicate that the 1950 flood was in the same category as the 1944 flood.

The data given in the tables have been plotted in Fig. 8. The 1944 flood was well above the 1950 flood even at Tlacotalpan, Ver., and Alvarado, Ver., below the lowest cutoff, in which part of the river it would be expected that higher levels would be obtained after the cutoffs were made than had existed before. Considering that a levee had been built on the left bank, it seems doubtful if the 1950 flood was as great as the 1947 flood. Assuming that it was in the category of the 1947 flood, which is possible, based on the stages at Papaloapan, Ver., and Alvarado, it appears that the Otatitlan cut resulted in a lowering of about 1 ft, which would probably have been greater without the levee. The variation of the 1950 profile of the top of the bank and from the profiles of previous floods below Novillero, Ver., is probably more the result of the levee than the cutoffs. It would appear from the data presented that the lower pilot cuts have probably not yet enlarged sufficiently to have an appreciable effect on the profile and that the problems of damages to navigation and banks have yet to be met.

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[4] "Mississippi River Cutoffs," by Gerard H. Matthes, *Transactions*, ASCE, Vol. 113, 1948, p. 3.

River engineering is admittedly still performed on the artisan, rather than the scientific, level. Advances in knowledge are slow, and great expenditures are required to secure data adequate for the semblance of a scientific approach—expenditures which the artisan can frequently point to as wasted. Compared to other fields of engineering, the practitioner finds very few records in publicly available literature to guide him. It is, therefore, of extraordinary importance that authors in this field present adequate data to support their conclusions. It is to be hoped that the author will present such additional information in his closing discussion.

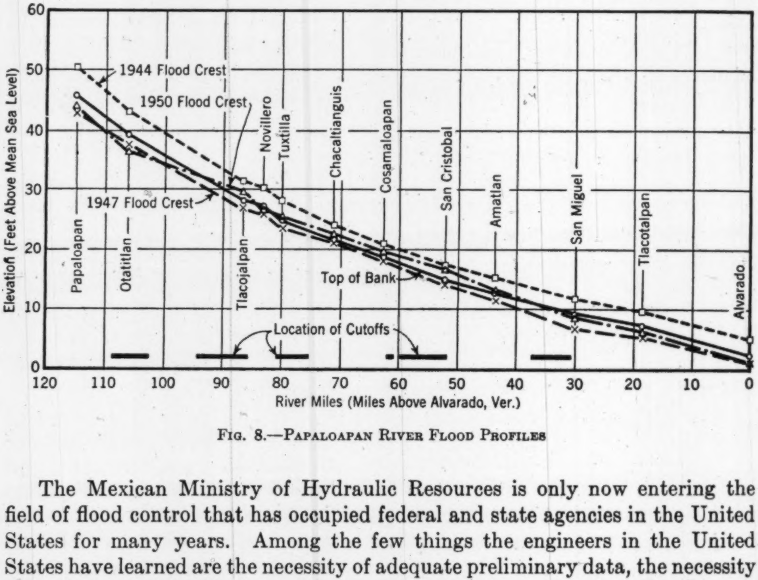

FIG. 8.—PAPALOAPAN RIVER FLOOD PROFILES

The Mexican Ministry of Hydraulic Resources is only now entering the field of flood control that has occupied federal and state agencies in the United States for many years. Among the few things the engineers in the United States have learned are the necessity of adequate preliminary data, the necessity of having adequate funds in sight for the completion of an over-all program, and a skeptical, rather than an optimistic, viewpoint toward comparison of floods.

SAMUEL SHULITS,[5] M. ASCE.—This valuable paper is an able demonstration of how carefully planned pilot cuts and cutoffs lower the river bed and thereby increase flood capacity. As an example of river rectification it is a much-needed contribution to the rather sparse supply of material on river engineering in the English language. This paucity of literature prompts some inquiry regarding the Papaloapan River Project, in the hope that in his closure the author might add to the knowledge of river behavior.

The immediate success of a series of cutoffs in keeping floods within banks or lowering flood stages is well known and well established; yet a discussion of the long-term consequences of the Papaloapan River rectification, say, 25 to 100 years hence, would be of great interest to river engineers.

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[5] Associate Prof., Michigan College of Mining and Technology, Houghton, Mich.

The only records known to the writer of the long-range results of a rectification apply to the Rhine River from Basel, Switzerland, to Mannheim, Germany. The downstream portion of this extensive correction consisted of eighteen cutoffs in the reach from Sandhofen to the Lauter River junction, executed between 1817 and 1842, reducing the thalweg from a length of 135 km (84 miles) to 85 km (53 miles), or 37%. Upstream to Basel, the rectification was accomplished by compelling the braided river to flow in a uniform bed. The entire correction caused a lowering of the bed in the period from 1820 to 1925, with the exception of a short stretch 20 km (12 miles) long. Of cogent significance is the conclusion[6] of K. Wittmann in 1927, after a study of more than 100 years of records, that the 250 miles of the Rhine River between Basel and Bingen, Germany, had still not been stabilized in 1925, even though some of the corrective works were then more than 60 years old.

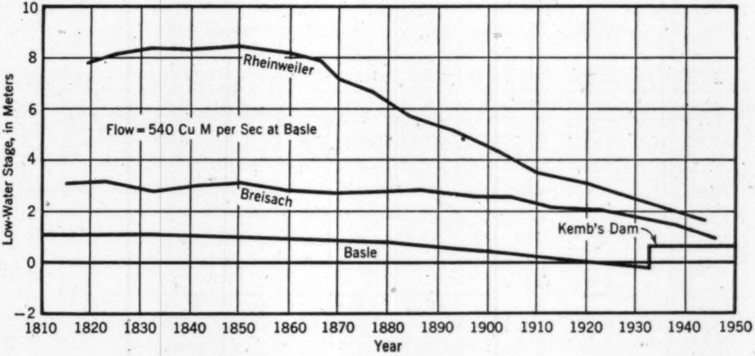

FIG. 9.—RECESSION OF LOW-WATER STAGE ON THE RHINE RIVER, GERMANY

In January, 1950, the writer inspected the Rhine River from Basel to Breisach, Germany, and learned that the degradation had not yet stopped and was progressing downstream. This is evidenced by Figs. 9 and 10, used with the permission of A. Kirchgaessner, Minister of Agriculture of South Baden, Germany. Although 70 years passed before the degradation reached Breisach, a damaging degradation had been in progress there since 1890. The Breisach low-water stage in 1947 was 2 m (6.6 ft) lower than in 1828. This lowering of low-stage profile between 1830 and 1947 is shown in Fig. 11, furnished by G. Schneider of the Freiburg Waterways Office, Freiburg, Germany.

Any apparent stabilization after rectification must be regarded with a wary eye. At Mannheim on the Rhine River, degradation started in 1842 and continued to 1854, after which no change occurred till 1869. From 1869 to 1874 the bed dropped further and then remained unchanged to 1887, and since then there has been uninterrupted degradation. Between 1825 and 1925 the low-water stage sank a total of 1.5 m (4.9 ft). This can be attributed primarily to the cutoffs and the interplay of their individual cycles of degradation and aggradation. Thus a steady-stage condition, even for 10 years, does not necessarily denote stabilization.

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[6] "Der Einfluss der Korrektion des Rheins zwischen Basel und Mannheim auf die Geschiebebewegung des Rheins," by K. Wittmann, *Deutsche Wasserwirtschaft*, 1927, Berlin, Germany.

There seems to be evidence of similar degradation phenomena on the Mississippi River. According to Mr. Ferguson,[7] between 1933 and 1937, from Glasscock Cutoff to Arkansas City, Ark., the mean depth at low water increased 0.7 ft (3.2%), while at bank-full stage it increased 2.8 ft (6.6%). This trend has persisted, as it was true, 12 to 16 years after the opening of the cutoffs in 1932–1933 that there had been a low-water lowering as well as a high-water lowering; the over-all slopes in each case were steeper than the original slopes before the cutoffs had been initiated; and the low-water plane for 150,000 cu ft per sec had been lowered less than 1 ft at Columbus, Ky., to all of 10 ft at Vicksburg, Miss.[8]

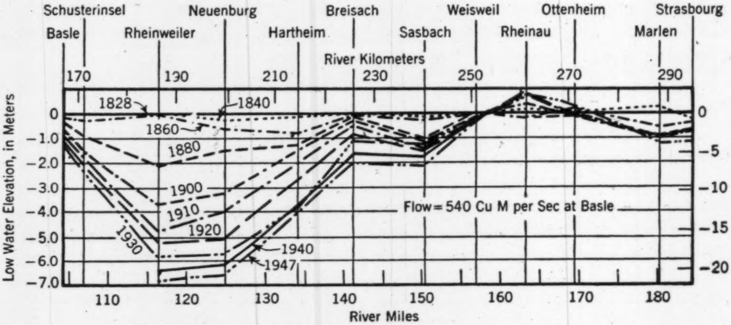

FIG. 10.—VARIATION OF LOW-WATER STAGE ON THE RHINE RIVER, GERMANY (1928 = BASE)

If one accepts, as the writer does not, that each river is a law unto itself, then comparison of the Rhine, Mississippi, and Papaloapan rivers is unwarranted. The contrast between the two methods of correction might be expressed thus: On the Mississippi and Papaloapan rivers excessive curvature is eliminated without straightening the river unduly, while the Rhine River was straightjacketed, perhaps unduly, into the desired rectified channel. The valid point, therefore, might be raised that different physiographic conditions and methods of execution of the rectification do not allow pertinent inferences; yet the very long record of the Rhine River and the short record of the Mississippi River, both pointing to progressive and extensive channel lowering, inspire this question: What will be the effect, 25 to 100 years hence, of the Papaloapan River rectification? Since this aspect has surely been studied, the inclusion of the forecast and the basic calculations would be of great value to river engineers.

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[7] "Effects of Mississippi River Cut-Offs," by Harley B. Ferguson, *Civil Engineering*, Vol. 8, 1938, p. 826.

[8] "Mississippi River Cutoffs," by Gerard H. Matthes, *Transactions*, ASCE, Vol. 113, 1948, p. 37.

In the upper Rhine region, near Breisach, some of the undesirable aftermaths of the depression of the river level are: A lowering of the ground-water table in Alsace, France, and Baden harmful to municipal ground-water supply systems, forest growth, and agriculture and a reduction of flood silt deposits, also harmful to agriculture and forest growth.

Should it be true that rectification could cause such sustained degradation, the projected reservoirs on the tributaries might intensify the lowering because of the well-known retrogression below reservoirs on silt-laden streams.

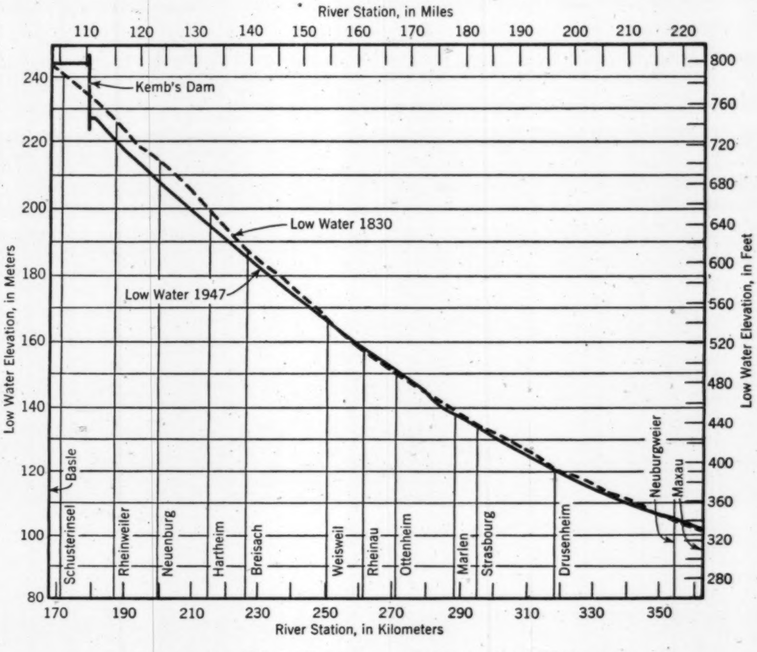

FIG. 11.—LOW-WATER PROFILE OF THE RHINE RIVER, GERMANY

Acknowledgment.—Muzzafarud Din of Afghanistan, a student at the Michigan College of Mining and Technology at Houghton, assisted the writer in the preparation of the illustrations.

REYNALDO SCHEGA[9].—The importance of adequate data and engineering information as a proper basis for the planning and execution of a program of river rectification is emphasized by Mr. Odom, who asks for more information in support of the conclusions. In the gradual development of Mexico it has not been possible to collect complete data of the rivers, particularly of the Papaloapan, even remotely comparable with the fund of data and information available for highly developed regions, such as the Mississippi River region. The Papaloapan data, collected from 1947 to 1950, are available in published form.[10,11]

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[9] Engr. in Executive Charge, Works in the Valley of Mexico, Ministry of Hydraulic Resources, Mexico, D. F., Mexico.

The peak of the flood of 1947 was 159,000 cu ft per sec, which is substantially less than the 220,000-cu-ft-per-sec peak of the 1950 flood. The 1944 flood was 353,000 cu ft per sec, as estimated from the high-water marks that could be identified. The comparison between flood stages at Tlacotalpan and Alvarado is affected not only by the tides, but also (*a*) by the discharge of the Rio Blanco into the Laguna de Alvarado west of the Laguna, (*b*) by the capacity of the outlet of the Laguna de Alvarado to the sea, and (*c*) by the discharge of the tributary rivers, Tesechoacan and San Juan, into the lower Papaloapan. The effect of the cutoffs on the flood stages is illustrated[12] in Fig. 12.

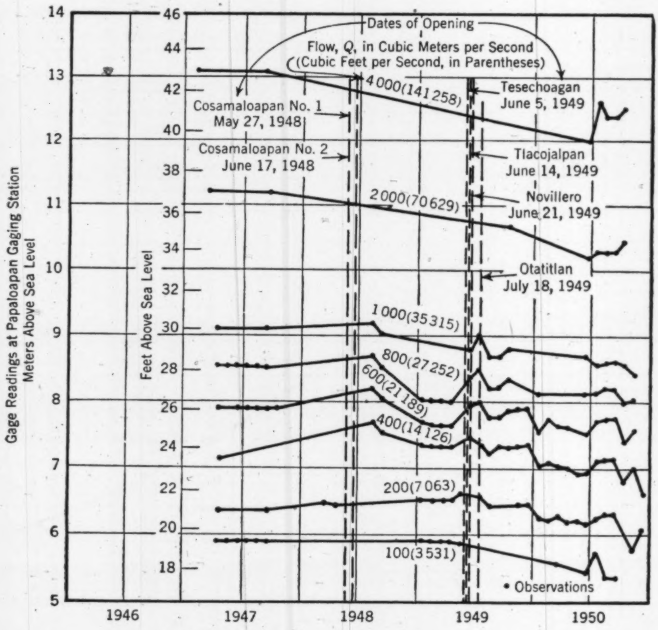

FIG. 12.

The complete flood-control program of the Papaloapan River will be a matter of gradual development as conditioned by the funds made available,

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[10] *Boletín Hidrológico No. 1*, Comisión del Papaloapan, Secretaría de Recursos Hidráulicos.

[11] *Ibid., No. 2*, Comisión del Papaloapan, Secretaría de Recursos Hidráulicos,

[12] *Ibid.*, Plate 43.

and contemplates the construction of multiple-purpose dams on the river's tributaries, with an aggregate capacity of 60% of the mean annual flow of the river. The left-bank levee, which also is the main highway along the river, was located at some distance from the zone of the river attack at low stage, for the purpose of reducing the maintenance expenses. The drainage basin of the Papaloapan River is not developed sufficiently to justify expenditures of the order that are justifiable in the lower Mississippi River basin. One multiple-purpose dam useful for flood control with a capacity of 4,864,000 acre-ft, on the Tonto River (which is one of the main tributaries of the Papaloapan) is being built, and it is hoped will be completed early in 1953.

Mr. Shulits presents very interesting data on the behavior of the Rhine River, based on very long records, which show a sustained degradation in more than 100 years. No forecast has been attempted of the effects of the cutoffs on the Papaloapan River, 25 years to 100 years hence, or earlier. Such a forecast would be uncertain considering the data available, as would be any forecast of the effect on the bed load due to the construction of the reservoirs.

Large-scale river rectification of the few big rivers of the Southeast is new in Mexico, and therefore, experience on this work is practically nonexistent. For this reason American practice is observed with much interest, and the benefit of the experience gained on the Mississippi River has been obtained in this early development, through Mr. Matthes. Observations will continue and reports on the developments on the Papaloapan River will be made available from time to time.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2505

PETENWELL HYDROELECTRIC PROJECT

By E. Montford Fucik,[1] A. M. ASCE

With Discussion by Messrs. L. F. Harza, and E. Montford Fucik

Synopsis

The problem of constructing and operating a water-retaining structure of considerable size on sand foundations is described in this paper. The Petenwell Hydroelectric Project consists of a concrete powerhouse and spillway and several miles of earth dams and dikes, all founded on a deep bed of sand. The concrete structures were built adjacent to the river channel, and the river then was diverted to complete the earth dam.

The spillway sections were the result of model tests to produce a design that would avoid scouring of the downstream channel. The construction and details of these spillway sections are described and several typical drawings are given. Various appurtenances necessary for successful operation of the project also are described.

General Description

The Petenwell Hydroelectric Project is located on the Wisconsin River, near Necedah, Wis., about 33 miles north of the famous Wisconsin Dells, as shown in Fig. 1. Construction was started in August, 1947, and the first unit was put into operation under reduced head on August 29, 1949. The reservoir was completely filled, and all units were operating in April, 1950.

The project is owned by the Wisconsin River Power Company, a company formed by the Consolidated Water Power and Paper Company, Wisconsin Rapids (Wis.), the Wisconsin Public Service Company, Green Bay (Wis.), and the Wisconsin Power and Light Company, Madison (Wis.).

The principal structures of the project, as shown in Fig. 2, are the concrete powerhouse and spillway flanked by approximately 9 miles of earth dams and dikes. The gross head is 42 ft with the reservoir at normal operating level and the full gate flow of 6,960 cu ft per sec through the units. The powerhouse contains four vertical, adjustable-blade Kaplan propeller turbines of 7,200 hp

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Note.—Published in July, 1951, as *Proceedings-Separate No. 81.* Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Vice Pres., Harza Engineering Co., Chicago, Ill.

each at full gate, with generators rated at 6,250 kva, 0.8 power factor, three-phase 60 cycle. The speed of the units is 163.6 rpm.

FOUNDATIONS

The foundation for the entire project consists of a deep bed of medium grained sand. Drilling has indicated this bed to be at least 100 ft deep with few, if any, lenses of silt or clay. As a result of these conditions, it was necessary to design all structures without attempting to provide any complete cutoff for leakage and to use the sand for all the earth structures. The nearest suitable material finer than sand was too far away to be used economically.

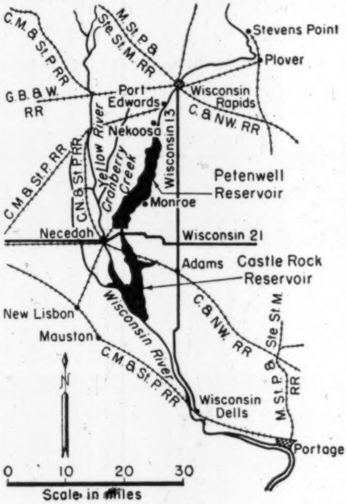

C.M.& St.P.RR
M.St.P.& Ste.St.M.RR
Stevens Point
Plover
G.B.& W. RR
Port Edwards
Wisconsin Rapids
C.& NW. RR
Nekoosa
Yellow River
Cranberry Creek
Wisconsin 13
C.M.& St.P.RR
C.N.& St.P.RR
Petenwell Reservoir
Monroe
Necedah
Wisconsin 21
Adams
Castle Rock Reservoir
New Lisbon
Mauston
Wisconsin River
C.& NW. RR
M.St.P.& Ste.St.M. RR
C.M.& St.P.RR
Wisconsin Dells
Portage
N
0 10 20 30
Scale in miles

FIG. 1.—PETENWELL PROJECT LOCATION MAP

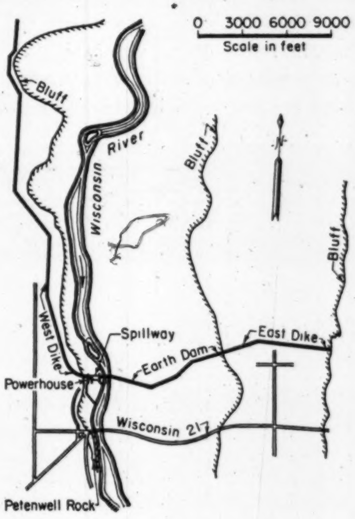

0 3000 6000 9000
Scale in feet
Bluff
River
Wisconsin
Bluff
N
Bluff
West Dike
Spillway
Earth Dam
East Dike
Powerhouse
Wisconsin 21
Petenwell Rock

FIG. 2.—PLAN OF PROJECT SITE

Typical grain size curves for the foundation materials are shown in Fig. 3. It can be seen that there are very few fines in this material, with the result that a considerable quantity of leakage had to be anticipated under the structures and through the earth dams. Laboratory permeability tests made on this sand indicated permeability coefficients ranging from $k = 93$ ft per day to $k = 555$ ft per day for void ratios varying from 0.46 to 0.60, respectively.

The natural density of the sand in the spillway and powerhouse foundations was found to vary from 95 lb per cu ft to 112 lb per cu ft.

ARRANGEMENT OF STRUCTURES

General Construction.—The powerhouse and spillway were constructed on the west bank of the river without encroaching on the normal river channel. This method of development saved the expense of elaborate cofferdams in the river and considerably reduced the pumping problem during construction.

All the material excavated for the tailrace back to the normal river course was used in the construction of the dams. Closure of the dam was made at the river section, while river diversion was accomplished by omitting the gravity (ogee) portion of the regulating bay of the spillway. Final closure was made by putting stop logs in front of the regulating bay and completing the ogee section behind these stop logs.

This plan of construction took full advantage of the sand foundations by permitting the powerhouse and spillway construction to proceed without requiring any work to be done in the normal river channel until closure of the dam. The only cofferdams required were an inexpensive sand fill around the area (cast up from, and a by-product of, the excavation) to protect against floods that would cause the river to overflow its banks, and a short crib cofferdam in front of the powerhouse. The latter cofferdam was necessary to protect the powerhouse work after diversion had been made because powerhouse construction was delayed by slow delivery of reinforcing steel.

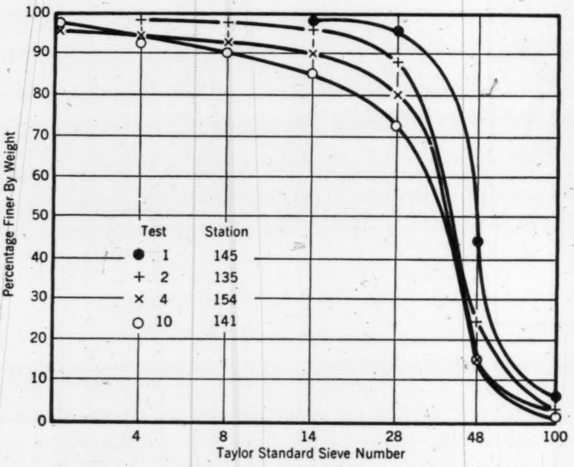


FIG. 3.—FOUNDATION MATERIAL GRAIN SIZE DISTRIBUTION

When the spillway was completed up to the crest level and the earth dams were at sufficient height, the river closure was made. This was done by building the earth fills up on each side of the river and then bulldozing the fill into the river channel from each side to close the gap in the middle. Riprap in sizes up to about 24 in. was used to aid in the initial closure made at the downstream toe of the dam. About 2,000 to 3,000 cu yd of material, mostly sand, was carried downstream, and this was almost the only extra cost involved. The simplicity of the entire procedure is evident in Fig. 4, showing the operation as closure was made.

The scheme of building the concrete structures away from the river channel, with the dam founded at the bench level, is relatively unique in the history of hydro power plants. Usually the spillway and the powerhouse are built in the river channel, thus requiring expensive cofferdams and dam foundations in the deepest section of the river. This site, however, with the bend in the river and the sand foundations, was admirably suited to such a scheme, and the result was a substantial saving in cost.

Underwatering.—Lowering the ground water in the excavation (which extended to a maximum depth of about 40 ft below river level) for the powerhouse and spillway was accomplished by a series of deep gravel-wall wells at the perimeter of the area, supplemented by well points in the deeper sections. The maximum total pumpage rate was about 11,500 gal per min.

Fig. 4.—Closure of Earth Dam

Procedure During Construction

Earth Structures.—The earth dam structures consist of the main dam, varying in height from 40 ft to 58 ft above natural ground, and the east and west dikes, both less than 22 ft in height. Fig. 5 shows the main dam, provided with a toe drain to collect the seepage and prevent raveling of the toe. Because of their lower heights, this feature was omitted from the dike structures. The freeboard of the dams is 5 ft above the maximum reservoir level for the design flood and 10 ft above normal reservoir level. In addition, a riprap parapet about 2 ft high is provided as additional wave protection during the period when the reservoir will be surcharged to pass the maximum flood.

The wave protection for the structures consists of a layer of riprap varying in thickness from 1.5 ft to 2.5 ft, depending on the depth of water in front of the dam. This riprap is placed on a filter blanket of gravel 12 in. thick. It extends to a berm at El. 127.0, 3 ft below low water. The material used as riprap has a maximum dimension of 27 in. and a minimum of 4 in., and the blanket material grades from about No. 8 sieve to 3 in. in size. A temporary riprap blanket 12 in. thick was placed on the main dam below El. 127.0 to prevent damage to the fill during the period when the reservoir was filling.

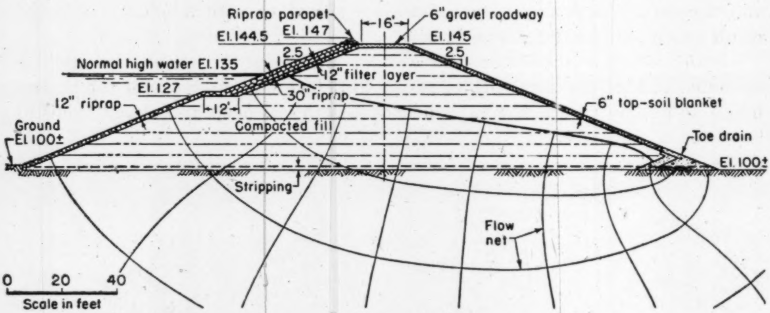
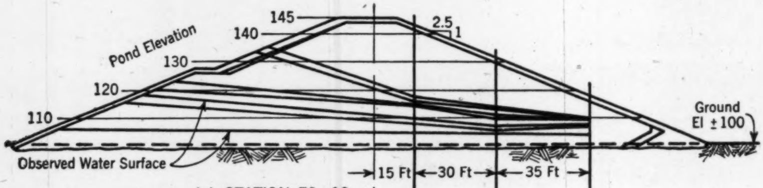
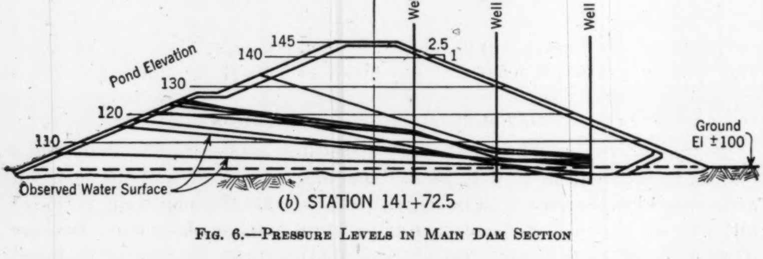

Riprap parapet
El. 144.5
El. 147
16'
6" gravel roadway
El. 145
2.5
1
2.5
1
Normal high water El. 135
12" filter layer
El. 127
30" riprap
12'
12" riprap
6" top-soil blanket
Ground
El. 100±
Compacted fill
Toe drain
El. 100±
Stripping
Flow net
0 20 40
Scale in feet

FIG. 5.—CROSS SECTION OF MAIN DAM


145
140
Pond Elevation
130
120
110
2.5
1
Ground
El ±100
Observed Water Surface
15 Ft
30 Ft
35 Ft

(a) STATION 79+10


Well A
Well B
Well C
145
140
Pond Elevation
130
120
110
2.5
1
Ground
El ±100
Observed Water Surface

(b) STATION 141+72.5

FIG. 6.—PRESSURE LEVELS IN MAIN DAM SECTION

Sand taken from areas adjacent to the structures was used to construct the main dam and dikes. All borrow for the main dam was taken from areas downstream from the dam. This was done in order not to disturb the topsoil upstream from the dam, thereby gaining the maximum use of this topsoil to reduce leakage through the foundation. Material for the dikes was taken from both sides of the structures, that on the upstream side being taken no closer than 135 ft from the dikes, to leave some topsoil to reduce seepage. Experience with the leakage under the dikes indicates that no difference in underseepage is evident, whether or not borrow was taken from the upstream side of the structures.

The dam and dikes were constructed by placing material in layers not thicker than 12 in. and compacting the material by routing the earth-moving equipment over the surface of the fill. Tests of the completed fill showed dry densities ranging from 97 lb per cu ft to 120 lb per cu ft, with an average density of about 107 lb per cu ft. This method of constructing the dam and dikes was satisfactory and inexpensive. The gradation of the completed fill material is essentially as shown in Fig. 3.

Control of Leakage.—After the diversion of the river through the spillway in December, 1948, the water level in the reservoir rose to El. 105 and then fluctuated with the flow of the river until March, 1949, when the closure of the spillway started. The pond was then raised to El. 117, the spillway crest, as rapidly as the river flow would permit, allowing a minimum discharge to keep the hydro plants downstream in operation. In May, 1949, the pond rose to El. 120 and in October to El. 125 (or 10 ft below the full reservoir level), and it remained at this level until the spring of 1950 when the reservoir was filled entirely.

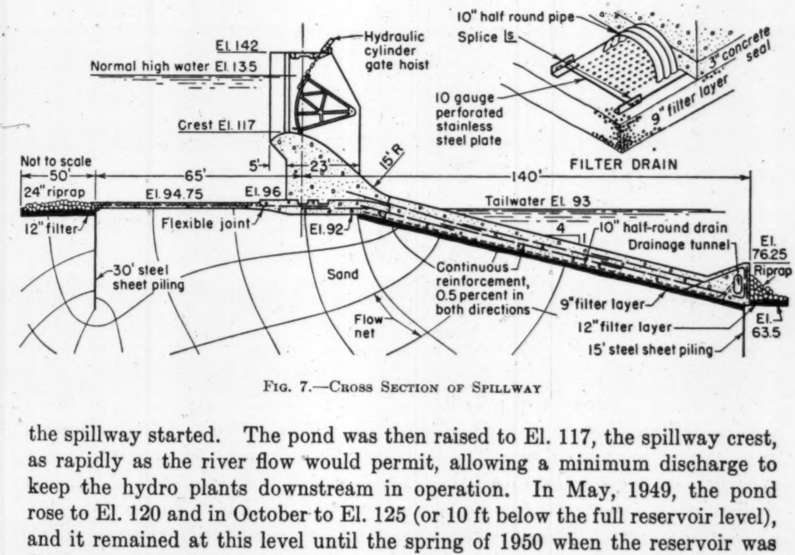

10" half round pipe
Splice ls
3" concrete seal
9" filter layer
10 gauge perforated stainless steel plate
FILTER DRAIN
El. 142
Hydraulic cylinder gate hoist
Normal high water El. 135
Crest El. 117
Not to scale
50'
65'
5'
23'
15'R
140'
24" riprap
El. 94.75
El. 96
Tailwater El. 93
12" filter
Flexible joint
El. 92
4
1
10" half-round drain
Drainage tunnel
El. 76.25
Riprap
30' steel sheet piling
Sand
Continuous reinforcement, 0.5 percent in both directions
9" filter layer
12" filter layer
El. 63.5
Flow net
15' steel sheet piling

FIG. 7.—CROSS SECTION OF SPILLWAY

As was expected, considerable leakage developed through and under the dam and dikes. By the latter part of March, 1949, general seepage increased, and boils appeared in a few places along the downstream toe of the dam. The larger boils and those close to the toe of the dam were blanketed with filter gravel to dissipate the flow and permit drainage without movement of sand. Because the structures were compacted to a density considerably higher than natural, the leakage through them was a great deal less than that through the foundation. Well points were installed in several sections of the dam and in the dikes to observe the seepage line. Fig. 6 shows the observed seepage lines for two of these sections. It can be seen that the top flow line is close to that found from the flow net study of the cross section made prior to construction (Fig. 5).

The leakage through and under the dikes was collected in a ditch located 50 to 75 ft from the toe. Flow net studies prior to construction indicated a total flow of about 108 cu ft per sec from the west dike, using an average coefficient of permeability of $k = 210$ ft per day for both the structure and the foundations. Measurements of the actual flow in the ditch indicated a flow of about 76 cu ft per sec. This agreement is well within the accuracy of measurement of the constants and quantities involved, especially because some of the seepage passed under the ditch and, therefore, was not measured. The east dike leakage was estimated prior to construction to amount to about 79 cu ft per sec, and measurements show an actual flow of about 24 cu ft per sec. The leakage through the main dam section could not be measured but is apparently about as expected.

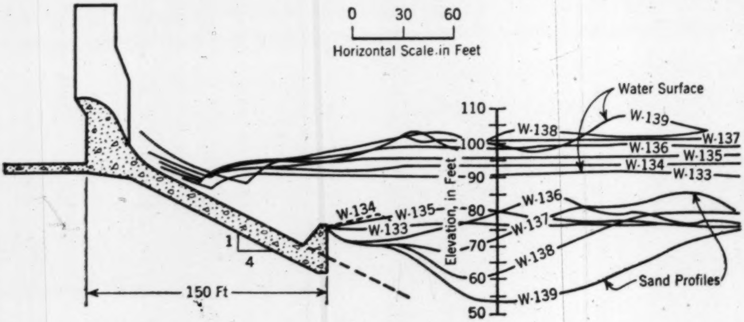


FIG. 8.—MODEL TEST OF SPILLWAY

When the reservoir was raised to full height in the early spring of 1950, considerable underground seepage developed outside of the original drainage ditch at the toe of the dike. The land outside of the dike is very flat, consequently the underseepage raised the ground-water table to the surface, forming shallow pools in low spots as far as a mile from the dike. To alleviate this condition, a second ditch was required, deeper than the original and located 200 to 500 ft from the dike. This satisfactorily intercepted the underseepage and dried up the surrounding land.

Spillway Construction.—The spillway contains 16 radial (tainter) gates, each 30 ft long and 18 ft high, designed to pass a flood of 134,000 cu ft per sec at normal reservoir elevation (135 Petenwell Datum) and capable of passing 200,000 cu ft per sec with a 5-ft surcharge on the reservoir. The maximum flood of record at this point in the river is about 95,000 cu ft per sec, having occurred in June, 1905. The drainage area of the river above the project is 5,800 sq miles.

The type of hydraulic design chosen for the spillway (Fig. 7) utilizes the principle of the sloping apron to provide the hydraulic jump. The foundation of the ogee section is at a very high level, just a few feet below the natural ground surface, with the sloping apron extending approximately 118 ft downstream to an elevation of 63.5, some 30 ft below the river level. Comparisons of the cost of this structure with one having a conventional horizontal apron, with or without baffle blocks, showed considerable saving in favor of the sloping apron design. Building the structure away from the river permits the foundation of the gravity section of the spillway to be about 10 ft higher than it would have been if built in the river bottom, with a considerable saving of concrete.

TABLE 1.—SPILLWAY MODEL TEST DATA

Run	Gate opening (feet)	ELEVATION (IN FEET)		FLOW (CU FT PER SEC)	
		Headwater	Tailwater	One gate	Sixteen gates
(1)	(2)	(3)	(4)	(5)	(6)
W-133	1.60	134.9	91.3	935	14,960
W-134	3.00	135.0	94.1	1,750	28,000
W-135	6.02	135.0	97.5	3,570	57,120
W-136	8.87	134.9	99.1	5,120	81,920
W-137	12.06	135.2	100.7	7,140	114,240
W-138	full	135.0	102.4	10,030	160,480
W-139	full	138.0	102.3	13,500	216,000

A series of model tests of the Petenwell spillway was carried out at the Hydraulic Laboratories of the University of Wisconsin at Madison. A model of a single bay was placed in a glass-walled flume, and tests were made with different lengths of apron, various sill shapes, and with and without baffles on the apron. The scale of this model was 1 to 25. The scour pattern with various flow and tailwater conditions was recorded for each variation of the model. Fig. 8 shows the scour patterns observed for various gate openings. The scour was continuous during this test; that is, the gate was opened a small amount, and a ½-hr run was made; then, without changing the sand configuration in the model, the gate was opened further, and another run was made.

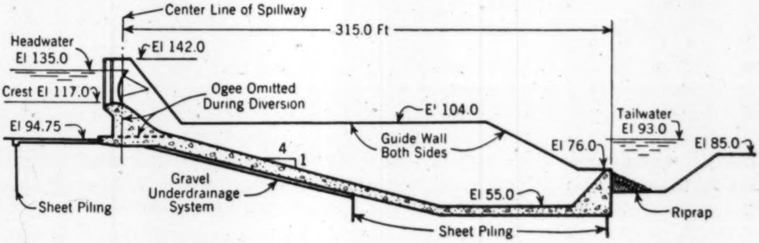

Center Line of Spillway
315.0 Ft
Headwater El 135.0
El 142.0
Crest El 117.0
Ogee Omitted During Diversion
El 104.0
Tailwater El 93.0
El 94.75
Guide Wall Both Sides
4
1
El 76.0
El 85.0
Gravel Underdrainage System
El 55.0
Sheet Piling
Riprap
Sheet Piling

FIG. 9.—CROSS SECTION OF REGULATING BAY

The test was continued in this way until the gate was wide open and the scour in the model had stabilized. Table 1 contains pertinent data on the test run shown in Fig. 8.

Fig. 8 shows the hydraulic action of this type of design. The sloping apron provides an increasing depth of tailwater as the overflow sheet proceeds downstream. The length of the apron required is such that the hydraulic jump will occur on the apron for all conditions of overflow and corresponding tailwater elevations. The sill at the end of the apron deflects the water upward and sets up a large ground roller wave that causes a strong upstream current along the bottom just downstream from the sill. This is the fundamental point in the design. The upstream current deposits material against the sill rather than scouring at that point. Both the length of the sloping apron and the shape and height of the sill were determined from the model studies.

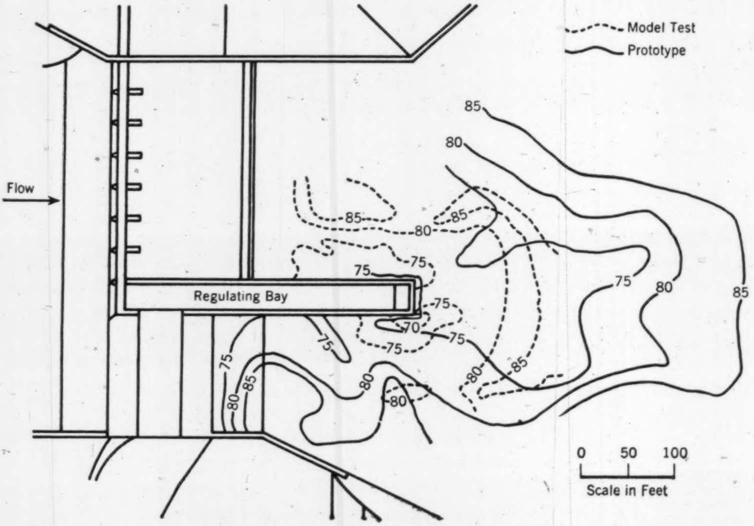


FIG. 10.—REGULATING BAY SCOUR PATTERN

Each of the models tested was operated for one run with the tailwater corresponding to an elevation about 5 ft below the normal river elevation for the corresponding flow. This was done to insure that satisfactory operation would be obtained if (*a*) the tailwater rating curve were not accurate and (*b*) if any tailwater degradation were to occur.

The spillway structure, as shown in Fig. 7, is founded directly on the sand, on a concrete slab 4 ft thick with $\frac{1}{2}$ of 1% continous reinforcing steel in both directions. The reinforcement runs through all joints. The ogee monoliths resting on the foundation slab are 35 ft long, separated by a contraction joint located in the middle of each gate opening. The slab under the gravity (ogee) section is tied into an upstream apron slab that has a steel sheet pile cutoff 30 ft deep at its upstream edge. The downstream sloping apron is also monolithic with the slab under the ogee section and is provided with a sheet pile cutoff 15 ft deep at its downstream edge. Sheet pile cutoffs are provided at each end of the structures, thus completely confining the sand in the foundations. All slabs have continuous reinforcing of 0.5% in both directions running through all joints.

The drainage provisions under the downstream sloping apron are of great importance because it is essential that no pressure from headwater be permitted to act against the apron. A drainage layer 9 in. thick was placed under the entire apron, as shown in Fig. 7, designed to act as a filter and prevent the foundation sand from being carried through. The leakage into the filter drain is collected in half-round pipes spaced 17.5 ft on centers at the bottom of the apron slab and then goes into 2-ft by 6-ft drainage tunnel in the toe of the apron. This tunnel discharges into the tailrace through openings in the training walls at each end.

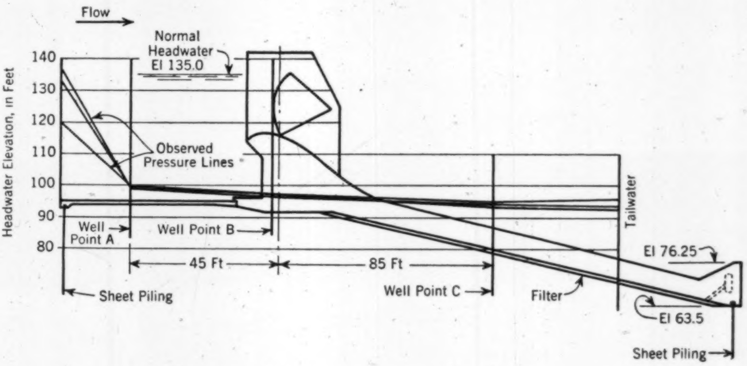


FIG. 11.—PRESSURE LEVELS IN SPILLWAY SECTION

The details of the joint between the ogee section and the upstream slab are also shown in Fig. 7. This joint must remain watertight, otherwise the effect of the slab is lost entirely. The joint was so designed that a positive bearing pressure exists at all times on the horizontal face. This pressure is a maximum before the water pressure acts against the ogee section and is reduced when the water load comes on. However, the design maintained a residual pressure of about 700 lb per linear ft after all the water load was against the dam. A poured hot rubber-asphalt seal was also placed at the joint to provide additional security against leakage.

The upstream, or heel, apron provides additional resistance against sliding of the structure. It is watertight, and, therefore, a net vertical force acts downward on the slab. This force is equal to the difference of the downward pressure of the full head and the upward pressure under the slab, which is progressively reduced in the downstream direction by friction loss in the foundation sand. The maximum calculated bearing pressure under the spillway is about 2,500 lb per sq ft, and the sliding factor is 0.4.

The sand bed of the river is so erodible that the spillway gates in the main section must all be opened at the same time and to an equal opening. If this is not done, eddies that would soon scour deep holes in the sand would be set up downstream from the spillway and jeopardize the safety of the structure. However, opening all the gates at once is not a satisfactory method of operation to pass flows only slightly in excess of the plant capacity. Ice or trash could not be passed through the dam since opening all gates wide enough to do this would waste so much water. As a result of these factors it was decided to provide the necessary stilling basin for one gate so that it could be opened by itself. This bay is called the regulating bay, and the section is shown in Fig. 9.

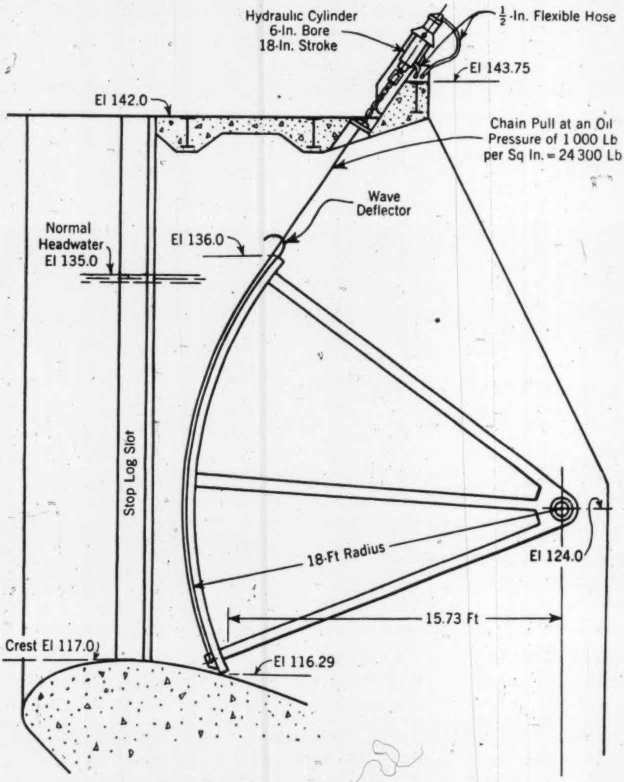

Hydraulic Cylinder
6-In. Bore
18-In. Stroke
$\frac{1}{2}$-In. Flexible Hose
El 143.75
El 142.0
Chain Pull at an Oil Pressure of 1 000 Lb per Sq In. = 24 300 Lb
Wave Deflector
Normal Headwater El 135.0
El 136.0
Stop Log Slot
18-Ft Radius
El 124.0
15.73 Ft
Crest El 117.0
El 116.29

FIG. 12.—DETAIL OF GATE MECHANISM

Because the flow through one bay causes so much lower tailwater than caused by opening the other 15 gates correspondingly, it was necessary to extend the sloping apron farther out for the regulating bay than for the remainder of the spillway. It was also necessary to confine the flow for a sufficient distance so the major portion of the energy of the water is dissipated by the time the water discharges to the sandy river bed. The slope of the retaining walls at the downstream edge causes a fanning out of the flow as it leaves the stilling basin and prevents undesirable eddies at the corners. The design of the regulating bay was also verified by model tests. Fig. 10 shows the contours of the scour pattern as observed in the model tests.

Diversion was carried out by omitting the ogee section of the regulating bay and permitting the river to pass through this bay. During the spring of 1949 a flow of more than 10,000 cu ft per sec was passed through this structure. This is considerably more flow than the 8,000 cu ft per sec that can pass through the gate of this bay with the ogee section in place, and it was a good test of the prototype. The scour caused by this condition was measured by sounding and observations by a diver, and these contours are also shown in Fig. 10. Comparison of the contours with the model test scour pattern indicates the correlations between model and prototype to be satisfactory.

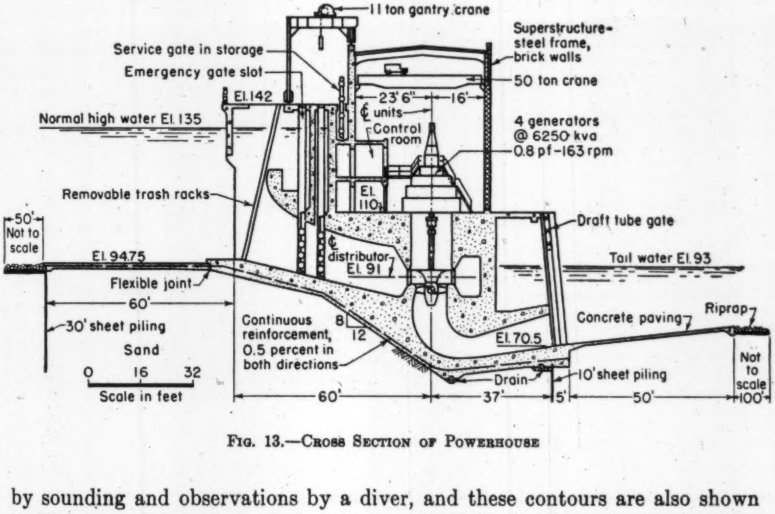

11 ton gantry crane
Superstructure-steel frame, brick walls
Service gate in storage
Emergency gate slot
50 ton crane
El. 142
23'6"
16'
℄ units
Control room
Normal high water El. 135
4 generators @ 6250 kva 0.8 pf-163 rpm
El. 110
Removable trash racks
Draft tube gate
50' Not to scale
℄ distributor El. 91
El. 94.75
Tail water El. 93
Flexible joint
60'
Riprap
Concrete paving
30' sheet piling
Continuous reinforcement, 0.5 percent in both directions
8
12
El. 70.5
Sand
Not to scale
0
16
32
Scale in feet
Drain
10' sheet piling
60'
37'
5'
50'
100'

FIG. 13.—CROSS SECTION OF POWERHOUSE

To determine the actual water pressure under the spillway structure and thus obtain information as to the adequacy of the upstream sheet piling cutoff and apron, well points were placed under the upstream slab at three sections of the structure. Pipes from the well points were brought up in the piers to permit easy observation. Fig. 11 shows the observed pressures at one of these sections (the others were similar) for various reservoir elevations. The large pressure drop between headwater pressure and the first well point (Point A on Fig. 11) shows that the sheet pile cutoff is functioning very well. It should be noted that this pressure drop is greater than would be anticipated from a flow net drawn on the basis of a coefficient of permeability equal in all directions. It indicates that the coefficient of permeability in the vertical direction is smaller (less pervious) than in the horizontal direction, a condition to be expected in an alluvial bed of sand. This condition also adds to the factor of safety against sliding.

SPILLWAY GATE HOISTS

A novel gate hoist mechanism (Fig. 12) was used to insure uniform operation of the 15 gates in the main spillway section. Each gate is raised by a pair of hydraulic cylinders, all of which are operated from a single pumping system. The maximum stroke of the cylinders is 18 in., resulting in a maximum lift of the gates of about 12 in. The gates are dogged after being lifted 12 in., and the hook is re-engaged to permit the next lift. This operation takes about 30 min for all gates. The flow permitted by the first 1-ft opening of all gates is about 9,000 cu ft per sec.

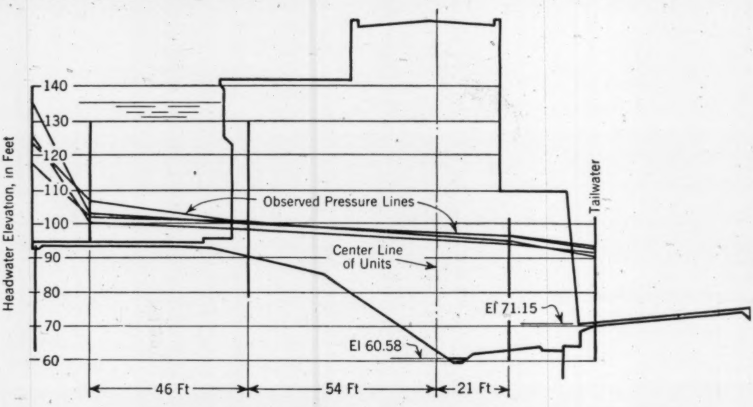


FIG. 14.—PRESSURE LEVELS IN POWERHOUSE SECTION

This hoisting system proved less than half as expensive as any other system that would permit uniform opening of all the gates at the same time, and it is operating satisfactorily. Some adjustment of the valving was required to balance the system, but otherwise no difficulties have been experienced. Also the gates can be raised individually if conditions require. The gate on the regulating bay is operated by a conventional motor operated hoist.

POWERHOUSE

Fig. 13 shows a cross section of the powerhouse. The same principles of foundation design were applied in this structure as in the spillway. A continous slab reinforced with 0.5% of steel in both directions forms the foundation. This foundation slab is integral with the spillway foundation slab, so in effect both structures are on a continous mat that is bounded on all sides by steel sheet piling. The drainage provisions are shown on the section.

The powerhouse substructure and the intake represent a single monolith over 140 ft long without contraction joints. Shrinkage cracks were held to a minimum by reducing the size of pours and building alternate units ahead of others. The intake bulkhead wall separating the generator room from headwater also contains 0.5% of steel in the horizontal direction. Its watertightness has proved very satisfactory.

Vertical-lift wheeled gates are provided for the intake. The gates are of all welded construction, and the wheels are equipped with roller bearings to reduce hoisting effort and eliminate by-pass, or filler, valves. The trashracks of all welded construction are built in sections that are self supporting and can be removed entirely, thus eliminating any fixed submerged members inaccessible to inspection.

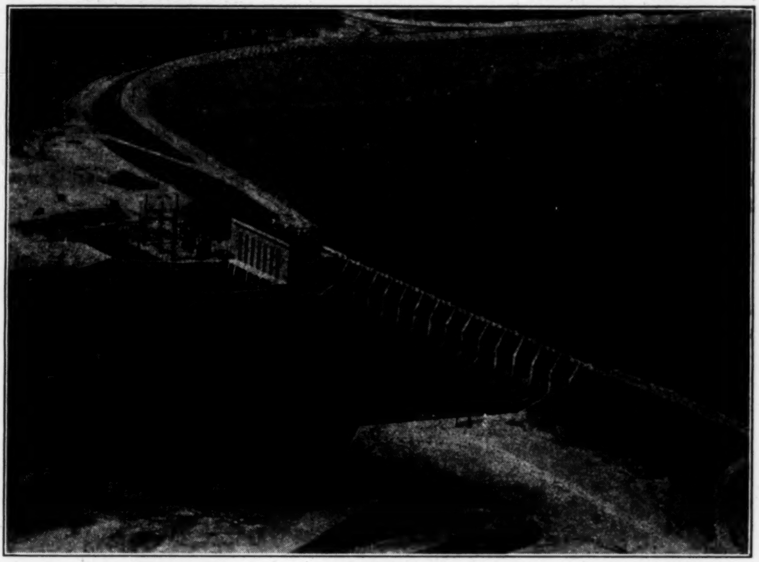


Fig. 15.—Aerial View of Completed Project

Water pressure readings have been taken under the powerhouse, similar to those for the spillway and main dam. The results of readings at a typical section are shown in Fig. 14. The large pressure drop caused by the sheet piling cutoff is clearly evidenced. Although the drainage provisions under the draft tube do not appear to be quite as effective as the drainage layer under the spillway apron, the pressures are well within the design limits, and observations of the flow from the drains indicate they function adequately.

In the case of a river flowing on sand as this one does, it was anticipated that the river levels below the dam would be lowered considerably after its completion because the reservoir traps all the bed load in the river and a new bed load will be picked up by the river after passing the dam. The turbines are set low to provide for this anticipated degradation.

Settlement records of the spillway and powerhouse have shown that very little, if any, settlement has occurred. The load spreading effect of the highly reinforced continuous foundation slab combined with the relatively low bearing pressures would lead to expectations of only insignificant settlements.

An aerial view of the completed project in operation is given in Fig. 15.

CONCLUSION

The successful construction and operation of this project has shown that water-retaining structures of considerable size can be built on sand foundations, providing adequate precautions to confine the sand and to handle the underseepage are incorporated in the design. The successful performance of the spillway to date indicates that the sloping apron type of design prevents harmful erosion of the sand in the river bed at a relatively low cost.

ACKNOWLEDGMENTS

The powerhouse and spillway were constructed by forces of the Wisconsin River Power Company under the direction of W. F. Thiele, Executive Vice President, and Ralph J. Kutchera, J. M. ASCE, project manager. Foley Bros.-Marsch-Peterson were the contractors for the earth dams.

Arno T. Lenz, A. M. ASCE, of the University of Wisconsin directed the model tests of the spillway.

The Harza Engineering Company was the consulting engineer on this work, which was done under the direction of L. F. Harza, M. ASCE, and the writer. C. E. Wattles was resident engineer on construction.

DISCUSSION

L. F. HARZA,[2] M. ASCE.—The paper sets forth the design and construction of a hydroelectric project on deep sand. This design takes advantage of the possibilities of convenience and economy offered by the sand foundation by making possible:

1. Complete construction of concrete spillway and powerhouse in a dragline trench away from the river channel, thus avoiding erection of a cofferdam and eliminating river construction hazards.
2. Founding of the spillway at riverbank instead of river-bottom elevation with only a sloping apron extending below river elevation.
3. Complete omission of bearing piles because of the low unit foundation load.
4. The use of a heel apron for lengthening the seepage travel distance and increasing the sliding resistance. The heel apron, of course, must contain enough steel to develop the full sliding resistance resulting from the water load on it. This is a very inexpensive means of accomplishing these two purposes.
5. Use of an homogeneous sand dam on sand foundation without resorting to sheet pile or other cutoffs, on the theory that leakage is to be expected; and providing for leakage in such a way as to make it harmless except as an economic loss.
6. Closure of the river by several bulldozers on each side operating on the sole principle of pushing sand into the river faster than the river could carry it away in spite of several feet of fall through the gap (as shown in Fig. 4). An attempt was made to dump loose rock ahead of the bulldozers as the work progressed, but the amount used is believed to have been so minor a proportion of the sand volume as to have had negligible value.

The Petenwell Project is one of three applications of most of the foregoing principles by the writer's firm and about the tenth example of the omission of bearing piles under powerhouse and heavy structures on sand foundations. Dependence is placed solely upon confining the foundation sand by sheet piling. All these applications have apparently proved successful. The settlement, if any, at Petenwell is so small as to be obscured by unavoidable errors of measurement.

These procedures will doubtless be criticized by some of the old timers who would never, under any circumstances, omit bearing piles under any structure on soft foundation or eliminate sheeting cutoff from any earth dam and to whom the seepage losses would cause heart failure.

E. MONTFORD FUCIK,[3] A. M. ASCE.—The design of this project was controlled by the nature of the sand foundation. The major features of this design, from this viewpoint, have been presented clearly by Mr. Harza.

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[2] Pres., Harza Eng. Co., Cons. Engrs., Chicago, Ill.

[3] Vice-Pres., Harza Eng. Co., Chicago, Ill.

The major feature that cannot be readily checked by visual observation is the erosion downstream from the spillway caused by floods passing over it. During the spring of 1951, a flood of approximately 55,000 cu ft per sec was passed over this spillway, requiring the gates to be raised about 6 ft. Although this flood was considerably less than the capacity of the spillway at normal headwater elevation (135,000 cu ft per sec), the owners decided to check the conditions at the toe of the spillway after the flood by having a diver make a thorough inspection. This inspection revealed no erosion along the length of the spillway, and the diver reported that the riprap downstream from the spillway was covered with enough sand to make walking very easy. This information, coupled with the extensive model tests which had been made on this type of sloping apron, would seem to indicate clearly the satisfactory operation of this design.

The closure operation, which was accomplished very simply, was the cause of some concern during the design stages of the project. Its successful accomplishment indicates that this method can be used whenever the head required for diversion is small, when a considerable amount of equipment is available, and where there is space for it all to operate at the same time.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2506

BUCKLING STRESSES FOR FLAT PLATES AND SECTIONS

By Elbridge Z. Stowell,[1] George J. Heimerl,[2] Charles Libove,[3] J. M. ASCE, and Eugene E. Lundquist,[4] A. M. ASCE

With Discussion by Messrs. P. P. Bijlaard; and Elbridge Z. Stowell, George J. Heimerl, Charles Libove, and Eugene E. Lundquist

Synopsis

The results of extensive studies in the buckling of flat unstiffened plates conducted in the field of aeronautics during and since World War II are surveyed in this paper. The presentation of a broad perspective, rather than a detailed picture, is intended. Therefore only a few important loadings are considered. For single plates, these loadings are compression, shear, and their combination, and for the integral flat-plate sections, compression only.

The buckling stresses for various types of plate under these several loadings are given in the form of a nondimensional chart for the elastic stress range. The use of the stress-strain curve to calculate buckling stresses in the plastic range is explained. A correlation between buckling strength and maximum strength is shown for the integral flat-plate sections. Good agreement is shown between theory and experiment in a few key cases. The theoretical and experimental techniques underlying the buckling investigation are discussed briefly in the appendixes.

The good agreement shown between theory and experiment indicates that the present status of the problem of calculating flat-plate buckling strengths over the entire range of stress is now satisfactory from an engineering viewpoint.

Note.—Published in July, 1951, as *Proceedings-Separate No. 77.* Positions and titles given are those in effect when the paper or discussion was received for publication.

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INTRODUCTION

The use of lightweight structures of thin-sheet metal for aircraft has stimulated much research on the buckling of plates. Considerable basic progress has been made in this field of engineering during and since World War II. The use of thin-walled members in civil engineering structures makes such progress of interest to civil engineers as well as to aeronautical engineers. The purpose of this paper, therefore, is to bring part of the recent developments (mostly that part due to the Structures Research Division of the National Advisory Committee for Aeronautics (NACA)) more directly to the attention of the former group.

The importance of knowing buckling stress lies in the fact that buckling represents the beginning of a radically different regime in the behavior of a structure. In an originally flat plate, buckling is the theoretically sudden appearance of small deflections perpendicular to the plane of the plate. In practice, however, the slight initial deviations from flatness result in deflections from the beginning of loading which accelerate in growth as the theoretical buckling load for the flat plate is approached. The post-buckling regime is characterized by reduced stiffness and a wrinkling whose physical (and sometimes psychological) effects may be undesirable. The physical processes that eventually lead to the failure of a plate are initiated at buckling, and for some structures, a knowledge of the buckling stress is a key to the calculation of the additional strength beyond buckling. Since buckling is not always equivalent to failure, appreciable economies in design may often be effected by permitting the plate to buckle and taking advantage of its post-buckling strength. In aircraft design, for example, standard practice frequently permits buckling within the range of working stresses. The philosophy behind such design is similar to the philosophy of limit design proposed for civil engineering structures.[5,6,7]

The problem chosen for discussion is that of calculating the buckling stresses of flat unstiffened plates and integral flat-plate sections (integral assemblies of such plates). The objective of this paper is to convey a broad perspective of the problem, rather than an exhaustive treatment. For this reason, only a few important loading conditions are considered. For single plates, these loadings are compression, shear, and combined compression and shear; for sections, only compression. The buckling stresses for these cases are given in the form of theoretical charts. Both plastic and elastic buckling are considered. Sections on correlation with experiment and on maximum strength beyond the buckling load are also included. The theories underlying the calculation of the charts are indicated briefly in Appendix I, in which reference is also made to the literature on plate-buckling theory. The experimental techniques are described in Appendix II.

Notation.—The letter symbols in this paper are defined where they first appear.

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[5] "Theory of Limit Design," by J. A. Van den Broek, *Transactions*, ASCE, Vol. 105, 1940, p. 638.

[6] "Theory of Limit Design," by J. A. Van den Broek, John Wiley & Sons, Inc., New York, N. Y., 1948.

[7] Review by F. R. Shanley of "Theory of Limit Design," by J. A. Van den Broek, *Aeronautical Engineering Review*, Vol. 7, April, 1948, p. 73.

ELASTIC BUCKLING

Form of Presentation of Theoretical Results.—A theoretical solution for the elastic buckling stress of a plate under compression load can be expressed by a numerical coefficient k_c (in which subscript c denotes "compression"), from which the buckling stress σ can be calculated by use of the formula,[8]

$$\sigma = k_c \frac{\pi^2 E}{12\,(1 - \mu^2)} \left(\frac{t}{b}\right)^2 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which E is Young's modulus; μ is Poisson's ratio; t is the plate thickness; and b is the plate width. Similarly, the shear buckling stress τ is obtained by substituting a numerical coefficient k_s (subscript s denoting "shear buckling stress") in the formula,

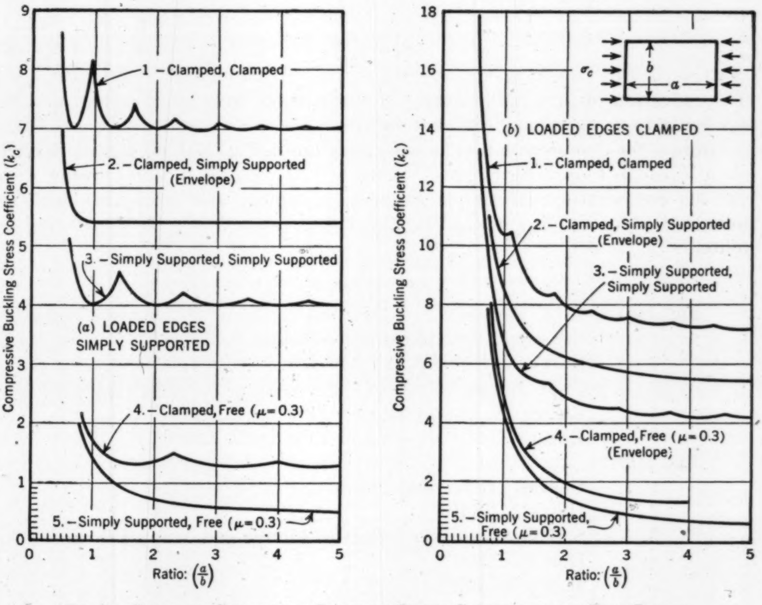

FIG. 1.—VALUE OF COMPRESSIVE BUCKLING STRESS COEFFICIENT FOR FLAT PLATES

$$\tau = k_s \frac{\pi^2 E}{12\,(1 - \mu^2)} \left(\frac{t}{b}\right)^2 \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

For given boundary conditions, the coefficients k_c and k_s generally depend upon the length-to-width ratio of the plate and can be plotted as a function of this ratio.

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[8] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1936, p. 331.

When both types of stress are present, the combinations that will cause buckling can be defined by combinations of values of the stress coefficients k_c and k_s to be used in Eqs. 1 and 2. For given boundary conditions and length-to-width ratio, a so-called "interaction curve" of k_c against k_s can be plotted. The interaction curves can also be plotted in terms of stress ratios R_c and R_s,[8] in which R_c is the ratio of the compressive stress σ_{cm} present at buckling under combined loading to the compressive stress σ_c required for buckling under simple compression loading; and R_s is defined similarly for shear. In equation form, the stress ratios can be defined as

$$R_c = \frac{\sigma_{cm}}{\sigma} \qquad (3)$$

and

$$R_s = \frac{\tau_{cm}}{\tau} \qquad (4)$$

The reduction of interaction curves to stress-ratio form often reveals a basic similarity in shape among different curves.

Results for Compression.—The inset diagram in Fig. 1 shows a plate loaded along breadth b on both edges. Values of k_c for plates in compression with various combinations of simple support, clamping, and freedom along the unloaded edges are summarized in Fig. 1, the respective authorities being as follows:

Curve	Authority
Fig. 1(*a*):	
1	Eugene E. Lundquist and Elbridge Z. Stowell[9]
2	Harry N. Hill,[10] A. M. ASCE
3	G. H. Bryan[11]
4	Eugene E. Lundquist and Elbridge Z. Stowell[9]
5	Eugene E. Lundquist and Elbridge Z. Stowell[12]
Fig. 1(*b*):	
1	Charles Libove and Manuel Stein[13]
2	Harry N. Hill[10]
3	S. Timoshenko[8] and Charles Libove and Manuel Stein[13]
4	Harry N. Hill[10]
5	Harry N. Hill[10]

The caption "clamped, simply supported" on Curve 2, Fig. 1(*a*) (as an example), means that one unloaded edge is clamped and the other is simply supported. Coefficient k_c is plotted for each combination of edge conditions as a function of the length-to-width ratio a/b of the plate.

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[9] "Critical Compressive Stress for Flat Rectangular Plates Supported Along all Edges and Elastically Restrained Against Rotation Along the Unloaded Edges," by Eugene E. Lundquist and Elbridge Z. Stowell, *Report No. 733*, NACA, 1942.

[10] "Chart for Critical Compressive Stress of Flat Rectangular Plates," by H. N. Hill, *Technical Note No. 773*, NACA, 1940.

[11] *Proceedings*, London Mathematical Soc., Vol. 22, 1891, p. 54.

[12] "Critical Compressive Stress for Outstanding Flanges," by Eugene E. Lundquist and Elbridge Z. Stowell, *Report No. 734*, NACA, 1942.

[13] "Charts for Critical Combinations of Longitudinal and Transverse Direct Stress for Flat Rectangular Plates," by Charles Libove and Manuel Stein, *Wartime Report No. L-224*, NACA, 1946, pp. 321–401.

The boundary conditions of simple support and clamping are seldom exactly realized in practice. Usually there is some degree of elastic restraint against rotation along a supported edge, such as those symbolized in Fig. 2. Values of the compressive buckling stress coefficient k_c for each of the conditions represented in Fig. 2 are given by the curves in Figs. 3 and 4, respectively: For a longitudinally compressed outstanding flange[12] (Fig. 3); for a longitudinally compressed long plate with both long edges equally restrained against rotation[10] (Fig. 4(a)); and for the same plate transversely compressed[14] (Fig. 4(b)).

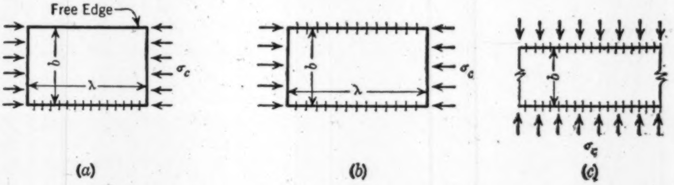

FIG. 2

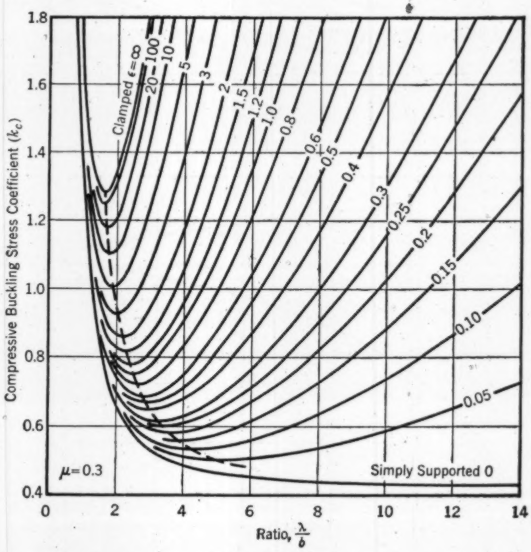

FIG. 3.—COMPRESSIVE BUCKLING STRESS COEFFICIENT k_c FOR A FLANGE WITH ITS SUPPORTED EDGE RESTRAINED ELASTICALLY AGAINST ROTATION ($\mu = 0.3$)

In Figs. 3 and 4 the magnitude of the elastic rotational restraint appears in the parameter ϵ, which is essentially the ratio of restraint stiffness to plate

---

[14] "A Method for Determining the Column Curve from Tests of Columns with Equal Restraints Against Rotation on the Ends," by Eugene E. Lundquist, Carl A. Rossman, and John C. Houbolt, *Technical Note No. 903*, NACA, 1943.

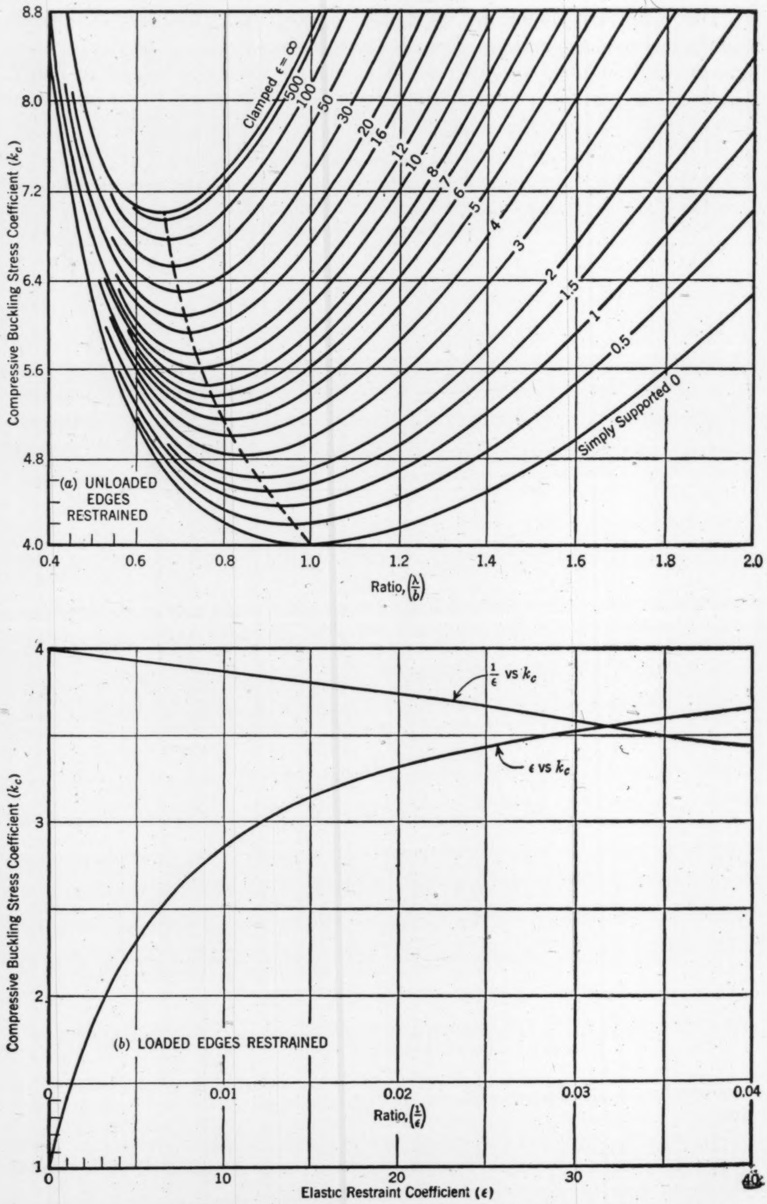

FIG. 4.—COMPRESSIVE BUCKLING STRESS COEFFICIENT FOR A LONG PLATE WITH A PAIR OF MUTUALLY OPPOSITE EDGES HAVING EQUAL ELASTIC RESTRAINT AGAINST ROTATION

stiffness and is defined[9] by the equation,

$$\epsilon = \frac{4\,S_o}{S_f/b}\text{.........................}(5)$$

in which S_e is the stiffness of the elastic restraining medium, expressed as moment per unit length per quarter radian of rotation, assumed constant along the edge of the plate; and the plate flexural stiffness is

$$S_f = \frac{E\,t^3}{12\,(1 - \mu^2)}\text{.....................}(6)$$

For simply supported edges, $S_e = \epsilon = 0$, and for clamped edges, $S_o = \epsilon = \infty$.

In Figs. 3 and 4(*a*), k_c is given not only as a function of ϵ but also as a function of λ, the buckle half-wave length or distance between successive nodes. (For the case of Fig. 4(*b*), λ is infinity.) The half-wave length λ was included as a parameter in Figs. 3 and 4(*a*) because the value of ϵ will depend upon λ for many types of elastic restraint, and also upon k_c if the restraining medium carries a load proportional to that on the plate. (The evaluation of ϵ in terms of λ and k_c for several types of elastic restraining medium is discussed subsequently.) Since λ and k_c are not known in advance, neither is ϵ. For a given plate and a given restraining medium, therefore, trial-and-error computation must be made. Mutually consistent sets of values of ϵ and λ must be assumed until a minimum value of k_c is obtained. A finite plate with simply supported edges will buckle with its length a divided into an integral number of half waves. The governing value of λ, therefore, must be sought in the sequence $a/1$, $a/2$, $a/3$, . . . and is that term which gives the lowest values of k_c.

For the academic case of a restraining medium in which rotation of one section does not influence rotations at adjacent sections (such as a series of discrete coil springs) ϵ will be independent of λ. Then, for an infinitely long plate, the buckle pattern will adjust itself to the value of λ at the minimum point of the appropriate ϵ-curve, and the plate will buckle at the corresponding stress.

Results for Shear.—Values of k_s for plates in shear with all edges either simply supported[15] or clamped[16] are given in Fig. 5(*a*). In addition, the value of k_s is given for a square plate with one pair of opposite edges clamped and the other pair simply supported.[17] For these latter boundary conditions the buckling stress will approach either the buckling stress of a plate with all edges clamped or all edges simply supported as the plate becomes more and more rectangular, depending upon which pair of edges becomes the longer.

The values of k_s for a long plate with both side edges having equal elastic restraint against rotation are given in Fig. 5(*b*),[18] which is similar to Fig. 4(*a*)

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[15] "Buckling Stresses of Simply Supported Rectangular Plates in Shear," by Manuel Stein and John Neff, *Technical Note No. 1222*, NACA, 1947.

[16] "Buckling Stresses of Clamped Rectangular Flat Plates in Shear," by Bernard Budiansky and Robert W. Connor, *Technical Note No. 1559*, NACA, 1948.

[17] "Die Knickung der rechteckigen Platte durch Schubkrafte," by S. Iguchi, *Ingenieur-Archiv*, Bd. IX, Heft 1, February, 1938, pp. 1–12.

[18] "Critical Shear Stress of an Infinitely Long Flat Plate with Equal Elastic Restraints Against Rotation Along the Parallel Edges," by Elbridge Z. Stowell, *Wartime Report No. L-476*, NACA, 1943.

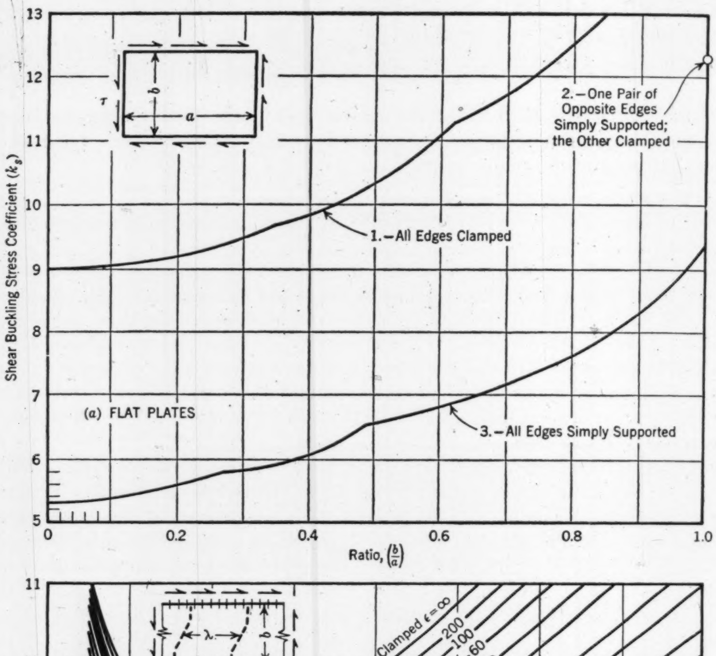

FIG. 5.—VALUE OF SHEAR-BUCKLING STRESS COEFFICIENT k_s

for compression. The discussion of ϵ and λ in connection with compression also applies in Fig. 5 for shear.

Results for Combined Compression and Shear.—Combinations of values of R_c and R_s for which a simply supported plate in combined compression and shear will buckle are given in Fig. 6.[19] The results are in the form of interaction curves of R_s versus R_c for different length-to-width ratios a/b. Fig. 6 shows that, when the compressive stress acts in a longitudinal direction ($a/b < 1$), the interaction curve is described by the parabola

$$R^2_s + R_c = 1 \qquad (7)$$

However, as the length-to-width ratio changes so that the compressive stress assumes a transverse nature ($a/b > 1$), the shape of the interaction curve changes markedly. An interesting characteristic of the curve for $a/b = \infty$ is the vertical part at $R_c = 1$, which reveals that an appreciable amount of shear stress may be applied to the plate before the transverse compressive stress required for buckling is reduced.

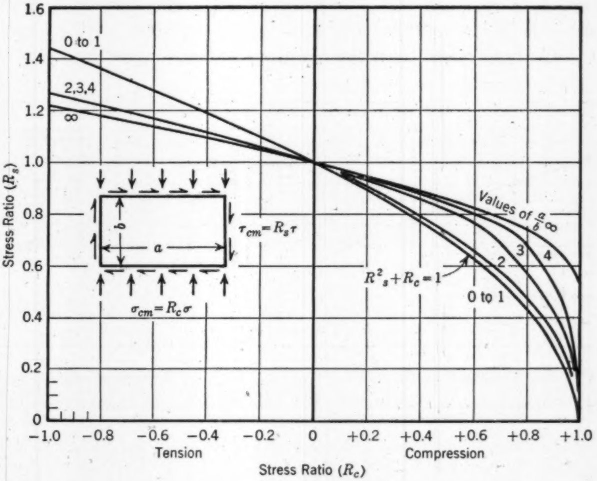

FIG. 6.—CRITICAL COMBINATIONS OF SHEAR AND DIRECT STRESS FOR A SIMPLY SUPPORTED FLAT PLATE

The effect of elastic restraint against rotation is shown in Fig. 7 for a long plate with both edges equally restrained against rotation. For the case of longitudinal compression and shear in Fig. 7(a),[20] the interaction curve for all degrees of restraint between simple support and clamping can still be described by Eq. 7. Although Fig. 7(a) and Eq. 7 were developed for the type of elastic restraint whose stiffness is unaffected by buckle wave length and load on the plate, the same parabolic relationship was also found to apply when the restraint is furnished by a stiffener whose stiffness S_o is affected by both these quantities.[20] For the case of transverse compression and shear shown in Fig. 7(*b*) different curves are obtained for different values of ϵ (independent of load and buckle wave length). All the curves have the vertical sections found to exist for the simply supported plate in Fig. 7(*b*).[21]

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[19] "Critical Combinations of Shear and Direct Stress for Simply Supported Rectangular Flat Plates," by S. B. Batdorf and Manuel Stein, *Technical Note No. 1223*, NACA, 1947.

[20] "Critical Stress for an Infinitely Long Flat Plate with Elastically Restrained Edges Under Combined Shear and Direct Stress," by Elbridge Z. Stowell and Edward B. Schwartz, *Wartime Report No. L-340*, NACA, 1943.

Evaluating the Stiffness of the Elastic Restraint.—The charts for the buckling of elastically restrained plates, described in the previous sections, apply to the case in which S_o, the stiffness of the elastic restraint, is a constant along the edge. In other words, the ratio of the bending moment intensity exerted by the buckled plate on the restraining medium to the rotation produced is assumed to be the same at all points along the edge of the plate. Fortunately, this condition is satisfied either exactly or approximately in many practical cases, particularly those involving long uniform plates with uniform elastic restraint. In these cases, both the bending-moment intensity and the rotation vary sinusoidally along the edge of the plate and are in phase with each other. Therefore, the ratio between the two is a constant.

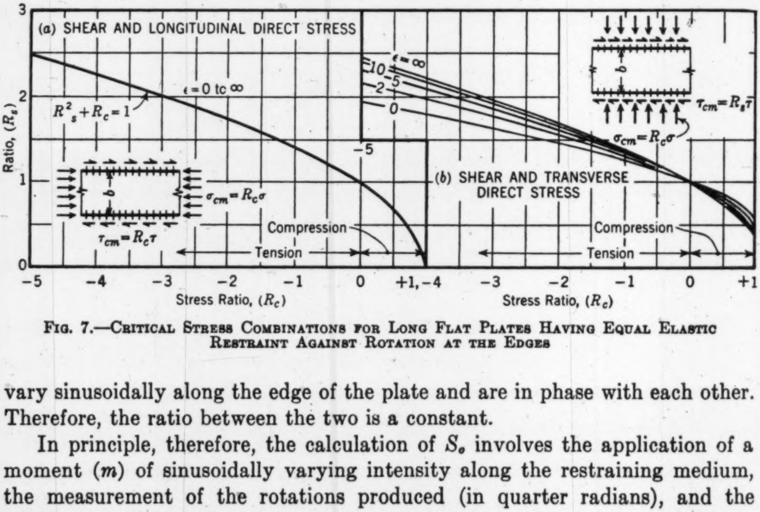

FIG. 7.—CRITICAL STRESS COMBINATIONS FOR LONG FLAT PLATES HAVING EQUAL ELASTIC RESTRAINT AGAINST ROTATION AT THE EDGES

In principle, therefore, the calculation of S_o involves the application of a moment (*m*) of sinusoidally varying intensity along the restraining medium, the measurement of the rotations produced (in quarter radians), and the calculation of the ratio between moment intensity and rotation as shown in Fig. 8.

If the restraint is furnished by a "sturdy stiffener,"—that is, a stiffener that suffers no cross-sectional distortion when it twists—its stiffness is given by the formula[22]

$$S_o = \frac{\pi^2}{4\,\lambda^2}\left(G\,J - \bar{\sigma}\,I_p + \frac{\pi^2}{\lambda^2}\,E\,C_{BT}\right) \ldots\ldots\ldots\ldots\ldots (8)$$

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[21] "Critical Combinations of Shear and Transverse Direct Stress for an Infinitely Long Flat Plate with Edges Elastically Restrained Against Rotation," by S. B. Batdorf and J. C. Houbolt, *Report No. 847*, NACA, 1946.

[22] "Restraint Provided a Flat Rectangular Plate by a Sturdy Stiffener Along an Edge of the Plate," by Eugene E. Lundquist and Elbridge Z. Stowell, *Report No. 735*, NACA, 1941.

in which λ is the half-wave length of a sinusoidally varying distributed twisting moment; G is the shear modulus of elasticity; J is the torsion constant of a stiffener; $G\,J$ is the torsional stiffness of a stiffener; $\bar{\sigma}$ is the average compressive stress in a stiffener; I_p is the polar moment of inertia of a stiffener sectional area about its axis of rotation; and C_{BT} is the torsion-bending constant of a stiffener sectional area about its axis of rotation at or near the edge of a plate.[23,24,25] The first term in parentheses in Eq. 8 gives the stiffness according to St. Venant; the second term gives the reduction in stiffness due to the compressive load in the stiffener; and the third term gives the stiffening effect due to the fact that the cross sections under a variable rate of twist are partly restrained against longitudinal warping.

If the plate being restrained is part of an assembly of plates, then the elastic restraining medium may itself be an adjoining plate under compression with its own far edge clamped, simply supported, free, or subjected to a moment equal and opposite to that at its near edge. In such a case, the stiffness of the restraining medium can be evaluated from tables of stiffness and carry-over factor for flat rectangular plates under compression, prepared by Wilhelmina D. Kroll in 1943.[26] In this table, the values S_o of a plate with its far edge clamped, simply supported, free, or subjected to a moment equal and opposite to that at the near edge are denoted by S, S^{II}, S^{III}, and S^{IV}, respectively, and are given as functions of the stress in the plate and the half-wave length of edge rotation.

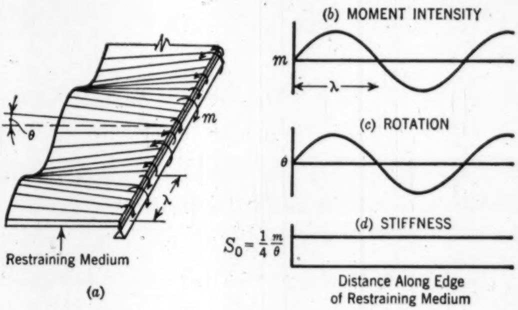
θ
m
λ
Restraining Medium
(a)
(b) MOMENT INTENSITY
m
λ
(c) ROTATION
θ
(d) STIFFNESS
$S_0 = \frac{1}{4}\frac{m}{\theta}$
Distance Along Edge of Restraining Medium

FIG. 8.—EVALUATION OF THE ROTATIONAL STIFFNESS S_o OF AN ELASTIC RESTRAINING MEDIUM

In some cases, the far edge of the restraining plate may itself be restrained elastically against rotation by one or more plates with a combined stiffness S_1.

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[23] "Torsion and Buckling of Open Sections," by Herbert Wagner, *Technical Memorandum No. 807*, NACA, 1936.

[24] "Torsion and Buckling of Open Sections," by H. Wagner and W. Pretschner, *Technical Memorandum No. 784*, NACA, 1936.

[25] "Twisting Failure of Centrally Loaded Open-Section Columns in the Elastic Range," by Robert Kappus, *Technical Memorandum No. 851*, NACA, 1938.

[26] "Tables of Stiffness and Carry-over Factor for Flat Rectangular Plates Under Compression," by W. D. Kroll, *Wartime Report No. L-398*, NACA, 1943.

In that case, the stiffness S_o of the first restraining plate may be expressed in terms of S_1 by the formula,[27]

$$S_o = \frac{S^{II}}{1 - C^2\left(\dfrac{S_1}{S^{II} + S_1}\right)} \qquad \text{(9)}$$

in which C is the carry-over factor of the first restraining plate and is tabulated with the stiffnesses by Miss Kroll;[26] and S^{II} is the stiffness that the first restraining plate would have if its far edge were simply supported. Eq. 9 is in the nature of a recurrence formula giving the stiffness S_o of any plate in terms of the restraint S_1 at its far edge. Successive application of Eq. 9 (starting with a plate for which the conditions at the far edge are known and working up to the plate under consideration) will yield the stiffness S_o to be used in conjunction with Figs. 3 and 4(a). This procedure is illustrated elsewhere.[27] In some cases, it may be necessary to assume simple support or clamping at some juncture one or two junctures away from the edge of the plate under consideration in order to shorten the process.

The k charts themselves (Figs. 3 and 4(a) for compression and Fig. 5(b) for shear) can sometimes give the stiffnesses of restraining plates. For example, Fig. 5(b) gives the external restraining stiffness S_o for which buckling will occur in shear at a given "load" k_s and wave length ratio λ/b. When buckling occurs, the total stiffness of the joint consisting of the juncture of plate and restraining medium is zero. The stiffness of the plate itself at buckling, therefore, is $-S_o$. Thus, Fig. 5(b), with the signs of the ϵ-values reversed, gives the stiffness of a long plate in shear corresponding to S^{IV} for a plate in compression.

Results for I-*Columns,* Z-*Columns, Channel-Columns, and Rectangular-Tube-Section Columns.*—By considering one of the component plates of the column to be elastically restrained by its neighbors and evaluating S_o with the aid of the stiffness tables,[26] it was possible to use Figs. 3 and 4(a) to calculate the local buckling stresses for columns of several simple cross-sectional shapes. The results are given in terms of a numerical parameter k_w in Fig. 9.[28] With k_w known, the buckling stress σ can be obtained from the formula,

$$\sigma = k_w \frac{\pi^2 E}{12\,(1 - \mu^2)} \left(\frac{t_w}{b_w}\right)^2 \qquad \text{(10)}$$

in which the subscript w refers to the web plate.

Method of Obtaining Results for Other Plate Sections.—For more general types of plate section, the joint-stiffness criterion[27] may be used to calculate the local buckling strength. This criterion states that when a plate assembly is carrying its local buckling load, the sum of the stiffnesses of all the members entering a joint is zero. To apply this principle, it is necessary to assume a series of values of buckling stress and for each one to calculate the stiffnesses

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[27] "Principles of Moment Distribution Applied to Stability of Structures Composed of Bars or Plates," by Eugene E. Lundquist, Elbridge Z. Stowell, and Evan H. Schuette, *Report No. 809*, NACA, 1945, Eq. 13.

[28] "Charts for Calculation of the Critical Stress for Local Instability of Columns with I-, Z-, Channel, and Rectangular Tube Section," by W. D. Kroll, Gordon P. Fisher, and George J. Heimerl, *Wartime Report No. L-429*, NACA, 1943.

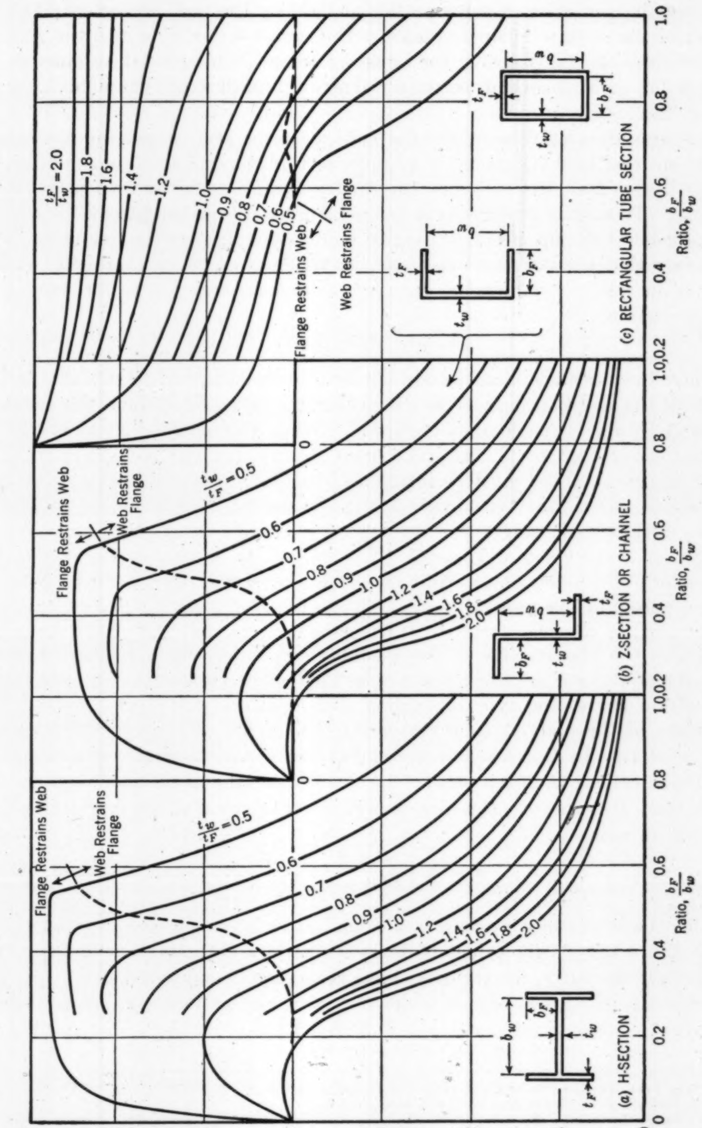

Fig 9.—Compressive Plate Buckling Stress Coefficient k_w

of the members meeting at some particular joint by the methods described at the end of the section on single plates. The lowest stress for which the sum of the stiffnesses equals zero is the buckling stress. This procedure must be repeated for each of several assumed values of λ until a minimum buckling stress is obtained.

This procedure applies only if the corners of the section remain straight lines when local buckling occurs. In open sections having small enough lips, such as lipped-**Z**-section columns, the lip may translate in its own plane at buckling. Two studies of this type of buckling have been completed.[29,30]

The general condition of buckling of plate assemblies, with some account of the fact that the intersections of the plates may not remain straight, was given in an earlier publication,[31] together with numerical results for various types of sections.

PLASTIC BUCKLING

When the buckling stress is low enough to remain on the straight line section of the stress-strain curves, the buckling is called elastic and the equations and figures discussed thus far apply. When the buckling stress is high enough to lie on the curved part of the stress-strain curve, the buckling is called plastic and new factors must be introduced to allow for the departure of the stress-strain curve from the original straight elastic section at these high stresses in the plastic range.

Form of Presentation of Theoretical Results.—Data on the plastic buckling stresses for plates under single loading have been presented in two forms. Both forms are used in this paper.

The first, or classical, form stems from the concept that Eqs. 1 and 2 may be used in the plastic range if the modulus of elasticity E is replaced by a reduced modulus $\eta\, E$, η being a function of the buckling stress, the shape of the stress-strain curve, the type of loading, the length-to-width ratio and edge conditions of the plate. (In the elastic range, $\eta = 1$ and above the elastic range, $\eta < 1$.) In this form of presenting the results, a curve of η versus buckling stress is given for a plate of specified material and specified boundary conditions. In terms of η the compressive buckling stress σ_e can be calculated from the formula,

$$\sigma = k_c \frac{\pi^2\, \eta_c\, E}{12\,(1 - \mu^2)} \left(\frac{t}{b}\right)^2 \dots\dots\dots\dots\dots\dots \text{(11)}$$

in which the subscript c is added to η to indicate a loading of compression and μ and E are the elastic values of Poisson's ratio and Young's modulus, respectively. Similarly, the shear buckling stress can be expressed as

$$\tau = k_s \frac{\pi^2\, \eta_s\, E}{12\,(1 - \mu^2)} \left(\frac{t}{b}\right)^2 \dots\dots\dots\dots\dots\dots \text{(12)}$$

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[29] "The Local Buckling Strength of Lipped **Z**-Columns with Small Lip Width," by Pai C. Hu and James C. McCulloch, *Technical Note No. 1335*, NACA, 1947.

[30] "Primary Instability of Open-Section Stringers Attached to Sheet," by Samuel Levy and Wilhelmina D. Kroll, *Journal of the Aeronautical Sciences*, Vol. 15, October, 1948, p. 581.

[31] "Theory of the Plastic Stability of Thin Plates," by P. P. Bijlaard, *Publications*, International Assn. for Bridge and Structural Eng., Vol. 6, 1940–41, p. 45.

Because η_c and η_s in Eqs. 11 and 12 are themselves given as functions of the unknown buckling stresses σ and τ, a trial-and-error calculation is generally necessary to obtain the buckling stress of a given plate. This difficulty is overcome in the second or alternate form of presenting the results.

The alternate presentation is essentially a plot of actual buckling stress σ (or τ) versus elastic buckling strain $\frac{\sigma}{E\,\eta_c}\left(\text{or}\,\frac{\tau}{G\,\eta_s}\right)$. Such a plot is directly obtainable from the plot of σ versus η_c (or τ versus η_s) and a knowledge of E (or G). The parameters of the buckling curve just described are dimensionally those of a stress-strain curve. The stress-strain curve of the plate material may therefore be superimposed on the same set of coordinates often revealing a useful correlation.

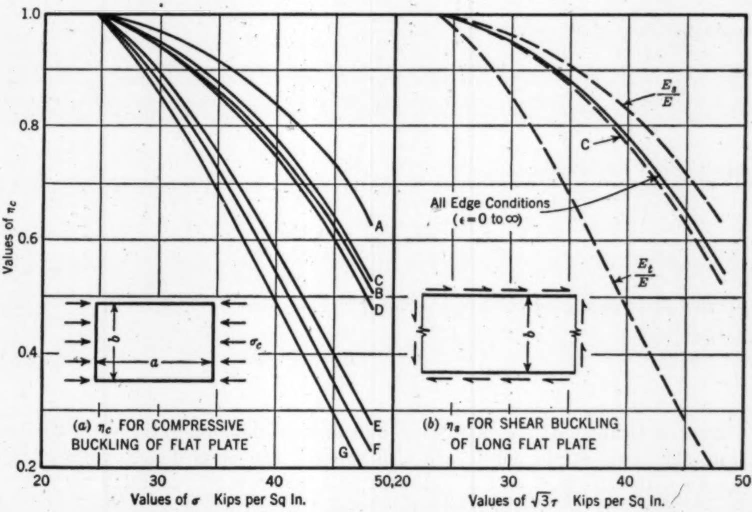

FIG. 10.—PLASTICITY REDUCTION FACTORS

For combined compression and shear, the stress ratios R_c and R_s used in the elastic range can also be used to present the results of buckling calculations in the plastic range.

Results for Compression.—Curves of η_c versus buckling stress σ are given in Fig. 10(a) for plates of 24S-T aluminum alloy. This alloy has a compressive yield stress of 46 kips per sq in. (0.2% offset). The loaded edges of the plate were simply supported, and the results for various types of plates are given in the following list[32] (see Fig. 10(a)):

Long hinged flange—

$$\eta_c = \frac{E_s}{E} \quad \text{(13a)}$$

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[32] "A Unified Theory of Plastic Buckling of Columns and Plates," by Elbridge Z. Stowell, *Report No. 898*, NACA, 1948.

long clamped flange—

$$\eta_c = \frac{E}{E}\left[0.330 + 0.670\sqrt{\frac{1}{4} + \frac{3}{4}\frac{E_t}{E_s}}\right] \dots\dots\dots\dots (13b)$$

long plate, hinged edges, simply supported—

$$\eta_c = \frac{E_s}{E}\left[\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{4} + \frac{3}{4}\frac{E_t}{E_s}}\right] \dots\dots\dots\dots (13c)$$

long plate, clamped edges—

$$\eta_c = \frac{E_s}{E}\left[0.352 + 0.648\sqrt{\frac{1}{4} + \frac{3}{4}\frac{E_t}{E_s}}\right] \dots\dots\dots\dots (13d)$$

wide plate as a short column $\left(\frac{a}{b} \leqq 1\right)$—

$$\eta_c = \frac{1}{4}\frac{E_s}{E} + \frac{3}{4}\frac{E_t}{E} \dots\dots\dots\dots (13e)$$

square plate loaded as a column $\left(\frac{a}{b} = 1\right)$—

$$\eta_c = 0.114\frac{E_s}{E} + 0.886\frac{E_t}{E} \dots\dots\dots\dots (13f)$$

narrow plate as a long column $\left(\frac{a}{b} \geqq 1\right)$—

$$\eta_c = \frac{E_t}{E} \dots\dots\dots\dots (13g)$$

Curves of secant modulus ratio E_s/E and tangent modulus ratio E_t/E, derived from the stress-strain curve, are also given. The coincidence of these two curves with curves A and G indicates that the reduced modulus for a long hinged flange is the secant modulus E_s, and that for a narrow plate-column is the tangent modulus E_t. Eqs. 13 are the basic formulas from which the η-curves can be computed for any material.

Three of the η-curves shown in Fig. 10(a) are replotted in Fig. 11 in the more directly usable form of buckling stress σ_c versus e (elastic), the strain at which buckling would occur in an elastic material.

Results for Shear.—A curve of η versus $\sqrt{3}\,\tau$ is given for long plates of 24S-T aluminum alloy in Fig. 10(b). This curve[33] applies to all degrees of equal elastic edge restraint against rotation between simple support and clamping. For comparison, curves of E_s/E and E_t/E (obtained from the compressive stress-strain curve with σ replaced by $\sqrt{3}\,\tau$) are also given. Also shown dotted, for comparison, is curve C, of Fig. 10(a), for a compressed simply supported plate. This curve practically coincides with the curve for η_s.

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[33] "Critical Shear Stress of an Infinitely Long Plate in the Plastic Region," by Elbridge Z. Stowell, *Technical Note No. 1681*, NACA, 1948.

Combined Longitudinal Compression and Shear.—Interaction curves for three long plates of 24S-T aluminum alloy are given in Fig. 12. The three plates were so chosen[34] that one would buckle in the elastic range for all stress combinations, one well in the plastic range, and one in the intermediate range. The three curves can be described approximately by the single parabola—

$$\left[R_s \frac{E_{s\tau}}{E_{sm}}\right]^2 + R_c \frac{E_{sc}}{E_{sm}} = 1 \qquad (14)$$

—in which R_s and R_c are the stress ratios previously defined in Eqs. 3 and 4, and all values of E_s are secant moduli obtained from the compressive stress-strain curve and defined as follows: E_{sm} is the secant modulus corresponding to a stress—

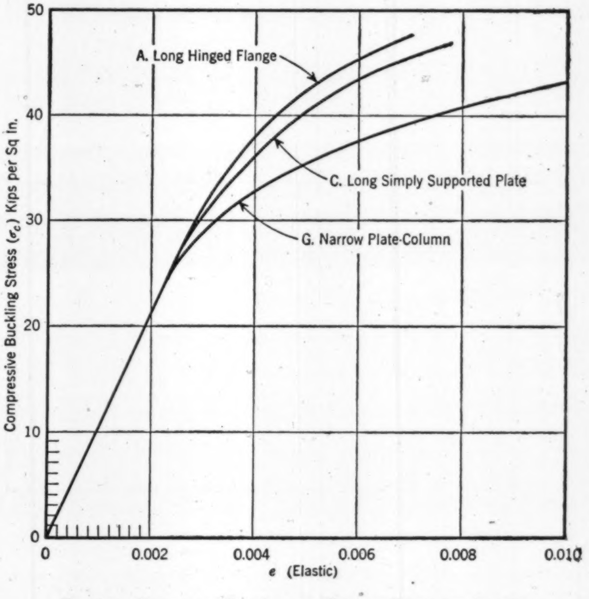

FIG. 11.—VALUES OF COMPRESSIVE PLASTIC BUCKLING STRESS

$$\sqrt{(\sigma_{cm})^2 + 3(\tau_{cm})^2} \qquad (15a)$$

—in which σ_{cm} and τ_{cm} are the stresses present at buckling; $E_{s\tau}$ is the secant modulus corresponding to a stress—

$$\sigma_i = \sqrt{3}\,\tau \qquad (15b)$$

---

[34] "Plastic Buckling of a Long Flat Plate Under Combined Shear and Longitudinal Compression," by Elbridge Z. Stowell, *Technical Note No. 1990*, NACA, 1949.

—in which τ_c is the stress required for buckling in shear alone; and E_{sc} is the secant modulus corresponding to a stress

$$\sigma_i = \sigma \quad \text{(15c)}$$

—in which σ is the stress required for buckling in compression alone.

The parameters used in Fig. 12 were chosen to show the parabolic relationship as indicated by Eq. 14. These interaction curves can also be replotted in terms of R_c and R_s alone. Such curves would have the same end points but would otherwise lie above the curves shown. Thus the use of the elastic interaction Eq. 7 would be conservative in the plastic range.

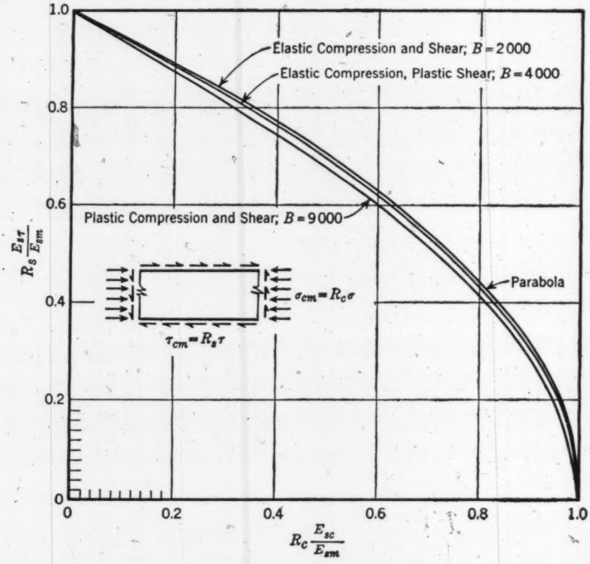

FIG. 12.—PLASTIC BUCKLING STRESSES IN COMBINED SHEAR AND COMPRESSION FOR LONG FLAT PLATES WITH SIMPLY SUPPORTED EDGES; $B = \frac{\pi^2 E\ b^2}{12(1-\mu^2)b^2}$

Results for H-*Section Columns.*—The basic theory for plastic buckling for single plates[32] has been applied to the more complex problem of the H-section column.[35] The calculated results for extruded 75S-T aluminum alloy are given in Fig. 13 as curves of buckling stress σ versus elastic buckling strain ϵ_e (elastic) for two families of cross sections $\left(\frac{t_w}{t_F} = 1; \frac{b_F}{b_w} = 0.6 \text{ to } 0.8\right)$. The compressive stress-strain curve for this material is also included in order to show how closely it follows the buckling curves.

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[35] "Plastic Buckling of Extruded Composite Sections in Compression," by Elbridge Z. Stowell and Richard A. Pride, *Technical Note No. 1971,* NACA, 1949.

COMPARISON OF THEORY AND EXPERIMENT

Compression.—The correlation of the compression test results with theory for both elastic and plastic buckling of the hinged flange (cruciform), the simple supported plate (square tube), and the H-section is illustrated in Fig. 14, with the test data taken from previous experiments.[32,36,37] The results are plotted in the form of experimental buckling stress σ_e versus calculated strain ϵ_e (elastic) at which buckling would occur if the material remained perfectly elastic. The compressive stress-strain curves are included for comparison. Appendix II illustrates the techniques used to detect buckling and to obtain the stress-strain curves.

In each case the agreement between theory and test results is good. For the hinged flange as represented by the cruciform section, both the theory and test results fall on the stress-strain curve, indicating that the effective modulus for this type of plate is the secant modulus. For the simply supported plate, as represented by the square tube, and for the H-section, the test results plot slightly below the stress-strain curve as predicted by theory, which means that the effective modulus for these specimens is slightly less than the secant modulus.

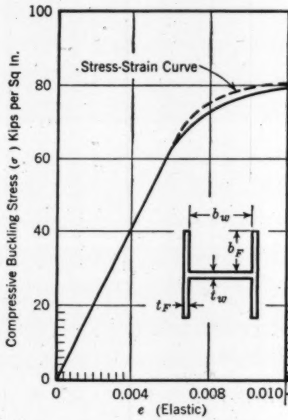

FIG. 13.—THEORETICAL COMPRESSIVE PLATE-BUCKLING CURVE

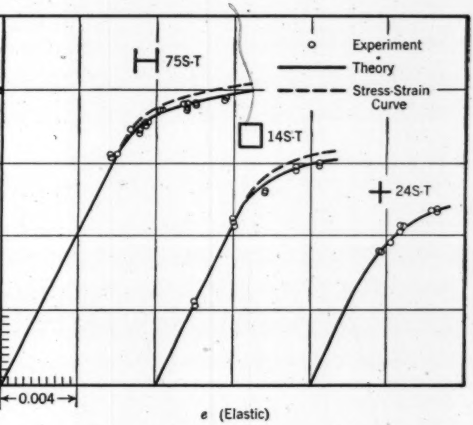

FIG. 14.—EXPERIMENTAL VERIFICATION OF COMPRESSIVE PLATE-BUCKLING THEORY FOR THREE ALUMINUM ALLOY SECTIONS

In order to show the generality of the close correlation between the test results and the stress-strain curve, the test data and stress-strain curves for a variety of aluminum and magnesium alloy H-sections, Z-sections, and channel-sections are shown in Fig. 15.[37] Because the average stress at maximum load,

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[36] "Plastic Buckling of Simply Supported Compression Plates," by Richard A. Pride and George J. Heimerl, *Technical Note No. 1817*, NACA, 1949.

[37] "Determination of Plate Compressive Strengths," by George J. Heimerl, *Technical Note No. 1480*, NACA, 1947.

$\bar{\sigma}_{\max}$, was also obtained, it has been included although only the buckling stress σ_e is of interest at this point. Two stress-strain curves are shown for each material to indicate the limits of variation in material properties found to exist for the sections tested. The test results for the **H**-sections fall somewhat closer to the stress-strain curve than do those for the **Z**-sections and channels. This is probably to be expected since the **H**-sections contain a higher percentage of flange material, and the stress-strain curve is theoretically correct only for a hinged flange.

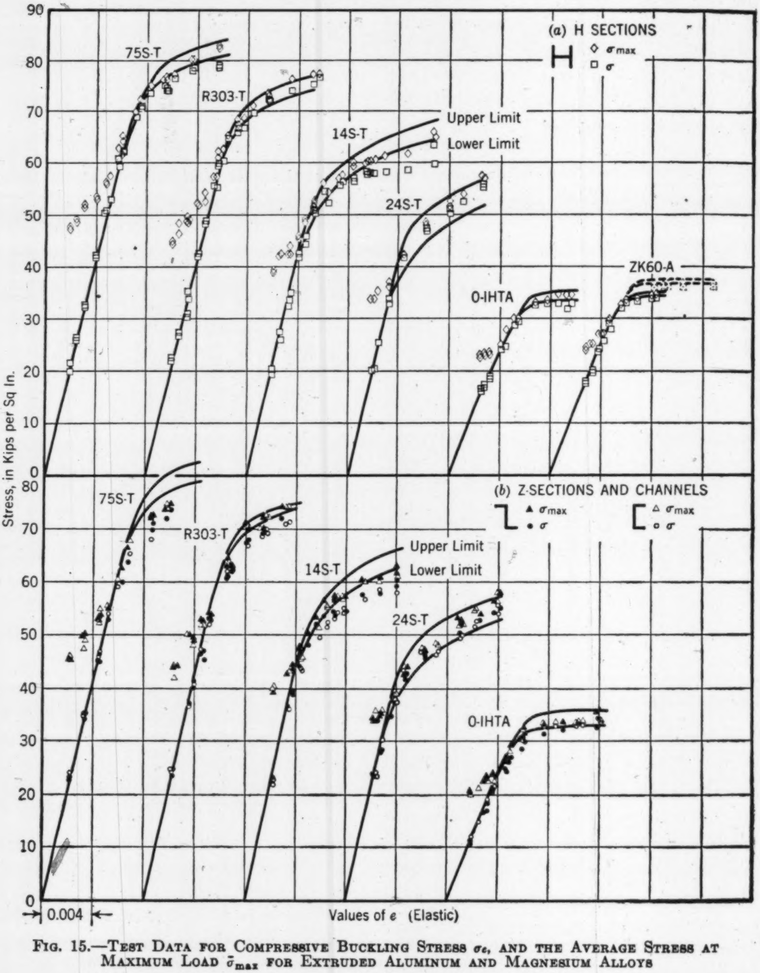

Fig. 15.—Test Data for Compressive Buckling Stress σ_e, and the Average Stress at Maximum Load $\bar{\sigma}_{\max}$ for Extruded Aluminum and Magnesium Alloys

In calculating the abscissa e (elastic) for each test specimen, an uncertainty of the order of magnitude of the plate thickness arose in measuring the plate widths. Therefore, a system of dimensioning was chosen for each type of specimen such that calculated and experimental values of the buckling stress were in good agreement in the elastic range. The same system was then also used in the plastic range. For the **H**-sections, **Z**-sections, channels, and square-tube sections, the use of inside face dimensions proved to be best. For the cruciforms, however, centerline dimensions were found to be preferable.

Shear.—Fig. 16, experimental data[38] are compared with the theoretical curve of Fig. 10(b). The shear stress-strain curve, added for comparison, was deduced from the compressive stress-strain curve by dividing the compressive stress by $\sqrt{3}$ to obtain the shear stress and by multiplying the compressive strain by $\sqrt{3}$ to obtain the shear strain.

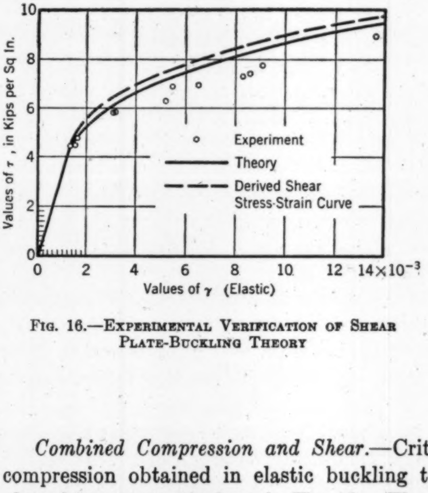

FIG. 16.—EXPERIMENTAL VERIFICATION OF SHEAR PLATE-BUCKLING THEORY

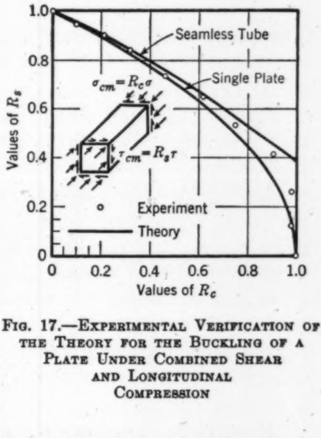

FIG. 17.—EXPERIMENTAL VERIFICATION OF THE THEORY FOR THE BUCKLING OF A PLATE UNDER COMBINED SHEAR AND LONGITUDINAL COMPRESSION

Combined Compression and Shear.—Critical combinations of torque and compression obtained in elastic buckling tests of a long square tube[39] are plotted in stress-ratio form in Fig. 17. These tubes were fabricated from four single plates connected by four corner angles. In Fig. 17 test results are compared with the parabolic interaction curve from Fig. 7(a) that should apply if the corner angles were heavy enough to isolate the walls so that they would buckle independently as four clamped plates. Since infinitely stiff angles would be required to accomplish this, the theoretical curve for a seamless square tube which takes into account the effect of adjoining walls is also included.[40] The test points, which represent averaged data from the four walls and two directions of torque, lie between the two theoretical curves.

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[38] "Critical Shear Stress of Plates Above the Proportional Limit," by George Gerard, *Journal of Applied Mechanics*, March, 1948, *Transactions*, ASME, Vol. 70, p. 7.

[39] "Buckling Test of Flat Rectangular Plates Under Combined Shear and Longitudinal Compression," by Roger W. Peters, *Technical Note No. 1750*, NACA, 1948.

[40] "Buckling of a Long Square Tube in Torsion and Compression," by Bernard Budiansky, Manuel Stein, and Arthur C Gilbert, *Technical Note No. 1751*, NACA, 1948.

MAXIMUM STRENGTH IN COMPRESSION

An accurate knowledge of the maximum strength of a structure is extremely important from the standpoint of both safety and economy. The second factor is especially important in aircraft, since the presence of unnecessary strength, and therefore unnecessary weight, will raise the operating costs. The introduction of the all-metal airplane has stimulated many studies of the maximum strength that plates develop after buckling.

Although maximum strength does not properly come under the subject of buckling, the importance of the subject and the close relationship observed between maximum strength and buckling strength warrant its discussion. The study of maximum strength in shear, through the medium of webs with transverse stiffeners, is too large a field to discuss except to call attention to some research in the field.[41] Only maximum strength in compression will be considered. The fact that the maximum strength of compressed plates may greatly exceed the buckling stress was first brought to the attention of American engineers by extensive tests at the National Bureau of Standards.[42]

Physical Action and Significant Parameters.—A simply supported plate would never fail in compression if the material remained perfectly elastic at all stresses. As the loading proceeds beyond buckling, the fraction of the total load taken by the central longitudinal fibers becomes less and less because of their buckled condition. The load shirked by the central fibers is carried by the less buckled fibers near the edges. Because of the finite strength of actual materials, a maximum load is reached when the load refused by the central fibers can no longer be taken by the edge fibers due to the fact that they are already working at stresses in the plastic region.

A precise theoretical analysis of the maximum strength of a plate would have to take into account large deflections and inelastic behavior of the material, both of which introduce complexities of nonlinearity. Such a theoretical analysis was not attempted until 1947, and then only for a relatively simple structure—the hinged flange.[43]

Even in the absence of a precise analysis, the parameters that might be significant in expressing the maximum compressive strength of a plate can be deduced from theoretical studies of the load-carrying capacity of purely elastic plates after buckling.

The first approximate analysis of this nature was given by Theodor von Kármán, M. ASCE[44] and summarized by S. P. Timoshenko.[45] Mr. von Kármán assumed that two longitudinal edge strips of combined width $m\,b$ carry the entire compression load, whereas the central strip of width $(1-\ m)b$ is free from stress. The maximum load is assumed to be reached when the uniform stress in the edge strips becomes equal to the yield stress of the material, σ_{cy}. The relationship that results from this analysis can be written in the form,

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[41] "Strength Analysis of Stiffened Beam Webs," by Paul Kuhn and James P. Peterson, *Technical Note No. 1364*, NACA, 1947.

[42] "Strength of Rectangular Flat Plates under Edge Compression," by L. Schuman and G. Back, *Report No. 356*, NACA, 1930.

[43] "The Compressive Strength of Flanges," by Elbridge Z. Stowell, *Technical Note No. 2020*, NACA, 1950.

[44] Strength of Thin Plates in Compression," by T. von Kármán, E. E. Sechler, and L. H. Donnell, *Transactions*, ASME, Vol. 54, 1932, p. 53.

[45] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1936, p. 395.

$$\frac{\sigma}{\sigma_{\max}} = \sqrt{\frac{\sigma}{\sigma_{cy}}} \qquad \text{(16)}$$

In later, more elaborate investigations[46,47] the same assumption (that the two edge strips act at a uniform stress σ_{edge}) was made, but it was further assumed that the stress in the central strip remains equal to its buckling value σ_c. The results of such studies can be put in the form,

$$\bar{\sigma} = m\,\sigma_{\text{edge}} + (1 - m)\,\sigma \qquad \text{(17)}$$

or

$$\frac{\sigma}{\sigma} = \frac{\dfrac{\sigma}{\sigma_{\text{edge}}}}{m + (1 - m)\dfrac{\sigma}{\sigma_{\text{edge}}}} \qquad \text{(18)}$$

in which $\bar{\sigma}$ is the average compressive stress; σ_{edge} is the compressive stress at the edge; σ_c is the critical buckling stress; and m is a constant, independent of the load, with a value between zero and unity.

If (as in Mr. von Kármán's analysis) the maximum strength is assumed to occur when σ_{edge} reaches some high value (say, the compressive yield stress σ_{cy}), then Eq. 18 can be written as

$$\frac{\sigma}{\sigma_{\max}} = \frac{\dfrac{\sigma}{\sigma_{cy}}}{m + (1 - m)\dfrac{\sigma}{\sigma_{cy}}} \qquad \text{(19)}$$

Eqs. 18 and 19, derived from different assumptions, are both based on elastic theory, and indicate that $\frac{\sigma}{\sigma_{\max}}$ and $\frac{\sigma}{\sigma_{cy}}$ might be significant parameters. Therefore, the results of the maximum strength tests were plotted in terms of these parameters.

Correlation Between Buckling Strength and Maximum Strength.—Maximum-strength data are plotted in Fig. 18 for various sections of aluminum and magnesium alloys. The data[37] are plotted in the form of curves of $\frac{\sigma}{\sigma_{\max}}$ versus $\frac{\sigma}{\sigma_{cy}}$.

For the cruciform section, the test data show good agreement with a theoretical curve calculated for a hinged flange utilizing the same plasticity relationships as those used in the plastic buckling analysis (see Appendix I). In the tests, the juncture of the four flanges of the cruciform section was observed to remain straight to and beyond the maximum load. The cruciform, therefore, may truly be regarded as four hinged flanges at maximum load as well as at the buckling load.

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[46] "The Apparent Width of the Plate in Compression," by Karl Marguerre, *Technical Memorandum No. 833*, NACA, 1942.

[47] "Bending of Rectangular Plates with Large Deflections," by Samuel Levy, *Technical Note No. 846*, NACA, 1942.

In Fig. 18, the calculated dashed curve for maximum strength of flanges just described is also superimposed on the experimental data for the **H**-sections **Z**-sections, channels, and square-tube sections. Although this curve strictly applies only to a hinged flange, it seems to describe fairly well the results for

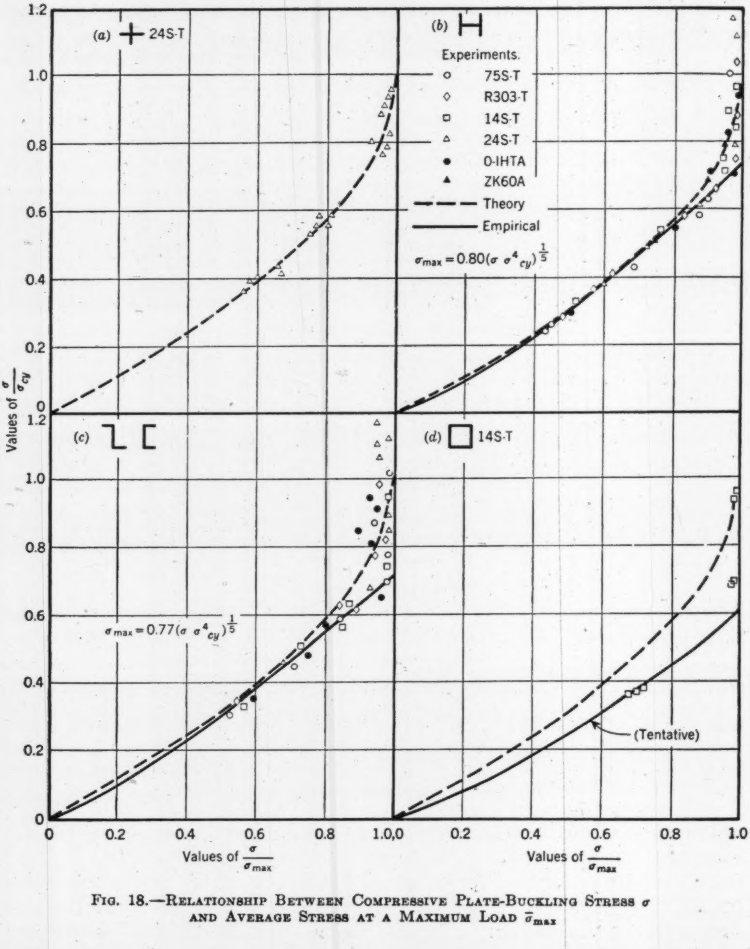

FIG. 18.—RELATIONSHIP BETWEEN COMPRESSIVE PLATE-BUCKLING STRESS σ AND AVERAGE STRESS AT A MAXIMUM LOAD $\bar{\sigma}_{max}$

these sections, for which flanges were primarily responsible for instability; but it misses widely the few results for the square-tube section, which does not possess flanges. Empirical curves that apply specifically to **H**-sections, **Z**-sections, and channel sections are also shown. The equations for these curves provide simple analytical expressions for engineering use. Although the data are very few for the square tube, and indicate a lower average stress at maximum load for the same ratio of buckling stress to yield stress $\frac{\sigma}{\sigma_{cy}}$, there are enough data to indicate that a similar empirical curve probably applies. One such curve, tentatively proposed, is shown in Fig. 18.

The grouping of all the experimental points close to a value $\frac{\bar{\sigma}}{\sigma_{\max}} = 1$, when the ratio $\frac{\sigma}{\sigma_{cy}}$ is greater than about 3/4, implies that for practical engineering purposes buckling at a stress greater than 3/4 of the yield stress is equivalent to failure. If the buckling stress is lower than about 3/4 the yield stress, the relation between the buckling stress and the maximum strength is nearly the same for all the flanged sections and all the materials. A similar but different relationship would be expected to hold for the square tubes. The statements in this paragraph apply to metals of the type of aluminum or magnesium alloys but may have to be modified for members made of mild steel, in view of the basically different shape of the stress-strain curve of this material.

It should be noted that the data in Fig. 18 are for integral plate sections with relatively sharp corners and without corner angles. The presence of curvature of connections angles at the corners can be expected to alter, somewhat, the relationships obtained.

CONCLUDING REMARKS

Charts have been presented for the calculation of elastic and plastic buckling stresses of flat unstiffened plates in compression, shear, and combined compression and shear, and for integral flat-plate sections in compression. Correlations have been shown between buckling strength and maximum strength for the integral flat-plate sections. Theoretical methods and experimental techniques for conducting buckling investigations are indicated briefly in Appendixes I and II.

The agreement shown between theory and experiment indicates that the status of the problem of calculating flat-plate buckling strength is satisfactory. Buckling stresses can be calculated with engineering accuracy in both the elastic and plastic stress ranges.

A survey of the various solutions available indicates that the different types of loading have not all been studied to the same extent. For most cases the solutions have stopped short of the plastic range and many have been concerned with only the simpler edge conditions. However, the foundations have now been laid for the gaps to be filled at least by extrapolation. In addition, solutions for a number of loading conditions other than those included in the paper are available in the literature or are in the process of being obtained.

A field that needs much work to lay a comparable foundation is that concerned with plate assemblies which are not integral, but fabricated with the aid of angles and other connections. Two uncertainties confront the analyst in his study of such plate assemblies. The first is the uncertainty as to the widths of the component plates due to the relatively large size of the connection angles; the second is the uncertainty as to the degree of continuity provided by the connections. The first uncertainty is also present to some extent in formed plate sections when the curved material created by forming is an appreciable part of the total cross-sectional area.

Additional problems include the determination of the size of permanent buckles resulting from loads above the buckling load, the effect of a permanent buckle on the ultimate strength, the fatigue life of a plate under repeated buckling, and the interaction of plate buckling with other types of instability involving the structure as a whole.

ACKNOWLEDGMENT

Valuable advice and assistance have been received in the preparation of this paper from associates of the writers in the Structures Research Division of NACA

APPENDIX I.—ELASTIC THEORETICAL BUCKLING ANALYSIS

Differential Equation Method.—An exact mathematical analysis to determine the buckling strength of a flat plate involves the integration of the partial differential equation,[13]

$$\frac{E\,t^3}{12\,(1-\mu^2)}\left[\frac{\partial^4 w}{\partial x^4}+2\,\frac{\partial^4 w}{\partial x^2\,\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right] = -\,t\left[\sigma_x\,\frac{\partial^2 w}{\partial x^2}+2\,\tau\,\frac{\partial^2 w}{\partial x\,\partial y}+\sigma_y\,\frac{\partial^2 w}{\partial y^2}\right]\,..(20)$$

in which w is the lateral deflection of the plate; x and y are the length and width coordinates, respectively; E is Young's modulus; μ is Poisson's ratio; t is the thickness; σ_x and σ_y are the middle plane compressive stresses in the x-direction and y-direction, respectively; and τ is the middle-plane shear in these directions. The stresses σ_x, σ_y, and τ are known from the external loading at buckling.

Detailed methods of solving the differential equations are too numerous to include in the paper.[48,49]

Energy Method.—Also useful to the calculation of buckling stresses is the energy method. The "potential energy" can be written as follows for plates with edges either free, simply supported, or clamped:[14]

$$\Delta W = \frac{E\,t^3}{24\,(1-\mu^2)}\int\!\!\int\left\{\sigma_x\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-\mu)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2\right]\right\}dx\,dy - \frac{t}{2}\int\!\!\int\left[\sigma_x\left(\frac{\partial w}{\partial x}\right)^2+2\,\tau\,\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}+\sigma_y\left(\frac{\partial w}{\partial y}\right)^2\right]dx\,dy\,..(21)$$

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[48] "Mathematical Theory of Elasticity," by I. S. Sokolnikoff, McGraw-Hill Book Co., Inc., New York, N. Y., 1946, Chapter V.

[49] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1936, Chapter VII.

in which the integration is performed over the length and width of the plate. The first integral in Eq. 21 represents the strain energy gain of the plate due to the bending and twist that occur during buckling. The second integral represents an energy loss associated with the middle-plane stresses due to the lateral deflection. If the plate edges are held fixed during buckling this integral represents a reduction in middle-plane stretching energy; if the edges may shift with respect to one another, it represents the work of the external loading.

The potential energy expression, Eq. 21 is useful by virtue of the fact that, of all functions w satisfying the geometric boundary conditions (conditions on deflection and slope), ΔW will be a minimum (equal to zero) for that function which also satisfies the differential equation.[48] Thus, of several functions of w satisfying the geometric boundary conditions but not necessarily the differential equation, the one which satisfies the differential equation best is the one for which ΔW is smallest. A special development of the energy method is due to Lord Rayleigh[50] and Walter Ritz.[51]

Although buckling stresses obtained by this approximate method may be very accurate, they are generally higher than the exact stresses. Thus, the energy method furnishes an upper limit to the buckling stress unless, by chance, the deflection pattern chosen for w is the true one—that is, the one that can satisfy the differential Eq. 20 exactly.

Lagrangian Multiplier Method.—By a slight but fundamental revision in the energy method, conservative results can be obtained. The modification consists in using a complete set of functions for w that does not satisfy the geometric boundary conditions term by term but is capable of satisfying them as a whole. If the set of functions is so chosen that they satisfy the boundary conditions only approximately, a lower limit to the buckling stress may be obtained.

The technique of relaxing boundary conditions was first used for buckling problems by E. Trefftz.[52] The method is described and refined by Bernard Budiansky, Pai C. Hu, and Robert W. Connor[53,54] and it is called the Lagrangian multiplier method. It is presented in such a way that both the upper-limit and lower-limit solutions can be obtained from the same mathematical setup.

PLASTIC THEORETICAL BUCKLING ANALYSIS

Buckling stresses obtained by the methods described in the first part of this appendix are correct only if they fall in the elastic range—that is, below the proportional limit of the plate material. This follows from the assumption of perfect elasticity upon which the derivation of Eqs. 20 and 21 is based.

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[50] "The Theory of Sound," by John William Strutt, Baron Rayleigh, Macmillan Co., New York, N. Y., 1929, Chapter IV.

[51] "Über eine Neue Methode Zur Lösung gewisser Variationprobleme der Mathematischen Physik," by Walter Ritz, *Journal für reine und angewandte Mathematik*, Vol. 135, 1909, p. 1.

[52] "Ein Gegenstück zum Ritzschen Verfahren," by E. Trefftz, *Proceedings*, 2d International Congress for Applied Mechanics, Zürich, 1926, pp. 131–137.

[53] "The Lagrangian Multiplier Method of Finding Upper and Lower Limits to Critical Stresses of Clamped Plates," by Bernard Budiansky and Pai C. Hu, *Technical Note No. 1103*, NACA, 1946.

[54] "Notes on the Lagrangian Multiplier Method in Elastic-Stability Analysis," by Bernard Budiansky, Pai C. Hu, and Robert W. Connor, *Technical Note No. 1558*, NACA, 1948.

Equations to replace Eqs. 20 and 21 for buckling in the plastic range have been proposed by various investigators.

The problem of plastic buckling analysis of plates is much more difficult than the corresponding problem for columns. For a long time, therefore, equations to replace Eq. 20 in the plastic range were written as intuitive generalizations of the corresponding plastic column buckling equation.[49,55] Eventually, more refined attempts were made to solve the plastic buckling problem through the use of theories for the plastic behavior of materials under combined stresses. The first attempts were made by P. P. Bijlaard,[31] M. ASCE, and A. A. Ilyushin.[56] The latter's analysis was modified by one of the writers[32,33,34] to incorporate the Shanley continuous-loading principle.[57,58] The resulting stresses show good agreement with experiment. This type of theory has received theoretical justification in the literature.[59]

The method of analysis proceeds from the assumption that in the plastic domain a unique stress-strain curve exists for combined loading that is a simple generalization of the stress-strain curve for a single loading. The existence of such a curve has been established approximately by experiment (and has also been justified theoretically) for the case in which the ratios of the various stresses are maintained constant as the loading proceeds[60]; unloading occurs elastically as is the usual case.

By making use of the generalized stress-strain curve and the corresponding stress-strain relations, equations for the plastic range were obtained to replace Eqs. 20 and 21, which hold in the elastic range.[32] Solutions of the equations were obtained by methods analogous to those in the corresponding elastic cases.

APPENDIX II.—EXPERIMENTAL BUCKLING ANALYSIS

Test Specimens.—Most of the experimental work conducted to check the theoretical results thus far described has been concerned with loading in compression. The test specimens used to obtain the data in Figs. 14 and 15 consisted of so-called cruciforms (+-sections), square tubes, and **H**-sections, **Z**-sections, and channel sections as shown in Fig. 19. All these sections

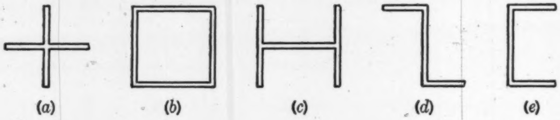

FIG. 19.—CROSS SECTIONS OF SPECIMENS USED IN COMPRESSIVE PLATE-BUCKLING TESTS

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[55] "Theorie und Berechnung der Eisernen Brücken," by Friedrich Bleich, Springer, Berlin, 1924, p. 216.

[56] "The Elasto-Plastic Stability of Plates," by A. A. Ilyushin, *Technical Memorandum No. 1188*, NACA, 1947.

[57] "The Column Paradox," by F. R. Shanley, *Journal of the Aeronautical Sciences*, Vol. 13, December, 1946, p. 678.

[58] "Inelastic Column Theory," by F. R. Shanley, *ibid.*, Vol. 14, May, 1947, p. 261.

[59] "Theories of Plastic Buckling," by S. B. Batdorf, *ibid.*, Vol. 16, July, 1949, p. 405.

[60] "Plastichnost," by A. A. Ilyushin, OGIZ, Moskva-Leningrad, 1948, pp. 69, 75, and 117.

were made by the extrusion process with the exception of the square tubes which were drawn.

The tests on the cruciforms (Fig. 19) and square tubes were for the purpose of obtaining data for single plates rather than for plate sections. In effect, the cruciform is composed of four flanges mutually supported along a common edge, as can be seen in Fig. 20(*a*). The square tube of constant wall thickness shown in Fig. 20(*b*) is similarly an integral assembly of four plates mutually supporting each other along both side edges.

The lengths were chosen short enough so that plate buckling would occur with the corners remaining straight, but not so short as to incur any appreciable increase in strength associated with short lengths. For the cruciform section the wave lengths developed on buckling were much longer in terms of plate widths than for the other sections as indicated in Fig. 20.

The test data shown in Fig. 16 for shear were obtained by George Gerard[38] from tests on panels of thin sheet material in a loading frame. Tests to check Fig. 7(*a*)—the interaction curve for combined compression and shear—have been made only in the elastic range.[39] The specimen used was a long square tube consisting of four thin flat plates joined by relatively heavy angles at the corners.

Compressive Stress-Strain Tests.—Compressive stress-strain curves, which are necessary in order to correlate the material properties with the plate compressive strength and for use in applying plasticity theory were obtained chiefly from tests of single-thickness specimens cut from the elements of the sections from which the buckling specimens were taken. The single-thickness specimens were tested in a modified form of the Montgomery-Templin type of compression fixture, utilizing longitudinally grooved supporting plates in place of rollers used in the original fixture. This type of fixture and associated techniques are described in several publications.[61]

Detection of Buckling.—In the theory of buckling, a plate is assumed to be perfectly flat; thus lateral deflection under the edge load will not occur until a sharply defined critical load is reached. In practical tests, however, plates (no matter how carefully they are fabricated) have some slight initial deviations from flatness which grow with increase in load. Thus ideal buckling is not realized, and the problem arises of how best to interpret experimental data so as to obtain the buckling loads.

One procedure that has been proposed for determining experimental plate-buckling loads is based on an extension of the Southwell plot method used for columns.[62,63,64] To be successful, the method requires that the theory of small deflection be applicable and, at the same time, that the loads at which measurements are taken be close to the theoretical buckling load. Although these two conditions are attainable for columns, they are very often mutually

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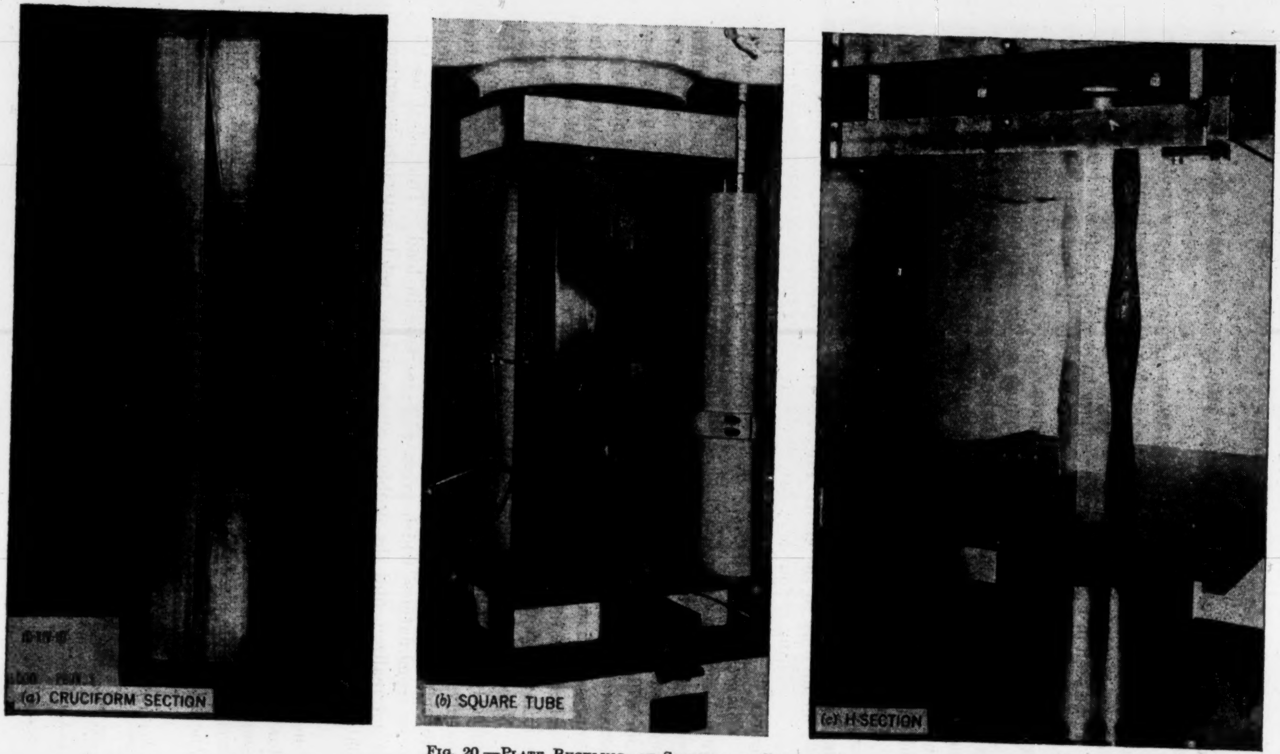
[61] "Determination and Presentation of Compressive Stress-Strain Data for Thin Sheet Metal," by Walter Ramberg and James A. Miller, *Journal of the Aeronautical Sciences*, Vol. 13, November, 1946, p. 569.

[62] "On the Analysis of Experimental Observations in Problems of Elastic Stability," by R. V. Southwell, *Proceedings*, Royal Soc. of London, Section A, Vol. 135, April, 1932.

[63] "Generalized Analysis of Experimental Observations in Problems of Elastic Stability," by Eugene E. Lundquist, *Technical Note No. 658*, NACA, 1938.

[64] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1936, pp. 321 and 401.



(a) CRUCIFORM SECTION

(b) SQUARE TUBE

(c) H-SECTION

FIG. 20.—PLATE BUCKLING OF SECTIONS IN COMPRESSION

contradictory for plates. In addition, precise measurements of strain or deflection were required. These difficulties, coupled with the fact that the method is strictly applicable to plates only in the elastic range, suggested that a more direct and simple engineering way of obtaining an experimental buckling load should be adopted.

Consequently, two more practical methods of defining experimental buckling have been considered. According to one—the strain-reversal method—the buckling load was taken as the load at which the strain on the convex side of a buckle stopped increasing and started decreasing. According to the other (the so-called "top-of-the-knee" method) the buckling load was the load corresponding to the top of the knee of a curve of load versus lateral deflection.

In order to assess the amount by which experimental buckling stresses determined by the strain-reversal or top-of-the knee methods might differ from the theoretical buckling stress of a perfectly flat plate, a theoretical study was made, by means of elastic large-deflection theory, of the behavior under edge compression of a simply supported square plate with slight initial deviations from flatness.[65] This study indicated that the top-of-the-knee method gives buckling loads below, but closer to, the theoretical load than the strain-reversal method.

Certain practical difficulties in using the strain-reversal method further strengthened the case for the top-of-the-knee method. These included inability to predict accurately the location of a buckle crest and, in some cases, the absence of strain reversal, particularly in the plastic buckling range. For these reasons, the experimental buckling stresses for compression given in this paper were determined by the top-of-the-knee method. For most of the tests, the load and deflection were recorded graphically.

The shear buckling stresses of Fig. 16 were based on the so-called strain-difference method.[38] The load readings were plotted against buckle curvature. The critical stress was defined by the intersection of two straight lines through the plotted test points. The buckling stresses obtained by this method generally lie between those given by the top-of-the-knee and strain-reversal methods.

For tests of the tube in combined compression and torsion to check the theoretical interaction curve of Fig. 7(a), the strain-reversal method was used to detect buckling. Although this would cause the actual buckling stresses to be low, conversion of the results to stress-ratio form was judged to eliminate most of the error due to using this definition of buckling.

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[65] "Effect of Small Deviations from Flatness on Effective Width and Buckling of Plates in Compression," by Pai C. Hu, Eugene E. Lundquist, and S. B. Batdorf, *Technical Note No. 1124*, NACA, 1946.

DISCUSSION

P. P. Bijlaard,[66] M. ASCE.—The extensive results obtained by the Structures Research Division of the National Advisory Committee for Aeronautics (NACA) are made more readily available by the publication of this paper. As a matter of course, the paper refers to applications to aluminum alloys and to aircraft structures. Similar computations referring to bridge engineering were presented by the writer[67,68,69,31,70] from 1938 to 1947.

As mentioned by the authors, the exact theory of buckling of plate assemblies[68,69,31] and that of the plastic buckling of plates[67,68,69,31] were developed by the writer. A short survey of the historical developments in this field has been given by G. Winter,[71] M. ASCE. As far as the first theory is concerned the NACA later made available very useful tables for the rotational stiffnesses S^{II}, S^{III}, and S^{IV} of flat rectangular plates under compression. These stiffnesses refer to an unloaded edge subjected to a sinusoidally varying moment, if the far edge is simply supported, free, or subjected to a moment equal and opposite to that at the near edge, respectively. These stiffnesses (or rather, specific angle rotations) were derived by the writer[68,69,31] in 1939 and 1940 and in 1943 by the NACA.[72,27] Using the NACA derivations, they were tabulated by W. D. Kroll[26] in 1943. The symbols in the paper, referring to quarter radians, differ somewhat from those preferred by the writer. In the elastic range, for S^{II}, S^{III}, and S^{IV}, the writer prefers $N/(4\,\theta)$, in which N is the flexural rigidity of the restrained plate, and θ is θ_a, θ_c, or θ_b, as defined elsewhere.[73] The tables by Miss Kroll[26] are a very definite help in finding values θ_a, θ_b, and θ_c, for the elastic range in the writer's equations, by means of the substitutions $N/(4\ S^{II})$, $N/(4\ S^{IV})$, and $N/(4\ S^{III})$, respectively. The writer's theory takes account of the fact that the intersection lines of the middle planes of the plates do not remain straight. This is of importance in case of flexural and torsional buckling of **T**-sections and angles.[31,74]

The writer contends[31] that, in connection with unavoidable eccentricities, for columns as well as for plates subject to buckling in the plastic range, it should be assumed that there is no unloading of initially plastically deformed regions; the plate should be assumed to remain entirely plastic.[75,76,77] However, at that time the concepts and codes that were current in 1939 and 1940[69,31] assumed local unloading of initially plastically deformed fibers for columns (Engesser-Kármán theory). The assumed stress-strain diagram for structrual steel was then derived from the assumed buckling stress-slenderness relation for columns. To find the buckling stress for plates, however, the writer[67,68,69,31] chose his formulas in such a way that the resulting buckling stresses were practically the same as would be the case for complete plastic behavior in columns and plates.[75] This was substantiated by additional data published in 1947.[76] For a given stress-strain diagram (as in the case of aluminum) these complications do not arise, so that the writer's computation, for plates that were completely plastic was straight forward and direct.[70,77] In 1947, F. R. Shanley[78] showed that even an initially straight column should be assumed to remain entirely plastic with incipient buckling. Mr. Shanley's principle leads to the same conclusion for a plate.

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[66] Prof. of Structural Eng., Cornell Univ., Ithaca, N. Y.; formerly Prof., Technical Universities of Delft, Holland, and Bandoeng, Netherlands East Indies.

[67] "A Theory of Plastic Stability and Its Application to Thin Plates of Structural Steel," by P. P. Bijlaard, *Proceedings*, Royal Netherlands Academy of Sciences, Amsterdam, Holland, Vol. 41, No. 7, September, 1938, pp. 731–743.

[68] "Exacte Berekening voor het Plooien van de Wanden der in de Bruggenbouw Gebruikelijke Staafprofielen," by P. P. Bijlaard, *De Ingenieur in Nederlandsch Indie*, Vol. 6, February, 1939, pp. I.17–I.23.

[69] "Afleiding van Eenvoudige Gebruiksformules en Grafieken ter Bepaling van het Plooigevaar van de Wanden van Vloeistalen Staafprofielen," by P. P. Bijlaard, *ibid.*, October, 1939, pp. I.239–I.256.

[70] "Some Contributions to the Theory of Elastic and Plastic Stability," by P. P. Bijlaard, *Publications*, International Assn. for Bridge and Structural Eng., Vol. 8, 1947, pp. 17–80.

[71] "Performance of Compression Plates as Parts of Structural Members," by G. Winter, *Engineering Structures Supplement*, Academic Press Inc., New York, N. Y., 1949, pp. 179–185.

[72] "Principles of Moment Distribution Applied to Stability of Structures Composed of Bars or Plates," by Eugene E. Lundquist, Elbridge Z. Stowell, and Evan H. Schuette, *Wartime Report No. L-326*, 1943.

[73] "Theory of the Plastic Stability of Thin Plates," by P. P. Bijlaard, *Publications*, International Assn. for Bridge and Structural Eng., Vol. 6, 1940–1941, Eqs. (48, 52), (56, 57), or (53, 54).

[74] "On the Torsional and Flexural Stability of Thin Walled Open Sections," by P. P. Bijlaard, *Proceedings*, Royal Netherlands Academy of Sciences, Amsterdam, Holland, Vol. 51, March, 1948, pp. 314–321.

The buckling stresses and wave lengths of buckles, predicted in 1938 by the writer's theory of plastic buckling,[67] then were confirmed by the writer in 1947[70,77] and in 1949,[79] as well as by experiments reported in 1946[80] by C. P. Kollbrunner. A summary of tests by NACA, which serves to confirm the writer's work, was published in 1948.[81] At first, the writer's theory received scant attention in the United States, but it was observed by A. A. Ilyushin[82] in Russia. The latter tried to simplify it by assuming Poisson's ratio for the elastic deformations equal to 0.5; but on the other hand, he assumed local unloading of initially plastically deformed regions and tried to find the exact solution for the boundary surface between the elastic and plastic region. However, he failed in this purpose by starting from a wrong assumption.[79,83] Elbridge Z. Stowell[32] changed Mr. Ilyushin's analysis by assuming the plate to remain entirely plastic, analogous to Mr. Shanley's principle for columns.[78] Hence, the only difference between Mr. Stowell's computations and those based on the writer's theory is that Mr. Stowell assumes Poisson's ratio for the elastic deformations equal to 0.5, instead of its actual value of about 0.3. It seems useful to state this relation between the foregoing methods of computation, since they are sometimes presented and thought of as entirely different theories.

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[75] "Theory of the Plastic Stability of Thin Plates," by P. P. Bijlaard, *Publications*, International Assn. for Bridge and Structural Eng., Vol. 6, 1940–1941 (see footnote 10).

[76] "Some Contributions to the Theory of Elastic and Plastic Stability," by P. P. Bijlaard, *ibid.*, Vol. 8, 1947, p. 28.

[77] "On the Plastic Stability of Thin Plates and Shells," by P. P. Bijlaard *Proceedings*, Royal Netherlands Academy of Sciences, Amsterdam, Holland, Vol. 50, September, 1947, pp. 765–775.

[78] "Inelastic Column Theory," by F. R. Shanley, *Journal of the Aeronautical Sciences*, Vol. 14, No. 5, 1947, pp. 261–268.

[79] "Theory and Tests on the Plastic Stability of Plates and Shells," by P. P. Bijlaard, *ibid.*, Vol. 16, No. 9, 1949, pp. 529–541.

[80] "Das Ausbeulen der auf Einseitigen, Gleichmássig Verteilten Druck Beanspruchten Platten im-Elastischen und Plastischen Bereich," by C. F. Kollbrunner, *Mitteilungen a.d. Institut f. Baustatik a.d. E.T.H.*, Zurich, Switzerland, No. 17, 1946.

[81] "Prediction and Reduction to Minimum Properties of Plate Compressive Curves," by E. H. Schuette and J. C. McDonald, *Journal of the Aeronautical Sciences*, Vol. 15, No. 1, 1948, pp. 23–27.

[82] "Stability of Plates and Shells Beyond the Proportional Limit," by A. A. Ilyushin, *Technical Memorandum No. 1116*, NACA, October, 1947 (Translation from Prikladnaya Matematika i Mekhanika, Vol. 8, 1944).

[83] "Buckling Stresses of Box-Beams under Pure Bending," by Sigge F. Eggwertz, *Meddelande No. 33*, Flygtekniska Försöksanstalten (FFA), Stockholm, Sweden, 1950.

Applications of his theory to aircraft structures were given by the writer in 1949,[79] 1950,[84,85] and 1951.[86] In a number of instances, the writer[67,79,87,88] has explained why, for plastic buckling of plates, the so-called deformation theory rather than the flow theory of plasticity should be used. The same was contended by S. B. Batdorf.[59] All of these views have been substantiated by experiments.[79,88] The writer's theory was especially mentioned as a sound basis for design in the conclusions, accepted by the Third Congress of the International Association for Bridge and Structural Engineering at Liege, Belgium, September, 1948.[89]

Elbridge Z. Stowell,[90] George J. Heimerl,[91] Charles Libove,[92] J. M. ASCE, and Eugene E. Lundquist,[93] A. M. ASCE.—The contribution by Mr. Bijlaard is welcome. Much of his work has appeared in foreign journals and has not been readily available. The first time that his work came to the writers' attention was in the paper by Mr. Ilyushin.[82]

A final acknowledgment is in order to the Column Research Council. It was at their suggestion that this paper was prepared and presented to the Society.

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[84] "Analysis of the Elastic and Plastic Stability of Sandwich Plates by the Method of Split Rigidities, I–II," by P. P. Bijlaard, *Preprint Institute of Aeronautical Sciences No. 259*, January, 1950.

[85] "Stability of Alclad Plates," by P. P. Bijlaard, *Journal of the Aeronautical Sciences*, Vol. 17, No. 4, 1950, p. 252.

[86] *Ibid.*, Vol. 18, 1951, No. 5, pp. 339–349 and No. 12, pp. 790–796, 829.

[87] On the Plastic Buckling of Plates," by P. P. Bijlaard, *ibid.*, Vol. 17, No. 11, 1950, pp. 742–743.

[88] "On the Plastic Buckling of Plates According to the Flow Theory," by P. P. Bijlaard, *ibid.*, No. 12, 1950, pp. 810–811.

[89] "Conclusions and Suggestions," Final Rep. of the Third Cong. of the International Assn. for Bridge and Structural Eng., Liege, Belgium, September, 1948, pp. 113–115.

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AMERICAN SOCIETY OF CIVIL ENGINEERS
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DESIGN OF LARGE COAL BUNKERS

By Paul Rogers,[1] M. ASCE

With Discussion by Messrs. George L. Blanchard, Alexander M. Turitzin, and Paul Rogers

Synopsis

Although coal bunkers of large capacity form an important part of power stations and similar plants, the technical literature contains little information concerning their practical design.

An attempt is made in this paper to present a rational design procedure based upon several coal bunkers actually constructed. Formulas for coal pressures on vertical and inclined walls are reviewed, and values are tabulated; bunker walls are presented as flexible plates; unbalanced forces due to asymmetry are analyzed; and continuity for the stiffeners is introduced.

Introduction

The coal bunkers of modern power stations (see Fig.1) have smooth walls with few vertical stiffeners widely spaced. If possible, these stiffeners should be placed on the inside, thus producing a neat outside appearance requiring little maintenance. There is objection to inside stiffeners, however, when the cyclone type of boilers is used, because the stored coal is composed of small particles and has an excessive tendency to arch. The hoppers of the bunkers have steeply sloped walls to insure the steady flow of coal and are reinforced by only one or two sets of horizontal stiffeners. Most bunkers have a 2½-in. or 3-in. gunite lining on the inside.

Coal Pressure

The pressures produced by the coal on the vertical and inclined walls of the bin can be computed by the shallow bin formulas or the deep bin formulas. The shallow bin theory is applicable to smaller bins where the plane of rupture does not intersect the opposite wall. For most large coal bunkers, however, the deep bin (or silo) theory is applicable. Final formulas for both theories are presented and useful tables are included. Fig. 2(*a*) shows the pressure conditions against the vertical walls of the bin.

Note.—Published in June, 1951, as *Proceedings-Separate No. 72*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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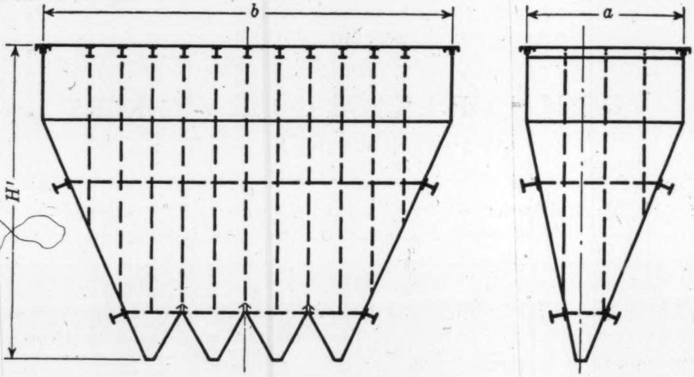
The Shallow Bin Theory.—Pressures in a shallow bin can be computed using the Coulomb theory and substituting commonly accepted constants. Pressure calculations will be based on a level coal surface in the bin. This is justified since most bunkers are loaded from one or more tripper conveyors with double spouts. In addition, it is assumed that the coal weight (w) is 50 lb per cu ft and that the angle of repose (ϕ) of the coal is 35°, which equals the angle of friction (ϕ') of the coal against the concrete bin lining.



(*a*) LONGITUDINAL SECTION (*b*) CROSS SECTION

Fig. 1.—Typical Coal Bunker

Pressure Considering Friction.—The equation for total pressure (P) on the bin walls, including friction, is[2]

$$P = \left[\tfrac{1}{2}\, w\, h^2\right] \left[\frac{\cos \phi}{(1 + \sqrt{2} \sin \phi)^2}\right] \tag{1a}$$

in which h is the depth from the top of the coal load to the point at which pressure is to be calculated. In this form Eq. 1*a* represents liquid pressure modified by a factor of effectiveness of the coal.

Substituting the assumed values in Eq. 1*a*

$$P = 0.25 \times 25\, h^2 \tag{1b}$$

The force (N) normal to the bin wall can be found by simple trigonometry to be

$$N = 0.205 \times 25\, h^2 \tag{2}$$

Pressure Not Considering Friction.—If friction is disregarded, the formula becomes[3]

$$N = P = \left(\tfrac{1}{2}\, w\, h^2\right) \left(\frac{1 - \sin \phi}{1 + \sin \phi}\right) \tag{3a}$$

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[2] "Structural Engineers' Handbook," by Milo S. Ketchum, McGraw-Hill Book Co., Inc., New York, N. Y., 1918, p. 230.

[3] *Ibid.*, p. 229.

Substituting the assumed constants

$$N = 0.27 \times 25\,h^2 \quad\text{(3b)}$$

Computation of the unit normal pressure (p_n) can be simplified by use of the factors found in Eq. 1*b* and Eq. 3*b* for considering friction

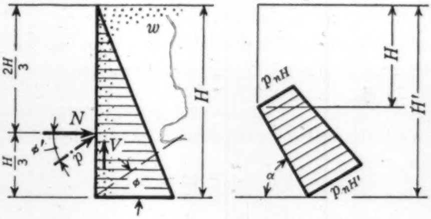
$$p_n = 0.205 \times w\,h \quad\text{(4a)}$$

or

$$p_n = 10.3\,h \quad\text{(4b)}$$

If friction is disregarded

$$p_n = 13.5\,h \quad\text{(4c)}$$



(*a*) VERTICAL SIDES (*b*) INCLINED SIDES

Fig. 2.—Pressure Diagrams for the Sides of a Coal Bunker

Consequently, in order to obtain the horizontal pressure in pounds per square foot at a depth h below the top of the coal, multiply p_n by h. For example if $h = 14$ ft, and if friction is considered, the normal pressure at $h = 14$ ft is equal to $10.3 \times 14 = 144.2$ lb per sq ft.

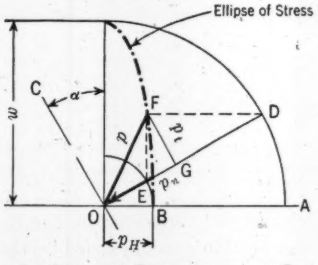


Fig. 3.—Graphical Determination of Coal Pressures in a Shallow Bin

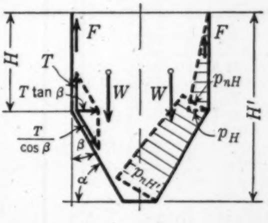


Fig. 4.—Pressures and Forces in a Shallow Coal Bunker

Coal Pressure on Inclined Walls.—Fig. 2(*b*) shows the pressure conditions on the inclined walls of a bin. The normal coal pressure on inclined walls can be computed most conveniently by use of the ellipse of stress. Theoretical derivation of the ellipse of stress can be found in standard texts,[4] but Fig. 3 indicates a direct application to coal bins. This diagram is constructed graphically by the selection of suitable scales for the linear quantities and following the procedure outlined:

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[4] "Design of Walls, Bins and Grain Elevators," by Milo S. Ketchum, *Engineering News Publishing Co.*, New York, N. Y., 2d Ed., 1911, p. 8.

1. With point 0 as a center, draw two quarter circles. Radius 0A represents the weight of 1 cu ft of coal (50 lb) and radius OB denotes the frictionless horizontal pressure of the coal per foot of depth (13.5 lb). It is customary to neglect friction on inclined walls of shallow bunkers because of exposure to impact during loading.
2. Draw line OC parallel to the inclined wall and line OD perpendicular to it. Angle α is the inclination of the bin wall from the vertical.
3. The coordinates of point F on the ellipse of stress are determined by drawing a horizontal line DF and a vertical line EF from the intersection of line OD with the inner and outer circles, respectively.
4. Line FO now represents the actual pressure p on the inclined wall due to a 1-ft depth of coal. The normal component of $P(p_n)$ is line OG and the tangential component (p_t) is line FG.

Pressures p_n and p_t can also be expressed in the algebraical form

$$p_n = 50 \cos^2 \alpha + 13.5 \sin^2 \alpha \qquad (5)$$

and

$$p_t = (50 - 13.5) \sin \alpha \cos \alpha \qquad (6)$$

Table 1 lists values of p_n for frequently used angles. Ordinarily there is no need to compute p_t as this force produces tension in the inclined walls, and this tension can be calculated easily and directly. Fig. 4 indicates the derivation of this tension force. Let W equal the weight of coal supported by the bottom and inclined wall, plus dead load; F equal the portion of coal carried by the wall through friction; and T equal the net vertical load at bend-line $(W - F)$. The fraction $\frac{T}{\cos \beta}$ is then the component of T parallel to the inclined wall, or the actual maximum tension in the inclined wall, where β is the angle of inclination of the bin from the vertical. A more complete analysis of these forces will be found in the subsequent section on design of stiffeners.

TABLE 1.—COAL PRESSURES p_n (IN POUNDS PER FOOT OF DEPTH) NORMAL TO WALLS INCLINED AT VARIOUS ANGLES α

α	p_n	α	p_n	α	p_n	α	p_n	α	p_n	α	p_n
45	31.8	51	28.0	57	24.3	63	21.0	69	18.2	75	16.0
46	31.1	52	27.3	58	23.8	64	20.5	70	17.8	76	15.6
47	30.5	53	26.7	59	23.2	65	20.1	71	17.4	77	15.4
48	29.8	54	26.1	60	22.6	66	19.6	72	17.0	78	15.1
49	29.2	55	25.6	61	22.1	67	19.1	73	16.7	79	14.8
50	28.6	56	25.0	62	21.6	68	18.6	74	16.3	80	14.6

The Deep Bin Theory.—The pressures exerted by the coal on the vertical walls of a deep bin can be conveniently computed by the use of the Janssen formula. A readily usable form of this equation[5] for circular bins is

$$p_h = \frac{w\,R}{\mu'}\,(1 - e^{-k\mu' j}) \text{.....................} (7)$$

in which (to simplify typography) $j = h/R$. In Eq. 7, furthermore R is the hydraulic radius, A/U; A is the area enclosed by a horizontal section through the bin; U is the perimeter of the same section; μ' is the tangent of the angle of friction, ϕ'; p_h is the unit horizontal pressure; e is the base of Naperian logarithms; and k is the ratio of horizontal pressure, p_h/p_v. The ratio k is approximately equal to $\frac{1 - \sin\phi}{1 + \sin\phi}$, although the actual value can only be obtained experimentally. Experiments have shown that the ratio of horizontal to vertical pressure varies with the depth and shape of the bin. For practical purposes, however, a constant value will be assumed.

To present Eq.7 in a convenient form for designers, it is assumed that $\phi' = 35°$ on concrete lining and $\phi' = 20°$ on steel plate; and $\mu' = 0.70$ on concrete and $\mu' = 0.37$ on steel. Angle ϕ equals 35°. Substituting assumed values the formula for horizontal pressure of coal on concrete bin walls is

$$p_h = 71.5R\,(1 - e^{-0.189j}) \text{...................} (8a)$$

The corresponding formula for coal pressure on steel plate is

$$p_h = 135\,R\,(1 - e^{-0.100j}) \text{...................} (8b)$$

Pressures p_h and p_v are functions of R and h. The quantity $1 - e^{-k\mu' j}$ is a function of h/R. Table 2 gives values of this quantity in Eq. 8a for a large variety of ratios h/R. Table 3 gives the actual values (pounds per square foot) of coal pressure for a variety of values of hydraulic radius R.

TABLE 2.—VALUES OF $1 - e^{-0.189j}$ FOR USE IN EQUATION 8a

h/R (1)	$1 - e^{-0.189j}$ (2)	h/R (1)	$1 - e^{-0.189j}$ (2)	h/R (1)	$1 - e^{-0.189j}$ (2)	h/R (1)	$1 - e^{-0.189j}$ (2)	h/R (1)	$1 - e^{-0.189j}$ (2)
0.3	0.058	1.4	0.237	3.0	0.435	5.5	0.646	11.0	0.877
0.4	0.068	1.5	0.244	3.2	0.451	6.0	0.680	11.5	0.888
0.5	0.086	1.6	0.259	3.4	0.473	6.5	0.708	12.0	0.897
0.6	0.104	1.7	0.274	3.6	0.493	7.0	0.733	12.5	0.906
0.7	0.122	1.8	0.288	3.8	0.512	7.5	0.758	13.0	0.915
0.8	0.139	1.9	0.302	4.0	0.528	8.0	0.779	13.5	0.922
0.9	0.156	2.0	0.316	4.2	0.546	8.5	0.800	14.0	0.928
1.0	0.173	2.2	0.343	4.4	0.564	9.0	0.817	14.5	0.936
1.1	0.190	2.4	0.362	4.6	0.581	9.5	0.835	15.0	0.941
1.2	0.205	2.6	0.387	4.8	0.597	10.0	0.849	16.0	0.950
1.3	0.221	2.8	0.411	5.0	0.609	10.5	0.862	17.0	....

The pressures computed by Eqs. 7 and 8 apply only to cylindrical cross-sections, and the pressure values must be multiplied by a coefficient in order to be applicable to square and rectangular cross sections. The hydraulic radius R is the same for a square as for its inscribed circle. The area of the square is

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[5] "Reinforced Concrete Water Towers, Bunkers ,Silos, and Gantries," by W. S. Gray, *Concrete Publications, Ltd.*, London, England, 2d Ed., 1944, p. 111.

27% greater, however, than that of the inscribed circle, which means that a square bin carries a 27% larger volume of coal than a circular one. Consequently, the values of p_h and p_v should be computed for the hydraulic radius R of the equivalent circle, the diameter of which is 13% larger than the side of the square. Thus the equivalent hydraulic radius (R') for a square bin is 1.13 $(a/4)$.

The pressures on the walls of a rectangular bin in the short direction (a) can be computed as for a square bin with sides a, but in the long direction(b) the pressures are larger and must be computed on the basis of an equivalent square bin with sides of $a' = \frac{2\,ab - a^2}{b}$. Setting b equal to $m\,a$, in which m is a coefficient always larger than unity, a' can be expressed as $= a\left(2 - \frac{1}{m}\right)$, or $a' = z\,a$.

TABLE 3.—Horizontal Pressures p_h (in

$R = A/U$	Depth h, in Feet											
	5	10	15	20	25	30	35	40	45	50	55	60
	(a) Coal on Concrete Lining											
3.0	60	100	130	155	170	180	190	200	200	205	210	210
3.5	60	105	140	165	185	200	210	220	230	235	240	240
4.0	60	110	145	175	200	215	230	245	250	260	265	270
4.5	60	110	150	185	210	230	250	265	275	280	290	295
5.0	60	115	155	190	220	240	260	280	290	305	315	320
5.5	60	115	160	195	230	250	275	295	310	325	335	340
6.0	65	120	165	200	235	265	290	310	325	340	350	365
6.5	65	120	165	205	240	270	300	325	340	360	370	385
7.0	65	120	165	210	245	280	310	335	350	370	390	400
7.5	65	120	170	215	250	285	315	340	365	385	400	420
8.0	65	120	170	215	255	290	320	350	375	400	415	435
8.5	65	120	175	220	260	295	330	360	385	410	430	450
9.0	65	120	175	220	265	300	335	370	395	420	440	460
9.5	65	120	175	225	270	305	340	370	400	430	450	475
10.0	65	120	180	225	270	310	345	380	410	440	460	490
10.5	65	120	180	230	270	315	350	390	420	450	470	500
11.0	65	120	180	230	275	320	355	390	425	455	480	505
11.5	65	120	180	230	275	320	360	400	430	460	490	515
12.0	65	120	180	235	280	325	365	400	435	465	500	530

These values can be easily tabulated:

m	z	m	z
1.00	1.00	2.25	1.56
1.25	1.20	2.50	1.60
1.50	1.33	2.75	1.64
1.75	1.43	3.00	1.67
2.00	1.50		

Coal Pressures on the Inclined Walls of Deep Bins.—The pressures on the inclined walls of deep bins can be computed easily by use of the ellipse of stress.

According to the Janssen theory, the horizontal pressure p_h is equal to k times the vertical pressure p_v; thus:

$$p_h = k\, p_v \dots\dots\dots\dots\dots\dots\dots\dots \quad (9a)$$

The ratio k has been determined previously and is equal to 0.27. Substituting this value in Eq. 9*a*, p_v is found in terms of p_h

$$p_v = 3.7\, p_h \dots\dots\dots\dots\dots\dots\dots\dots \quad (9b)$$

POUNDS PER SQUARE FOOT) BY EQUATIONS 8

DEPTH *h*, IN FEET												$R = A/U$
5	10	15	20	25	30	35	40	45	50	55	60	
(*b*) Coal on Steel Plates												
65	115	160	200	230	255	280	300	315	330	340	350	3.0
65	120	165	205	240	270	300	320	340	360	375	390	3.5
65	120	170	210	250	285	315	340	365	385	400	420	4.0
65	120	170	220	260	295	330	360	385	405	425	445	4.5
65	120	175	225	265	305	340	370	400	425	450	470	5.0
65	125	175	225	270	310	350	385	415	445	470	495	5.5
65	125	180	230	275	320	355	395	430	460	485	510	6.0
65	125	180	235	280	325	365	405	440	470	505	530	6.5
65	125	185	235	285	330	370	410	450	480	515	545	7.0
65	125	185	240	285	335	375	420	455	490	525	555	7.5
65	125	185	240	290	340	385	425	465	500	540	570	8.0
65	125	185	240	295	340	390	430	470	510	550	585	8.5
65	125	190	240	295	340	390	435	480	515	555	595	9.0
65	130	190	245	295	350	395	440	485	525	565	600	9.5
65	130	190	245	300	350	400	445	490	530	570	610	10.0
65	130	190	245	300	350	400	445	495	535	575	615	10.5
65	130	190	245	300	350	405	450	500	540	580	620	11.0
65	130	190	250	300	355	405	450	500	545	590	630	11.5
65	130	190	250	305	360	410	460	510	555	600	635	12.0

A graphical solution for the bin pressures provides the most convenient approach to the problem. This procedure is outlined here, with reference to Fig. 5.

a. Plot a cross section of the bin to any convenient scale. Alongside the bin erect line AC and subdivide this line into 5-ft scale intervals, starting at the top. Extend the subdivision lines through the bin and compute the hydraulic radius at each 5-ft interval.

b. By computation (or by Table 3), determine the horizontal pressure (p_h) at each level. These pressures are plotted to a convenient scale perpendicular to line AC (line BD, for example). The horizontal pressure diagram (line ADC) is determined by connecting successive values of p_h.

c. The pressure on the inclined walls is determined by constructing an ellipse of stress as described previously. If radius OA of Fig. 3 is drawn to a value of 3.7, representing p_v, and line OB is made equal to a value of 1.0 at the same scale, representing p_h, then a coefficient value for p_n can be obtained for the given slope of bin wall. The horizontal pressures at each 5-ft level in Fig. 5 are multiplied by this coefficient to obtain the absolute value of p_n.

d. The total normal pressure on the inclined wall is obtained by adding the normal component of the dead load of the bunker plate and lining to the pressure due to the coal.

Table 4 gives values of p_n based on $p_h = 1$ and $p_v = 3.7$ for a number of common wall slopes. To obtain transverse loading on stiffeners, multiply the total normal pressure by the stiffener spacing. This loading produces bending in the stiffeners.

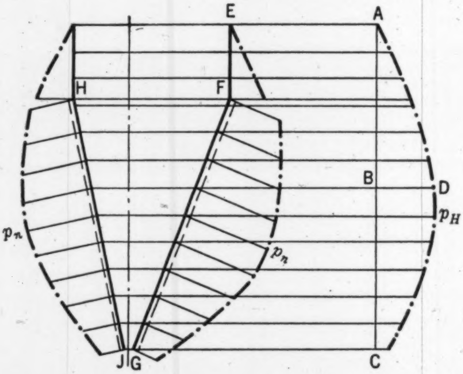

FIG. 5.—GRAPHICAL DETERMINATION OF COAL PRESSURES IN A DEEP BIN

A standard form for use in tabulating pressures is a useful analytical tool. The suggested table contains eight columns and is simple yet complete. The columns to be used are:

Column No.	Caption
1	Depth.—The location of the point of analysis below the top of the bin.
2	Bin Ratio.—The ratio of length to width of the bin (b/a).
3	z Values.—Determined from listing in previous section.
4	Hydraulic Radius in Short Direction.—This equals 1.13 $a/4$ as previously explained.
5	Hydraulic Radius in Long Direction.—Equals z times the value from Col. 4.
6	Correction for Inclination of Walls.—Determined from Table 4.
7	Normal pressure in the short direction.
8	Normal pressure in the long direction.

DESIGN OF FLEXIBLE BUNKER PLATES

The conventional flexural theory is not adaptable to flexible plates that have deflections exceeding half of the plate thickness under normal loading conditions. Flexible plates can carry much heavier loads than ordinary theory would permit because the steel area around the neutral axis is utilized to carry additional loads through pure tension in addition to the areas near the outside fibers which carry loads by flexural tension and compression.

TABLE 4.—COAL PRESSURE COEFFICIENT p_n TO DETERMINE THE PRESSURE NORMAL TO WALLS INCLINED AT VARIOUS ANGLES α

α	p_n	α	p_n	α	p_n	α	p_n	α	p_n	α	p_n
45	2.36	51	2.07	57	1.80	63	1.56	69	1.35	75	1.18
46	2.31	52	2.02	58	1.76	64	1.52	70	1.32	76	1.16
47	2.26	53	1.98	59	1.72	65	1.49	71	1.29	77	1.14
48	2.21	54	1.93	60	1.68	66	1.45	72	1.26	78	1.12
49	2.17	55	1.89	61	1.64	67	1.42	73	1.24	79	1.10
50	2.12	56	1.85	62	1.60	68	1.38	74	1.21	80	1.08

The analysis of flexible plates is given in many books of advanced strength of materials. The values of allowable loads for plates of different thicknesses and a variety of spans, with both free and clamped edges, are given in Table 5, which is based on formulas and charts presented by S. Timoshenko.[6] The values are calculated for 20,000 lb per sq in. allowable stress.

TABLE 5.—ALLOWABLE LOADS (IN POUNDS PER SQUARE FOOT) ON FLEXIBLE PLATES

PLATE THICKNESS		SPANS, IN FEET								
Inches	End condition	2.0	3.0	3.5	4.0	4.5	5.0	5.5	6.0	7.0
¼	Simply supported	980	890	870	855	840	....	....	....	....
	Fixed	640	310	240	200	180	....	....	....	....
⅜	Simply supported	....	985	940	910	890	880	870	855	835
	Fixed	....	655	560	380	300	260	225	200	165
½	Simply supported	....	....	1.025	980	930	920	905	890	870
	Fixed	....	....	830	640	420	420	360	310	240
⅝	Simply supported	....	....	....	1,050	1,015	980	950	935	900
	Fixed	....	....	....	940	780	640	530	460	340

Table 5 indicates that, because of the increased resistance to bending, flexible plates (unlike ordinary slabs) carry much less load with the ends fixed than with ends simply supported.

Bunker plates, riveted or welded to stiffeners, are neither fixed nor simply supported, and an intermediate value should be chosen. Ordinarily ⅜-in. plate thickness is sufficient for all practical purposes. Neither the increase of plate thickness nor of the span produces any pronounced variation in the value of the allowable load.

No allowance was made for the gunite lining, due to its possible deterioration and wear, although it is obvious that it adds materially to the load-carrying capacity of the plates.

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[6] "Theory of Plates and Shells," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1940.

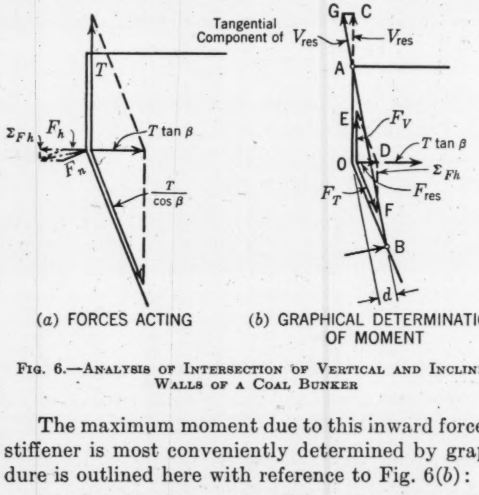
THE ANALYSIS OF THE INTERSECTION OF VERTICAL AND INCLINED STIFFENERS

It was shown in Fig. 4 that there is an inward force acting at the intersection of the vertical and inclined walls equal to $T \tan \beta$.

The coal pressures on the vertical and inclined walls produce outward forces F_h and F_n as shown in Fig. 6(*a*).

The ideal case occurs when the sum of $T \tan \beta$ plus F_h, and the horizontal component of F_n equal zero. This can be accomplished by establishing the location of the horizontal stiffener so as to produce the proper reaction F_n at the intersection. Quite often, however, the location of the horizontal stiffener is set by other requirements, and consequently a resultant inward force is created at the intersection.

This inward force can be resisted by a horizontal member at the intersection or by designing the broken-back vertical stiffeners to resist the bending created by this force.



(*a*) FORCES ACTING (*b*) GRAPHICAL DETERMINATION OF MOMENT

FIG. 6.—ANALYSIS OF INTERSECTION OF VERTICAL AND INCLINED WALLS OF A COAL BUNKER

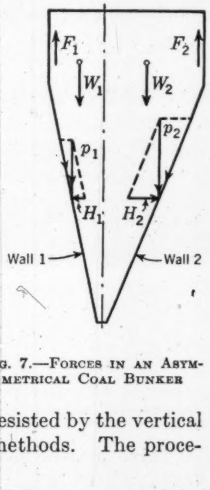


FIG. 7.—FORCES IN AN ASYMMETRICAL COAL BUNKER

The maximum moment due to this inward force to be resisted by the vertical stiffener is most conveniently determined by graphical methods. The procedure is outlined here with reference to Fig. 6(*b*):

(1) Graphically, resolve the resultant inward force (F_{res}), represented by line OD into two components, one parallel to the vertical stiffener (OE) and one parallel to the inclined stiffener (OF).

(2) Draw line BA connecting the horizontal stiffener at point B with the horizontal support at point A.

(3) Lay off the force F_v (line OE) above point A (line AC) and determine the force component parallel to the connecting line BA.

(4) The maximum moment is determined by multiplying the parallel component of F_v (line AG) by the perpendicular distance (*d*) from line BA to point O.

Design the vertical stiffeners to resist the combined effect of this moment, the negative moment due to the coal pressure, and the maximum tension

$T/\cos\beta$. The bunker plate and the stiffeners act as one unit; thus the stiffeners should be designed as **T**-beams.

ANALYSIS OF UNBALANCED FORCES

In case the inclined walls are of different slopes, the bunker will tend to assume a symmetrical shape. This tendency to overcome asymmetry produces an unbalanced horizontal force. The amount of this horizontal force can be computed as shown on Fig. 7.

Divide the bunker into two parts by means of a vertical line through the bottom opening, and compute the coal weights (W_1 and W_2) and the weight of coal carried by the vertical walls (F_1 and F_2).

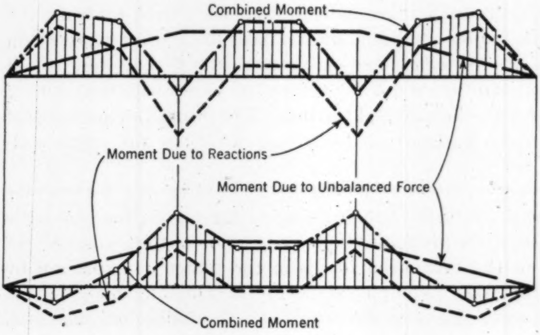


FIG. 8.—MOMENT DIAGRAM FOR HORIZONTAL GIRDER

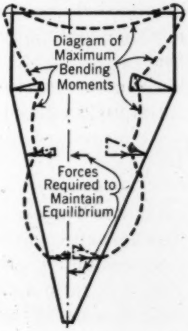


Diagram of Maximum Bending Moments
Forces Required to Maintain Equilibrium

FIG. 9.—MOMENT DIAGRAM FOR DESIGN OF VERTICAL STIFFENERS

The force on the inclined wall is the sum of the coal weight and dead load minus the weight of coal carried by the vertical wall. If this value is computed for both walls and the horizontal components found, the difference in horizontal components of the two walls is the unbalanced force to be resisted.

This can be accomplished by providing one or more horizontal girders on the inclined walls. These girders will support the reactions of the vertical stiffeners, which automatically include the unbalanced forces. Long bunkers ordinarily have horizontal struts at specific intervals connecting the horizontal girders. These struts have the effect of creating continuous beams out of the horizontal girders.

In the case of asymmetry, however, the reactions of the horizontal girders on the struts are opposite but not equal, and the difference between these reactions creates concentrated loads on the girder that spans between endwalls.

Fig. 8 shows the moment diagram of a typical horizontal girder with two struts. This diagram indicates moments for bending due to the reactions of vertical girders and for the differential reactions of the struts. The two moments then are superimposed in order to obtain the maximum moments.

Design of the Vertical Stiffeners.—The two sets of vertical stiffeners at any cross section should be designed by the use of continuity principles. In this method the upper horizontal girder, which supports the flooring and the coal unloading tripper, should be included.

It may be necessary to make several analyses, for instance with, and without tripper loading, or with the bunkers fully and partly loaded. A simplified analysis for such cases has been presented by the writer.[7]

A typical moment diagram for an asymmetrical bunker is given in Fig. 9. These moments are used in the design of the vertical stiffeners.

LIMITATIONS OF THE COAL BUNKER DESIGN

The foregoing theory and analysis of large coal bunkers furnishes practical solutions for safe and economical design. Investigations and field measurements conducted on a large coal bunker indicate that the actual pressures did not vary materially from values computed on the basis of theories expressed in this paper. Extensive studies conducted in France[8] imply that the ratio of horizontal to vertical pressure (k) is not a constant but varies with the depth of the stored material and with the shape of the bin. The filling and emptying of the bin produce different pressures which do not seem to follow any regular pattern.

The actual stresses in the plates and beams are probably much less than the computed values. The effect of the gunite lining on the plates, the composite T-beam action of the beams, the probable diaphragm action of the walls, and the stress-reduction effect of the tangential forces in the plates as expressed by Poisson's ratio all characterize a complex structure in which all members act simultaneously, thus greatly reducing the stresses in the individual members. Actual strain-gage measurements on a large bunker suggested the presence of extremely low stresses in the bunker plates.

The use of the foregoing analysis, with the exception of formulas for coal pressures, is not recommended for the design of reinforced concrete bunkers. Concrete bunkers with rigid plates and no stiffeners for walls are most conveniently designed as hipped plate construction.[9]

SUMMARY

Design formulas for the practical analysis of large-capacity coal bunkers have been presented. Several figures and tables are included in order to facilitate analysis and to reduce computations to the minimum. It is believed that this paper will be a useful tool for the office designer.

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[7] "Simplified Analysis for Live Loads," by Paul Rogers, *American Concrete Institute Journal*, Vol. 20, October, 1949, p. 144.

[8] "Recherches nouvelles sur les efforts exercés par les matières pulvérulentes ensilées sur les parois des silos," by M. Reimbert, *Annales de l'Institut Technique du Batiment et des Travaux Publics*, Paris, France, Série I, May 10, 1943.

[9] "Hipped Plate Construction," by George Winter and Minglung Pei, *American Concrete Institute Journal*, Vol. 18, January, 1947, p. 505.

DISCUSSION

George L. Blanchard,[10] M. ASCE.—The first recent attempt to make available to the designer a modern and practical solution to the problem of coal bunker design is presented in this paper. The author refers to studies made in France and, in particular, uses one of the formulas proposed by M. Reimbert made to extend the cylindrical deep silos pressures equations to square or rectangular bunkers.

The writer wishes to emphasize the fact mentioned by Mr. Rogers that those studies,[11] which were originally based on experimental research, have proved that the exponential expression as used by Mr. Janssen was inadequate for the computation of pressures in the upper zones of a bin, and gave results far below those measured experimentally.[12] Mr. Albert Caquot,[13] through rather elaborate and complete mathematical developments, has evolved an original method and practical formulas, the results of which appear to corroborate closely those obtained with Mr. Reimbert's method. The writer has had the occasion to use both the Reimbert and Caquot methods in specific cases of design. It has been found that the percentage of discrepancies between the two was so small as to be unimportant. On the contrary, the use of the Janssen formulas has led, for example, to an underestimation of pressures and tangential forces at wall faces of more than 33% less at depths of 10 ft from the top to 10% less only at 80 ft down from the top.[14] According to reliable authorities, there lies the main reason for certain types of disorders observed in the upper zones of deep silos built in reinforced concrete. The paper deals with structural steel bunkers specifically. Therefore, the remarks may appear somewhat as a superfluous refinement, considering the arbitrary assumptions, with questionable degree of accuracy, which are often accepted as regards the characteristics of the material to be handled and the conditions of the wall's face. It is obvious, however, that an important underestimation of the lateral pressures and forces should be avoided consciously, particularly in the upper part of such a complex structure as a coal bunker of large capacity, where the solicitations of the structural elements are multiple and intense under constantly variable service conditions. The purpose of the present comment is to draw the attention on this particular point and bring up some additional precisions to the author's note.

The needs of more elaborate and precise methods of structural design cannot be emphasized too forcibly to prevent the temptation of the designer to compensate for the risk of uncertainties by an empirical and unjustified increase in the structural sizes or in the quantities of critical materials, particularly at the present time.

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[10] Consulting Engr., Algiers, Chicago, Ill.

[11] "Recherches nouvelles sur les efforts exercés par les matières pulvérulentes ensilées sur les parois des silos," by M. Reimbert, *Annales de l'Institut Technique du Batiment et des Travaux Publics*, Paris, France, Série I, May, 1943, pp. 1–60.

[12] "Etude expérimentale et théorique des contraintes dans les Silos," by Maurice Buisson, *Travaux*, Paris, France, April, 1943, p. 129, August, 1943, p. 290.

[13] "Traité de la Mécanique des Sols," by Albert Caquot and R. Kérisel, Gauthier-Villars, Paris, France, 1949, Chapter VII ("Silos"), p. 350.

[14] Discussion by G. L. Blanchard of "Design of Hexagonal Bins," by Paul Rogers, *Journal*, A. C. I., December, 1950, Part 2, pp. 34–46.

The author's paper, in that respect, is a valuable contribution to the improvement of methods of design accessible to a larger number of structural designers in the tremendously important field of power production.

Alexander M. Turitzin,[15] M. ASCE.—In addition to the statement (under the heading, "Coal Pressure") that—

> "The shallow bin theory is applicable to smaller bins where the plane of rupture does not intersect the opposite wall. For most large coal bunkers, however, the deep bin (or silo) theory is applicable."

the author should have given a definite criterion or a procedure to follow in determining the theory applicable to any bin size and shape.

After the capacity and the shape of the coal bunker is determined, the designer is confronted with the problem of determining whether he should follow the shallow bin theory or the deep bin theory to compute the pressures exerted by the coal on the bin walls. An analysis of the plane of rupture and the derivation of the criteria for both a shallow bin and a deep bin will indicate when to use one or the other of the two theories. It is generally conceded that when the plane of rupture intersects the surface of the coal, the bin is called a shallow bin. It is assumed that the mass of coal is unlimited, or at least of sufficient extent so that the outline of the sliding wedge (that is, a wedge having the wall as one side and the plane of rupture as the other side) is not interrupted.

The closer the walls, the more erroneous the shallow bin theory is; and when the walls are so close together that the plane of rupture does not intersect the surface of the coal but falls on the opposite wall, the weight of the coal is largely supported on the sides by friction. The deep bin theory must be used to determine the pressures exerted on the bin walls. According to the Coulomb theory, the plane of rupture lies between the angle of repose of the filling and the back of the wall. For a wall without surcharge (surface horizontal), the plane of rupture bisects the angle between the plane of repose and the back of the wall.

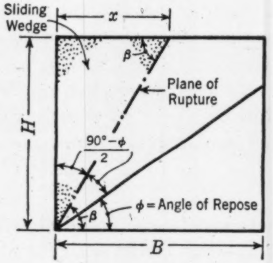

Fig. 10

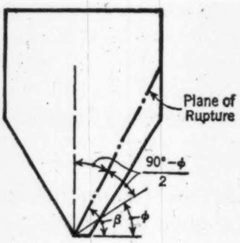

Fig. 11

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[15] Structual Engr., Devenco Inc., New York, N. Y.

Fig. 10 illustrates the relation between the angle of repose α, for such a case, and the angle β that determines the plane of rupture.

From the definition previously given, the criterion for a shallow bin is

$$x < B \qquad (10)$$

From Fig. 10,

$$\tan \beta = \frac{H}{x} \qquad (11)$$

Solving for x in Eq. 11 and substituting in Eq. 10,

$$\frac{H}{\tan \beta} < B \qquad (12)$$

From Fig. 10,

$$\beta = \phi + \frac{90° - \phi}{2} = \frac{\phi}{2} + 45° \qquad (13)$$

Substituting the value of β from Eq. 13 into Eq. 12,

$$\frac{H}{\tan\left(\frac{\phi}{2} + 45°\right)} < B \qquad (14)$$

When the angle of repose ϕ of the coal is equal to 35°,

$$\frac{H}{\tan\,(62°\ 30')} < B \qquad (15)$$

and

$$\frac{H}{\mathrm{B}} < 1.9 \qquad (16)$$

Eq. 16 shows that when the ratio of the height to the width of a coal bin is less than 1.9, the coal bin is called a shallow bin and therefore a shallow bin formula should be used to determine the pressure.

For a hopper bin, a graphical solution is the most convenient. The procedure to follow in determining whether a coal bin is a shallow bin or a deep bin, with reference to Fig. 11, is as follows:

1. Draw a transverse cross section of a bin to any convenient scale.
2. From the lower left corner of the bin, plot with a protractor an angle of 62° 30′; the plane thus formed is the plane of rupture.
3. If the plane of rupture does not intersect the wall of the bin, the hopper bin is a shallow bin. A deep bin is shown in Fig. 11.

Paul Rogers,[16] M. ASCE.—The need for more accurate analysis in connection with the deep bin theory is emphasized by Mr. Blanchard, who refers to the works of the outstanding French engineers, M. Reimbert,[11] M. Buisson,[12] and A. Caquot and R. Kerisel.[13] It is true that the Janssen formulas had been developed for circular bins only, and their use, without corrections, for square and rectangular cross sections is definitely fallacious. This is not recognized easily, however, because, through a mathematical incongruity, the hydraulic radius of a square is identical with that of the inscribed circle.

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[16] Structural Design Engr., Sargent & Lundy, Chicago, Ill.

With the advancement of science and the measuring technique, it should become obvious that Mr. Janssen's pioneering formulas are not entirely accurate. It is believed that the findings and advanced theories of the French scientists should be presented to the American engineers. This is a problem, however, in the realm of the engineering mechanics and cannot be included in the scope of a practical paper dealing with actual design.

Further experiments have been conducted by the Institut Technique du Batiment et des Travaux Publics, under the direction of Mr. Reimbert, with particular emphasis on dynamic and impact forces acting on the stored material in the bins. Much is to be learned yet about the internal stresses, especially in very deep bunkers where a 50-ft or 60-ft "head" may materially change the density of the coal. The density of "cyclone" boiler coal is considerably greater than the ordinary kind, probably because of the dust particles, and it is advisable to increase all tabulated values by 20% when such coal is used. Accurate information as to the value of the internal friction and the angle of friction between the coal and the wall should also be forthcoming. Mr. Turitzin emphasizes the necessity for a clear definition between the shallow and deep bins. His suggestions are practicable for bins with vertical walls and, in accordance with the classical theory of equivalent fluid pressure, for granular materials. However, the reduction of the coal pressure resulting from friction between the coal and the wall is applicable even for shallow bins.

The governing factor in the design of coal bunkers is not the coal pressure but the tendency of the bend line to straighten. This tendency is partly counteracted by the coal pressure. Thus the assumption of unreasonably large coal pressure may not produce a safe design because it may reduce the inward acting forces unrealistically.

The importance of these inward forces cannot be overemphasized. Not only does analysis show their existence, but also the measurements and visible movements of models and prototypes definitely prove their actions. Several cases are known whereby the opposite bend lines were connected by limber "ties," presumably to restrain outward forces. In each case, these slender members failed in buckling. The distribution of the forces and moments in bunkers is not unlike inverted gable frames where the moments are produced by the deviation of the pressure line from the actual shape of the frame.

The foregoing discussion would imply that the ideal bunker is what is commonly known as the catenary or suspension bunker because, under ideal loading conditions, no moments are present—only pure tension in the bunker walls.

There are numerous objections to these catenary bunkers, however. There is no accurate theory giving the shape of the bunker. The commonly used triangular loading, as it is given in older textbooks, is definitely inadequate because it disregards the coal pressures. Should one attempt to obtain a reasonable funicular polygon, by a cut-and-try method, based both on vertical and horizontal forces, there still would be many uncertainties as to the value of such forces. Catenary bunkers are not self-cleaning and there is a constant danger of instantaneous combustion of the coal.

As the outlets must necessarily be about 12 ft to 16 ft apart, the coal must be directed toward these outlets by timber bulkheads or lean-mix concrete saddles. A large part of the capacity is thus lost, and the bunker is considerably stiffened instead of being elastic. The fabrication is costly; the rolling of the steel plates and stiffeners to the exact catenary shape is quite difficult; and substitutions by circular arcs are faulty.

When catenary bunkers are loaded, the sides bulge outward and the bottom moves upward. Consequently, the outlet connections must be flexible to allow for these vertical movements, which may amount to several inches. Unsymmetrical cross sections are impossible with catenary bunkers, thus imposing a serious restriction on the layout.

The foregoing difficulties indicate that the disadvantages of the catenary bunkers far outweigh the advantages due to reduction of bending moments. Considerable savings can be achieved by proper layouts of regular bunkers composed of straight sides. If the side walls are on a column-row, such walls also act as deep girders between columns and there is no need for supporting beams. More than a dozen large bunkers (up to 2800 ton capacity) have been designed and built based on the principles expressed in the paper, and all of them were operating very successfully in 1952.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 2508

WEDGE-BEAM FRAMING

By Arsham Amirikian,[1] M. ASCE

With Discussion by Messrs. G. C. Ernst, J. J. Hromadik, and G. R. Swihart; Herbert A. Sawyer, Jr.; J. J. Polivka; and Arsham Amirikian

Synopsis

Appreciable savings in materials and in the cost of a structural framing can be assured by the use of elements having tapering depth or flanges. "Wedge beams" can be substituted for members of constant cross section, as normally utilized in conventional framing, by introducing a series of hinges or joints of limited moment capacity. In a rectangular framing, the hinges are placed at midspan of the horizontal members and at the base of the columns, and the sections are tapered from a maximum at the rigidly connected joints to a minimum at the hinges.

The analysis of the "wedge-beam" framing remains in the domain of statically indeterminate structures. However, in comparison with rigid or continuous framing, it involves the solution of a relatively small number of unknowns, consisting of hinge reactions in the alternate bays of a given frame.

In this paper suggested arrangements of wedge-beam framing are outlined, the range of application is discussed, and a method of analysis is presented. Formulas and tables, as well as a simplified procedure of solution of simultaneous equations, are also given to facilitate the necessary computations in practice.

Introduction

The flexural pattern of a member in conventional arrangement of framing, utilizing either rigid-end or simple-end connections, follows an irregular course. The variations occur not only in the shape of the bending moment diagram but in the location of the maximum bending moment as well, in accordance with the condition and the position of the imposed loading. In the case of rigid framing, the flexural pattern is further affected by the changes in the relative stiffness of the various members. Consequently, it is not economically practicable, often not even possible, to shape the member for a close fit to its stress

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Note.—Published in January, 1951, as *Proceedings-Separate No. 53*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Head Designing Engr., Bureau of Yards and Docks, U. S. Dept. of the Navy, Washington, D. C.

pattern. As a result, material could not be used efficiently in conventional arrangements of framing. This fact, in turn, suggests a re-examination of present concepts and practice of structural design.

The problem of stress control (making the stress pattern as favorable as possible for proportioning the several sections) is not as hopeless as might first appear. A possible solution is already suggested by the flexural behavior of a member in a continuous framing. When the sections of the member, originally having a constant cross section along the span, are increased near its supports, the moment ordinates under a given lateral loading will increase correspondingly toward the ends of the span and decrease at the middle. Furthermore, if the increase in the end sections is accompanied with reductions in section at the middle of the member, the resulting changes in the moment ordinates become still more pronounced. From these observations it is not difficult to conclude that, as the shape of the member is changed from its original outline of constant cross section to that of wedge, the bending moment diagram will change correspondingly from the initial irregular outline of reversed flexure to the familiar pattern of single flexure—as obtained from two simple cantilevers.

The concept of wedge-beam framing is predicated on this consideration. It provides a method of establishing a consistent stress pattern to which the framing is closely fitted. The resulting arrangement assures more advantageous use of the materials of framing and thus yields appreciable savings in weight and economies in the cost of construction.

PART I. FRAMING

In general, wedge-beam framing may be defined as a system composed of wedge-shaped members arranged to provide a consistent or controllable stress pattern. For this purpose use is made of a hinged connection, or a joint of small moment capacity, at the tapered or narrow end of each member, to form a series of fixed inflection points in the flexural path of a framing under any given condition of loading. To assure the stability of the system, the connec-

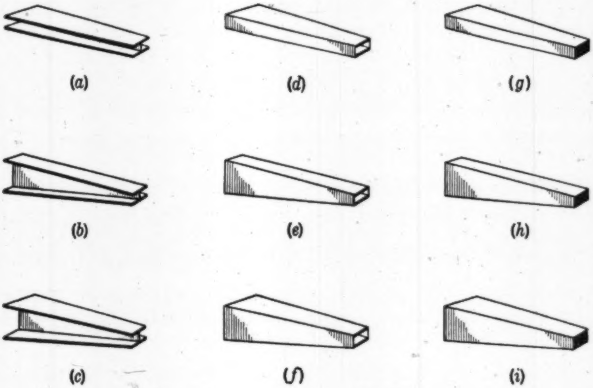


FIG. 1.—TYPES OF WEDGE BEAMS

tion at the butt or wide end of each member is made rigid. Thus the resulting arrangement is not fully continuous, as in rigid framing, nor is it like a simple framing where joints are assumed to have no moment resistance.

The Basic Element of Framing

A wedge beam is essentially a framing member of variable cross section. The variation may be due to either a single taper or a double taper, affecting the outlines of the web and the flanges of the beam. That is, a wedge beam may have either a tapering web or a tapering flange only, or it may have a tapering web and tapering flanges as well. In general, the taper will conform to a straight line; however, in some cases, a curved outline may also be used. In cross section, wedge beams may have the same shapes as those normally utilized in structural framing—namely, an I-section, a channel section, and a rectangle of either solid or hollow section. Some of these shapes and outlines are shown in Fig. 1.

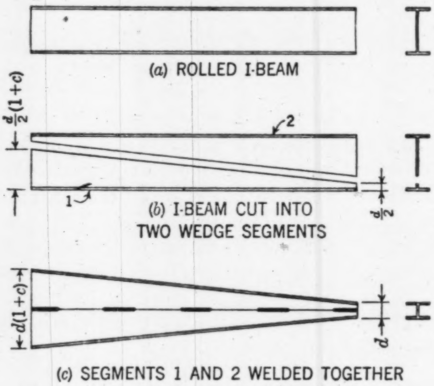

(a) ROLLED I-BEAM
$\frac{d}{2}(1+c)$
2
1
(b) I-BEAM CUT INTO TWO WEDGE SEGMENTS
$\frac{d}{2}$
$d(1+c)$
d
(c) SEGMENTS 1 AND 2 WELDED TOGETHER

Fig. 2.—Method of Forming a Wedge Beam from an I-Beam

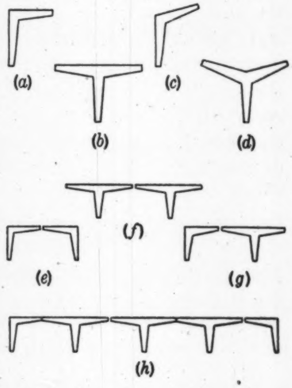


Fig. 3.—Subassemblies and Assemblies of Wedge-Beam Framing

A wedge beam may be constructed of steel, concrete, or timber. In the case of steel, any desired shape and outline can readily be obtained by welded plating. Rolled steel I-beam sections can also be used in building a wedge of constant width and of tapering depth, as indicated in Fig. 2. For concrete construction, hollow and solid rectangular sections can be utilized in obtaining the desired wedge, the former section being particularly adaptable to precast work.

Arrangement of Framing

Components of framing and the manner of assembly are shown diagrammatically in Fig. 3. For reference purposes, the main assembly may be considered to be composed of a series of statically determinate frames called "simple frames." A simple frame, in turn, is composed of one or two subassemblies each of which is formed by two or three wedge members rigidly joined at their butt ends. A two-member subassembly conforms to an inverted

L-outline, and is thus called an "**L**-segment." The arm of an **L**-segment may extend either horizontally, as in Fig. 3(*a*), or may be placed on an upward slope as in Fig. 3(*c*). A three-member subassembly has the outline of either a **T** (Fig. 3(*b*)), or a **Y** (Fig. 3(*d*)) and is, accordingly, called a "**T**-segment," or a "**Y**-segment." Figs. 3(*e*), 3(*f*), and 3(*g*) are three types of simple frames, composed, respectively, of two **L**-segments, two **T**-segments, and a combination of an **L**-segment and a **T**-segment. Fig. 3(*h*) shows a framing assembly consisting of a **T**-segment linked to a simple frame at each side.

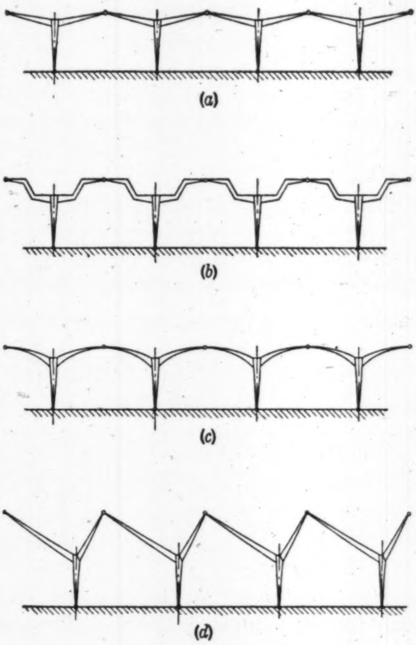


FIG. 4.—TYPES OF WEDGE-BEAM FRAMING

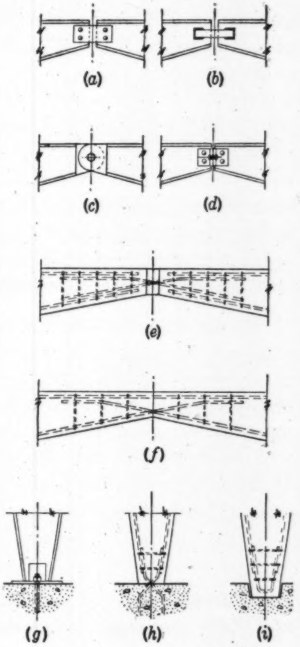


FIG. 5.—TYPES OF CONNECTIONS

In general, the wedge members are designed to provide a symmetrical arrangement in each bay of the framing. However, in certain cases it may be necessary to use an unsymmetrical arrangement—in elevation as well as in cross section—to conform to a given building outline (such as the unsymmetrical gable framing shown in Fig. 4(*d*)) or to obtain a more economical design.

In elevational outline, wedge-beam framing is adaptable also to buildings where the roof may have an outline differing from that of a flat or sloping roof. Two examples of this case are shown in Figs. 4(*b*) and 4(*c*).

CONNECTIONS

As stated in the preceding section, the members in a wedge-beam framing are connected by a rigid joint at the butt end and a hinge at the tapered end of each wedge. Theoretically, the hinged connections should have no moment resistance since their use is intended to provide fixed inflection points in the stress pattern of a frame. In certain types of framing, it will be feasible to design a connection that will nearly satisfy this condition of restraint. However, if the detail for such a joint should entail much expense, then use may be made of a so-called flexible connection or a joint of limited rigidity. A joint of this type will usually yield and rotate when its small moment resisting capacity is overcome. Some suggested details conforming to this concept are given in Fig. 5. Details (*a*) and (*d*), Fig. 5, utilize riveted or welded connection gussets and clip angles, respectively. Detail (*b*) consists of a welded link bar of narrow width. Detail (*c*) is a simplified hinge connection, consisting of two overlapping end plates and an ordinary bolt. Detail (*g*) illustrates a simple anchor-bolt connection suitable for use at the bases of steel columns. Details (*e*), (*f*), (*h*), and (*i*), Fig. 5, are applicable in concrete construction. Of these, details (*e*) and (*h*) are similar to the semiflexible hinge of A. Mesnager.[2] Detail (*f*) has the advantage of a simplified construction but it is also subject to cracking; and it may be used where appearance is not of prime importance. Detail (*i*), Fig. 5, is suitable for use at column bases where no net uplift is present.

Range of Application

Wedge-beam framing can be utilized in a variety of building frames of steel, concrete, or timber. It is adaptable to riveted and conventionally poured-in-place concrete construction and, more particularly, to welded-steel and precast concrete construction. It can be used in tier-type, multistory frames and, still more advantageously, in single-story industrial buildings. These applications will result in large reductions in the weight of the framing materials, as well as in appreciable savings in construction costs. In addition, wedge-beam framing provides a framing outline of pleasing appearance.

PART II. ANALYSIS

A simple frame, as defined in Part I and sketched in Figs. 3(*e*), 3(*f*), and 3(*g*), is a statically determinate, two-legged, three-hinged bent. However, when two or more simple frames are linked together to form an assembly, the resulting arrangement becomes statically indeterminate. Accordingly, the analysis of a wedge-beam framing of two or more bays falls in the category of statically indeterminate structures. However, in comparison to rigid framing, the number of unknowns in a wedge-beam framing is relatively small. These unknowns consist of the two hinge reactions at each joint of linkage of adjacent simple frames. To determine the values of these hinge reactions, it is first necessary to establish their interrelation. This is obtained from the simple consideration that two adjacent simple frames will have the same deflections at a common link. The next step then is to solve the resulting series of simultaneous equations. More specifically, the procedure to be followed in the analysis will consist of five steps, as follows:

---

[2] "Design of Hinges and Articulations in Reinforced Concrete Structures," by George C. Ernst, *Transactions*, ASCE, Vol. 106, 1941, p. 862.

1. Subdivide the main assembly or framing into its component simple frames, and indicate the two unknown hinge reactions at each joint of interconnection or linkage. This step will result in a series of statically determinate simple frames, each frame subjected to unknown H-forces and V-forces on each side, in addition to its own loading.
2. Draw the equilibrium or simple moment diagram of each simple frame due to each unknown H-force and V-force, as well as one under the given loading of the frame. This will result, in general, in five diagrams.
3. Using the moment diagrams in step 2, write an expression for the horizontal and vertical deflection at each joint of linkage of each simple frame. In general, four equations per simple frame will result.
4. Equate the deflections of adjacent simple frames. By so equating, a set of simultaneous equations (the number of which will correspond to the number of the unknown hinge reactions) results.
5. From the equations in step 4, obtain the values of the unknown hinge reactions. This procedure will now be elaborated in its applications to various types of rectangular frames.

DETACHMENT OF SIMPLE FRAMES AND THE DETERMINATION OF THE NUMBER OF UNKNOWN HINGE REACTIONS

The subdivision or the cutting of an assembly into its component simple frames does not require special rules. In most cases, the separation is made in every other bay of an assembly, beginning in the second bay. For assemblies having an odd number of bays, such as in Fig. 6, the separation will yield $\left(\frac{n-1}{2}\right)$-cuts, n indicating the number of bays. If the loading is unsymmetrical, as in Figs. 6(d), 6(e), and 6(f), the number of unknown hinge reactions at the cut joints will become $n - 1$. If the loading and framing are symmetrical, Figs. 6(a), 6(b), and 6(c), the number of the unknown hinge reactions is reduced to $\frac{n-1}{2}$.

When the assembly is composed of an even number of bays and the loading is unsymmetrical, Figs. 7(d), 7(e), and 7(f), the severance of hinges in alternating bays will result in $n/2$ simple frames and one L-segment having two interdependent hinge reactions. Accordingly, the number of unknown hinge reactions will become $n - 1$. If the loading and the framing are symmetrical, the separations will be made as indicated in Figs. 7(a), 7(b), and 7(c). To maintain the symmetry of arrangement and thus reduce the number of the corresponding unknown hinge reactions to a minimum, the assembly is cut in two adjacent bays at the center to form a T-segment. The number of unknown hinge reactions for this group of assemblies is given by $n/2$.

In the case of multistory assemblies, each story may be considered as a separate frame and treated as outlined in the preceding paragraphs. The arrangement, as applied to a three-story frame, is shown in Fig. 8.

DEFLECTION EQUATIONS

Development.—In general, two equations can be derived from the two deflections at each connecting joint of adjacent simple frames. As stated in the

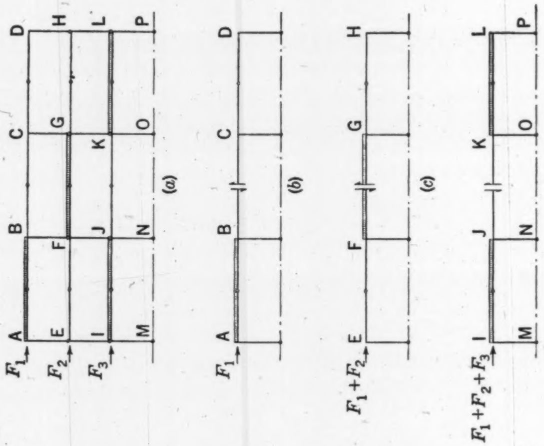

F1
A
B
C
D
F2
E
F
G
H
F3
I
J
K
L
M
N
O
P
(a)
F1
A
B
C
D
(b)
F1+F2
E
F
G
H
(c)
F1+F2+F3
I
J
K
L
M
N
O
P
(d)

FIG. 8.—SUBDIVISION OF A MULTISTORY FRAME

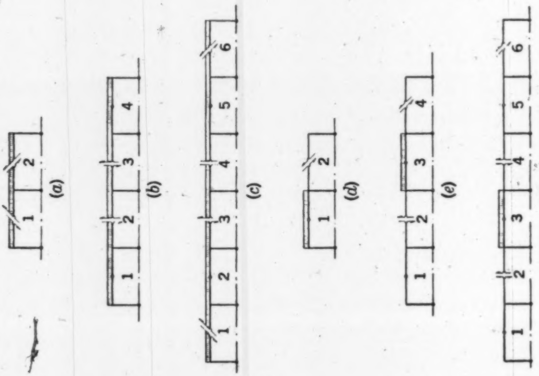

1
2
(a)
1
2
3
4
(b)
1
2
3
4
5
6
(c)
1
2
(d)
1
2
3
4
(e)
1
2
3
4
5
6
(f)

FIG. 7.—SUBDIVISION OF AN EVEN NUMBER OF BAYS

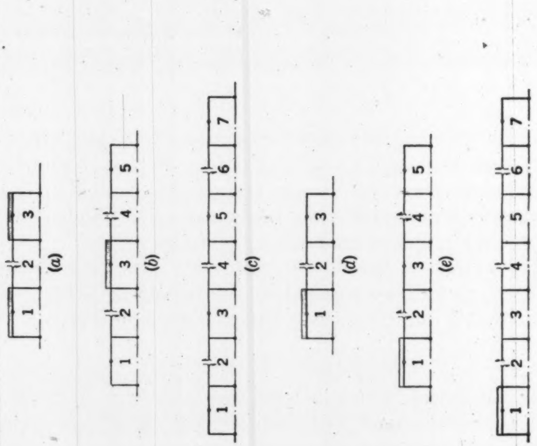

1
2
3
(a)
1
2
3
4
5
(b)
1
2
3
4
5
6
7
(c)
1
2
3
(d)
1
2
3
4
5
(e)
1
2
3
4
5
6
7
(f)

FIG. 6.—SUBDIVISION OF AN ODD NUMBER OF BAYS

preceding section, these are obtained by equating the horizontal and vertical deflections of the joint from one frame to the corresponding expressions in the other frame. The deflection of a simple frame is given by the familiar expression:

$$\Delta_D = \int \frac{M_s\, m_s\, \delta s}{E\, I_x} \quad \ldots\ldots (1)$$

in which Δ_D is the deflection of a joint D, measured in the desired direction; M_s is the simple frame moment due to the applied loading; m_s is the simple frame moment due to a unit load applied at the joint in the desired direction of the deflection; δs is an element of length; E and I are the modulus of elasticity and the moment of inertia, respectively; and the summation includes all the members of the frame.

Since each member of the framing has a rigid joint at one end and a hinged connection at the other, the bending moment diagram of each member is similar to that of a cantilever beam. The moment may be due to an end force, such as the hinge reaction normal to the axis or end shear, or to a given loading applied between the two ends of a member. Let P_M represent the end force, or the normal hinge reaction, which produces the simple moment M_s in a member, and let p_m represent the end force producing a moment m_s. Then, the contribution of a member to the deflection of the simple frame, Eq. 1, may be considered as the deflection, Δ, of a cantilever subjected to a load $P_M\, p_m$ at its end:

$$\Delta = P_M\, p_m \frac{l^3}{E\, I} k \quad \ldots\ldots (2)$$

in which k is a constant depending on the shape of the member; l is the length; and I is the moment of inertia at the tapered end of the member. Formulas for k corresponding to typical loadings are given in Tables 1 to 5.

The deflection of a member due to a loading applied between the ends is the same as that of a simple cantilever. For concentrated load P, the expression is

$$\Delta_p = \frac{P\, l^3}{E\, I} k_p \quad \ldots\ldots (3a)$$

in which k_p is a constant depending on the shape of the member and the position of the load. Similarly, for a uniformly distributed loading w, the expression is

$$\Delta_w = \frac{w\, l^4}{E\, I} k_w \quad \ldots\ldots (3b)$$

k_w being the shape and the load constant for this case.

With this concept, to obtain the deflection δ_D of a simple frame, one need only sum up the deflections Δ of its component members, as given by Eqs. 2 and 3.

Special Case.—To illustrate the procedure, consider first the relatively simple cases of framing that contain only a few bays, such as: (*a*) The three-bay frame with symmetrical framing and loading; (*b*) the three-bay frame, with symmetrical framing and unsymmetrical loading; (*c*) the two-bay frame with unsymmetrical framing and loading; and (*d*) the four-bay frame with symmetrical framing and loading.

(*a*) Three-Bay Frame, Symmetrical Framing and Loading.—The frame in Fig. 9(*a*) illustrates this case. Because of the symmetry of framing and load-

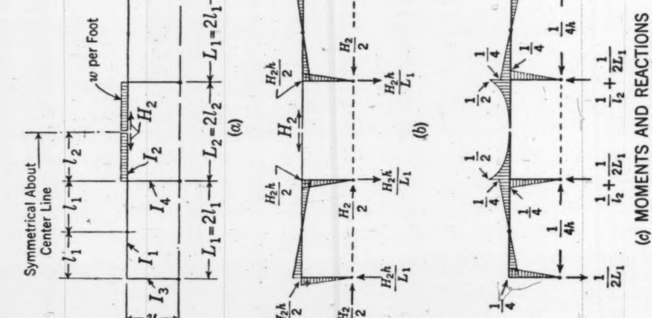

Symmetrical About Center Line
w per Foot
l_1
l_2
I_1
I_2
I_3
I_4
H_2
h
$L_1=2l_1$
$L_2=2l_2$
(a)
$\frac{H_2h}{2}$
$\frac{H_2}{2}$
$\frac{H_2h}{L_1}$
(b)
$\frac{1}{4}$
$\frac{1}{2}$
$\frac{1}{4h}$
$\frac{1}{2L_1}$
$\frac{1}{l_2}+\frac{1}{2L_1}$
(c) MOMENTS AND REACTIONS IN TERMS OF wl^2_2

FIG. 9.—THREE-BAY FRAME WITH SYMMETRICAL FRAMING AND LOADING

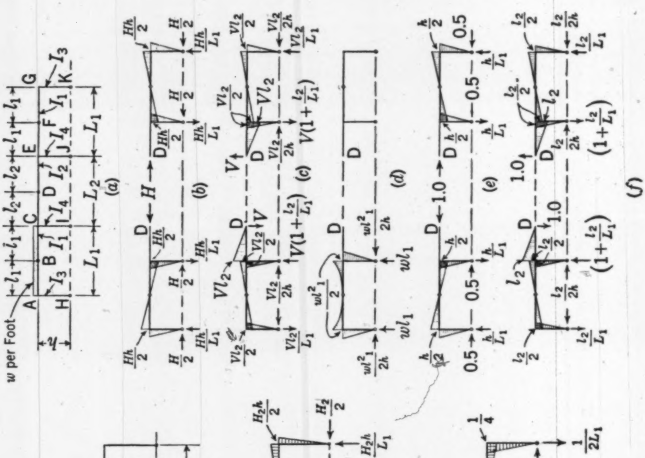

w per Foot
A
B
C
D
E
F
G
H
J
K
l_1
l_2
I_1
I_2
I_3
I_4
h
L_1
L_2
(a)
$\frac{Hh}{2}$
$\frac{H}{2}$
$\frac{Hh}{L_1}$
H
(b)
Vl_2
$\frac{Vl_2}{2}$
$\frac{Vl_2}{2h}$
$\frac{Vl_2}{L_1}$
V
$V(1+\frac{l_2}{L_1})$
(c)
$\frac{wl^2_1}{2}$
$\frac{wl^2_1}{2h}$
wl_1
(d)
$\frac{h}{2}$
$\frac{h}{L_1}$
0.5
1.0
(e)
l_2
$\frac{l_2}{2}$
$\frac{l_2}{2h}$
$\frac{l_2}{L_1}$
$(1+\frac{l_2}{L_1})$
(f)

FIG. 10.—THREE-BAY FRAME WITH SYMMETRICAL FRAMING AND UNSYMMETRICAL LOADING

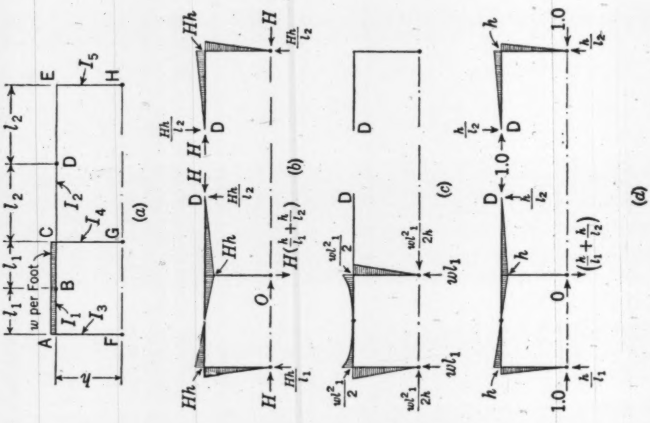

w per Foot
A
B
C
D
E
F
G
H
l_1
l_2
I_1
I_2
I_3
I_4
I_5
h
(a)
Hh
H
$\frac{Hh}{l_1}$
$\frac{Hh}{l_2}$
O
$H(\frac{h}{l_1}+\frac{h}{l_2})$
(b)
$\frac{wl^2_1}{2}$
$\frac{wl^2_1}{2h}$
wl_1
(c)
h
1.0
$\frac{h}{l_1}$
$\frac{h}{l_2}$
$(\frac{h}{l_1}+\frac{h}{l_2})$
(d)

FIG. 11.—TWO-BAY FRAME WITH UNSYMMETRICAL FRAMING AND LOADING

ing, the vertical hinge reaction at the center of the middle bay beam is zero, and the horizontal hinge reaction, H_2, at the same location remains the only unknown force to be determined. This force may be obtained from the equation of horizontal deflection, Δ_{H2}, at the center hinge. For this purpose, the simple frame moment diagrams due to H_2 and w, are sketched as shown in Figs. 9(*b*) and 9(*c*). By substituting 1 for H_2, Fig. 9(*b*) will also give the simple frame moments due to a unit horizontal load at the center hinge. Then, to determine the contribution of the five members in one half of the frame to the deflection at the center hinge (by Eqs. 2 and 3):

For member 1—

$$\Delta_1 = 2\,\frac{H_2\,h}{2\,l_1}\,\frac{h}{2\,l_1}\,\frac{l^3_1}{I_1}\,k_1 - 2\,\frac{w\,l^2_2}{4\,l_1}\,\frac{h}{2\,l_1}\,\frac{l^3_1}{I_1}\,k_1 \ldots\ldots\ldots\ldots \tag{4a}$$

for member 2—

$$\Delta_2 = 0 \ldots\ldots\ldots\ldots \tag{4b}$$

for member 3—

$$\Delta_3 = \frac{H_2}{2}\,\frac{1}{2}\,\frac{h^3}{I_3}\,k_3 - \frac{w\,l^2_2}{4\,h}\,\frac{1}{2}\,\frac{h^3}{I_3}\,k_3 \ldots\ldots\ldots\ldots \tag{4c}$$

and for member 4—

$$\Delta_4 = \frac{H_2}{2}\,\frac{1}{2}\,\frac{h^3}{I_4}\,k_4 + \frac{w\,l^2_2}{4\,h}\,\frac{1}{2}\,\frac{h^3}{I_4}\,k_4 \ldots\ldots\ldots\ldots \tag{4d}$$

Since $\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 = \Delta_{H2} = 0$,

$$H_2 = \frac{w\,l^2_2}{2}\;\frac{2\,\dfrac{l_1}{h}\,\dfrac{k_1}{I_1} + \dfrac{k_3}{I_3} - \dfrac{k_4}{I_4}}{2\,l_1\,\dfrac{k_1}{I_1} + h\left(\dfrac{k_3}{I_3} + \dfrac{k_4}{I_4}\right)} \ldots\ldots\ldots\ldots \tag{5}$$

The moment sign is governed by the direction of rotation of the forces about the rigid joint of a member. A clockwise rotation is plus (+) and a counterclockwise rotation is minus (−).

(*b*) Three-Bay Frame, Symmetrical Framing, Unsymmetrical Loading.—The frame in Fig. 10(*a*), illustrating this case, carries a uniformly distributed load in only the first bay. Because of the dissymmetry of loading, there are two unknown hinge reactions, H and V, at the center hinge D. The simple frame moment diagrams due to these two forces are shown in Figs. 10(*b*) and 10(*c*). The simple frame moments due to the loading in the first bay are as indicated in Fig. 10(*d*). Moments caused by a unit horizontal and vertical force at hinge D are given in Figs. 10(*e*) and 10(*f*), respectively. With these values, and applying Eqs. 2 and 3, the horizontal deflection at hinge D is

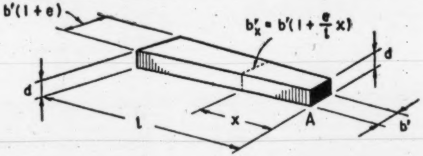
$$\Delta_{1H} = 4\,\frac{H\,h}{2\,l_1}\,\frac{h}{2\,l_1}\,\frac{l^3_1}{I_1}\,k_1 + V \times 0 + 0 \ldots\ldots\ldots\ldots \tag{6a}$$

$$\Delta_{2H} = 0 \ldots\ldots\ldots\ldots \tag{6b}$$

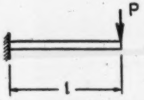
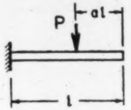
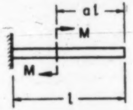
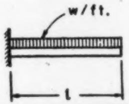
$$\Delta_{3H} = 2\,\frac{H}{2}\,\frac{1}{2}\,\frac{h^3}{I_3}\,k_3 + V \times 0 + \frac{w\,l^2_1}{2\,h}\,\frac{1}{2}\,\frac{h^3}{I_3}\,k_3 \ldots\ldots\ldots\ldots \tag{6c}$$

TABLE 1.—Deflection Formulas for Tapered-Flange Solid Rectangular Beams

$I_x = I(1 + \frac{e}{l}x)$



$I_A = \frac{1}{12} b'd^3 = I$

LOADING	DEFLECTION
	$\Delta_A = \frac{Pl^3}{EI} \cdot \frac{1}{e^2}\left[\frac{e}{2} - 1 + \frac{1}{e}\log_e(1+e)\right]$ $= \frac{Pl^3}{EI} \cdot k$
	$\Delta_A = \frac{Pl^3}{EI} \cdot \frac{1}{e^2}\left[a - 1 + \frac{e}{2} + ea(\frac{a}{2} - 1) + (a + \frac{1}{e})\log_e \frac{1+e}{1+ea}\right]$ $= \frac{Pl^3}{EI} \cdot k_p$
	$\Delta_A = \frac{Ml^2}{EI} \cdot \frac{1}{e^2}\left[e(1-a) - \log_e \frac{1+e}{1+ea}\right]$ $= \frac{Ml^2}{EI} \cdot k_m$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2e^3}\left[\frac{e(2e-3)+6}{6} - \frac{1}{e}\log_e(1+e)\right]$ $= \frac{wl^4}{EI} \cdot k_w$

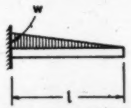
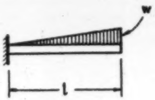
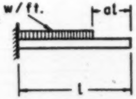
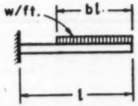
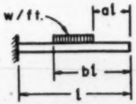
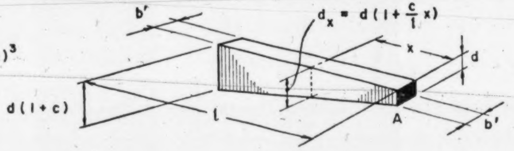
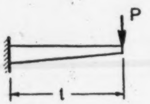
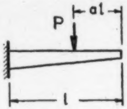
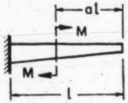
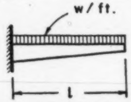
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{6e^4}\left[\frac{e(3e^2-4e+6)-12}{12} + \frac{1}{e}\log_e(1+e)\right]$ $= \frac{wl^4}{EI} \cdot k_{w1}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{6e^4}\left[\frac{e(9e^2-14e+30)+12}{12} - (3+\frac{1}{e})\log_e(1+e)\right]$ $= \frac{wl^4}{EI} \cdot k_{w2}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2e^4}\left\{-e(1+e)(1-a)(ea+3) + \frac{1+e}{6}\left[e(2e+13)+11\right]\right.$ $\left. - \frac{1+ea}{6}\left[ea(2ea+13)+11\right] - (ea+1)^2\log_e\frac{1+e}{1+ea}\right\} = \frac{wl^4}{EI} \cdot k_{w3}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2e^4}\left\{-\frac{11}{6} - eb(1+e)\left[e(b-1)+3\right] + (1+eb)\left[(eb+2)^2-1\right] - \frac{(1+eb)^2}{6}(4eb+7)\right.$ $\left. - (1+eb)^2\log_e(1+eb) + eb(eb+2)\log_e(1+e)\right\} = \frac{wl^4}{EI} \cdot k_{w4}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2e^4}\left\{-e(1+e)(b-a)\left[e(a+b-1)+3\right] - (1+ea)\left[(ea+2)^2-1\right]\right.$ $+ (1+eb)\left[(eb+2)^2-1\right] + \frac{(1+ea)^2}{6}(4ea+7) - \frac{(1+eb)^2}{6}(4eb+7)$ $\left. + e\left[e(b^2-a^2)+2(b-a)\right]\log_e\frac{1+e}{1+eb} - (ea+1)^2\log_e\frac{1+eb}{1+ea}\right\} = \frac{wl^4}{EI} \cdot k_{w5}$

TABLE 2.—DEFLECTION FORMULAS FOR TAPERED-WEB SOLID RECTANGULAR BEAMS

$I_x = I\left(1 + \frac{c}{l}x\right)^3$



$I_A = \frac{1}{12} b'd^3 = I$

LOADING	DEFLECTION
	$\Delta_A = \frac{Pl^3}{EI} \cdot \frac{1}{c^2}\left[-\frac{2+3c}{2(1+c)^2} + \frac{1}{c}\log_e(1+c)\right]$ $= \frac{Pl^3}{EI} \cdot k$
	$\Delta_A = \frac{Pl^3}{EI} \cdot \frac{1}{c^3}\left[\frac{2+ca}{1+c} - \frac{3+2ca}{2(1+ca)} - \frac{1+ca}{2(1+c)^2} + \log_e\frac{1+c}{1+ca}\right]$ $= \frac{Pl^3}{EI} \cdot k_p$
	$\Delta_A = \frac{Ml^2}{EI} \cdot \frac{1}{c^2}\left[-\frac{1}{1+c} + \frac{1}{1+ca} + \frac{1}{2(1+c)^2} - \frac{1}{2(1+ca)^2}\right]$ $= \frac{Ml^2}{EI} \cdot k_m$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2c^3}\left[\frac{c(2c+9)+6}{2(1+c)^2} - \frac{3}{c}\log_e(1+c)\right]$ $= \frac{wl^4}{EI} \cdot k_w$

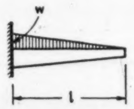
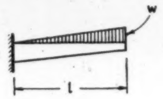
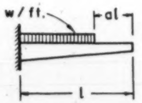
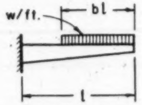
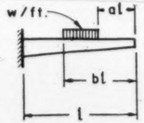
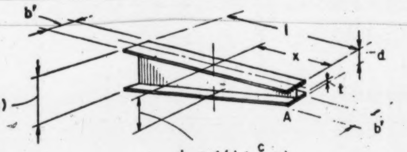
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{6c^4} \left[\frac{c(c^2-4c-18)-12}{2(1+c)^2} + \frac{6}{c} \log_e (1+c) \right]$ $= \frac{wl^4}{EI} \cdot k_{w1}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{6c^4} \left[\frac{c(5c^2+31c+36)+12}{2(1+c)^2} - (9+\frac{6}{c}) \log_e (1+c) \right]$ $= \frac{wl^4}{EI} \cdot k_{w2}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2c^2} \left\{ \frac{1-a}{c} - \left[a(a+\frac{4}{c}) + \frac{3}{c^2} \right] \left[\frac{1}{1+c} - \frac{1}{1+ca} \right] \right.$ $\left. + \frac{(1+ca)^2}{2c^2} \left[\frac{1}{(1+c)^2} - \frac{1}{(1+ca)^2} \right] - \frac{1}{c}(2a+\frac{3}{c}) \log_e \frac{1+c}{1+ca} \right\} = \frac{wl^4}{EI} \cdot k_{w3}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2c^2} \left\{ \frac{b(4+cb)}{c(1+c)} - b(\frac{1}{c}+\frac{b}{2}) \left[\frac{1}{(1+c)^2} - \frac{1}{(1+cb)^2} \right] \right.$ $\left. - \frac{1}{2c^2} \left[1 - \frac{1}{(1+cb)^2} \right] - \frac{3}{c^2} \log_e (1+cb) + \frac{2b}{c} \log_e \frac{1+c}{1+cb} \right\} = \frac{wl^4}{EI} \cdot k_{w4}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2c^2} \left\{ \frac{b-a}{c} + (b-a)(a+b+\frac{4}{c}) \left[\frac{1}{1+c} - \frac{1}{1+cb} \right] + \left[a(a+\frac{4}{c}) + \frac{3}{c^2} \right] \left[\frac{1}{1+ca} - \frac{1}{1+cb} \right] \right.$ $- (b-a)(\frac{1}{c} + \frac{a+b}{2}) \left[\frac{1}{(1+c)^2} - \frac{1}{(1+cb)^2} \right] - \frac{(1+ca)^2}{2c^2} \left[\frac{1}{(1+ca)^2} - \frac{1}{(1+cb)^2} \right]$ $\left. + \frac{2(b-a)}{c} \log_e \frac{1+c}{1+cb} - \frac{1}{c}(2a+\frac{3}{c}) \log_e \frac{1+cb}{1+ca} \right\} = \frac{wl^4}{EI} \cdot k_{w5}$

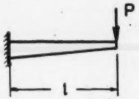
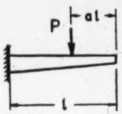
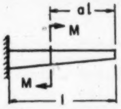
TABLE 3.—Deflection Formulas for Tapered-Web I-Beams

$$I_x = I\left(1+\frac{c}{l}x\right)^2$$



$$I_A = \frac{b' t d^2}{2} = I \text{ (approx.)}$$

$$d_x = d\left(1+\frac{c}{l}x\right)$$

LOADING	DEFLECTION
	$\Delta_A = \frac{Pl^3}{EI}\cdot\frac{1}{c^2}\left[\frac{2+c}{1+c}-\frac{2}{c}\log_e(1+c)\right]$ $= \frac{Pl^3}{EI}\cdot k$
	$\Delta_A = \frac{Pl^3}{EI}\cdot\frac{1}{c^2}\left[\frac{(1-a)(2+c)}{1+c}-\frac{2+ca}{c}\log_e\frac{1+c}{1+ca}\right]$ $= \frac{Pl^3}{EI}\cdot k_p$
	$\Delta_A = \frac{Ml^2}{EI}\cdot\frac{1}{c^2}\left[\frac{1}{1+c}-\frac{1}{1+ca}+\log_e\frac{1+c}{1+ca}\right]$ $= \frac{Ml^2}{EI}\cdot k_m$

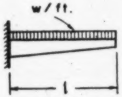
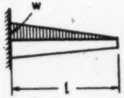
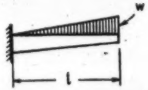
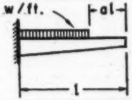
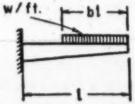
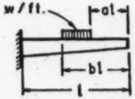
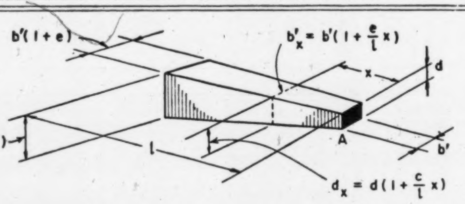
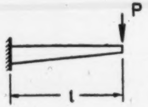
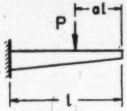
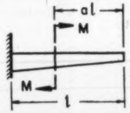
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2c^3}\left[\frac{c(c-3)-b}{2(1+c)} + \frac{3}{c}\log_e(1+c)\right]$ $= \frac{wl^4}{EI} \cdot k_w$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{6c^4}\left[\frac{c(c^2-2c+6)+12}{3(1+c)} - \frac{4}{c}\log_e(1+c)\right]$ $= \frac{wl^4}{EI} \cdot k_{w1}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{6c^4}\left[\frac{c(7c^2-23c-66)-24}{6(1+c)} + (9+\frac{4}{c})\log_e(1+c)\right]$ $= \frac{wl^4}{EI} \cdot k_{w2}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2c^2}\left\{(1-a)(\frac{1}{2}-\frac{1}{c}-\frac{3}{2}a) - \frac{1-a}{1+c}(a+1+\frac{2}{c})\right.$ $\left. + \left[a(a+\frac{4}{c})+\frac{3}{c^2}\right]\log_e\frac{1+c}{1+ca}\right\} = \frac{wl^4}{EI} \cdot k_{w3}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2c^2}\left[b(2-\frac{1}{c}-\frac{3b}{2}) - \frac{b}{1+c}(b+\frac{2}{c}) + \frac{3}{c^2}\log_e(1+cb)\right.$ $\left. - b(b+\frac{4}{c})\log_e\frac{1+c}{1+cb}\right] = \frac{wl^4}{EI} \cdot k_{w4}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2c^2}\left\{(b-a)\left[2-\frac{1}{c}-\frac{3}{2}(a+b)\right] - \frac{b-a}{1+c}(a+b+\frac{c}{2})\right.$ $\left. + \left[a(a+\frac{4}{c})+\frac{3}{c^2}\right]\log_e\frac{1+cb}{1+ca} - (b-a)(a+b+\frac{4}{c})\log_e\frac{1+c}{1+cb}\right\} = \frac{wl^4}{EI} \cdot k_{w5}$

TABLE 4.—DEFLECTION FORMULAS FOR TAPERED-FLANGE AND TAPERED-WEB SOLID RECTANGULAR BEAMS

$I_x = I\left(1+\frac{e}{l}x\right)\left(1+\frac{c}{l}x\right)^3$


$b'(1+e)$
$b'_x = b'\left(1+\frac{e}{l}x\right)$
x
d
$d(1+c)$
l
A
b'
$d_x = d\left(1+\frac{c}{l}x\right)$

$I_A = \frac{1}{12}b'd^3 = I$

LOADING	DEFLECTION
	$\Delta_A = \frac{Pl^3}{EI}\cdot\left[\frac{e-2c}{c(e-c)^2(1+c)} - \frac{2+c}{2c(e-c)(1+c)^2} + \frac{1}{(e-c)^3}\log_e(1+e) - \frac{1}{(e-c)^3}\log_e(1+c)\right] = \frac{Pl^3}{EI}\cdot k$
	$\Delta_A = \frac{Pl^3}{EI}\cdot\left\{\frac{c(ca+2)-e}{c^2(e-c)^2}\left[\frac{1}{1+c}-\frac{1}{1+ca}\right] + \frac{1+ca}{2c^2(e-c)}\left[\frac{1}{(1+c)^2}-\frac{1}{(1+ca)^2}\right] + \frac{1+ea}{(e-c)^3}\log_e\frac{1+e}{1+ea} - \frac{1+ea}{(e-c)^3}\log_e\frac{1+c}{1+ca}\right\} = \frac{Pl^3}{EI}\cdot k_p$
	$\Delta_A = \frac{Ml^2}{EI}\cdot\left\{-\frac{1}{(e-c)^2}\left[\frac{1}{1+c}-\frac{1}{1+ca}\right] - \frac{1}{2c(e-c)}\left[\frac{1}{(1+c)^2}-\frac{1}{(1+ca)^2}\right] - \frac{e}{(e-c)^3}\log_e\frac{1+e}{1+ea} + \frac{e}{(e-c)^3}\log_e\frac{1+c}{1+ca}\right\} = \frac{Ml^2}{EI}\cdot k_m$

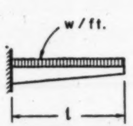
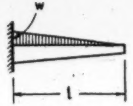
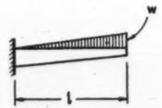
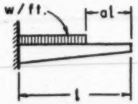
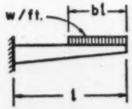
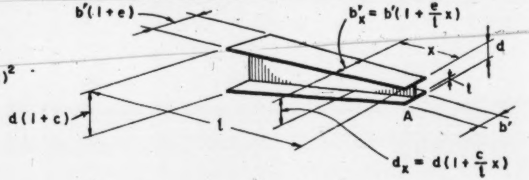
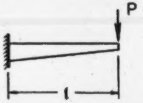
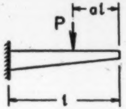
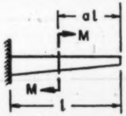
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2} \left[\frac{3c-2e}{c^2(e-c)^2(1+c)} + \frac{2+c}{2c^2(e-c)(1+c)^2} - \frac{1}{e(e-c)^3} \log_e(1+e) + \frac{e^2-3c(e-c)}{c^3(e-c)^3} \log_e(1+c) \right] = \frac{wl^4}{EI} \cdot k_w$
	$\Delta_A = \frac{wl^5}{EI} \cdot \frac{1}{6} \left[\frac{3e-4c}{c^3(e-c)^2(1+c)} - \frac{2+c}{2c^3(e-c)(1+c)^2} + \frac{1}{e^2(e-c)^3} \log_e(1+e) - \frac{6c^2-8ec+3e^2}{c^4(e-c)^3} \log_e(1+c) + \frac{1}{ec^3} \right] = \frac{wl^5}{EI} \cdot k_{w1}$
	$\Delta_A = \frac{wl^5}{EI} \cdot \left[k_w - k_{w1} \right]$ $= \frac{wl^5}{EI} \cdot k_{w2}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2} \left\{ -\frac{3c-2e+2ca(2c-e)+c^3a^2}{c^3(e-c)^2} \left[\frac{1}{1+c} - \frac{1}{1+ca} \right] - \frac{(1+ca)^2}{2c^3(e-c)} \left[\frac{1}{(1+c)^2} - \frac{1}{(1+ca)^2} \right] - \frac{(1+ea)^2}{e(e-c)^3} \log_e \frac{1+e}{1+ea} + \frac{e^2-3c(e-c)+ac^3(2+ea)}{c^3(e-c)^3} \log_e \frac{1+c}{1+ca} \right\} = \frac{wl^4}{EI} \cdot k_{w3}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \left[k_w - k_{w3}, \; a = b \right]$ $= \frac{wl^4}{EI} \cdot k_{w4}$

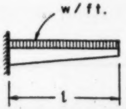
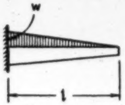
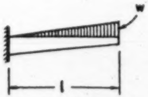
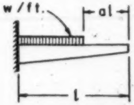
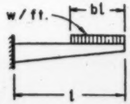
TABLE 5.—Deflection Formulas for Tapered-Flange and Tapered-Web I-Beams

$I_x = I(1+\frac{e}{l}x)(1+\frac{c}{l}x)^2$



$I_A = \frac{b'td^2}{2} = I$ (approx.)

LOADING	DEFLECTION
	$\Delta_A = \frac{Pl^3}{EI}\cdot\left[\frac{-1}{c(e-c)(1+c)} + \frac{1}{e(e-c)^2}\log_e(1+e) - \frac{2c-e}{c^2(e-c)^2}\log_e(1+c)\right]$ $= \frac{Pl^3}{EI}\cdot k$
	$\Delta_A = \frac{Pl^3}{EI}\cdot\left\{\frac{(1+ca)}{c^2(e-c)}\left[\frac{1}{1+c} - \frac{1}{1+ca}\right] + \frac{1+ea}{e(e-c)^2}\log_e\frac{1+e}{1+ea} + \frac{e-c(2+ca)}{c^2(e-c)^2}\log_e\frac{1+c}{1+ca}\right\}$ $= \frac{Pl^3}{EI}\cdot k_p$
	$\Delta_A = \frac{Ml^2}{EI}\cdot\left\{\frac{-1}{c(e-c)}\left[\frac{1}{1+c} - \frac{1}{1+ca}\right] - \frac{1}{(e-c)^2}\log_e\frac{1+e}{1+ea} + \frac{1}{(e-c)^2}\log_e\frac{1+c}{1+ca}\right\}$ $= \frac{Ml^2}{EI}\cdot k_m$

	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2}\left[\frac{1}{ec^2} + \frac{1}{c^2(e-c)(1+c)} - \frac{1}{e^2(e-c)^2}\log_e(1+e) + \frac{3c-2e}{c^3(e-c)^2}\log_e(1+c)\right]$ $= \frac{wl^4}{EI} \cdot k_w$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{6}\left[\frac{-1}{c^3(e-c)(1+c)} + \frac{c(e-2)-4e}{2e^2c^3} + \frac{1}{e^3(e-c)^2}\log_e(1+e)\right.$ $\left. - \frac{4c-3e}{c^4(e-c)^2}\log_e(1+c)\right] = \frac{wl^4}{EI} \cdot k_{w1}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{6}\left[\frac{3c+1}{c^3(e-c)(1+c)} + \frac{c(5e+2)+4e}{2e^2c^3} - \frac{3e+1}{e^3(e-c)^2}\log_e(1+e)\right.$ $\left. + \frac{3c(3c-2e)+(4c-3e)}{c^4(e-c)^2}\log_e(1+c)\right] = \frac{wl^4}{EI} \cdot k_{w2}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \frac{1}{2}\left\{\frac{1-a}{ec^2} - \frac{(1+ca)^2}{c^3(e-c)}\left[\frac{1}{1+c} - \frac{1}{1+ca}\right] - \frac{(1+ea)^2}{e^2(e-c)^2}\log_e\frac{1+e}{1+ea}\right.$ $\left. + \frac{ca\left[c^2a+2(2c-e)\right]+3c-2e}{c^3(e-c)^2}\log_e\frac{1+c}{1+ca}\right\} = \frac{wl^4}{EI} \cdot k_{w3}$
	$\Delta_A = \frac{wl^4}{EI} \cdot \left[k_w - k_{w3}\,(a=b)\right]$ $= \frac{wl^4}{EI} \cdot k_{w4}$

and

$$\Delta_{4H} = 2\,\frac{H}{2}\,\frac{1}{2}\,\frac{h^3}{I_4}\,k_4 + V \times 0 - \frac{w\,l^2_1}{2\,h}\,\frac{1}{2}\,\frac{h^3}{I_4}\,k_4 \ldots\ldots\ldots\ldots (6d)$$

Since the deflection of the left side of the joint minus the deflection of the right side of joint equals zero,

$$\Delta_{1H} + \Delta_{2H} + \Delta_{3H} + \Delta_{4H} = \Delta_{HD} = 0 \ldots\ldots\ldots\ldots (7)$$

and

$$H = \frac{w\,l_1}{2}\,\frac{\dfrac{k_4}{I_4} - \dfrac{k_3}{I_3}}{2\,\dfrac{k_1}{I_1} + \dfrac{h}{l_1}\left(\dfrac{k_3}{I_3} + \dfrac{k_4}{I_4}\right)} \ldots\ldots\ldots\ldots (8)$$

Similarly, the vertical deflection at hinge D is

$$\Delta_{1V} = 4\,\frac{V\,l_2}{2\,l_1}\,\frac{l_2}{2\,l_1}\,\frac{l^3_1}{I_1}\,k_1 + H \times 0 + 0 \ldots\ldots\ldots\ldots (9a)$$

$$\Delta_{2V} = 2\,V \times 1 \times \frac{l^3_1}{I_2}\,k_2 + H \times 0 \ldots\ldots\ldots\ldots (9b)$$

$$\Delta_{3V} = 2\,\frac{V\,l_2}{2\,h}\,\frac{l_2}{2\,h}\,\frac{h^3}{I_3}\,k_3 + H \times 0 - \frac{w\,l^2_1}{2\,h}\,\frac{l_2}{2\,h}\,\frac{h^3}{I_3}\,k_3 \ldots\ldots\ldots\ldots (9c)$$

and

$$\Delta_{4V} = 2\,\frac{V\,l_2}{2\,h}\,\frac{l_2}{2\,h}\,\frac{h^3}{I_4}\,k_4 + H \times 0 - \frac{w\,l^2_1}{2\,h}\,\frac{l_2}{2\,h}\,\frac{h^3}{I_4}\,k_4 \ldots\ldots\ldots\ldots (9d)$$

As before, since $\Delta_{1V} + \Delta_{2V} + \Delta_{3V} + \Delta_{4V} = \Delta_{VD} = 0$,

$$V = \frac{w\,l_1}{2}\,\frac{\dfrac{k_3}{I_3} + \dfrac{k_4}{I_4}}{2\,\dfrac{l_2}{h}\left(\dfrac{k_1}{I_1} + 2\,\dfrac{l_2}{l_1}\,\dfrac{k_2}{I_2}\right) + \dfrac{l_2}{l_1}\left(\dfrac{k_3}{I_3} + \dfrac{k_4}{I_4}\right)} \ldots\ldots\ldots\ldots (10)$$

(*c*) Two-Bay Frame, Unsymmetrical Framing and Loading.—The framing in Fig. 11(*a*) is composed of a simple frame and an L-segment. The two hinge reactions at the cut joint D being interrelated, there is only one unknown force (namely, *H*) that needs to be determined. Using the simple moment diagrams shown in Figs. 11(*b*), 11(*c*), and 11(*d*), and applying Eqs. 2 and 3,

$$\Delta_{1D} = 2\,\frac{H\,h}{l_1}\,\frac{h}{l_1}\,\frac{l^3_1}{I_1}\,k_1 + 0 \ldots\ldots\ldots\ldots (11a)$$

$$\Delta_{2D} = 2\,\frac{H\,h}{l_2}\,\frac{h}{l_2}\,\frac{l^3_2}{I_2}\,k_2 \ldots\ldots\ldots\ldots (11b)$$

$$\Delta_{3D} = H \times 1 \times \frac{h^3}{I_3}\,k_3 + \frac{w\,l^2_1}{2\,h}\,1\,\frac{h^3}{I_3}\,k_3 \ldots\ldots\ldots\ldots (11c)$$

$$\Delta_{4D} = 0 \ldots\ldots\ldots\ldots (11d)$$

and

$$\Delta_{5D} = H \times 1 \times \frac{h^3}{I_5}\,k_5 \ldots\ldots\ldots\ldots (11e)$$

Since $\Delta_{1D} + \Delta_{2D} + \Delta_{3D} + \Delta_{4D} + \Delta_{5D} = \Delta_{DD} = 0$,

$$H = -\frac{w\,l_1}{2}\,\frac{\dfrac{k_3}{I_3}}{2\,\dfrac{k_1}{I_1} + 2\,\dfrac{l_2}{l_1}\,\dfrac{k_2}{I_2} + \dfrac{h}{l_1}\left(\dfrac{k_3}{I_3} + \dfrac{k_5}{I_5}\right)} \ldots\ldots\ldots\ldots (12)$$

(*d*) Four-Bay Frame, Symmetrical Framing and Loading.—The framing in Fig. 12(*a*) illustrating this case is composed of two simple frames and a T-segment. There are only two unknown hinge reactions (namely, *H* and *V*) at joint D. Their solution is obtained from horizontal and vertical deflection equations at the same joint, using the simple moment diagrams in Figs. 12(*b*), 12(*c*), and 12(*d*). Formulas for the horizontal deflection at hinge D are

$$\Delta_{1H} = 2\,\frac{H\,h}{2\,l_1}\,\frac{h}{2\,l_1}\,\frac{l^3{}_1}{I_1}\,k_1 - 2\,\frac{V\,l_2}{2\,l_1}\,\frac{h}{2\,l_1}\,\frac{l^3{}_1}{I_1}\,k_1 - \frac{2\,w\,l^2{}_2}{4}\,\frac{h}{2\,l_1}\,\frac{l^3{}_1}{I_1}\,k_1 \ldots (13a)$$

$$\Delta_{2H} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13b)$$

$$\Delta_{3H} = \frac{H}{2}\,\frac{1}{2}\,\frac{h^3}{I_3}\,k_3 - \frac{V\,l_2}{2\,h}\,\frac{1}{2}\,\frac{h^3}{I_3}\,k_3 + \frac{w}{2\,h}\left(l^2{}_1 - \frac{l^2{}_2}{2}\right)\frac{1}{2}\,\frac{h^3}{I_3}\,k_3 \ldots\ldots (13c)$$

$$\Delta_{4H} = \frac{H}{2}\,\frac{1}{2}\,\frac{h^3}{I_4}\,k_4 + \frac{V\,l_2}{2\,h}\,\frac{1}{2}\,\frac{h^3}{I_4}\,k_4 - \frac{w}{2\,h}\left(l^2{}_1 - \frac{l^2{}_2}{2}\right)\frac{1}{2}\,\frac{h^3}{I_4}\,k_4 \ldots\ldots (13d)$$

and

$$\Delta_{5H} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13e)$$

Since $\Delta_{1H} + \Delta_{2H} + \Delta_{3H} + \Delta_{4H} + \Delta_{5H} = \Delta_{HD} = 0$, the values of *H* and *V* at the hinge are found by combining Eqs. 13. Likewise, formulas for the vertical deflection at hinge D are

$$\Delta_{1V} = 2\,V\,\frac{l_2}{2\,l_1}\,\frac{l_2}{2\,l_1}\,\frac{l^3{}_1}{I_1}\,k_1 - 2\,\frac{H\,h}{2\,l_1}\,\frac{l_2}{2\,l_1}\,\frac{l^3{}_1}{I_1}\,k_1 + \frac{2\,w\,l^2{}_2}{4}\,\frac{l_2}{2\,l_1}\,\frac{l^3{}_1}{I_1}\,k_1 \ldots (14a)$$

$$\Delta_{2V} = 3\,V\,1\,\frac{l^3{}_2}{I_2}\,k_2 - \frac{w\,l^2{}_2}{2}\,\frac{l^3{}_2}{I_2}\,k_{2w} \ldots\ldots\ldots\ldots\ldots\ldots (14b)$$

$$\Delta_{3V} = V\,\frac{l_2}{2\,h}\,\frac{l_2}{2\,h}\,\frac{h^3}{I_3}\,k_3 - \frac{H}{2}\,\frac{l_2}{2\,h}\,\frac{h^3}{I_3}\,k_3 - \frac{w}{2\,h}\left(l^2{}_1 - \frac{l^2{}_2}{2}\right)\frac{l_2}{2\,h}\,\frac{l_2\,h^3}{I_3}\,k_3 \ldots (14c)$$

$$\Delta_{4V} = V\,\frac{l_2}{2\,h}\,\frac{l_2}{2\,h}\,\frac{h^3}{I_4}\,k_4 + \frac{H}{2}\,\frac{l_2}{2\,h}\,\frac{h^3}{I_4}\,k_4 - \frac{w}{2\,h}\left(l^2{}_1 - \frac{l^2{}_2}{2}\right)\frac{l_2}{2\,h}\,\frac{l_2\,h^3}{I_4}\,k_4 \ldots (14d)$$

and

$$\Delta_{5V} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14e)$$

With $\Delta_{1V} + \Delta_{2V} + \Delta_{3V} + \Delta_{4V} + \Delta_{5V} = \Delta_{VD} = 0$, the values of *H* and *V* at the hinge are found by combining Eqs. 14.

General Case.—The more general case is represented by the deflection equations at a hinge linking two simple frames in the interior bays of a multibay assembly. The bent in Fig. 13(*a*) is used to illustrate this case. In writing

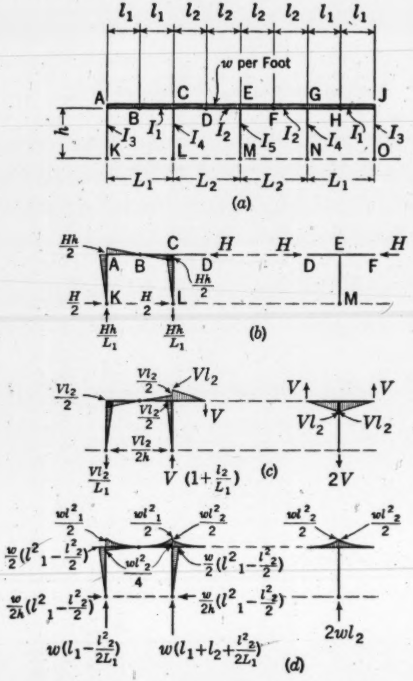

l_1 l_1 l_2 l_2 l_2 l_2 l_1 l_1
w per Foot
A B C D E F G H J
K L M N O
I_3 I_1 I_4 I_2 I_5 I_2 I_4 I_1 I_3
h
L_1 L_2 L_2 L_1
(a)
$\frac{Hh}{2}$ $\frac{H}{2}$ $\frac{Hh}{L_1}$ H
(b)
$\frac{Vl_2}{2}$ Vl_2 V $\frac{Vl_2}{2h}$ $\frac{Vl_2}{L_1}$ $V(1+\frac{l_2}{L_1})$ 2V
(c)
$\frac{wl_1^2}{2}$ $\frac{wl_2^2}{2}$ $\frac{wl_2^2}{4}$ $\frac{w}{2}(l_1^2-\frac{l_2^2}{2})$ $\frac{w}{2h}(l_1^2-\frac{l_2^2}{2})$
$w(l_1-\frac{l_2^2}{2L_1})$ $w(l_1+l_2+\frac{l_2^2}{2L_1})$ $2wl_2$
(d)

FIG. 12.—FOUR-BAY FRAME WITH SYMMETRICAL FRAMING AND LOADING

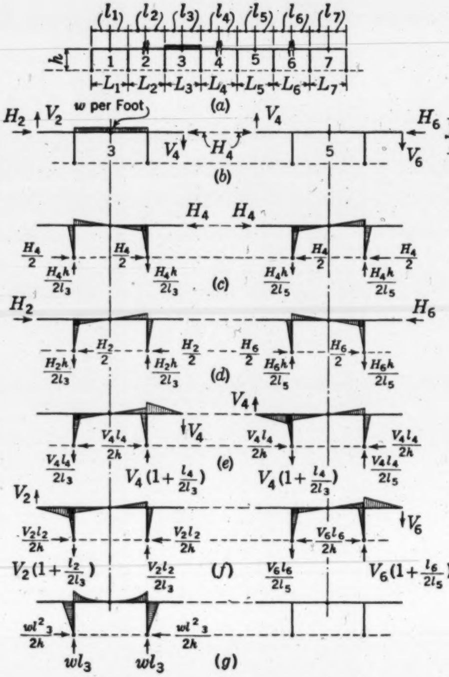

l_1 l_2 l_3 l_4 l_5 l_6 l_7
1 2 3 4 5 6 7
h
L_1 L_2 L_3 L_4 L_5 L_6 L_7
(a)
H_2 V_2 w per Foot V_4 H_4 H_6 V_6
3 5
(b)
$\frac{H_4}{2}$ $\frac{H_4h}{2l_3}$ $\frac{H_4h}{2l_5}$
(c)
$\frac{H_2}{2}$ $\frac{H_2h}{2l_3}$ $\frac{H_6}{2}$ $\frac{H_6h}{2l_5}$
(d)
$\frac{V_4l_4}{2h}$ $\frac{V_4l_4}{2l_3}$ $\frac{V_4l_4}{2l_5}$ $V_4(1+\frac{l_4}{2l_3})$
(e)
$\frac{V_2l_2}{2h}$ $\frac{V_2l_2}{2l_3}$ $V_2(1+\frac{l_2}{2l_3})$ $\frac{V_6l_6}{2h}$ $\frac{V_6l_6}{2l_5}$ $V_6(1+\frac{l_6}{2l_5})$
(f)
$\frac{wl_3^2}{2h}$ wl_3
(g)

FIG. 13.—GENERAL CASE, THE MULTIBAY ASSEMBLY

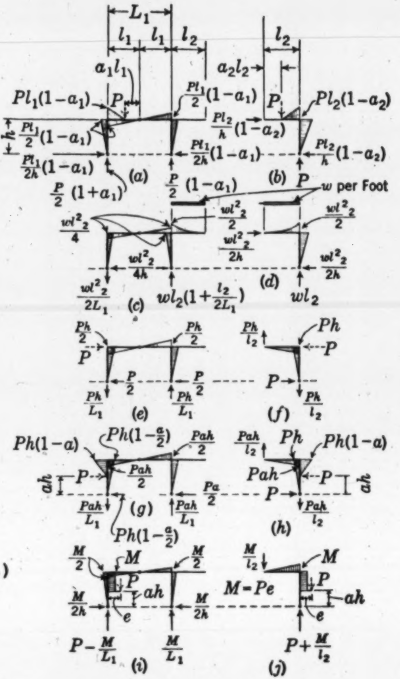
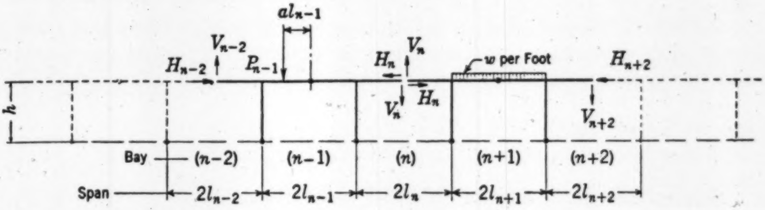

L_1 l_1 l_1 l_2 a_1l_1 a_2l_2 P h
$Pl_1(1-a_1)$ $\frac{Pl_1}{2}(1-a_1)$ $\frac{Pl_1}{2h}(1-a_1)$ $\frac{P}{2}(1+a_1)$ $\frac{P}{2}(1-a_1)$
(a)
$Pl_2(1-a_2)$ $\frac{Pl_2}{h}(1-a_2)$
(b)
w per Foot $\frac{wl_2^2}{4}$ $\frac{wl_2^2}{2}$ $\frac{wl_2^2}{2h}$ $\frac{wl_2^2}{4h}$ $\frac{wl_2^2}{2L_1}$ $wl_2(1+\frac{l_2}{2L_1})$ wl_2
(c) (d)
$\frac{Ph}{2}$ P $\frac{P}{2}$ $\frac{Ph}{L_1}$ $\frac{Ph}{l_2}$ Ph
(e) (f)
$Ph(1-a)$ $Ph(1-\frac{a}{2})$ $\frac{Pah}{2}$ $\frac{Pa}{2}$ $\frac{Pah}{L_1}$ $\frac{Pah}{l_2}$ Pah ah
(g) (h)
$\frac{M}{2}$ M $\frac{M}{2h}$ P ah e $M=Pe$ $\frac{M}{l_2}$ $P-\frac{M}{L_1}$ $\frac{M}{L_1}$ $P+\frac{M}{l_2}$
(i) (j)

FIG. 14.—SIMPLE MOMENT DIAGRAMS, GENERAL CASES

the deflection equations at the hinge in the fourth or connecting bay, the effects of the six unknown hinge reactions, and the given loading in the third bay, must be considered as indicated in Fig. 13(*b*). The corresponding simple moment diagrams are shown in Figs. 13(*c*) to 13(*g*). To obtain the horizontal deflection, Δ_{H4}, it will be necessary to combine the moment ordinates of these diagrams with those resulting from unit horizontal force at hinge 4. As a guide in setting up the desired deflection equation, the contribution of each member to the deflection at the joint under study is given in Table 6. Symbols C in Col. 2 denote the quantities—

$$C = k\,h\,\frac{l}{I} \qquad (15a)$$

TABLE 6.—CONTRIBUTION OF MEMBERS TO THE DEFLECTION AT A GIVEN JOINT n



Member (1)	$C^{(a)}$ (2)	$h\,H_n$ (3)	$l_n\,V_n$ (4)	$h\,H_{n-2}$ (5)	$l_{n-2}\,V_{n-2}$ (6)	$h\,H_{n+2}$ (7)	$l_{n+2}\,V_{n+2}$ (8)	$w_{n+1}\,l^2_{n+1}$ (9)	$(1-a)\,l_{n-1}\,P_{n-1}$ (10)
(*a*) COEFFICIENTS FOR THE HORIZONTAL DEFLECTION Δ_{H_n} AT JOINT n									
Beam:									
$(n-1)_{L,R}$	C_1	$r_{n(n)1}$	$s_{n(n)1}$	$r_{n(n-2)1}$	$s_{n(n-2)1}$				$t_{n(n-1)1}$
$n_{L,R}$	C_2								
$(n+1)_{L,R}$	C_3	$r_{n(n)3}$	$s_{n(n)3}$			$r_{n(n+2)3}$	$s_{n(n+2)3}$		
Column:									
$(n-2)-(n-1)$	C_4	$r_{n(n)4}$	$s_{n(n)4}$	$r_{n(n-2)4}$	$s_{n(n-2)4}$				$t_{n(n-1)4}$
$(n-1)-n$	C_5	$r_{n(n)5}$	$s_{n(n)5}$	$r_{n(n-2)5}$	$s_{n(n-2)5}$				$t_{n(n-1)5}$
$n-(n+1)$	C_6	$r_{n(n)6}$	$s_{n(n)6}$			$r_{n(n+2)6}$	$s_{n(n+2)6}$	$t_{nw(n+1)6}$	
$(n+1)-(n+2)$	C_7	$r_{n(n)7}$	$s_{n(n)7}$			$r_{n(n+2)7}$	$s_{n(n+2)7}$	$t_{nw(n+1)7}$	
(*b*) COEFFICIENTS FOR THE VERTICAL DEFLECTION Δ_{V_n} AT JOINT n									
Beam:									
$(n-1)_{L,R}$	C'_1	$r'_{n(n)1}$	$s'_{n(n)1}$	$r'_{n(n-2)1}$	$s'_{n(n-2)1}$				$t'_{n(n-1)1}$
$n_{L,R}$	C'_2		$s'_{n(n)2}$						
$(n+1)_{L,R}$	C'_3	$r'_{n(n)3}$	$s'_{n(n)3}$			$r'_{n(n+2)3}$	$s'_{n(n+2)3}$		
Column:									
$(n-2)-(n-1)$	C'_4	$r'_{n(n)4}$	$s'_{n(n)4}$	$r'_{n(n-2)4}$	$s'_{n(n-2)4}$				$t'_{n(n-1)4}$
$(n-1)-n$	C'_5	$r'_{n(n)5}$	$s'_{n(n)5}$	$r'_{n(n-2)5}$	$s'_{n(n-2)5}$				$t'_{n(n-1)5}$
$n-(n+1)$	C'_6	$r'_{n(n)6}$	$s'_{n(n)6}$			$r'_{n(n+2)6}$	$s'_{n(n+2)6}$	$t'_{nw(n+1)6}$	
$(n+1)-(n+2)$	C'_7	$r'_{n(n)7}$	$s'_{n(n)7}$			$r'_{n(n+2)7}$	$s'_{n(n+2)7}$	$t'_{nw(n+1)7}$	

[a] $C = k\,h\,\frac{l}{I}$; $C' = k\,l_n\,\frac{l}{I}$.

—when applied to horizontal deflections (Table 6(*a*)), and—

$$C' = k\, l_n \frac{l}{I} \qquad \text{(15b)}$$

—when applied to vertical deflections (Table 6(*b*)). The terms r, s, and t in each row designated by a member, when multiplied by the coefficient C and the respective column heading, will represent the contribution to deflection by that member. The sum of these contributions, when equated to zero, will result in the typical deflection expression containing the six unknown hinge reactions. The subscripts L and R in Col. 1 refer to the left and right segments respectively, of horizontal members, the two segments in each bay being combined into a single item. The expression for horizontal deflection (Table 6(*a*)) may be written in general form:

$$\begin{aligned}\Delta_{Hn} = \alpha_n H_n + \beta_n V_n + \alpha_{n-2} H_{n-2} \\ + \beta_{n-2} V_{n-2} + \alpha_{n+2} H_{n+2} + \beta_{n+2} V_{n+2} + Q_n = 0 \quad \text{(16a)}\end{aligned}$$

Similarly, from the data in Table 6(*b*) for vertical deflection at joint n,

$$\begin{aligned}\Delta_{Vn} = \beta'_n V_n + \alpha'_n H_n + \beta'_{n-2} V_{n-2} + \alpha'_{n-2} H_{n-2} \\ + \beta'_{n+2} V_{n+2} + \alpha'_{n+2} H_{n+2} + Q'_n = 0 \quad \text{(16b)}\end{aligned}$$

In Eqs. 16 the symbols α and β indicate the total coefficients of the corresponding unknown reactions H or V, obtained by summing up the $(h\, C\, r)$-values or the $(l\, C\, s)$-values in the respective columns of Table 6(*a*) and Table 6(*b*), and Q is the total load coefficient, obtained by adding the $(C\, w\, l^2\, t)$-values and the $(C\, l\, P\, t)$-values of the load columns—thus:

$$\left.\begin{aligned}\alpha_n &= h\,[C_1 r_{n(n)1} + C_3 r_{n(n)3} + C_4 r_{n(n)4} + C_5 r_{n(n)5} \\ &\qquad + C_6 r_{n(n)6} + C_7 r_{n(n)7}] \\ \beta_n &= l_n\,[C_1 s_{n(n)1} + C_3 s_{n(n)3} + C_4 s_{n(n)4} + C_5 s_{n(n)5} \\ &\qquad + C_6 s_{n(n)6} + C_7 s_{n(n)7}] \\ \alpha_{n-2} &= h\,[C_1 r_n(_{n-2)1} + C_4 r_n(_{n-2)4} + C_5 r_{n(n-2)5}] \\ \beta_{n-2} &= l_{n-2}\,[C_1 s_{n(n-2)1} + C_4 s_{n(n-2)4} + C_5 s_{n(n-2)5}] \\ \alpha_{n+2} &= h\,[C_3 r_{n(n+2)3} + C_6 r_{n(n+2)6} + C_7 r_{n(n+2)7}] \\ \beta_{n+2} &= l_{n+2}\,[C_3 s_{n(n+2)3} + C_6 s_{n(n+2)6} + C_7 s_{n(n+2)7}] \\ Q_n &= w_{n+1}\, l^2{}_{n+1}\,[C_6 t_{nw(n+1)6} + C_7 t_{nw(n+1)7}] \\ &\qquad + (1 - a)\, l_{n-1}\, P_{n-1}[C_1 t_{n(n-1)1} + C_4 t_{n(n-1)4} + C_5 t_{n(n-1)5}]\end{aligned}\right\} \quad \text{(17)}$$

in which r, s, and t are numerical coefficients, and $C = h\, l \frac{k}{I}$ (see Eq. 15*a*). The symbols α', β', and Q' designate the corresponding values for the V_n-equation, and are similarly obtained from the data in Table 6(*b*).

The beam framing in each bay (that is, the two beams constituting the horizontal member) is assumed to be symmetrical. In general, there will be little reason for shaping the two wedge beams in a given interior bay differently. However, should the shapes differ, then each beam will have a different C-value. In addition, if the hinge is not located at the center of the span, the r-values and the s-values will also differ for the two beams in the bay. It is also to be noted that the components of Q will vary with the loading. For loads other than those shown in the sketch in Table 6, additional components from other affected members will be needed. The simple moment diagrams for some other conditions of loading are shown in Fig. 14. The components of Q due to such loading may be obtained from these diagrams.

SIMPLIFIED PROCEDURE

For any given framing, r, s, and t can be determined by following the general procedure as outlined and illustrated previously. Once these coefficients are determined, the deflection equations can be set directly. Table 7 contains these coefficients for a typical interior bay of a rectangular framing. The same data for end bays in two arrangements of rectangular framing are given in Tables 8 and 9. With the aid of these tables, it is possible to write the deflection equations of the unknown hinge reactions of a frame without the necessity of supplementary computations. All that is needed for this purpose is a sketch of the frame indicating the properties of the members and the locations of the unknown hinge reaction. Then Eqs. 16 are written by using Table 6 as a guide to formulate the α-coefficient and the β-coefficient defined by Eqs. 17.

Example.—Obtain the deflection equations for the frame shown in Fig. 13(*a*), assuming equal spans and shapes for all members. Omitting C from the expressions (since it has the same value for all members), and using Table 8 for equations of H_2 and V_2,

$$\begin{aligned}
&h\,H_2\left(\tfrac{1}{2}+\tfrac{1}{2}+\tfrac{1}{4}+\tfrac{1}{4}+\tfrac{1}{4}+\tfrac{1}{4}\right) \\
&\qquad - h\,H_4\left(\tfrac{1}{2}+\tfrac{1}{4}+\tfrac{1}{4}\right) + l\,V_2\left(-\tfrac{1}{2}+\tfrac{1}{2}-\tfrac{1}{4}+\tfrac{1}{4}-\tfrac{1}{4}+\tfrac{1}{4}\right) \\
&\qquad\qquad + l\,V_4\left(\tfrac{1}{2}+\tfrac{1}{4}-\tfrac{1}{4}\right) + w\,l^2\left(-\tfrac{1}{4}+\tfrac{1}{4}\right) = 0; \\
&h\,H_2\left(-\tfrac{1}{2}+\tfrac{1}{2}-\tfrac{1}{4}+\tfrac{1}{4}-\tfrac{1}{4}+\tfrac{1}{4}\right) - h\,H_4\left(\tfrac{1}{2}-\tfrac{1}{4}+\tfrac{1}{4}\right) \\
&\qquad + l\,V_2\left(\tfrac{1}{2}+2+\tfrac{1}{2}+\tfrac{1}{4}+\tfrac{1}{4}+\tfrac{1}{4}+\tfrac{1}{4}\right) \\
&\qquad\qquad + l\,V_4\left(\tfrac{1}{2}-\tfrac{1}{4}-\tfrac{1}{4}\right) + w\,l^2\left(\tfrac{1}{4}+\tfrac{1}{4}\right) = 0.
\end{aligned}$$

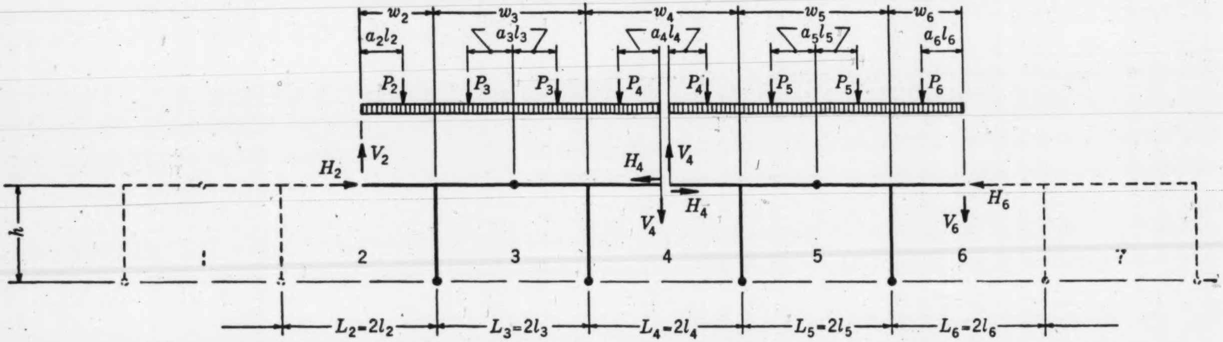
Equations of H_4, V_4, H_6, and V_6 are obtained from Table 7 as follows:

$$\begin{aligned}
&-h\,H_2\,(1) + h\,H_4\,(2) - h\,H_6\,(1) - l\,V_2\left(\tfrac{1}{2}\right) \\
&\qquad + l\,V_4\,(0) + l\,V_6\left(\tfrac{1}{2}\right) + w\,l^2\,(0) = 0; \\
&h\,H_2\left(\tfrac{1}{2}\right) + h\,H_4\,(0) - h\,H_6\left(\tfrac{1}{2}\right) - l\,V_2\,(0) \\
&\qquad + l\,V_4\,(4) + l\,V_6\,(0) - w\,l^2\left(\tfrac{1}{2}\right) = 0; \\
&-h\,H_4\,(1) + h\,H_6\,(2) - l\,V_4\left(\tfrac{1}{2}\right) + l\,V_6\,(0) = 0; \text{ and} \\
&h\,H_4\left(\tfrac{1}{2}\right) + h\,H_6\,(0) - l\,V_4\,(0) + l\,V_6\,(4) = 0.
\end{aligned}$$

Collecting terms,

$$\left.\begin{aligned}
2\,h\,H_2 - h\,H_4 + \tfrac{1}{2}\,l\,V_4 &= 0 \\
-\tfrac{1}{2}\,h\,H_4 + 4\,l\,V_2 - \tfrac{1}{2}\,w\,l^2 &= 0 \\
-h\,H_2 + 2\,h\,H_4 - h\,H_6 - \tfrac{1}{2}\,l\,V_2 + \tfrac{1}{2}\,l\,V_6 &= 0 \\
\tfrac{1}{2}\,h\,H_2 - \tfrac{1}{2}\,h\,H_6 + 4\,l\,V_4 - \tfrac{1}{2}\,w\,l^2 &= 0 \\
-h\,H_4 + 2\,h\,H_6 - \tfrac{1}{2}\,l\,V_4 &= 0 \\
\tfrac{1}{2}\,h\,H_4 + 4\,l\,V_6 &= 0
\end{aligned}\right\}\cdots\cdots\cdots(18)$$

TABLE 7.—CONTRIBUTION OF MEMBERS TO DEFLECTION OF A JOINT IN INTERIOR BAYS OF RECTANGULAR FRAMES (JOINT 4)


w_2
w_3
w_4
w_5
w_6
a_2l_2
a_3l_3
a_4l_4
a_5l_5
a_6l_6
P_2
P_3
P_3
P_4
P_4
P_5
P_5
P_6
H_2
V_2
H_4
V_4
H_4
V_4
H_6
V_6
h
1
2
3
4
5
6
7
$L_2=2l_2$
$L_3=2l_3$
$L_4=2l_4$
$L_5=2l_5$
$L_6=2l_6$

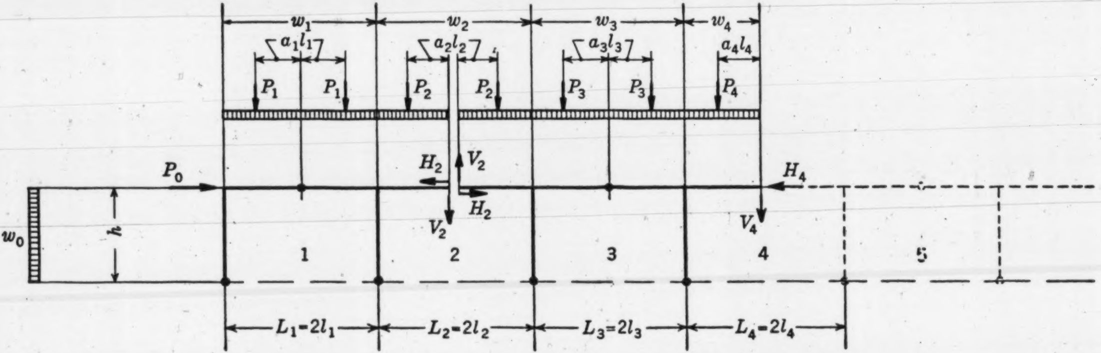
Member	$C^{(a)}$	COEFFICIENTS H AND V FOR Δ_{H4} AND Δ_{V4}						LOAD COEFFICIENTS FOR Δ_{H4} AND Δ_{V4}									
		$h\,H_2$	$h\,H_4$	$h\,H_6$	$l_2\,V_2$	$l_4\,V_4$	$l_6\,V_6$	$(1-a_2)\,l_2\,P_2$	$(1-a_3)\,l_3\,P_3$	$(1-a_4)\,l_4\,P_4$	$(1-a_5)\,l_5\,P_5$	$(1-a_6)\,l_6\,P_6$	$w_2\,l^2_2$	$w_3\,l^2_3$	$w_4\,l^2_4$	$w_5\,l^2_5$	$w_6\,l^2_6$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
(*a*) COEFFICIENTS FOR HORIZONTAL DEFLECTION Δ_H AT JOINT 4																	
Beam:																	
$3_{L,R}$	C_3	$-\frac{1}{2}$	$\frac{1}{2}$		$-\frac{1}{2}$	$-\frac{1}{2}$		$\frac{1}{2}$		$-\frac{1}{2}$			$\frac{1}{4}$		$-\frac{1}{4}$		
$4_{L,R}$	C_4																
$5_{L,R}$	C_5		$\frac{1}{2}$	$-\frac{1}{2}$		$\frac{1}{2}$	$\frac{1}{2}$			$-\frac{1}{2}$		$\frac{1}{2}$			$-\frac{1}{4}$		$\frac{1}{4}$
Column:																	
2–3	$C_{2,3}$	$-\frac{1}{4}$	$\frac{1}{4}$		$\frac{1}{4}$	$-\frac{1}{4}$		$-\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$			$-\frac{1}{8}$	$\frac{1}{4}$	$-\frac{1}{8}$		
3–4	$C_{3,4}$	$-\frac{1}{4}$	$\frac{1}{4}$		$-\frac{1}{4}$	$\frac{1}{4}$		$\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$			$\frac{1}{8}$	$-\frac{1}{4}$	$\frac{1}{8}$		
4–5	$C_{4,5}$		$\frac{1}{4}$	$-\frac{1}{4}$		$-\frac{1}{4}$	$\frac{1}{4}$			$\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$			$\frac{1}{8}$	$-\frac{1}{4}$	$\frac{1}{8}$
5–6	$C_{5,6}$		$\frac{1}{4}$	$-\frac{1}{4}$		$\frac{1}{4}$	$-\frac{1}{4}$			$-\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$			$-\frac{1}{8}$	$\frac{1}{4}$	$-\frac{1}{8}$

TABLE 7.—(*Continued*)

Member	$C^{(a)}$	COEFFICIENTS H AND V FOR Δ_{H4} AND Δ_{V4}						LOAD COEFFICIENTS FOR Δ_{H4} AND Δ_{V4}									
		$h\,H_2$	$h\,H_4$	$h\,H_6$	$l_2\,V_2$	$l_4\,V_4$	$l_6\,V_6$	$(1-a_2)\,l_2\,P_2$	$(1-a_3)\,l_3\,P_3$	$(1-a_4)\,l_4\,P_4$	$(1-a_5)\,l_5\,P_5$	$(1-a_6)\,l_6\,P_6$	$w_2\,l^2_2$	$w_3\,l^2_3$	$w_4\,l^2_4$	$w_5\,l^2_5$	$w_6\,l^2_6$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
(*b*) COEFFICIENT FOR VERTICAL DEFLECTION Δ_V AT JOINT 4																	
Beam:																	
$3_{L,R}$	C'_3	$\frac{1}{3}$	$-\frac{1}{3}$		$\frac{1}{3}$	$\frac{1}{3}$		$-\frac{1}{3}$		$\frac{1}{3}$			$-\frac{1}{4}$		$\frac{1}{4}$		
$4_{L,R}$	C'_4					2											
$5_{L,R}$	C'_5		$\frac{1}{3}$	$-\frac{1}{3}$		$\frac{1}{3}$	$\frac{1}{3}$			$-\frac{1}{3}$		$\frac{1}{3}$			$-\frac{1}{4}$		$\frac{1}{4}$
Column:																	
2–3	$C'_{2,3}$	$\frac{1}{4}$	$-\frac{1}{4}$		$-\frac{1}{4}$	$\frac{1}{4}$		$\frac{1}{4}$	$-\frac{1}{3}$	$\frac{1}{4}$			$\frac{1}{8}$	$-\frac{1}{4}$	$\frac{1}{8}$		
3–4	$C'_{3,4}$	$-\frac{1}{4}$	$\frac{1}{4}$		$-\frac{1}{4}$	$\frac{1}{4}$		$\frac{1}{4}$	$-\frac{1}{3}$	$\frac{1}{4}$			$\frac{1}{8}$	$-\frac{1}{4}$	$\frac{1}{8}$		
4–5	$C'_{4,5}$		$-\frac{1}{4}$	$\frac{1}{4}$		$\frac{1}{4}$	$-\frac{1}{4}$			$-\frac{1}{4}$	$\frac{1}{3}$	$-\frac{1}{4}$			$-\frac{1}{8}$	$\frac{1}{4}$	$-\frac{1}{8}$
5–6	$C'_{5,6}$		$\frac{1}{4}$	$-\frac{1}{4}$		$\frac{1}{4}$	$-\frac{1}{4}$			$-\frac{1}{4}$	$\frac{1}{3}$	$-\frac{1}{4}$			$-\frac{1}{8}$	$\frac{1}{4}$	$-\frac{1}{8}$

[a] $C = k\,h\,\frac{l}{I}$; $C' = k\,l_4\,\frac{l}{I}$.

TABLE 8.—CONTRIBUTION OF MEMBERS TO DEFLECTION OF A JOINT (JOINT 2) IN END BAYS OF RECTANGULAR FRAMES WITH AN ODD NUMBER OF BAYS


w_1
w_2
w_3
w_4
a_1l_1
a_2l_2
a_3l_3
a_4l_4
P_1
P_1
P_2
P_2
P_3
P_3
P_4
P_0
H_2
V_2
H_2
V_2
H_4
V_4
w_0
h
1
2
3
4
5
$L_1=2l_1$
$L_2=2l_2$
$L_3=2l_3$
$L_4=2l_4$

Member	$C^{(a)}$	Coefficients H and V for Δ_{H2} and Δ_{V2}				Load Coefficients for Δ_{H2} and Δ_{V2}									
		$h\ H_2$	$h\ H_4$	$l_2\ V_2$	$l_4\ V_4$	$h\ P_0$	$(1-a_1)\ l_1\ P_1$	$(1-a_2)\ l_2\ P_2$	$(1-a_3)\ l_3\ P_3$	$(1-a_4)\ l_4\ P_4$	$w_0\ h^2$	$w_1\ l^2_1$	$w_2\ l^2_2$	$w_3\ l^2_3$	$w_4\ l^2_4$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
		(a) COEFFICIENTS FOR HORIZONTAL DEFLECTION Δ_{H2} AT JOINT 2													
Beam:															
$1_{L,R}$..	C_1	$\frac{1}{2}$		$-\frac{1}{2}$		$-\frac{1}{2}$		$-\frac{1}{2}$			$-\frac{1}{4}$		$-\frac{1}{4}$		
$2_{L,R}$..	C_2														
$3_{L,R}$..	C_3	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$			$-\frac{1}{2}$		$\frac{1}{2}$			$-\frac{1}{4}$		$\frac{1}{4}$
Column:															
0–1...	$C_{0,1}$	$\frac{1}{4}$		$-\frac{1}{4}$		$-\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$			$[k_w/(2\ k)]-\frac{3}{8}$	$\frac{1}{4}$	$-\frac{1}{8}$		
1–2...	$C_{1,2}$	$\frac{1}{4}$		$\frac{1}{4}$		$-\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$			$-\frac{1}{8}$	$-\frac{1}{4}$	$\frac{1}{8}$		
2–3...	$C_{2,3}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$			$\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$			$\frac{1}{8}$	$-\frac{1}{4}$	$\frac{1}{8}$
3–4...	$C_{3,4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$			$-\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$			$-\frac{1}{8}$	$\frac{1}{4}$	$-\frac{1}{8}$

TABLE 8.—(*Continued*)

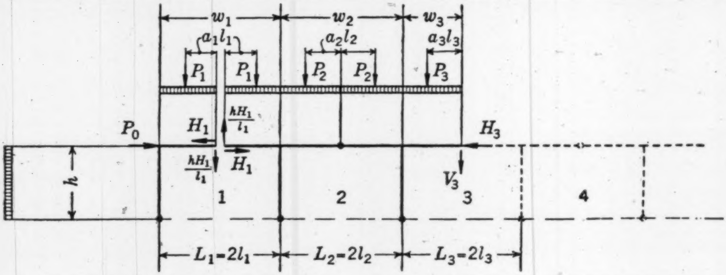
Member	$C^{(a)}$	COEFFICIENTS H AND V FOR Δ_{H2} AND Δ_{V2}				LOAD COEFFICIENTS FOR Δ_{H2} AND Δ_{V2}									
		$h\,H_2$	$h\,H_4$	$l_2\,V_2$	$l_4\,V_4$	$h\,P_0$	$(1-a_1)\,l_1\,P_1$	$(1-a_2)\,l_2\,P_2$	$(1-a_3)\,l_3\,P_3$	$(1-a_4)\,l_4\,P_4$	$w_0\,h^2$	$w_1\,l^2_1$	$w_2\,l^2_2$	$w_3\,l^2_3$	$w_4\,l^2_4$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
(*b*) COEFFICIENTS FOR VERTICAL DEFLECTION Δ_{V2} AT JOINT 2															
Beam:															
$1_{L,R}$ . .	C'_1	$-\frac{1}{2}$		$\frac{1}{2}$		$\frac{1}{2}$		$\frac{1}{2}$			$\frac{1}{4}$		$\frac{1}{4}$		
$2_{L,R}$ . .	C'_2			2											
$3_{L,R}$ . .	C'_3	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$			$-\frac{1}{2}$		$\frac{1}{2}$			$-\frac{1}{4}$		$\frac{1}{4}$
Column:															
0–1 . . .	$C'_{0,1}$	$-\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$			$\frac{1}{8}-[k_w/(2\,k)]$	$-\frac{1}{4}$	$\frac{1}{8}$		
1–2 . . .	$C'_{1,2}$	$\frac{1}{4}$		$\frac{1}{4}$		$-\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$			$-\frac{1}{8}$	$-\frac{1}{4}$	$\frac{1}{8}$		
2–3 . . .	$C'_{2,3}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$			$-\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$			$-\frac{1}{8}$	$\frac{1}{4}$	$-\frac{1}{8}$
3–4 . . .	$C'_{3,4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$			$-\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$			$-\frac{1}{8}$	$\frac{1}{4}$	$-\frac{1}{8}$

[a] $C = k\,h\,\frac{l}{I}$; $C' = k\,l_2\,\frac{l}{I}$.

APPROXIMATE DESIGN

In the selection of sections for an initial or approximate analysis and design, it will be satisfactory to use the simple moments of the component simple frames of an assembly. In most cases, the wedge-beam sections obtained from these moments will not differ materially from those required by the corrected moments of the final or exact analysis. This fact, in itself, constitutes a great advantage seldom afforded to the designer in the analysis of rigid or continuous frames. In other words, it is possible to prepare an approximate design of sufficient accuracy with the ease and speed generally obtained in the analysis of statically determinate structures.

TABLE 9.—CONTRIBUTION OF MEMBERS TO DEFLECTION OF A JOINT IN END BAYS OF RECTANGULAR FRAMES WITH AN EVEN NUMBER OF BAYS (HORIZONTAL DEFLECTION AT JOINT 1)



Member	$C^{(a)}$	COEFFICIENTS[b] H AND V			LOAD COEFFICIENTS FOR Δ_{H1}							
		$h\ H_1$	$h\ H_3$	$l_3\ V_3$	$h\ P_0$	$(1-a_1)\ l_1\ P_1$	$(1-a_2)\ l_2\ P_2$	$(1-a_3)\ l_3\ P_3$	$w_0\ h^2$	$w_1\ l^2_1$	$w_2\ l^2_2$	$w_3\ l^2_3$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Beam:												
$1_{L,R}$..	C_1	2			−2				−1			
$2_{L,R}$..	C_2	2	−1	1	−1			1	$-\frac{1}{3}$			$\frac{1}{3}$
$3_{L,R}$..	C_3											
Column:												
0–1...	$C_{0,1}$	1			−1	1			$\frac{k_w}{k}-1$	$\frac{1}{3}$		
1–2...	$C_{1,2}$											
2–3...	$C_{2,3}$	1	$-\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2}$		1	$-\frac{1}{3}$	$-\frac{1}{4}$		$\frac{1}{3}$	$-\frac{1}{4}$

[a] $C = k\,h\,\frac{l}{I}$. [b] Coefficients H and V for Δ_{H1}.

SOLUTION OF SIMULTANEOUS EQUATIONS

Perhaps the greatest drawback to the more widespread use of indeterminate structures in general design practice is that the analysis is difficult. The aversion of the average engineer to the so-called "intricacies of refined analysis" and, more particularly, to the task of solving simultaneous equations incidental to the work is well known. Still another adverse factor is the needed accuracy of operations which necessitates the use of computing machines. Since the analysis of wedge-beam framing remains essentially in the realm of indeterminate structures, the extent of its popularity by most designers will no doubt be subject to the same considerations as those for continuous frames.

The simultaneous equations derived in the analysis of wedge-beam framing are relatively of a simple form. In most cases of design, where the arrangement contains only a few bays, or where the framing is symmetrical and the loading uniform, the group may consist of less than three equations. In such cases the solution of the correspondingly small number of unknowns will involve little difficulty. However, in other arrangements of framing or conditions of loading which result in the setting of a greater number of simultaneous equations, the solution of the unknowns by the use of ordinary methods of computation may become rather burdensome. To obviate this difficulty, and thus render the new framing more acceptable by the average engineer, a more simple procedure is suggested.

Simplified Solution.—Consider the typical expression (see Eq. 16*a*):

$$\alpha_4 H_4 + \beta_{44} V_4 + \alpha_{42} H_2 + \beta_{42} V_2 + \alpha_{46} H_6 + \beta_{46} V_6 + Q_4 = 0 \ldots (19)$$

which was obtained by equating the relations for horizontal deflection at two sides of an interior bay joint. It is characteristic of this expression (as will be noted by examination of data in the corresponding tables) that the coefficient of H_4—that is, the horizontal reaction at the joint under consideration—is always numerically larger than the coefficients of the other unknowns of the equation. Then, for simplicity of reference, the relation can be referred to as the H_4-expression. Furthermore, H_4 can be considered as the principal unknown and the other reactions as the secondary unknowns of the expression. Similarly, each of the latter forces, in turn, will become the principal unknown of the corresponding equation in the group of equations written for a given framing.

In solving for the value of H_4, it will be assumed that its final value, q_4, is obtained by the summation of a series of increments of q, defined as follows:

$$H_4 = q_{41} + q_{42} + q_{43} + \cdots + q_{4n} = q_4 \ldots\ldots\ldots\ldots (20)$$

in which

$$q_{41} = -\frac{Q_4}{\alpha_4} \ldots\ldots\ldots\ldots\ldots\ldots (21a)$$

$$q_{42} = -\frac{1}{\alpha_4}(\beta_{44} q'_{41} + \alpha_{42} q_{21} + \beta_{42} q'_{21} + \alpha_{46} q_{61} + \beta_{46} q'_{61}) \ldots (21b)$$

$$q_{43} = -\frac{1}{\alpha_4}(\beta_{44} q'_{42} + \alpha_{42} q_{22} + \beta_{42} q'_{22} + \alpha_{46} q_{62} + \beta_{46} q'_{62}) \ldots (21c)$$

and

$$q_{4n} = -\frac{1}{\alpha_4}[\beta_{44} q'_{4(n-1)} + \alpha_{42} q_{2(n-1)} + \beta_{42} q'_{2(n-1)} + \alpha_{46} q_{6(n-1)} + \beta_{46} q'_{6(n-1)}] \ldots (21d)$$

In Eqs. 20 and 21, also, $q'_{41}, q'_{42}, \cdots, q'_{4(n-1)}$ are the successive increment values of V_4; $q_{21}, q_{22}, \cdots, q_{2(n-1)}$ are those of H_2, etc., similarly obtained from the respective expression and corresponding cycle of increment operation.

Since the quotient of the numerical coefficient of the secondary unknowns to that of the principal unknown is a small fraction, the values of the successive increments decrease rather rapidly and become negligible quantities in a few cycles of multiplication. Accordingly, only a few increments will suffice to provide a reasonably accurate solution. Furthermore, the operations will not require the use of computing machines, the desired accuracy being obtained by ordinary slide rule work.

Numerical Example.—Consider the group of six equations (Eqs. 18) derived for the bent in Fig. 13. Here, again it will be assumed that $h = l$, and all members are of like shapes. Then,

$$\left.\begin{aligned}
&2\,H_2 - H_4 + \tfrac{1}{2}\,V_4 = 0\\
&4\,V_2 - \tfrac{1}{2}\,H_4 + \frac{w\,l}{2} = 0\\
&2\,H_4 - H_2 - H_6 - \tfrac{1}{2}\,V_2 + \tfrac{1}{2}\,V_6 = 0\\
&4\,V_4 + \tfrac{1}{2}\,H_2 - \tfrac{1}{2}\,H_6 - \frac{w\,l}{2} = 0\\
&2\,H_6 - H_4 - \tfrac{1}{2}\,V_4 = 0\\
&4\,V_6 + \tfrac{1}{2}\,H_4 = 0
\end{aligned}\right\}\ldots\ldots\ldots\ldots(22)$$

Eqs. 22 can be rewritten in the form:

$$\left.\begin{aligned}
H_2 &= \tfrac{1}{2}\,(H_4 - \tfrac{1}{2}\,V_4)\\
V_2 &= \tfrac{1}{4}\,(\tfrac{1}{2}\,H_4 - \tfrac{1}{2}\,w\,l)\\
H_4 &= \tfrac{1}{2}\,(H_2 + H_6 + \tfrac{1}{2}\,V_2 - \tfrac{1}{2}\,V_6)\\
V_4 &= \tfrac{1}{4}\,(-\,\tfrac{1}{2}\,H_2 + \tfrac{1}{2}\,H_6 + \tfrac{1}{2}\,w\,l)\\
H_6 &= \tfrac{1}{2}\,(H_4 + \tfrac{1}{2}\,V_4)\\
V_6 &= \tfrac{1}{4}\,(-\,\tfrac{1}{2}\,H_4)\,.
\end{aligned}\right\}\ldots\ldots\ldots\ldots(23)$$

For initial values of the unknowns, or for the first increments,

$$\left.\begin{aligned}
H_{21} &= \tfrac{1}{2}\,(0) = 0\\
V_{21} &= \tfrac{1}{4}\,(-\,\tfrac{1}{2}\,w\,l) = -\,\tfrac{1}{8}\,w\,l\\
H_{41} &= \tfrac{1}{2}\,(0) = 0\\
V_{41} &= \tfrac{1}{4}\,(\tfrac{1}{2}\,w\,l) = \tfrac{1}{8}\,w\,l\\
H_{61} &= \tfrac{1}{2}\,(0) = 0\\
V_{61} &= \tfrac{1}{4}\,(0) = 0
\end{aligned}\right\}\ldots\ldots\ldots\ldots(24)$$

From the values in Eqs. 24, the second increments become

$$\left.\begin{aligned}
H_{22} &= \tfrac{1}{2}\,(1 \times 0 - \tfrac{1}{2} \times \tfrac{1}{8}\,w\,l) = -\,\tfrac{1}{32}\,w\,l\\
V_{22} &= \tfrac{1}{4}\,(\tfrac{1}{2} \times 0) = 0\\
H_{42} &= \tfrac{1}{2}\,(1 \times 0 + 1 \times 0 + \tfrac{1}{2} \times -\,\tfrac{1}{8}\,w\,l - \tfrac{1}{2} \times 0) = -\,\tfrac{1}{32}\,w\,l\\
V_{42} &= \tfrac{1}{4}\,(-\,\tfrac{1}{2} \times 0 + \tfrac{1}{2} \times 0) = 0\\
H_{62} &= \tfrac{1}{2}\,(1 \times 0 + \tfrac{1}{2} \times \tfrac{1}{8}\,w\,l) = \tfrac{1}{32}\,w\,l\\
V_{62} &= \tfrac{1}{4}\,(-\,\tfrac{1}{2} \times 0) = 0
\end{aligned}\right\}\ldots(25)$$

Similarly, the third increments, obtained from the second increments, are as follows:

$$\left.\begin{aligned}
H_{23} &= \tfrac{1}{2}\,(1 \times -\,\tfrac{1}{32}\,w\,l - \tfrac{1}{2} \times 0) = -\,\tfrac{1}{64}\,w\,l\\
V_{23} &= \tfrac{1}{4}\,(\tfrac{1}{2} \times -\,\tfrac{1}{32}\,w\,l) = -\,\tfrac{1}{256}\,w\,l\\
H_{43} &= \tfrac{1}{2}\,(1 \times -\,\tfrac{1}{32}\,w\,l + 1 \times \tfrac{1}{32}\,w\,l + \tfrac{1}{2} \times 0 - \tfrac{1}{2} \times 0) = 0\\
V_{43} &= \tfrac{1}{4}\,(-\,\tfrac{1}{2} \times -\,\tfrac{1}{32}\,w\,l + \tfrac{1}{2} \times \tfrac{1}{32}\,w\,l) = \tfrac{1}{128}\,w\,l\\
H_{63} &= \tfrac{1}{2}\,(1 \times -\,\tfrac{1}{32}\,w\,l + \tfrac{1}{2} \times 0) = -\,\tfrac{1}{64}\,w\,l\\
V_{63} &= \tfrac{1}{4}\,(-\,\tfrac{1}{2} \times -\,\tfrac{1}{32}\,w\,l) = \tfrac{1}{256}\,w\,l
\end{aligned}\right\}\ldots(26)$$

Likewise, the fourth, fifth, and sixth increments are obtained:

$$\begin{array}{lll}
H_{24} = -\dfrac{1}{512}\,w\,l & H_{25} = -\dfrac{9}{1{,}024}\,w\,l & H_{26} = -\dfrac{1}{8{,}192}\,w\,l \\[2ex]
V_{24} = 0 & V_{25} = -\dfrac{9}{4{,}096}\,w\,l & V_{26} = 0 \\[2ex]
H_{44} = -\dfrac{9}{512}\,w\,l & H_{45} = 0 & H_{46} = -\dfrac{81}{8{,}192}\,w\,l \\[2ex]
V_{44} = 0 & V_{45} = \dfrac{1}{2{,}048}\,w\,l & V_{46} = 0 \\[2ex]
H_{64} = \dfrac{1}{512}\,w\,l & H_{65} = -\dfrac{9}{1{,}024}\,w\,l & H_{66} = \dfrac{1}{8{,}192}\,w\,l \\[2ex]
V_{64} = 0 & V_{65} = \dfrac{9}{4{,}096}\,w\,l & V_{66} = 0
\end{array}$$

From the sixth increments it appears that no further operations are needed. The total values, corresponding to six increments, then become

$$\left.\begin{aligned}
H_2 &= \left(0 - \frac{1}{32} - \frac{1}{64} - \frac{1}{512} - \frac{9}{1{,}024} - \frac{1}{8{,}192}\right) w\,l \\
&= -\frac{473}{8{,}192}\,w\,l = -\,0.05774\,w\,l \\
V_2 &= \left(-\frac{1}{8} + 0 - \frac{1}{256} + 0 - \frac{9}{4{,}096} + 0\right) w\,l \\
&= -\frac{1{,}074}{8{,}192}\,w\,l = -\,0.13110\,w\,l \\
H_4 &= \left(0 - \frac{1}{32} + 0 - \frac{9}{512} + 0 - \frac{81}{8{,}192}\right) w\,l \\
&= -\frac{481}{8{,}192}\,w\,l = -\,0.05872\,w\,l \\
V_4 &= \left(\frac{1}{8} + 0 + \frac{1}{128} + 0 + \frac{1}{2{,}048} + 0\right) w\,l = \frac{1{,}092}{8{,}192}\,w\,l = 0.13330\,w\,l \\
H_6 &= \left(0 + \frac{1}{32} - \frac{1}{64} + \frac{1}{512} - \frac{9}{1{,}024} + \frac{1}{8{,}192}\right) w\,l \\
&= \frac{73}{8{,}192}\,w\,l = 0.00891\,w\,l \\
V_6 &= \left(0 + 0 + \frac{1}{256} + 0 + \frac{9}{4{,}096} + 0\right) w\,l \\
&= +\frac{50}{8{,}192}\,w\,l = 0.00610\,w\,l
\end{aligned}\right\}\ldots(27)$$

For further simplification, the necessary operations of the solution may be arranged in a tabular form, as indicated in Fig. 15.

	H_2	V_2	H_4	V_4	H_6	V_6
α_n & β_n	2	4	2	4	2	4
$-\frac{\alpha_{nm}}{\alpha_n}$ or $-\frac{\beta_{nm}}{\alpha_n}$	$\frac{1}{2}H_4$ $-\frac{1}{4}V_4$	$\frac{1}{8}H_4$	$\frac{1}{2}H_2$ $\frac{1}{4}V_2$ $\frac{1}{2}H_6$ $-\frac{1}{4}V_6$	$-\frac{1}{8}H_2$ $\frac{1}{8}H_6$	$\frac{1}{2}H_4$ $\frac{1}{4}V_4$	$-\frac{1}{8}H_4$
$-Q_n$	0	$-\frac{1}{2}wl$	0	$\frac{1}{2}wl$	0	0
$q_{n1} = \frac{-Q_n}{\alpha_n}$	0	$-\frac{1}{8}$	0	$\frac{1}{8}$	0	0
$q_{n2} = -\frac{1}{\alpha_n}\int \alpha_{nm} \cdot q_{n1}$	$\frac{1}{2}\cdot 0 = 0$ $-\frac{1}{4}\cdot\frac{1}{8} = -\frac{1}{32}$	$\frac{1}{8}\cdot 0 = 0$	$\frac{1}{2}\cdot 0 = 0$ $\frac{1}{4}\cdot -\frac{1}{8} = -\frac{1}{32}$ $\frac{1}{2}\cdot 0 = 0$ $-\frac{1}{4}\cdot 0 = 0$	$-\frac{1}{8}\cdot 0 = 0$ $\frac{1}{8}\cdot 0 = 0$	$\frac{1}{2}\cdot 0 = 0$ $\frac{1}{4}\cdot\frac{1}{8} = \frac{1}{32}$	$\frac{1}{8}\cdot 0 = 0$
	$-\frac{1}{32}$	0	$-\frac{1}{32}$	0	$\frac{1}{32}$	0
$q_{n3} = -\frac{1}{\alpha_n}\int \alpha_{nm} \cdot q_{n2}$	$\frac{1}{2}\cdot -\frac{1}{32} = -\frac{1}{64}$ $-\frac{1}{4}\cdot 0 = 0$	$\frac{1}{8}\cdot -\frac{1}{32} = -\frac{1}{256}$	$\frac{1}{2}\cdot -\frac{1}{32} = -\frac{1}{64}$ $\frac{1}{4}\cdot 0 = 0$ $\frac{1}{2}\cdot\frac{1}{32} = \frac{1}{64}$ $-\frac{1}{4}\cdot 0 = 0$	$-\frac{1}{8}\cdot -\frac{1}{32} = \frac{1}{256}$ $\frac{1}{8}\cdot\frac{1}{32} = \frac{1}{256}$	$\frac{1}{2}\cdot -\frac{1}{32} = -\frac{1}{64}$ $\frac{1}{4}\cdot 0 = 0$	$-\frac{1}{8}\cdot -\frac{1}{32} = \frac{1}{256}$
	$-\frac{1}{64}$	$-\frac{1}{256}$	0	$\frac{1}{128}$	$-\frac{1}{64}$	$\frac{1}{256}$
$q_{n4} = -\frac{1}{\alpha_n}\int \alpha_{nm} \cdot q_{n3}$	0 $-\frac{1}{512}$	0	$-\frac{1}{128}$ $-\frac{1}{1024}$ $-\frac{1}{128}$ $-\frac{1}{1024}$	$\frac{1}{512}$ $-\frac{1}{512}$	0 $\frac{1}{512}$	0
	$-\frac{1}{512}$	0	$-\frac{9}{512}$	0	$\frac{1}{512}$	0
$q_{n5} = -\frac{1}{\alpha_n}\int \alpha_{nm} \cdot q_{n4}$	$-\frac{9}{1024}$ 0	$-\frac{9}{4096}$	$-\frac{1}{1024}$ 0 $\frac{1}{1024}$ 0	$\frac{1}{4096}$ $\frac{1}{4096}$	$-\frac{9}{1024}$ 0	$\frac{9}{4096}$
	$-\frac{9}{1024}$	$-\frac{9}{4096}$	0	$\frac{1}{2048}$	$-\frac{9}{1024}$	$\frac{9}{4096}$
$q_{n6} = -\frac{1}{\alpha_n}\int \alpha_{nm} \cdot q_{n5}$	0 $-\frac{1}{8192}$	0	$-\frac{9}{2048}$ $-\frac{9}{16384}$ $-\frac{9}{2048}$ $-\frac{9}{16384}$	$\frac{9}{8192}$ $-\frac{9}{8192}$	0 $\frac{1}{8192}$	0
	$-\frac{1}{8192}$	0	$-\frac{81}{8192}$	0	$\frac{1}{8192}$	0
$q_n = q_{n1} + q_{n2} + q_{n3}$ $+ q_{n4} + q_{n5} + q_{n6}$	$-\frac{473}{8192}wl$	$-\frac{1074}{8192}wl$	$-\frac{481}{8192}wl$	$\frac{1092}{8192}wl$	$\frac{73}{8192}wl$	$\frac{50}{8192}wl$
	$-.05774wl$	$-.13110wl$	$-.05872wl$	$.13330wl$	$.00891wl$	$.00610wl$

FIG. 15.—SOLUTION OF SIMULTANEOUS EQUATIONS (FOR THE BENT IN FIG. 11)

CONCLUSION

The wedge-beam framing system was developed as an economy measure to conserve critical materials of construction. Its presentation here is predicated on that same consideration. It is hoped that full advantage will be taken of the potentialities of the system to assure important savings and, at the same time, to alleviate the present conditions of critical shortage of materials in structural framing. Design formulas and numerical values for typical loadings are presented in Tables 1 to 5.

DISCUSSION

G. C. **Ernst**,[3] M. ASCE, J. J. **Hromadik**,[4] and G. R. **Swihart**,[5] J. M. ASCE.—A novel adaptation of the "dummy" or "unit" load method of determining deflections is proposed for solving the redundant crown forces in a connected series of three-hinged frames. The statical determinateness of these three-hinged units results in simple expressions for the forces that produce moments in the several members. This is one of the evident advantages of the analysis; but a more important advantage is the possible economy of material obtained in shaping the members to fit the stress pattern advantageously. It may be expected, however, that the unit cost of such construction will be considerably higher than for normal types until contractors become accustomed to the differences in details.

In the case of precast units of reinforced concrete the writers feel that the Considère hinge will have an advantage over the Mesnager hinge with regard to assembly, although either could be used. The advantage gained in using the Considère hinge lies in the fact that the bars need not cross, the steel may be welded, and a covering can be cast more readily after the subassemblies are in place, as shown in Fig. 16. The resulting appearance need not be different from that obtained with a Mesnager hinge.

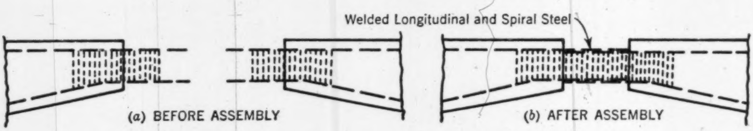


Fig. 16.—Assembly of the Considère Hinge

Rotational resistance of such hinges, although substantially greater than the Mesnager hinge, is sufficiently low to permit an assumption of a hinged condition for the angular rotations met in practice. Design charts have been prepared for Considère hinges, based upon the use of a limiting deformation that will prevent unsightly cracking.[6]

Furthermore, the use of the flexible connection or joint of limited rigidity as proposed in Part I under "Connections" should logically raise a question as to the additional economy possible by the use of sections of predetermined capacity. It is just as logical to use a section of predetermined capacity, having a magnitude of resistance other than virtually zero, as it is to use the Mesnager or Considère articulations, and there is neither more nor less likeli-

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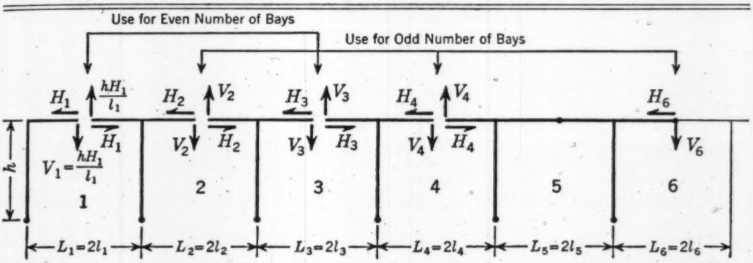
[6] "Tests of Considère Hinges Under Direct Stress, Bending, and Shear," by G. C. Ernst, *Proceedings*, A. C. I., Vol. 36, 1940, p. 49.

TABLE 10.—CONTRIBUTION OF MEMBERS TO DEFLECTION OF JOINTS IN END OR INTERIOR BAYS OF RECTANGULAR FRAMES FOR AN EVEN OR ODD NUMBER OF BAYS

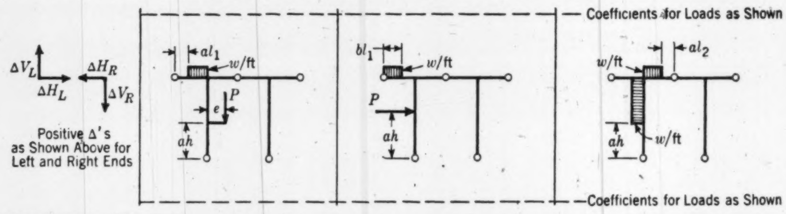

Use for Even Number of Bays
Use for Odd Number of Bays
H_1
$\frac{hH_1}{l_1}$
H_1
$V_1 = \frac{hH_1}{l_1}$
H_2
V_2
V_2
H_2
H_3
V_3
V_3
H_3
H_4
V_4
V_4
H_4
H_6
V_6
h
1
2
3
4
5
6
$L_1=2l_1$
$L_2=2l_2$
$L_3=2l_3$
$L_4=2l_4$
$L_5=2l_5$
$L_6=2l_6$

APPLICABLE VALUES OF Δ						COEFFICIENTS H AND V FOR ΔH_1 IN END BAY OF EVEN NUMBER OF BAYS			COEFFICIENTS H AND V FOR Δ_{H2} AND Δ_{V2} IN END BAYS OF ODD NUMBER OF DAYS					
										$h\ H_2$	$h\ H_4$		$l_2\ V_2$	$l_4\ V_4$
									COEFFICIENTS H AND V FOR Δ_{H3} AND Δ_{V3} IN INTERIOR BAYS OF EVEN NUMBER OF BAYS					
									$h\ H_1$	$h\ H_3$	$h\ H_5$	$l_1\ H_1$	$l_3\ V_3$	$l_5\ V_5$
Δ_{H1}, Δ_{H2}, Δ_{V2}		Δ_{H3}, Δ_{V3}		Δ_{H4}, Δ_{V4}					COEFFICIENTS H AND V FOR Δ_{H4} AND Δ_{V4} IN INTERIOR BAYS OF ODD NUMBER OF BAYS					
Member (1)	C^a (2)	Member (3)	C (4)	Member (5)	C (6)	$h\ H_1$ (7)	$h\ H_3$ (8)	$l_3\ V_3$ (9)	$h\ H_2$ (10)	$h\ H_4$ (11)	$h\ H_6$ (12)	$l_2\ V_2$ (13)	$l_4\ V_4$ (14)	$l_6\ V_6$ (15)
(*a*) COEFFICIENTS FOR HORIZONTAL DEFLECTION														
Beam:		Beam:		Beam:										
$1_{L,R}$	C_1	$2_{L,R}$	C_2	$3_{L,R}$	C_3	2			−½	½		−½	−½	
$2_{L,R}$	C_2	$3_{L,R}$	C_3	$4_{L,R}$	C_4	2	−1	−1						
$3_{L,R}$	C_3	$4_{L,R}$	C_4	$5_{L,R}$	C_5					½	−½		½	½
Column:		Column:		Column:										
0–1	$C_{0,1}$	1–2	$C_{1,2}$	2–3	$C_{2,3}$	1			−¼	¼		¼	−¼	
1–2	$C_{1,2}$	2–8	$C_{2,3}$	3–4	$C_{3,4}$				−¼	¼		−¼	¼	
2–3	$C_{2,3}$	3–4	$C_{3,4}$	4–5	$C_{4,5}$	1	−½	−½		¼	−¼		−¼	¼
3–4	$C_{3,4}$	4–5	$C_{4,5}$	5–6	$C_{5,6}$					¼	−¼		¼	−¼
(*b*) COEFFICIENTS FOR VERTICAL DEFLECTION														
Beam:		Beam:		Beam:										
$1_{L,R}$	C'_1	$2_{L,R}$	C'_2	$3_{L,R}$	C'_3				½	−½		½	½	
$2_{L,R}$	C'_2	$3_{L,R}$	C'_3	$4_{L,R}$	C'_4								2	
$3_{L,R}$	C'_3	$4_{L,R}$	C'_4	$5_{L,R}$	C'_5					½	−½		½	½
Column:		Column:		Column:										
0–1	$C'_{0,1}$	1–2	$C'_{1,2}$	2–3	$C'_{2,3}$				¼	−¼		−¼	¼	
1–2	$C'_{1,2}$	2–3	$C'_{2,3}$	3–4	$C'_{3,4}$				−¼	¼		−¼	¼	
2–3	$C'_{2,3}$	3–4	$C'_{3,4}$	4–5	$C'_{4,5}$					−¼	¼		¼	−¼
3–4	$C'_{3,4}$	4–5	$C'_{4,5}$	5–6	$C'_{5,6}$					¼	−¼		¼	−¼
(*c*) TOTALS[b]														
Δ_{H1}						6	−1½	½						
Δ_{H2}										2	−1		0	½
Δ_{H3} and Δ_{H4}									−1	2	−1	−½	0	½
Δ_{V2}										0	−½		4	0
Δ_{V3} and Δ_{V4}									½	0	−½	0	4	0

[a] $C = k\,h\,\frac{l}{I}$; $C' = k\,l_n\,\frac{l}{I}$. [b] Use for same-shaped members, equal spans, and $h = l$ only.

TABLE 11.—CONTRIBUTION OF MEMBERS TO DEFLECTION

Member (1)	Common dimensional multiplier (2)	Load multiplier k (3)	Δ_L (4)	Δ_R (5)	Load multiplier k (6)	Δ_L (7)	Δ_R (8)	Load multiplier k (9)	Δ_L (10)	Δ_R (11)
(*a*) LOAD COEFFICIENTS FOR ΔH FOR BEAMS AND COLUMNS,										
1			0	0		0	0		0	0
2_L	$h\,l_2/I_{2L}$	$(1-a)^2 l^2_1 w k$	$-\frac{1}{8}$	$+\frac{1}{8}$	$\left(1-\frac{b}{2}\right) b\,l^2_1 w k$	$-\frac{1}{4}$	$+\frac{1}{4}$	$\begin{cases}(1-a)^2 l^2_2 w k \\ l^2_2 w k_{w3}\end{cases}$	$+\frac{1}{8}$ $-\frac{1}{2}$	$-\frac{1}{8}$ $+\frac{1}{2}$
2_R	$h\,l/I_{2R}$	$(1-a)^2 l^2_1 w k$	$-\frac{1}{8}$	$+\frac{1}{8}$	$\left(1-\frac{b}{2}\right) b\,l^2_1 w k$	$-\frac{1}{4}$	$+\frac{1}{4}$	$(1-a)^2 l^2_2 w k$	$+\frac{1}{8}$	$-\frac{1}{8}$
3			0	0		0	0		0	0
1–2	$h^2/I_{1,2}$	$(1-a)^2 l^2_1 w k$	$+\frac{1}{8}$	$-\frac{1}{8}$	$\left(1-\frac{b}{2}\right) b\,l^2_1 w k$	$+\frac{1}{4}$	$-\frac{1}{4}$	$(1-a)^2 l^2_2 w k$	$-\frac{1}{8}$	$+\frac{1}{8}$
2–3	$h^2/I_{2,3}$	$(1-a)^2 l^2_1 w k$	$-\frac{1}{8}$	$+\frac{1}{8}$	$\left(1-\frac{b}{2}\right) b\,l^2_1 w k$	$-\frac{1}{4}$	$+\frac{1}{4}$	$(1-a)^2 l^2_2 w k$	$+\frac{1}{8}$	$-\frac{1}{8}$
(*b*) LOAD COEFFICIENTS FOR ΔV FOR BEAMS AND COLUMNS,										
1	l_1/I_1	$l^3_1 w k_{w3}$	-1	0	$l^3_1 w k_{w4}$	-1	0		0	0
2_L	$l_2/I_2\Delta$	$(1-a)^2 l^2_1 w k$	$-\frac{1}{8} l_1$	$-\frac{1}{8} l_3$	$\left(1-\frac{b}{2}\right) b\,l^2_1 w k$	$-\frac{1}{4} l_1$	$-\frac{1}{4} l_3$	$\begin{cases}(1-a)^2 l^2_2 w k \\ l^2_2 w k_{w3}\end{cases}$	$+\frac{1}{8} l_1$ $-\frac{1}{2} l_1$	$+\frac{1}{8} l_3$ $-\frac{1}{2} l_3$
2_R	l_2/I_{2R}	$(1-a)^2 l^2_1 w k$	$-\frac{1}{8} l_1$	$-\frac{1}{8} l_3$	$\left(1-\frac{b}{2}\right) b\,l^2_1 w k$	$-\frac{1}{4} l_1$	$-\frac{1}{4} l_3$	$(1-a)^2 l^2_2 w k$	$+\frac{1}{8} l_1$	$+\frac{1}{8} l_3$
3			0	0		0	0		0	0
1–2	$h/I_{1,2}$	$(1-a)^2 l^2_1 w k$	$-\frac{1}{8} l_1$	$+\frac{1}{8} l_3$	$\left(1-\frac{b}{2}\right) b\,l^2_1 w k$	$-\frac{1}{4} l_1$	$+\frac{1}{4} l_3$	$(1-a)^2 l^2_2 w k$	$+\frac{1}{8} l_1$	$-\frac{1}{8} l_3$
2–3	$h/I_{2,3}$	$(1-a)^2 l^2_1 w k$	$-\frac{1}{8} l_1$	$+\frac{1}{8} l_3$	$\left(1-\frac{b}{2}\right) b\,l^2_1 w k$	$-\frac{1}{4} l_1$	$+\frac{1}{4} l_3$	$(1-a)^2 l^2_2 w k$	$+\frac{1}{8} l_1$	$-\frac{1}{8} l_3$

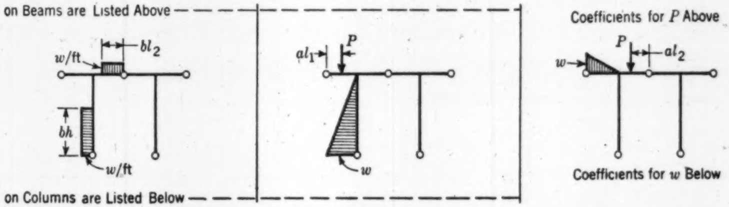

— Coefficients for Loads as Shown
ΔV_L
ΔH_L
ΔH_R
ΔV_R
Positive Δ's as Shown Above for Left and Right Ends
al_1
w/ft
P
e
ah
bl_1
w/ft
P
ah
w/ft
al_2
ah
w/ft
—Coefficients for Loads as Shown

(*c*) LOAD COEFFICIENTS FOR ΔH FOR BEAMS AND COLUMNS,										
1			0	0		0	0		0	0
2_L	$h\,l_2/I_{2L}$	$P_y k$	$+\frac{1}{4}$	$-\frac{1}{4}$	$P a h k$	$+\frac{1}{4}$	$-\frac{1}{4}$	$(1-a^2) w h^2 k$	$+\frac{1}{8}$	$-\frac{1}{8}$
2_R	$h\,l_2/I_{2R}$	$P_y k$	$+\frac{1}{4}$	$-\frac{1}{4}$	$P a h k$	$+\frac{1}{4}$	$-\frac{1}{4}$	$(1-a^2) w h^2 k$	$+\frac{1}{8}$	$-\frac{1}{8}$
3			0	0		0	0		0	0
1–2	$h^2/I_{1,2}$	$P_y k_m$	$\begin{cases}-\frac{1}{4} \\ +\frac{1}{2}\end{cases}$	$+\frac{1}{4}$ $-\frac{1}{2}$	$\left(1-\frac{a}{2}\right) P h k$ $P h k_p$	$+\frac{1}{2}$ $-\frac{1}{2}$	$-\frac{1}{2}$ $+\frac{1}{2}$	$\begin{cases}(1-a)(3-a) w h^2 k \\ w h^2 k_{w3}\end{cases}$	$+\frac{1}{8}$ $-\frac{1}{2}$	$-\frac{1}{8}$ $+\frac{1}{2}$
2–3	$h^2/I_{2,3}$	$P_y k$	$+\frac{1}{4}$	$-\frac{1}{4}$	$P a h k$	$+\frac{1}{4}$	$-\frac{1}{4}$	$(1-a^2) w h^2 k$	$+\frac{1}{8}$	$-\frac{1}{8}$
(*d*) LOAD COEFFICIENTS FOR ΔV FOR BEAMS AND COLUMNS,										
1	l_1/I_1		0	0		0	0		0	0
2_L	l_2/I_{2L}	$P_y k$	$+\frac{1}{4} l_1$	$+\frac{1}{4} l_3$	$P a h k$	$+\frac{1}{4} l_1$	$+\frac{1}{4} l_3$	$(1-a^2) w h^2 k$	$+\frac{1}{8} l_1$	$+\frac{1}{8} l_3$
2_R	l_2/I_{2R}	$P_y k$	$+\frac{1}{4} l_1$	$+\frac{1}{4} l_3$	$P a h k$	$+\frac{1}{4} l_1$	$+\frac{1}{4} l_3$	$(1-a^2) w h^2 k$	$+\frac{1}{8} l_1$	$+\frac{1}{8} l_3$
3			0	0		0	0		0	0
1–2	$h/I_{1,2}$	$\begin{cases}P_y k \\ P_y k_m\end{cases}$	$+\frac{1}{4} l_1$ $-\frac{1}{2} l_1$	$-\frac{1}{4} l_3$ $+\frac{1}{2} l_3$	$\left(1-\frac{a}{2}\right) P h k$ $P h k_p$	$-\frac{1}{2} l_1$ $+\frac{1}{2} l_1$	$+\frac{1}{2} l_3$ $-\frac{1}{2} l_3$	$(1-a)(3-a) w h^2 k$ $w h^2 k_{w3}$	$-\frac{1}{8} l_1$ $+\frac{1}{2} l_1$	$+\frac{1}{8} l_3$ $-\frac{1}{2} l_3$
2–3	$h/I_{2,3}$	$P_y k$	$+\frac{1}{4} l_1$	$-\frac{1}{4} l_3$	$P a h k$	$+\frac{1}{4} l_1$	$-\frac{1}{4} l_3$	$(1-a^2) w h^2 k$	$+\frac{1}{8} l_1$	$-\frac{1}{8} l_3$

OF A JOINT CAUSED BY APPLIED LOADS ON BEAMS AND COLUMNS

Load multiplier k (12)	Δ_L (13)	Δ_R (14)	Load multiplier k (15)	Δ_L (16)	Δ_R (17)	Load multiplier k (18)	Δ_L (19)	Δ_R (20)
with Loading on Beams Only as Shown Below								
	0	0		0	0		0	0
$\left(1-\frac{b}{2}\right) b\, l_2^2\, w\, k$	$+\frac{1}{4}$	$-\frac{1}{4}$	$(1-a)\, l_1\, P\, k$	$-\frac{1}{4}$	$+\frac{1}{4}$	$(1-a)\, l_2\, P\, k$	$+\frac{1}{4}$	$-\frac{1}{4}$
$l_2^2\, w\, k_{w4}$	$-\frac{1}{2}$	$+\frac{1}{2}$				$l_2\, P\, k_p$	$-\frac{1}{2}$	$+\frac{1}{2}$
$\left(1-\frac{b}{2}\right) b\, l_2^2\, w\, k$	$+\frac{1}{4}$	$-\frac{1}{4}$	$(1-a)\, l_2\, P\, k$	$-\frac{1}{4}$	$+\frac{1}{4}$	$(1-a)\, l_2\, P\, k$	$+\frac{1}{4}$	$-\frac{1}{4}$
	0	0		0	0		0	0
$\left(1-\frac{b}{2}\right) b\, l_2^2\, w\, k$	$-\frac{1}{4}$	$+\frac{1}{4}$	$(1-a)\, l_2\, P\, k$	$+\frac{1}{4}$	$-\frac{1}{4}$	$(1-a)\, l_2\, P\, k$	$-\frac{1}{4}$	$+\frac{1}{4}$
$\left(1-\frac{b}{2}\right) b\, l_2^2\, w\, k$	$+\frac{1}{4}$	$-\frac{1}{4}$	$(1-a)\, l_2\, P\, k$	$-\frac{1}{4}$	$+\frac{1}{4}$	$(1-a)\, l_2\, P\, k$	$+\frac{1}{4}$	$-\frac{1}{4}$
with Loading on Beams Only as Shown Below								
	0	0	$l_1^2\, P\, k_p$	-1	0		0	0
$\left(1-\frac{b}{2}\right) b\, l_2^2\, w\, k$	$+\frac{1}{4} l_1$	$+\frac{1}{4} l_3$	$(1-a)\, l_1\, P\, k$	$-\frac{1}{4} l_1$	$-\frac{1}{4} l_3$	$(1-a)\, l_2\, P\, k$	$+\frac{1}{4} l_1$	$+\frac{1}{4} l_3$
$l_2^2\, w\, k_{w4}$	$-\frac{1}{2} l_1$	$-\frac{1}{2} l_3$				$l_2\, P\, k_p$	$-\frac{1}{2} l_1$	$-\frac{1}{2} l_3$
$\left(1-\frac{b}{2}\right) b\, l_2^2\, w\, k$	$+\frac{1}{4} l_1$	$+\frac{1}{4} l_3$	$(1-a)\, l_1\, P\, k$	$-\frac{1}{4} l_1$	$-\frac{1}{4} l_3$	$(1-a)\, l_2\, P\, k$	$+\frac{1}{4} l_1$	$+\frac{1}{4} l_3$
	0	0		0	0		0	0
$\left(1-\frac{b}{2}\right) b\, l_2^2\, w\, k$	$+\frac{1}{4} l_1$	$-\frac{1}{4} l_3$	$(1-a)\, l_1\, P\, k$	$-\frac{1}{4} l_1$	$+\frac{1}{4} l_3$	$(1-a)\, l_2\, P\, k$	$+\frac{1}{4} l_1$	$-\frac{1}{4} l_3$
$\left(1-\frac{b}{2}\right) b\, l_2^2\, w\, k$	$+\frac{1}{4} l_1$	$-\frac{1}{4} l_3$	$(1-a)\, l_1\, P\, k$	$-\frac{1}{4} l_1$	$+\frac{1}{4} l_3$	$(1-a)\, l_2\, P\, k$	$+\frac{1}{4} l_1$	$-\frac{1}{4} l_3$

on Beams are Listed Above



on Columns are Listed Below

Coefficients for P Above

Coefficients for w Below

with Loading on Columns Only as Shown Above								
	0	0		0	0		0	0
$w\,(b\,h)^2\,k$	$+\frac{1}{8}$	$-\frac{1}{8}$	$w\,h^2\,k$	$+\frac{1}{24}$	$-\frac{1}{24}$	$w\,l_1^2\,k$	$-\frac{1}{12}$	$+\frac{1}{12}$
$w\,(b\,h)^2\,k$	$+\frac{1}{8}$	$-\frac{1}{8}$	$w\,h^2\,k$	$+\frac{1}{24}$	$-\frac{1}{24}$	$w\,l_1^2\,k$	$-\frac{1}{12}$	$+\frac{1}{12}$
	0	0		0	0		0	0
$(4-b)\,w\,b\,h^2\,k$	$+\frac{1}{8}$	$-\frac{1}{8}$	$w\,h^2\,k$	$+\frac{5}{24}$	$-\frac{5}{24}$	$w\,l_1^2\,k$	$+\frac{1}{12}$	$-\frac{1}{12}$
$w\,h^2\,k_{w4}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$w\,h^2\,k_{w2}$	$-\frac{1}{2}$	$+\frac{1}{2}$			
$w\,(b\,h)^2\,k$	$+\frac{1}{8}$	$-\frac{1}{8}$	$w\,h^2\,k$	$+\frac{1}{24}$	$-\frac{1}{24}$	$w\,l_1^2\,k$	$-\frac{1}{12}$	$+\frac{1}{12}$
with Loading on Columns Only as Shown Above								
	0	0		0	0	$w\,l_1^2\,k_{w2}$	-1	0
$w\,(b\,h)^2\,k$	$+\frac{1}{8} l_1$	$+\frac{1}{8} l_3$	$w\,h^2\,k$	$+\frac{1}{24} l_1$	$+\frac{1}{24} l_3$	$w\,l_1^2\,k$	$-\frac{1}{12} l_1$	$-\frac{1}{12} l_3$
$w\,(b\,h)^2\,k$	$+\frac{1}{8} l_1$	$+\frac{1}{8} l_3$	$w\,h^2\,k$	$+\frac{1}{24} l_1$	$+\frac{1}{24} l_3$	$w\,l_1^2\,k$	$-\frac{1}{12} l_1$	$-\frac{1}{12} l_3$
	0	0		0	0		0	0
$(4-b)\,w\,b\,h^2\,k$	$-\frac{1}{8} l_1$	$+\frac{1}{8} l_3$	$w\,h^2\,k_{w2}$	$-\frac{5}{24} l_1$	$+\frac{5}{24} l_3$	$w\,l_1^2\,k$	$-\frac{1}{12} l_1$	$+\frac{1}{12} l_3$
$w\,h^2\,k_{w4}$	$+\frac{1}{2} l_1$	$-\frac{1}{2} l_3$	$w\,h^2\,k$	$+\frac{1}{2} l_1$	$-\frac{1}{2} l_3$			
$w\,(b\,h)^2\,k$	$+\frac{1}{8} l_1$	$-\frac{1}{8} l_3$	$w\,h^2\,k$	$+\frac{1}{24} l_1$	$-\frac{1}{24} l_3$	$w\,l_1^2\,k$	$-\frac{1}{12} l_1$	$+\frac{1}{12} l_3$

hood of exceeding the elastic properties to a detrimental degree in either case. Obviously, such an extension would necessitate a careful development of procedure.

A significant contribution has been made by the author in presenting a practical procedure for the proposed method of analysis. However, for those situations in which it would be desirable to vary the dimensions of the members or for occasions when symmetry of load within a bay does not occur, the substitution in the equations for the *k*-value (a relatively minor operation) is likely to deter many from taking full advantage of such framing. Although it is true that conditions of load symmetry and equal amounts of taper for the members remove the need to compute values of *k*, it is not likely that such a condition can always be used. The equations of Tables 1 through 5 may be made more useful by placing them in chart form, such as the example presented in Fig. 17 prepared for a section of Table 1. Such charts could be readily plotted for the remaining parts of Table 1 and for Tables 2 through 5 and would markedly increase the value of the author's proposed method.

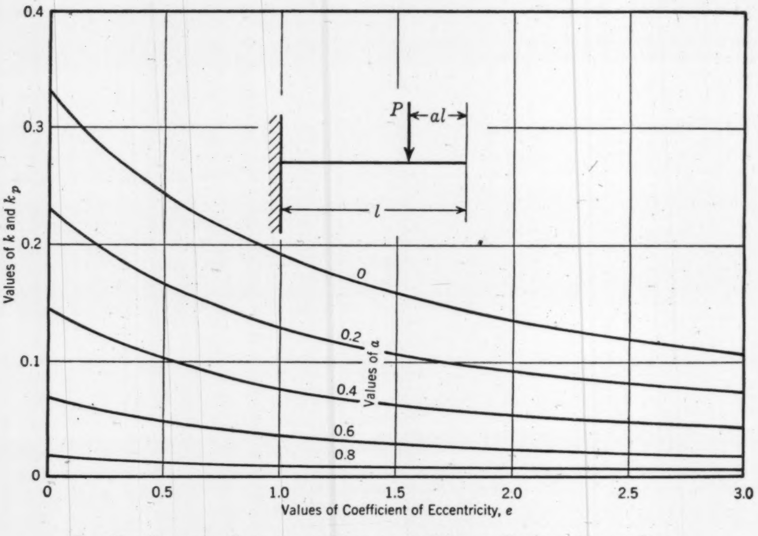


FIG. 17.—GRAPH FOR SOLUTION OF DEFLECTION FORMULA TAPERED FLANGE BEAM

The writers feel that a valuable further simplification of the tables of coefficients for the contributions of members to the deflection of a joint may be developed. The coefficients for H and V for values of ΔH and ΔV do not change with variations in applied loading, and the coefficients shown in Table 7 may be used for the end bays of frames with an odd number of bays. Therefore, it is possible to combine parts of Tables 7, 8, and 9 into a single table as shown in Table 10.

It is also evident that the load coefficients shown in Tables 7, 8, and 9 are applicable only to the type of loading shown and applicable only if symmetry of loading within a bay is maintained, although all bays need not be loaded. The writers are of the opinion that data in the form of Table 11 are more useful and, of course, cover a broader range of loading conditions. An arrangement similar to Table 11 could also be developed for the end bays of frames with an even number of bays. When used in conjunction with charts, such as those proposed herein for Tables 1 through 5, Tables 10 and 11 will provide an effective design tool for the author's proposed method of analysis.

TABLE 12.—EXACT METHOD OF SOLUTION FOR THE SIMULTANEOUS EQUATIONS FROM THE FRAME IN FIG. 13(*a*)

Equation No.	Joint No.	H_2	V_2	H_4	V_4	H_6	V_6	Constant in terms of *wl*
(*a*) TABULATION OF COEFFICIENTS OF ORIGINAL EQS. 22								
1	2	+2		−1	+0.5			0
2			+4	−0.5				−0.5
3	4	−1	−0.5	+2		−1	+0.5	0
4		+0.5			+4	−0.5		+0.5
5	6			−1	−0.5	+2		0
6				+0.5			+4	0
(*b*) EXACT SOLUTION BY SUCCESSIVE ELIMINATION OF UNKNOWNS								
1′		+1.0		−0.5	+0.25			0
4′		+1.0			+8.00	−1.0		+1.0
3′		−1.0	−0.5	+2.0		−1.0	+0.5	0
7				+0.5	+7.75	−1.0		+1.0
8			−0.5	+1.5	+0.25	−1.0	+0.5	0
5′				−0.5	−0.25	+1.0		0
9			−0.5	+1.0	−7.50		+0.5	−1.0
10					+7.50			+1.0
6′				+1.0			+8.0	0
9′			−0.5	+1.0			+0.5	0
2′			+8.0	−1.0				−1.0
11			+8.0				+8.0	−1.0
12			+7.5				+0.5	−1.0
11′			+1.0				+1.0	−0.125
12′			+15.0				+1.0	−2.000
13			+14.0					−1.875
Final Values *H* and *V*		−0.0690	−0.1339	−0.0714	0.1333	−0.0024	0.0089	

It is not clear to the writers why the method of summation of successive increments is proposed for the solution of the simultaneous equations. The equations take the form of secondary stress equations and in some respects might well be considered as such. The method of successive approximations, frequently used in the solution of secondary stress equations, could be adapted to the wedge-beam framing equations more advantageously than the summation of successive increments. Comparison of the results obtained by an exact solution of Eqs. 22 by successive elimination of unknowns with those obtained by successive approximations, as well as the author's results, are shown in Tables 12 and 13. The method of successive approximations removes the need of adding increments to obtain the final answer and will converge more rapidly on the true values for the equations shown. Furthermore, convergence is most rapid if substitution is made as soon as approximate values are obtained, as may be observed at the bottom of Table 13.

The writers feel that the author's proposed method of tabular coefficients is of greatest advantage when it is necessary (as is usually the case) to carry through solutions for a given frame for different loadings. Maximum stresses resulting from various combinations of loads—such as wind load, crane load, conveyor system load, and roof load—may be readily obtained after the *H*- and *V*-values are known. Considerable saving may be noted in the fact that the coefficients for *H* and *V* do not change for the various loadings, and the equations may be solved in the same routine for all constants. The charts and Tables 10 and 11 proved distinctly valuable in setting up the equations for such an analysis.

TABLE 13.—SOLUTION BY SUCCESSIVE APPROXIMATIONS

Quantity	Approximations:						Actual values	Author's values
	First	Second	Third	Fourth	Fifth	Sixth		
(*a*) ASSUMING EQUAL VALUES FOR THE UNKNOWNS IN DETERMINING THE FIRST APPROXIMATION								
H_2	0	−0.0313	−0.0492	−0.0489	−0.0590	−0.0627	−0.0690	−0.0577
V_2	−0.1429	−0.1250	−0.1295	−0.1289	−0.1314	−0.1324	−0.1339	−0.1311
H_4	0	−0.0358	−0.0313	−0.0516	−0.0489	−0.0588	−0.0714	−0.0587
V_4	+0.1250	+0.1250	+0.1328	+0.1328	+0.1333	+0.1333	+0.1333	+0.1333
H_6	0	+0.0313	+0.0133	+0.0176	+0.0074	+0.0040	−0.0024	+0.0088
V_6	0	0	+0.0045	+0.0039	+0.0065	+0.0076	+0.0089	+0.0061
(*b*) ZERO ASSUMED FOR ALL, EXCEPTING THE PRINCIPAL UNKNOWN, UNTIL VALUES ARE OBTAINED FOR IMMEDIATE SUBSTITUTION IN SUBSEQUENT OPERATIONS								
H_2	0	−0.0469	−0.0577	−0.0627	−0.0655	−0.0671	−0.0690	−0.0577
V_2	−0.1250	−0.1289	−0.1311	−0.1323	−0.1331	−0.1335	−0.1339	−0.1311
H_4	−0.0313	−0.0489	−0.0588	−0.0644	−0.0675	−0.0693	−0.0714	−0.0587
V_4	0.1250	−0.1328	0.1333	0.1333	0.1333	0.1333	0.1333	0.1333
H_6	0.0156	0.0087	0.0039	0.0011	−0.0004	−0.0013	−0.0024	0.0089
V_6	0.0039	0.0061	0.0074	0.0081	0.0084	0.0087	0.0089	0.0061

Solution by successive elimination of unknowns is advocated, although successive approximations would be advantageous in the case of a large number of bays.

The author is to be congratualted upon his approach to the problem, and it is hoped that further simplifications will be forthcoming.

HERBERT A. SAWYER, JR.,[7] A. M. ASCE.—The primary reason advanced (in the paragraph headed "Range of Application") for the introduction and use of wedge-beam framing is economy in material and construction costs.

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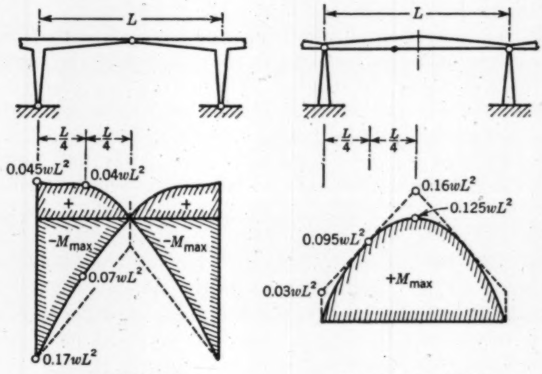
Engineering practice has long demonstrated that, for given loads and spans, a beam with varying strength along its length will weigh less than a beam of constant strength; and, if the added fabrication cost and change in erection cost of this variable strength beam are not greater than its material savings it is certainly more economical.

Another related, yet more elusive, source of economy of material is to change the effective span lengths of beams by a change in articulation or type of support, in effect changing the moment diagrams. Possibilities for change in articulation range from the classic post-and-lintel construction, with hinges over the columns, to the only slightly less classic cantilever construction, with hinges at midspan. A compromise between these extremes is the more modern continuous construction, with "hinges" or points of contraflexure usually within the second and fifth sixths of the span length. This paper considers only cantilever construction because (as stated in the "Introduction") the arrangement of two simple cantilevers "* * * yields appreciable savings in weight * * *"



(a) CANTILEVER (b) POST AND LINTEL

FIG. 18.—COMPARISON OF TAPERED BEAM REQUIREMENTS OF FRAMING

The writer realizes it is dangerous to attempt to generalize on the relative economy of the various types of articulation on the basis of a single analysis, because no single analysis can account for all the many types of loads, materials, and stresses. However, one analysis offers some evidence, so in Fig. 18 an attempt has been made to compare the beam requirements for post-and-lintel and for cantilever construction. The curves of maximum moment are for a moving uniform gravity load equal to *w* per unit length, on a frame of many bays, and the half-span cantilever beam is assumed to have the same stiffness as the column. As can be seen, the curves of maximum moment have been enveloped by a dotted line representing the maximum resisting moments of a tapered beam, showing graphically the minimum strength beam required to resist the maximum moments. The resisting moment of this beam was assumed to vary linearly from a maximum down to a value of one fifth the maximum, corresponding quite closely to the type of strength variation of beams of the author's Tables 1, 3, and 5. Dead load moments have been neglected because they would have about the same effect on both types, although actually the small component of dead load moment from the weight of the beam itself would be somewhat less for the cantilever type since its weight is concentrated nearer the supports.

According to this analysis, cantilever construction requires a slightly heavier beam than post-and-lintel construction. However, other important factors, not apparent from Fig. 18, should be considered. The maximum shear occurs at the deepest section of the cantilever beam and at the shallowest section of the lintel beam. Thus, extra material could well be required to resist shear in the lintel beam, even though an important part of the shear is resisted by the nonparallel flanges. On the other hand, large gravity-load moments occur in the columns of the cantilever frame but not in the columns of the post-and-lintel frame. Also, unlike the lintel beam moments, the cantilever beam moments will be reversible (unless the dead load is improbably high), resulting in extra tensile steel in reinforced concrete.

A practical consideration affecting construction costs is that, in post-and-lintel construction, field joints can be at the hinges and the resulting pieces are convenient in shipping and erection. However, for cantilever construction the engineer is faced with the choice of shipping awkward T-segments or having a field splice near the point of maximum moment and shear.

A consideration of engineering effort (unfortunately not very important economically) would certainly favor the statically determinate post-and-lintel analysis over the statically indeterminate cantilever analysis described in the paper.

The conclusion of this analysis is that for the conditions investigated there is probably no significant difference in the amount of material required for post-and-lintel framing and cantilever framing using tapered beams. Therefore, the cantilever framing as described seems to have no important point of superiority over the more traditional type.

Possibly, the compromise between the two types here investigated—continuous construction—could be shown to be the most economical materially. The differences, however, are surely not great, and such an attempt is beyond the scope of this discussion.

If an analysis of cantilevered framing is required, it is interesting to note that the deflection formulas presented in Tables 1 to 5 may be used to solve any of the illustrative examples by the method of moment distribution devised by Hardy Cross,[8] Hon. M. ASCE. If moment distribution is used, the mathematics and simultaneous equations of the latter part of the paper are avoided.

To use moment distribution, the carry-over factors, fixed-end moments, and relative stiffnesses for all spans of a wedge-beam frame may be found using Tables 1 to 5 and Eqs. 28, 30, and 31 which follow. Moments may then be distributed, as described by Mr. Cross, to obtain the final moments.

---

[8] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, ASCE, Vol. 96, 1932, p. 1.

Considering the typical wedge-beam span of Fig. 19 the carry-over factor from point C to point D (R_{CD}) is statically determinate

$$R_{CD} = \frac{M_D}{M_C} = \frac{l_D}{l_C} \qquad \text{(28)}$$

in which M_C is the moment at point C accompanying a rotation of support C, and M_D is the moment induced at point D from this rotation.

The fixed-end moment at point C from any loads on span CD (or M_{FC}) is equal to the moment at point C considering the member CA as a simple cantilever (M_{SC}) plus the moment at C from V_A, the shear at hinge A

$$M_{FC} = M_{SC} + l_C\, V_A \qquad \text{(29)}$$

The shear at point A may be found by the common method of removing the connecting hinge at point A and dividing the resulting difference in deflection of the two simple cantilevers at point A by the difference in deflection of the two cantilevers loaded only with opposite unit loads acting at point A. Replacing the shear by the quotient of these deflections, as obtained from Tables 1 to 5 and assuming that the values of $E\,I$ near the hinge on both sides are equal (as shown by Fig. 5), Eq. 29 becomes

$$M_{FC} = M_{SC} + \frac{m_C - m_D\, R^2{}_{CD}}{(k)_C + (k)_D\, R^3{}_{CD}} \qquad \text{(30)}$$

in which k is the k (without subscript) of Tables 1 to 5, and m is equal to $k_p\, P\, l$, $k_m\, M$, or $k_w\, w\, l^2$, all as defined by Tables 1 to 5. For sign convention, both terms in the denominator are always positive; m is positive for downward deflection at point A of the simple cantilever, and M_{SC} and the resulting M_{FC} are both negative for a moment that produces tension in the upper fiber.

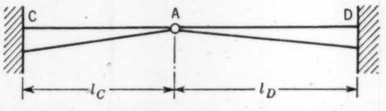


FIG. 19.—TYPICAL WEDGE-BEAM SPAN

The relative stiffness of member CD for rotation at point C (K_{CD}) or the moment at point C for a unit rotation at this point may easily be shown to be

$$K_{CD} = \frac{E\,I}{l_C\,[(k)_C + (k)_D\, R^3{}_{CD}]} \qquad \text{(31)}$$

in which I is as defined in Tables 1 to 5.

For the column members of wedge-beam framing, l_D equals zero, and R_{CD} becomes zero in all the foregoing equations.

The author's illustrative examples involve spans symmetrically proportioned and symmetrically loaded about the hinges. The carry-over factors, fixed-end moments, and relative stiffnesses would be 1, M_{SC}, and $E\,I/(k\,L)$, respectively, for these spans, and 0, 0 and $E\,I/(k\,l)$, respectively, for the columns.

The analysis of two-dimensional frames in this paper neglects the restraints of the roofing or flooring and adjacent framing in the third dimension in preventing sidesway. Moment distribution, on the other hand, assumes complete restraint from sidesway. This condition is nearer the actual action of framing in a building of many bays. For a building of a very few bays the effect of sidesway should usually be checked. Using moment distribution, this correction need only be made for one loading for a given frame, the correction for any other loading being found by direct proportion from the first determination.

Solutions for types of framing with beams whose axes depart significantly from the horizontal, such as those of Fig. 4, were not included in the paper. For these types, the sidesway of each joint should usually be accounted for in the analysis, as is done by the author's method of simultaneous equations. Using moment distribution, shears as well as moments must be distributed, as described in the paper by Mr. Cross and in the discussions of the paper. Tables 1 to 5 are helpful in determining the constants required by either method, for the frame types of Fig. 4(*a*) and Fig. 4(*d*) only. Constants must be derived for other frame types, such as those of Fig. 4(*b*) and Fig. 4(*c*).

J. J. Polivka,[9] M. ASCE.—When the great economy of precast concrete structures is finally recognized, this paper becomes especially valuable. The serious drawback of tedious analysis is considerably alleviated by the author's simplified solution, although the computation of the values of the factor (*k*) in Tables 1 to 5 requires considerable time and great accuracy. For this reason these factors for various shapes of tapered beams and various ratios of the moments of inertia at both ends of the beams can be tabulated. A simpler method of computing elastic deformations of any cross section and shape of a beam can be demonstrated[10] by reference to the special case (*a*) the three-bay frame with symmetrical framing and loading, shown in Fig. 9. The moment of the exterior corner (X) is considered to be redundant (Fig. 20). Equaling the deflections of the columns, the following equation determines X:

$$\Delta_L = G_3\, s_3\, d_3\, \frac{X}{h} + G_1\, s_1\, d_1\, \frac{X}{l_1} = \Delta_R = -\, G_1\, s_1\, d_1\, \frac{X}{l_1} + \frac{(M - X)}{h}\, G_4\, s_4\, d_4 \,.\,.\,(32)$$

and

$$X = \frac{M\, G_4\, s_4\, d_4}{G_3\, s_3\, d_3 + \dfrac{2\,h}{l_1}\, G_1\, s_1\, d_1 + G_4\, s_4\, d_4} \ldots\ldots\ldots\ldots\ldots (33)$$

in which M is the cantilever moment of the center bay ($M = 0.5\, w\, l^2$); G is the elastic weight of the individual tapered members with elastic centroids O; and fixed points D, s, and d are the distances from the hinge points, respectively. No matter what cross section, shape, or irregularities the structural members have, the values G, s, and d can be accurately and reliably found as shown in Fig. 21 and outlined in the following sections.

Determination of the Elastic Weight G.—Using the author's notations, $I_A = I$, and reducing the uniform moment area $M = 1$, constant along the length of the tapered beam, by the ratio I_x/I (the ordinates at 4 or 5 points will be sufficient to draw the reduced moment area), the centroidal axis of the reduced moment area determines point O, and the reduced moment area represents G.

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[9] Cons. Engr., Berkeley, Calif.; Lecturer, Stanford University, Stanford, Calif.

[10] Discussion by J. J. Polivka of "An Investigation of Steel Rigid Frames," by Inge Lyse and W. E. Black, *Transactions*, ASCE, Vol. 107, 1942, p. 176.

Determination of Fixed Point D.—Proceed in the same manner as for the determination of G using, however, a triangular moment area with maximum ordinate = 1 as a basis. The reduced area represents the elastic rotation of the beam at the hinge due to the force producing $M = 1$ at the other end of the beam.

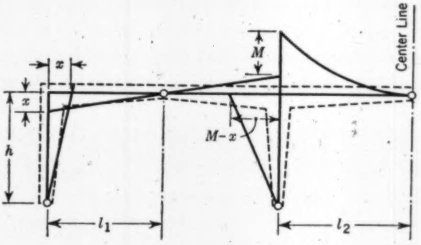

FIG. 20.—THREE-BAY FRAME WITH SYMMETRICAL FRAMING AND LOADING

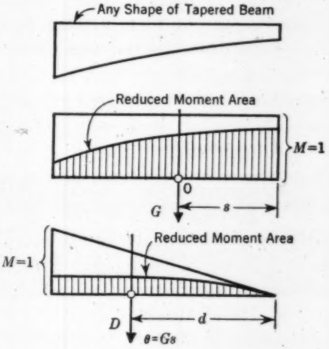

FIG. 21.—ANALYSIS OF ODD-SHAPED BEAM

The writer has demonstrated that continuous rigid frames involving any number of redundants can be solved directly without equations. Similar methods can be developed for frameworks consisting of tapered, hinged members with sufficient accuracy for economical design.

ARSHAM AMIRIKIAN,[11] M. ASCE.—The supplementary data and some of the tabulations submitted by Messrs. Ernst, Hromadik, and Swihart will unquestionably be helpful in the applications of the method. The presentation of typical hinge details in the paper was made merely as a suggestion, leaving the selection or the development of differing details to the designer's choice. Undoubtedly, there are a number of other types, including the Considère hinge, which can be used advantageously in various framing arrangements. However, in the case of hinges of that type, the desirability of the suggested utilization of the limited moment capacity is open to question because of the resulting complications in the analysis. In design practice, the omission of aid provided by such moments may be considered as an added measure of safety.

The writer fully realized that the derived mathematical expressions for k, given in Tables 1 to 5, would not be sufficient for a practical use of the method. With this thought, he also compiled extensive numerical tables and plotted curves similar to those shown in Fig. 17, covering a great number of load conditions. Unfortunately, imposed space limitations necessitated their omission from the text. However, it is hoped, perhaps through another medium, an opportunity may be had for their eventual presentation in a future elaborated version of the paper.

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[11] Chf. Designing Engr., Bureau of Yards and Docks, U. S. Dept. of the Navy, Washington, D. C.

The advantage in brevity obtained by the suggested combination of Tables 7, 8, and 9 into a single form, Table 10, must be weighed against the resulting loss of clarity of presentation of the various coefficients and the greater care needed in the use of the table. On the other hand, Table 11 provides considerable additional data pertaining to some of the unusual conditions of loading not completely treated in the paper.

Regarding the relative merit of the various methods of solving the simultaneous equations which result in the analysis, the writer's preference for the so-called "successive increments" method, which he has developed and used advantageously in a wide variety of design work, is predicated on the following practical considerations:

1. Compared to classical methods, it is a much simpler procedure.
2. It does not need the use of computing machines to assure a reasonable degree of accuracy, the use of an ordinary slide rule being amply adequate.
3. It provides a much greater operational flexibility, since the number of cycles or increments may vary in accordance with the desired accuracy.
4. Its accuracy is less sensitive to possible operational errors, since each successive increment is only a part of the total value, and thus an error in a given cycle would be only feebly reflected in the succeeding increments.

Mr. Sawyer has made a limited study in his attempt to appraise the economic merit of wedge-beam framing. For this purpose, a partial comparison is made with a so-called "post and lintel" framing. From the point of view of shop fabrication, it is interesting to note that the elements of framing used in the latter system do not appreciably differ from those of wedge-beam framing, since they consist of tapered sections. However, in arrangement, because of reversal of the directions of taper, two entirely dissimilar patterns of stress are obtained. Thus, the points of maximum strength in the "post and lintel" framing are at the middle of the lintels or beams and at the bottom of the posts. Obviously, such a system will be unstable under lateral loading when used in a framing more than one story high. Even in the case of a single-story framing, the lateral resistance will be solely a function of the moment capacity at the base or foundation.

As in the case of all comparisons, conclusions drawn from partial studies are of little value. Reliable comparative data can be obtained only from complete designs of actual framing. The writer had a number of opportunities of making such investigations in connection with the desired choice of a framing. In all these studies, the total material required by the wedge-beam system was found to be appreciably less than that obtained either for simple or rigid framing, the extent of material savings varying from 20% to as much as 40%.

Mr. Sawyer's suggested use of the moment distribution method in the analysis raises an interesting question for discussion. In the minds of most engineers, this method is associated with a marvelous magic without which no procedure or solution is deemed acceptable or practicable.

In the analysis of any statically indeterminate system of framing there are a minimum number of unknowns, usually consisting of moments and reactions, which must be solved for determining the magnitude of stresses in the component members. For this purpose, it is necessary to set an equal number of relations between the unknowns and solve for their values. Regardless of what method of analysis is used, this is the work that must be accomplished—directly or indirectly. In the case of moment distribution (which, in reality, is an application of slope deflection) the required equations, consisting of the summations of moments at each joint, are set indirectly,[12] and the values of the unknowns are obtained by an arithmetical routine of solution similar to that outlined in Fig. 15. Except for the simple cases of frame analysis, such as given in the original presentation of the method,[8] the claimed advantage of operation is open to serious question. This is particularly true when the factors of interrelation (distribution and carry-over factors) cannot be readily formulated. The proposed application of the method to wedge-beam framing falls within this category. With the very simple procedure outlined in the paper, available for obtaining the equations for the redundant reactions, the writer would not choose to follow a devious procedure for attaining the same objective. A comparative solution of moments for any one of the frames used as illustrative examples in the paper by the suggested method will suffice to demonstrate this point.

Mr. Polivka suggests a semigraphical solution for frames composed of members of differing shape. In such cases, assuming also that the numerical values of k are not available or readily obtainable, a better procedure would be to utilize the well-known and simpler concept of moment area, as expressed by Eq. 17. For this purpose it will suffice to draw only one moment diagram for each member, reduce the moment ordinates by the ratios of I_x/I, obtain the moment area thus reduced, and determine its centroid. If the moments are induced by end forces only—that is, hinge reactions, which is the case of members 1, 2, 3, and 4 in Fig. 20—the lower diagram in Fig. 21, containing the values of D and d, is all that will be needed for the desired deflections and, hence, the middle or G-diagram becomes superfluous. This will be evident also from the consideration that

$$G\,s = p\,\frac{l^2}{E\,I} = D \qquad (34)$$

in which p is the reduced moment area of a cantilever of unit length under a unit load at its end.

Mr. Polivka's statement regarding the solution of redundants without equations is not clear. Assumedly this pertains to an unexplained procedure where the needed relations are formulated indirectly. However, this has not been done in his example, since Eq. 32 is the needed relation for the single redundant of the frame, which equation is derived and solved in a manner similar to that given in the paper.

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[12] "Analysis of Rigid Frames," by A. Amirikian, U. S. Govt. Printing Office, Washington, D. C., 1942, p. 83.

Gable Framing.—In the case of framing arrangements where the axes of the members deviate from a horizontal or vertical (mentioned by Mr. Sawyer), it becomes necessary to consider the effect of both vertical and horizontal reactions at the joints. As a result, the deflection equations will contain additional terms from some of these members. Except for this modification, the analysis follows the same procedure as that given for rectangular frames. The application can be illustrated by two examples.

Example 1. Symmetrical Framing.—The three-bay gable bent shown in Fig. 22(*a*) represents a simple case of symmetrical framing. Under the indicated unsymmetrical loading there are two unknown hinge reactions at joint D. The two deflection equations necessary for the solution of *H* and *V* are obtained as follows:

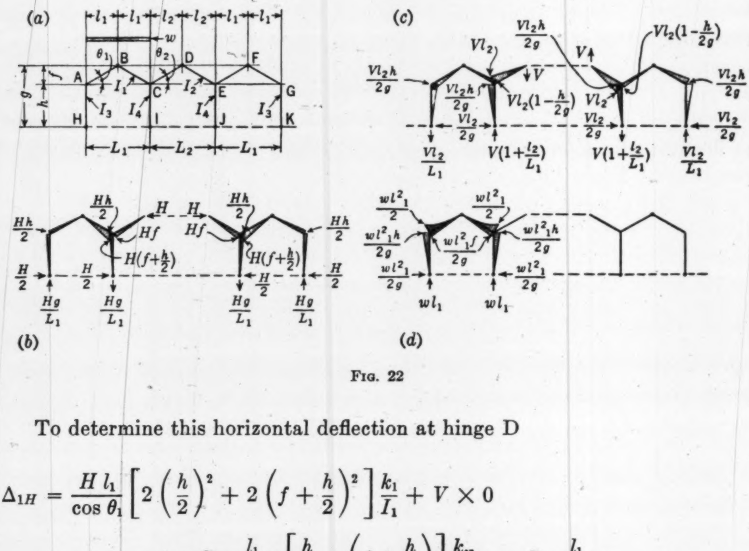

(a)
l_1
l_1
l_2
l_2
l_1
l_1
w
θ_1
B
θ_2
D
F
A
f
g
h
I_1
C
I_2
E
G
I_3
I_4
I_4
I_3
H
I
J
K
L_1
L_2
L_1
(c)
$\frac{Vl_2h}{2g}$
$Vl_2(1-\frac{h}{2g})$
Vl_2
V
V
$\frac{Vl_2h}{2g}$
$\frac{Vl_2h}{2g}$
$\frac{Vl_2h}{2g}$
$Vl_2(1-\frac{h}{2g})$
Vl_2
$\frac{Vl_2}{2g}$
$\frac{Vl_2}{2g}$
$\frac{Vl_2}{2g}$
$\frac{Vl_2}{L_1}$
$V(1+\frac{l_2}{L_1})$
$V(1+\frac{l_2}{L_1})$
$\frac{Vl_2}{L_1}$
$\frac{Hh}{2}$
H
H
$\frac{Hh}{2}$
$\frac{Hh}{2}$
Hf
Hf
$\frac{Hh}{2}$
$H(f+\frac{h}{2})$
$H(f+\frac{h}{2})$
$\frac{H}{2}$
$\frac{H}{2}$
$\frac{H}{2}$
$\frac{H}{2}$
$\frac{Hg}{L_1}$
$\frac{Hg}{L_1}$
$\frac{Hg}{L_1}$
$\frac{Hg}{L_1}$
(b)
$\frac{wl^2_1}{2}$
$\frac{wl^2_1}{2}$
$\frac{wl^2_1h}{2g}$
$\frac{wl^2_1f}{2g}$
$\frac{wl^2_1h}{2g}$
$\frac{wl^2_1}{2g}$
$\frac{wl^2_1}{2g}$
wl_1
wl_1
(d)

FIG. 22

To determine this horizontal deflection at hinge D

$$\Delta_{1H} = \frac{H\,l_1}{\cos\theta_1}\left[2\left(\frac{h}{2}\right)^2 + 2\left(f+\frac{h}{2}\right)^2\right]\frac{k_1}{I_1} + V\times 0$$

$$+\, w\,l^2{}_1\,\frac{l_1}{\cos\theta_1}\left[\frac{h}{2} - \left(f+\frac{h}{2}\right)\right]\frac{k_w}{I_1} - w\,l^2{}_1\,\frac{l_1}{\cos\theta}$$

$$\times\,\frac{f}{2\,g}\left[\frac{h}{2} - \left(f+\frac{h}{2}\right)\right]\frac{k_1}{I_1}\,.\,.\,(35a)$$

$$\Delta_{2H} = 2\,H\,\frac{l_2}{\cos\theta_2}\times f^2\times\frac{k_2}{I_2} + V\times 0 + w\times 0\,.\,.\,.\,.\,.\,.\,.\,.\,(35b)$$

$$\Delta_{3H} = 2\,H\,h\left(\frac{h}{2}\right)^2\frac{k_3}{I_3} + V\times 0 + w\,l^2{}_1\,\frac{h^3}{4\,g}\times\frac{k_3}{I_3}\,.\,.\,.\,.\,.\,.\,.\,.\,(35c)$$

and

$$\Delta_{4H} = 2\,H\,h\left(\frac{h}{2}\right)^2\frac{k_4}{I_4} + V \times 0 - w\,l^2_1\,\frac{h^3}{4\,g} \times \frac{k_4}{I_4}\,\ldots\ldots\ldots(35d)$$

also

$$\Delta_{1H} + \Delta_{2H} + \Delta_{3H} + \Delta_{4H} = \Delta_H = 0\ldots\ldots\ldots\ldots\ldots(36)$$

from which

$$H = \frac{w\,l^2_1\left[\dfrac{l_1}{\cos\theta_1} \times \dfrac{f}{I_1}\left(k_w - \dfrac{f\,k_1}{2\,g}\right) + \dfrac{h^3}{4\,g}\left(\dfrac{k_4}{I_4} - \dfrac{k_3}{I_3}\right)\right]}{\dfrac{l_1}{\cos\theta_1}\,(f^2 + g^2)\,\dfrac{k_1}{I_1} + \dfrac{2\,l_2\,f^2}{\cos\theta_2}\,\dfrac{k_2}{I_2} + \dfrac{h^3}{2}\left(\dfrac{k_3}{I_3} + \dfrac{k_4}{I_4}\right)}\ldots\ldots(37)$$

To determine the vertical deflection at hinge D

$$\begin{aligned}\Delta_{1V} = V\,\frac{l_1}{\cos\theta_1}&\left[2\left(\frac{l_2\,h}{2\,g}\right)^2 + 2\,l^2_2\left(1 - \frac{h}{2\,g}\right)^2\right]\frac{k_1}{I_1} + H \times 0\\ &+ w\,l^2_1\,\frac{l_1}{\cos\theta_1}\left[\frac{l_2\,h}{2\,g} - l_2\left(1 - \frac{h}{2\,g}\right)\right]\frac{k_w}{I_1} + w\,l^2_1\,\frac{l_1}{\cos\theta_1}\\ &\times \frac{f}{2\,g}\left[\frac{l_2\,h}{2\,g} - l_2\left(1 - \frac{h}{2\,g}\right)\right]\frac{k_1}{I_1}\ldots(38a)\end{aligned}$$

$$\Delta_{2V} = 2\,l^2_2\,\frac{l_2}{\cos\theta_2} \times \frac{k_2}{I_2} + H \times 0 + w \times 0\ldots\ldots\ldots\ldots(38b)$$

$$\Delta_{3V} = 2\,V\,h\left(\frac{l_2\,h}{2\,g}\right)^2 \times \frac{k_3}{I_3} + H \times 0 - w\,l^2_1\,\frac{l_2\,h^3}{4\,g^2} \times \frac{k_3}{I_3}\ldots\ldots(38c)$$

$$\Delta_{4V} = 2\,V\,h\left(\frac{l_2\,h}{2\,g}\right)^2 \times \frac{k_4}{I_4} + H \times 0 - w\,l^2_1\,\frac{l_2\,h^3}{4\,g^2} \times \frac{k_4}{I_4}\ldots\ldots(38d)$$

also, similar to Eq. 36,

$$\Delta_{1V} + \Delta_{2V} + \Delta_{3V} + \Delta_{4V} = \Delta_V = 0\ldots\ldots\ldots\ldots\ldots(39)$$

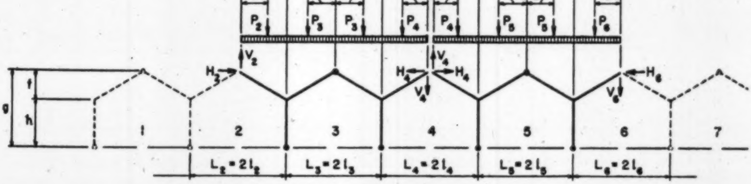


FIG. 23

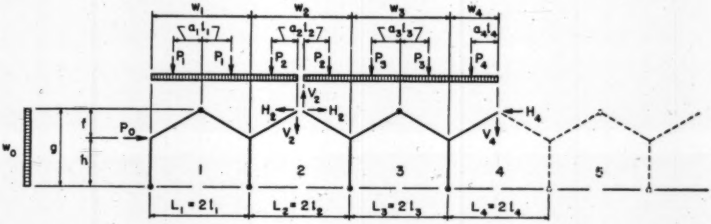
TABLE 14.—CONTRIBUTION OF MEMBERS TO THE DEFLECTION OF A JOINT (JOINT 4, FIG. 23) IN INTERIOR BAYS OF GABLE FRAMES

HORIZONTAL DEFLECTION AT JOINT 4																	
MEMBER	$C = k\frac{l}{I}h$	H-V COEFFICIENTS FOR Δ_{H4}						LOAD COEFFICIENTS FOR Δ_{H4}									
		hH_2	hH_4	hH_6	l_2V_2	l_4V_4	l_6V_6	$(1-a_2)l_2P_2$	$(1-a_3)l_3P_3$	$(1-a_4)l_4P_4$	$(1-a_5)l_5P_5$	$(1-a_6)l_6P_6$	$w_2l_2^2$	$w_3l_3^2$	$w_4l_4^2$	$w_5l_5^2$	$w_6l_6^2$
BEAM $3_{L,R}$	C_3	$\frac{f^2-g^2}{2h^2}$	$\frac{f^2+g^2}{2h^2}$		$\frac{f^2-g^2}{2gh}$	$-\frac{f^2+g^2}{2gh}$		$\frac{g^2-f^2}{2gh}$	$\frac{f^2}{gh}-\frac{fk_p}{h(1-a_3)k}$	$-\frac{f^2+g^2}{2gh}$			$\frac{g^2-f^2}{4gh}$	$\frac{f^2}{2gh}-\frac{fk_w}{hk}$	$-\frac{f^2+g^2}{4gh}$		
$4_{L,R}$	C_4		$\frac{2f^2}{h^2}$							$-\frac{2fk_p}{h(1-a_4)k}$					$-\frac{2fk_w}{hk}$		
$5_{L,R}$	C_5		$\frac{f^2+g^2}{2h^2}$	$\frac{f^2-g^2}{2h^2}$		$\frac{f^2+g^2}{2gh}$	$\frac{g^2-f^2}{2gh}$			$-\frac{f^2+g^2}{2gh}$	$\frac{f^2}{gh}-\frac{fk_p}{h(1-a_5)k}$	$\frac{g^2-f^2}{2gh}$			$-\frac{f^2+g^2}{4gh}$	$\frac{f^2}{2gh}-\frac{fk_w}{hk}$	$\frac{g^2-f^2}{4gh}$
COL. 2-3	$C_{2,3}$	$-\frac{1}{4}$	$\frac{1}{4}$		$\frac{h}{4g}$	$-\frac{h}{4g}$		$-\frac{h}{4g}$	$\frac{h}{2g}$	$-\frac{h}{4g}$			$-\frac{h}{8g}$	$\frac{h}{4g}$	$-\frac{h}{8g}$		
3-4	$C_{3,4}$	$-\frac{1}{4}$	$\frac{1}{4}$		$-\frac{h}{4g}$	$\frac{h}{4g}$		$\frac{h}{4g}$	$-\frac{h}{2g}$	$\frac{h}{4g}$			$\frac{h}{8g}$	$-\frac{h}{4g}$	$\frac{h}{8g}$		
4-5	$C_{4,5}$		$\frac{1}{4}$	$-\frac{1}{4}$		$-\frac{h}{4g}$	$\frac{h}{4g}$			$\frac{h}{4g}$	$-\frac{h}{2g}$	$\frac{h}{4g}$			$\frac{h}{8g}$	$-\frac{h}{4g}$	$\frac{h}{8g}$
5-6	$C_{5,6}$		$\frac{1}{4}$	$-\frac{1}{4}$		$\frac{h}{4g}$	$-\frac{h}{4g}$			$-\frac{h}{4g}$	$\frac{h}{2g}$	$-\frac{h}{4g}$			$-\frac{h}{8g}$	$\frac{h}{4g}$	$-\frac{h}{8g}$
VERTICAL DEFLECTION AT JOINT 4																	
MEMBER	$C' = k\frac{l}{I}l_4$	H-V COEFFICIENTS FOR Δ_{V4}						LOAD COEFFICIENTS FOR Δ_{V4}									
		hH_2	hH_4	hH_6	l_2V_2	l_4V_4	l_6V_6	$(1-a_2)l_2P_2$	$(1-a_3)l_3P_3$	$(1-a_4)l_4P_4$	$(1-a_5)l_5P_5$	$(1-a_6)l_6P_6$	$w_2l_2^2$	$w_3l_3^2$	$w_4l_4^2$	$w_5l_5^2$	$w_6l_6^2$
BEAM $3_{L,R}$	C'_3	$\frac{g^2-f^2}{2gh}$	$-\frac{f^2+g^2}{2gh}$		$\frac{g^2-f^2}{2g^2}$	$\frac{f^2+g^2}{2g^2}$		$\frac{f^2-g^2}{2g^2}$	$\frac{fk_p}{g(1-a_3)k}-\frac{f^2}{g^2}$	$\frac{f^2+g^2}{2g^2}$			$\frac{f^2-g^2}{4g^2}$	$\frac{fk_w}{gk}-\frac{f^2}{2g^2}$	$\frac{f^2+g^2}{4g^2}$		
$4_{L,R}$	C'_4					2											
$5_{L,R}$	C'_5		$\frac{f^2+g^2}{2gh}$	$\frac{f^2-g^2}{2gh}$		$\frac{f^2+g^2}{2g^2}$	$\frac{g^2-f^2}{2g^2}$			$-\frac{f^2+g^2}{2g^2}$	$\frac{f^2}{g^2}-\frac{fk_p}{g(1-a_5)k}$	$\frac{g^2-f^2}{2g^2}$			$-\frac{f^2+g^2}{4g^2}$	$\frac{f^2}{2g^2}-\frac{fk_w}{gk}$	$\frac{g^2-f^2}{4g^2}$
COL. 2-3	$C'_{2,3}$	$\frac{h}{4g}$	$-\frac{h}{4g}$		$-\frac{h^2}{4g^2}$	$\frac{h^2}{4g^2}$		$\frac{h^2}{4g^2}$	$-\frac{h^2}{2g^2}$	$\frac{h^2}{4g^2}$			$\frac{h^2}{8g^2}$	$-\frac{h^2}{4g^2}$	$\frac{h^2}{8g^2}$		
3-4	$C'_{3,4}$	$-\frac{h}{4g}$	$\frac{h}{4g}$		$-\frac{h^2}{4g^2}$	$\frac{h^2}{4g^2}$		$\frac{h^2}{4g^2}$	$-\frac{h^2}{2g^2}$	$\frac{h^2}{4g^2}$			$\frac{h^2}{8g^2}$	$-\frac{h^2}{4g^2}$	$\frac{h^2}{8g^2}$		
4-5	$C'_{4,5}$		$-\frac{h}{4g}$	$\frac{h}{4g}$		$\frac{h^2}{4g^2}$	$-\frac{h^2}{4g^2}$			$-\frac{h^2}{4g^2}$	$\frac{h^2}{2g^2}$	$-\frac{h^2}{4g^2}$			$-\frac{h^2}{8g^2}$	$\frac{h^2}{4g^2}$	$-\frac{h^2}{8g^2}$
5-6	$C'_{5,6}$		$\frac{h}{4g}$	$-\frac{h}{4g}$		$\frac{h^2}{4g^2}$	$-\frac{h^2}{4g^2}$			$-\frac{h^2}{4g^2}$	$\frac{h^2}{2g^2}$	$-\frac{h^2}{4g^2}$			$-\frac{h^2}{8g^2}$	$\frac{h^2}{4g^2}$	$-\frac{h^2}{8g^2}$

from which

$$V = \frac{w\, l_1^2 \left[\frac{l_1}{\cos \theta_1} \times \frac{f}{g\, I_1} \left(k_w + \frac{f}{2\, g} k_1 \right) + \frac{h^3}{4\, g^2} \left(\frac{k_3}{I_3} + \frac{k_4}{I_4} \right) \right]}{l_2 \left[\frac{l_1}{\cos \theta_1} \left(\frac{f^2 + g^2}{g^2} \right) \frac{k_1}{I_1} + \frac{2\, l_2}{\cos \theta_2} \times \frac{k_2}{I_2} + \frac{h^3}{2\, g^2} \left(\frac{k_3}{I_3} + \frac{k_4}{I_4} \right) \right]} \quad \ldots (40)$$

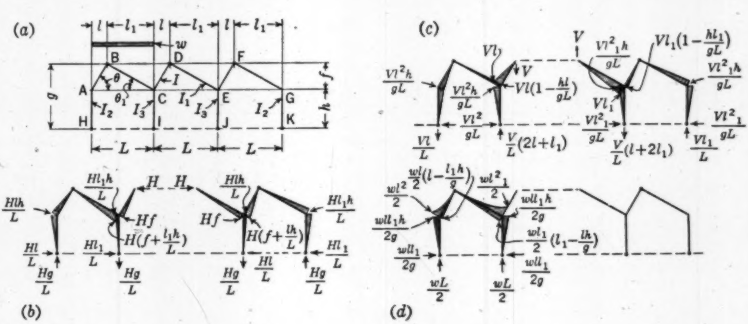
As in the case of rectangular framing, the deflection equations can be obtained directly from Eqs. 16. The data necessary for the determination of the various coefficients of those equations are given in Tables 14 (with Fig. 23), 15 (with Fig. 24), and 16.



W1
W2
W3
W4
a1l1
a2l2
a3l3
a4l4
P1
P2
P3
P4
H2
V2
H4
V4
P0
W0
f
g
h
1
2
3
4
5
L1 = 2l1
L2 = 2l2
L3 = 2l3
L4 = 2l4

FIG. 24

Example 2. Unsymmetrical Framing.—The three-bay bent shown in Fig. 25(*a*) illustrates this case. Here, as a result of the dissymmetry, in writing the deflection expressions, the two roof beams in each bay are no longer considered in pairs but are treated separately, as independent members, similar to the case in which the members are of differing shape or where the loading in a bay is unsymmetrically applied. The equilibrium diagrams for *H* and *V* (the two unknown reactions at hinge *D*) and for the loading in the first bay, obtained as simple-frame moments, are given in Fig. 25(*b*) to Fig. 25(*d*), respec-



(a)
(b)
(c)
(d)
A
B
C
D
E
F
G
H
I
J
K
w
θ
θ1
I
I1
I2
I3
l
l1
L
f
g
h
Hl1h/L
H
Hlh/L
Hf
H(f+l1h/L)
H(f+lh/L)
Hl/L
Hl1/L
Hg/L
V
Vl
Vl²h/gL
Vl²/gL
Vl(1−hl/gL)
Vl1(1−hl1/gL)
Vl²1h/gL
Vl²1/gL
Vl1
Vl/L
Vl1/L
V/L(2l+l1)
V/L(l+2l1)
wl²/2
wl/2(l−l1h/g)
wl²1/2
wll1h/2g
wl1/2(l1−lh/g)
wll1/2g
wL/2

FIG. 25

TABLE 15.—CONTRIBUTION OF MEMBERS TO THE DEFLECTION OF A JOINT (JOINT 2, FIG. 24) IN END BAYS OF GABLE FRAMES WITH AN ODD NUMBER OF BAYS

HORIZONTAL DEFLECTION AT JOINT 2															
MEMBER	$C=k\frac{l}{I}h$	H-V COEFFICIENTS FOR Δ_{H2}				LOAD COEFFICIENTS FOR Δ_{H2}									
		hH_2	hH_4	l_2V_2	l_4V_4	hP_0	$(1-a_1)l_1P_1$	$(1-a_2)l_2P_2$	$(1-a_3)l_3P_3$	$(1-a_4)l_4P_4$	w_0g^2	$w_1l_1^2$	$w_2l_2^2$	$w_3l_3^2$	$w_4l_4^2$
BEAM $1_{L,R}$	C_1	$\frac{f^2+g^2}{2h^2}$		$-\frac{f^2+g^2}{2gh}$		$-\frac{f+g}{2g}$	$\frac{f^2}{gh}-\frac{fk_p}{h(1-a_1)k}$	$-\frac{f^2+g^2}{2gh}$			$\frac{flk_w}{2g^2k}-\frac{f+g}{4g}$	$\frac{f^2}{2gh}-\frac{fk_w}{hk}$	$-\frac{f^2+g^2}{4gh}$		
$2_{L,R}$	C_2	$\frac{2f^2}{h^2}$						$-\frac{2fk_p}{h(1-a_2)k}$					$-\frac{2fk_w}{hk}$		
$3_{L,R}$	C_3	$\frac{f^2+g^2}{2h^2}$	$\frac{f^2-g^2}{2h^2}$	$\frac{f^2+g^2}{2gh}$	$\frac{g^2-f^2}{2gh}$			$-\frac{f^2+g^2}{2gh}$	$\frac{f^2}{gh}-\frac{fk_p}{h(1-a_3)k}$	$\frac{g^2-f^2}{2gh}$			$-\frac{f^2+g^2}{4gh}$	$\frac{f^2}{2gh}-\frac{fk_w}{hk}$	$\frac{g^2-f^2}{4gh}$
COL. 0-1	$C_{0,1}$	$\frac{1}{4}$		$-\frac{h}{4g}$		$-\frac{f+g}{4g}$	$\frac{h}{2g}$	$-\frac{h}{4g}$			$\frac{h^2k_w}{2g^2k}-\frac{3h}{8g}$	$\frac{h}{4g}$	$-\frac{h}{8g}$		
1-2	$C_{1,2}$	$\frac{1}{4}$		$\frac{h}{4g}$		$-\frac{h}{4g}$	$-\frac{h}{2g}$	$\frac{h}{4g}$			$-\frac{h}{8g}$	$-\frac{h}{4g}$	$\frac{h}{8g}$		
2-3	$C_{2,3}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{h}{4g}$	$\frac{h}{4g}$			$\frac{h}{4g}$	$-\frac{h}{2g}$	$\frac{h}{4g}$			$\frac{h}{8g}$	$-\frac{h}{4g}$	$\frac{h}{8g}$
3-4	$C_{3,4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{h}{4g}$	$-\frac{h}{4g}$			$-\frac{h}{4g}$	$\frac{h}{2g}$	$-\frac{h}{4g}$			$-\frac{h}{8g}$	$\frac{h}{4g}$	$-\frac{h}{8g}$

VERTICAL DEFLECTION AT JOINT 2															
MEMBER	$C'=k\frac{l}{I}l_2$	H-V COEFFICIENTS FOR Δ_{V2}				LOAD COEFFICIENTS FOR Δ_{V2}									
		hH_2	hH_4	l_2V_2	l_4V_4	hP_0	$(1-a_1)l_1P_1$	$(1-a_2)l_2P_2$	$(1-a_3)l_3P_3$	$(1-a_4)l_4P_4$	w_0h^2	$w_1l_1^2$	$w_2l_2^2$	$w_3l_3^2$	$w_4l_4^2$
BEAM $1_{L,R}$	C'_1	$-\frac{f^2+g^2}{2gh}$		$\frac{f^2+g^2}{2g^2}$		$\frac{g^2-f^2}{2g^2}$	$\frac{fk_p}{g(1-a_1)k}-\frac{f^2}{g^2}$	$\frac{f^2+g^2}{2g^2}$			$\frac{g^2-f^2}{4h^2}-\frac{flk_w}{2ghk}$	$\frac{fk_w}{gk}-\frac{f^2}{2g^2}$	$\frac{f^2+g^2}{4g^2}$		
$2_{L,R}$	C'_2			2											
$3_{L,R}$	C'_3	$\frac{f^2+g^2}{2gh}$	$\frac{f^2-g^2}{2gh}$	$\frac{f^2+g^2}{2g^2}$	$\frac{g^2-f^2}{2g^2}$			$-\frac{f^2+g^2}{2g^2}$	$\frac{f^2}{g^2}-\frac{fk_p}{g(1-a_3)k}$	$\frac{g^2-f^2}{2g^2}$			$-\frac{f^2+g^2}{4g^2}$	$\frac{f^2}{2g^2}-\frac{fk_w}{gk}$	$\frac{g^2-f^2}{4g^2}$
COL. 0-1	$C'_{0,1}$	$-\frac{h}{4g}$		$\frac{h^2}{4g^2}$		$\frac{g^2-f^2}{4g^2}$	$\frac{h^2}{2g^2}$	$\frac{h^2}{4g^2}$			$\frac{3}{8}-\frac{hk_w}{2gk}$	$-\frac{h^2}{4g^2}$	$\frac{h^2}{8g^2}$		
1-2	$C'_{1,2}$	$\frac{h}{4g}$		$\frac{h^2}{4g^2}$		$-\frac{h^2}{4g^2}$	$\frac{h^2}{2g^2}$	$\frac{h^2}{4g^2}$			$-\frac{1}{8}$	$-\frac{h^2}{4g^2}$	$\frac{h^2}{8g^2}$		
2-3	$C'_{2,3}$	$-\frac{h}{4g}$	$\frac{h}{4g}$	$\frac{h^2}{4g^2}$	$-\frac{h^2}{4g^2}$			$-\frac{h^2}{4g^2}$	$\frac{h^2}{2g^2}$	$-\frac{h^2}{4g^2}$			$-\frac{h^2}{8g^2}$	$\frac{h^2}{4g^2}$	$-\frac{h^2}{8g^2}$
3-4	$C'_{3,4}$	$\frac{h}{4g}$	$-\frac{h}{4g}$	$\frac{h^2}{4g^2}$	$-\frac{h^2}{4g^2}$			$-\frac{h^2}{4g^2}$	$\frac{h^2}{2g^2}$	$-\frac{h^2}{4g^2}$			$-\frac{h^2}{8g^2}$	$\frac{h^2}{4g^2}$	$-\frac{h^2}{8g^2}$

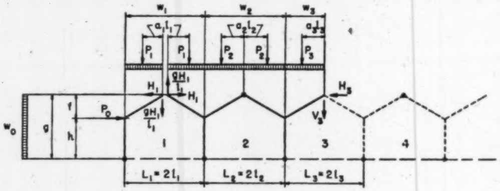
tively. Then, from the consideration of horizontal deflection at point D,

$$\begin{aligned}\Delta_H = {} & \frac{H\,l}{\cos\theta} \times \frac{k}{I}\left[\left(\frac{l\,h}{L}\right)^2 + f^2 + \left(f + \frac{l\,h}{L}\right)^2\right] + \frac{H\,l_1}{\cos\theta_1} \times \frac{k_1}{I_1}\left[\left(\frac{l_1\,h}{L}\right)^2\right.\\
& \left. + f^2 + \left(f + \frac{l_1\,h}{L}\right)^2\right] + H\,h^3\,\frac{k_2}{I_2}\left[\frac{l^2}{L^2} + \frac{l^2_1}{L^2}\right] + H\,h^3\,\frac{k_3}{I_3}\left[\frac{l^2}{L^2} + \frac{l^2_1}{L^2}\right]\\
& - \frac{V\,l}{\cos\theta} \times \frac{k}{I}\left[\frac{l^3\,h^2}{L^2\,g} + f\,l - \left(f + \frac{l\,h}{L}\right)\left(l_1 - \frac{h\,l^2_1}{L\,g}\right)\right] + \frac{V\,l_1}{\cos\theta_1}\\
& \times \frac{k_1}{I_1}\left[f\,l_1 - \left(f + \frac{h\,l_1}{L}\right)\left(l - \frac{h\,l^2}{L\,g}\right) + \frac{h^2\,l_1^3}{L^2\,g}\right]\\
& + V\,h^3\,\frac{k_2}{I_2}\left[\frac{l^3_1}{L^2\,g} - \frac{l^3}{L^2\,g}\right] + V\,h^3\,\frac{k_3}{I_3}\left[\frac{l_1\,l^2}{L^2\,g} - \frac{l^2_1\,l}{L^2\,g}\right]\\
& + \frac{w\,l^3}{\cos\theta} \times \frac{k_w}{I}\left[\frac{h\,l}{L}\right] - \frac{w\,l^3_1}{\cos\theta_1} \times \frac{k_{w1}}{I_1}\left[\left(f + \frac{h\,l_1}{L}\right)\right]\\
& + w\,h^3\,\frac{k_2}{I_2}\left[\frac{l^2\,l_1}{2\,L\,g} - w\,h^3\,\frac{k_3}{I_3}\,\frac{l\,l^2_1}{2\,L\,g}\right] - \frac{w\,l}{\cos\theta} \times \frac{k}{I}\left[\frac{h\,l^2}{2\,L}\left(l - \frac{h\,l_1}{g}\right)\right]\\
& + \frac{w\,l_1}{2\cos\theta_1} \times \frac{k_1}{I_1}\left[\left(f + \frac{h\,l_1}{L}\right)\left(l^2_1 - \frac{h\,l\,l_1}{g}\right)\right] = 0 \ldots\ldots\ldots\ldots\ldots\ldots (41)\end{aligned}$$

Similarly, consideration of the vertical deflection at the same joint will yield the following relation:

$$\begin{aligned}\Delta_V = {} & \frac{V\,l}{\cos\theta} \times \frac{k}{I}\left[\left(\frac{h\,l^2}{g\,L}\right)^2 + l^2 + l^2_1\left(1 - \frac{h\,l_1}{g\,L}\right)^2\right] + \frac{V\,l_1}{\cos\theta_1} \times \frac{k_1}{I_1}\left[\left(\frac{h\,l^2_1}{g\,L}\right)^2\right.\\
& \left. + l^2_1 + l^2\left(1 - \frac{h\,l}{g\,L}\right)^2\right] + V\,h^3\,\frac{k_2}{I_2}\left[\left(\frac{l^2}{g\,L}\right)^2 + \left(\frac{l^2_1}{g\,L}\right)^2\right]\\
& + V\,h^3\,\frac{k_3}{I_3}\left[\left(\frac{l^2}{g\,L}\right)^2 + \left(\frac{l^2_1}{g\,L}\right)^2\right] - \frac{H\,l}{\cos\theta}\\
& \times \frac{k}{I}\left[\frac{h^2\,l^3}{g\,L^2} + f\,l - l_1\left(1 - \frac{h\,l_1}{g\,L}\right)\left(f + \frac{h\,l}{L}\right)\right] - \frac{H\,l_1}{\cos\theta_1}\\
& \times \frac{k_1}{I_1}\left[l\left(1 - \frac{h\,l}{g\,L}\right)\left(f + \frac{h\,l_1}{L}\right) - f\,l_1 - \frac{h^2\,l^3_1}{g\,L^2}\right]\\
& - H\,h^3\,\frac{k_2}{I_2}\left[\frac{l^3}{g\,L^2} - \frac{l^3_1}{g\,L^2}\right] - H\,h^3\,\frac{k_3}{I_3}\left[\frac{l\,l^2_1}{g\,L^2} - \frac{l_1\,l^2}{g\,L^2}\right]\\
& - w\,\frac{k_w}{I} \times \frac{h\,l^2}{g\,L} \times \frac{l^3}{\cos\theta} + w\,\frac{k}{I} \times \frac{h\,l^2}{g\,L} \times \frac{l}{2}\left(l - \frac{h\,l_1}{g}\right)\\
& \times \frac{l}{\cos\theta} + w\,\frac{k_{w\,l}}{I_1} \times l\left(1 - \frac{h\,l}{g\,L}\right)\frac{l^3_1}{\cos\theta_1} - w\,\frac{k_1}{I_1}\\
& \times l\left(1 - \frac{h\,l}{g\,L}\right)\frac{l_1}{2}\left(l_1 - \frac{h\,l}{g}\right)\frac{l_1}{\cos\theta_1} - w\,\frac{k_2}{I_2}\\
& \times \frac{l^3\,l_1}{2\,g^2\,L}\,h^3 - w\,\frac{k_3}{I_3} \times \frac{l^3\,l_1}{2\,g^2\,L}\,h^3 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)\end{aligned}$$

TABLE 16.—CONTRIBUTION OF MEMBERS TO THE DEFLECTION OF A JOINT IN END BAYS OF GABLE FRAMES WITH AN EVEN NUMBER OF BAYS (HORIZONTAL DEFLECTION AT JOINT 1)


w_1
w_2
w_3
$a_1 l_1$
$a_2 l_2$
$a_3 l_3$
P_1
P_1
P_2
P_2
P_3
$\frac{gH_1}{l_1}$
H_1
H_1
H_3
P_0
$\frac{gH_1}{l_1}$
V_3
w_0
g
f
h
1
2
3
4
$L_1 = 2l_1$
$L_2 = 2l_2$
$L_3 = 2l_3$

HORIZONTAL DEFLECTION AT JOINT 1												
MEMBER	$C = k\frac{l}{I}h$	H-V COEF'S FOR Δ_{D1}			LOAD COEFFICIENTS FOR Δ_{D1}							
		hH_1	hH_3	l_3V_3	hP_0	$(1-a_1)l_1P_1$	$(1-a_2)l_2P_2$	$(1-a_3)l_3P_3$	w_0g^2	$w_1l_1^2$	$w_2l_2^2$	$w_3l_3^2$
BEAM $1_{L,R}$	C_1	$\frac{2(f^2+g^2)}{h^2}$			$-\frac{2g}{h}$	$\frac{2f^2}{gh} - \frac{2fk_p}{h(1-a_1)k}$			$\frac{flk_w}{g^2k} - \frac{g}{h}$	$\frac{f^2}{gh} - \frac{2fk_w}{hk}$		
$2_{L,R}$	C_2	$\frac{2(f^2+g^2)}{h^2}$	$\frac{f^2-g^2}{h^2}$	$\frac{f+g}{g}$	$-\frac{f^2+g^2}{gh}$		$\frac{2f^2}{gh} - \frac{2fk_p}{h(1-a_2)k}$	$\frac{f+g}{g}$	$-\frac{f^2+g^2}{2gh}$		$\frac{f^2}{gh} - \frac{2fk_w}{hk}$	$\frac{f+g}{2g}$
$3_{L,R}$	C_3											
COL. 0-1	$C_{0,1}$	1			-1	$\frac{h}{g}$			$\frac{h^2k_w}{g^2k} - \frac{h}{g}$	$\frac{h}{2g}$		
1-2	$C_{1,2}$											
2-3	$C_{2,3}$	1	$-\frac{1}{2}$	$-\frac{h}{2g}$	$-\frac{h}{2g}$		$\frac{h}{g}$	$-\frac{h}{2g}$	$-\frac{h}{4g}$		$\frac{h}{2g}$	$-\frac{h}{4g}$

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REDESIGN OF MAJOR AIRPORT TERMINALS

By Herbert H. Howell,[1] M. ASCE

Synopsis

The development of electronic devices for all-weather flying will make it necessary to increase the capacity of many major airports. The trend will be toward the redevelopment of existing airports rather than the construction of new fields.

The requirements of an adequate design are discussed and illustrated by reference to a typical redesign as worked out for the Lambert-St. Louis Municipal Airport (St. Louis, Mo.). The advantages of the improved layout are described and related to the general problem of airport redesign.

Introduction

In the years following World War II, many great improvements in devices to aid in the navigation and landing of aircraft in inclement weather were developed. The program of the Civil Aeronautics Administration (CAA) in connection with this type of operation follows, as a guide, a report[2] that proposes certain criteria for a common system of aids to navigation. The CAA is working toward the early completion of the so-called Transition Period Common System of Air Traffic Control and Navigation. To re-evaluate the usefulness of the Common System, a special group, known as the Operational Policy Group, was appointed by the Air Coordinating Committee's Air Traffic Control and Navigation Panel. This group has completed its work and issued a report[3] that re-emphasizes the value and necessity of a common system. This report should be studied by all airport engineers.

The essential features of the Common System are: (*a*) Very high frequency omnidirectional radio range; (*b*) distance measuring equipment; (*c*) surveillance radar; (*d*) precision approach radar; (*e*) instrument landing

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Note.—Published in November, 1951, as *Proceedings-Separate No. 104*. Positions and titles given are those in effect when the paper was received for publication.

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[2] *SC-31 Report*, by Special Committee 31 of the Radio Technical Committee for Aeronautics, Washington, D. C., May 12, 1948.

[3] "Air Traffic Control and the National Security," Report of the Operational Policy Group, Air Coordinating Committee, Washington, D. C., December, 1950.

system; and (*f*) high intensity approach lights. The purpose of these navigational and landing aids is to accommodate a greatly increased number of aircraft in the flight lanes and to guide them more quickly to safe landings at their destination. The federal government cannot undertake the entire job alone, and each local community must be in partnership with the government in the effort to handle a constantly increasing volume of air traffic.

Most of the major air transport centers of the United States have already developed their airports, and only a few new major airports are going to be developed in the foreseeable future. In most of our major cities it is logical, therefore, to assume that activity will continue at the existing airport. Four main reasons for this assumption are: (1) The investment in the existing facility must be considered; (2) the normal resistance to relocation must be overcome; (3) the growth of most major cities tends to force the location of a new airport to open country, generally an excessive distance from the center of the city; and (4) the element of cost in which even the first stage of developing a new airport can represent a rather staggering amount of money. Therefore, this paper considers only redesign as opposed to new development.

AIRPORT CAPACITY

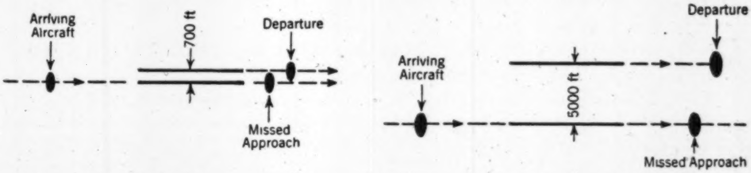
Effect of Electronic Development.—The electronic developments of the transition period, as previously mentioned, will do much to make all-weather flying a reality. They will permit a very high percentage factor of flight operations to be achieved. Actually, on major airports, it is logical to think of the capacity of the airport as the number of aircraft that can be handled during restrictive weather. This concept follows from the fact that the major portion of stable traffic on the average airport is that generated by scheduled air carriers plus those nonscheduled civil, cargo, and military aircraft that regularly fly, regardless of the weather. Thus, any increase in airport capacity for this segment of the aviation industry must be measured in terms of capacity during instrument weather. The fact has affected the thinking of the industry regarding airport layouts, the spacing of parallel runways, the design of taxiway systems, and the parking of aircraft at loading areas. Also, one point is becoming more and more obvious—the ability to use electronic developments to guide aircraft to the threshold of the runway results in a capacity that exceeds the acceptance rate of most airports. In other words, it appears practicable to bring in aircraft at the rate of 1 every 1.5 min or at a steady rate of 40 per hr.

Factors Affecting Capacity.—There are few airports that can accept aircraft at the rate of 40 per hr, because of two factors:

a. Inability of planes to clear the runway in sufficient time; and
b. Lack of adequate ramp space for parking.

Before attempting to solve the problem, it should be thoroughly understood. The landing of aircraft 1.5 min apart, during weather of restricted visibility, is really a rapid rate when considering that the pilot does not have visual contact with the ground and the airport traffic controller does not have visual contact with the aircraft. In order to maintain this rate the landing aircraft must, of necessity, clear the runway at the earliest possible instant. This requires that the runway must have taxiways at frequent intervals throughout its length so that aircraft can get off the runway as soon as the speed has been reduced sufficiently to permit a turn to be executed. If the taxiways lead from the runway at an angle of 30° or 45°, naturally, the aircraft can turn at a higher rate of speed than if a 90° or more acute turn must be made.

To land aircraft at the rate of 40 per hr means that one runway must be restricted exclusively to landings. A second runway must be available for take-off, permitting aircraft to leave the airport at the same rate of 40 per hr. Under instrument conditions, it is doubtful that parallel runways can be used simultaneously unless they are separated by a minimum distance of 3,000 to 5,000 ft (Fig. 1(*a*)). A simultaneous departure on one parallel runway and a missed approach on the corresponding landing runway would put both aircraft in the air very close together along the projected center lines of the parallel runways. The horizontal separation would be inadequate for safe operations. Fig. 1(*b*) shows the condition of parallel runways having adequate runway separation. The use of intersecting runways, one for landing and one for take-off, is preferable to parallel runways without adequate clearance, inasmuch as the projected center lines of the runways diverge, resulting in a constantly increasing separation.



(*a*) RUNWAYS WITH 700 FT SEPARATION (*b*) RUNWAYS WITH 5000 FT SEPARATION

FIG. 1.—REQUIRED SEPARATION FOR PARALLEL RUNWAYS

Coupled with the runway and taxiway requirements is the problem of parking aircraft after they have landed. It is obvious that aircraft cannot be landed at the rate of 40 per hr unless the available parking areas will provide adequate accommodations for them. The Airport Operators Council has conducted surveys in this regard, and the CAA is attempting to establish basic criteria for determining the requirements for gate positions on loading ramps. Only a few airports in the United States have loading aprons adequate to take care of scheduled arrivals at the rate of 40 per hr.

FEATURES OF DESIGN

Several basic requirements must be met if an airport is to achieve maximum efficiency when handling near-capacity volume of traffic. These requirements may be listed as follows:

1. All airports in a community should be located so their respective instrument traffic patterns do not overlap or conflict with each other;

2. Adjacent airports served by instrument approach aids should be located so that their respective instrument traffic patterns for both arriving and departing aircraft do not overlap or conflict with each other;

3. Landing, taxiing, and take-off must be considered as independent operations, and allowance must be made for their accomplishment without mutual interference;

4. Where practicable, the runways should be separated so that simultaneous use may be effected without conflict of traffic patterns. This will greatly increase the capacity and permit more direct approaches and departures and thus save aircraft time and reduce traffic congestion;

5. The airport should be so designed that the air traffic control tower can achieve flexibility in the choice of take-off sequence without regard to the sequence of taxiing from the ramp to the take-off runway;

6. Taxiways should be located at various points along the runway to permit landing aircraft to turn off the runway as quickly as possible to clear it for other aircraft and to provide minimum taxi distances to the service or ramp area on completion of the landing roll;

7. Taxiing aircraft should be able to pass other aircraft that are parked or holding on loading aprons on near runway entrances;

8. Taxi exits should be marked in such a manner that the pilot can anticipate the turnoff point in sufficient time to enter the taxi exit easily under day or night conditions; and

9. The control tower must provide excellent visibility of the entire airport area used by aircraft and all the approach and departure areas.

These requirements can easily be met in a new airport design. It is much more difficult, but generally possible, to incorporate the essential features into existing airports, with a minimum of compromise, assuming, of course, that the airports are properly located and susceptible of modification.

Redesign of Lambert-St. Louis Municipal Airport

The writer recently participated in a study of the master plan of Lambert-St. Louis Municipal Airport. Lambert Field is one of the major airports in the United States. It was initially developed in the early 1920's and has been expanded from time to time as changing aeronautical conditions required. The airport has a complete variety of traffic including flight training, charter work, executive transport, military, cargo, and scheduled airlines. There is an adjacent aircraft factory that manufactures jet planes. Therefore, the airport operates at high capacity during good weather conditions. This requirement for capacity drops off considerably during restricted visibility weather.

Fig. 2 shows the master plan of Lambert Field, developed in 1944 and prepared on the basis of the best thinking current at that time. The old portion of the field is toward the western side. By 1950 a major grading project was in progress on the eastern portion of the airport.

This master plan was based on the time-proved concept of a three-runway layout with one runway usable at a time, depending upon wind direction. Provision was made for future parallel runways when traffic density required.

There is nothing wrong with such a concept, except that the expense of parallel runways is considerable and unnecessary if the airport can be designed so that all runways can bear traffic simultaneously.

Fig. 2. also shows suggested revisions to the master plan. The changes represent adaptations that conform with advanced thinking in airport design

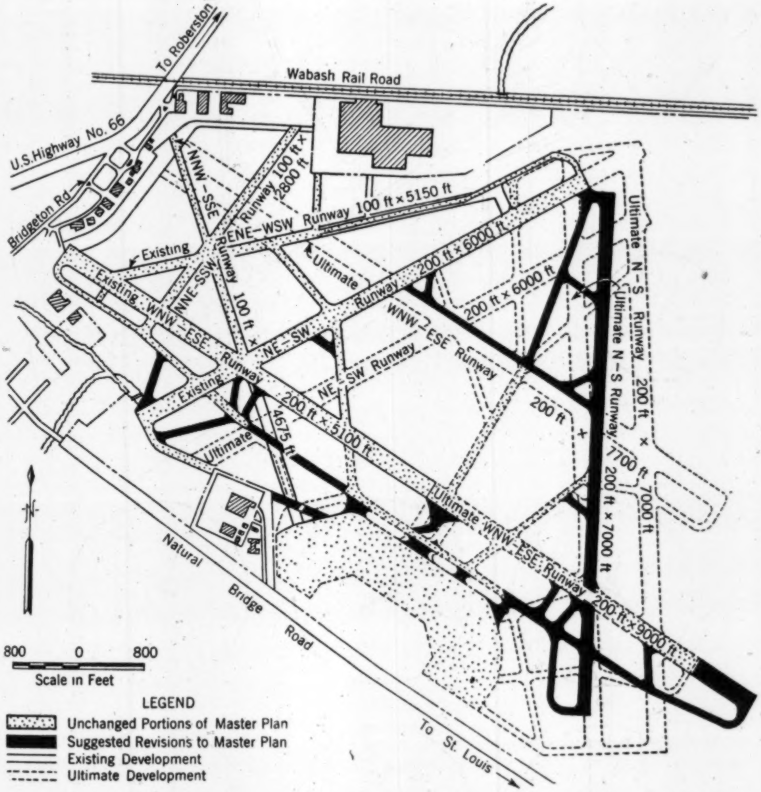

To Roberston
Wabash Rail Road
U.S.Highway No. 66
Bridgeton Rd.
NNW–SSE
Runway 100 ft × 2800 ft
ENE–WSW Runway 100 ft × 5150 ft
Existing
Ultimate
Existing
Existing WNW–ESE Runway
NNE–SSW Runway 100 ft ×
NE–SW Runway 200 ft × 6000 ft
WNW–ESE Runway
200 ft × 6000 ft
NE–SW Runway
Existing
200 ft × 5100 ft
4675 ft
Ultimate
200 ft × 7700 ft
Ultimate N–S Runway 200 ft × 7000 ft
Ultimate N–S Runway 200 ft × 7000 ft
Ultimate WNW–ESE Runway 200 ft × 9000 ft
N
Natural Bridge Road
To St. Louis
800 0 800
Scale in Feet
LEGEND
Unchanged Portions of Master Plan
Suggested Revisions to Master Plan
Existing Development
Ultimate Development

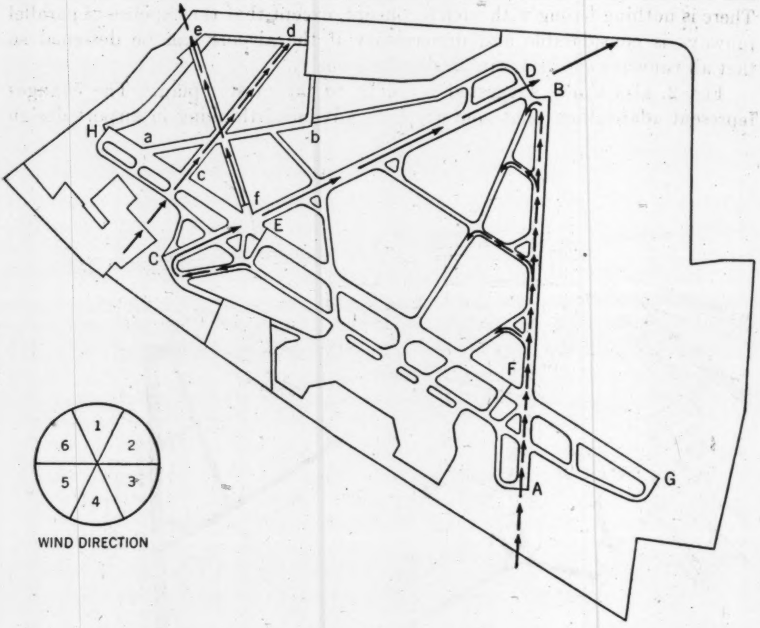
FIG. 2.—LAMBERT-ST. LOUIS MUNICIPAL AIRPORT RECOMMENDATIONS FOR DEVELOPMENT (1950)

and give the airport its maximum traffic capacity without providing parallel runways. The suggested revisions include the following:

(*a*) An increase in the ultimate length of northwest-southwest runway from 9,000 ft to 9,800 ft to provide a minimum of 7,000 ft of usable runway in either direction and still permit other runways to operate;

(*b*) Relocation of the proposed north-south runway by rotating it about 5° to the east, thus eliminating the intersection of the north-south and northeast-southwest runways and permitting their simultaneous use;

TABLE 1.—TRAFFIC PATTERNS AT LAMBERT-ST. LOUIS MUNICIPAL AIRPORT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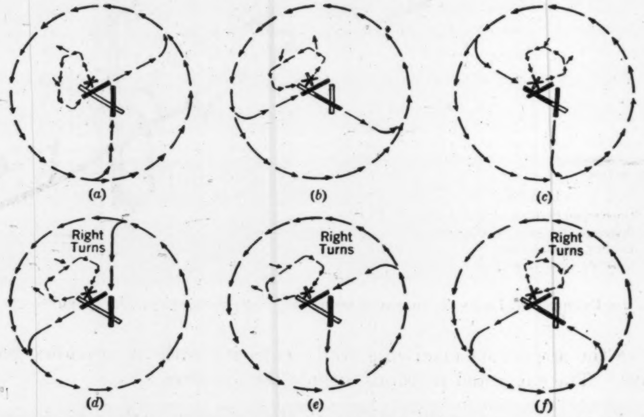
WIND DIRECTION


Right Turns
Right Turns
Right Turns

(a) (b) (c)

(d) (e) (f)

Wind direction	Landing Runway		Take-off Runway		Area pattern	Percentage of time applicable[a]
	Light aircraft	Other aircraft	Light aircraft	Other aircraft		
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	cd	AB	fe	CD	a	10.2
2	ab	CD	cd	EG	b	9.9
3	ef	HF	ab	BA	c	12.9
4	dc	BA	ba	DC	d	16.6
5	dc	DC	ba	BA	e	17.6
6	ba	GE	fe	DC	f	19.8

[a] Not including calm wind periods.

(*c*) Revision of the taxiway system; and
(*d*) Provision for bleed-off taxiways.

The object of the suggested revisions is to increase the potential capacity of the airport by making two major runways traffic-bearing (one for take-off and one for landing) simultaneously.

Table 1 indicates the traffic patterns that will develop peak capacity under various wind conditions. In each case the runway most nearly alined with the wind is used for landing, and a runway quartering into the wind is used for take-off. Both can be used simultaneously. Provision is also made for light aircraft on the west side of the field, using existing runways. These runways intersect but permit nonconflicting traffic patterns, and under proper control they offer greater capacity than a single runway. Vicinity patterns show the lack of conflict between light and heavy aircraft patterns.

The resulting capacity approximates that of parallel runways. The potential saving of approximately $3,000,000 probably influenced the city officials of St. Louis, who received this study enthusiastically.

These patterns are developed for contact flying conditions. Instrument use of the airport would, of course, be restricted to the major runway system. At the present time the northeast-southwest runway is the instrument runway. If this continues to be the instrument runway, it can be used exclusively for landing, with take-offs from the western portion of the northwest-southeast runway that does not interfere with the use of the instrument landing runway.

If the northwest-southeast runway becomes the instrument landing runway (with approach from the southeast) the northeast-southwest runway can be used for simultaneous take-off, or if the approach is from the northwest, the north-south run can be used for take-off.

The ultimate apron provides thirty parking positions for aircraft, which appear to be sufficient to permit capacity operation of the airport.

Studies indicate that the capacity of the airport for both instrument and contact weather will be sufficient to satisfy the needs of St. Louis for at least the foreseeable future.

CONCLUSION

In the case of St. Louis, it was possible, by relatively minor revision of the master plan, to eliminate the intersection of runways. The test of a good master plan is "its ability to accommodate itself to changes and advances in technological developments."[4] On this basis the St. Louis master plan of 1944 was a good one. No master plan for an airport can be a static one, for the aviation industry sees frequent and far-reaching technological changes.

Fig. 3 shows the layout of Stapleton Airfield at Denver, Colo. Here the east-west runway is to be the instrument runway with approach from the east. For several reasons it is not possible to expand the airport sufficiently to permit any two runways to be nonintersecting. For capacity operation, however, aircraft can take off to the north, on a runway that intersects the east-west runway near its western end. Properly controlled, the traffic capacity could approach that of parallel or nonintersecting runways, provided that the capacities of the taxiway system and parking ramp are increased.

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[4] "Master Plan-Chicago International Airport," by Ralph H. Burke, Chicago, Ill., November, 1950.

Further analysis of various layouts would indicate that properly designed airports can generally be redesigned to meet changing conditions. At first glance, the almost daily changes in the industry present a hopeless problem to the airport engineer, but the advancement that the aviation industry has made is adequate testimony to the ability of engineers to meet and solve these problems.

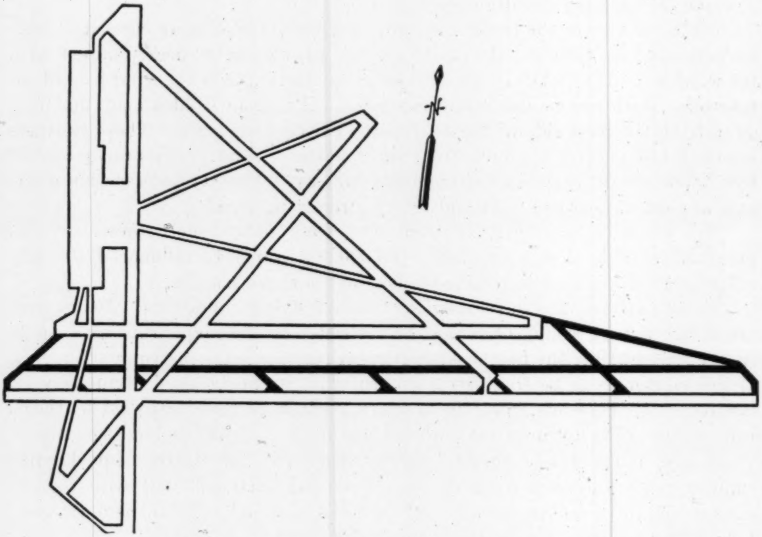

Fig. 3.—Stapleton Airfield, Denver, Colo.—Recommendations for Development

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 2510

DIRECTIONAL REQUIREMENTS FOR AIRPORT RUNWAYS

BY RALPH H. BURKE[1] AND HARRY OTIS WRIGHT, JR.,[2]
MEMBERS, ASCE

SYNOPSIS

The layout of airport runways involves many problems that can be solved only by a thorough analysis of requirements at each specific location. The important factor of runway orientation can be influenced by the type of landing gear used on the airplanes. The effect of this factor and also the effect of cross wind are evaluated and discussed.

Attainment of maximum utilization of an airport is to a great extent dependent on the selection of a suitable layout. The variation of utilization factor with cross-wind velocity and landing gear characteristics is tabulated.

Suggestions are made concerning runway organization and the ability of an airport to meet the imposed traffic requirements.

INTRODUCTION

The purpose of this paper is to review the problem of directional requirements from the standpoint of the designer of airports. On the one hand, full cognizance must be taken of the policy of the Civil Aeronautics Administration (CAA) limiting runway construction to that required by capacity without consideration of wind directions. On the other hand, every effort must be made to meet the objective of the airlines which is to provide uninterrupted and dependable schedules of operation. A further element of importance to the designer of municipal airports is the balance of all the economic factors in order that the investment of public funds in airports will be fully justified on the basis of public benefits derived from their construction. A logical set of airport standards is necessary to meet this requirement.

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AIRPORT STANDARDS

Airport requirements have undergone a series of radical changes during the relatively short history of aviation. In 1903, when the Wright brothers, Orville and Wilbur, launched the airplane "Kitty Hawk," and for the next decade or so, there were no airports. During the succeeding years, as speed, weight, and carrying capacity of airplanes were increased, prepared places to land became a necessity. Aircraft characteristics changed so rapidly that airports were outmoded almost as soon as they were built. Many localities had the sad experience of constructing airports that became obsolete or relegated to secondary use long before the expiration of a reasonable amortization period.

The CAA performed a long-needed service to aviation in adopting, as it did in 1947, standards as to length and load capacity of runways for the several classes of airports throughout the United States. Here, finally, was a yardstick to be used by the airport designer to measure the required facilities, and here also was a definite limit on the aircraft designers.

The so-called "one runway" policy of the CAA will further clarify the design problem for airports and aircraft. Stated in its simplest terms, the basis of the CAA policy is the theory that, if all aircraft were equipped with castering landing gears, the wind direction could be disregarded entirely; then only such runways should be built as would accommodate the volume of traffic at any certain airport. Since it would obviously be cheaper to equip all aircraft with castering gears than to build multiple runways at all airports, such a policy results in over-all economy.

HISTORY OF THE PROBLEM

The entire problem of runway direction centers around the type of landing gear used on the aircraft, so a brief review of the history of the development of landing gear is pertinent at this point. It should be understood that landing gear is a necessary evil. This gear is a detriment to any aircraft while in the air. Its sole function is to safely permit loss of speed on landing and acceleration on take-off and to permit some measure of maneuverability for the aircraft in taxiing on the ground.

Greased rails were used for the take-off in the first aeronautical experiments. Then skids, with the addition of a pair of wheels, were tried. Later, about 1910, 3-wheeled or 4-wheeled arrangements were used. These later applications overcame the excessive ground frictions of the earlier skids on take-off, but were inefficient in decelerating on landing.

In 1924 E. Stinson pioneered the installation of brakes on the main wheels, thus contributing a great advance in ground control. Skids were no longer needed, either for drag or directional stability. Prior to World War II the tricycle gear was revived, since it was less susceptible to ground looping and produced the desirable characteristic of a level cabin floor while the airplane was on the ground. The tricycle gear, with steerable nose wheel, has become the most commonly used landing gear on transport aircraft. The latest development is the castering gear. It has been built and tested for small aircraft of various types and also for the DC–3. Record of its application to larger aircraft has not yet appeared.

Granting the feasibility of the universal application of castering gear to all aircraft, there remains an analysis of the over-all economy, dependability of service, and capacity of the airport to determine whether such an application justifies the acceptance of the CAA policy.

CROSS–WIND PROBLEM

Cross wind is of significance only when the airplane is in contact with the ground. When flying, wind direction or intensity in the absence of sudden gusts does not affect the maneuverability or control of an airplane. As the airplane approaches the ground, however, it is flying at some angle of crab depending upon the magnitude and direction of the cross wind and the speed and direction of flight. If the resultant angle of crab is small (less that 15°), no problem exists, for at the instant of contact the noncastering landing gear will track immediately and the wheels will roll readily. If the resultant angle of crab is large, the wheels of the landing gear must skid sideways causing severe overloads and possible failure of the mechanism.

To illustrate, a plane with a speed of 100 miles per hr at touchdown, operating under a 20 miles per hr cross-wind component, is flying at an angle of crab of 11½°. This is a relatively safe condition. If the plane has a speed of only 75 miles per hr and encounters a cross wind of 50 miles per hr, the angle of crab is 43°, and a dangerous condition exists.

Since 1938 the cross-wind problem has become less critical in regard to adequate control after contact. New developments, such as tricycle landing gear with steerable nosewheels in multi-engine planes, the ability to use more power on one side than on the other, and reversible pitch propellers have made it much easier to control the plane, and the margin of safety has been raised considerably. The Air Transport Association permits aircraft with noncastering landing gear to operate in cross winds with velocity up to 20 miles per hr at a 90° angle to the runway.

Under this ruling the utilization of any airport depends upon the number of runways and the percentage of the time when the cross-wind component relative to any available runway exceeds 20 miles per hr. Assuming the wind rose at Chicago, Ill., to be fairly typical, a single runway airport could be used 96% of the time and a 2-runway airport (with the runways at right angles) could be used 98.1% of the time. A standard 4-runway airport could be used 99.9% of the time.

As a result of exhaustive tests, the CAA will permit operation of planes of DC-3 type equipped with castering landing gear in cross winds up to 40 miles per hr. Using this criterion, with wind conditions that prevail in Chicago, a single-runway airport could be used 99.0% of the time and a 2-runway airport 99.9% of the time. A 4-runway airport would be useable practically 100% of the time. The percentage of utilization of various types of airports with noncastering and castering gears at various cross winds is given in Table 1.

The differences in utilization of an airport as shown in Table 1 may seem small, yet a difference of only 4% will affect regularity and dependability of service to such a degree that, in scheduled flights at least, it spells the difference between an acceptable mode of travel and a totally inadequate service.

Therefore, in the large airports, including all those at which scheduled passenger service is maintained, practically 100% utilization is a necessity. The question remains whether the result can be most economically obtained by multiple runways or by the universal application of castering gears. The CAA has estimated that it would cost about $10,000,000 (1950) to equip all of the approximately 1,100 planes now in scheduled operation with castering gears. This figure does not include lost income during the conversion period. This estimate seems low, but even if the figure were doubled the situation would not be seriously altered. This figure must be compared with the average total cost of a runway of $3,000,000 including land acquisition, drainage, grading, and other incidentals. Since there are over 780 airports of Class IV or larger in the United States, the evident conclusion is that the universal application of castering gears will amount in cost to only a small fraction of the cost of added runways.

TABLE 1.—Useable Percentage of Total Time at an Airport[a]

Allowable cross wind (miles per hr)	Type of landing gear	Number of Runway Directions		
		One	Two	Four
(a) Aircraft Landing at Approximately 40 Miles per Hr				
10.0	Noncaster	61.2	74.4	91.6
22.5	Caster	98.6	99.4	99.9
(b) Aircraft Landing at Approximately 55 Miles per Hr				
15.0	Noncaster	91.4	95.1	99.9
32.5	Caster	98.9	99.8	100.0
(c) Aircraft Landing at Approximately 70 Miles per Hr				
20.0	Noncaster	96.0	98.1	99.9
40.0	Caster	99.0	99.9	100.0

[a] Based on conditions at Chicago, Ill.

Airport Utilization

As seen in Table 1, a 4-runway airport can be used by aircraft with noncastering gear 99.9% of the time in Chicago. Castering gears would raise the utilization to practically 100%. In fact, Chicago Midway Airport in all its history has never been closed to traffic because of high adverse winds alone. If Chicago had a single direction runway airport, even aircraft with castering gear could operate only 99.0% of the time. This difference, even though appearing small when expressed in percentage, is vital in determining dependability of air travel, for it means that for a total of over 3 days per yr the Chicago airport would be unable to operate as far as air travel is concerned. These occurrences are of sufficient duration as to affect the continuity and seriously disrupt scheduled operations. The policy of single directional runways cannot be seriously considered for such airports as Chicago.

Applying the results of Table 1 to the operation of small aircraft, such as the type that can safely operate with noncastering gear at cross winds of 10 to 15 miles per hr, a vastly different picture appears. On single-runway airports (such as Meigs Airport, Chicago's downtown air strip) the addition of castering gears to the light planes that use the field raises the utility of the airport from 61.2% to 98.6% for aircraft limited to a 10 miles per hr cross-wind landing with noncastering gear. Similar increases in utilization are noted in the case of aircraft having higher landing speeds. These increases are significant and will result in marked improvement of efficiency of the air strip. Meigs Airport is licensed for operation only when the cross wind is below 15 miles per hr.

TRAFFIC REQUIREMENTS

Under the CAA policy, the salvation of larger airports might very well lie in the words "Additional runways will be approved only where required to handle the volume of traffic." Assuming a traffic requirement of 60 plane movements per peak hr at a major airport (and few airports exceed this figure), how many runways are needed? In fair weather 1 runway can handle one plane movement per min, or 60 movements per hr. An analysis of peak hour requirements, however, must include the characteristics of the peak and the weather conditions. Records show that 60% or 36 movements will occur 20 min and will consist, on the average, of 24 arrivals and 12 departures.

Under adverse weather conditions, even with such navigational aids as ground-controlled approach, instrument landing system, and high intensity lighting, the landing rate drops to 1 airplane every 3 min. Thus, the 24 arrivals in 20 min will require 4 runways, although 1 more runway is needed for the 12 departures. A total of 5 runways are, therefore, needed to accommodate 60 plane movements per peak hr. A further argument for multiple runways is the fact that air traffic is expanding, and it can reasonably be expected to result ultimately in at least 3 times the present number of plane movements. On the other hand, the further development of electronic aids will doubtlessly decrease the interval between plane movements. These two elements tend to compensate for one another.

Considering all the elements, multiple runways should be built to meet traffic requirements under adverse weather conditions, and such runways should be nonintersecting, so as to allow their simultaneous use. Far from being wasted, such runways will not only provide adequate wind coverage for planes that are not equipped for safe operation under high cross winds, but also will provide the extra capacity for the inevitable growth of aviation when future planes are equipped so that cross winds can safely be disregarded.

SUMMARY

A definite trend exists toward a greater tolerance as to cross winds. This has been brought about partly by higher landing speeds of large aircraft, resulting in a decreased angle of crab in landing and partly by the general use of tricycle landing gear with steerable nose wheel. This trend will be widened as the use of castering gear expands. Castering gear appears of greater advantage to small planes. Its general use on small planes will greatly increase the utilization of single-strip airports. When castering gear has been demonstrated as practical for larger planes, it will increase their ability to serve single-strip runways, but not to a point at which definite schedules can be guaranteed.

Major airports at which the importance of dependable scheduled operation is paramount and at which the CAA authorizes such navigational aids as ground-controlled approach, instrument landing system, and high intensity lighting to provide uninterrupted service still require multiple runways for safe operation under peak load conditions. Multiple runways in a nonintersecting pattern will provide for present safety and high utilization and also will prove adequate for future traffic expansion. The CAA policy if applied realistically as to traffic requirements at major airports will, therefore, be beneficial. It will especially aid small aircraft operating on single runway airports and will hasten the day when improved landing gear on large aircraft will increase the capacity of major airports.

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Paper No. 2511

MISSISSIPPI RIVER VALLEY GEOLOGY RELATION TO RIVER REGIME

BY HAROLD N. FISK[1]

WITH DISCUSSION BY MESSRS. LEO M. ODOM, STAFFORD C. HAPP, AND HAROLD N. FISK

SYNOPSIS

The Mississippi River alluvial valley developed concurrently with the advance and retreat of the final stage of continental glaciation. Sea level dropped as the ice accumulated, and the Mississippi River system eroded a deeply entrenched valley system. A rise in sea level when the ice melted reduced stream gradient, the river was forced to drop its load, and the valley system became filled with a gradational sequence of alluvium. Characteristics of the alluvial deposits show that the river was an overloaded braided stream most of the time that sea level was rising and that the river did not take on its meandering habit throughout the entire length of the valley until sea level reached a stand about 5,000 years ago. The river has developed and abandoned several courses, the positions of which are still visible in traces of river migration or well-defined zones of meander belts. Each ancient course is marked by an alluvial ridge along its meander belt and by a characteristic pattern of distributary streams in its delta area.

Remaining alluvial features show that the river has maintained a close adjustment between its load, slope, and the bed and bank materials. The river is said to be poised because it has shown no tendency either to aggrade or degrade its channel. The adjustment to bed and bank materials is shown by the local and regional characteristics of the channel and by its rate of migration. A broad, shallow, shifting channel is developed in sandy bed and bank materials, whereas a narrow, deep, slowly-migrating channel takes shape in fine-grained deposits. Each of several course diversions has left a clear record of slow and orderly development. The modern Atchafalaya River-Mississippi River relationship suggests an early stage in course diversion.

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INTRODUCTION

A great deal of information concerning the surface and subsurface features of the alluvial valley of the lower Mississippi River was collected and analyzed in connection with a geological investigation of the region conducted for the Mississippi River Commission. The project was undertaken to determine the origin and development of the alluvial valley and, from the geological relationships, to evaluate the factors controlling Mississippi River behavior. Results have been published in two reports,[2,3] and those points which demonstrate the relationship between geological features and the river's regimen are briefly summarized in this paper.

THE ALLUVIAL VALLEY

The alluvial valley of the lower Mississippi, a broad lowland varying in width from 25 miles to 125 miles, extends for more than 600 miles across the Gulf coastal plain from Cape Girardeau, Mo., to the Gulf of Mexico (Fig. 1) and covers an area of approximately 50,000 sq miles. This valley is primarily a region of alluviation which has developed as a result of long-continued depositional action of the Mississippi River and its tributaries. Throughout almost the entire length of the valley, the lowland surface, or alluvial plain, is bordered by abrupt walls which gradually lose elevation in a southward direction and disappear where the upland surface slopes beneath the coastal marshlands of Louisiana. The valley walls are breached where the flat-surfaced alluvial plains of tributary valleys merge with the surface of the Mississippi valley.

The continuity of the plain is broken by two ridges: (1) Crowleys Ridge, centrally located in the northern part of the valley, rises 200 ft above the plain and extends for 200 miles south of Commerce, Mo.; and (2) Macon Ridge rises to a maximum of 40 ft above the valley lowlands and extends from Eudora, Ark., to Sicily Island, La., a distance of approximately 100 miles. The plain has an elevation of more than 300 ft at the north end of the valley and has a very low regional slope to the Gulf of Mexico. The slope gradually decreases southward and is practically nonexistent south of central Louisiana.

GENERAL GEOLOGIC FEATURES

Data from thousands of scattered borings, supplemented by detailed information from several trans-valley lines of close interval borings, show the surface of the alluvial valley to be underlain by a thick mass of alluvial deposits that grade upward from a substratum of sands and gravels into a top stratum of silty sands and silty clays (Fig. 2). The substratum comprises most of the mass at the north end of the valley, where it almost reaches the surface, but near the Gulf of Mexico the substratum is deeply buried beneath the top stratum (Fig. 3). The alluvium buries a rugged valley system in which trenches of the major tributaries extend inland from their junctions with the master trench. The valley walls are the surface manifestations of the margins

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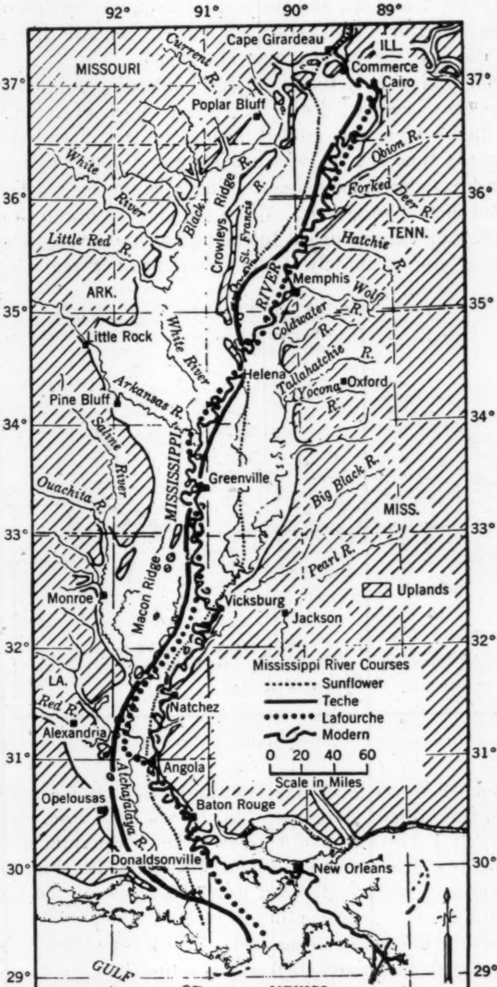
[2] "Geological Investigation of the Alluvial Valley of the Lower Mississippi River," by Harold N. Fisk, Mississippi River Comm., Vicksburg, Miss., December, 1944.

[3] "Fine-Grained Alluvial Deposits and Their Effect on Mississippi River Activity," by Harold N. Fisk, Waterways Experiment Station, Vicksburg, Miss., July, 1947.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MISSOURI
ILL.
Cape Girardeau
Commerce
Cairo
Current R.
Poplar Bluff
Obion R.
White River
Black R.
Ridge
Crowleys Ridge
St. Francis R.
Forked Deer R.
Little Red R.
Hatchie R.
TENN.
Memphis
Wolf R.
ARK.
RIVER
Coldwater R.
Little Rock
White River
Tallahatchie R.
Helena
Oxford
Yocona R.
Arkansas R.
Pine Bluff
Saline River
MISSISSIPPI
Greenville
Ouachita R.
Big Black R.
MISS.
Pearl R.
Macon Ridge
Uplands
Monroe
Vicksburg
Jackson
Mississippi River Courses
Sunflower
Teche
Lafourche
Modern
LA.
Red R.
Natchez
Alexandria
0 20 40 60
Scale in Miles
Angola
Atchafalaya R.
Opelousas
Baton Rouge
Donaldsonville
New Orleans
GULF OF MEXICO

FIG. 1.—THE MISSISSIPPI RIVER ALLUVIAL VALLEY

of the main stream trenches. Crowleys Ridge marks the divide between the upper Mississippi and Ohio river trenches, and Macon Ridge marks the divide between the Arkansas and lower Mississippi river trenches. Divides between minor tributary valleys are all buried beneath the alluvial mass, although they lie near the surface in several areas.

The alluvial valley is actually a valley-in-valley feature reflecting the cyclical events that occurred during the latest epoch of geologic time. The entrenched valley system was eroded during the last glacial stage (late Wisconsin) during the time the sea was being lowered 450 ft as water was withdrawn to form huge glacial ice masses. These ice masses reached their climax of growth about 30,000 years ago. When the glaciers began to disappear, the waters from their melting returned to the oceans and caused the sea to reach its present level approximately 5,000 years ago. Stream gradients were relatively steep when sea level was low, and sands and gravels were transported beyond the present shoreline. As the sea level rose, stream gradients were progressively decreased, and a great wave of alluvium spread upstream, resulting in the deposition of progressively finer particles (Fig. 3).

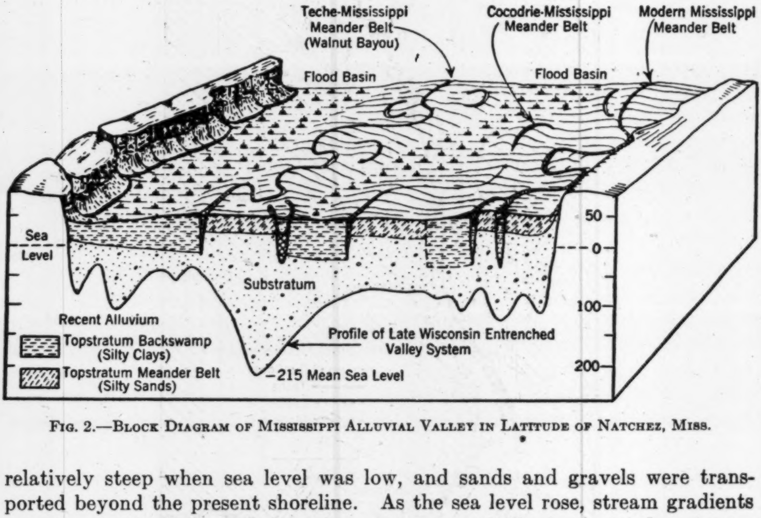
Teche-Mississippi Meander Belt (Walnut Bayou)
Cocodrie-Mississippi Meander Belt
Modern Mississippi Meander Belt
Flood Basin
Flood Basin
50
Sea Level
0
Substratum
100
Recent Alluvium
Topstratum Backswamp (Silty Clays)
Topstratum Meander Belt (Silty Sands)
Profile of Late Wisconsin Entrenched Valley System
200
-215 Mean Sea Level

FIG. 2.—BLOCK DIAGRAM OF MISSISSIPPI ALLUVIAL VALLEY IN LATITUDE OF NATCHEZ, MISS.

The sedimentary mass laid down near the shore of the Gulf of Mexico during and subsequent to the rise in sea level is much thicker than any that could result from a 450-ft rise in sea level. Both top stratum and substratum are present and their combined thickness is more than 600 ft. The excessive thickness of the section indicates that the river mouth region has been sinking during the period of sedimentation.

THE MISSISSIPPI RIVER

The lower Mississippi River starts at Cairo, Ill., at the junction of the upper Mississippi-Missouri rivers and the Ohio River and follows a crooked

1,000-mile path to reach the Gulf of Mexico. Its course lies close to the eastern wall of the alluvial valley except in the central part of the region, south of Memphis, Tenn., where it swings far to the west of Greenville, Miss. Its flow is divided near Angola, La., where the principal distributary, the Atchafalaya River, starts a relatively short course to the Gulf of Mexico. The main Mississippi River flow continues past New Orleans, La., and then separates into several minor distributaries (or passes) which form the river mouth delta. The master stream is joined on the western side of the valley near Greenville by waters of the Arkansas and White river systems, and near Angola by those of the Red and Ouachita river systems (Fig. 1). Tributaries from the eastern side of the valley are all small, the Yazoo River—which joins the Mississippi near Vicksburg, Miss.—being the largest.

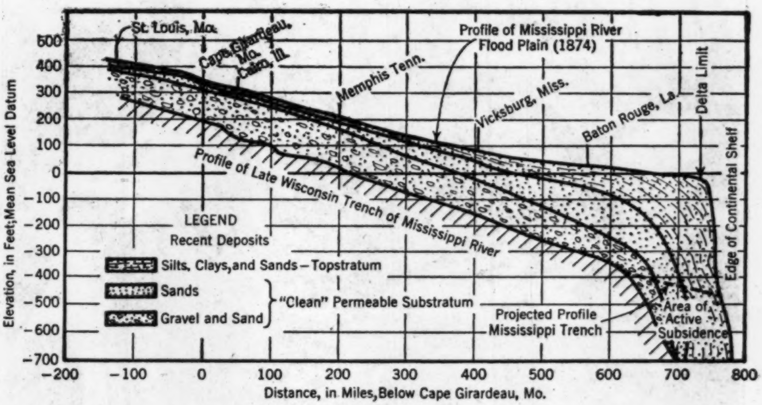

FIG. 3.—GENERALIZED LONGITUDINAL CROSS SECTION OF THE ALLUVIAL MISSISSIPPI VALLEY

An outstanding feature of the Mississippi River is the active migration of its channel. Bank surveys dating from 1765 show that the channel has migrated rapidly and over wide areas in the northern part of the valley. Below Donaldsonville, La., its migration has been very slow, and the shift in river position has been slight. Inequalities in the erosional and depositional activity of the flowing water mass are directly responsible for channel migration. The highest water velocities and deepest channel are at points of bend (bendways) where the thalweg lies close to the concave river bank. Bendways are separated by "crossings" in which the channel is comparatively broad and shallow and the velocities highly variable. Scouring is most intense in the thalweg of bendways, and bank retreat occurs when the concave bank becomes oversteepened and caves into the river. Coarse sediments derived from river bank caving are transported downstream and deposited on sand bars in the shallow slackwater areas along the convex bank of the downstream bend. Thus, the retreat of the concave bank is accompanied by concurrent advance of the convex bank as a result of accretion. Irregularities in the retreat of the concave river bank are caused by normal changes in channel alinement, resulting from variation in river stage and by local resistance to bank caving. The advance of the convex bank is also irregular and is marked by a succession of sand bar ridges and intervening swales. A bend enlarges as the bank-caving and bar-building processes continue, and the channel may eventually form a meander loop. The history of development of the loop is faithfully recorded in the arrangement of point bar ridges and swales (the accretion topography) within the loop.

Shortening of the river is accomplished by a neck "cutoff" when the upstream and downstream arms of the loop intersect. The ends of the cutoff channel become plugged with sediment, and the remaining portion of the channel forms an oxbow lake which eventually becomes filled with a "clay plug" of silts and other fine-grained alluvial materials. Shortening of the channel is also brought about by "chute" cutoffs which follow a short channel (or chute) across the point bar. Such channels are originally established in swale lowlands between sand bar ridges and gradually enlarge to carry the main river flow. This type of cutoff is more common above Memphis, where it gives rise to a number of large islands.

MISSISSIPPI RIVER

Fig. 4.—Aerial Photograph of False River, La.

Channel migration occurs within wide zones called meander belts, the limits of which can be accurately established from aerial photographs and topographic maps. Within such a zone channel migration is indicated by abandoned channels, accretion topography, oxbow lakes, and similar phenomena. Many cutoff meander loops line the channel north of Donaldsonville, and their associated accretion topography provides an excellent tool for interpreting the history of river migration. One of the larger meander loops, False River, La., which was cut off in 1722, is shown in Fig. 4, the aerial photograph mosaic.

The river experienced wide variations in level and under natural conditions frequently overflowed its banks. When the river topped its bank, sediments were piled up in natural levees (wedge-like ridges extending landward from the river bank). These levees are better drained than other flood-plain features and form a large part of the cultivated lands in the valley. A typical one borders the cutoff meander loop occupied by False River. Natural levees line the entire course of the river and, as a result, the stream flows within an alluvial ridge which rises a few feet above the lowland surface.

South of Donaldsonville the ridge is a simple feature made up of broad continuous landward-sloping levees on either side of the channel. To the north, however, it is a complex feature made up of a large number of natural levee elements which border abandoned channel segments within the meander belt, as well as the active channel.

THE ALLUVIAL PLAIN

The surface of the alluvial valley is a wide plain of very low relief that has been constructed through depositional activity of the Mississippi River and its tributaries. The nature and distribution of drainage lines and of such topographic features as natural levees, bar accretions, and old channels provide evidence of long-continued stream action and demonstrate that each of the rivers has occupied and abandoned several courses while the plain has been under construction. Most prominent of the depositional features are the meander belt ridges marking ancient courses of the Mississippi River. Each ridge now holds traces of river activity comparable to that of the modern Mississippi, and each course branches near the coast into a pattern similar to that formed by the Mississippi distributaries. The generalized position of the axial lines of three of the old Mississippi courses is shown in Fig. 1. Although ancient meander belts of the Red, Arkansas, and White rivers, and other large tributaries are minor features, they are well defined and easily traceable on aerial photographs.

The various alluvial ridges formed by the Mississippi River separate the valley into flood-basin lowlands called backswamps (Fig. 2). Typical of these is the Yazoo River Basin (or "delta") in which floodwaters are impounded between the eastern valley wall and the Mississippi meander belt ridge between Memphis and Vicksburg. Low alluvial ridges, such as the one marking the ancient Mississippi-Sunflower course separate the basin into minor lowland elements. The extensive Atchafalaya Basin in southern Louisiana lies between the alluvial ridges of the ancient Teche-Mississippi course and the modern river. It is separated into minor basins by the alluvial ridge of the Lafourche-Mississippi course and by a ridge along the Atchafalaya River. The heavily forested flood basins hold a complicated network of drainage lines which serve both to introduce floodwaters and to release ponded waters as a flood subsides. Near the coast the basins merge with grass-covered marshlands and are connected with the Gulf of Mexico through a series of minor bayous and lakes.

The three main sections distinguishable in the alluvial plain are shown in Fig. 5, together with the sedimentary deposits which underlie them. In Fig. 5, the meander belt deposits include bar accretions, channel fillings, and natural levees. The alluvial fan deposits are locally overlain with thin backswamp deposits. Included in these backswamp deposits are fresh water sediments laid down in flood basins. The deltaic plain deposits consist of backswamp, marsh, and marine deposits. The flood plain is the most extensive of these sections and covers a total area of 22,000 sq miles. This flood plain includes all the meander belt ridges and flood-basin lowlands in the valley that were subject to flooding under natural conditions. The deltaic plain is the most continuous section of the valley. It includes the near sea level lands, south of the head of the Atchafalaya River, which bear traces of river distributaries marking the deltas of the several courses of the river. The deltaic plain embraces an area of 13,000 sq miles and merges with the flood plain around its northern margin. The alluvial fan section lies in the northwestern and northern parts of the valley and rises slightly above normal flood level. This section consists of approximately 15,000 sq miles of slightly dissected lands that bear traces of braided stream courses and are the remnants of the old alluvial fans which were built into the valley surface while sea level was rising.

THE TOP STRATUM ALLUVIUM

The distribution and lithology of the top stratum are closely related to environments within the valley. The deltaic plain, flood basins, alluvial fans, and several units in the meander belts, all receive deposits of fine-grained sediments and have developed in the course of valley aggradation.

Viewed as a longitudinal unit extending along the valley through its principal environments of deposition, the top stratum is a wedge that thickens in a seaward direction, composed of relatively impermeable deposits grading from thin silty sands at the north end of the valley into a thick, predominantly silty clay mass near the Gulf of Mexico. This wedge is evident in Fig. 3. The top stratum materials everywhere grade downward into the "clean" sands and gravels of the substratum, reflecting the gradual changes which occurred in each of the environments as sea level rose. Along the Mississippi meander belts, where the river scours into the substratum over much of its course, scouring action and river migration have eroded the original top stratum and replaced it with meander belt deposits in which the top stratum element is less widespread and generally much thinner. As a result of meander belt development, the top stratum over much of the valley is divided into a series of long, thick prisms with intervening thinner and less regular masses, as shown in Fig. 2, the cross section of the narrow part of the valley near Natchez, Miss. Thicknesses of the regional top stratum wedge have been measured at several points where this wedge forms the river bank at the edge of the modern meander belt. The elevation of the base of the top stratum at these points is shown by

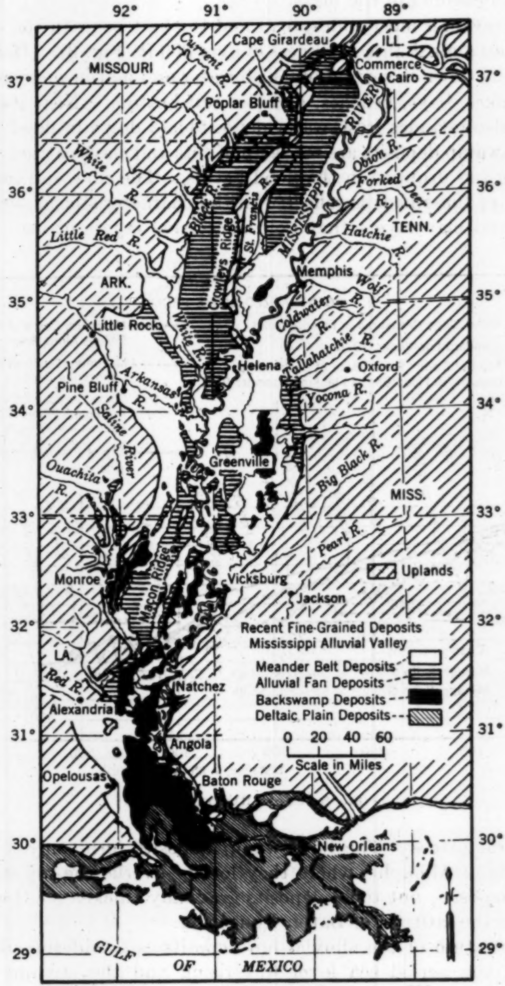
92°
91°
90°
89°
37°
36°
35°
34°
33°
32°
31°
30°
29°
MISSOURI
ILL.
Cape Girardeau
Current R.
Commerce
Cairo
Poplar Bluff
MISSISSIPPI RIVER
White R.
Obion R.
Forked Deer R.
Black R.
St. Francis R.
Crowleys Ridge
TENN.
Hatchie R.
Little Red R.
ARK.
Memphis
Wolf R.
Coldwater R.
Little Rock
White R.
Tallahatchie R.
Helena
Oxford
Pine Bluff
Arkansas R.
Yocona R.
Saline River
Greenville
Big Black R.
Ouachita R.
MISS.
Pearl R.
Macon Ridge
Monroe
Vicksburg
Jackson
Uplands
Recent Fine-Grained Deposits Mississippi Alluvial Valley
Meander Belt Deposits
Alluvial Fan Deposits
Backswamp Deposits
Deltaic Plain Deposits
LA.
Red R.
Natchez
Alexandria
0 20 40 60
Scale in Miles
Angola
Opelousas
Baton Rouge
New Orleans
GULF OF MEXICO

FIG. 5.—GENERALIZED DISTRIBUTION OF TOP STRATUM DEPOSITS IN THE MISSISSIPPI ALLUVIAL VALLEY

the line marked "top of clean sand" on the diagram (Fig. 6), demonstrating the seaward thickening of the mass.

The top stratum is most widespread and thickest beneath the deltaic plain, where it reaches a maximum thickness of over 300 ft (Fig. 3). Here it consists of materials laid down in several brackish water and marine environments, such as marshes, lakes, bays, beaches, and the gulf bottom. Near its margins the top stratum also includes fresh water deposits laid down in flood basins.

The backswamp deposits are made up of silts and clays carried in suspension by floodwater and dropped in the flood basins. The slow and long-continued accumulation of these beds is reflected in the characteristic sequence of homogeneous silt and clay layers that hold successive layers of logs and roots of trees in several places. The backswamp deposits form only a thin mantle in the latitude of Memphis where they locally overlie the top stratum of the alluvial fan deposits, but these deposits gradually thicken southward to more than 100 ft in the latitude of Baton Rouge, La.

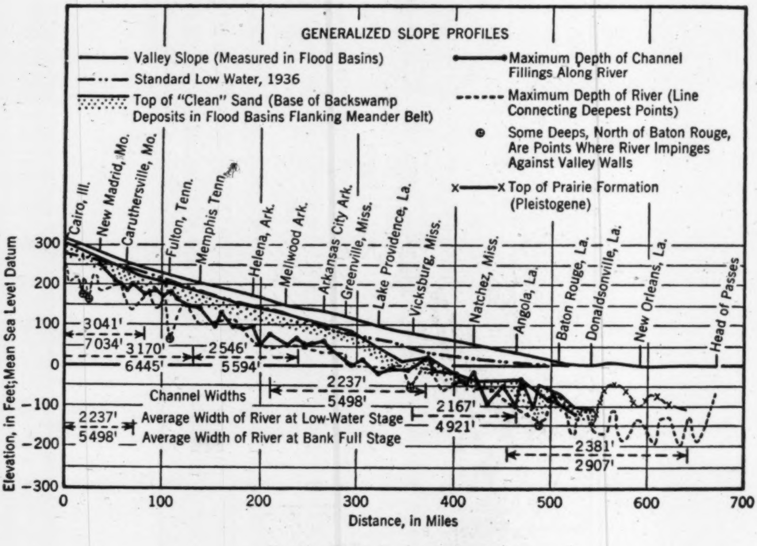

FIG. 6.—SLOPE RELATIONSHIPS

The top stratum of the alluvial fan deposits is a widespread mantle laid down during the period sea level was rising and the streams had steeper gradients. It is relatively coarse and consists primarily of silty sands with local channel fillings of silty clay. The entire mass is thin, generally not more than 20 ft thick.

Most complicated of the top stratum materials are those laid down in natural levees, point bar accretions, and abandoned channel fillings of meander belts. In the southern part of the valley natural levees are composed mainly of silty clays and reach a maximum thickness of 25 ft. At the north end of the valley the natural levees are composed of silty sands, and few are more than 15 ft thick. The point bar accretions are blanketed with an irregular thickness of silty sands and silty clays, sand bar ridges are overlain with silty sands, and swales are underlain by somewhat thicker silty clays.

The abandoned channel fillings (or clay plugs) are thicker than any other meander belt top stratum deposit. They are thickest in the position of the channel thalweg in bendways and in the area across the ends of the channel arms in which a deep scour pool forms when the cutoff is accomplished. Fig. 7 indicates generalized thicknesses of the top stratum within the channel and over the scour pool of the abandoned Lake Chicot meander, in which the clay plug filling has been adequately tested by borings. The clay plugs have been mapped within the modern meander belt throughout the valley and their thickness, as determined from borings, is shown in Fig. 6. The maximum thickness of the plugs conforms to the maximum depths of abandoned channels and the thickness increases from an average of 60 ft in the north end of the valley to over 120 ft near Donaldsonville.

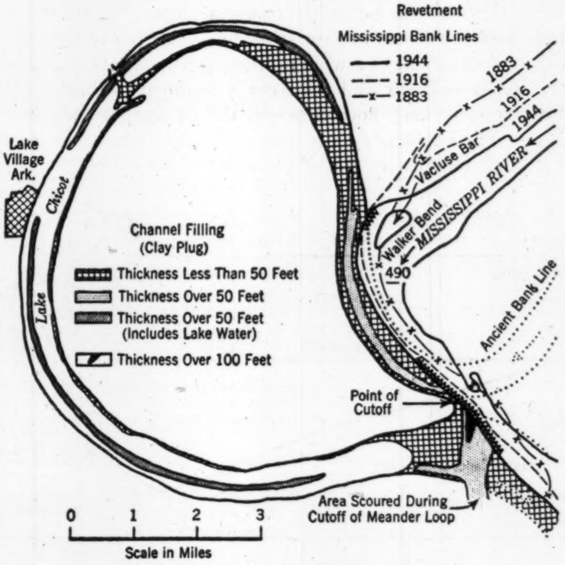
Revetment
Mississippi Bank Lines
1944
1916
1883
1883
1916
1944
Vacluse Bar
Walker Bend
MISSISSIPPI RIVER
490
Ancient Bank Line
Lake Village Ark.
Lake Chicot
Channel Filling (Clay Plug)
▨ Thickness Less Than 50 Feet
▨ Thickness Over 50 Feet
▨ Thickness Over 50 Feet (Includes Lake Water)
▨ Thickness Over 100 Feet
Point of Cutoff
Area Scoured During Cutoff of Meander Loop
0
1
2
3
Scale in Miles

FIG. 7.—LAKE CHICOT CUTOFF MEANDER

CONTROL OF RIVER BEHAVIOR BY ALLUVIUM

The characteristics of the Mississippi River reflect the adjustment of its load and great volume of flow to the valley slope and to the bed and bank materials in which its channel is scoured. The river everywhere scours into the alluvial mass, and its adjustment to the bed and bank materials is well shown by both the local and regional characteristics of the channel and by the rate of stream migration.

The relationship between high stage and low stage channel widths and bed and bank materials is summarized in the profiles of Fig. 6. The channel is wide and shallow at the north end of the valley above Memphis, where the slope is relatively steep and substratum sands and gravels form most of the bed and bank materials below the low waterline. The channel gradually narrows and deepens in a southward direction on the intermediate valley slopes between Memphis and Donaldsonville. These channel changes can be correlated with the gradual increase in thickness of the top stratum throughout this section that extends below the low waterline. Where the river scours into the substratum, its rate of migration has been relatively rapid and has resulted in the development of a wide meander belt. South of Donaldsonville, where the valley slope is low and the river is confined to a thick mass of fine-grained alluvium and valley-floor deposits, the channel is deep and comparatively narrow. The extremely slow rate of channel migration in this section is reflected in the comparative straightness of the stream course.

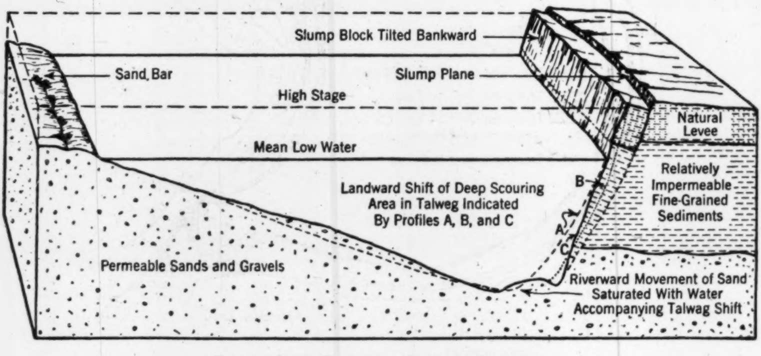
Slump Block Tilted Bankward
Sand Bar
Slump Plane
High Stage
Natural Levee
Mean Low Water
Relatively Impermeable Fine-Grained Sediments
B
Landward Shift of Deep Scouring Area in Talweg Indicated By Profiles A, B, and C
A
C
Permeable Sands and Gravels
Riverward Movement of Sand Saturated With Water Accompanying Talwag Shift

FIG. 8.—BANK RECESSION THROUGH SLUMPING

Top stratum materials effectively hinder local as well as regional channel migration. Thick channel fillings have long been known to form "hard points" that resist bank caving, and thereby alter the shape of the channel cross section and that of river bends, and bring about changes in the river alinement. The greater resistance offered to bank caving by the top stratum as compared to the resistance of sandy bed and bank materials reflects the physical properties of its mass. Inasmuch as fine-grained silty and clayey beds are more cohesive than sands and are relatively impermeable, they are not greatly influenced by fluctuations in hydrostatic pressure resulting from changes in river stage. The top stratum reacts as a unit when the bank caves. Large masses, called slump blocks, slip into the river, rotate, and are tilted bankward (Fig. 8). The resultant indentations give a scalloped bank line. Conversely, bank lines in sandy sediments are smooth because sands behave as a collection of individual particles and sandy banks continually retreat under river attack through a gradual process called sloughing.

The primary cause of bank caving is known to be scouring in the thalweg of bendways. This has been proved by dredging operations. In order to correct river alinement in some bendways, the Corps of Engineers, United States Army, have dredged in the thalweg and forced bank recession of the concave bank. The principle has also been proved experimentally through careful studies with large models.[4]

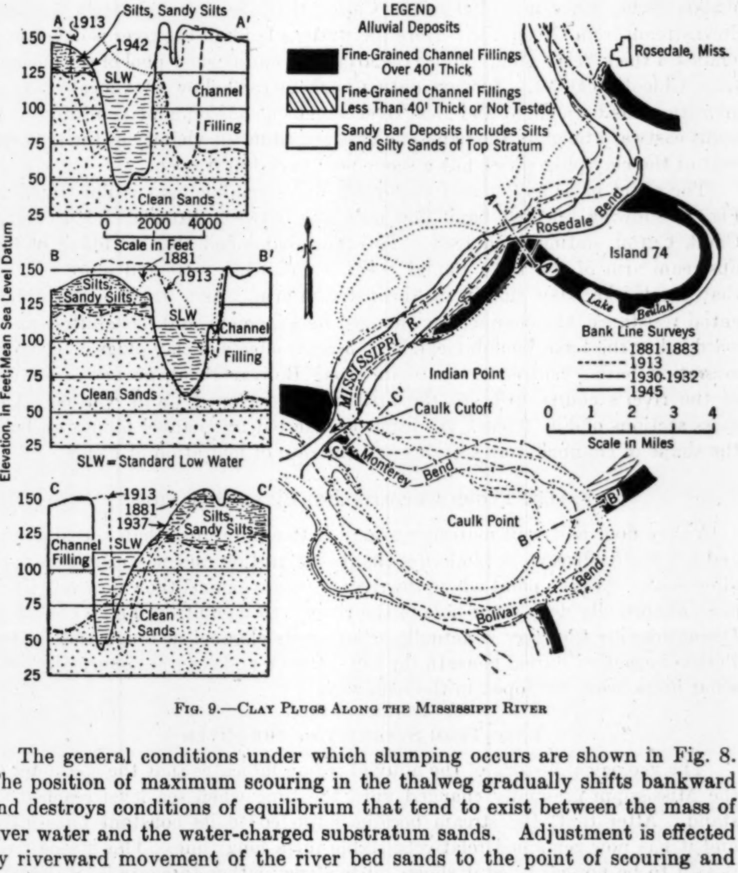
A
1913
Silts, Sandy Silts
A'
1942
SLW
Channel
Filling
Clean Sands
0 2000 4000
Scale in Feet
B
1881
1913
B'
Silts, Sandy Silts
SLW
Channel
Filling
Clean Sands
SLW = Standard Low Water
C
1913
1881
1937
C'
Silts, Sandy Silts
Channel Filling
SLW
Clean Sands
150
125
100
75
50
25
Elevation, in Feet; Mean Sea Level Datum
LEGEND
Alluvial Deposits
Fine-Grained Channel Fillings Over 40' Thick
Fine-Grained Channel Fillings Less Than 40' Thick or Not Tested
Sandy Bar Deposits Includes Silts and Silty Sands of Top Stratum
Rosedale, Miss.
Rosedale Bend
Island 74
Lake Beulah
MISSISSIPPI R.
Indian Point
Caulk Cutoff
Monterey Bend
Caulk Point
Bolivar Bend
Bank Line Surveys
1881-1883
1913
1930-1932
1945
0 1 2 3 4
Scale in Miles

FIG. 9.—CLAY PLUGS ALONG THE MISSISSIPPI RIVER

The general conditions under which slumping occurs are shown in Fig. 8. The position of maximum scouring in the thalweg gradually shifts bankward and destroys conditions of equilibrium that tend to exist between the mass of river water and the water-charged substratum sands. Adjustment is effected by riverward movement of the river bed sands to the point of scouring and results in an undermining of the bank. Riverward movement of sands is possible because the sands are saturated with water that may be under a higher hydrostatic stress than the water in the river. The difference in hydrostatic stress may be caused by a falling river stage, a high ground-water level, turbulence in the river, or a combination of these factors. If the bank consists primarily of sandy materials, adjustment is translated upward through the grains, and sloughing occurs immediately. If the bank is formed of the cohesive substratum, however, scouring may proceed until the channel is greatly overdeepened, and the withdrawal of the material from beneath the top stratum causes the bank to collapse and to slump.

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[4] "A Laboratory Study of the Meandering of Alluvial Rivers," by Joseph F. Friedkin, Mississippi River Comm., Vicksburg, Miss., 1945.

Many areas illustrate the control exerted on river migration by thick top stratum materials in local bank areas. An excellent example is afforded by Walker Bend (river mile 490 below Cairo) that lies approximately 5 miles downstream from Greenville. Here the western bank of the river is developed against a thick "clay plug," filling the arms and cutoff scour pool of the ancient Lake Chicot meander. Surveys dating back to 1883 show that local channel migration has been slight and local river alinement has remained constant. A slight eastward bulge in the bank line has formed in the vicinity of the thickest part of the clay plug which fills a scour pool over 100 ft deep.

The effect of widely spaced clay plugs on river alinement is illustrated in Fig. 9, showing selected bank line positions between Rosedale (Miss.) and Caulk Cutoff, dating from 1881. The thin abandoned channel filling of the upstream arm of the Lake Beulah meander loop has retreated more rapidly than the thicker clay plug in the downstream arm. As a result of this differential resistance, the channel has changed its alinement and has shifted westward below the Lake Beulah meander. Similar effects of clay plugs on channel migration have occurred downstream along Bolivar Bend, an active section of the river's course prior to the artificially developed Caulk Cutoff. The cross sections of Fig. 9 show the influence exerted by channel fillings on both the shape of channel cross sections and the rate of migration of bends.

VALLEY FLOOR DEPOSITS AND RIVER BEHAVIOR

Valley floor and wall materials are all better indurated than the alluvium and are more effective in hindering channel migration and in controlling river alinement. At each point where the river impinges against the valley wall there is an abnormally deep scour hole in the river. (See points on Fig. 6.) South of Donaldsonville the river is actually scouring its channel in tough soils of the Prairie formation buried beneath the top stratum, and several abnormally deep scour holes have developed in the bendways.

LONG-TERM STABILITY OF THE RIVER

The geological history of the alluvial region indicates that the character of the Mississippi gradually changed until the time when sea level reached its stand. After that, the stream became adjusted to its constant base level, and it has now remained relatively stable for a long time. This stable river is said to be poised[5] since it shows no tendency either to aggrade or degrade its channel. The absence of appreciable quantities of silt and clay in the substratum and the presence of many lentils of well-sorted gravels indicate that the Mississippi River was a comparatively fast-flowing, overloaded stream that occupied many shallow channels as sea level rose. At that time the behavior of the river on the steep valley slope was in all probability similar to that of modern braided streams such as the Platte River. The braided character of abandoned channel scars of Mississippi origin may still be seen on the surface of the alluvial fans at the north end of the valley west of Cairo (Fig. 5). The river did not begin to meander until the sea had almost reached its present level. The meandering habit started on the relatively low slope of the valley near the river mouth and gradually developed upstream as valley aggradation progressed and the upstream slope was lowered. Eventually sea level reached its stand, and a permanent adjustment was reached between slope, volume of flow, and load introduced by tributaries. After that the meandering channel of the lower Mississippi developed throughout the entire length of the valley.

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[5] "Basic Aspects of Stream Meanders," by Gerard H. Matthes, *Transactions*, Amer. Geophysical Union, Vol. 22, 1941, p. 632.

RIVER DIVERSIONS

Each time the Mississippi was diverted from a long-occupied course, traces of river activity were left in the various meander belts, and the cause of each diversion can be interpreted from flood-plain features. New courses became established in flood basins and developed because their shorter paths to the sea provided a gradient advantage. They appear to have developed slowly and in an orderly manner.

Most of the ancient diversions occurred when an actively migrating Mississippi meander loop reached an adjacent flood basin and intersected a tributary stream that had a channel sufficiently deep and open to carry low-stage flow from the river. Typical of such changes was the shift from the Teche-Mississippi to the modern course near Vicksburg when a migrating meander loop extended to the channel of the Yazoo River. Somewhat different conditions led to the development of the Atchafalaya River that carries a large volume of water from the Mississippi. Mississippi waters enter the Atchafalaya by way of Old River, a short stream which follows a lower arm of an ancient meander loop of the Mississippi. The Red River enters the upstream arm of the same loop and originally joined the Mississippi at the head of Old River. When the meander loop was breached, waters from both the Red River and Old (Mississippi) River flowed seaward through a network drainage channel within the Atchafalaya Basin. It is probable that the original enlargement of the Atchafalaya was accomplished primarily by the scouring action of the Red River rather than by that of the Mississippi. As the channel enlarged, an increasing quantity of Mississippi water passed through it.

Ancient courses of the Mississippi, such as the Teche, Lafourche, or Sunflower (Fig. 1), all present excellent evidence of diminishing flow prior to complete abandonment. The full-flow course may be seen outlined on photographs by natural levees and other distinctive features, but, in addition, the channel itself is filled with narrow accretion ridges. These depositional features have caused a narrowing and local straightening of the channel with a few abnormally sharp bends. Because there is no evidence of channel migration accompanying the filling of the channel, it is clear that the stream flow must have been greatly decreased before deterioration set in.

No record of the widening of the new channel is preserved, but the increase in flow is recorded in progressive course lengthening across the deltaic plain. The course grows seaward only after the Gulf bottom has been built to sea level and low natural levees can confine the flow. Many distributaries develop as sedimentation continues, but only those with favored alinements continue to carry water from the river. The final pattern of streams in the delta area is composed of a few large distributaries branching from a main channel near the coast with traces of many abandoned distributaries forking from the channel farther inland. Some evidence exists in this stream pattern that the oldest distributaries had narrow channels and were abandoned in the initial stages of course development during the time the river was carrying less volume.

Judging from ancient diversions, the Atchafalaya-Mississippi relationship illustrates an early stage in course diversion. The Atchafalaya is known to be actively enlarging its channel, and its course is being lengthened seaward. No channel deterioration effects, such as those which mark older courses, are as yet visible downstream in the Mississippi, and it is probable that they will not develop until the main low water flow follows the Atchafalaya.

Acknowledgment

The geological work in this paper was done under the direction of the writer. It was initiated in May, 1941, by Maj.-Gen. M. C. Tyler, M. ASCE, President of the Mississippi River Commission (Vicksburg, Miss.), and was under the general supervision of Gerard H. Matthes, Hon. M. ASCE. The work was continued during the administration of Maj.-Gen. R. W. Crawford during which time it was supervised by Charles Senour, M. ASCE. The project was concluded in July, 1948.

DISCUSSION

Leo M. Odom,[6] M. ASCE.—The geological report on the alluvial valley of the Mississippi River, prepared by the author for the Mississippi River Commission, that is outlined briefly in this paper, is a true milestone in the work of collecting and analyzing information concerning the great river and its valley. This collection and analysis has been in progress since the earliest settlements were located along its banks.

During past years students have arrived at various conclusions regarding the geology of the valley. Sir Charles Lyell in 1845, for instance, deduced that the valley plan of the Mississippi has been subjected to great oscillation of level varying from 100 ft below its present level to its former position. Each theory has been more or less discounted or has been proved to be insufficient in scope by subsequently available data.

The author's investigation was made at a time when it appears that practically everything necessary for a study had finally been secured. The geology of the continent is now well understood. Thousands of borings, many of them of great depth, had been made and logged; but of probably the greatest value, a great portion of the valley had been mapped by aerial photography, furnishing a new tool for the study of land forms more accurate and much more detailed than had ever been available before.

The need for a new comprehensive geological study in the light of the wealth of information accumulated since the last effort was one of the prime problems that faced Major-General Tyler when he became president of the Mississippi River Commission. The cutoff program having been largely completed, stabilization of the river was the next big problem. An over-all picture of the characteristics of bed and banks could only be obtained by such a study. General Tyler was fortunate to obtain the services of a man with the competence and vision of the author for this study. The geological investigation carried on concurrently with a laboratory investigation of the meandering of alluvial rivers and various researches on type and effects of revetments, dikes, dredging, and other factors have provided a greatly expanded volume of dependable information for the guidance of engineers engaged in river work.

The geological background of the river with which it is necessary to deal today, as developed by the author, would have been beyond the scope of the training and imagination of most engineers. The fact that it offers a reasonable and productive concept of the formation of the valley must be gratefully accepted. The portion of the paper devoted to the mechanics of meandering and alluviation is, however, within the field of every-day experience and treats of phenomena that have been the subject of engineering investigations since the earliest days of work on the control of rivers.

It is granted that the generalized statements concerning scour, fill, and meandering are probably sufficient for the purposes and scope of the paper.

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[6] Cons. Engr., Baton Rouge, La.

For engineering usage, however, no such positive rules can be accepted. For example, there are many locations on the river at which the deepest holes and maximum caving are far removed from the concave bank. At Point Menoir (Mile 253 above Head of Passes) and Free Nigger Point (Mile 234 above Head of Passes) there are deep scour holes caused by eddies very near the convex bank. In the bights of these bends depths near the concave bank are relatively small. These bends come under the class of those mentioned by the author in the section on "Control of River Behavior by Alluvium" as occurring where the river impinges against the valley wall. However, he does not mention the locations of the scour holes. At many other locations the greatest depth in bends is near the center of the river. Off Algiers Point, for instance, where there is no impingment against valley walls, the greatest depths are near the convex bank and are in excess of 200 ft below mean sea level.

In studying the processes that mould and modify the river today, the geologist's field laps with that of the engineer. It is to be hoped that a cooperative study of the processes will be carried on until definite answers are obtained.

The author's use of the obscure term "point of bend" brings to mind the paucity and unsatisfactory character of the terminology used in river engineering. The writer has often found it very difficult in preparing reports to reserve the terms "point" and "crossing," for instance, for the phenomena for which they are the only accepted names. Geologists are noted for the invention of names, so much so that their vocabulary has become a study in itself. The writer for one would be grateful if they would evolve definitive words to denominate river phenomena.

STAFFORD C. HAPP[7].—The importance of the studies, ably and conveniently summarized in this paper, is not limited to the lower Mississippi engineering problems, important as those are, but without doubt will have much wider influence in both the engineering and geological fields. The recent geological investigations of the Mississippi alluvial valley have provided a major advance in the knowledge of river behavior, and related geomorphic principles. The complex alluvial deposits and river processes have not previously been so concisely and clearly described. This paper will be especially useful in bringing to wider attention the studies that have been previously described in more detail, but in media of limited circulation. It is still to be hoped that more of the original, basic field data will be published for critical study in the geological profession.

The author interprets the geological history essentially in terms of river entrenchment following late Pleistocene lowering of sea level by glacial ice accumulation, and subsequent aggradation due to post-glacial rise of sea level. These basic concepts will probably not be seriously questioned, but some further details of his interpretation of the alluviation are less convincing, and alternate hypotheses deserve more consideration than they are accorded in print. The questionable details include:

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[7] Geologist, Corps of Engineers, U. S. Dept. of the Army, Kansas City, Mo.

(1) Postulate of a braided-channel river to account for accumulation of the substratum of some 100 ft to 300 ft of clean sands and gravels;

(2) A gradual change in the nature of deposition to produce an "original top stratum" of finer-textured sediment;

(3) Beginning of the present meandering habit on the lower slope near the mouth only after the sea had almost reached its present level, with meandering developing upstream as aggradation progressed and the slope was lowered; and

(4) Adjustment of the river to "its constant base level," and thereby attaining a "poised" condition in which it is neither aggrading nor degrading, but only reworking its older alluvial deposits within meander belts of several successive courses in various parts of the alluvial valley.

It is not necessary to postulate a braided channel phase in order to account for the sand and gravel substratum, or a change in deposition to account for the top stratum. There is no reason why a meandering river cannot aggrade its bed and flood plain, and accumulate a considerable substratum of sand and gravel beneath a silt and clay top stratum. Sand and gravel are usually bed load materials and are sorted and left behind in point-bar ridges, or lateral accretion deposits, as a meandering channel shifts within its flood plain. In time the bed load deposits are covered by finer materials deposited from shallower overflow waters, usually slowed by vegetation, and thus a top stratum is formed over sands and gravels of only slightly greater age. From a geological viewpoint, flood plain top stratum and substratum are formed concurrently and are of the same age. If more bed load reaches the river than it can carry away, the lateral accretion deposits will gradually be built to higher elevations, and thus a thickening substratum will accumulate and be covered by the top stratum which is being eroded away on the outer sides and replaced on the inner sides of channel bends. The relatively thick sandy substratum, and finer top stratum, thus could have been produced by the same river processes which Mr. Fisk has described as operative in the present Mississippi valley, without requiring any changes in base level, or in river behavior.

The only direct evidence for former braided channels appears to be the surface pattern of terraces in the upper part of the valley. These are quite different from the pattern of the meander belts, and the terraces are reported to slope beneath the flood plain in a downstream direction; but the surface features may reflect only a late, minor phase in terrace development, or possibly even slight surface erosion. The Rio Grande is an aggrading river above Elephant Butte Reservoir in central New Mexico, and has a braided channel, but during the prolonged major flood of 1941 most of its flow was concentrated in a relatively narrow, deepened, winding course within the wider sands of the usual braided channel. Aggradation is comparatively rapid on the Rio Grande, relative to meandering, and considerable parts of the top stratum are covered with bed load sands by river avulsions during floods; yet meander patterns are prominent on the flood-plain surface. Where aggradation is relatively less rapid than meandering, the sand substratum will be thickened while the top stratum is repeatedly reworked and replaced at higher elevations, rather than being buried. It seems probable that meandering would have been predominant on the lower Mississippi, which generally has not aggraded its bed more than about 100 ft since the late Pleistocene entrenchment.

The postulate that meandering began on the flatter slope near the mouth, and gradually developed upstream, also seems questionable. The relatively straight and stable present course below Donaldsonville, is ascribed to resistance to meandering by the thick clay top stratum. This implies relatively straight original courses on relatively flat slopes, which is in accord with conditions observable on the new deltas of artificial lakes. If the present river does not meander on its flatter slopes, however, it appears illogical to postulate that the ancestral river first developed meandering on the flatter slope near its mouth. It seems that meandering would be more likely to develop farther upstream, where steeper slope and less resistant sandy banks would be more conducive to lateral bank cutting and channel migration. Individual meanders certainly shift in a downstream direction, and this would facilitate downstream, but not upstream, extension of a meandering pattern.

It is suggested, as an alternate hypothesis apparently in accord with the geological evidence, that the lower Mississippi was probably a meandering river throughout much, if not all, its period of aggradation, and that its past deposits and future history are most reliably interpreted in terms of its present observable processes of behavior. This implies that meandering has been sufficiently active to remove and replace top stratum deposits before they would otherwise be buried by aggradation, except in the flatter delta where aggradation presumably has been largely a thickening of the top stratum. This hypothesis does not require a former braided channel condition, nor any "change in deposition" to account for an "original topstratum." The present thicker flood-basin or backswamp deposits are accepted as those parts of the flood plain which, by chance, have remained longest outside the meander belts. The lower elevation of the sands under such areas may be evidence of progressive aggradation. Finally, and most important, this hypothesis implies that the present river is the product of a long period of aggradation, rather than being the product of a new and more stable "poised" regime.

The concept of the "poised" river is useful, and the terminology is apt. It is important, however, to understand that it is based on lack of recognizable aggradation or degradation during the comparatively short period of accurate measurements. Levee building, revetment to protect eroding banks, dredging of crossings and shoals, removal of snags and clearing of banks, artificial river cutoffs, partial flood control by reservoirs, and changes in surface runoff and increased soil erosion due to clearing of forests and agricultural practices, all have affected river behavior to some extent, but in varying ways, within the period of records. It is doubtful whether geologic trends can yet be determined from data affected by so many factors. The Mississippi delta will continue to grow into the Gulf of Mexico, however, and further aggradation of the river is a logical result to be expected; but the rate and extent of such aggradation, and the effects of other factors, are uncertain. The poised condition may be sufficiently well verified to indicate that neither aggradation nor degradation is rapid enough to affect engineering plans within periods of a few decades, or so; but it would appear unwise, from present data, to conclude that aggradation will not require consideration within the near future.

Harold N. Fisk[8].—The remarks of Mr. Odom concerning the value of the writer's geological investigation of the alluvial valley of the lower Mississippi River are greatly appreciated. It is the writer's belief that no generalization based on either geological or engineering studies of over-all river activity can be formulated as a hard and fast rule for interpreting a local engineering problem. Although cooperative work between geologists and engineers is highly desirable for a complete understanding of river phenomena, the answer to any local question must rest on the accumulation of sufficient data necessary for proper analysis. Field evidence has fully supported the writer's generalizations that deep scour pools occur within bendways in areas where the river impinges against the valley walls, or in other areas, such as those near New Orleans, which Mr. Odom mentions, where realtively impermeable bed and bank materials strongly resist bank caving. It has long been known that the position of a scour hole varies slightly from time to time in a bendway and that its position varies with each bend. In order to explain satisfactorily the location of an individual scour hole in a bendway, it is necessary to obtain detailed data concerning such factors as changes in river alinement associated with stage variation and with bend migration, and the local character of bed and bank materials.

In directing attention to the lack of satisfactory terminology for Mississippi River phenomena Mr. Odom has touched on a subject which plagues both engineers and geologists. There is a great need for definitive and descriptive terms since many of those in use today originated with early boatmen. Although such terms may have sufficed for the needs of men navigating the river, they are certainly inadequate for detailed expression required in the technical description of river activity.

In his thought-provoking discussion, Mr. Happ expresses disagreement with the writer's views that during the epoch of valley alluviation the Mississippi River has changed from a braided to a meandering stream. He suggests that the Mississippi River probably has maintained its meandering habit during the time the valley was being aggraded. All the data on which the writer's concept of the Mississippi River activity has been based are summarized in the paper under discussion, and those pertinent to questions raised by Mr. Happ will be restated here.

Perhaps the most convincing proof that the Mississippi River was a braided stream until late in the history of valley development can be obtained from the aerial photographs. These disclose many old braided channel scars on the surface of the large low alluvial fans which slope away from the mouths of each tributary valley. The fact that fans are in places buried by modern flood-plain deposits indicates they were constructed in large part while valley slopes were somewhat steeper than at present. Like similar features in other areas, extension of the foot of each of these fans was accompanied by entrenchment of headward areas. As a result the upstream part of each fan surface now rises as a low terrace above the flood plain while, downstream, the slope of the surface carries it below flood-plain level. Braided stream scars that converge toward the mouth of the upper Mississippi valley can be traced downstream into areas where the alluvial fan and flood-plain surfaces merge. In some areas individual braided channels can be seen to change gradually into meanders. Braided courses have also been mapped for other major tributaries, such as the Arkansas, White, and the Ohio rivers, demonstrating that the change from braided to meandering habit was not limited to the Mississippi River, but was brought about by regional changes affecting river activity.

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An outstanding feature of the Mississippi valley alluvium is the absence of silts and clays in those parts of the substratum away from the Mississippi River meander belts or below depths of river scour in those belts. Samples in these areas from thousands of borings which have completely penetrated the substratum show only "clean" sands or gravels. The matrix of the substratum is composed of sand grains which decrease in size upward in the section and down valley. As interpreted by the writer, these changes in grain size record a gradual loss in stream carrying capacity resulting from a progressive decrease in slope during valley aggradation. They also show that at any given time of alluviation the valley slope was lower in the river mouth areas than upstream. The matrix sands enclose thin lenticular bodies of pebbles, cobbles, and boulders of very local extent. Particles comprising these masses also decrease in size seaward; pea-sized grains are the coarsest recorded in the substratum near the modern shoreline. Lentils are distributed throughout the substratum and are found characteristically developed in areas where the "clean" sands extend close to the surface of the alluvial fans. In such areas, the shape, thickness, and distribution of the lentils can be closely correlated with the width, depth, and other characteristics of the shallow braided stream courses. Borings show that the sand accretions of the meandering Mississippi are large thick bodies in comparison with the lenticular masses in the substratum. The point bar deposits within the meander belt accretions typically incorporate thick swale fillings of fine sands, silts and clays. In addition, these deposits include extensive fine-grained fillings for cutoff meander channels. In so far as no such fine-grained deposits are found in the substream below or away from the meander belts, it is difficult to relate the buried "clean" sands to meandering stream action. These relationships form a firm basis for assuming that the Mississippi River was a braided stream throughout much of the history of valley aggradation. They do not support the hypothesis proposed by Mr. Happ that the substratum represents bed load of sands and gravels which were sorted and left behind in point bar ridges or accretion deposits as the meandering Mississippi shifted with its flood plain.

In concluding his discussion, Mr. Happ expresses doubt that the poised condition of the Mississippi River has existed long enough to establish a geologic trend. He apparently believes that the only available critical evidence that the river is neither aggrading nor degrading its channel is confined to observational data obtained by engineers during the period of historical records. It is true that in presenting the concept of a poised stream, Mr. Matthes used engineering data and observation of river activity over the past 150 years. However, one of the principal conclusions reached by the writer as a result of his alluvial valley studies was that the Mississippi River has been maintaining its poised character since the sea reached its present level.

Since that time the river has constructed and abandoned its meander belt along the ancient Bayou Teche and Lafourche courses and has developed its present channel below Donaldsonville. Careful comparison of the lengths of the ancient courses, their natural levee heights, and the widths and depths of their channels, with similar features of the modern Mississippi River, demonstrates that the poised condition has existed for at least 2,000 years.

The writer appreciates this opportunity to clarify and strengthen interpretations presented in the paper. He feels that the wealth of data, the extensive borings, the comprehensive photographic coverage, and the detailed mapping of the Mississippi valley so amply substantiate the conclusion drawn as to preclude the necessity for appealing to alternate hypotheses as suggested by Mr. Happ.

Acknowledgment.—The illustrations used in the paper were taken, directly or slightly modified, from the figures and plates originally presented in publications of the Mississippi River Commission and Waterways Experiment Station.[2,3,4]

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 2512

STAGE PREDICTIONS FOR FLOOD CONTROL OPERATIONS

BY RALPH E. KING[1]

WITH DISCUSSION BY MESSRS. ROBERT BUEHLER AND W. S. HOEHL; AND RALPH E. KING

SYNOPSIS

The problems relative to the prediction of stages in the lower Mississippi River Valley during flood periods are the subject of this paper. The development of various forecasting techniques and the advantages and limitations of these techniques with respect to use on the Mississippi River are briefly outlined. Methods used by the Stage Prediction Section of the Mississippi River Commission are explained, with emphasis being given to the method used for reaches which are influenced by inflows from major tributaries. Conclusions as to the type of forecasting procedure necessary to obtain reliable stage predictions on the lower Mississippi River are presented.

INTRODUCTION

Principles of Stage Forecasting.—The coming of spring brings a promise of relief to the people who inhabit the colder portions of this country, but to the residents of the lower Mississippi River Valley spring brings a threat of disaster. From the confluence of the Mississippi River and the Ohio River at Cairo, Ill., a thousand miles southward to the Gulf of Mexico the adequacy of the levee system to confine the upward surging floodwaters is of universal concern. To the engineers responsible for the safety of the valley an accurate forecast of the crest stages that will be reached is of vital importance. Stage forecasters, armed with the records of past floods and a sublime faith in the future, suddenly become either important citizens or public enemies, depending on the accuracy of their predictions.

The forecasting of flood stages has been separated into the following classes by Merrill Bernard,[2] M. ASCE:

NOTE.—Published in July, 1951, as *Proceedings-Separate No. 79*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Engr., Mississippi River Commission, Vicksburg, Miss.

[2] "Recent Developments in Flood-Forecasting," by Merrill Bernard, *Transactions*, Am. Geophysical Union, Vol. 20, 1939, p. 187, Table 1.

a. Prediction of the maximum flood for the purpose of establishing the capacity limits of control works.

b. Prediction of the probable flood for the purpose of evaluating the accepted tolerances in the design of control works.

c. Prediction of the individual flood for the purpose of preventing loss of life and reducing damage to a minimum.

The forecast for the maximum flood is expressed in terms of magnitude and duration; the forecast for the probable flood in terms of magnitude, duration, and frequency; and the forecast for the individual flood in terms of magnitude, duration, and expected time of occurrence. This paper deals only with the individual-flood type of forecast.

During the flood season almost every operation of the Corps of Engineers, United States Army, throughout the Mississippi River alluvial valley is for the purpose of controlling or minimizing the destructive forces of the floodwaters. Levees must be patrolled constantly. Forces must be mobilized for immediate action, and materials and equipment for emergency operations must be available at the time and place needed. Headwater reservoirs must be operated so as to secure the greatest reduction in the crest of the main stream, and at the same time the operation of floodways and the flooding of leveed backwater areas must be carefully planned. Areas which will be inundated by such operations must be evacuated to prevent loss of life, and levees must be raised or strengthened in critical reaches. Detailed observations of stage and discharge of the current flood must be made so that information will be available for future study. All of these operations are based primarily on stage forecasts.

Factors Affecting the Forecast.—Stage forecasts used in planning and executing flood-fighting activities must possess a degree of accuracy sufficient to insure the completion of all essential operations and the avoidance of useless operations. The mere undertaking of a flood-fighting operation is of very little value, since the successful completion prior to the occurrence of the crest stage is the only criterion by which the operation can be judged. Consequently, a crest forecast must not only be sufficiently accurate as to the stage expected but also must be made far enough in advance of the crest so that time will be available for mobilization and the successful completion of the operation. Needless to say, the accuracy of any forecast decreases as the number of days for which the forecast is made increases.

Stages in any reach of a river are affected by the flow received from the adjacent upstream reach and by the volume of water introduced into the reach by tributaries and rainfall or lost through outlet channels. On the lower Mississippi River the effect of channel storage (including wedge storage) is very pronounced, and studies of past floods on the lower Mississippi River show that the effect of storage varies considerably for different floods. These studies indicate that the amount of water going into storage is a function of both the stage height and the rate of change in stage. The operation of floodways is based on stage forecasts which must then be revised to reflect the effect of such operation. Planned variations in flow releases from flood control reservoirs must be taken into account by the stage forecaster. Since the operations of flood control reservoirs for the purpose of decreasing crests on the main river must be properly timed with the hydrograph of the main river, these operations must be based on stage forecasts for the main river. This mutual dependence forces the reservoir operator and the stage predictor to work hand in hand.

Other factors that may seriously impair the usefulness of stage forecasts are the accuracy of the observed stages and discharges used as a basis for forecasting and changes that occur in the regimen of the river. If stage predictions are to be successfully used for the planning and execution of emergency operations during flood periods, all of these influencing factors must be considered and their effects evaluated within reasonable limits of accuracy.

DEVELOPMENT OF STAGE RELATION CURVES

The earliest forecasting methods were probably simple "rules of thumb" similar to the one that estimated the crest stage at Vicksburg, Miss., to be 2 ft below the Cairo crest on the first rise of the flood season and the same as the Cairo crest on subsequent rises. Another similar rule states that the crest stage at Cairo would not exceed 50 ft unless the Ohio River stage at Cincinnati, Ohio, exceeded 50 ft. The limitations of the first rule are readily apparent since it can only apply to average conditions. The second rule was definitely disproved in 1943 when a crest stage of 36.0 ft at Cincinnati combined with one of the larger upper Mississippi River floods to produce a stage of 53.0 ft at Cairo. The floods of 1944 and 1945 also produced crests that demonstrated the fallacy of the second statement.

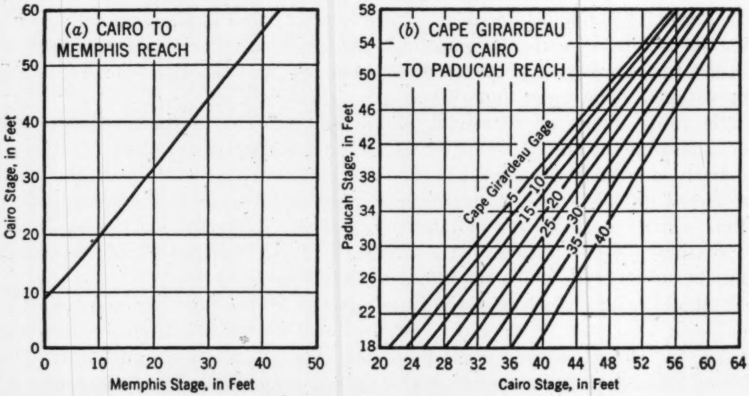

FIG. 1.—STAGE PREDICTION CURVES—MISSISSIPPI RIVER

Single Line Curves.—The inadequacy of these rules of thumb to yield results of the degree of accuracy required led to the development of stage prediction methods that incorporated the records of previous floods. One of the simplest of these was the construction of single-line stage relation curves, similar to the one for the Cairo to Memphis, Tenn., reach shown in Fig. 1 (*a*). This type of diagram, containing only two factors (the stage at the upper end of the reach and the corresponding stage at the lower end of the reach) is usually constructed by plotting stages observed from past crests and troughs that occurred under conditions essentially the same as those existing at the period for which the chart is to be used. Similar charts for determining the time required for flows to pass through the reach may be constructed by plotting the upstream stages against the observed time between the corresponding upstream and downstream gages.

The fact that single-line stage relation curves, especially for the higher stages, usually take the form of straight or nearly straight lines led to the custom of expressing the graphical relationship by an equation of the form

$$h_L = a\,h_U + b \qquad (1)$$

in which a and b are constants and h_L and h_U are the corresponding stages at the lower and upper ends of the reach. Equations of this type were used for predicting stages on the Rhine River,[3] Germany, as early as 1903, and it was claimed that with some exceptions the variation between floods predicted on the Rhine and floods that actually occurred did not exceed 20 cm. Curtis McD. Townsend, a former president of the Mississippi River Commission, states[4]

> "Since the Rhine rises in the Alps, and since its discharge is regulated by Lake Constance, it frequently has a flood when its lower tributaries are discharging little water. Hence the primary wave is more readily determined than in the rivers of the United States, whose tributaries, flowing at all stages with great irregularity, interfere with the primary wave flowing down the main stream. On the long rivers of the United States this method would not give as satisfactory results. Gage stations on the Rhine are less than 20 kilometers apart on the average; while on the Ohio River and on the Mississippi River they are frequently more than 100 miles apart, and it is necessary to make predictions over river distances of from 300 to 1500 miles. An error of 20 centimeters between stations on the Rhine may readily correspond to an error of from 10 to 20 feet on the Ohio between Cincinnati and Cairo, since the principal tributaries of the Rhine empty into the main stream within a distance of 150 kilometers."

Multiple Line Charts.—Since the single-line diagram is applicable only to reaches of the main stream in which, or immediately below which, no important tributary enters, the development of multiple-line charts, such as the one presented by E. W. Lane,[5] M. ASCE, was necessary. With these charts, for instance, the stages at Paducah, Ky., on the Ohio River and Cape Girardeau, Mo., on the upper Mississippi River are used to determine the stage at Cairo for the following day. Fig. 1(*b*) is an example of this type of chart.

The entrance of more than one tributary into a reach of the main river further complicates the use of stage relations for stage forecasting. The entrance of the White River and the Arkansas River into the Mississippi River just above the Arkansas City (Ark.) gaging station provides a good example of the effect of tributary flow.

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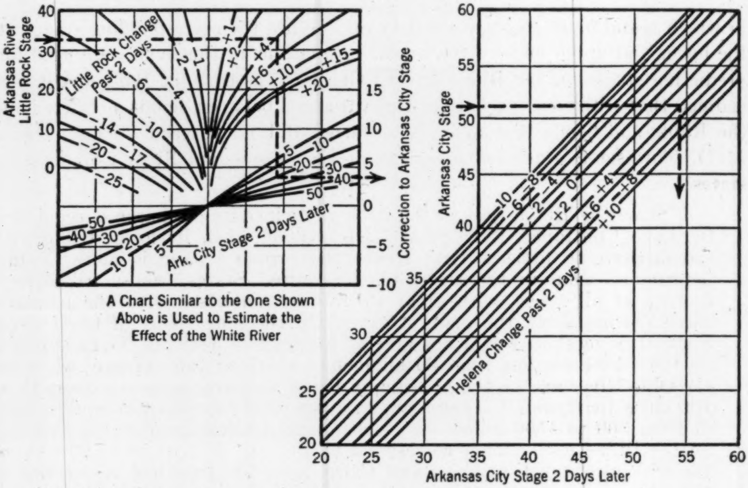
[3] "Étude hydrologique du Rhin allemand et du Main, les crues, et leur prévision," par M. Ed. Maillet, *Annales des Ponts et Chaussées, Mémoires et Documents*, 1903, p. 200.

[4] "The Hydraulic Principles Governing River and Harbor Construction," by Curtis McD. Townsend, The Macmillan Co., New York, N. Y., 1922, p. 42.

[5] "Predicting Stages for the Lower Mississippi," by E. W. Lane, *Civil Engineering*, Vol. 7, February, 937, p. 122.

In 1927 the combined crest discharges of the White River and Arkansas River amounted to approximately 40% of the total Mississippi River flow at Arkansas City. Unless the effect of these tributaries on main river stages is considered, reliable forecasts for Arkansas City and the stations downstream are impossible. In 1944 Max A. Kohler, A. M. ASCE, developed a forecasting procedure[6] that requires the development of a chart for determining the normal relationship between gages on the main river and of auxiliary charts (one for each tributary) that are used to determine the corrections to be applied to the predicted normal stages. Charts of the type shown in Fig. 2 were developed by Mr. Kohler for use in predicting stages at Arkansas City.



A Chart Similar to the One Shown Above is Used to Estimate the Effect of the White River

FIG. 2.—ROUTING CURVES—HELENA TO ARKANSAS CITY REACH

Methods of this type permit the predictions of future stages without the use of discharge observations or stage discharge curves. If discharge data are available, the method can be simplified by use of a single multiple-line chart which evaluates the effects of tributary flow on main river stages. Fig. 3 used for predicting crest stages at Red River Landing, La., is an example of this type of chart. For day-by-day stage forecasting the effects of changes in tributary flow on main river stages, either positive or negative, can be considered by using charts of the type shown in Fig. 4.

FORECASTING TECHNIQUES

Limitations of Forecasting.—Gaging stations on tributaries are usually located a considerable distance above the mouth to avoid the influence of backwater from the main river. Successful stage forecasting requires consideration of the time necessary for fluctuations in stage or discharge at the tributary station to reach the main river.

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[6] "A Forecasting Technique for Routing and Combining Flow in Terms of Stage," by Max A. Kohler, *Transactions*, Am. Geophysical Union, Vol. 25, 1944, p. 1030.

Stage relation diagrams give the average relation between gages. The variation in the rate of storage in the overbank areas and in the channel itself, as the water surface rises or falls, is not taken into account. For that reason stage relation diagrams are more accurate in reaches where the capacity for storage is relatively small. The Mississippi River below Red River Landing is closely confined by levees, and storage is a minor factor. Stage relation diagrams give very good results in this reach. The reach between Helena, Ark., and Red River Landing contains several large areas which are subject to overflow by backwater from the river. Storage is an important factor in this reach, and the use of stage relation diagrams does not give reliable results.

Application of Techniques to the Mississippi River.—Forecasting techniques that use selected headwater stations as indexes for predicting stages in the main valley have been evolved. Three days in advance M. Belgrand was able to predict flood heights at Paris, France, within 40 cm by using this method.[3] Other forecasters have frequently used tabulations that listed the crest rise in feet per inch of rainfall for large storms on headwater basins. Because of the tremendous drainage area of the Mississippi River, neither the index station method nor the unit crest method is applicable.

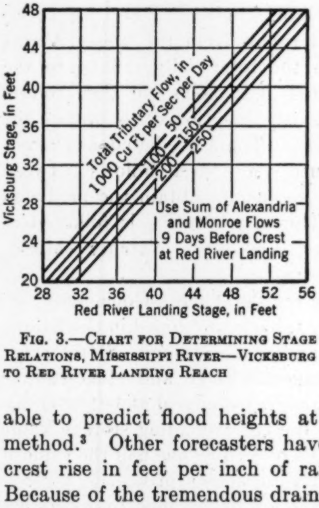

FIG. 3.—CHART FOR DETERMINING STAGE RELATIONS, MISSISSIPPI RIVER—VICKSBURG TO RED RIVER LANDING REACH

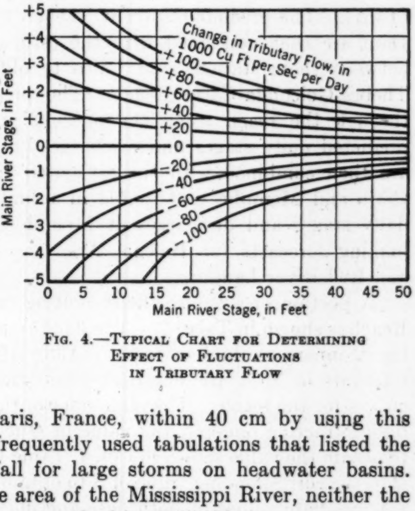

FIG. 4.—TYPICAL CHART FOR DETERMINING EFFECT OF FLUCTUATIONS IN TRIBUTARY FLOW

The method used by the Stage Prediction Section of the Mississippi River Commission for forecasting flood stages on the lower Mississippi River is a modification of the graphical method of flood routing developed by Louis G. Puls,[7] A. M. ASCE. During flood periods daily discharge observations are made at the principal gaging stations. The results of these observations are used to develop stage-discharge relation curves that are applicable to the individual flood only. For forecasting purposes these stage-discharge curves are extended as far beyond the limits of the observed data as necessary, the extrapolation being shaped by comparison with data from previous floods.

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[7] "Flood Regulation of the Tennessee River," by Louis G. Puls, *House Document No. 185*, 70th Congress, 1st Session, Appendix B, Part II, U. S. Govt. Printing Office, Washington, D. C., 1928.

These curves are then used to determine the daily inflows and outflows at the upper and lower ends of each reach. Similar curves are used to determine the daily contributions from the tributaries. Runoff from rainfall occuring over the ungaged areas below the tributary gaging stations is computed by the use of unit graphs. The observed inflows (main river and tributaries) are then combined with the runoff computed from local rainfalls and reverse routed, using the observed reach outflows, to develop stage-storage curves for the various reaches. These storage curves are then extended beyond the limit of the available data by comparison with similar storage curves for previous floods.

During the progress of a flood, five-day forecasts of the discharges to be expected at Thebes, Ill., on the upper Mississippi River, at Golconda, Ill., on the Ohio River, and at Smithland, Ky., on the Cumberland River, are made by the Corps of Engineers offices responsible. In addition to these forecasts there is also available a five-day forecast of anticipated releases from Kentucky Dam on the Tennessee River furnished by the Tennessee Valley Authority (TVA). The predicted flows at Thebes, Golconda, Smithland, and Kentucky Dam are then combined with the estimated runoff from the ungaged area between these stations and Cairo to obtain the total daily inflows to the Thebes-Golconda-Cairo Reach. The predicted daily flows are then routed through the reach using the method previously mentioned to obtain the predicted daily stages and discharges at Cairo. These predicted discharges are then combined with the estimated tributary and local inflows between Cairo and Memphis and routed in a similar manner to obtain the predicted daily stages and discharges at Memphis. The same routine is followed in making forecasts for Helena, Arkansas City, Vicksburg, Natchez (Miss.), and Red River Landing.

A portion of a typical flood routing table for the Thebes-Golconda-Cairo Reach is shown in Table 1. The flood routing table is developed in the following manner: Values of reach outflow (*Q*) and reach storage (*S*) for stage intervals of 1 ft are obtained from the stage-discharge and stage-storage curves for the reach. The remainder of the table is developed at 1-ft intervals by subtracting one-half the value of *Q* from the corresponding storage value to obtain the value shown in Col. 4, Table 1, and by adding one half the value of *Q* to the corresponding value of *S* to obtain the value shown in Col. 5, table 1. Intermediate values of both base and indication are determined by interpolation between the values computed at the 1-ft intervals.

The routing table thus developed is used as shown in Table 2. For a stage at Cairo of 50.0 ft at the beginning of the first time period, the base value of 2,303,000 cu ft per sec per day is obtained from the routing table (Table 1). The average inflow of 1,332,000 cu ft per sec per day during the first time period added to the base value, results in a total of 3,625,000 cu ft per sec per day. The value in the indication column of the routing table that most nearly agrees with this total is 3,633,000 cu ft per sec per day, corresponding to a stage of 50.4 ft. This is the stage at the end of the first time period and at the beginning of the second time period. The process is repeated for subsequent time periods.

It is axiomatic that the ability to reproduce an observed hydrograph is the principle criterion for evaluating any method for flood routing. Comparisons of the observed hydrographs of the 1944 and 1945 floods at Cairo and Arkansas City with the hydrographs computed by the method described are shown in Fig. 5.

As previously mentioned the reach below Red River Landing is not seriously affected by storage or inflow from tributaries. Consequently single-line stage relation diagrams are used to predict stages in this reach with very close agreement between forecasted and observed stages being obtained. In considering the use of stage-discharge diagrams for forecasting in the lower reaches of the river, it should be noted that errors of as much as 1 ft in forecasts are quite frequent if a diagram representative of average conditions is used. As a means of reducing errors to a minimum, observed stages are plotted daily during the progress of the flood and the resulting relationship extended in accordance with similar curves for previous floods. By using this procedure, the forecasts are based on the relationship existing for the individual flood instead of on the mean relationship determined from previous floods.

Table 1.—Routing Table for the Thebes-Golconda to Cairo Reach

Stage (feet)	Flow Characteristics, in Thousands of Cubic Feet per Second per Day			
	Outflow, Q	Storage, S	Base $S - Q/2$	Indication $S + Q/2$
(1)	(2)	(3)	(4)	(5)
50.0	1,242	2,924	2,303	3,545
50.1			2,320	3,567
50.2			2,337	3,589
50.3			2,354	3,611
50.4			2,371	3,633
50.5			2,388	3,655
50.6			2,405	3,677
50.7			2,422	3,699
50.8			2,439	3,721
50.9			2,456	3,743
51.0	1,292	3,119	2,473	3,765
51.1			2,491	3,789
51.2			2,509	3,813
51.3			2,527	3,837
51.4			2,545	3,861
51.5			2,563	3,885
51.6			2,581	3,909
51.7			2,599	3,933
51.8			2,617	3,957
51.9			2,635	3,981
52.0	1,352	3,329	2,653	4,005

Table 2.—Sample Routing for the Thebes-Golconda to Cairo Reach

Period	Average inflow, in thousands of cubic feet per second per day	Base plus average inflow	Routed Cairo stage (feet)
(1)	(2)	(3)	(4)
1			50.0
	1,322	3,625	
2			50.4
	1,355	3,726	
3			50.8
	1,370	3,809	
4			51.2
	1,365	3,874	
5			51.5
	1,352	3,915	
6			51.6
	1,320	3,901	
7			51.6
	1,270	3,851	
8			51.4

During the flood period all pertinent data are carefully analyzed as soon as received. Stage-discharge curves are revised if the latest discharge observations indicate that the curves being used have been extended erroneously. If observed stages show a serious variation from the forecast stages, the reach storage curves and routing tables are revised to reflect more nearly the conditions existing during the flood.

By using the method outlined, sufficiently accurate seven-day forecasts for flood stages at Cairo can be made. Stages in the lower reaches of the river have been forecast within reasonable limits for periods of three weeks.

Throughout the critical season very close cooperation is maintained with the TVA, the offices of the United States Weather Bureau throughout the Mississippi River Basin, and the various districts and divisions of the Corps of Engineers, all of which furnish data essential to the success of the forecasting procedure. The accuracy obtained by the procedure used is not perfect, but the results are consistently better than those obtained by using any of the other methods described or by using any of the many other forecasting procedures that have been investigated throughout the years. Needless to say continuing investigation and testing of techniques is going on to give greater accuracy and reduce the time and effort required to make sufficiently accurate forecasts.

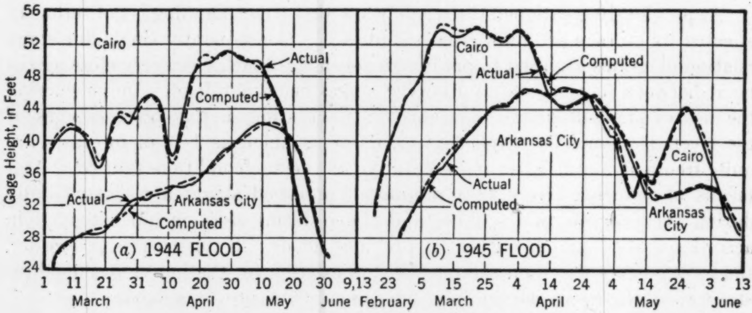

FIG. 5.—ACTUAL AND COMPUTED STAGES OF THE MISSISSIPPI RIVER AT CAIRO AND ARKANSAS CITY

CONCLUSIONS

Extensive study of the problem of flood-stage forecasting for the lower Mississippi River has led to the following conclusions:

1. Reliable forecasts for the lower Mississippi River cannot be made by using any rigid formula or procedure;
2. Procedures which do not evaluate the effects of current channel conditions, tributary or diversion flows, storage, and precipitation will not give reliable forecasts;
3. Procedures dependent on the use of volumetric principles will give better results than those which are based entirely on stage heights; and
4. Stages on the lower Mississippi River can be forecast with sufficient accuracy to warrant the use of these forecasts as the basis for planning and executing flood-fighting activities.

DISCUSSION

Robert Buehler,[8] A. M. ASCE and W. S. Hoehl[9].—In the paragraph preceding "Conclusions," Mr. King states: "Throughout the critical season very close cooperation is maintained with the TVA***"

Routing methods similar to those of the Stage Prediction Section of the Mississippi River Commission are used during a flood by the Flood Control Branch of the TVA as an aid to project operation. Such methods are also used (as rapidly as preliminary basic inflows and outflows can be received) to determine Cairo stages for natural Tennessee River flows as a measure of the effectiveness of the TVA system. After the flood is over, when provisional basic inflows and outflows are available, more refined estimates are prepared. The preliminary flows are usually instantaneous morning rates received daily by teletype as the flood is in progress. The provisional flows at most locations are the midnight-to-midnight average daily discharges which are scheduled for publication later in the appropriate yearly pamphlets of the United States Geological Survey (USGS) or the Mississippi River Commission. Slight variations in procedure allow for the difference in time of observation for the two classes of data.

In addition to studies that show the effect, at Cairo, of actual TVA reservoir operation, the Flood Control Branch uses routing procedures to compute the effect of theoretical reservoir regulation. Ideal operation is one such type which, if there is a sufficient volume of water, makes full use of all storage up to gatetop levels to reduce Cairo to its lowest level crest stage. It is an optimum operation against which the effectiveness of other regulations can be measured.

As does the Mississippi River Commission, the TVA prepares storage curves and Cairo rating curves for each flood from the actual data collected or forecast as the flood progresses. During a flood TVA also extends these curves somewhat beyond the data when they are serving to help guide reservoir operations. They are also extended after the flood peak has passed to reach the higher stages and flows that would have resulted without the TVA system. The departure of average rating curves and routing curves from somewhat scattered data usually results in little indicated error in computed Cairo stage. Alternate trial positions of curves with respect to data to test this effect have resulted in Cairo stage differences of only a few tenths of a foot.

The main purpose of this discussion is to demonstrate that in at least one application of the routing method to Cairo, appreciable differences in the results are caused by using different inflow stations. Contributing to Cairo floods are flows from the Mississippi and Ohio Rivers. Discharges are obtained on the Mississippi River at Thebes, a short distance above Cairo, and below major tributaries on the Ohio River at Metropolis, Ill. Metropolis flows are composed of contributions from the Tennessee and Cumberland Rivers and from the Ohio River proper above these tributaries. Suitable discharge stations on these streams, respectively, are Kentucky Dam, Dover (or Smithland), and Golconda. In addition there are unmeasured local contributions between gages.

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[8] Civ. Engr. V, Hydr., TVA, Knoxville, Tenn.

[9] Civ. Engr. III, Hydr., TVA, Knoxville, Tenn.

Applications of the TVA routing procedure to Cairo embodies changing the contribution from the Tennessee River only. This may involve the use of natural Tennessee River flows instead of actual regulated flows to determine the effect of regulation, or it may involve hypothetical regulated flows. In any event, TVA routing is in effect the passage of altered Tennessee River flows through storage to learn the effect at Cairo.

Storage and routing curves prepared with Dover, Golconda, Kentucky Dam, and Thebes as inflow stations include the natural valley storage below these locations down to Cairo, all lumped together. When the curves then are used in a computation which seeks only to show the effect of modifying Tennessee River flows, these modified flows are being routed through storage lying along streams in which Tennessee waters actually do not travel. Specifically, these modified flows are being routed through combined storage which incorrectly includes portions along the Mississippi River from Thebes to Cairo, along the Cumberland River below Dover, and along the Ohio River from Golconda to the mouth of the Tennessee River. The incorrect inclusions minimize the computed effect at Cairo of altering Tennessee River flows.

Changing the inflow stations to Metropolis on the Ohio River and to Thebes on the Mississippi River is not a complete remedy, but has been found to give results which are believed to be better. When these inflow stations are used, the effect of modifying Tennessee River flows is introduced by altering Metropolis flows by the extent of the change at Kentucky Dam. This method correctly includes the valley storage below Metropolis, incorrectly excludes storage along the Tennessee and Ohio Rivers between Kentucky Dam and Metropolis, and incorrectly includes valley storage on the Mississippi River below Thebes. These two inaccuracies cause opposite effects. The omission of Tennessee and Ohio River valley storage exaggerates the computed effect at Cairo of altering Tennessee River flows. The inclusion of Mississippi River valley storage minimizes the effect. The influence that dominates may depend on the relative size of contributions in the basic flood used to prepare storage curves. If the Mississippi River at Thebes is unusually high compared to other contributions, a large volume of natural valley storage may be built into the combined storage curve and this minimizing influence may dominate; or, unusually large Ohio or Tennessee flows used to prepare the combined storage curves may cause the exaggerating influence to dominate.

It may be argued that a more correct appraisal could be made by routing altered Tennessee River flows from Kentucky Dam to the mouth, combining these with Ohio River flows routed to the same point, and then routing these combined flows to Cairo. Here the Ohio flows would be combined with the flows routed down the Mississippi River from Thebes. Although this appraisal would be more correct, the task would involve so much time and study that, with present methods, it is prohibitive.

To determine which of the two stated inflow station locations would give the more nearly correct computed Cairo stage, approximate corrections for storage errors, excepting on the Mississippi below Thebes, have been prepared for the January, 1950, flood. Data were not conveniently available to make this Mississippi River correction. All comparisons have been computed using identical inflows and outflows for any given selection of inflow stations, and using a variation of the routing procedure described by Mr. King for the river reach immediately above Cairo. This variation of method first was checked to be sure it had no influence on results.

For the Golconda-Dover-Kentucky Dam-Thebes selection of inflow stations the routed Cairo crest stage, if there had been no Tennessee River regulation, was 56.7 ft. When adjustments were applied to correct for having routed altered Tennessee River flows through natural storage on the Cumberland and upper Ohio Rivers this Cairo crest was increased to 57.5.

For the Metropolis-Thebes selection of inflow stations the routed Cairo crest stage, if there had been no Tennessee River regulation, was 57.4 ft. Corrections for having ignored natural Tennessee River storage below Kentucky Dam, and some Ohio River natural storage, reduced this Cairo crest slightly to 57.3 ft.

These results show that the selection of inflow stations in the January, 1950, flood had a substantial effect on the estimated Cairo crest stage when no regulation was assumed in the Tennessee River. In this 1950 flood, errors that are evidently present when using Golconda-Dover-Kentucky Dam-Thebes inflows are greater in their effect on Cairo than are the errors present when Metropolis-Thebes inflows are used. It does not necessarily follow that this will be true in all floods. Uncorrected error for storage along the Mississippi River from Thebes to Cairo is present in both comparisons.

Because the system using inflows at Golconda-Dover-Kentucky Dam-Thebes appears to have a chance of error always in one direction, it seems preferable to use the other selection, Metropolis-Thebes, in which erroneous influences are opposite, and have at least some chance of canceling each other.

It should be mentioned that corrections for storage errors are quite involved and would not be prepared except on an experimental basis or in case of unusually important Cairo floods.

Ralph E. King[10].—The writer is fully in accord with the argument advanced by Messrs. Buehler and Hoehl that a more correct appraisal of the effects of modifying flows from the Tennessee River could be made if the flows from the individual rivers could be routed separately to the points of confluence and then combined. He also agrees with their contention that the time and study which would be required make this method of computation impractical.

Since routing of the separate flows is not feasible, the selection of inflow stations on several streams which contribute to the total flow at Cairo, becomes of primary importance. The choice of inflow stations to be used during the progress of a flood, for the purpose of predicting future stages at stations downstream, is limited by the availability of such stations and by the degree of accuracy with which flows at the different stations can be predicted.

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[10] Engr., Mississippi River Commission, Vicksburg, Miss.

For predicting flood stages at Cairo, the choice of inflow stations must of necessity be either the Golconda-Smithland-Kentucky Dam-Thebes combination, as used by the Mississippi River Commission office, or the Metropolis-Thebes combination as suggested by Messrs. Buehler and Hoehl. From a cursory inspection the Metropolis-Thebes selection would appear to be the better of the two because of the more simplified routing procedure required. If this selection of stations is used for predicting flood stages, the error of excluding natural storage along the Tennessee and Ohio Rivers, between Kentucky Dam and Metropolis, is not made since the total Ohio River contribution is estimated for the Metropolis station. However, experience with past floods, during which both groups of inflow stations have been used, has indicated that the predictions of flood flows at Golconda are less subject to error than are similar predictions for Metropolis. Because the minor amount of additional computations required is more than compensated for by the increased accuracy of the Cairo forecasts, the Stage Prediction Section of the Mississippi River Commission prefers to use the Golconda-Smithland-Kentucky Dam-Thebes combination of inflow stations for making predictions during the occurrence of a flood.

Although determination of the effect on Cairo stages of modifying flows from the Tennessee River was not discussed in the paper, the writer is grateful to Messrs. Buehler and Hoehl for discussing this application of the basic method. Any value which the original paper might possess will be considerably enhanced by the inclusion of another practical application of the same routing method.

Introducing the effect of modifying Tennessee River flows by altering Metropolis flows to the extent of the change at Kentucky Dam has the following disadvantages:

a. Valley storage along the Ohio and Tennessee rivers between Kentucky Dam and Metropolis is incorrectly excluded;

b. Valley storage along the Ohio and its tributaries from the mouth of the Tennessee River upstream to the limit of the backwater effect produced by the flows from the Tennessee River is incorrectly excluded; and

c. Variations in Ohio River flows above the mouth of the Tennessee River, caused by modification of flows from Kentucky Dam, are not included.

The disastrous flood on the Missouri river in 1951 produced a crest stage of 40.6 ft at Cairo on July 28. The stage tendencies at stations on the lower Ohio River from July 2 to the date of the Cairo crest, except for minor fluctuations, are shown in Table 3.

Flow from the upper Mississippi River (which includes the flood flow from the Missouri River) caused valley storage to accumulate along the Ohio River from Cairo to a point upstream from Golconda. Although the flow from the upper Mississippi River does not pass through this reach, the successful routing of this flood requires the inclusion of valley storage along the Ohio River below the selected inflow station which could be either Golconda or

Metropolis. Further confirmation of the fact that the accumulation of valley storage along the Ohio River was caused by the floods from the upper Mississippi River is given by an inspection of the releases from Kentucky Dam during the same period. The average daily flow from Kentucky Dam, which was 52,000 cu ft per sec on July 2, 1951, increased to 73,000 cu ft per sec on July 4, decreased at a fairly uniform rate to 20,000 cu ft per sec on July 21, and then gradually increased to 45,000 cu ft per sec on July 28. Because valley storage along the Ohio River below Golconda was increasing from July 4 to July 21 (the period in which outflow from Kentucky Dam was decreasing) flow from the Tennessee River is eliminated as a possible contributing factor.

TABLE 3.—STAGE TENDENCIES OBSERVED IN THE LOWER OHIO RIVER

Gaging station	Station mile[a]	Stage tendency	Total change in stage (ft)
Cairo, Ill.	2.2	Rising	+5.5
Dam 53, near Mound City, Ill.	19.1	Rising	+5.0
Dam 52, near Metropolis, Ill.	42.8	Rising	+4.5
Paducah, Ky., near the mouth of the Tennessee River	47.2	Rising	+4.7
Dam 51, near Golconda, Ill.	78.6	Rising	+0.8
Dam 50, near Fords Ferry, Ky.	104.9	Falling	−3.5

[a] Mile above the confluence of the Ohio River and the Mississippi River.

Since flows from the upper Mississippi River affect valley storage along the Ohio River, it follows that flows from the Ohio River will produce a similar effect on the upper Mississippi River above Cairo and that flows from the Tennessee River can affect valley storage on the Ohio River above the mouth of the Tennessee River. For this reason the writer is unable to agree with the contention of Messrs. Buehler and Hoehl that the selection of Metropolis as the inflow station on the Ohio River incorrectly includes valley storage on the Mississippi River below Thebes, or that the selection of Golconda results in the incorrect inclusions of the Mississippi River below Thebes, the Cumberland River below Smithland (or Dover), and the Ohio River from Golconda to the mouth of the Tennessee River.

Studies of the effect on Cario stages of modifying the Tennessee River flows during the January, 1950, flood were made by the writer using both Metropolis and Golconda as inflow stations. The use of the Metropolis-Thebes selection resulted in a routed crest stage at Cairo of 57.4 ft, assuming that there had been no Tennessee River regulation. For the Golconda-Smithland-Kentucky Dam-Thebes selection the routed Cairo crest stage, if there had been no Tennessee River regulation, was 57.1 ft.

Because the routing that used the Metropolis-Thebes selection of inflow stations did not allow for either the inclusion of valley storage on the Ohio River from Metropolis to the upstream limit of the backwater influence of the Tennessee River, or for the decrease in Ohio River flow which would result from an increase in the flow from the Tennessee River, the routed stage of 57.4 ft at Cairo is higher than the stage that would have been routed if the proper corrections were feasible.

The Cairo stage of 57.1 ft, which was routed using the Golconda-Smithland-Kentucky Dam-Thebes selection of stations, is subject to error because no reductions, for the effect of increased flow from the Tennessee River, were made in the flows at Golconda and because the valley storage on the Ohio River, from the mouth of the Tennessee River to Golconda, would be slightly increased by the backwater effect of the increased flow from the Tennessee River. Correction of the routing method to allow for these factors would probably result in a slightly different routed crest stage at Cairo. The word "probably" is used because any reduction in flow at Golconda would tend to reduce the valley storage in the reach, and any raise in stages caused by flow from the Tennessee River would tend to increase the valley storage in the reach. Since these two corrections have opposite effects, it is doubtful whether their incorporation in the routing procedure would result in any appreciable difference in the routed Cairo stages.

Based on the foregoing analysis of the various factors that affect the combining and routing of flood flows from the various streams involved, in order to determine stages at Cairo, it is believed that the use of the Golconda-Smithland-Kentucky Dam-Thebes combination of inflow stations is preferable to the Metropolis-Thebes selection, for either predicting stages during a flood or determining the effect of modifying flows from the Tennessee River. This belief is further strengthened by the results of model tests conducted by the Waterways Experiment Station at Vicksburg. These tests (which were made in 1947 for the purpose of determining the effects on Cairo crest stages of modifying Tennessee River flows for the 1945, 1946, and 1947 floods) gave results which were in substantial agreement with the results obtained from routings that used the Golconda-Smithland-Kentucky Dam-Thebes combination of inflow stations.

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AIRCRAFT DESIGN AS RELATED TO AIRPORT STANDARDS

BY MILTON W. ARNOLD[1]

WITH DISCUSSION BY MR. JOHN H. GEISSE

SYNOPSIS

The point in aviation has been reached at which a much closer correlation must be sought between the design of aircraft and the design of airports, and also between these two factors and the efforts to achieve all-weather operation. Aviation history will reveal that the airplane was designed first and then the airports were constructed to satisfy the needs of the airplanes as built. As the goal of all-weather operating conditions is approached, electronic equipment that will overcome the deficiencies in both airplane and airport design is an urgent need. Unless a most intimate relationship between the design and construction of our airports, airplanes, and electronic equipments can be established, the cost of providing adequate airports, airplanes, and service can become economically prohibitive.

Somewhere along the line there is an area of diminishing returns, in which the cost of reducing cancellations and interruptions to scheduled operations makes the undertaking no longer profitable. The answer is to be found in the design of aircraft and airports and in the selection and installation of airborne and ground equipments to accomplish all-weather flight. Failure in any one of these fields of development can easily dictate failure of the all-weather objective.

Unless aircraft are designed to fit airports and to overcome certain flight and weather phenomena, and unless airport and equipment costs are kept down and maximum use made of all equipment that can be economically supplied, the over-all cost of providing all-weather operation is greater than the value of the benefits to be derived from such operation.

NOTE.—Published in October, 1951, as *Proceedings-Separate No. 93*. Positions and titles given are those in effect when paper or discussion was received for publication.

[1] Vice Pres., Operations and Eng., Air Transport Association of America.

ALL-WEATHER OPERATION

Very definite progress has been made since 1946 in reducing cancellations and delays and improving flight regularity. In 1946, the scheduled airlines sustained a loss of revenue due to delays and cancellations of more than $40,000,000. In 1947, this loss was even greater due to the disastrous winter of 1946–1947 that so disrupted public confidence in the airplane as a medium of travel that load factors dropped to an all-time low. A conservative estimate places the 1947 loss at an excess of $50,000,000. In 1948, a number of new electronic aids and certain operational improvements reduced the loss to $32,000,000. In 1949, in spite of substantially increased operations and schedules, the number of cancellations and delays was further reduced. The result was a loss of revenue estimated at $30,000,000. The total expenditure over the period 1946–1950 for improved terminal navigational aids necessary to bring about this reduction is something less than $20,000,000. The most important reason for this reduction in revenue losses is that the aids installed on the ground and in airplanes have permitted lowering of landing and operating minima from 400 ft of ceiling and 1 mile of visibility to 200 ft of ceiling and $\frac{1}{2}$-mile visibility. This reduction in operating limits eliminated 56.5% of the previously nonflyable weather.

It must be readily apparent that progress since 1946 has paid a very handsome return on the capital investment; and this progress has been realized without any really significant change in the concept of aircraft or airport design. Progressive reductions in minimum operating limits will probably go through at least three stages: (1) A reduction 200 ft of ceiling and $\frac{1}{2}$-mile visibility to 100 ft and $\frac{1}{4}$ mile; (2) a further reduction to 50 ft and $\frac{1}{8}$ mile; and (3) finally from there to an all-weather operation. There could be an intermediate stage involving a reduction from 50 ft of ceiling and $\frac{1}{8}$-mile visibility to a zero ceiling and $\frac{1}{16}$-mile visibility.

As these progressive reductions are made, each stage eliminates less unflyable weather, yet costs more. It will also be found that if this cost is to be kept to a minimum, some change will be required in aircraft and airport design, and that there must be a close liaison in aircraft, airport, and electronic design and planning.

By way of illustration, as previously stated, the first reduction from 400 ft of ceiling to 200 ft eliminates 56.5% of the nonflyable weather. At 11 major terminals in the so-called weather areas of the United States, visibilities of $\frac{1}{8}$ mile or less have been recorded as occurring only about 0.5% of the time. Although no figures are available for visibilities of $\frac{1}{16}$ of a mile or less, such extreme conditions are relatively rare—probably occurring no more than 0.1% of the time. Obviously, the number of cancellations and interruptions to service occurring as a result of visibility conditions below $\frac{1}{8}$ of a mile will be small by comparison to those that would occur in the range between $\frac{1}{8}$ and $\frac{1}{2}$ of a mile.

The question, then, can and undoubtedly will be raised—is the attainment of the all-weather goal a profitable one? The answer is probably an affirmative one, but it is dependent upon careful planning in the field of aircraft and airport design and in the field of electronics and visual aids.

INSTALLATION OF EQUIPMENT

It is possible to define the equipment necessary to achieve all-weather flight and to show the method of accomplishing this goal. As much as $50,000 can be spent per aircraft and $2,000,000 per runway for equipment to meet the anticipated standards for scheduled airline operations to be established by the airlines and the Civil Aeronautics Administration (CAA). Such an expenditure is certainly prohibitive and will undoubtedly prove unnecessary. This point can be developed, however, as an illustration of one path, and an unproductive one, that can be followed, but which, if followed, would dictate the necessity for a change in concept in aircraft and airport design and planning.

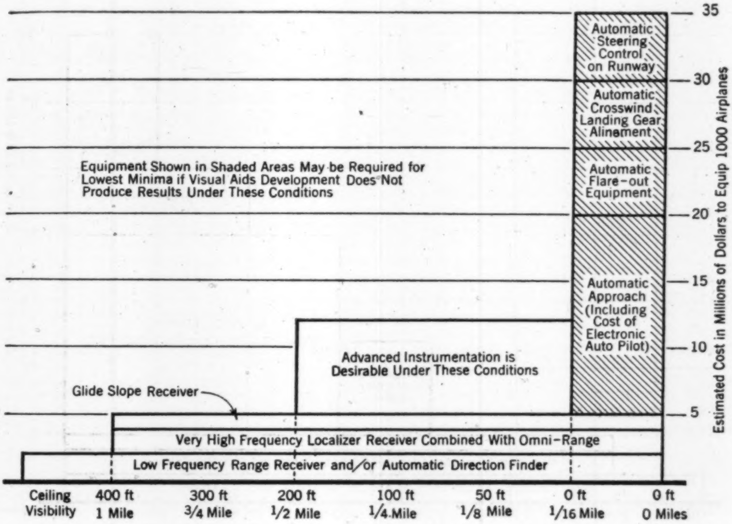

FIG. 1.—COST OF AIRBORNE EQUIPMENT WITH MAXIMUM USE OF VISUAL AIDS

Airborne Equipment.—First consider the airplane aspects of the cost of all-weather operations. Figs. 1 and 2 show the distribution of costs for equipment. The costs given do not include maintenance, which will be considerable, nor do they include the cost of carrying the heavy equipment that would be required for all-weather operation. This latter may become a considerable factor when it is realized that it costs an average of $6,250 to transport 100 lb of equipment in one airline aricraft for 1 year (1950).

To equip an entire airline fleet of approximately 1,000 aircraft for 100% all-weather operation would cost approximately $35,000,000. Since most aircraft equipment is amortized on a 5-year basis, this means that this cost would recur every 5 years or, on a prorated basis, is equal to a recurring yearly expense of $7,000,000 for airborne equipment alone. Thus, to do the job by automatic electronic means alone the cost would appear to be unreasonable. It is unknown with any reasonable degree of certainty whether such equipment as the automatic approach, the automatic flare-out, the automatic cross-wind gear alinement, and the automatic steering control on the runway will be a requirement for a zero operating condition. There is a real basis, however, for a determination of airborne equipment requirements and airplane design for operation down to 1/16 of a mile. A combination of visual ground aids and simpler airborne equipment will certainly permit operation down to 1/16 of a mile and perhaps to zero. Such a plan entails a cost for airborne equipment amounting to about $15,000 per aircraft (1950), and fleet costs for 1,000 airplanes would be reduced from $35,000,000 to $15,000,000—a difference of $20,000,000—repeatable each 5 years. This computation certainly dictates a careful analysis and evaluation of the over-all costs for all-weather operation. Such an analysis can be made only by starting at the airport.

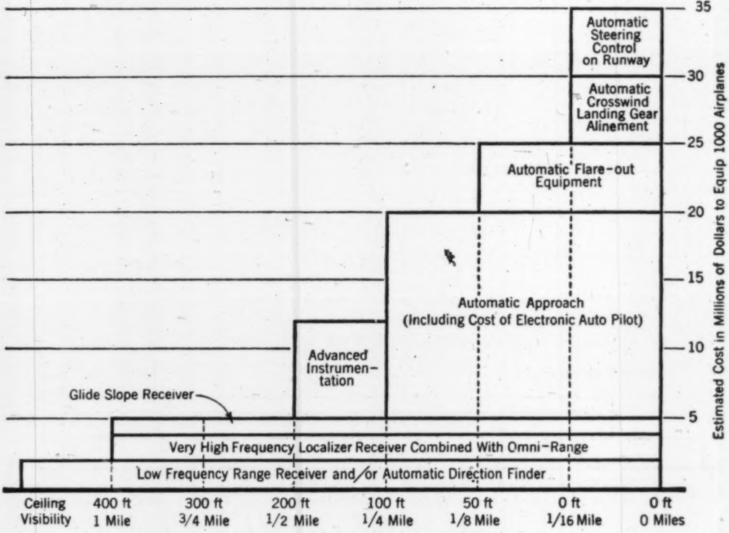

FIG. 2.—COST OF AIRBORNE EQUIPMENT USING ELECTRONIC DEVICES

Ground Equipment.—The airport problem must be considered in two phases: (1) The equipment requirements to properly implement a runway for instrument operations and the cost of such implementation; and (2) a consideration of the airport layout and design. The two are related and when combined are certainly related to airplane design.

Equipping a single runway for all-weather operation will cost a minimum of $350,000 (1950) and may run as high as $1,150,000 if it is found that some Fog Dispersal Unit, such as FIDO is required for operation below 1/16 of a mile visibility (see Fig. 3). This is mentioned because it is very doubtful if, for economic reasons alone, more than 1 or 2 runways will be equipped for complete instrument operations. Regardless of the economics of the question, such factors as the proximity of other airports, terrain obstructions, other flight patterns, and the like make it operationally improbable that more than 2 runways would be used during instrument operations.

AIRPORT REQUIREMENTS

Airports have become extremely expensive facilities. As progress has been made in the field of aircraft design, increased weight and speed of aircraft have imposed stringent airport requirements that show up to a marked degree in the cost and size of the terminals. As the weight, speed, and wing-loadings of the aircraft have increased, larger terminals and longer runways have been required. As greater land areas were required, it has been necessary to move the airports farther from the centers of the municipalities that the airport was intended to serve.

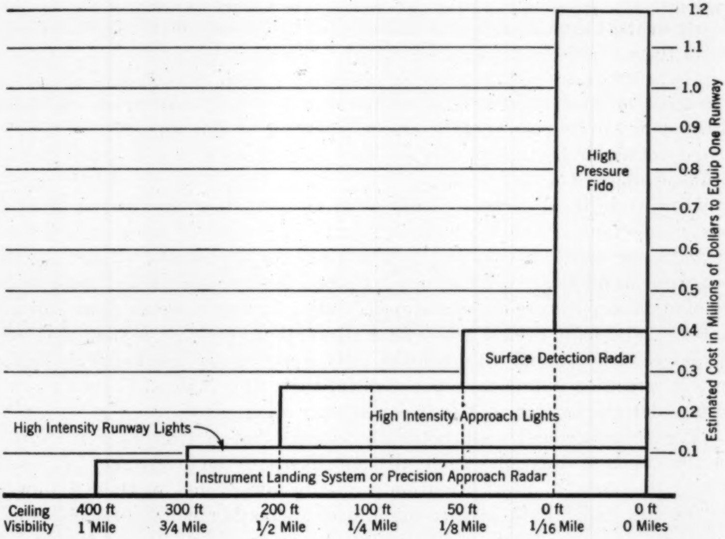

FIG. 3.—COST OF GROUND EQUIPMENT

Here again is found an area of diminishing returns. Land areas of such dimensions are required that necessary expansion and land acquisition is possible only in outlying areas. Aviation is unlikely to attract a passenger when the ground time to reach an airplane from the downtown area equals or exceeds the time required to conduct the flight. Obviously, the longer the flight, the further the airport may be from centers of population. The great untapped market of commerical aviation, however, lies in the short-haul field of 1,000 miles or less. Hence, airports must be kept within a reasonable distance of the downtown area, and such a requirement dictates a change in the design of our aircraft. As long as our concept of airports embodies a multi-runway pattern, the realization of this requirement becomes more and more remote.

Our commercial airports today are monstrosities from the standpoint of size and area. The cost of a single runway designed in 1950 to support aircraft the size of a Boeing 377 almost equals the cost of a complete airport in 1935. Such a runway costs approximately $10.50 per sq yd or $1,500,000. A medium heavy aircraft, such as the DC-6 and the Constellation, requires a runway costing on the order of $7.50 per sq yd or $750,000 (1950).

When these basic runway costs are multiplied by the number of runways normally comprising the airport and when this sum is added to the cost of implementing parallel runways for 100% all-weather operation, it becomes clearly evident that aircraft and airports must be designed to make maximum use of these expensive facilities.

Aircraft design, airport design and implementation, and all-weather operation must be considered as a basically single problem. Attention should be directed to careful cost analysis and comparison of this analysis with anticipated return from capital investments, like airports and facilities for instrument operation.

Multi-runway airports are built with clear-weather approaches from at least 6 directions. Yet, during instrument weather conditions only 2 of these runways are used—one for landings and one for departures. There are two basic reasons for this. First, other air traffic, obstructions, terrain, and proximity of other airports are the operational limiting factors. Second, the economic factor of equipping even 1 runway for all-weather operation is such that only the larger cities can afford a runway fully equipped with instrument landing system, precision approach radar, high intensity approach and runway lights, surface detection radar, and similar necessary equipments.

The economic and operational limitations that restrict an airport to a parallel or 2-runway operation are not likely to be overcome. It seems fitting, therefore, to direct attention primarily toward making these 2 runways do the job under both instrument and clear-weather conditions that the 6 runways do in clear weather. If this can be accomplished, then a tremendous step has been taken toward the reduction of ground or plant costs.

AIRCRAFT DESIGN

The dual objective of lowered ground costs and further inroads into the short-haul travel market can be realized only if there are changes in aircraft design. Some progress has been made in this direction. The tricycle landing gear has increased the allowable cross-wind component, reversible-pitch propellers have increased the allowable down-wind component, and there will be other improvements. But this fact is clear—aircraft design must be oriented in the direction of meeting airport requirements just as in the past the airports have been designed to meet aircraft requirements.

Here, basically, is what is required in aircraft design: (*a*) Cross-wind or caster-gear landing structures must be installed on the airplane; (*b*) slower landing speeds must be achieved, along with shorter take-off runs, and improved braking characteristics; and (*c*) aircraft designed primarily for short-haul operation must be designed capable of operation into smaller airports close in to the downtown area.

THE PARALLEL RUNWAY AIRPORT

If the conditions of aircraft design are met, then the case for the parallel runway airport becomes more clear. Such an airport is illustrated in Fig. 4. This airport employs the continuous flow principle, and its outstanding features are summarized as follows:

1. There are no crossing intersections;
2. High speed turnoffs are provided to clear the runways;
3. All aircraft move in the same direction on the taxiways;
4. There are equal facilities for both directions of approach;
5. Landing directions may be reversed without disrupting operations;
6. All taxi areas and runways are visible to the tower or ground surveillance radar; and
7. A minimum of taxi distance is required.

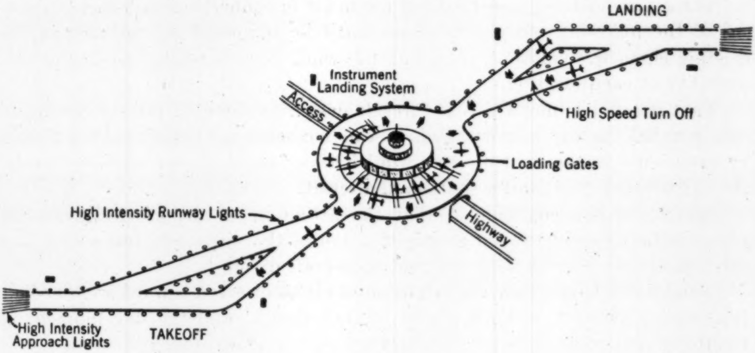
LANDING
Instrument Landing System
Access
High Speed Turn Off
Loading Gates
High Intensity Runway Lights
Highway
High Intensity Approach Lights
TAKEOFF

FIG. 4.—EXAMPLE OF CONTINUOUS FLOW AIRPORT

If the parallel runway airport can be accepted and used, then aviation can afford an all-weather operation. The operational advantages of this type airport are outlined as follows:

a. Regularity of schedules is occasionally dependent upon factors other than weather. Snow removal, for instance, can cause hours of delay. The ability to concentrate on the removal of snow from a single or dual runway could, in many cases, cut delays from this cause.

b. In congested areas, the number of airports that can be used during instrument conditions is dependent to a large extent on the circling radius of aircraft. By polarizing all runways and traffic patterns in such an area, it should be possible to achieve much closer spacing of airports with no decrease in safety.

c. Simplified traffic patterns resulting from single-axis approaches should go a long way toward unsnarling the traffic problem. Airport surveillance radar could be much more effective under these conditions than with the random picture seen on radar screens today (1950).

d. Airport surface traffic control, which is becoming more and more of a problem as minima are lowered, would be greatly simplified by a much more efficient and simpler taxiway pattern. The airport layout would enable the tower to be placed so that better visibility of the operational area could be achieved.

e. Access to the airport and the location of the airport adjacent to principal highways and population areas would be facilitated. Fully automatic landing does not appear practical until the extremely difficult problem of the drift angle at the touchdown point is overcome. The only technically feasible solution to this problem appears to lie in the use of the caster or cross-wind landing gear, and for this reason this factor has been placed foremost in the aircraft design problem. A parallel runway airport is feasible only if this development is achieved.

RECOMMENDATIONS

Three principal recommendations are made in connection with this subject. All of the following recommendations must be tempered by considering the existing climatic conditions, including the wind rose, as related to ceiling and visibility at each location.

First it is recommended that aircraft design be directed toward operation into parallel runway airports and, in this connection, that the development be intensified in the field of cross-wind landing gears, slower landing speeds, shorter take-off runs, and a short-haul aircraft.

Second, it is recommended that the attention of airport designers and planners be directed toward a simplification of the airport layout and a correlation of this activity with the field of aircraft design.

Finally it is urged that the attention of all be directed toward a closer-knit relationship between aircraft design, airport design, and the development of electronic and visual aids for the common goal of all-weather operation.

If these three conditions are met, then all-weather operation becomes a very feasible goal and will prove to be a profitable undertaking. If they are not met, then that goal becomes a most expensive one, and in all probability will prove to be economically unjustified.

DISCUSSION

JOHN H. GEISSE[2].—By his endorsement of the cross-wind undercarriage for airplanes (under the heading, "Aircraft Design") the author has earned the thanks of all those who pay for and use airports. By his persistence in the face of known opposition he is emulating his famous namesake, General "Hap" Arnold. It was the latter who, with equal foresightedness and courage, endorsed the tricycle undercarriage in 1934 against similar opposition.

There are interesting similarities between the two developments which are probably not well known. Both the cross-wind gear and the tricycle gear antidated the so-called conventional gear with two fixed wheels forward of the center of gravity. The tricycle gear was in general use some time prior to World War I and Louis Bleriot had a castered cross-wind undercarriage on his airplane which made the first crossing of the English Channel in 1909. The tricycle gear was reintroduced in 1933 by the Bureau of Air Commerce and the castered gear was reintroduced by its successor, the CAA, in 1945.

There is also one important dissimilarity. In 1934, the Bureau of Air Commerce abandoned its promotion of the tricycle gear and stopped work on two contracts for airplanes with castered wheels. Although there was a determined opposition to the action, in 1949, the CAA abandoned—not the cross-wind gear but—the construction of any more airports of the windrose type.

The loss to the taxpayer and to the industry by this early abandonment of the cross-wind gear will never be fully appreciated. Under the heading, "Airport Requirements," the author states that the land areas required for multiple-runway airports has forced their construction in outlying areas. He also stated that the great untapped market for commercial aviation lies in the short-haul field of 1,000 miles or less. He was being ultra-conservative in this statement. Even under present conditions the average air-line trip is only about 450 miles. The great untapped market is in still shorter hauls, and the ability of the airplane to compete with other means of transportation over these short distances is wholly a function of the distance of the airports from the population centers. Percentagewise, it can save just as much time over short hauls as over long hauls if time to and from the airport does not have to be considered.

The author's argument can be well illustrated by one example: The distance from Chicago, Ill., to Cleveland, Ohio, is not much less than the present trip average; yet between these two cities a DC-3 operating between downtown airstrips could provide faster and cheaper transportation than supersonic airplanes operating between the existing airports.

Perhaps even more pronounced has been the effect of this abandonment on private flying. In 1940 the CAA made two studies of private flying. One disclosed that only 15% of those who started flying continued with it. The other disclosed that the major reason for this poor showing was the inaccessability of airports. This was a combination of airports too far out and airports completely nonexistent because of their high cost.

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[2] Consulting Engr., Washington, D. C.

More tangible than the foregoing example is the sum of money that was spent needlessly on airports between 1935 and 1945. In this period approximately $800,000,000 was spent on the construction of civil airports alone. Some years ago the writer studied the distribution of costs on all classes of airports and found that an average saving of 50% could be made by eliminating wind coverage. Assuming this cost to be twice too high, the saving that could have been made ($200,000,000) would have been more than the present net value of all the flight equipment of all domestic airlines operating in the United States.

The author states (under the heading, "Installation of Equipment: Airborne Equipment") that it costs $6,250 per year to transport 100 lb of equipment in one airline aircraft. This factor must be given consideration when recommending any alteration in this equipment. However, it is quite possible that adding a cross-wind gear in the original design can subtract weight. In all transport equipment, the wheels are mounted on collapsible shock struts which can provide perfectly good caster spindles if the wheels are in caster relationship therewith. The added weight would then be the weight of a caster restraint and shimmy damping mechanism, less the weight of the jackknives now used to prevent rotation in the strut.

As against this slight increase in weight a substantial decrease in the weight of the entire landing gear would be made possible by the reduction in the side loads on the gear. With castered wheels this side load needs to be only that required for adequate taxi control—not more than one third the weight of the airplane. With fixed wheels the entire undercarriage, including the wheels, must be designed to take an inwardly directed side load equal to 0.8 times the verticle load and an outwardly directed side load of 0.6 times the verticle.

The paper presents such an excellent argument for the advantages to be gained by the use of omnidirectional airports that one may wonder what are the disadvantages that have delayed their acceptance. As far as the writer is aware there are none.

There are two major reasons for the delay. One is the question of which should come first, the change in the airports or the change in the airplanes: Why build an airport adapted primarily to a nonexistent airplane? Why build an airplane to use airports that do not exist? The other reason, and a cogent one, is that any added costs of the conversion must be paid for by the airplane owners whereas the immediate savings go to the airport owners. In view of this situation the fact that the savings would far exceed the costs has little weight.

Opposition to initiating the change in the airports stemmed from various roots. There were the owners of airplanes who thought that the taxpayer and others should continue their airport subsidization of their flying rather than ask them to spend some of their money to make this unnecessary. Then there were those who, politically or otherwise, had no desire to see the cost of airports drastically reduced.

In the face of this opposition the Civil Aeronautics Administrator in 1949 made his courageous decision to refuse any further federal funds for the construction of runways whose only purpose was to provide wind coverage. The effect of this decision will become evident only as new airports are constructed but it is to be hoped that the author's endorsement of the cross-wind undercarriage will hasten its consideration by the airplane manufacturers and users. These airports will be constructed and in use long before airplanes now on order, or on the drawing board, have been retired.

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TRANSACTIONS

Paper No. 2514

LATERAL FORCES OF EARTHQUAKE AND WIND

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With Discussion by The Seismic Group, Architectural Institute of Japan; Messrs. Reuben W. Binder; R. R. Martel, G. W. Housner, and J. L. Alford; S. K. Guha and Gurdas Ram; and Arthur W. Anderson, John A. Blume, Henry J. Degenkolb, Harold B. Hammill, Edward M. Knapik, Henry L. Marchand, Henry C. Powers, John E. Rinne, George A. Sedgwick, and Harold O. Sjoberg

Synopsis

In this paper the writers have drafted a lateral force provision which constitutes a reasonable requirement for building codes. Although it was prepared specifically for California conditions, the code is generally applicable, possibly with some quantitative modifications to suit different local conditions. The report includes the development of a more rational criterion for design seismic forces than has been attempted heretofore in lateral force codes.

Introduction

Earthquake design forces stipulated in California codes vary from a maximum of 13.3% of vertical loads for the Los Angeles, Calif., formula to a minimum of 2% of vertical loads for the state law known as the Riley Act. Criteria by which these forces are established are generally empirical and not always

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Note.—The authors of this paper constitute a Joint Committee of the San Francisco, Calif. Section, ASCE, and the Structural Engineers Association of Northern California, as appointed April 2, 1948. Mr. Rinne served as Joint Committee chairman. Published in April, 1951, as *Proceedings-Separate No. 66*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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consistent or rational. They do not account for the dynamic characteristics that determine how structures respond to earthquake ground motion. Advances in scientific knowledge have not been reflected in current building codes.

On April 2, 1948, the San Francisco (Calif.) Section, ASCE, and the Structural Engineers Association of Northern California appointed a Joint Committee on Lateral Forces to evaluate the current understanding of wind and earthquake forces and to draft a model lateral force provision for the building codes in California. The writers constituted the personnel of that Joint Committee. It is believed that, with some modification for local conditions, the resulting code (see Appendix I) will be applicable to a larger field.

As part of its study the Joint Committee has:

1. Reviewed the historical record of earthquake damage;
2. Noted the limited accurate recording of ground accelerations in earthquakes;
3. Extended the mathematical response of simplified structures subjected to known earthquake acceleration patterns into a practicable dynamic approach to earthquake design forces on engineering structures;
4. Indicated a number of problems that require further study, acknowledging that there is still much to be learned about earthquakes and earthquake effects on structures; and
5. Prepared the model lateral force provision.

A statement of the function of the lateral force provision is necessary so that it is understood and agreed what is to be accomplished by the provision. It is the province of the building code to specify design and construction requirements which will result in structures safe against major structural damage and the loss of life in the event of precedented winds or earthquakes. The lateral force provision is primarily structural. Panic and fire hazards, for example, are covered in other sections of the building code. The design requirements are considered to be the minimum consistent with the general objective. If unfavorable conditions of a particular site, such as the proximity of known faults, extreme exposure to high winds, or the importance of continued operation of a facility in the event of unprecedented winds or earthquakes, suggest the advisability of using design lateral forces in excess of those specified, this is left to the judgment of the owner on the advice of his engineer. The basis of such judgment is a matter of evaluating the calculated risk and is beyond the scope of the minimum provisions of a building code.

The relationship between wind and earthquake for design purposes is well recognized. Both wind and earthquake involve primarily lateral force considerations. Wind forces relate to areas, whereas earthquake forces relate to masses. However, the principle of determining the larger of the two forces and being governed thereby is accepted design practice, and is the basis for specifying both wind and earthquake minimum design forces in a code section entitled "Lateral Forces."

Wind forces have been studied extensively by others (see Appendix II); design criteria are well established and well in advance of corresponding criteria for earthquake forces. The principal problem of the Joint Committee was the determination of earthquake forces, and these, therefore, have been developed in greater detail than have wind forces. Considerations have been confined to the establishment of the design forces; no attempt has been made to specify methods of providing for these forces.

EARTHQUAKE HISTORY

References to the occurrence of earthquakes are sprinkled liberally throughout written history. To a lesser extent, observations of damage have been recorded, but few scientific records of ground motion have been made.

California has a comparatively short historical record. Some of the earliest records are the diaries of the padres and the early Spaniards which tell of heavy earthquakes in the 18th century. In Northern California alone, since 1800, there have been no less than seventy shocks of Intensity VIII or higher by the Rossi-Forel scale. Continued seismic activity, recently on a minor scale, portends future shocks which there is no reason to believe will be any more or any less severe than those experienced in the past. The earthquake of 1906 along the San Andres fault in the vicinity of San Francisco may be accepted as a reasonable maximum shock for design purposes in California. This earthquake is rated as one of the three strongest known to have occurred in California; but, unfortunately, the ground motion data are very limited and of questionable accuracy. The ground motion of more recent shocks, although of lower intensity, have been recorded more consistently.

In any event, there is no justification in the historical record for complacency or indifference to the design for earthquake forces. The brief seismic history of the United States as portrayed (54)[11], in Fig. 1, indicates that an awareness of the hazards of earthquakes and the application of the basic principles of seismic design should not be confined to California.

The year 1906 marks the beginning of a period of intensive study of earthquakes and research in earthquake-resistive design. Observations of the effects of the 1906 San Francisco earthquake have been well documented by several investigators. Records are also available of the following subsequent major shocks:

Location	Year
Tokyo, Japan	1923
Santa Barbara, California	1925
Long Beach, California	1933
Helena, Montana	1935
El Centro, California	1940
Fukui, Japan	1948

It is not necessary to relate extensively the recorded history of earthquake damage. A brief review of the nature of the damage will be given; more detailed information is available by reference to Appendix II.

DAMAGE TO STRUCTURES

Generally, the records confirm that buildings of sound design and good construction fare well and that those of poor design and poor construction, from the standpoint of lateral forces, fare badly. The advantages of flexibility coupled with strength were apparent to those reporting the 1906 shock and also

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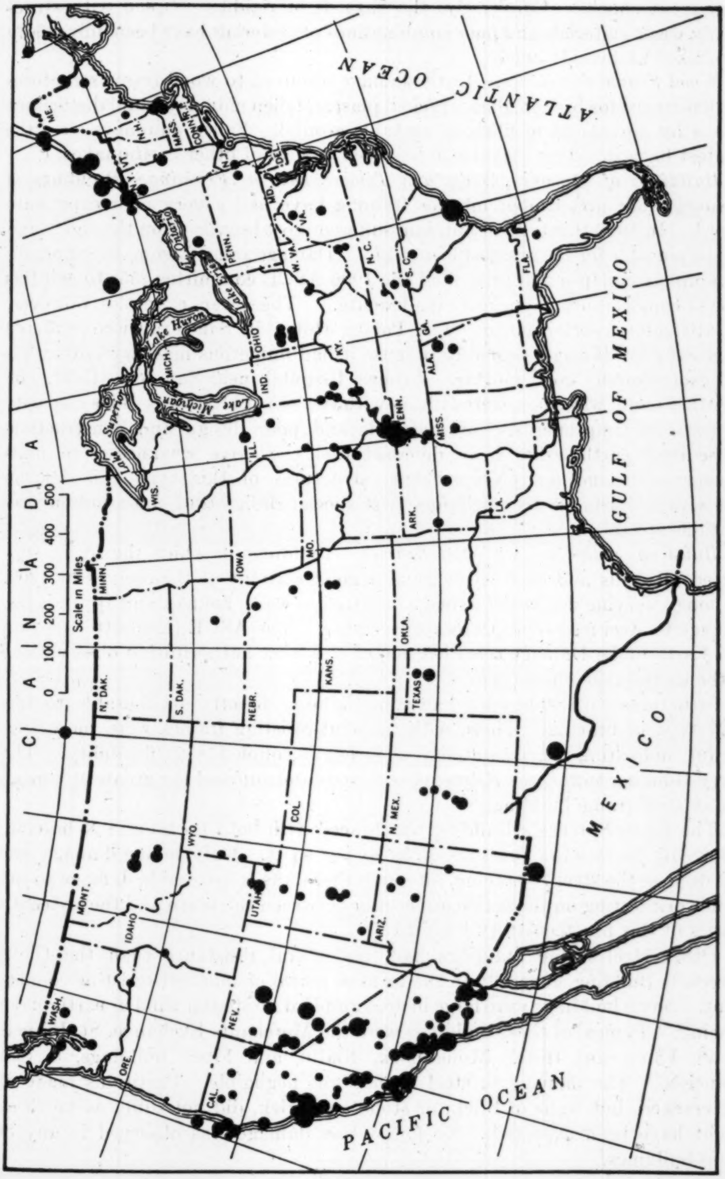
[11] Numerals in parentheses, thus: (54), refer to corresponding items in the Bibliography (see Appendix II).


CANADA
ATLANTIC OCEAN
GULF OF MEXICO
MEXICO
PACIFIC OCEAN
Scale in Miles
0
100
200
300
400
500
Lake Superior
Lake Michigan
Lake Huron
Lake Erie
Lake Ontario
ME.
N.H.
VT.
MASS.
CONN.
R.I.
N.Y.
N.J.
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MD.
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MICH.
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MISS.
ILL.
WIS.
MINN.
IOWA
MO.
ARK.
LA.
N. DAK.
S. DAK.
NEBR.
KANS.
OKLA.
TEXAS
MONT.
WYO.
COL.
N. MEX.
IDAHO
UTAH
ARIZ.
WASH.
ORE.
NEV.
CAL.

FIG. 1.—DESTRUCTIVE AND NEAR DESTRUCTIVE EARTHQUAKES IN THE UNITED STATES THROUGH 1946

subsequent shocks. Conversely, the hazards of rigidity coupled with structurally weak materials and poor combinations of materials have been universally recognized and condemned.

Wood Frame Structures.—Little damage occurred to wood frame structures in 1906 except for broken glass, cracked plaster, fallen chimneys, and distortions caused by movement of filled or unstable ground. Fallen chimneys were the greatest hazard. This experience has been typical of other earthquakes.

Buildings of Masonry Walls and Timber Interior Framing.—Buildings of masonry walls and timber interior framing have had a very poor experience record. In 1906 this type of building suffered most heavily from the shock and was responsible for the greatest loss of life. Damage ranged from minor cracks to complete collapse and prompted the 1906 ASCE committee (55) to call this type of construction "hopelessly inadequate." There were examples, however, of satisfactory performance. The Palace Hotel, in San Francisco, suffered practically no damage, probably because it had numerous masonry cross walls and exceptionally good mortar. Agnews Hospital, near San Jose (Calif.), on the other hand, was a complete failure, with the loss of 112 lives. This disparity suggests the improper use of materials and poor design and construction. Subsequent earthquakes have demonstrated the same weaknesses in unit-masonry construction. Nevertheless, structures of this type can develop reasonable resistance to earthquakes, if special design and construction precautions are taken.

Buildings with Complete Steel Frames.—Buildings in which the frame supported all walls and floor loads gave a good account of themselves in 1906. Although varying degrees of damage occurred to walls, facings, and appendages, damage to steel frames appeared to be minor. The ASCE Committee on Fire and Earthquake Damage recorded one of the most authoritative observations of the earthquake effects (55).

Structures with X-bracing had some failures directly attributable to this rigid type of bracing. Those with moment-resisting frames were more successful, indicating the desirability of strength coupled with flexibility. The Ferry Building tower was X-braced with rods and suffered the greatest damage of any steel frame building.

The Claus Spreckels Building was braced with both portal and X-bracing, reportedly for a wind pressure of 50 lb per sq ft. Only minor damage was apparent in the steel X-bracing, although there was considerable damage to the stonework resting on the frame, especially betweeen the tenth and the sixteenth stories of this nineteen-story building.

Other steel frame buildings had less lateral resistance than the Claus Spreckels Building, generally in the form of portal or moment-resisting connections. Some buildings were large in area and had no special wind or earthquake bracing. Typical of these buildings were the Merchants Exchange, St. Francis Hotel, Fairmount Hotel, Monadnock, Rialto, and Flood buildings, in San Francisco. The damage to steel frames was negligible. Partitions made of tile cracked, but walls of brick, or stone and brick, did not suffer as much as might have been expected. No foundation damage was observed in any of these buildings.

Some rivets were sheared in the Flood Building, but this damage is generally regarded to have been minor. According to current methods of analysis, and with generous assumptions, this building will resist no more wind pressure than 10 lb per sq ft or a lateral load of 0.6% gravity.

The cracking of brick walls (St. Francis Hotel, Merchants Exchange, Atlas, New Chronicle, and Monadnock buildings, as examples) indicates a need, not only for an adequate bonding of the walls to the steel frame but also for a recognition of the rigidities of these structural elements.

The record in Tokyo in 1923 was somewhat similar, although evidence at hand seems to indicate that this earthquake was much more severe than the San Francisco earthquake of 1906. Quoting from a paper by John R. Freeman (2*a*),

> "There were 16 large steel-framed buildings in Tokyo. Of these, six were absolutely undamaged by the earthquake, the remaining ten being more or less severely damaged, one to the point of incipient collapse."

This last building, the Tokyo Kaikan, has been described as follows (2*b*):

> "The steel frame was very light. The steel columns were so small in cross-section and the bracing so little that this can hardly be classified as a steel frame building, the construction being far below both the Japanese and American standards for steel frame buildings."

The Nippon Kogyo Bank, in Tokyo, designed by Tachu Naito for a lateral force of g/15, was among the buildings practically undamaged.

Among the more severely damaged buildings of the steel frame type were the Marunouchi, Yuraku, Japanese Oil, Yusen, Mitsukoshi, Tokyo Kaijo, and Chiyoda buildings in Tokyo. Of these buildings, Mr. Freeman states, "* * * the question of bracing was more or less incidental, and probably subordinated to other considerations." From the meager details available, the undamaged steel buildings seemingly had reinforced concrete filler walls, whereas those that were damaged had brick or tile walls. Without more complete details, it is impossible to appraise properly the damage to these buildings.

Reinforced Concrete.—In 1906 reinforced concrete construction was in its infancy, but well-designed structures came through excellently. At Santa Barbara in 1925, reinforced concrete frame buildings with concrete floors and hollow tile filler walls suffered fractured concrete columns and more or less complete destruction of the tile walls. Other all-reinforced concrete buildings fared better, although there was some cracking of walls and piers. Tokyo contained a large number of concrete buildings in 1923. Of some 700 buildings of this type, approximately 75% were undamaged, 2% failed completely, and 23% were damaged in varying degree. The experience of composite construction with weak-bonded filler walls of another material, such as brick or terra cotta tile, has not been very satisfactory.

Building Appurtenances.—Taken as a whole, a very small percentage of buildings has suffered complete collapse. Considerable damage and loss of life, however, have been attributed to lack of attention to certain design and construction details. Fallen chimneys have been mentioned. Damage to parapets, cornices, filler walls, gable end walls, and various appendages has been very common in all major earthquakes. Such damage is inexcusable, since, with reasonable attention to the details of the fastening of these parts to the structure, the damage and attendant injury and loss of life need not have occurred.

Structures Other Than Buildings.—Structures, such as bridges, towers, and chimneys, have also suffered damage. The records contain much data on damage to bridges which is not related here, since bridges are not generally built under the jurisdiction of the building codes. They are a subject for separate study and report. Industrial structures, such as free-standing towers and chimneys, however, do come under the building codes.

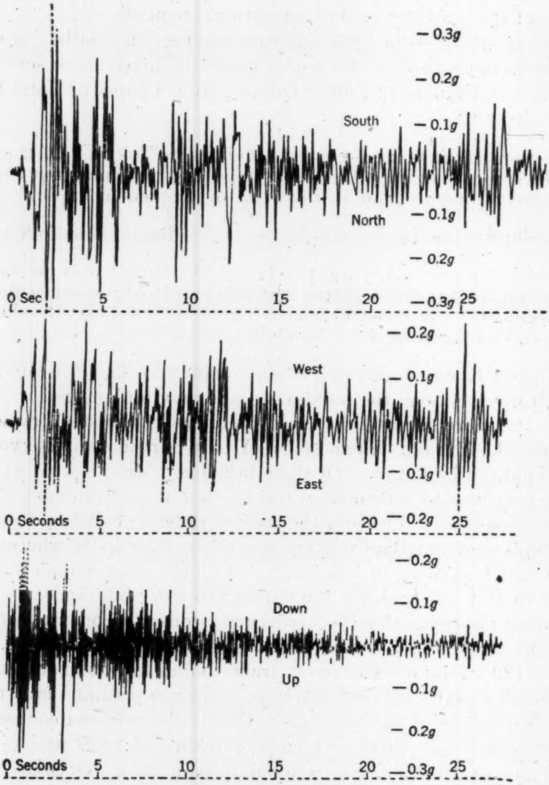


FIG. 2.—EL CENTRO ACCELEROGRAPH RECORD OF GROUND ACCELERATIONS; IMPERIAL VALLEY EARTHQUAKE OF MAY 18, 1940

Tank towers of the rod-braced type have been extremely vulnerable to damage. One reason given is the relative rigidity of the X-bracing and its relative weakness compared to the "reserve" strength of the vertical load-carrying members. Following failure of the rigid bracing, the tower deflects under the influence of the ensuing ground motion to the point where the remaining frame fails and collapses. Typical was the experience at El Centro, in 1940, where two elevated tanks collapsed, four tanks were seriously damaged, and two others were undamaged. The two undamaged tanks had been designed or rebraced for lateral forces of 0.1 g.

Frequent partial or complete collapse of chimneys (mostly brick, but also some concrete) has also occurred. An observation that the failures generally come well above the base indicates a weakness not properly covered in design by the conventional wind or static lateral force methods.

GROUND MOTION

Knowledge of the effects of earthquakes on structures is much more extensive than knowledge of the ground motion responsible for these effects. To account for the damage produced in San Francisco (1906) and in Tokyo (1923), for example, there are only rough estimates of the accelerations based upon observed effects on simple structures such as tombstones.

In 1932 the United States Coast and Geodetic Survey (USCGS) initiated a strong-motion program in California to obtain instrumental records of ground motion in destructive areas. The Long Beach Earthquake of March 10, 1933, produced the first results. They were imperfect, however, because the acceleration exceeded that for which the instruments were designed. The program was extended with instruments of more rugged design, and the El Centro record (51) of May 18, 1940, shown in Fig. 2, represents the most significant result to date. Table 1(*a*) summarizes instrumental data from Long Beach and Los Angeles, 1933; El Centro, 1940; and some of the aftershocks at Helena, 1935. Also tabulated in Table 1(*b*) are noninstrumental estimates of ground motion at San Francisco, Tokyo, and Fukui. The data in Table 1 should not be used without consulting the complete record cited in Col. 15. The velocities, accelerations, and displacements are maxima and do not necessarily correspond one with the other.

The strong-motion seismographs in general use are called accelerographs because, for all practical purposes, the record produced is that of three mutually perpendicular components of the ground acceleration. The acceleration record may be integrated to obtain the velocities and displacements against time which are shown in Figs. 3 and 4 for the El Centro record (51).

Aside from these instrumental records, the practice has been (and in the main continues to be) to rate earthquakes on the basis of their destructiveness, or on a so-called intensity scale. About 1883 the Rossi-Forel (37) scale was established, ranging through ten divisions of intensity. In 1931 the USCGS adopted the Modified Mercalli scale (48) ranging through twelve divisions, Intensity I being an earthquake barely felt under favorable conditions, and Intensity XII (the other extreme) being the most severe ever experienced. Even

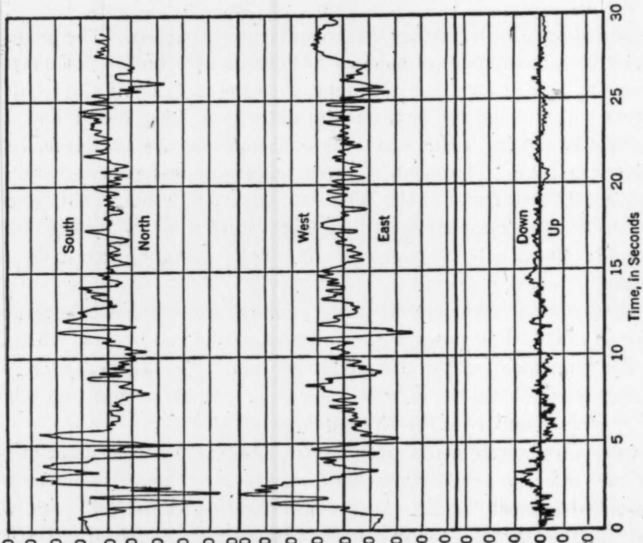


FIG. 3.—GROUND VELOCITIES FROM THE EL CENTRO ACCELERATION RECORD OF MAY 18, 1940

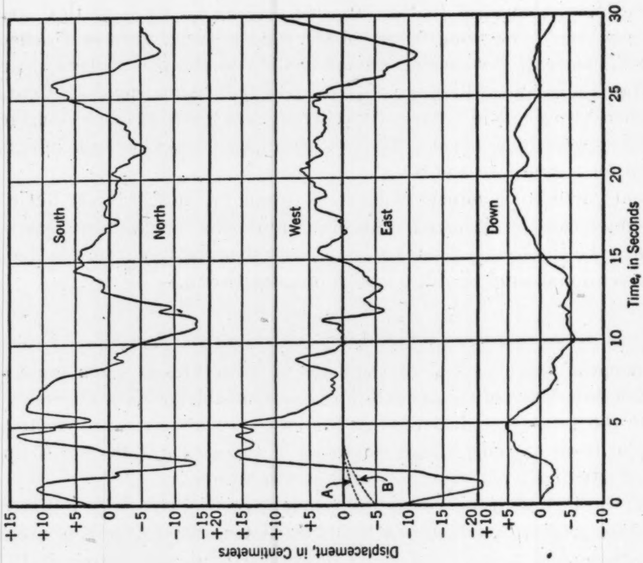


FIG. 4.—GROUND DISPLACEMENTS FROM THE EL CENTRO ACCELERATION RECORD OF MAY 18, 1940

where instrumental records of ground motion are obtained, the observation of damage and the establishment of the intensity of the earthquake by a damage scale is a necessary supplement to the records for an engineering understanding of the earthquake and its effects.

TABLE 1.—SUMMARY OF GROUND MOTION

Line	Description	MAXIMUM ACCELERATION								MAXIMUM HORIZONTAL VELOCITY		MAXIMUM HORIZONTAL DISPLACEMENT		Reference[a]
		Horizontal				Vertical								
		$a\left(\frac{\text{cm}}{\text{sec}^2}\right)$	$\frac{a}{g}$	P Period (sec)	Displacement (cm)[b]	$a\left(\frac{\text{cm}}{\text{sec}^2}\right)$	$\frac{a}{g}$	P Period (sec)	Displacement (cm)[b]	$\left(\frac{\text{cm}}{\text{sec}}\right)$	Period (sec)	(cm)	Period (sec)	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
	(a) Strong-Motion Instrument Records													
	Long Beach, Calif., 1933:													
1	Utilities Bldg.[c]	230	0.24	0.30	0.50	250	0.26	0.11	0.07	25	1.50	6.0	1.5	(48*a*)
2	C M D Bldg.[e,d]	210	0.215	0.70	2.6	50	0.05	0.13	0.02	24	0.70	6.0	2.5	(48*a*)
3	Subway Terminal[e]	60	0.06	1.0	1.5	20	0.02	0.20	0.02	15	2.50	6.0	2.5	(48*a*)
4	Helena, Mont., 1935	115	0.12	0.13	0.05	78	0.08	0.125	0.03	9.6	1.2	1.8	1.2	(49*a*)
5	El Centro, Calif., 1940	320	0.33	0.3[f]	1.3	220	0.23	0.10	0.07	30	4.0	19.0	4.0	(50)
	(b) Noninstrumental Records													
	San Francisco, Calif., 1906:													
6	Rock	...	0.025	...	...	...	...	...	...	...	...	5	1.0	...
7	Marsh	...	0.30	...	...	...	...	...	...	...	...	10	1.0	...
8	Tokyo, Japan, 1923	...	0.33	...	...	...	...	...	...	...	...	18	2	...
9	Fukui, Japan, 1948	...	0.60	...	...	...	...	...	...	...	...	...	1.5	...

[a] Reference numbers corresponding to those in the Bibliography, Appendix II. [b] As calculated by the formula $\frac{a P^2}{4 \pi^2}$. [c] Long Beach. [d] Central Manufacturing District Bldg. [e] Los Angeles. [f] 0.3 sec to 0.5 sec.

EARTHQUAKE FORCES

Various assumptions have been used to determine, from the ground motion, the forces acting on a structure. The simplest and most extensively used analysis assumes the acceleration of all parts of the structure to conform to the acceleration of the ground. The transformation of accelerations to forces is made by applying Newton's law: Force is proportional to mass times acceleration. This assumption requires a structure of infinite rigidity and fails to consider the elastic characteristics of real structures. The case of the Nippon Kogyo Bank is cited to illustrate the inadequacy of such a simple assumption; the building was practically undamaged although the estimated ground acceleration was five times the acceleration corresponding to the design lateral force.

Dynamic analytical methods based on the elastic theory of deformations of a uniform cantilever have been proposed by several earlier investigators (56)

(59). Lacking accurate ground motion records, it was impractical to complete these analyses rigorously. The assumption of simple harmonic ground motion was generally made, and the results reflected the steady-state condition of vibration. The conclusions thus established were of limited value, for sinusoidal ground motion does not occur in the destructive zone of an earthquake; nor can steady-state response of the structure be established.

In the absence of sustained harmonic vibrations, which is apparent from the El Centro record, more elaborate dynamic theories are necessary. These are based on the transient response of elastic structures to the irregular ground motion that actually occurs and which has been recorded accurately for a few earthquakes, starting with that at Long Beach, in 1933.

DYNAMIC APPROACH TO EARTHQUAKE FORCES

The behavior of structures in earthquakes has generally been recognized as a dynamic vibration phenomenon of a transient nature. Although rigorous solutions are possible for particular ground motions applied to particular structures, these solutions are too involved and of too limited significance to be of direct practical value to the structural engineer. The more rigorous methods, however, should be encouraged to guide the thinking toward less rigorous but more practicable methods.

It is the purpose herein to outline an approach to earthquake forces which, although not new in its basic concept, has been extended to a rational dynamic method for establishing the design forces on a structure. The method involves: First, the determination of the total lateral force or the base shear transmitted into the structure from the ground; and, second, the distribution of that shear as equivalent forces applied to the structure.

Determination of Base Shear.—The base shear for any structure subjected to ground motion is the sum of components each of which corresponds to a mode of vibration. Each mode may be represented by a simple one-mass system as regards the base shear for that mode. Detailed analysis indicates, however, that, for practical application only, the first or fundamental mode need be considered.

Mathematically, the response of a simple, undamped one-mass system to an earthquake acceleration pattern, as measured by the base shear at a particular time, t, is given by the formula:

$$V_t = \frac{-W\,k}{g} \int_0^t f(z) \sin k\,(t - z)\,dz \;\ldots\ldots\ldots\ldots\ldots (1)$$

in which V_t is the base shear, in pounds; W is the weight, in pounds; k is equal to $2\pi/T$; T is the period of the system, in seconds; g is the acceleration of gravity equal to 32.2 ft per sec per sec; t is the time, in seconds; z is the variable of integration of the nature of time; and $f(z)$ is the applied acceleration function of time. The base shear V_t will vary in an irregular manner with time, depending on the irregular nature of the acceleration pattern, and will have both positive and negative values. However, for design purposes the important values of V_t are the maximum. It is convenient to determine the envelope $\bar{V}_t$ that will contain V_t, as follows:

$$\bar{V}_t = \frac{W\,k}{g}\,(X^2 + Y^2)^{\frac{1}{2}} \ldots\ldots (2)$$

In Eq. 2

$$X = \int_0^t f(z) \sin (k\,z)\, dz \ldots\ldots (3a)$$

and

$$Y = \int_0^t f(z) \cos (k\,z)\, dz \ldots\ldots (3b)$$

The maximum value of the envelope, V, is the absolute maximum base shear that will develop in the one-mass system of period T during the particular earthquake being analyzed. It is called the spectral value of V_t, and if $V = \frac{W\,\bar{a}}{g}$, then $\bar{a}$ is the spectral acceleration, or the spectral intensity of the earthquake for the period T. If a number of values of $\bar{a}$ are determined for varying periods, T, and if $\bar{a}$ is plotted against T, the resulting curve is the acceleration spectrum and is characteristic of the particular earthquake.

Although the El Centro record exhibits an irregular pattern and lacks any semblance of a simple harmonic motion and although the spread of acceleration periods is wide, there are definitely dominating periods between 0.1 sec and 0.3 sec. Other minor and moderate earthquakes recorded seem to have peak acceleration periods in this same general range. The accelerations produced by explosive shocks, which simulate minor earthquakes, also exhibit this short period predominance. It might be expected, therefore, that a one-mass system subjected to actual earthquake acceleration patterns would build up a greater response or resonance effect when its natural period, T, is in the range of the dominant earthquake acceleration periods; and it might also be expected that when the period, T, is removed from the dominant range of the earthquake, the response would decrease. Available data confirm this view.

The response of an undamped one-mass system can be determined analytically or experimentally as solutions of Eq. 1 or Eq. 2. Experimentally, a model such as a torsion pendulum can be subjected to an angular disturbance of the same pattern as the recorded ground motion, and the resulting maximum angular response is the spectral acceleration for the particular period of the torsion pendulum. This is directly proportional to the maximum base shear, V. By a series of such tests, the acceleration spectrum may be obtained, which is indicative of the response or the maximum base shear transmitted from the ground into one-mass systems of varying period, T.

M. A. Biot, using this model method (20) (43) (63) determined the spectra of a number of earthquake patterns, including the Helena aftershocks (1935) and those recorded at Ferndale (Calif.) on medium intensity shocks (1935, 1937, and 1938). From these recordings, an average response curve was obtained which he calls a "standard acceleration spectrum."

Calculations based upon the El Centro record, made at the Watson Scientific Computing Laboratory, New York, N. Y., on the recommendation and under the direction of E. C. Robison (33), produced similar results. Plotted to a qualitative scale in which the maximum response or base shear is represented by 100, Mr. Biot's standard acceleration spectrum and Mr. Robison's El Centro data are shown in Fig. 5. The two curves have remarkably similar shapes, with peaks at 0.2 sec and 0.25 sec periods, respectively.

Following the Biot curve, if a one-mass system of 0.2-sec natural period develops a base shear of 100 kips, a one-mass structure of the same weight but with a natural period of 0.73 sec would develop a base shear of only 30 kips under the influence of the same earthquake pattern.

A lateral force coefficient, C, may be defined by the formula:

$$V = C\,W \text{ ....} \quad (4)$$

If W is constant, the lateral force coefficient C is directly proportional to V, the maximum base shear. In effect then Fig. 5 also represents the variation of the lateral force coefficient, C, with the period of the one-mass system; and since the one-mass system represents the real structure in its response, these conclusions apply equally well to the real structure.

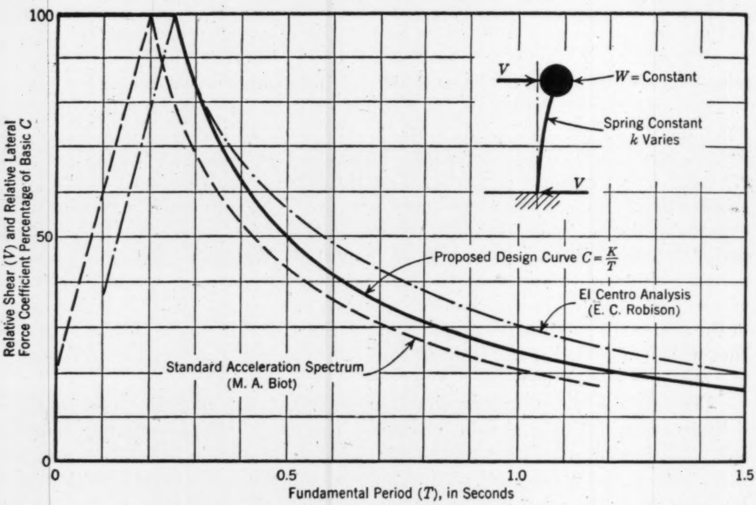


FIG. 5.—RESPONSE OF SIMPLIFIED STRUCTURES TO EARTHQUAKES

The calculated quantitative responses of the idealized one-mass systems are considerably higher than experience indicates is necessary in the design of real structures. Mr. Biot's standard acceleration spectrum has a maximum value of 1.0 g at $T = 0.2$ sec. All the effects of such factors as damping, tilting, and slipping in real structures tend to reduce the shear transmitted from the ground. The net result is that the spectra must be treated as qualitative curves and the assignment of quantitative coefficients must be based on the damage experience in earthquakes, with special reference to structures of conventional design which have withstood an earthquake of major proportions without significant damage.

In practical application, it is inadvisable to reduce the lateral force coefficient for periods less than that corresponding to the maximum coefficient. Therefore, it is proposed to require design for the maximum coefficient for periods from 0 to 0.25 sec. For periods higher than $T = 0.25$ sec, it is proposed to reduce the coefficient along a curve $C = K/T$, in which K is a constant depending on the maximum coefficient selected; and T is the natural period of the structure in the fundamental mode. This qualitative design curve is shown in Fig. 5 and falls between the Biot curve and the Robison curve. A minimum coefficient should also be established which will apply for all periods higher than that for which $C = K/T = C_{\text{min}}$. The quantitative maximum and minimum coefficients and the corresponding value of K will be discussed in detail after other aspects of the problem are considered. For the present it is sufficient to note that the base shear has been established by two formulas, Eq. 4 and

$$C = \frac{K}{T} \qquad (5)$$

Distribution of Base Shear as Applied Forces.—The base shear V having been determined, the distribution of that shear as a number of applied lateral forces on the structure becomes the second part of this approach to earthquake forces. For this consideration, two types of simplified structures are analyzed, both being cantilevers of uniform weight per unit of height: Case I for which the deflection is entirely due to bending; and Case II for which the deflection is entirely due to shear, or story-to-story bending combined with shear. Both these simplified structures are capable of vibrating in an infinite number of modes. For any particular mode, the deflection curve also represents the lateral force curve or the distribution of the base shear as applied lateral forces up through the structure. The deflections are given by the formulas (4*a*): Case I—

$$y = B\,(\cos k\,x - \cosh k\,x) + D\,(\sin k\,x - \sinh k\,x) \qquad (6a)$$

Case II—

$$y = \bar{a} \sin m\,x \qquad (6b)$$

in which y is the deflection at height x; x is the height above the base; k and m are constants depending upon the mode of vibration and physical properties of the structure; and $\bar{a}$, B, and D are constants depending on end conditions. For Case I the lateral force is proportional to the fourth derivative of y with respect to x, $\left(\frac{d^4y}{dx^4}\right)$, which has exactly the same form as the equation for y. For Case II the lateral force is proportional to the second derivative of y with respect to x, $\left(\frac{d^2y}{dx^2}\right)$, which again has exactly the same form as the equation for y.

The distribution of a unit base shear ($V = 1$) applied to a structure of unit height ($H = 1$), vibrating in the fundamental mode is shown in Fig. 6(*a*) for Case I bending deflection, Case II shear deflection, and Case III conventional static lateral load proportional to weight distribution. For most structures, the distribution of the unit base shear falls between the curves for Cases I and

II as limiting conditions of deflection. For slender structures, it will be in accord with Case I curve; for squatty structures, it will be in accord with Case II curve. The Case III curve is an empirical criterion for establishing a lateral force, adequate in some cases but not technically logical.

If the deflection δ at the top of the structure, vibrating in the fundamental mode, is half due to shear and half due to moment, this combination is shown in Fig. 6(*b*), the mean between w_M and w_V, for $\delta_M = \delta_V$ being

$$w' = \tfrac{1}{2}(w_M + w_V) \qquad (7)$$

This curve approximates very closely a triangular loading, zero at the base and twice the average lateral load at the top. For structures approximating uniform weight per unit height and for which not more than half the deflection results from moment, the triangular lateral load gives a conservative basis for the distribution of the base shear. For structures in which moment deflection predominates, the triangular loading is still adequate although greater refinement can be used by distributing the base shear directly in accordance with curve I of Fig. 6(*a*). A more general form of distribution, which assumes linear deflection but which permits the variable distribution of the masses or weights of the structure, is given by the formula,

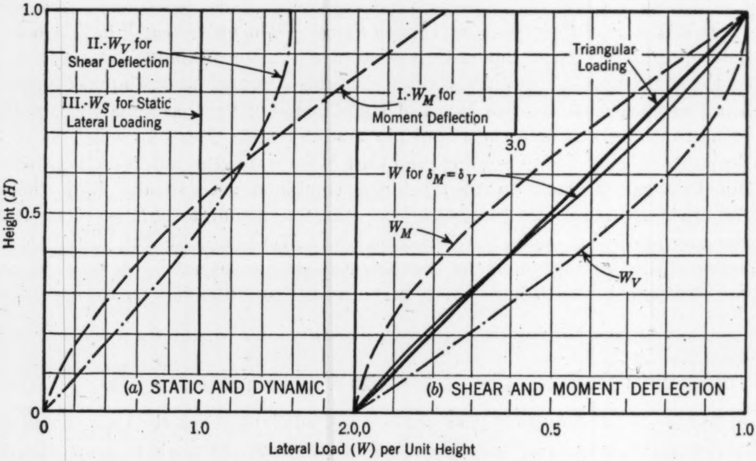


Fig. 6.—Lateral Load Distribution

$$F_x = V\left[\frac{w_x h_x}{\sum (w\,h)}\right] \qquad (8)$$

For uniform weight per unit height, assumed in developing the curves in Fig. 6, Eq. 8 reverts to the triangular lateral loading.

The triangular loading or the formula for F_x (Eq. 8) determines the vertical distribution of the base shear. To complete the application of these forces, they must be distributed over the area of the structure at level x. This should be done in accordance with the mass distribution on that level.

Shear Curves.—The integration of the lateral force curves of Fig. 6 for a unit base shear results in the shear curves (numbered 1, 2, 3, 4) in Fig. 7. The triangular loading shear curve, curve 1 is closer to the shear curve for shear deflection, curve 2, than it is to the shear curve for moment deflection, curve 4. This is reasonable for the average structure. Tests made by J. A. Blume, M. ASCE, on the Alexander Building in San Francisco (22), show that this rather tall, slender structure (60 ft by 68 ft by 196 ft high) deflects roughly half in shear and half in moment, excluding the effects of foundation rotation. Foundation rotation also produces a triangular lateral force distribution. Since most buildings are relatively squattier, the proposed triangular lateral loading or the linear deflection curve is generally on the safe side.

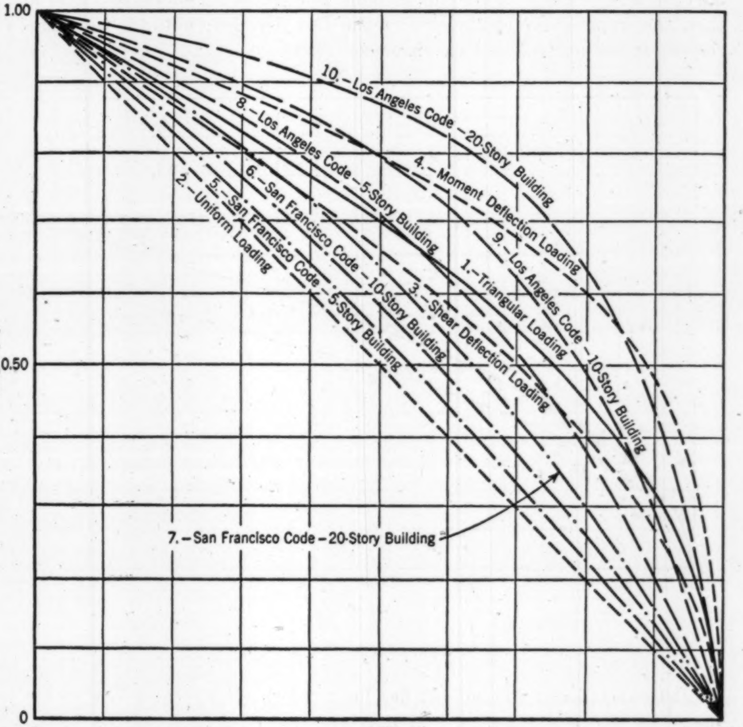


FIG. 7.—SHEAR DISTRIBUTION COMPARED TO EXISTING CODE REQUIREMENTS

A comparison of the shape of the triangular loading shear curve with the shapes of the shear curves for five-story, ten-story, and twenty-story buildings in accordance with the current codes in San Francisco and Los Angeles is shown in Fig. 7. It appears that the San Francisco code, irrespective of the actual quantitative lateral force coefficients, tends to reduce the shears too rapidly up through the structure. All three curves for five-story, ten-story, and twenty-story buildings fall below the shear curve for shear deflection only, which curve should be the lower limit of base shear distribution. On the other hand, the Los Angeles code seems to carry the shears too high, particularly for the taller buildings. Perhaps concern for the effects of higher modes of vibration prompted this standard. However, if the shape of the fundamental mode shear curve is accepted as an adequate envelope to contain the effects of higher modes, then the upper-story coefficients in the Los Angeles code are too high with respect to the over-all or base shear coefficient.

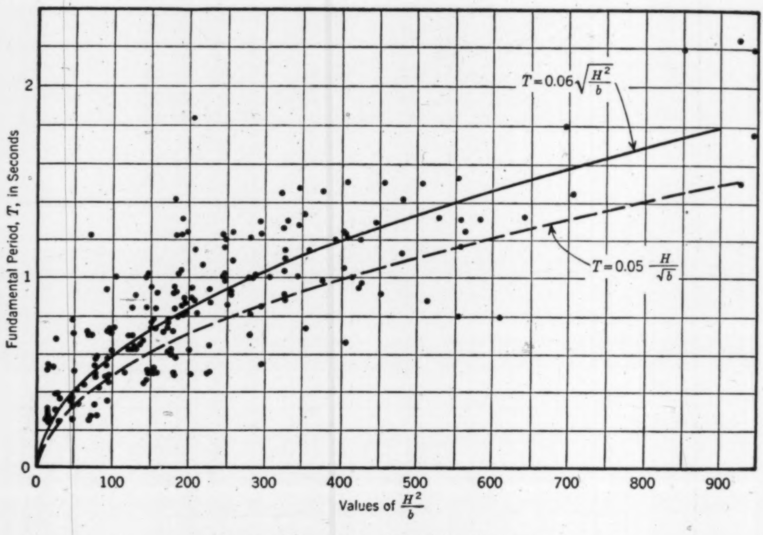


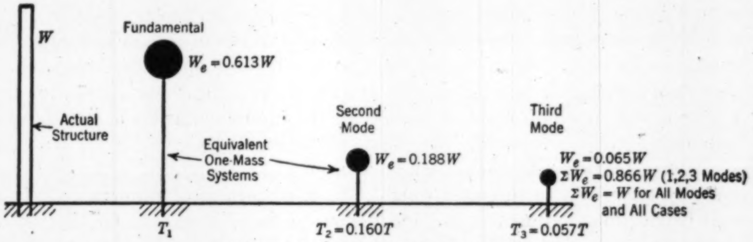
FIG. 8.—FUNDAMENTAL PERIOD OF BUILDINGS

Determination of Period.—It has been shown (Eqs. 4 and 5) that the base shear can be calculated from $V = C\,W$ and that $C = K/T$. In order to determine C, some reasonable means must be devised for predetermining the period T of a structure. It will have to be known (approximately, at least) from the basic dimensions and possibly some of the other physical characteristics of the structure. Comparison with actual similar structures of known period is one approach to this determination. The USCGS has made many measurements of the period for buildings and other structures. Up to July 1, 1949, approximately 1,600 vibration observations had been made in 430 buildings, 150 observations on 42 elevated water tanks, 250 special observations, as well as more than 600 ground vibration measurements.

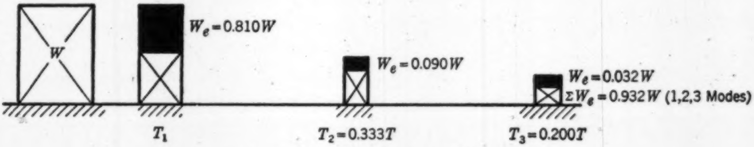
Plotting representative but random building periods observed against H^2/b (the height H squared over the width b in the direction considered), Fig. 8

was obtained. The correlation is not entirely satisfactory, but it is the best of several other simple functions tried. From Fig. 8, an average curve is

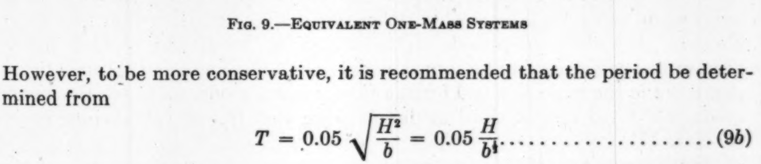
$$T = 0.06\sqrt{\frac{H^2}{b}} = 0.06\,\frac{H}{b^{\frac{1}{2}}}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9a)$$


W
Fundamental
$W_e = 0.613W$
Actual Structure
Equivalent One-Mass Systems
Second Mode
$W_e = 0.188W$
Third Mode
$W_e = 0.065W$
$\Sigma W_e = 0.866W$ (1,2,3 Modes)
$\Sigma W_e = W$ for All Modes and All Cases
T_1
$T_2 = 0.160T$
$T_3 = 0.057T$

(*a*) CASE I MOMENT DEFLECTION ONLY


W
$W_e = 0.810W$
$W_e = 0.090W$
$W_e = 0.032W$
$\Sigma W_e = 0.932W$ (1,2,3 Modes)
T_1
$T_2 = 0.333T$
$T_3 = 0.200T$

(*b*) CASE II SHEAR DEFLECTION ONLY


W
$W_e = 0.712W$
$W_e = 0.139W$
$W_e = 0.049W$
$\Sigma W_e = 0.900W$ (1,2,3 Modes)
T_1
$T_2 = 0.247T$
$T_3 = 0.129T$

(*c*) CASE III SHEAR AND MOMENT DEFLECTION (AVERAGE OF CASES 1 AND 2)

FIG. 9.—EQUIVALENT ONE-MASS SYSTEMS

However, to be more conservative, it is recommended that the period be determined from

$$T = 0.05\sqrt{\frac{H^2}{b}} = 0.05\,\frac{H}{b^{\frac{1}{2}}}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9b)$$

—unless more direct and more accurate determinations of period are available. Of the points plotted in Fig. 8, 80% fall above (on the conservative side) of the recommended curve, 10% are subject to a possible error of less than 25% on the unsafe side, and the remaining 10% are in error between 25% and 40%. From this standpoint, the simple, easily-applied formula (Eq. 9*b*) is reasonable for the present. Further research should be undertaken to confine the period predetermination within closer limits without sacrifice of simplicity.

Effect of Higher Modes.—Equivalent one-mass systems can be established for complex structures vibrating in the fundamental mode, and also for higher modes. In each of these one-mass systems the response (as measured by the base shear transmitted from the ground) corresponds to the base shear transmitted from the ground into the real structure in the particular mode of vibration being considered. In each case, the period of the one-mass system corresponds to the period of the complex structure. If the shape of the fundamental shear curve constitutes a good envelope to contain the shear curves for the higher modes, it is reasonable to design for the fundamental mode and thereby take into account adequately the effects of higher modes.

Milton Ludwig (32) has determined ratios of the weight of equivalent one-mass systems, W_e, to the total weight, W, of structures of uniform weight per unit height, for the fundamental and higher modes of vibration, for the case of deflection due entirely to bending, and for the case of deflection due entirely to shear (see Table 2(*a*)). Diagrammatically, this is shown in Fig. 9. These two cases correspond exactly to the cases considered under the distribution of the base shear as equivalent applied forces. Case III is an average of the equivalent weights W_e of Cases I and II.

TABLE 2.—RELATIVE BASE SHEARS IN FUNDAMENTAL SECOND AND THIRD MODES OF VIBRATION

Mode	(*a*) RATIOS W_e/W			(*b*) RATIOS T/T_f			(*c*) RELATIVE BASE SHEARS	
	Case I Bending	Case II Shear	Case III Average	Case I Bending	Case II Shear	Case III Average	Computation	V
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Fundamental	0.613	0.810	0.712	1.000	1.000	1.000	$V_1 = 0.247 \times 0.712\,W$	$= 0.176\,W$
Second	0.188	0.090	0.139	0.160	0.333	0.247	$V_2 = 1.00 \times 0.139\,W$	$= 0.139\,W$
Third	0.065	0.032	0.049	0.057	0.200	0.129	$V_3 = 1.00 \times 0.049\,W$	$= 0.049\,W$

The relationship between the fundamental and higher mode periods for these same cases has been established as shown in Table 2(*b*). Averaging the ratios for Cases I and II is perhaps a rather arbitrary way to arrive at some in-between value that represents deflection due to both bending and shear. However, the observed periods in the Alexander Building, for which the deflection was determined to be approximately half due to moment and half due to shear, are in the ratio of 1.000 for the fundamental mode, 0.252 for the second mode, and 0.132 for the third mode, checking very closely the average values for Case III.

If the response of the equivalent one-mass systems follows Fig. 5, the relative influence of the higher modes—particularly the second mode— will be greatest in structures of fundamental period 1.03 sec, second mode period 0.250 sec, and third mode period 0.130 sec. This establishes maximum coefficients for the second and third modes and a low coefficient for the fundamental mode. The relative base shears developed in these three modes will be as shown in

Table 2(*c*). These base shears and their distribution are plotted over the height of the structure, using the triangular distribution for the fundamental mode, to obtain Fig. 10. The second mode distribution corresponds to the average of the shear and moment deflection curves in this mode, and the third mode distribution corresponds to the shear deflection curve.

The higher modes, particularly the second mode, can have a very important part in shear and moment and stress distribution in the height of the structure (see Fig. 10); but the significant point is that the shape of the fundamental mode shear curve supplies a reasonable envelope to contain the effect of the higher modes. If a structure has been designed for a practical base shear coefficient and if this base shear is distributed in the triangular fashion, the resulting structure will have reasonably well-balanced strength to resist the combination of shears of various modes to which the structure will actually be subjected in an earthquake. A factor that has not been considered in Fig. 10 is damping, which reduces the response of the fast-moving higher modes more than it does the fundamental. The effect of higher modes is less for buildings and structures for which the shear deflection predominates, or conversely, the fundamental mode is even more important.

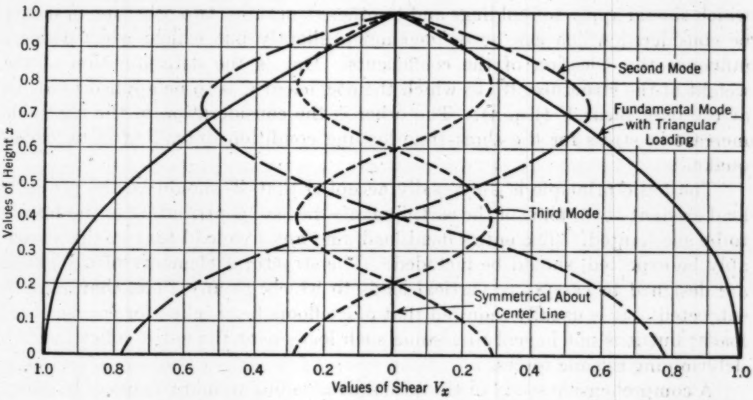


FIG. 10.—RELATIVE SHEARS FOR THE FUNDAMENTAL, THE SECOND, AND THE THIRD MODES

Experiments conducted with a dynamic building model on a shaking table at Stanford University (42) indicated that shears for the second and third modes of vibration were considerably greater than for the fundamental mode for a uniformly rigid model. These experiments have made a notable contribution to the over-all understanding of the dynamic effects of earthquakes. However, the simplifying assumptions which had to be made in the model experiments magnify the influence of the higher modes. A constant duration of 2.2 sec was employed and also a constant initial velocity of the ground or table. The ensuing ground motion was a damped harmonic vibration of prearranged period. With the constant duration, if the fundamental mode represented 1.63 cycles of applied harmonic vibration, the second mode was 5.61 cycles, and the third mode, 11.82 cycles for the model of the Alexander Building, San Francisco. While there is a dominance of higher frequencies in an actual earthquake motion, it is doubtful whether the ratios of cycles would ever reach 1.63: 5.61: 11.82 in any 2.2 sec interval.

Further experiments with simplified models, including equivalent one-mass systems, would be helpful in establishing or corroborating the relative importance of fundamental and higher modes. The models should be subjected to actual recorded earthquake motion and provision should be made for the effects of damping.

ESTABLISHMENT OF COEFFICIENT *C*

Determination of W.—Before establishing the quantitative coefficient *C* which should apply to buildings and to other structures, two other items should be considered which can be defined independently but which, when defined, influence the selection of the coefficients. One is the determination of the weight of the structure, *W*, to which the coefficient *C* is to be applied to arrive at the base shear *V* (Eq. 4). The other is the consideration of the allowable increase in stress for the short-time loading condition prevailing in an earthquake.

The basic principle is universally accepted that *W* should be the probable load existent at the time of the earthquake—the load contributing to the masses to be accelerated. The entire dead load, and any live load that might reasonably be expected, should be included. The structural elements of a building are designed to carry any vertical loads to which, at any time, they may be subjected. It is usually required that office floors be checked for concentrated loads; but it is not logical to assume such loads over the entire office floor for determining seismic forces.

A comprehensive study of the actual live loading in many types of buildings was made by the Department of Commerce Building Code Committee (28) in 1923. The actual average live load in office buildings, hospitals, hotels, and residences was approximately 10 lb per sq ft. Retail stores averaged about 25 lb per sq ft. Public corridors may be loaded up to 50 lb per sq ft or even 75 lb per sq ft, but in such cases the adjoining rooms, in all probability, would not be occupied. Storage areas, warehouses, etc., naturally vary greatly in their loading, dependent on such factors as the materials and method of storing. However, due to the necessary aisles, passageways, and unloaded areas, an average live load in warehouses of 50% of the design live load is adequate for seismic design. In view of these considerations, the load *W* to be used in lateral design for buildings is defined as the total weight of the structure above the base including

> "* * * all dead load plus 50% of the design live load for storage and warehouse floors, 25% of the design live load for all other floors, and no live load for roofs."

For structures other than buildings, *W* is defined as dead load plus normal operating live loads.

Allowable Stress Increase.—Practically every current code allows one-third increase in normal design stresses for combined stresses from lateral forces and vertical loads, provided the resulting section is not less than that required for the dead loads and live loads alone. Also, for members carrying lateral force stresses only, the permissible stress is one third greater than normal.

The one-time proposed California Building Code which was sponsored by the State Chamber of Commerce (29) included provisions for stress increase of 100% for combinations including earthquake loading, except that for columns the maximum increase permitted was 75%.

There is a sound basis for these higher increases in allowable working stresses under earthquake conditions. They provide for a better balanced design. Members which normally carry full stress under vertical loading and members which carry virtually no stress under vertical loading approach failure together as extreme lateral forces are applied. The moderate one-third increase in allowable working stress results in unbalance such that certain members are stronger than others. The weakest members are those which normally carry no stress and the strongest are those which are fully stressed under vertical loading. This is a well-known fact and can be demonstrated mathematically. Further study should be made of the interrelation between the allowable stress increase, the balanced character of the design, and the adopted seismic coefficients.

Pending such study, however, the Joint Committee has retained the presently accepted one-third allowable stress increase and has set the lateral force coefficients on this basis.

Quantitative Coefficient C.—In the present state of knowledge, the choice of the maximum and minimum lateral force coefficients must be based on engineering judgment and experience. Correlated factors, such as the definition of the weight W and the allowable increase in stress for combined lateral and vertical loading, have been discussed and the coefficients chosen are necessarily related to these factors.

To arrive at these coefficients, the Joint Committee has considered in detail the following basic points:

1. It is the province of the building code to specify design and construction requirements which will result in structures safe against major structural damage and loss of life in the event of precedented earthquakes.

2. The extensive review of earthquake history and damage, and the calculated resistance of buildings and structures which have successfully gone through a major shock, have not revealed major structural damage in any California building that had a lateral resistance corresponding to $C = 0.02$ or greater based on current design practices. In fact, many buildings with less lateral resistance than $C = 0.02$ have successfully withstood major earthquakes. The Flood Building with a calculated resistance less than $C = 0.006$ has been mentioned. Other buildings designed for wind forces, again corresponding to low earthquake coefficient forces by current design practices, also withstood major earthquakes successfully.

3. Damage and loss of life that did occur in such buildings was caused by inadequate anchorage of elements such as parapets, cornices, filler walls, parti-

tions, stair and elevator walls, stair treads and stringers; and in appurtenant structures such as roof tanks. The anchorage of such items requires special consideration.

4. An unnecessarily high coefficient for slender buildings requires a type of bracing that is inherently very rigid, and this rigidity increases the probability of damage.

The Joint Committee was unanimous in its selection of $C = 0.06$ as the maximum base coefficient for buildings and $C = 0.02$ as the minimum base coefficient. Hence, K in the formula $C = K/T$ must be 0.015, as shown graphically in Fig. 11.

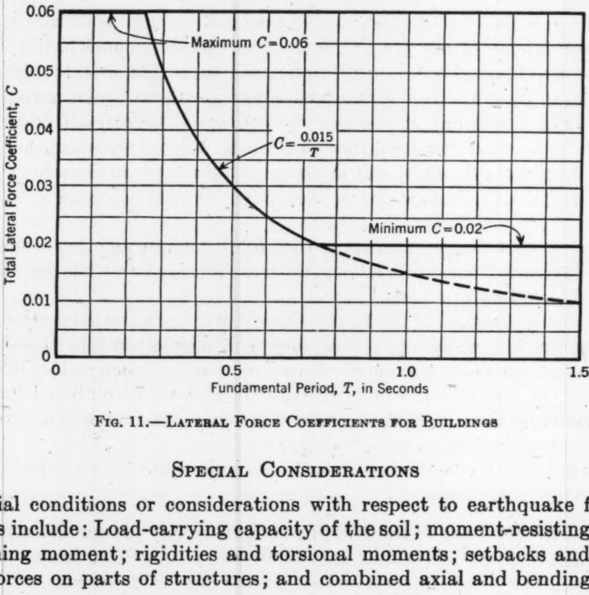


FIG. 11.—LATERAL FORCE COEFFICIENTS FOR BUILDINGS

SPECIAL CONSIDERATIONS

Special conditions or considerations with respect to earthquake forces on buildings include: Load-carrying capacity of the soil; moment-resisting frames; overturning moment; rigidities and torsional moments; setbacks and towers; lateral forces on parts of structures; and combined axial and bending stresses in columns.

Load-Carrying Capacity of the Soil.—It has been generally (although not consistently) observed that damage in earthquakes has been more severe on alluvial or filled ground than on firm ground or rock. Records show greater movement and larger accelerations in the softer ground. However, it has also been observed that certain rigid buildings fared better on the soft ground than they did on the firmer soils. This was the case in Tokyo. Kyoji Suyihiro and Mishio Ishimoto (64) stated that "* * * there is ample evidence to show that rigidly constructed buildings withstood the earthquake better downtown than uptown," although the relative number of buildings destroyed proved conclusively that the earth motion was more severe downtown where the soil was alluvial and more unstable than uptown. They further state:

> "* * * the view that the foundations of rigidly constructed buildings move in just the same manner as the surrounding ground seems to us to be erroneous. If the surrounding ground were of hard rock and the foundations rigidly built upon it, the foundations would follow the ground. However, for buildings standing on soft alluvial ground like downtown Tokyo, the condition should be different. In this case, the rigidity of the buildings being far greater than that of the ground on which they stand, the buildings should behave like pebbles placed on gelatin and there is possibility that they actually rock as a whole upon the soft ground."

It has been found that the rotation of a building on its base, in vibrating, has a very considerable effect in lengthening the natural periods of vibration of the building, especially the fundamental period. This rotation is of greater effect on a rigid building than on a flexible one. The rotation of a building on its foundation has a cushioning effect, which decreases the dynamic shears and moments throughout the building in all modes.

The observed greater damage in alluvial ground is apparently due to other factors as much as or more than to the soft ground itself. Consideration must be given to the types of construction on the two areas—the soft soil and the firm soil—and it may be found that inferior construction occurred in the poorer soil area. Again, settlement damage that has occurred in alluvial soils in earthquakes cannot be corrected by increasing the lateral force factors.

Several codes already recognize that current knowledge is insufficient to justify different lateral force factors for different soil conditions. The Joint Committee took this same view and established only one set of coefficients.

Moment-Resisting Frames.—The history of earthquake damage indicates that buildings with moment-resisting frames, designed usually for wind without earthquake considerations, have had a very good record. A frame encased in more rigid walls can resist only a very small part of the lateral forces to which a building is subjected. If cracks develop in the more rigid walls, the frame comes into action and provides the structural support for the building "clothing." For earthquakes of long duration, this is especially important.

Recognizing this experience, provisions have been made to allow for the added safety feature of moment-resisting frames, provided that such frames have a resistance not less than 25% of the total lateral force requirement. Any resistance built into the frame may be subtracted from the total lateral force requirement to arrive at the resistance to be carried by other elements; but the building as a whole (that is, in accordance with rigidities) must be required to resist a lateral load not less than that given by $C = 0.02$.

For buildings higher than 135 ft, a moment-resisting frame is made obligatory. This frame must be designed for a lateral base shear corresponding to $C = 0.01$, distributed in accordance with Eq. 8, or the prescribed wind force, whichever is the greater.

Overturning Moment.—In recent years the provisions to guard against the overturning of a structure as a whole, or the overturning of lateral force carrying elements of the structure, have varied all the way from no provision to full provision for the overturning effects of the design lateral forces about a fixed base. Without doubt, earthquakes induce moments in buildings; that is, the type of moment by which the floors rotate. On the other hand, there is general agreement that the requirement for full overturning moment effects on a tall, slender structure is more severe than can be justified by any research to date. This type of structure will tend to vibrate in combinations of higher modes with the fundamental. The full design shears of the fundamental mode (which, as explained, is taken to be the envelope of shear requirements) do not infer that the corresponding overturning moments are realized. The vibrations in the higher modes will cause reversals, adding to and subtracting from each other and the fundamental vibration. Down a certain distance from the top, it is conceivable that the full overturning effect might be realized; but not all the way down—not for the tall slender structures, at least.

The Joint Committee recommends that these overturning moments be calculated for the design lateral forces for the top ten stories of buildings, or for the top 120 ft of other structures, for the full cantilever effect and that moments be considered constant from that point to the foundations. The full overturning effect of the stipulated wind forces must be provided for. Furthermore, the over-all structure and its foundation must have a dead-load resisting moment not less than one and one half times the overturning moment due to these wind forces.

Rigidities and Torsional Moments.—The provision that the forces be distributed in accordance with rigidities requires the designer to distribute the specified lateral loads properly. Rigidities of both horizontal diaphragms and vertical elements carrying lateral load must be considered. Recognition of horizontal torsional moment follows from rigidities. If the rigidities of the elements on any horizontal plane are such as to produce a variation between the center of gravity of the loads and the center of rigidity of the resisting elements, then provision must be made for the twisting or torsional effect induced by the lateral loads on the structure.

Setbacks and Towers on Buildings.—Setbacks and towers on buildings affect the period T. After consideration of the effect of setbacks on the period, both from the standpoint of shear and moment deflections, the Joint Committee concluded that the best solution is to require the calculation of an average height H, found by projecting an elevation of the building on a plane parallel to the direction considered and dividing the area of this projected elevation by the over-all width b. Thereafter, the building is treated as a building without setback. For tower area greater than 25% of the building area below, this method gives reasonable results. For smaller areas, the shears, expressed as a percentage of the tower weight, increase very abruptly. Consequently, an optional method is introduced permitting the designer to consider small towers to be buildings on top of buildings, each designed independently for its own height, width, and weight, but with the shear from the tower applied as an additional lateral force on the top of the building below.

Lateral Forces on Parts of Structures.—It has been mentioned that considerable damage in earthquakes and the injury and loss of life have been attributed to inadequate ties and lack of attention to details. It has been common practice in seismic design and seismic codes to stipulate larger lateral forces on various parts of a building than on the building as a whole. The Joint Committee is in agreement with the coefficients as they are required by current earthquake codes.

Combined Axial and Bending Stress in Columns.—The point in a member, subject to both axial and bending stresses, at which the combined stress should be checked by the formula—

$$\frac{f_a}{f_{aa}} + \frac{f_b}{f_{ab}} \leqq 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

—needs clarification. Adjacent to the support, the full allowable bending stress should be allowed. Based on its judgment, the Joint Committee has indicated that Eq. 10 needs to be checked at and between the quarter points of the unsupported length of the column and that the allowable bending stress f_{ab} shall be permitted between the quarter point and the support.

STRUCTURES OTHER THAN BUILDINGS

For structures other than buildings, the Joint Committee recommends the same general approach as that outlined for buildings. However, due to the generally simpler structures involved, with less damping action and little if any reserve strength such as is usually available in buildings, it is recommended that higher coefficients be used. The maximum C proposed is 0.10, the minimum C is 0.03, and K is 0.025. No formula is given for determining the period T, which must be calculated by recognized methods.

Because of experience with braced tank towers, their lack of reserve strength, the rigidity of bracing, and the susceptibility of the bracing to impact stresses, it is stipulated that these towers be designed for a C-factor of not less than 0.10. Considering the importance of keeping a water supply intact in event of an earthquake, more conservative design appears to be in order.

ILLUSTRATIVE EXAMPLES

The application of the recommended code to determine the design earthquake forces in a rectangular five-story building (Fig. 12(*a*)) and in an L-shaped, twelve-story building with a setback (Fig. 12(*b*)) is shown in Table 3. The following are the steps taken in the direction of each principal axis:

a. Determine the period of the structure, T;

b. Using this period, calculate the base shear coefficient, C;

c. Apply this coefficient to the total weight of the building to determine the base shear, V;

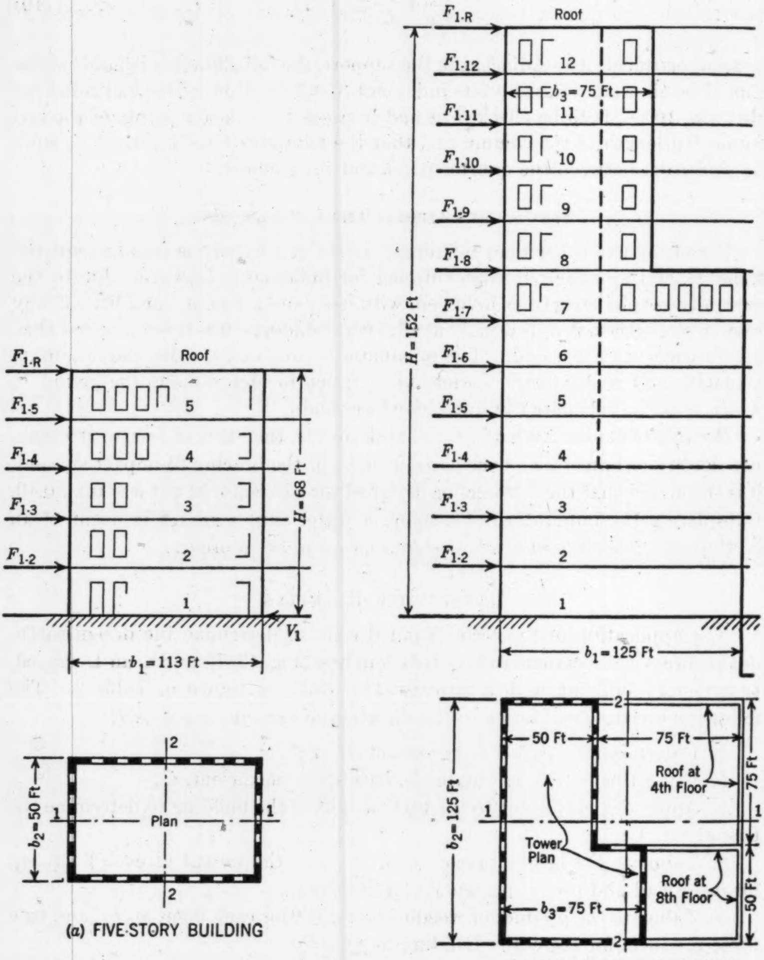
d. Tabulate the height to each floor, h_x, and the weight at each floor, w_x, for dead load and proper percentage of live load;

e. Tabulate the product of weight and height for each floor, $w_x h_x$, and sum these products for the entire building to $\sum(w\,h)$;

f. Determine the ratio $\dfrac{V}{\sum(w\,h)}$ by which each product $w_x h_x$ is to be multiplied to determine F_x, the portion of the base shear V to be applied at that floor; and

g. Tabulate the F_x for all floors, and sum these to check with the base shear V.

In Table 3, Cols. 7 and 9, a further step is shown in recording the ratio of the forces F_x to the weight w for each floor. This is not normally required but is shown here to illustrate that the equivalent coefficients in the top stories, as defined in current codes, are approximately double the base shear coefficients. Thus in the five-story building, the equivalent top-story coefficients in the two directions are 7.95%, and 5.62% compared to the over-all or base shear coeffi-


$F_{1\text{-}R}$
Roof
$F_{1\text{-}5}$
$F_{1\text{-}4}$
$F_{1\text{-}3}$
$F_{1\text{-}2}$
5
4
3
2
1
$H = 68$ Ft
V_1
$b_1 = 113$ Ft
2
1
Plan
$b_2 = 50$ Ft
$F_{1\text{-}R}$
$F_{1\text{-}12}$
$F_{1\text{-}11}$
$F_{1\text{-}10}$
$F_{1\text{-}9}$
$F_{1\text{-}8}$
$F_{1\text{-}7}$
$F_{1\text{-}6}$
$F_{1\text{-}5}$
$F_{1\text{-}4}$
$F_{1\text{-}3}$
$F_{1\text{-}2}$
12
$b_3 = 75$ Ft
11
10
9
8
7
6
5
4
3
2
1
$H = 152$ Ft
$b_1 = 125$ Ft
50 Ft
75 Ft
Roof at 4th Floor
75 Ft
Tower Plan
Roof at 8th Floor
50 Ft
$b_2 = 125$ Ft
$b_3 = 75$ Ft

(a) FIVE-STORY BUILDING

(b) TWELVE-STORY BUILDING

FIG. 12.—BUILDINGS SELECTED FOR ILLUSTRATIVE EXAMPLES (SEE TABLE 3)

cients of 4.68% and 3.31%, respectively. Similarly, in the twelve-story building, the equivalent top-story coefficients are 5.45% and 4.69%, compared with base shear coefficients of 2.62% and 2.21%, respectively. The effect of the setback is to require somewhat higher lateral forces in the upper parts of the structure, as measured by the equivalent story coefficients. The top-story coefficients in Fig. 12(b) (Table 3(b)) are slightly more than double the base shear coefficient, whereas the corresponding ratio for a twelve-story uniform plan building would be slightly less than double.

TABLE 3.—COMPUTATIONS FOR ILLUSTRATIVE EXAMPLES FOR 50 BUILDINGS SHOWN IN FIGURE 12.

Floor (1)	Δh (ft) (2)	h (ft) (3)	w (kips) (4)	$w\,h$[a] (5)	F_1 (kips) (6)	$\frac{F_1}{w}$[b] (7)	F_2 (kips) (8)	$\frac{F_2}{w}$[b] (9)
(a) Five-Story Building								
R	→ 12	68	500	34	39.7	7.95	28.1	5.62
5	→ 12	56	1,000	56	65.4	6.54	46.4	4.64
4	→ 12	44	1,000	44	51.3	5.13	36.4	3.64
3	→ 12	32	1,000	32	37.3	3.73	26.5	2.65
2	→ 20	20	1,000	20	23.3	2.33	16.6	1.66
1		0	150	0	0	0	0	0
Totals			4,650 $= W$	186 $= \Sigma(wh)$	217.0 $= V_1$	...	154.0 $= V_2$	...
(b) Twelve-Story Building								
R	→ 12	152	750	114	40.8	5.45	35.2	4.69
12	→ 12	140	1,500	210	75.2	5.01	64.9	4.32
11	→ 12	128	1,500	192	68.8	4.58	59.3	3.95
10	→ 12	116	1,500	174	62.3	4.15	53.7	3.58
9	→ 12	104	1,500	156	55.8	3.72	48.2	3.21
8	→ 12	92	1,760	161	57.6	3.29	49.7	2.84
7	→ 12	80	2,000	160	57.4	2.87	49.4	2.47
6	→ 12	68	2,000	136	48.6	2.43	42.0	2.10
5	→ 12	56	2,000	112	40.1	2.01	34.6	1.73
4	→ 12	44	2,562	113	40.5	1.58	34.9	1.36
3	→ 12	32	3,125	100	35.7	1.14	30.9	0.99
2	→ 20	20	3,125	62	22.2	0.71	19.2	0.61
1		0	300	0	0	0	0	0
Totals			23,612 $= W$	1,690 $= \Sigma(w\,h)$	605.0 $= V_1$	...	522.0 $= V_2$	...

Computations (10)

Five-Story Building:

$$T_1 = \frac{0.05\,H}{\sqrt{b_1}} = \frac{0.05 \times 68}{\sqrt{113}} = 0.320$$

$$T_2 = \frac{0.05 \times 68}{\sqrt{50}} = 0.453$$

$$C_1 = \frac{0.015}{T_1} = \frac{0.015}{0.320} = 0.0468$$

$$C_2 = \frac{0.015}{0.453} = 0.0331$$

$$V_1 = C_1\,W = 0.0468 \times 4{,}650 = 217$$

$$V_2 = 0.0331 \times 4{,}650 = 154$$

$$F_{1-x} = \frac{V_1\,w_x\,h_x}{\Sigma(w\,h)} = \frac{217}{186}\,w_x\,h_x = 1.17\,w_x\,h_x$$

$$F_{2-x} = \frac{154}{186}\,w_x\,h_x = 0.828\,w_x\,h_x$$

Twelve-Story Building:

$$H_1 = \frac{92 \times 125 + 60 \times 75}{125} = 128$$

$$H_2 = H = 152$$

$$T_1 = \frac{0.05\,H_1}{\sqrt{b_1}} = \frac{0.05 \times 128}{\sqrt{125}} = 0.571$$

$$T_2 = \frac{0.05 \times 152}{125} = 0.678$$

$$C_1 = \frac{0.015}{T_1} = \frac{0.015}{0.571} = 0.0262$$

$$C_2 = \frac{0.015}{0.678} = 0.0221$$

$$V_1 = C_1\,W = 0.0262 \times 23{,}612 = 605$$

$$V_2 = 0.0221 \times 23{,}612 = 522$$

$$F_{1-x} = \frac{V_1\,w_x\,h_x}{\Sigma\,(w\,h)} = \frac{605}{1{,}690}\,w_x\,h_x = 0.358\,w_x\,h_x$$

$$F_{2-x} = \frac{522}{1{,}690}\,w_x\,h_x = 0.309\,w_x\,h_x$$

[a] Thousands of foot-kips or millions of foot-pounds. [b] Percentages.

WIND

Wind forces and their effects have been reported ably by other investigating committees. This Joint Committee reviewed much of the work which has been done in this field in recent years, and concluded that no major changes from the specification of current codes were necessary.

The basic measure of wind pressure on a structure can be approximated for engineering design purposes by the formulas:

$$P = c\,q \qquad (11)$$

and

$$q = 0.00256\,v^2 \qquad (12)$$

in which v is the design velocity of the wind, in miles per hour; q is the velocity pressure, in pounds per square foot; c is the shape factor of the part or whole of the structure; and P is the design pressure, in pounds per square foot.

The 5-min average design velocity, plus a gust factor, is obtained from records of the United States Weather Bureau. The American Standards Association (ASA) recommends gust factors varying with height above the ground (36). At 30 ft, the factor is 1.5; at 500 ft, 1.3; at 1,000 ft, 1.23; at 1,500 ft and higher, 1.2. After extensive investigations at the University of Michigan, in Ann Arbor, R. H. Sherlock, M. ASCE, recommends (25) a gust factor of 1.385 for low buildings compared with 1.5 by the ASA (36). The 1.385 factor corresponds to a 3-sec gust, the minimum time necessary to develop the maximum pressure on a building as a whole.

For San Francisco, 78 years of observation give a maximum 5-min average velocity of 50 miles per hr, 112 ft above the ground. Using the relationship that the wind velocity varies as the one-seventh power of the height, this gives a velocity pressure for this area by Eq. 12. Using the more conservative gust factor of 1.5, the wind pressure is $q = 0.00256 \left[1.5 \times 50 \times \left(\frac{30}{112} \right)^{1/7} \right]^2 = 9.88$ lb per sq ft at 30 ft above ground. This accords with the ASA velocity pressure recommendation for Northern California, which is 9 to 10 lb per sq ft.

The shape factor varies considerably. For structural shapes, such as small angles, channels, squares, H-beams, H-columns, and flat plates, it varies from 1.62 to 2.19 when the wind is free to go all around the member. For buildings and structures of "average" size on the ground, it is generally assumed that the shape factor c is 1.3, consisting of 0.8 pressure on the windward side and 0.5 suction on the leeward side.

For San Francisco conditions, this value would give a design pressure P, the total windward and leeward effect, of $P = 1.3 \times 9.88 = 12.85$ lb per sq ft at an elevation of 30 ft. Based on the Sherlock gust factor of 1.385, this design pressure would be less; that is, 10.95 lb per sq ft instead of 12.85 lb per sq ft. Also for heights less than 30 ft above ground, the pressure would be further reduced, so that a building 30 ft high with a design pressure at the top of 12.85 lb per sq ft would have an average lateral wind force of less than 10 lb per sq ft.

Thus, there is considerable evidence to support a reduction of the design pressure for low buildings and structures. From the standpoint of safety alone this could be done. However, the Joint Committee felt that to provide reasonable rigidity against deflection, current wind pressures specified in this area should be continued. These are 15 lb per sq ft on structures up to 60 ft high and 20 lb per sq ft above this point. For purposes of design of the structure as a whole, these pressures can be applied entirely on the windward side because lateral forces on parts of the structure include an inward or outward force on walls not less than 20% of the wall weight nor less than 15 lb per sq ft.

For small roof structures, subject to higher wind velocities and higher gust factors, 30 lb per sq ft wind pressure is specified. Also on open-frame structures for which the shape factor is higher and for which the leeward side framing is exposed to a reduced velocity, wind pressure of 30 lb per sq ft on one and one-half times the area of the members in the windward exposure is recommended.

It should be emphasized that, although the lateral forces specified are generally satisfactory and conservative for most of Northern California, there are locations of extreme exposure, fetch, and height which require higher factors. Point Reyes Lighthouse, for example, reported a 5-min average velocity estimated at 84 miles per hr in 1914, with a continuous record from 1911 to 1944. This would correspond to a wind design pressure of 36 lb per sq ft, compared to the 12.85 lb per sq ft for the San Francisco record. This record at Point Reyes, however, is not typical of the other record stations. It would not be in order to penalize a large area by excessive design wind pressures because of a localized condition. Designers should be cautioned, however, to consider the local conditions, wind record, exposure, etc., in determining the design wind forces.

Recommendations for Future Study

In the development of the recommended code, several problems were encountered which require further research either to substantiate the current recommendations, or possibly to modify or refine them. Although they are not all-inclusive, suggestions for future research include:

1. More extensive, accurate records of ground motion, encompassing on a uniform basis not only continental United States but all of the more active seismic regions of the world;
2. Additional response curves for one-mass systems, including the effects of damping, preferably to be made a part of the analysis of the ground motion records and reported therewith;
3. More complete examination of the periods of vibration of buildings and other structures, and the factors affecting these periods, and establishing a more accurate yet practicable criterion for determining these periods for lateral force design purposes;
4. Study of the effects of soil conditions on ground motion and the response of structures, including the effects of rotation in softer soils; and the influence of these factors on the lateral force coefficient C;
5. Additional model and shaking table studies, particularly relating the response of one-mass systems and equivalent one-mass systems to the responses of multi-mass prototypes;

6. Study of the rigidity criterion for the distribution of design lateral forces for various types of structures and conditions encountered, preferably to be reported as a design practice;

7. Further study of allowable increases in working stresses for short-time lateral loading combined with normal vertical loading to include the basic structural materials and, if increases higher than the current practice of 33% are in order, to consider the effects of these upon the lateral force coefficient C;

8. Establishment of a design criterion for the overturning effects in earthquakes, based upon dynamic considerations;

9. Cataloging and indexing of buildings and other structures which have been designed for seismic forces, with basic structural data, so that in the event of a major earthquake, an early report of damage or lack of damage and its relationship to the design can be compiled;

10. Further study of wind effects on low buildings for California conditions, and consideration of modification of lateral wind pressures for these structures, if this is found warranted; and

11. Study of earthquake effects on bridges that are not covered in this paper because they do not normally come under the jurisdiction of the building code.

ACKNOWLEDGMENTS

The Joint Committee on Lateral Forces was appointed April 2, 1948, by the Boards of Directors of the San Francisco Section, ASCE, and the Structural Engineers Association of Northern California. This paper, based on the report of the Joint Committee, is presented under the joint authorship of the members of this committee.

APPENDIX I.—LATERAL FORCE CODE

The following code recommendations are numbered for adaptation to a larger code. For example, if "Lateral Forces" were included as Sect. 8 of the more comprehensive code, Sect. 01 would then become Sect. 801, etc.

SECT. 01.—GENERAL REQUIREMENTS

Every building and other structure, in all its parts and details, shall be designed and constructed to resist and withstand lateral forces herein described from any direction due to wind forces or earthquake forces, whichever are greater. The forces considered shall be not less than those described in Sections 02, 03, and 04. In general, every building or structure shall be so designed that all its elements calculated to resist lateral forces are tied and bonded together.

SECT. 02.—WIND PRESSURE

(*a*) For purposes of design the wind pressure shall be taken on gross area of the vertical projection of buildings and structures at not less than 15 lb per sq ft for those areas of the building or structure less than 60 ft above ground and at not less than 20 lb per sq ft for areas more than 60 ft above ground. Roofs and their anchorages shall be designed for an uplift force of 10 lb per sq ft.

(*b*) The wind pressure on roof tanks, roof signs, or other exposed roof structures and their supports, shall be taken as not less than 30 lb per sq ft of the gross area of the plane surface, acting in any direction.

(*c*) In calculating the wind pressure on circular tanks, towers, or stacks, these pressures shall be assumed to act on six-tenths of the projected area.

(*d*) On open-framed structures the area used in computing wind pressure shall be one and one-half times the net area of the framing members in the side exposed to the wind using a base pressure of 30 lb per sq ft.

SECT. 03.—MINIMUM EARTHQUAKE FORCES FOR BUILDINGS

(*a*) *Total Lateral Force.*—Every building shall be designed and constructed to withstand minimum total lateral forces, determined independently in the directions of the principal axis of the building, as given by the formula,

$$V = C/W$$

in which V is the total lateral force or shear at the base; C is the numerical coefficient as given hereafter; and W is the total weight of the building above the base, including dead load plus a percentage of the live load hereinafter specified.

(*b*) *Coefficient C.*—

1. In the building as a whole the coefficient C shall be

$$C = \frac{0.015}{T}$$

in which T is the fundamental period of vibration of the building in seconds in the direction considered. The required value of C shall not be less than 0.02 nor more than 0.06.

2. Qualified persons may submit technical data to substantiate the period T obtaining for a contemplated building. In the absence of such data, it may be assumed that

$$T = 0.05 \frac{H}{\sqrt{b}}$$

in which H is the height of the main portion of the building, in feet, measured above the base which shall be the level at which the structure is positively connected to the ground; and b is the width, in feet, of the main portion of the building in the direction considered.

(*c*) *The Weight W.*—The weight W of buildings shall include all dead load plus 50% of the design live load for storage and warehouse floors, 25% of the design live load for all other floors, and no live load for roofs.

(*d*) *Applied Lateral Forces.*—

1. The total lateral force V shall be distributed over the height of the building in accordance with the following formula:

$$F_x = V \frac{w_x h_x}{\sum(w\,h)}$$

in which F_x is the lateral force applied to any level x; w_x is the weight at level x; h_x is the height of level x above the base; and $\sum(w\,h)$ is the summation of the products $w\,h$ for the building.

2. At each level x, the force F_x shall be applied over the area of the building, in accordance with the mass distribution on that level.

(*e*) *Lateral Forces on Parts of the Building.*—

1. Parts of buildings and their anchorage shall be designed for lateral forces determined from the formula,

$$F_p = C_p\,W_p$$

in which F_p is the lateral force on the part and in the direction under consideration; W_p is the weight of the part; and C_p is a coefficient selected from Table 4.

TABLE 4.—COEFFICIENTS C_p FOR USE IN SECT. 03(*e*)1

Part	C_p	Comment	Direction
Exterior walls and other nonresisting elements not otherwise classified	0.20	With a minimum of 15 lb per sq ft	Normal to surface of wall
Interior walls and partitions	0.10	With a minimum of 5 lb per sq ft	....
Parapet walls, exterior and interior ornamentations	0.50	....	Any direction horizontally
Towers and tanks, including contents, chimneys, smokestacks, and penthouses when connected to or part of a building	0.20	....	Any direction horizontally

2. The distribution of these forces shall be according to the gravity loads pertaining thereto.

SECT. 04.—MINIMUM EARTHQUAKE FORCES FOR OTHER STRUCTURES

(*a*) All structures other than buildings, except tower-supported structures, shall be designed and constructed to withstand minimum total lateral forces determined as for buildings (Sect. 03) except that C shall be determined by

$$C = \frac{0.025}{T}$$

in which the required value of C shall not be less than 0.03 nor more than 0.10. Qualified persons may submit technical data to substantiate the period T obtaining for a contemplated structure; otherwise T shall be calculated by recognized methods assuming fixed base conditions.

(*b*) In all tower-supported structures, such as elevated tanks, C shall not be less than 0.10.

(*c*) The total lateral force shall be distributed over the height of the structure in the same manner as for buildings, Sect. 03(*d*).

(*d*) The weight W shall include all dead load plus normal operating live load.

(*e*) The provisions of Sect. 03(*e*) shall apply also to parts of structures other than buildings.

SECT. 05.—DISTRIBUTION OF HORIZONTAL SHEAR

The total horizontal shear at any horizontal plane shall be distributed to the various resisting elements in proportion to rigidities.

SECT. 06.—HORIZONTAL TORSIONAL MOMENTS

Proper provision shall be made for horizontal torsional moments unless the center of rigidity of the resisting structural units is coincident with the center of gravity of the vertical loads in the building or other structure.

SECT. 07.—PROVISIONS AGAINST OVERTURNING

(*a*) The dead load moment of stability of every building or other structure shall not be less than one and one-half times the overturning moment caused by wind pressure.

(*b*) Provision for overturning moment shall be made for the specified earthquake forces in the top ten stories of buildings or the top 120 ft of other structures, and the moments shall be assumed to remain constant from these levels into the foundations.

SECT. 08.—BUILDINGS HAVING SETBACKS

(*a*) Buildings with setbacks shall be designed by the methods previously specified in Sect. 03 except that, for purposes of determining the period T, the coefficient C, and the base shear V, the height H shall be the average derived by dividing the area of the elevation of the building, as projected on a plane parallel to the direction considered, by the base width b.

(*b*) Towers having a plan area less than 25% of the plan area of the lower part of the building may be designed as separate structures for their own height, width, and weight. In this case the resulting total shear from the tower is to be applied at the top of the lower part of the building which shall be otherwise considered separately for its own height, width, and weight.

SECT. 09.—DESIGN REQUIREMENTS

(*a*) *Structural Frames.*—The lateral force-resisting capacity of any moment-resisting structural frame may be deducted from the total required lateral force to determine the required resistance of other lateral force-resisting elements, provided that such a frame has a resistance of at least 25% of the total lateral forces hereinbefore required, and provided that the building or structure as a whole shall be designed to resist a lateral force resulting from a coefficient C equal to not less than 0.02.

(*b*) *High Buildings.*—In buildings higher than 135 ft a moment-resisting frame shall be provided, designed to resist a lateral force resulting from a coefficient C equal to 0.01, or the wind force specified in Sect. 02, whichever is the greater.

(*c*) *Increase of Unit Stresses for Horizontal Forces.*—

1. For combined stresses due to lateral loads caused by earthquakes or wind, and for the vertical design dead and live loads as defined elsewhere, the permissible unit stresses may be increased one third, provided the section thus required is not less than that required for dead and live loads alone. For members carrying

stresses due only to lateral forces caused by earthquakes or wind the permissible unit stresses may be increased one third.

2. In foundations due consideration shall be given to the capacity of the soil under combined vertical and short time lateral loading, but any increase shall not exceed 100% of the normal bearing value.

(*d*) *Combined Axial and Bending Stresses in Columns.*—

1. Maximum allowable extreme fiber stress in columns at the intersection of a column with floor beams or girders, for combined axial and bending stresses, shall be the allowable bending stress for the material used. Within the center one half of the unsupported length of the column, the combined axial and bending stresses shall be such that

$$\frac{f_a}{f_{aa}} + \frac{f_b}{f_{ab}} \leqq 1$$

in which f_a is the axial stress; f_{aa} is the allowable axial stress; f_b is the bending stress; and f_{ab} is the allowable bending stress.

2. When stresses are due to a combination of vertical and lateral loads, the allowable unit stresses may be increased as provided in Sect. 09(*c*).

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DISCUSSION

Seismic Research Group, Architectural Institute Of Japan.—Although Japan has experienced frequent violent earthquakes in the past, no seismograms useful to engineers have been obtained, except at the Earthquake Research Institute of Tokyo University, Tokyo, Japan, during the 1923 Kanto Earthquake. Japanese engineers have been unable to obtain the necessary support to establish and maintain strong-motion seismographs at the various meteorological stations throughout Japan.

The proposal of the authors has shown the way for theoretical determination of the design coefficient by fully utilizing strong-motion seismograms. It is a remarkable achievement and deserves the highest praise. The Seismic Research Group wishes to congratulate the authors for their contribution and to express admiration for their development of the dynamic method of determining lateral force.

Although there are no reliable strong-motion seismograms to use for definite comparison, it is hoped that what data have been accumulated, together with Japanese experience, will make this discussion useful.

Effect of Soil Type.—The condition of the soil is considered to be the most influential factor in the consideration of seismic force upon a building. A building founded on hard rock normally suffers little damage unless it is of very fragile construction. For hard rock, the seismic design coefficient can be very small. As the soil becomes softer the possibility of damage increases and differences that result from the use of differing construction materials become visible. The design coefficient may be varied because of varying conditions of the nature of the site soil. K. Muto (see "Acknowledgment") has proposed reduction percentages to be applied to the design coefficient for the various values of soil bearing capacity. Table 5 shows the proposed reduction values which soon will be adopted into the Japanese National Building Code.

TABLE 5.—Percentage Effect of Soil Type on Seismic Force

Soil	Type of Construction			Corresponding soil bearing power (kips per sq ft)
	Wood and light steel	Reinforced concrete and concrete block	Masonry	
Soft	100	100	100	1.0 or less
Intermediate	80	90	100	....
Hard	60	80	100	4.0 or more

According to surveys made by R. Tanahashi and others on damages caused by past violent earthquakes, ordinary wooden buildings with heavy roof framing members and tile are certain to collapse when built upon soft soil.

This is true unless such structures are designed for an ultimate design coefficient not less than 0.3 g to 0.4 g. Within the area of violent earthquake motions, the observed damage rate to wooden structures decreases quickly when the structure has been built on intermediate soils. On hard soils the observed damage rate to wooden buildings is usually very small and of a minor nature. The average natural period of these wooden buildings is around 0.5 sec within the small vibrational period range.

The damage surveys on reinforced concrete buildings, either with or without structural steel composite reinforcing, after the Kanto Earthquake of September, 1923, reveal that the over-all damage rate (including all minor cracking) was least for buildings situated on the soft soil in the downtown (Marunouchi) area of Tokyo. However, recent extensive surveys conducted by T. Kitazawa on all damage reports submitted for this earthquake shows that, when the damage rate is compared on the basis of damage to main structural members, the buildings located upon the downtown soft soil areas suffered considerably more than did those located on the uptown (Hongo) intermediate soil areas of Tokyo. Hard and intermediate soils are much more preferred for this type of construction than is soft soil.

TABLE 6.—PERIOD OF BUILDINGS IN TOKYO, JAPAN

Building	Height, in feet	EAST-WEST DIRECTION				NORTH-SOUTH DIRECTION			
		Width b, in feet	Period, in Seconds			Width b, in feet	Period, in Seconds		
			Actual, T_A		Computed[b]		Actual, T_A		Computed[b]
			Normal	1923[a]			Normal	1923[a]	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Marunouchi . . .	110	280	0.67	1.11[c]	0.33	330	0.71	1.18[c]	0.30
N Y K.	100	160	0.69	0.90	0.40	300	0.77	0.80	0.30
Tokyo Kaikan. .	86	144	0.72	1.30	0.36	144	0.54	1.20	0.36
Nippon Kogyo.	99	155	0.65	. . . .	0.40	131	0.61	. . . .	0.43

[a] Period after the September, 1923, earthquake. [b] Computed values of T_s using Eq. 9*b*. [c] Prior to the earthquake of April, 1922, the periods were 0.87 east-west and 0.94 north-south.

Small earthquakes produce more damage on stone or masonry structures on hard soil than on soft soil. However, violent earthquakes usually inflict severe damage on this type of structure even though it is built on soft soil. There are some notable exceptions to this general observation but generally the softer soils are the safest for this type of construction.

Effect of Period.—In Japan, buildings with a natural period of about 1 sec are considered as having a critical period when built upon soft soils. The damage to buildings in the downtown soft soil area of Tokyo in the 1923 Kanto Earthquake was greatest to those buildings having a natural period of about 1 sec. Table 6 has been compiled to give comparative information on the measured natural periods of several buildings in this soft soil area with the period value calculated using the authors' proposed period formula. It was found, for buildings in an undamaged state, that the period determined by use of the formula is much shorter than the measured values.

Vibration tests on full-size and half-size test structures have confirmed conclusively the fact, already observed previously, that a structure's natural period of vibration changes in an earthquake. The natural period, even in an undamaged structure, elongates when subject to the larger amplitudes and accelerations of an earthquake. The natural period of a structure is, therefore, a transient value.

The Seismic Research Group, working in conjunction with the Building Research Institute of the Construction Ministry and the Earthquake Research Institute, has been conducting statical and vibrational tests on specially built full-size and half-size test structures. The Committee used a specially constructed large vibration generator (Fig. 13) on six types of structures and succeeded in attaining accelerations as large as 0.4 g to 0.7 g at the roof slab levels of these structures.

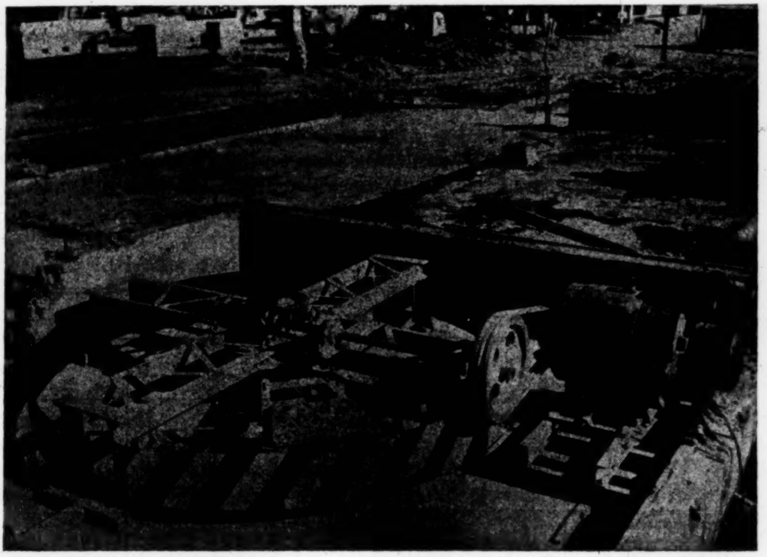

FIG. 13.—LARGE VIBRATION GENERATOR

A portion of the test data is given in Table 7. Initial vibration tests were made with a vibrator that had the following characteristics: 4 vertical eccentric disks were used having a total weight of 46 kg, maximum eccentric distance of 11 cm powered by a 2-hp electric motor and exerting a maximum force of 4.7 tons at 15 rps. The final vibration tests were made with the larger vibrator that had the following characteristics: One 380-kg weight was used on a horizontal rotating arm 90 cm in length powered by a 10-hp electric motor and exerting a maximum force of 10 tons at 5 rps (overloaded 100% during final tests).

The final observed periods using the large vibrator are compared in Table 7 with the initial periods that were observed using the small vibrator. The periods of vibration in two mutually perpendicular directions and corresponding accelerations measured at the top and bottom of the buildings are tabulated. The soil in these tests was Tokyo loam (red clay). This soil is in the intermediate class, having a design bearing capacity of between 3 and 4 kips per sq ft.

The observed periods in the final tests showed the following elongation percentages over the observed periods in the initial tests:

Type of construction	Percentage period elongation
Concrete block	140 to 230
Brick masonry	180
Reinforced concrete	200
Prefabricated reinforced concrete	300

All the buildings in the test program had a fairly large height-to-width ratio, and the tests on each structure were repeated several times. This accounts for predominant soil effects, especially tilting. However, it must be remembered that the soil was good intermediate soil.

TABLE 7.—RESULTS

Type of construction	SIZE OF BUILDING INITIAL						
	Dimensions, feet	Height, feet	Stories	Period, seconds		Acceleration	
				N-S	E-W	Ground line	
						N-S	E-W
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Japanese style concrete block[a]	13 by 20	29	3	0.238	0.255	0.011	0.013
Japanese style concrete block[b]	12 by 18	18	2	0.185	0.159	0.027	0.035
American standard concrete block[c]	11 by 14	19	2	0.174	0.163	0.023	0.027
Medium-burned clay brick[d]	14 by 15	16	2	0.196	0.193	0.036	0.030
Reinforced concrete	11 by 21	15	3	0.122	0.130	....	....
Prefabricated reinforced concrete	13 by 13	20	2	0.205	0.256	....	....

[a] Hollow concrete block 18 in. × 19 in. × 6.7 or 8 in., using vertical and horizontal reinforcement. [b] Channel block 16 in. × 8 in. × 8 in. using vertical reinforcement. [d] Brick 8 in. × 4 in. × 2 in. without reinforcement.

The peak period of the frequency spectrum in the paper appears to be caused by the short period element of the ground motion. This coincides with the period that most Japanese seismologists recognize for intermediate soil areas. When the natural vibration period of the surface soil strata is excited, the value attained depends on the nature of the soil. The prevailing primary period of the soft soil in the downtown Tokyo area is 0.6 sec, and the secondary period is 0.3 sec.

After the 1923 Kanto Earthquake, the Imperial Damage Committee headed by R. Sano debated earnestly and at length as to the advisability of using the prevailing periods as the basis for the design of structures in this area. If the prevailing period plays an important part in contributing to damage at the resonant stage, it was then argued that the buildings in this area were subjected to the most unfavorable conditions. However, numerous important buildings with natural periods very close to the primary period of this area were not damaged in the least. One of these was the Nippon Kogyo Bank Building, designed by T. Naito, that stands as a contradictory manifestation to any such theory. Other factors against the use of prevailing periods were the knowledge that the natural period of the soil may elongate and vary during violent earthquakes and the known difficulty of transmission of free vibration periods from the soft soil to the building. The adoption of period consideration into practical design was, therefore, abandoned. There must be some definite law, however, that governs all cases concerning hard soils.

Acceleration Spectrum.—The standard acceleration spectrum proposed by the authors in the section on "Earthquake Forces" has been commented on by interested engineers in Japan. Mr. Sano states that the period of maximum ground acceleration chosen (0.25 sec) seems to be too small based on past Japanese earthquake experience. Accounting for all soil factors, he believes a period of about 0.50 sec is more reasonable as an over-all period. S. Ban believes that a large acceleration with a larger period is especially critical but is not represented by the proposed spectrum. T. Taniguchi feels that the

OF VIBRATION TESTS

TEST RESULTS		FINAL TEST RESULTS						
(Ratio to g)		Period, seconds		Acceleration (Ratio to g)				Wall damages
Roof				Ground line		Roof		
N-S (9)	E-W (10)	N-S (11)	E-W (12)	N-S (13)	E-W (14)	N-S (15)	E-W (16)	(17)
0.039	0.046	0.325	0.348	0.171	0.110	0.610	0.404	Hair cracks
0.150	0.133	0.476	0.338	0.121	0.161	0.717	0.598	Moderate cracks
0.115	0.113	0.377	....	0.206	0.154	0.608	0.392	Moderate cracks
0.121	0.099	....	0.355	0.164	0.131	0.439	0.595	Large cracks
....	....	....	0.267	0.104	0.245	0.180	0.725	No damage
0.067	0.100	....	0.690	0.100	0.035	0.343	0.552	Failure

type concrete block 36 in. × 36 in. × 3 or 6 in., using vertical and horizontal reinforcement. [e] Hollow concrete Wall 8 in. thick. Reinforced concrete floors.

spectrum represents conditions considered proper for intermediate soils in the near epicentral area of an intermediate earthquake. The peak would be expected to shift to the right for soft soils even comparatively remote from the epicentral area of a large intensity earthquake. Although Biot's spectrum for minor earthquake response had its peak at 0.2 sec, K. Kanai notes that Robison's spectrum for the El Centro Earthquake has its peak at 0.25 sec. If the El Centro seismogram is analyzed by Mr. Ishimoto's method, the predominant period is found to be exactly 0.25 sec. Judging from Japanese experience the predominant period becomes longer the more the earthquake intensity increases and the peak period can be expected to shift to the right to a larger value during high intensity earthquakes. Mr. Naito comments that the El Centro seismogram plays the key role in preparing this spectrum and influencing the authors' proposal. The El Centro record can be typical only if all earthquakes and all soil conditions are similar, which cannot be expected.

Mr. Muto states that the spectrum (Fig. 5) applies to perfectly elastic structures. It is necessary, therefore, before application for design use to make an adequate correction to the curve for damping. This would decrease and round out the peak values of the curve, decrease the rate of reduction, and increase the values for the larger periods. Messrs. Naito and Taniguchi have commented that the proposal states that it is sufficient to design for the fundamental mode only. It is very probable that for many of the tall, slender buildings in the United States, the second mode of vibration is the most effective and the additive effect of the first and second modes should be considered.

Based on the concept that at least the second mode of vibration should be considered, Mr. Muto and H. Umemura made calculations on two-story, five-story, and infinite-storied buildings based on the proposed spectrum. The results show that, in the case of the combined first and second modes, approximately 10% excess of the total base shear may occur. This seems to be well within the allowable excess for practical design.

Lateral Force Coefficients.—The value of the proposed lateral force coefficient *C* has also aroused Japanese comment. Past damage surveys and records indicate that on low, simple buildings force coefficients of not less than 0.2 were experienced in moderately severe earthquakes. Almost universal agreement was reached on the necessity for the reduction of the seismic force for tall buildings.

Mr. Sano stated that, although there may be some dispute for over-all areas, such values as proposed by the authors seem to be acceptable for the western United States. However, he does not believe that the formula is readily applicable to all buildings of intermediate period, say, 0.25 to 0.75 sec. The formula merely defines a change between maximum and minimum values. It would not make an appreciable difference even if the formula defined a linear or hyperbolic change. Messrs. Ban and Taniguchi comment that it is questionable whether the selection of 0.02 as the minimum value of *C* without regard for soil conditions was made on a sound basis. The choice seems to have been based on the fact that many buildings with less calculated resistance than this were undamaged in past major earthquakes. What were the measured resistances of these same buildings in a major earthquake? On any building the only sure way of finding the actual resistance is by measurement during an earthquake. Mr. Kanai believes that the strong-motion seismograms should be basis enough for determining building response and that they represent a more exact approach than the use of engineering judgment only. Y. Otsuki feels that use of the vibrational period of a building to determine its force coefficient without considering the damping nature of the particular building makes for an incomplete cycle. Low, flexible buildings, in which the danger of phase exists, the possibility of reversal with an increase in the rate of deformation should receive careful consideration.

Messrs. Tanahashi and Ishizaki note that the magnitude of seismic force is not always proportional to the mass but rather theoretically proportional to the square root of the mass. Statistical studies of damage data prove this statement. Accordingly, if an appropriate design seismic coefficient is determined for heavy reinforced construction, it may be unsafe for lighter structures having the same natural period of vibration.

Distribution of Base Shear.—The proposed distribution of base shear has evoked comment in Japan, some of which is reported here. General agreement has been shown with the proposed triangular distribution. Some disagreement occurs in regard to the application to tall slender buildings, and Mr. Sano feels that the adoption of triangular distribution of the total shear may be excessively on the safe side. Mr. Naito notes that, when the building has basement floors, the nature of the soil may affect the vibrational characteristics of the building; and the distribution of the base shear may become different from those buildings without basements. However, Mr. Ban agrees with the proposed method of shear distribution over the building. Mr. Otsuki holds that the actual ground motion includes components of all periods. Every component has a definite number of frequencies; hence even an undamped system would never come to resonance with any of these components. This is the meaning of the frequency spectrum. However, every structure is loaded critically only when it is shaken by the component of the resonant period. As far as this concept holds, the proposed triangular distribution works satisfactorily.

Mr. Muto and Mr. Umemura, as a result of study on lateral force distribution with the higher modes, find the triangular distribution to be insufficient especially when the period relation is such that the secondary mode is deeply excited. In certain cases it becomes necessary for the loading at the top story, as found by triangular distribution, to be increased by 20%. In ordinary buildings, foundations cannot be considered to be fixed perfectly. With foundation rotation and rocking taken into account, the period of the fundamental mode becomes somewhat longer, even with the slightest amount of foundation rotation. In the case of ground motion, the secondary mode tends to occupy a more prominent role relative to the case of a fixed foundation. The base shear is distributed best in some cases triangularly, in some cases uniformly, and in some cases in reverse directions for the lower and upper stories.

When a structure having a comparatively long period is shaken by rapidly alternating violent accelerations with a large number of frequencies, the shear distribution would be X-type. The modification of the triangular method could be made by utilizing good average values for ground coefficient, C_g, the base shear found by the authors' proposal for the fundamental mode, C; and the base shear found for the second mode, C_2. Fig. 14 shows the development of the final line of distribution. Although not a simple operation, it does eliminate the necessity for a separate consideration of the overturning moment. It is well known by engineers that if the triangular or uniform distribution is used, the overturning moment always becomes excessive and provision for its consideration is necessary. X-type distribution reserves more shear for the upper stories than does triangular distribution. It uses the same value of C but limits the overturning moment within a reasonable range as compared with the design shear.

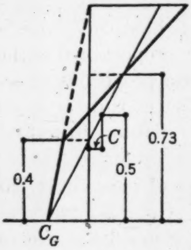

FIG. 14

Determination of Period.—The natural period of a building varies for small acceleration vibrations as well as for large acceleration vibrations. It also varies as the nature of the soil changes. The authors' proposal represents the period simply by a formula that depends entirely on the building's geometry, although the measured data vary widely. Japanese engineers have expressed some disagreement with determinining the C value solely on this basis. For instance, Mr. Naito states that the determination of the period formula based on the data contained in Fig. 8 is sound. However, if these data were assembled from the results of small acceleration vibration tests, the data should be modified for larger accelerations, such as are measured in a severe earthquake. K. Takeyama notes that the proposed formula gives results that coincide with test data obtained from small vibration tests conducted on numerous buildings by the Building Research Institute of the Construction Ministry. However, he questions the feasibility of direct adoption of this period for use with Fig. 5.

Increase of Allowable Unit Stresses.—Japanese building codes divide the allowable unit stresses into two catagories: Long-time or permanent loads (dead load and live load) and short-time or temporary loads. The allowable unit stresses for long-time loadings are increased for the short-time loadings by the ratio of 2 for wood and concrete, and 1.5 for steel. The proposal follows the current American practice of allowing an increase of 33⅓% for combined loading stresses. However, the Japanese engineers are convinced that a well-balanced design will be achieved by further increasing the allowable stresses for combined loading together with the increase of the so-called load factors.

Summary.—The authors' proposal is based quite conclusively upon an overall basis period of ground motion of 0.25 sec. Seismological records in Japan indicate that the basic natural periods of ground motion have about the following values: 0.1 sec for rock, 0.3 sec for intermediate soils, and about 0.6 sec for one particular soft soil area—the Marunouchi area of Tokyo. Perhaps there is some basic difference between earthquake wave motions in the United States and in Japan. Geologically, however, there is no sound basis for such a conclusion.

The Japanese records, indicating a basic natural period of 0.3 sec for intermediate soils, show close agreement with the 0.25 sec for the basic natural period of earthquake motion in the El Centro area. The evident similarity of these two periods suggests El Centro soil is in the class of intermediate soils. If this is true, it shows the obvious need for special consideration in the soft soil areas in the United States.

The basic natural period can be expected to vary between any two soft soil areas. It varies for any one soft soil area as the intensity of the earthquake varies. Adding to this effect the fact that a building's natural period of vibration varies with the intensity of earthquake motion complicates the problem further.

If it were possible in any soft soil area to fix exactly a basic period of ground motion that was generally applicable, the problem would be solved easily by merely modifying the spectrum for the period and magnitude of the coefficient. However, many complications must be considered, such as foundations piled to firm strata, foundation rotation, and energy dissipation.

It can be argued that, since a building on soft soil does not have the same elastic reaction to the ground motion as a building on hard soil, the energy transmitted to the building is reduced, thereby obviating the need for any special consideration. Other factors indicate a reason for not accepting this argument in its entirety. It is now accepted generally in Japan that the possibility of damage to main structural members of reinforced concrete buildings in a violent earthquake is greater for buildings on soft soils than on hard soils. D. W. Butler believes, therefore, that during some period of the violent distortional wave motion, the building, even though located on soft soils, is subjected to large accelerations and displacements. He believes this happens during the first few cycles of distortional wave motion. At this time the soil, before failure, must act elastically. The energy transmission rate to the building is at a maximum, and the energy dissipation rate from the building is at a minimum. The damping qualities of the construction have a minimum effectiveness at this time.

Not all buildings in the Marunouchi area reacted in this way to the 1923 earthquake. Some were undamaged. It is to be noted that the Nippon Kogyo Building was one of the undamaged buildings and that it was designed for an acceleration of 1/15 g. The ground acceleration was several times this value. However, the excellence of the details and construction and the location of seismic bearing walls gives this building a balance and strength that would no doubt indicate a measured capacity of resistance many times this force coefficient.

In addition to the occurrence of a longer basic natural period of ground motion, the initial tendency of the building to react with the ground motion suggests that the period and the magnitude of the *C* values must be increased for safe use for buildings on soft soils.

Japanese engineers are unanimous in expressing their warmest praise and admiration for a real accomplishment—the furtherance of knowledge as to the treatment of a building so that it will better resist dynamic lateral forces. It is believed the authors' proposal is ideal for structures on hard and intermediate soils and is the most economical that can be devised. The problem of estimating the amount of reactive lateral force for important buildings on soft soils appears to be a matter in which factors other than the period of the building can be of fundamental importance.

Acknowledgment.—The Seismic Research Group of the Architectural Institute of Japan comprises the following members: M. Kobayashi and T. Taniguchi of the Tokyo Institute of Technology; K. Muto, R. Sano, K. Kanai, and H. Umemura of Tokyo University; T. Naito of Waseda University in Tokyo; S. Ban of Tokyo University; and K. Takeyama of the Ministry of Construction for Japan. Y. Otsuki serves as translator for the group and D. W. Butler serves as editor.

REUBEN W. BINDER,[12] A. M. ASCE.—This well-organized, clear, and concise paper is an outstanding contribution, covering the many phases of engineering seismology involved in the development of a lateral force code.

The authors have worked into usable form the concept presented by Mr. Biot in a paper (63*a*)[12a] on engineering seismology. Mr. Biot stated

> "Spectrum curves presented herein indicate that the effectiveness of an earthquake is inversely proportional (roughly) to the period, so that increasing the period means an increase in safety. Application of the methods developed in this paper should be helpful as a guide in extending safety rules and interpreting earthquake data."

In an earlier paper (43), Mr. Biot cautioned that

> "* * * when we possess a collection of earthquake spectrums at a given location, it is suggested that a simplified envelope should be used as a spectrum in that region. Further investigation is needed to establish such reliable spectrum envelopes and correct them for the influence of the damping and other causes of stress reduction."

The writer believes that the authors of the paper fully realize that their conclusions must be open to continued study and that they recognize the importance of obtaining considerably more statistical data. Much of the valuable information collected to date has been furnished by the Seismological Division of the United States Coast and Geodetic Survey (USCGS). Too much emphasis cannot be placed on the importance of the Survey's continued activity in this phase of the work. It can be assumed that the authors realized that arguments concerning their proposals will be centered around these factors: (1) The method used to obtain the period of the structure; (2) the character of the expected ground motion; (3) the elastic and damping properties of the structure; and (4) the duration and random character of the earthquake. Depending on the supposition made, almost any kind of a case can be built up with endless discussion resulting. Nevertheless, the materials have been presented in such a manner as to form a valuable contribution. It is apparent that the authors make no special claims for the proposals they have presented in the section on "Dynamic Approach to Earthquake Forces" because they state

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> "The behavior of structures in earthquakes has generally been recognized as a dynamic vibration phenomenon of a transient nature. Although rigorous solutions are possible for particular ground motions applied to particular structures, these solutions are too involved and of too limited significance to be of direct value to the structural engineer. The more rigorous methods, however, should be encouraged to guide the thinking toward less rigorous but more practicable methods."

When, in addition, consideration is given to the authors' recommendations for future study and accumulation of data, it is clear that the rules that they propose must be considered as entirely empirical.

In the introduction to the paper, reference is made to the Los Angeles code, and it is stated that previous codes do not account for the dynamic characteristics that determine how structures respond to earthquake ground motion and that advances in scientific knowledge have not been reflected in current building codes. The Los Angeles City Code requires that the lateral force on buildings shall be determined by the formula,

$$F = \frac{60}{N + 4.5} \qquad \text{(13)}$$

in which F is the lateral force on the part and in the direction under consideration, and N is the number of stories above the story under consideration. The factor F is applied to the summation of all required loads above the story under consideration. Prior to the establishment of this formula considerable analytical and experimental work, sponsored by the Los Angeles County Department of Building and Safety and undertaken by the California Institute of Technology, was directed toward improvement of the then current design rules. The resulting report was turned over to the Los Angeles County Building and Safety Department and pointed the way to replacing the use of a constant acceleration curve with one based on equivalent acceleration, which could take into account some of the many important dynamic considerations. The formula of Eq. 13, while admittedly empirical, incorporated the use of equivalent acceleration coefficients and shear distribution based on equivalent acceleration. In adopting this formula, recognition was given to the seismic coefficient, shear envelope (effect of various modes), and the evidence of observed damage, as well as to the influence of damping and other causes of stress reduction. Since the City of Los Angeles has a height regulation limiting buildings to 150 ft, the formula did not envisage very tall buildings. The values used in this formula recognized among other things: (*a*) The importance of higher factors for the low period buildings; and (*b*) the evidence of damage to roof and penthouse structures. The writer believes that one of the important contributions of the authors is in separating the equivalent acceleration and the shear distribution into two steps.

Fig. 7 shows the shear distribution for uniform weight per unit height for various existing codes. Fig. 15 shows the effect of varying the mass at the various floor levels for one height of building and may be helpful for comparison purposes. It is interesting to note that the late Leon Nishkian, M. ASCE, in an unpublished paper presented to the Seismological Society of America in 1936, suggested that a triangular distribution be used for the lower two thirds of the height and that from that point up the shear be made constant.

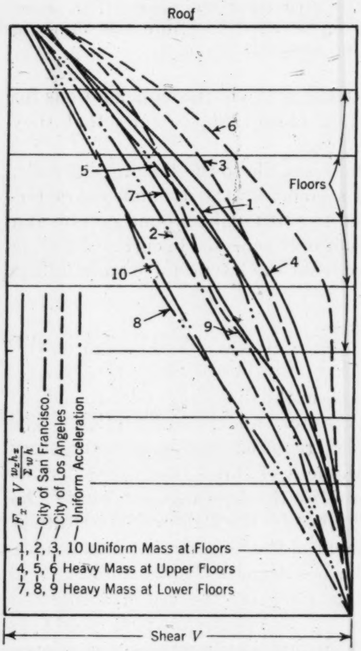

FIG. 15.—SHEAR DISTRIBUTION CURVES

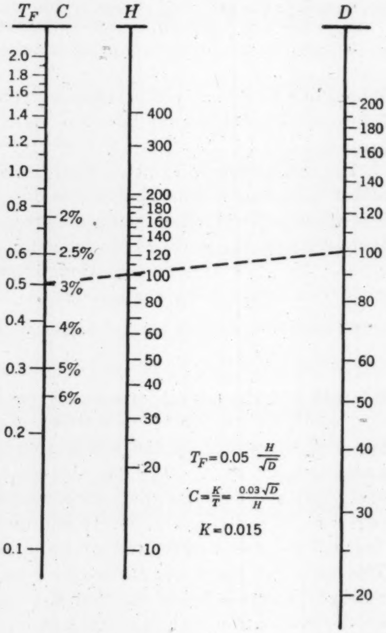

FIG. 16.—NOMOGRAPH FOR DETERMINATION OF PERIOD AND COEFFICIENT C

The free period of a structure has sometimes been considered, on a rule of thumb basis, to be from one tenth to one twentieth per second. Fig. 16 of this discussion was prepared to give rapid answers to the several formulas proposed by the authors. It may be of use for checking quickly previously made assumptions, or results obtained by calculations or experimentation with the empirical values proposed.

It is interesting to note that the authors were unanimous in their selection of $C = 0.06$ as the maximum base coefficient for buildings and of $C = 0.02$ as the minimum base coefficient. It should be noted also that the weight used (with the seismic coefficient) in obtaining the lateral force includes a percentage of the live load. Obviously, the authors in selecting these values took into account many factors. It is assumed that one of the most important considerations was the effect of damping, on which subject much work remains to be done.

The writer considers that, in general, on the basis of the present information, the authors have taken a sound position in not establishing variable values for different types of soils.

In specifying that any resistance built into the frame (no less than 25%) may be substracted from the total lateral force requirements in order to arrive at the resistance to be carried by the other elements, the authors, in the writer's opinion, have shown considerable engineering courage and pointed the way to better structures.

For the important consideration of overturning moment the authors' recommendation is noted with interest. It has been observed by the writer that some designers, although using equivalent acceleration for the design of the structure itself, use uniform acceleration for design in connection with the foundations.

The authors' statement regarding damage to structures (in the section on "Earthquake History") deserves careful attention. It is in essence an observation that places emphasis on the importance of:

1. The provision that inertia forces should be distributed in accordance with the principles of relative rigidities; and
2. Probable high stress concentrations that may occur anywhere within a structure.

From observation of earthquake damage, important general rules may be formulated but unfortunately they cannot be quantitative. Although the observation of earthquake damage furnishes a significant yardstick, it should be remembered that with multiple variables the damage may be interpreted in many different ways.

Design rules are the result of work such as is presented in this paper and of actual observation of earthquake damage. Usually damage has been observed on structures that have not been designed for seismic forces and in which little or no provision has been made for lateral forces. It should be noted that observed damage, in general, was on box-like structures; there has been little or no experience with earthquake damage in connection with what is known as modern architecture. The importance of considering special rules for seismic design in connection with so-called modern architecture deserves considerable attention.

The authors call attention to the need of clarification for combined axial and bending stresses in columns. It should be mentioned that considerable work on this subject has been accomplished by the Column Research Council of the Engineering Foundation and it is hoped that in the near future some of the findings may be crystallized into design rules.

Since this excellent presentation might well serve as a standard reference on the subject of earthquake provisions, the writer suggests that the authors extend the bibliography given in the paper.

R. R. Martel,[13] M. ASCE, G. W. Housner,[14] A. M. ASCE, and J. L. Alford[15].—The subject of the paper is of interest to engineers in many parts of the world. The problem of earthquake-resistant design is of particular concern in the western portions of North and South America, Japan, Australia, Italy, Greece, Turkey, and portions of India and China. The paper recommends design values for lateral forces that differ from those specified in the earthquake-design provisions of existing building codes (1951), and the question arises as to whether the differences are justifiable; that is, are the recommendations presented in the paper the best earthquake-design provisions? The authors, in their paper, present some arguments, data, and analysis on which they base their recommendations, but in some instances these are faulty and invalidate the conclusions. The general problem of earthquake analysis and design is very complex, and the problem is too involved to be treated satisfactorily in a short discussion, so these remarks are limited to pointing out what appear to be shortcomings in the authors' analysis of the problem.

The conclusions of the paper are based in large part upon the so-called "standard spectrum curves," shown in Fig. 5. As the authors state, these curves actually show accelerations that are too large to be reconciled with observed earthquake damage. Therefore, one would expect that some pertinent factor has been overlooked in the preparation of the curves. The writers have recently completed a research study sponsored by the Office of Naval Research, in which some 80 spectrum curves have been calculated from 28 strong-motion seismograph records that were specially prepared by the USCGS. These curves were calculated with various amounts of damping; and typical sets of curves for horizontal components of strong earthquakes are shown in Fig. 17. It will be seen that the spectrum curves for zero damping agree with the standard spectrum curves of Fig. 5 if the sharp peaks are smoothed out. In fact, referring to the original publications it is found that these spectrum curves were calculated for zero damping, and the sharp peaks were smoothed out to obtain the so-called standard spectrum curves. Comparison of Biot's original curves with those of the writers' indicates that the former's results were subject to approximately 1% of critical damping. From Fig. 18 it can be seen that with as little as 2% of critical damping the spectrum curve is much reduced and that with 10% of critical damping the spectrum has a shape very different from that of the zero-damping curve. In a test program sponsored by the Earthquake Engineering Research Institute, the damping in a four-story, monolithic, reinforced concrete building was measured and was found to be 7.5% of critical damping (70). This probably is in the lowest range for damping in ordinary buildings, and those that are not monolithic undoubtedly have greater damping. For example, a concrete frame building with tile filler walls was found to have approximately 14% of critical damping (52*a*). With these amounts of damping, the maximum accelerations exhibited by the spectrum curve are much reduced from those given by the zero-damping curve. It is indicated, therefore, that damping is an important factor in the response of structures to earthquakes and that the undamped standard spectrum curve used by the authors is an upper bound that should be modified when applied to ordinary buildings. The curve for the lateral force coefficient shown in Fig. 11 is based on the undamped spectrum and is not correct for structures with damping.

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The authors state that there are dominating periods in the range 0.1 to 0.3 sec, but it is shown in Fig. 18 that when damping is taken into account, it cannot be said that these periods are in any sense dominant.

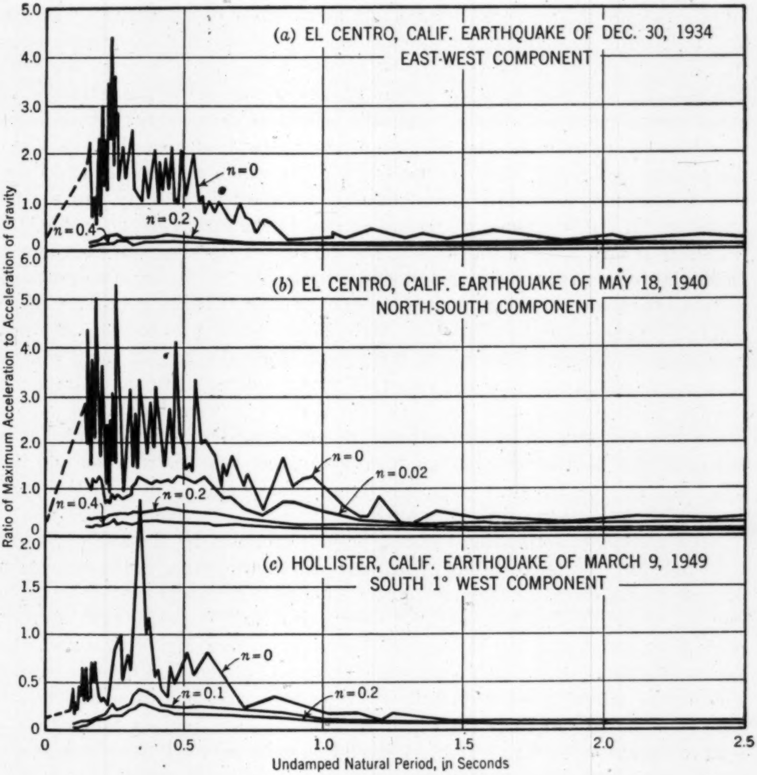

Fig. 17.—Acceleration Spectrums

The authors assert that it is reasonable to assume the motion of the average building as 50% shear deflection and 50% bending deflection, which gives them a straight-line shear distribution. This, however, does not agree with experience. If buildings move with shear deflection, the fundamental periods of vibration are proportional to the height, whereas if the deflections are in bending, the periods vary as the square of the height. On the basis of this criterion, experience indicates that the average building moves essentially with shear deflections, and only a minority of rather exceptional buildings have appreciable bending deformation. This is borne out by the authors' observations on building periods, which they conclude vary as the height.

The assertion is made that only the first mode of vibration need be taken into account, and the recommendations are based upon this. It is stated that the higher modes have greater damping than the first mode and hence are quickly damped out. Experiments do not confirm this. In the aforementioned building test of a 40-ft-wide four-story, reinforced concrete building, vibrations were induced with a maximum acceleration of 8% gravity. It was found that the damping of the second mode was only slightly greater than that of the first, being 8.0% of critical for the second and 7.5% for the first. The period of the fundamental mode in these tests was 0.4 sec, and the period of the second mode was 0.2 sec. It is the opinion of the writers that modes of vibration higher than the first should not be arbitrarily disregarded. In fact, the example presented by the authors in Fig. 10, which was intended to show that the second and third modes can be disregarded, actually proves the opposite. If the first three modes are excited to the extent shown in Fig. 10, the shear at any point in the structure is the sum of the shears of each mode. During an earthquake the phase angles between the modes are not constant, so that sometimes the shears will reinforce each other, and the maximum shear will be the sum of the absolute values of the shears of the three modes. If these maximum shears are added up at each point along the height, there will be obtained a maximum shear diagram that is much different in shape from the shear of the first mode only. So, it is clear that for very small damping, which is the only case studied by the authors, the higher modes are important. It is the opinion of the writers that the question of the shear distribution and the effect of higher modes must be answered on the basis of a much more thorough analysis than is presented in the paper.

The method of estimating building periods proposed by the authors seems to be too crude for its proposed application. As shown by Fig. 8, the estimation of periods has errors as high as 100%. It does not appear proper to use a method of design that is sensitive to the period and then use such a crude method of estimating the period. For example, using Fig. 8 to obtain the period of a building in which $H^2/b = 75$, there is obtained $T = 0.4$ sec. From Fig. 11 the lateral force coefficient is 3.7% of gravity, but from Fig. 8 it is seen that actual buildings (in which $H^2/b = 75$) have periods ranging from 0.22 to 1.2 and according to Fig. 11 should be designed for lateral force coefficients ranging from 2% to 6% gravity.

The recommendation in this paper for magnitude of lateral force coefficients does not seem to be well founded. Aside from the fact that the derivation of the curve shown in Fig. 11 is faulty, as explained previously, the magnitudes of the coefficients seem too low. For example, according to this recommendation a typical five-story building would be designed for less than 3% gravity since it has been found that typical five-story buildings have periods of the order of 0.5 sec. Of course, the proper design values cannot be accurately determined until more experience is gained with strong earthquakes, but the concensus of opinion as reflected, for example, in the building codes in Southern California, Chile, and Japan is that the magnitude of lateral force coefficient for a typical five-story building should be of the order of 10% gravity rather than under 3%. It is also clear that following the authors' proposals would permit flexible designs, so that buildings over four stories in height would need to be designed for only 2% gravity if desired. In fact, the proposals are practically equivalent to saying that the majority of ordinary buildings over five stories in height need be designed for wind only as regards their structural frame and that the earthquake forces can be neglected. This would be a very marked reduction from design requirements in force in regions of high seismic activity.

The authors state that the rotation of a building on its base, that is, foundation yielding, has a very considerable effect upon the period of vibration of a building and reduces the dynamic shears. It is true that if appreciable foundation yielding does occur, it will affect the vibration of a building; but to the writers' knowledge there exist no reliable data that show that, for actual buildings of average type, the amount of foundation yielding that does occur has an appreciable effect on the period or on the magnitude of shear.

It is recommended in the paper that a relatively small, slender tower on top of a building should be designed as if it were a structure resting on the ground, quite independent of the fact that it is actually supported on top of a building. It is certain that this is erroneous, for the motion of the top of a building is not at all similar to the motion of the ground during an earthquake. The actual motion of the top of a building during an earthquake is essentially composed of simple harmonic components with periods the same as the lower modes of vibration. It follows that a relatively light structure set on top of the building is subjected essentially to a harmonic forced vibration. The most important point involved is that the superimposed structure be designed so that resonance does not occur.

Because of the points brought out in the preceding comments, the writers are not in agreement with the arguments and conclusions presented in the paper. It is their opinion that the authors extended their remarks and conclusions to cover more than was justified by the amount of data and analysis that was at their disposal. It is also the opinion of the writers that all points of earthquake-resistant design that cannot be established on the basis of reliable empirical data or incontrovertible analysis should be treated with caution and that the design should be on the conservative side.

S. K. GUHA[16] AND GURDAS RAM[17].—In designing for lateral forces, such as seismic movements, codes have been developed mostly on static, rather than on dynamic considerations. This approach obscures a realistic perspective of the true situation in effect during an actual earthquake; and it never permits taking into account the various factors involved in such a rapidly changing state.

It is true that considerable field and laboratory data concerning earthquakes have been assembled in the past century; but there is still an acute need for studying the types of complex ground motions associated with destructive shocks. The dynamical theory ordinarily developed on the basis of sustained and simple harmonic motion of the ground is not considered sufficient to take into account all the hazards that arise during an earthquake. Figs. 3 and 4 show the extreme complexity of the ground motion near the destructive zones, and it is realized that, at greater distances from the epicentral region, these complexities would be simplified as the very high frequency oscillations become dampened and the remaining waves become lengthened. A structure can naturally withstand a smooth and sustained vibration better than it can withstand the complex nonsystematic oscillations depicted in Figs. 3 and 4.

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It is also necessary to know the prevalent seismic spectrum of a zone—depending mostly on the geology of the region. Such a spectrum, for California, was prepared by Beno Gutenberg, et al (52), from analyses of a large number of seismograms of different observatories. In design, this regional influence on the applied, actual seismic spectrum should also be incorporated because different regions may react to produce widely different seismic spectrums.

The seismic damage to structures described in this paper has not been corrected to the prevailing geology of the region, although the importance of such correction has been emphasized, in a general way. It is quite possible that a consolidated foundation would reduce the seismic intensity, and consequently the damage. The detailed isoseismal study of the 1906 earthquake in California revealed this fact clearly.

Syun'itira Omote (71) and others have studied the effect of geology on damage rate in great detail and have concluded that the depth of overburden has a predominant effect. In the case of soil, they found that the damage rate increases with depth of alluvium, reaching a critical value at 35 m (115 ft). Similar conclusions were also reached in the case of mud with a critical depth of about 6 m (20 ft). It is possible that the existence of critical values during seismic movements signifies the presence of some kind of resonant vibration in the overburden layer (at a critical depth). Although Mr. Omote's study was limited to wooden structures, the general conclusions apply to other types of structures.

There has been a distinct difference in the damage sustained by structures with and without frames, especially steel frames. Possibly, such differences are due to the dynamic characteristics of the framing. It is possible that the dynamic characteristics of the filler walls, masonry or concrete, are obscured by those of the framing during transit vibrations, with the frame responding much more quickly than the filler wall material. Of course, a complete dynamic analysis can be made only when the elastic properties and the bond between the filler walls and the frame is known. These factors determine the dynamic behavior and the potential damage to a structure during a transient vibration. The difference in damage with and without the frame can be systematized or assessed only when the foregoing properties of building frames are known.

It is also possible that the filler walls in a framed structure act simply as loads on the framing during vibration. This assumption ignores the existence of the wall material even as a weakly participating element working with the frame. The excellent record of framed structures during earthquakes, both in the United States and in Japan, argues for the tenability of this concept especially for the transient impulses of an earthquake. However, for sustained and harmonic vibration, it is likely that the filler walls may actually contribute something to the resistance of a structure.

During an earthquake, a masonry or concrete structure with a steel frame is expected to respond in a manner completely different from that of a structure without framing. The slow reaction of masonry material in a framed structure is due to the quick transmission and response of the elastic energy by the framing material, especially steel. This difference in behavior becomes more acute for higher structures.

In general design analysis, the base of the building is assumed to be clamped; but Kiyoshi Kanai observed (72) that they act as clamped bases in the relatively longer vibration period and as hinged bases in the shorter periods. Thus, the foregoing dynamical picture envisaged for a building may need to be changed, considering the applied frequency of the seismic waves. In the hinged condition, the reception and transmission of the waves will not be similar to that prevailing in a clamped state. The authors have not considered, in sufficient detail, the effect on the dynamic design of diverse seismic spectrums.

In any appraisal of seismic damage, it is suggested that proper attention be given to the direction of ground motion with respect to the structure. A given seismic spectrum may actuate the structure in one direction whereas it may fail to do so rationally in the other direction, depending on the natural frequencies in the respective directions, and on the degree of interaction between the two. In an asymmetric system, rotation and translation are each induced by a translatory ground motion. The rotation is caused by the interaction of a structure in its several planes, caused by asymmetry. Generally, the motion along one axis is accompanied by motions along the remaining two co-ordinate axes. Thus, the direction of the ground motion with respect to the structure plays a dominant part in its response. Of course, for design purposes, one must apply standards for determining the maximum response in a given structure under the seismicity that is prevalent in the region.

The distortions produced in a structure, under the foregoing conditions, are influenced by both flexural and shear modes of vibration; the predominance of one or the other is determined by the height-width ratio (H/b) of the structure. It is generally believed (and recognized theoretically) that distortions are caused mostly by flexural modes for higher values of H/b and by shear modes for lower values. For values of H/b greater than 4, distortions can be assumed to be mostly flexural. The predominance of the one mode or the other, for various values of H/b, will defeat all efforts to systematize the natural periods of buildings as a whole. Fig. 8 suggests that the authors gave preference to the flexural mode of vibration, which is directly related with H^2/b. For complete shear distortion, the period will be directly proportional to H. The final correlation that the authors report for the natural period, and H and b, suggests that data taken as a whole would not show either flexural or shear distortion but a combination of both. This situation could probably be avoided if T was found as a function of H^2/b for buildings having H/b-ratios greater than 4 and, as a function of H for buildings having H/b-ratios less than 4. Of course, the value 4 has been selected rather arbitrarily. It is realized that there is a zone for H/b that will overlap both T_1 and T_2 to a certain extent because, under the circumstances, there cannot be any sharp division between shear and flexural distortion in structures, in terms of the ratio H/b.

The vibrational characteristics of a structure will certainly be affected by the elastic yielding of the foundation. The resultant effect will be an increase in the deflection of the structure and, consequently, an increase in the period. The yielding affects the fundamental periods of translation as well as those of torsion very appreciably. The effects on the overtones are also felt in this respect. It should be an inherent part of dynamic design procedure to take into consideration the relative elastic behavior of foundation and structure.

From a general dynamic viewpoint, it would seem basic that the fundamental mode should maintain a comfortable envelope for higher harmonics in respect to shear distribution. Although the authors have called attention to increases in shear in higher parts of structures, for higher harmonics, it is recognized that the fundamental mode may still have a good envelope for higher harmonics, in dynamic design. The absolute value of impressed shear in those parts is not considerable. This has been demonstrated by statistical analyses of the usual damage to buildings of different heights; and it has been found that, in buildings, serious damage occurs generally below the mid-height of the structure. This confirms the observation that the fundamental mode is of predominant importance in the dynamic approach because, as such, extreme precautions may not be necessary in higher parts. Sometimes, of course, it is found that higher parts of a structure suffer from small cracks, which may be the result of elastic nonhomogeneity in the structure producing differential movement near the region. Although model experiments show increases in shear for third harmonics and second harmonics at higher levels, it is felt that this is an exaggerated effect characteristic of the model, and may not have much bearing in the prototype.

Model experiments and theory sometimes seem to indicate that a structure will react with very high dynamic shear and flexural stress values, especially when the known seismic spectrum has prevalent frequencies approximating the natural frequency of the structure; but the unexpectedly excellent record of moderate structures, in seismic zones such as Japan, has given rise to a belief that damping due to internal friction in the structure, and to dissipation of energy in the elastic foundation, will prevent the structures from attaining large vibrational energy and thus will help in reducing dynamic shears and vibrational stress. Although the authors have recognized the dynamic influence of the damping process, they have not shown, clearly, a practical method of incorporating this factor in design to avoid great expense as compared with the cost of other shock-proof measures.

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[20] Structural Engr., J. J. Gould, Cons. Engr., San Francisco, Calif.

[21] Cons. Civ. Engr., San Francisco, Calif.

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M. ASCE; Henry L. Marchand;[23] Henry C. Powers;[24] John E. Rinne,[25] M. ASCE; George A. Sedgwick,[26] A. M. ASCE; and Harold O. Sjoberg,[27] A. M. ASCE.—The Joint Committee appreciates the study which the discussers have made on the controversial subject of earthquakes. The controversial aspects arise from the two basic sources: (1) Incomplete understanding of earthquake phenomena; and (2) criticism of simplifications which lack complete rigor.

The first source is being resolved by continuing research and the Joint Committee has indicated several specific problems needing further study. Included was the subject of damped spectra for which Messrs. Martel, Housner, and Alford are developing some data. Need for world-wide recording of ground motions in moderate and strong earthquakes, on an accurate and uniform basis, was also acknowledged. Research along these lines should be pursued more intensively.

The second source comes from the fact that, even under idealized conditions, earthquake phenomena are very complex. Completely rigorous analyses are not warranted in the day-to-day application to structural designs. Simplifications are necessary and it must be recognized that when these are made there is room, and perhaps justifiable room, for criticism because simple rules raise exceptions and complex rules become overbearing.

The discussion by Messrs. Martel, Housner, and Alford requires special comment. When some important aspects of the paper are properly understood, the arguments presented in this discussion do not "invalidate the conclusions." The analysis of some eighty spectra, completed after the April, 1951, publication of the paper, is a fine contribution. However, these spectra can be shown to justify rather than refute the recommendations in the paper. It must also be recognized that where "reliable empirical data" and "incontrovertible analysis" are lacking, judgment backed by available facts and engineering experience must be relied upon to meet the demands for building designs which cannot wait for such data. A purely academic approach is impractical and probably will be for some time to come. Future studies should either corroborate and reinforce the recommendations or indicate that modifications are required to keep engineering practice in step with scientific developments. Until considerable new factual data are reported, however, it is necessary to apply engineering judgment to the data at hand.

The discussions have not revealed any aspects of earthquake design forces which had not been considered carefully by the Joint Committee. Existing earthquake codes are unrealistic in some important respects and too severe to be justified as minimum requirements of law. In fact, certain code requirements may be more harmful than beneficial.

The following comments relate to specific aspects of the earthquake problem as covered in the discussions.

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Ground Motion.—The Japanese discussions emphasize variations in "natural periods" of the ground. The conclusion is that the peak of the spectra should shift to the right, going from firmer to weaker foundation materials possibly with peaks at 0.1 sec for rock, 0.2 sec for intermediate soils, and as high as 0.6 sec for soft soils. Spectra available to the Joint Committee to date do not exhibit this much variation.

The Joint Committee is not familiar with the details of the instruments currently used in Japan for measuring ground motion, and more specifically, ground accelerations needed to determine the earthquake spectra. Assuming these instruments are comparable to those used by the USCGS, records of acceleration should be obtained for different soil conditions. Spectra can then be determined using an electric analog such as that at the California Institute of Technology, in Pasadena, or by other analytical or experimental methods.

Messrs. Guha and Ram state that a structure can withstand a smooth sustained ground motion of a distant earthquake better than the complex nonsystematic oscillations in the epicentral area. There are two conflicting considerations here, namely, the higher intensities near the epicenter and the greater possibilities for resonance effects under the influence of the smoothed, but less intense ground motion at greater distances. The usual experience has been that the greatest damage is inflicted in the epicentral area, indicating that intensity normally governs, but resonance effects at some distance might be damaging to particular structures.

Spectra.—The development of damped spectra by Messrs. Martel, Housner, and Alford is a valuable contribution. Mr. Binder and the Japanese investigators were also aware of the need for such data. Messrs. Guha and Ram emphasize the need for spectra in particular zones which may vary depending especially upon the geology of the region.

Mr. Martel's complete test data have been made available to the Joint Committee and the following observations are the result of an engineering review of all these data:

(1) The undamped and damped spectra were obtained using an electric analogy. This is an excellent method for quickly and conveniently determining spectra from ground-acceleration records obtained with the modern strong-motion seismograph. It is strongly recommended that such spectra be determined promptly for all future records of ground motion.

(2) It is unfortunate that more acceleration records of major earthquakes are not avilable. For a modified Mercalli scale intensity of *X*, the El Centro record of May 18, 1940, is the only acceleration record extant. It is questionable whether spectra determined from records of minor intensity can be interpreted to represent conditions in a major earthquake. This is especially significant for damped spectra

(3) Damping spectacularly reduces the spectral values of all of the earthquake motions investigated, especially in the region of short periods. The ratio of maximum value (at short periods) to the minimum value (at long periods) is also reduced.

(4) With damping of 0.10, the maximum spectral value of the El Centro earthquake of 1940 is of the order of 1.0 g, indicating that the idealized damping analysis still does not reduce the quantitative spectral values to coefficients practical or necessary for design. Therefore, the damped spectra like the undamped spectra, must be considered as qualitative curves to which practical design coefficients must be assigned by engineering judgment. This will be influenced by observations of earthquake damage and structures undamaged, the allowable working stresses, and other design considerations which the practicing structural engineer is most competent to evaluate.

Periods of Structures.—The method for establishing design earthquake forces on structures proposed by the Joint Committee requires the predetermination of the fundamental period of the structure. The factors entering into the period are complex, and a rigorous determination does not appear warranted generally. The formula involving only the geometry of the building was intentionally selected to give periods usually on the low and conservative side. Further study should be made to establish a better formula. Until this is done, there is little to be gained by making a nominal distinction suggested by Messrs. Guha and Ram of relating the period to the height H for low buildings and to H^2/b for taller buildings. A more exact formula will involve at least some of the following factors—soil conditions; elastic properties of the materials of construction, dimensions of the structural elements; number, size, and spacing of openings; number and construction of partitions; and mass and its distribution.

The present Los Angeles, San Francisco, and the Uniform Building codes of the Pacific Coast Building Officials Conference (PCBOC) specify coefficients varying only with the number of stories of a building, a procedure justified by dynamic considerations which basically involve the period. The introduction of the period as the criterion for the lateral force coefficients acknowledges the importance of the proportions of a building and is a forward if not exact step.

Messrs. Martel, Housner, and Alford state that

> "* * * there exists no reliable data that show that, for actual buildings of average type, the amount of foundation yielding that does occur has an appreciable effect on the period or on the magnitude of shear."

This is contrary to many expressed views, as well as the known behavior of structures (22) (59) (63) (68).

The shortcomings in the method of predetermining the period does not invalidate the general approach. It would appear logical and practical to introduce the criterion of relating the lateral forces of earthquake to period, and then set about to develop more reasonable means for predetermining the period—reasonable both as to accuracy and workability. The nomograph solution of the equation for period and the corresponding coefficients as presented by Mr. Binder, are useful tools pending the development of a more accurate formula.

Distribution of Base Shear: Effects of Higher Modes.—Mr. Martel suggests that the triangular distribution of the base shear arbitrarily disregards modes higher than the first. Such is not the case. The point that is made by the writers is that the shape of the fundamental mode shear curve—that is, the shear curve corresponding to the triangular distribution of the base shear—is a reasonable envelope to contain the higher mode shears. The Joint Committee is well aware that the actual response of a structure, especially a tall multi-mass structure, is a composite of the fundamental and higher modes. Under the most adverse conditions, these modal responses add to produce a shear diagram with an hour-glass shape; but, from a practical view, present knowledge does not warrant the design of a multi-storied building with other than an increasing shear resistance from the top to the bottom of the building. The inherent increasing lateral resistance built into a structure to take care of the increasing vertical load makes this both practicable and desirable. Furthermore, this accords with the observations of Messrs. Guha and Ram and others that, almost without exception, building damage in earthquakes has been in the lower half. It is the Joint Committee's contention that the triangular distribution of the base shear will provide sound structures for higher modes as well as for the fundamental.

In arriving at the triangular distribution, the case of a building deflecting half in shear and half in moment was assumed. This was not represented to be an average building. In fact, it was acknowledged to be a slender building of the Alexander Building type in San Francisco. Most buildings are not so slender and would deflect more in shear than in moment. Therefore, the triangular distribution is conservative with respect to the fundamental mode and provides some excess resistance in the upper parts of a multi-storied building to take care of the second and higher modes which have secondary maximum values of shear in these parts.

Messrs. Muto and Umemura suggest combining the shear curves in the fundamental and second modes, resulting in the aforementioned design shear curve with the hour-glass shape. An advantage in this is that the arbitrary restriction on overturning moment considerations need not then be specified. The moments would automatically decrease near the base, compared to the values computed from the triangular distribution of the lateral forces. As new research is undertaken, this approach may have distinct possibilities. For the present, the writers prefer to design for increasing shear from the top to the bottom of the building and to introduce a limitation on overturning considerations.

Coefficients.—The Japanese group is strongly inclined toward different sets of coefficients for different soil conditions, with higher factors for weaker soils. Although some codes have this feature, most engineers feel that this refinement is not justified for California conditions. The Uniform Building Code of PCBOC, for example, varies the coefficients with the seismicity of the region but not with the soil conditions. There are conflicting effects of soil conditions on the coefficients and until more data are found to establish how these are interrelated, a single set of coefficients is specified.

The actual values assigned to the design coefficients depend upon a number of factors including the seismicity of the area, the definition of the loads to which the coefficients are to apply (percentage of design live load), the allowable increase in working stresses, the earthquake damage experience, the review of structures which have successfully withstood major earthquake, and the basic consideration of what these design coefficients are to accomplish, assuming they are properly applied.

The Joint Committee disagrees with Mr. Martel's comments on the coefficients. Japanese conditions cannot be compared directly with California conditions. In Japan, the seismicity is higher, the allowable working stresses are higher, and it is understood that arbitrary allowances are made for noncomputed elements of the building which are recognized to add to its lateral resistance.

Even for Japanese conditions, the writers understand that it was the practice of Mr. Naito to design buildings for lateral coefficients of one fifteenth of gravity ($C = 0.067$) previous to the 1923 Tokyo Earthquake. Little damage was experienced in these buildings, the Nippon Kogyo Bank, for example, being virtually undamaged although surrounded by almost complete destruction of other buildings.

In California in areas now specifying the Los Angeles formula for the shear coefficients, these coefficients are not 0.10 for a five-story building, but 0.071 for Zone 3 areas and 0.036 for Zone 2 areas. These coefficients are applied to dead load only. For comparison,the Joint Committee's coefficients on a typical five-story building are given in Table 3, and are 0.0468 in the long direction of the building, and 0.0331 in the short direction, applied to dead load plus 25% of live load (or 50% of live load for storage buildings). Thus, the coefficients recommended are both somewhat lower and somewhat higher than those now applicable in Southern California. It is true that the code recommended in the paper recognizes flexibility by providing different coefficients in the narrow and long direction of the building, but this has a rational basis as the periods in the two directions are not normally equal. It is very doubtful that the coefficient would ever be as low as 0.02 for a four-story building. By the formula for period given in the paper, this would require a building 7.1 ft to 12 ft wide, corresponding to heights of 40 ft to 52 ft which is the range that might be expected in a four-story building.

The Joint Committee is satisfied that the proposed coefficients and code provisions are adequate and reasonable minimum legal requirements for California conditions. The establishment of these coefficients is not an academic problem but requires the judgment of those members of the profession who are regularly engaged in the practice of structural engineering.

Allowable Stresses.—The Joint Committee was guided with some reluctance by the prevailing practice in the United States of using allowable increase in stress of one third for abnormal combinations of vertical and lateral load. A better balanced design is obtained by increases in stress and higher coefficients—the practice followed in Japan. These increases have been recommended for further study.

Small Towers on Buildings.—The comment by Messrs. Martel, Housner, and Alford, that small towers on top of buildings are subjected essentially to a forced harmonic vibration of periods corresponding to the lower modes of the building below, is correct. The Joint Committee would prefer to specify higher coefficients for the towers rather than introduce an elusive statement about avoiding periods in the tower which might be in resonance with the periods of the several modes of the building below.

It must be emphasized that present codes make no provisions for towers or how they should be handled. Under some codes, the towers may even be treated as additional stories and so reduce the design lateral load on the entire building, erroneous though this procedure is.

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FRIENDSHIP INTERNATIONAL AIRPORT

By Benjamin Everett Beavin,[1] Sr., M. ASCE

With Discussion by Mr. Charles O. Landrum

Synopsis

The design of Friendship International Airport at Baltimore, Md., embodied many advances in the theory of airfield layout. The airport was considered to consist of two parts, the airfield and the terminal, and each of these parts was designed to function with a maximum of convenience and efficiency.

The factors affecting design and construction of the airfield are outlined and discussed, and a brief description of the construction phase is given.

The design factors influencing the layout and orientation of the terminal are presented, along with their effect on normal operations. Passenger handling is described, and the service facilities in the terminal are outlined in their relation to the traveler and the airline operation and maintenance.

Introduction

Friendship International Airport was designed and constructed in a period of 5 years between 1945 and 1950. The first 2 years were devoted largely to research covering many air, bus, railroad, and marine terminals, surveys, conferences with aviation officials, airlines, travelers and prospective concessionaires, and planning and preparation of the master plan. This paper has been prepared for the purpose of making available to the profession any knowledge that may have been gained during this 5-year period.

In order to simplify the problem, the design of an airport may be considered to consist of 2 parts: (1) The part most intimately connected with flight, such as the air above and around the airport, approach and landing facilities, runways, taxiways, and aprons that for convenience are called the airfield; and (2) the access road, interior roads, fuel storage, auto parking areas, hangars, shops, industrial areas, and, last but not least, the terminal building. This second part, mainly devoted to earthbound activities, is called the terminal.

The division of the airport into 2 major parts leads to the realization that the length of the boundary line between the two is one important index of the facil-

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Note.—Published in October, 1951, as *Proceedings-Separate No. 94*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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ities provided. This boundary line may be defined as the apron frontage line. The total cost of Friendship International Airport, including land, legal, engineering, utilities, and administrative expense, and all construction contracts was approximately $15,000,000. Dividing the total cost by the 3,100 lin ft of apron frontage presently constructed (1950) indicates a cost of approximately $5,000 per front ft. Friendship Airport has a potential apron frontage of 7,700 lin ft. A great deal of apron area has been graded but not paved; therefore, it is not included as completed apron frontage.

Obviously, this apron frontage is far too valuable to be dedicated permanently to any activity that does not contribute heavily to aviation. All airport design should be carried out with this thought in mind.

Airfield Design and Construction

Design Factors.—Having established two parts of an airport, it is logical to consider first the design of the airfield.

There are many criteria for airfield design, but the most important are: (*a*) Safety; (*b*) economy of construction; (*c*) economy of airline operation; and (*d*) economy of maintenance.

(*a*) *Safety.*—Many factors contribute to a safe design. Among others it is important to consider uncrowded airways, unobstructed approaches, adequate navigation aids, zoning protection, wide and long runways, ample taxiways, and freedom from fog and wind vagaries.

(*b*) *Economy of Construction.*—By careful site selection, avoidance of unnecessary approach-zone damage to near-by property, exploitation of on-site and near-site materials, careful subsurface surveys, and preparation of specifications that take as many risks from the contractor as possible, it is feasible to secure the maximum utilization of the available funds. Under the best of conditions, the contractor is still subject to plenty of risks, and bid prices are bound to reflect them.

(*c*) *Economy of Airline Operation.*—Many factors affect this phase of design, including expeditious landings, favorable meteorological conditions, short taxiway distance, ample apron facilities for quick turn-around, and lack of restrictions on take-off weights. An approximate measure of the direct cost per ton-mile of airborne persons or cargo may be obtained by dividing the airplane cost of operation per hour by the product of velocity between terminals and the pay load. Restrictions due to airfield deficiencies affect not only the landing and take-off at the airport but may limit the load for the entire trip, thereby increasing the cost per ton-mile. Friendship Airport is fortunate in having no such weight limitations.

(*d*) *Economy of Maintenance.*—Careful drainage design, adequate sub-base construction, wind and water erosion control, and pavements designed for resistance to heavy loads, weather, and scuffing will reduce maintenance costs.

Details of Site.—With these general criteria well in mind, the Baltimore City Department of Aviation selected a 3,200-acre site for Friendship International Airport 9 miles southwest of the center of the city, alongside the Baltimore-Washington Expressway, which is now partially completed. The airport abuts the Pennsylvania Railroad and is not far from the Baltimore and Ohio Railroad. Its surface transportation facilities, when completed, will be ideal. It is located on a plateau some 140 ft above sea level. The soil is sandy, drainage is away from the airfield in all directions, and the site is remarkably free from fog and smoke.

Design and Construction.—The details have been rather thoroughly documented[2,3] but the important highlights of the airfield design and construction will be presented as background for the terminal design. Economy of construction demands that all materials on a given site be utilized in so far as possible in the construction of the airfield. Few sites are blessed with abundant supplies of sub-base or pavement material deposited in such large quantities that they may be economically salvaged and used. More often, deposits are found scattered throughout the area with a greater or lesser amount of overburden. The latter was the case at Friendship Airport. Many small deposits of sand gravel, suitable as aggregate for sand-gravel asphalt, were encountered as the grading operation proceeded.[4,5] Materials from these deposits were routed to stock piles located near the future site of the asphaltic concrete hot mix plants.[6]

Culverts under the older highways were seldom long enough to cause dangerous acceleration of the water flowing through. This is not the case, however, with culverts under modern dual highways or under airport runways and taxiways. In the long culverts at Friendship, velocities were found to increase from 5 ft per sec at entrance to 15 to 25 ft per sec at exit. With soils that erode at velocities of about 5 to 6 ft per sec, these high exit velocities could not be tolerated The solution adopted was to flatten the lower end of the culverts so as to cause the hydraulic jump to occur within the structure, thus reducing the exit velocities to a safe figure.

Where fills over culverts were comparatively low, concrete cradles were necessary to enable the culverts to withstand the heavy loads from the expected use of the 200-ton pneumatic roller[7,8] and the 150,000-lb wheel load for which the airfield is designed. Concrete made of imported aggregates would have cost $30.00 per cu yd in place. By mixing 3 bags of cement with a cubic yard of sand-gravel aggregate taken from the trench side, the cost was reduced to $8.00 per cu yd. Tests indicated a crushing strength of approximately 900 lb per sq in., which was considered adequate for cradle construction.

The light sandy soil at the site was very susceptible to wind and water erosion. Therefore, soil erosion control measures were made a part of the very first and all succeeding grading contracts. Weather permitting, as soon as an area was finally graded, it was fertilized, seeded, and mulched. In

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[2] "Master Plan Report, Friendship International Airport," submitted to Baltimore City Aviation Commission, June 24, 1946, by Whitman, Requardt-Greiner Co. and Associates.

[3] "Baltimore Breaks Airport Precedent," *The American City*, Vol. 63, November, 1948, p. 101.

[4] "50,000 cu. yd. Daily for Baltimore Airport," *Engineering News-Record*, Vol. 139, 1947, p. 532.

[5] "Baltimore Airport Grading . . . Three Types of Rigs for Various Hauls," *Construction Methods*, Vol. 30, January, 1948, p. 102.

[6] "Asphaltic Surfacing Details at Baltimore Airport," by B. E. Beavin, *Roads and Streets*, Vol. 92, April, 1949, p. 58.

[7] "Super Compactor Cuts Airport Construction Costs," *Civil Engineering*, Vol. 18, 1948, p. 140.

[8] "Soil Compaction at Friendship International Airport," by B. Everett Beavin, *The Military Engineer*, Vol. 40-41, July-August, 1949, p. 264.

this way, a healthy stand of grass usually was obtained before any gulleying had started. There was not sufficient topsoil on the site to build a high-grade turf over the entire disturbed area; therefore, topsoil was applied to those areas immediately adjoining runways, taxiways, and aprons, where the blasts from airplane propellers might be felt and where planes might occasionally run off the pavement. In other areas the mineral sand was fertilized, limed, and planted in rye and vetch. The following spring the rye and vetch was "disced" in and the areas replanted with grass. The resulting humus helped to keep the ground moisture within reach of the grass roots, and the stand attained has been nearly as good as that found in the topsoiled areas. In steeper and more remote areas, at the suggestion of the Soil Conservation Service (SCS), a new type grass (the African Weeping Love Grass) was planted directly upon the bare clay and sandy surfaces with very gratifying success. The seeds of this grass are so small that only 2 to 3 lb are required per acre. Being an annual it has the disadvantage that it may only be planted in the spring and early summer if it is expected to survive permanently. A total of approximately 1.5 sq miles of land were successfully tied down with vegetation so as to resist wind and water erosion.

The utilities serving the airfield are mostly conventional in their design and construction. However, even at the time of construction of Friendship Airport it seemed probable that aircraft would not always be dependent upon wind direction in landing and taking off and that in the future some of the runways might become superfluous. The matter was discussed with the Civil Aeronautics Authority (CAA) and it was decided that the city's proper course would be to proceed with the construction of three runways and to make all possible provision for future conversion of one or more of the runway areas into sites for industrial or other activity closely related to aviation. The power, communications, and water distribution systems were designed with this thought in mind, and at every intersection of runways and taxiways there were installed spare ducts for high and low tension wires and capped-off sections of water main. After supercompaction and before the ends were backfilled, mandrels were run through the duct lines and a pressure test was applied to the water main sections to make sure that no defects existed. These precautions will be of considerable assistance in adapting the design of the airport to accommodate the rapidly changing needs of aviation.

Sufficient aggregate was accumulated from grading operations at the site to provide 800,000 sq yd of sand gravel asphalt, 7 in. thick. The top 3 in. of pavement were composed of imported sand and crushed stone having an abrasion loss of less than 40 based on the Los Angeles abrasion test (American Society for Testing Materials Designation: C131–47). Stabilities of 1,400 were obtained, as measured with Marshall Stability Apparatus,[9,10] enabling the pavements to withstand rutting from tire pressures of approximately 250 lb per sq in. Although damage from jet fuel spillage is no longer a very serious problem, the present tendency in aircraft design is to turn jet exhausts

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[9] "Formulas and Procedures for Design and Control of Asphalt Paving Mixtures," Waterways Experiment Station, Vicksburg, Miss., June, 1948.

[10] "Investigations of Design and Control of Asphalt Paving Mixtures," *Technical Memorandum No. 3-254*, Waterways Experiment Station, Vicksburg, Miss.

downward. This will increase the damage to paving of all kinds. The cost of the pavement was $3.00 per sq yd (1949) and would have been considerably higher if all the materials for the entire 10 in. of asphaltic concrete had been imported.

The most perfectly designed and constructed airfield will not endure very long unless its approaches are clear and protected from the encroachment of man-made hazards. Friendship Airport is protected by zoning regulations[11] that will maintain forever the 50:1 approach glide angles, a turning zone 8-miles in diameter, and 150-ft clearance above average elevation of the field. The layout of the airfield is shown in Fig. 1.

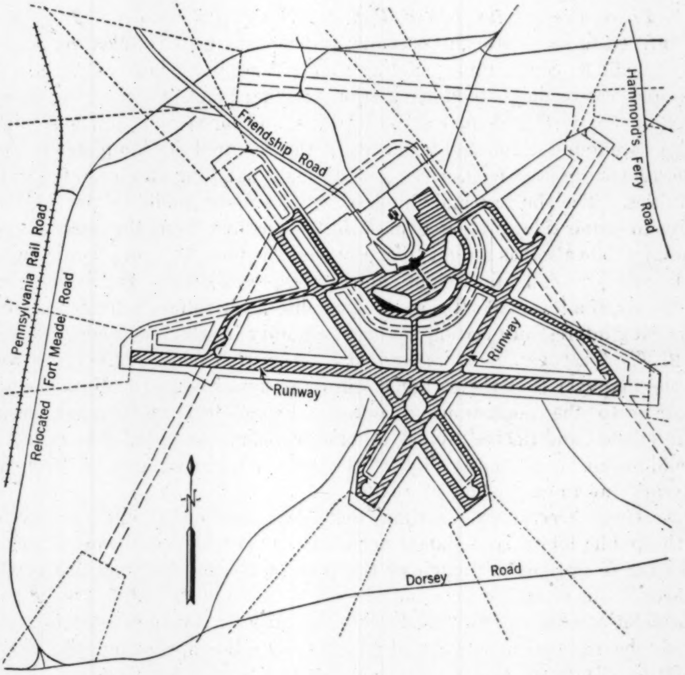

Fig. 1.—Layout of Friendship International Airport, Baltimore, Md.

Terminal Design

Design Factors.—The question of terminal design involves far more complex factors than the airfield. On the airfield, the needs of aircraft are paramount and all other interests must take second place. In the case of the terminal, however, needs of many varied and often conflicting interests come into focus and must be given the consideration that each deserves. The more important of these interests are as follows:

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[11] "Zoning Protection for Friendship International Airport," by B. E. Beavin, *The Military Engineer*, Vol. XLII, January-February, 1950, p. 35.

1. Passengers and the general public;
2. The taxpayer as represented by the airport management;
3. The airline ticketing, operating, and repair facilities; and
4. Related activities such as the CAA, United States Weather Bureau, and the Customs, Health, and Immigration departments.

1. *Passengers and the General Public.*—Those who use airports expect and are entitled to swift, safe access roads; ample parking space; pleasing grounds; a commodious, comfortable building that will accommodate surges in traffic; a simple, easily understood circulation scheme; efficient ticket and baggage handling; comfortable lounge and rest rooms; and concessions and conveniences such as are found in railroad terminals. Also required are complete separation of passenger facilities from baggage and freight handling and aircraft servicing activities. The shortest and simplest walking route should be provided from lobby to plane side, protected in so far as possible from the weather. The general public also has a right to expect that the terminal will provide facilities to handle air mail, air freight, and air express, swiftly and economically. Obviously, if a terminal satisfies these conditions, it will do a great deal toward increasing interest in aviation in the community in which it is constructed.

2. *The Taxpayer, as Represented by the Airport Management.*—It is expected by the taxpayer that the terminal will be designed so as to utilize the apron frontage to the maximum advantage. Economical construction, ease of maintenance, and the inclusion of all practicable revenue producing concessions should result in the possibility of the terminal becoming a self-liquidating enterprise at an early stage of its life.

3. *Airline Operations.*—Airlines ticketing operations should be convenient to the public lobby, and should permit the possibility of future consolidated ticketing operations, although at the present time the airlines are unwilling to accept this obviously economical system as being workable. Some partial consolidations are in effect at Friendship Airport. Airlines operation offices should be conveniently situated in regard to the apron and the ticketing facilities. Hangars and repair shops should be located to make the most efficient use of the apron frontage allotted them. For many years to come, air cargo will be used as supplementary loading of aircraft that are not fully loaded with passengers. The day is not yet in sight when entirely separate passenger and freight terminals will be justified in the average city. Therefore, convenient and integral cargo handling facilities must be provided.

4. *Related Activities.*—The needs of related governmental activities are fairly well established and usually may be met satisfactorily without serious conflict with the needs of passengers, airlines, and taxpayers.

It would be interesting to chart the dozens of design criteria stemming from the various interested groups and to show the manner in which they overlap and sometimes conflict. However, such a discussion is far beyond the scope of this paper which will be limited to describing some of the more important criteria and then a description of the actual operation at Friendship International Airport.

A single plane loading position utilizes 158 ft of apron frontage. Except in the case of a small airport, the only way in which passenger walking distance may be kept within a reasonable limit, and the maximum amount of apron frontage be obtained from any given layout, is to load and unload planes on both sides of piers or fingers extending out from the terminal into the apron area. The piers increase the capacity of the terminal and conserve apron frontage in exactly the same manner as piers increase the capacity of a harbor. In this type of layout, the use factor of the loading positions is increased by not assigning loading positions permanently to any one airline but rather to require that incoming planes use the open position nearest to the main lobby.

The only practicable way to separate passengers from the service activities is to have these two interests occupy different levels of the terminal. This may not be necessary in the case of small- to medium-sized airports, but there seems to be no doubt that for the larger airports vertical separation is the answer.

In the old style horseshoe-shaped terminals (in reality a string of small terminals) 32 plane-loading positions would occupy 1 mile of apron frontage. In addition to squandering the precious commodity of apron frontage, this type of layout also made it impossible to derive any substantial amount of nonaviation revenue because only a very small percentage of the total number of passengers ever had occasion to pass through the administration building. With such a low revenue potential, it is practically impossible to provide concessions and conveniences comparable to the luxurious travel facilities furnished by the airlines. Therefore, it is highly desirable that 100% of all passengers and visitors to the airport pass through the main lobby and the concession area.

Economy of operation can be aided materially by careful design. Buildings should be arranged so that most of the cleaning and other maintenance functions may be done with labor saving machinery. Heating plant, electrical shops, garages, fire department, and police headquarters should be located adjoining each other in order that fire and emergency duties may be shared by all. Boilers should be studied to determine the effect of size upon the type of licensed employee required for operation. The installation of one small steam engine may result in the airport's being required to hire 4 licensed engineers for 3 shifts and 1 relief. These are only a few of the many items in which care in design will result in reasonable operation costs.

Perhaps the greatest fault in the older terminals of the United States was the lack of provision for expansion of the various major activities. Many of them were overcrowded when they opened for business. Very few had electrical and utility shafts capable of receiving the many new mechanical and electrical contrivances that have developed along with aviation. A terminal building will not remain adequate for any considerable length of time unless it is designed so that every major activity is free to expand without infringing upon the space of adjoining major activities. This may be accomplished by designing the building for both horizontal and vertical expansion and by allowance for certain activities to enlarge by encroaching upon cargo space. Cargo space may be converted to other usages with very little lost investment.[13]

AIRPORT OPERATION

Passenger Handling.—In order to see how some of these precepts have been incorporated into the design of Friendship International Airport, it is merely necessary to follow the route of a typical passenger leaving Baltimore by limousine or bus for the airport. The conveyance will turn off the Washington-Baltimore Expressway at Friendship Interchange and proceed into the airport via the mile-long access highway. The passenger will leave the vehicle at the lobby floor and proceed directly into the spacious main lobby. If by chance he had driven his own car, he would have parked it on the parking lot (capacity 1,400 cars) and entered directly into the lower lobby, thus avoiding the necessity of crossing the main flow of vehicular traffic. He then would take the moving stairs to the upper lobby. As he proceeds along the main lobby toward the ticket counters, which are immediately visible, he sees on either side of him a branch bank, a game room, a dining room, cocktail lounge, coffee shop, interesting display cases, and a merchandise mart in which many articles, attractive or useful to travelers, are for sale. In addition, just off the main lobby are located the men's and women's lounges, barber shop, cobbler shop, telephone room, and roomettes that may be rented by weary travelers. After checking in at the ticket counter, the passenger's baggage is conveyed by a spiral chute to the floor below, where it is picked up by airline employees and carried to the plane. The passenger waits in the main lobby and has an opportunity to patronize the concessions that he has seen as he came into the lobby. He may, if he has time to spare, proceed to the second floor and walk out upon the 500-ft long promenade deck that is open to the public upon payment of an admission fee of 10 cents. This promenade deck permits an intimate view of the entire airfield operation. Over 300,000 visitors used the deck during the first year of operation.

Upon hearing his plane departure announced, the passenger proceeds via 1 of the 3 piers to the loading position and as presently constructed (see following section on aircraft loading) goes down a flight of stairs to the apron level and proceeds across the apron to the plane. No fences or gates as such are provided. All passenger checking operations are conducted indoors. Friendship Airport has 8 fully equipped loading positions and 4 more that may be used during rush hours. Eventually, 40 or more positions may be provided. However, the passenger will never have to search for more than 1 out of 3 main doorways (Pier A, Pier B, or Pier C), all of which are visible from the main lobby.

Aircraft Loading.—In an effort to reduce the amount of small equipment that usually clutters up the apron of an airport, as many as possible of the utilities have been installed in the service pits. These pits contain a sewer connection, a storm water drain, a 1.5-in. cold water line, ducts for future communication lines, receptacles for 110-volt alternating current portable lights, 26-volt direct current for starting plane engines, and conditioned air ducts to warm the planes in winter and cool them in summer. In so far as it is deemed practicable, a passage through the airport has been made pleasant, simple, and efficient.

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[13] "World's Best Airport—Baltimore's Friendship Field," *Engineering News-Record*, Vol. 146, Feb. 1, 1951, p. 32.

It is proposed to equip some or all of the plane loading positions with adjustable covered walkways reaching from the plane door to the lobby floor level of the pier, thus obviating the necessity for certain ramp equipment and for the passengers to walk across the apron (sometimes in bad weather) and to climb up a set of stairs into the plane. Window openings and landings are designed to fit such equipment when installed.

Arrival Procedure.—A passenger arriving at Friendship Airport on board a domestic plane will proceed through 1 of the same piers, in a reverse manner, and along the walkway into the main lobby. Here he may either wish to patronize some of the concessions or facilities available or he may wish to proceed by moving stairs immediately to the lower lobby to pick up his baggage. All buses, limousines, taxicabs, and private cars leave from this level.

A passenger arriving from overseas will disembark at the International Port, located at the extreme end of the central pier known as Pier B. Two plane loads of 60 passengers each, or one plane load of 120 passengers, may be accommodated at any one time. A comfortable waiting room with a snack bar and rest rooms is provided. The passenger from overseas passes, in order, first through the public health examination area, thence through the immigration examination rooms, and, finally, through the customs area. After clearing through customs, he walks along Pier B directly into the main lobby where he has the same opportunity to patronize concessions and other facilities as does the domestic passenger and uses the same facilities for transportation and communication.

Service Areas

On the floor above the main lobby is located a serving kitchen, a roof garden, banquet and conference rooms, business offices used by local concerns, additional roomettes and, as previously mentioned, the public promenade or observation deck. The third floor is devoted to appropriate offices for the Baltimore City Department of Aviation, its director and his assistants, the airport manager, the CAA, and the U. S. Weather Bureau.

The tower extends from the fourth to the ninth floor and is devoted to CAA activities, such as a chief controller's office, radio and radar rooms, and the control cab. From the control cab all parts of the airfield and terminal are visible. A utilities shaft, sealed off at every floor for fire control purposes, extends from the service floor up to the under side of the control cab. Ample space has been provided for the installation of any future developments in electronics or other electrical navigation aids. A large freight and passenger elevator runs from the service floor to the third floor, and a smaller elevator runs from the third floor up through the tower to the seventh floor.

The service floor, at apron level, is occupied by airlines operations offices, baggage handling facilities, the lower lobby unloading docks, merchandise receiving and storage rooms, a large kitchen for the preparation of in-flight meals and another section devoted to the preliminary preparation of foods to be finally prepared in the kitchens above. Refrigeration and air conditioning machinery are also found at this level, as well as truck unloading docks for air mail, air cargo, and air express, and adequate warehouse space. At the extreme end of Pier B is located an engine room for fire and crash equipment, plant quarantine rooms, and a bonded ware-room. At the present writing (1951) all rentable space is under lease.

Friendship Airport differs widely from any other airport now in existence. However, it has been designed and constructed as closely as practicable in accordance with the fundamental precepts outlined herein, and it is believed that it will be able to take care of the changing needs of aviation in Baltimore and its vicinity for many years to come.

ACKNOWLEDGMENT

Much of the credit for the successful completion of Friendship International Airport is due to businesslike administration by the Baltimore City Department of Aviation through two former directors, Maj.-Gen. Julian L. Schley, U.S.A., Retired, and Maj.-Gen. Cecil R. Moore, U.S.A., Retired (M. ASCE), and the present director, Maj.-Gen. Donald H. Connolly, U.S.A., Retired (M. ASCE). The master plan, design, and supervision of construction for Friendship Airport were conceived and carried out by Whitman, Requardt-Greiner Company and Associates, Consulting Engineers. The author formerly was an associate of this company and project engineer for the airport.

DISCUSSION

Charles O. Landrum[14].—To those who are familiar with the development of the Friendship International Airport, this paper will be recognized as very well balanced, in scope and in technical matter. This airport, which is of the inter-continental express type, is a major unit in the United States civil aviation system. Much of the airport itself, and a major part of the supplementary operations sections would apply specifically to a major airport project designed to handle more than 250,000 locally originated travelers per year.

Smaller airports could not justify the planning of such extensive terminal facilities and aircraft handling areas in the vicinity of the terminal. Their layout must be much more simple and less extensive—generally with only a one-level terminal building to accommodate both passengers and services

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[14] Cincinnati, Ohio.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2516

BASE COURSE DRAINAGE FOR AIRPORT PAVEMENTS

By A. Casagrande,[1] M. ASCE, and W. L. Shannon,[2] A. M. ASCE

With Discussion by Messrs. Edward S. Barber, D. P. Krynine, and A. Casagrande and W. L. Shannon

Synopsis

Base courses beneath airport pavements may become saturated under some conditions. The principal causes for base saturation were determined from observations on a number of airfields. An approximate theoretical analysis for the drainage of a saturated base course was developed and compared with the results of laboratory model tests and full-scale field tests. These comparisons show that the formulas which are proposed are satisfactory for the design of base drainage of wide pavements.

Introduction

It is generally recognized that saturated base courses are detrimental to the life of pavements because they are not as strong as drained bases. Water forced upward through the pavements at joints and cracks by traffic is known to produce early disintegration of bituminous mixtures and pumping at joints of concrete pavements. Therefore, base course drainage is often necessary to remove a major cause of failure in airport pavements. It is the purpose of this paper to analyze the conditions under which base courses become saturated and to develop and test a procedure for the design of base drainage.

Observations on Airport Pavements

To determine the conditions under which base courses may become saturated, observations were made from 1945 to 1947 by the Corps of Engineers, United States Department of the Army, at airfields in the continental United States. These observations showed that base courses of airfield pavements may become fully saturated under certain conditions. Most of the observations were

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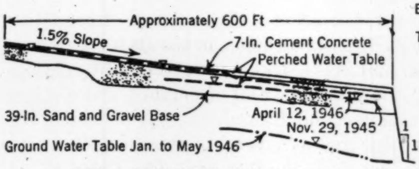
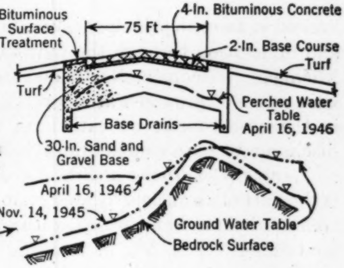
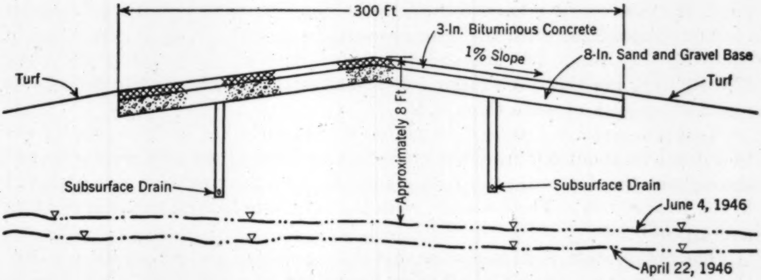

Approximately 600 Ft
1.5% Slope
7-In. Cement Concrete
Perched Water Table
39-In. Sand and Gravel Base
April 12, 1946
Nov. 29, 1945
1
1
Ground Water Table Jan. to May 1946

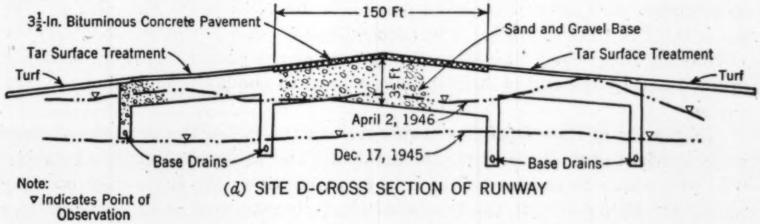
(*a*) SITE A-CROSS SECTION OF APRON


Bituminous Surface Treatment
75 Ft
4-In. Bituminous Concrete
2-In. Base Course
Turf
Turf
Perched Water Table April 16, 1946
Base Drains
30-In. Sand and Gravel Base
April 16, 1946
Nov. 14, 1945
Ground Water Table
Bedrock Surface

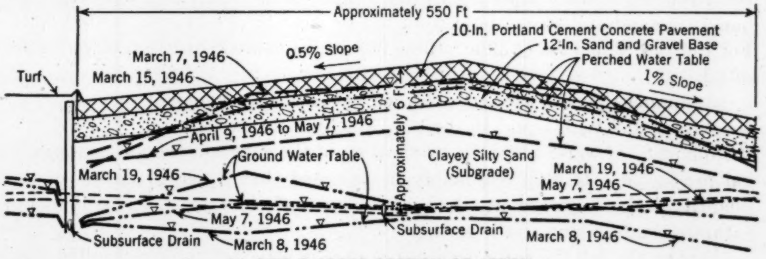
(*b*) SITE B-CROSS SECTION OF TAXIWAY


300 Ft
3-In. Bituminous Concrete
1% Slope
8-In. Sand and Gravel Base
Turf
Turf
Approximately 8 Ft
Subsurface Drain
Subsurface Drain
June 4, 1946
April 22, 1946

(*c*) SITE C-CROSS SECTION OF RUNWAY


150 Ft
3½-In. Bituminous Concrete Pavement
Sand and Gravel Base
Tar Surface Treatment
Tar Surface Treatment
Turf
Turf
3½ Ft
April 2, 1946
Dec. 17, 1945
Base Drains
Base Drains
Note:
▽ Indicates Point of Observation

(*d*) SITE D-CROSS SECTION OF RUNWAY


Approximately 550 Ft
10-In. Portland Cement Concrete Pavement
12-In. Sand and Gravel Base
Perched Water Table
March 7, 1946
0.5% Slope
1% Slope
Turf
March 15, 1946
Approximately 6 Ft
April 9, 1946 to May 7, 1946
Ground Water Table
Clayey Silty Sand (Subgrade)
March 19, 1946
March 19, 1946
May 7, 1946
May 7, 1946
Subsurface Drain
Subsurface Drain
March 8, 1946
March 8, 1946

(*e*) SITE E-CROSS SECTION OF APRON

FIG. 1.—PERCHED WATER LEVEL AND GROUND-WATER LEVEL AT OBSERVATION SITES

limited to the two principal causes for the saturation of base courses—frost action and infiltration through the pavement. At six airfields, in the states of Maine, Michigan, Wisconsin, North Dakota, and South Dakota, detailed observations were made of ground-water levels in the subgrade, and of perched water tables in the base courses beneath both concrete and bituminous pavements. The discharge of base course drains was measured at several of these fields.

General Conditions at Site A.—In Fig. 1(*a*) the test area designated site A is the part of an airfield located in northern Maine. The base course consists of a well-graded sand and gravel, with material larger than about 3 in. removed, and containing 3% to 5% of material smaller than a No. 200 mesh sieve. The coefficient of permeability of the base course material averages 27×10^{-4} cm per sec. No base drains are provided. However, on the low edge of the pavement, the base course is carried through the shoulder, thus permitting drainage.

The subgrade beneath the base course is fill, about 5 ft deep, consisting of compact, silty sand, and gravel, with a coefficient of permeability averaging 1×10^{-6} cm per sec. A similar soil underlies this fill, and shale bedrock is found at a depth of more than 20 ft.

Test pits excavated during the winter of 1945–46 indicated frost penetration to a depth of about 6 ft from the pavement surface. Ice lenses were observed throughout the depth of frozen subgrade, but no ice segregation was observed in the base course. The frost action in the subgrade heaved the pavement an average of 0.2 ft.

The observation wells indicated that during the fall and winter the depth of the ground-water table in the subgrade fluctuated from 7 ft to 12 ft below pavement surface. During the fall, a perched water table was observed in the base course, as shown in Fig. 1(*a*). In the spring as soon as the base had thawed, the observation wells again indicated a perched water table in the base and a rising water table in the subgrade. The perched water table in the base rose quickly as thawing progressed, almost to the surface of the pavement; then, slowly it drained and by the end of June it had dropped to about the middle of the base course.

Discussion of Base Drainage at Site A.—In the fall of the year the average water content of the base above the perched water table was found to be about 6%, which may be assumed to be held by capillarity. At an average unit dry weight of 140 lb per cu ft, the base would be fully saturated at an average water content of 8%. Thus, only 2% by dry weight would be required to change this bank-run sand and gravel base from the drained to the fully saturated state. For the base thickness of 39 in., these 2% are equivalent to 10 gal of water per sq yd.

Assuming that the average observed heave of the pavement surface during the winter (0.2 ft) is equal to the total thickness of ice lenses in the subgrade, the quantity of water liberated by thawing of the subgrade would be about 13 gal per sq yd. It may reasonably be assumed that a major portion of this quantity drained upward into the base. Thus, sufficient water was available to saturate the base during the spring.

During the fall the saturation of the base is believed to have resulted from infiltration of water through joints and cracks in the pavement. An estimate of the quantity of infiltration may be made by assuming that the joint width is 0.1 in. and that atmospheric pressure is maintained at the under side of the pavement. Assuming laminar flow, the quantity that would flow through the joint is computed to be 0.15 cu ft per sec per ft of joint length. At this rate and with a joint spacing of 10 ft, the base would become saturated in about 1 min. A similar computation using a joint width of 0.01 in. indicates that the base would become saturated in about 20 hr.

The preceding discussion indicates that saturation of the base at this site is possible, during the spring, by water from melting ice lenses in the subgrade, and that saturation can occur at other times by infiltration through joints and cracks in the pavement.

Observations at Site B.—At site B (Fig. 1(*b*)), also located in northern Maine, observations were made of the ground-water table and perched water table in the base course beneath a taxiway with a bituminous concrete surface. The base course and subgrade were similar to site A, except that the height of filled subgrade was less. Subsurface drains were provided at the edge of the pavement. At this site the highest and lowest positions of the ground-water table in the subgrade, over a period of one year, were above and almost parallel to the surface of the underlying bedrock. During the spring, immediately after the base course thawed, a perched water table was observed in the base. This water table gradually dropped, and within three weeks after thawing was complete, the base course was drained.

Frost action in the subgrade resulted in a heave of the pavement surface of about 0.25 ft. Over most of its area the bituminous concrete pavement was free from cracks. Therefore, it is believed that the perched water table in the base resulted from upward seepage of water from the thawing subgrade.

Observations at Site C.—Site C (Fig. 1(*c*)), located in Massachusetts, was a part of a bituminous concrete paved runway with a sand and gravel base constructed on a clean sand subgrade. Before construction, the ground-water table was within about 1 ft of the ground surface. During construction the water table was lowered by means of open ditches. Those ditches, located along runway edges, were later replaced with closed joint pipe with surface inlets, whereas those located in areas between runways were maintained as open ditches. In addition, subsurface drains were installed beneath the pavement as shown in Fig. 1(*c*) to control the ground-water table.

Observations of the ground-water table and of the subsurface drains during the spring of 1946 indicated that the ground-water table never rose above the invert of these drains, as shown by highest and lowest positions of the ground-water profiles in Fig. 1(*c*). For this subgrade (which has a coefficient of permeability of about 10×10^{-4} cm per sec) the open ditches in the areas between runways are apparently adequate to maintain a reasonably constant ground-water table. The nearest open ditch to the location of the observations was at a distance of about 1,000 ft.

Observations at Site D.—Site D (Fig. 1(*d*)), in central Maine, was a runway paved with bituminous concrete with a gravel-shoulder surface-treated with tar. The base course beneath the pavement and in the treated shoulder consists of a well-graded, slightly silty sand and gravel. The subgrade is a gray-blue, silty clay. At the location of the observations, the bottom of the base course is several feet below the original ground surface. Four lines of subsurface drains are installed.

Observations of the ground-water table in 1946 and 1947 indicated that during the summer, fall, and winter the water table became progressively lower, reaching the lowest position in December. During the spring, as the subgrade was thawing, a water table was observed in the base course, as shown for April 2 in Fig. 1(*d*). In the shoulder area the water table sometimes rose as high as the ground surface. When the subgrade was completely thawed, the ground-water table, for a short period, was at about the same elevation as the water table in the base during thawing. Then the ground-water table gradually dropped, and by August both the base course beneath the pavement and the surface-treated shoulder was substantially drained.

During the spring, the water in the base in the shoulder areas frequently stood at a higher elevation than in the base beneath the paved area, as shown by the ground-water surface for April 2. In addition a close correlation was observed between precipitation and discharge of subsurface drains in the shoulder. Therefore, it may be concluded that the high water table beneath the shoulder area was due largely to the infiltration of precipitation into the base through the bituminous surface treatment. On the other hand, it is believed that the water in the base beneath the paved area is principally water released by melting ice in the subgrade, augmented perhaps by some infiltration through cracks in the pavement.

Observations at Site E.—Site E(Fig. 1(*e*)), in Michigan, consists of a part of a Portland cement concrete apron with a sand and gravel base course and a clayey-silty sand subgrade. Subsurface drains extend around the periphery and beneath the apron.

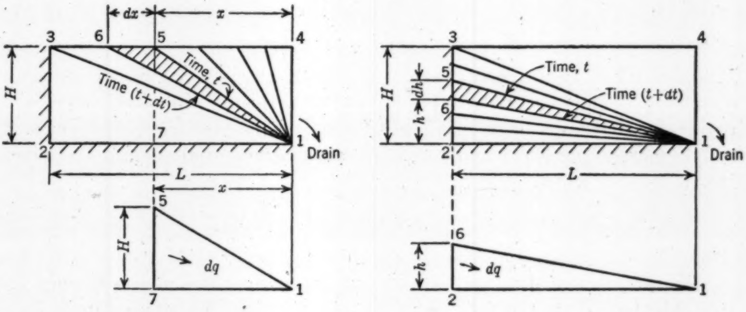
Observations of the water table in the subgrade indicated that it varied from slightly above the elevation of the subsurface drains to the elevation of the lowest invert. The water table was highest following the frost-melting period. No ice segregation was observed in the subgrade; however, heaving of the pavement surface, averaging 3/4 in., was measured. A perched water table was observed in the base course during the thawing period, as shown for March 7 and March 15. Therefore, it is believed that most of the water in the base during the thawing period must have been caused by infiltration of surface water through joints and cracks in the pavement. Actually there was an almost continuous flow of water across the pavement during this period, resulting from the melting of a large snow bank piled near the crown of the apron.

Conclusions from Field Observations.—Based on these and other observations it was concluded that during the thawing period ice segregation in a subgrade may be the cause of saturation of an overlying, free-draining base. It was also concluded from some of the preceding observations that infiltration of surface water through pavement cracks, joints, or through a pervious pavement, such as certain types of penetration macadam pavements, may cause saturation of a free-draining base overlying a relatively impervious subgrade. Other causes for the saturation of bases may be inundation of the pavement, such as might occur at an airport located in an area that might become flooded, or where the natural water table may rise above the bottom of the base course.

THEORETICAL ANALYSIS OF BASE DRAINAGE

Drainage of a saturated base is essentially a problem in nonsteady flow (also called transient flow) with a free surface. Present development of mathematical tools does not permit a rigorous solution of such problems. Therefore, any attempt at a mathematical solution must be based on simplifying assumptions.

Analysis of Horizontal Base.—The assumptions on which the mathematical analysis is based are illustrated in Fig. 2. The center line of the base course and the bottom of the base course are considered replaced by impervious boundaries as shown by the cross-hatched sides of the rectangle. Open discharge is assumed at the right side. At the start of drainage (elapsed time of zero) the base course is assumed saturated, and the right side suddenly opened for free drainage. The free water surface is assumed to remain a straight line that changes with time, as illustrated.



(*a*) *U* EQUAL TO OR GREATER THAN 50%

(*b*) *U* EQUAL TO OR LESS THAN 50%

FIG. 2.—ASSUMED PROGRESS OF FREE WATER SURFACE—HORIZONTAL BASE

For the purpose of analysis, the drainage process is divided into two parts: (1) the first part, Fig. 2(*a*) in which the free surface gradually changes from position 1–4 to position 1–3; and (2) the second part, Fig. 2(*b*) in which the free surface changes from position 1–3 to position 1–2. For the first part of the drainage process, the differential equation can be set up by considering the position of the free surface at elapsed time *t* and then at time $(t + dt)$. In the time element dt, the quantity dq discharged per unit width is equal to the area of the narrow, shaded triangle 1–5–6, multiplied by the effective porosity n_e. The effective porosity n_e is the ratio of the volume of voids that can be drained to the total volume of soil. It is assumed constant, that is, independent of the height above the impervious boundary.

$$dq = \frac{H\, n_e}{2}\, dx \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The flow through volume 1–5–7 is computed by means of Darcy's law. The simplest assumption that could be made is to use $H/2$ as the average area per unit of width through which flow takes place and to assume an average or effective hydraulic gradient of H/x. Then the rate of flow could be expressed by

$$\frac{dq}{dt} = k\,\frac{H}{2}\,\frac{H}{x} = k\,\frac{H^2}{2\,x} \qquad (2a)$$

In this form, the equation is identical with Dupuit's formula for steady seepage through a homogeneous earth dam.

A greater flexibility can be introduced in the solution of this problem by using an effective hydraulic gradient equal to $\frac{H}{c_1\,x}$ and an average area per unit of width equal to H/c_2, in which c_1 and c_2 are quantities that will be assumed constant during the entire drainage process. Substituting $c = c_1\,c_2$, one obtains

$$\frac{dq}{dt} = k\,\frac{H^2}{c\,x} \qquad (2b)$$

Whereas in Eq. 2*a* the constant c is equal to 2, a known—or rather assumed—quantity, it should be noted that in Eq. 2*b* the quantity c represents an unknown parameter that must be determined by other means—for instance, from model tests. Combining Eq. 1 and Eq. 2*b*, a simple differential equation is derived, the solution of which leads to

$$t = \frac{c\,n_e\,x^2}{4\,k\,H} \qquad (3)$$

The progress of drainage is best defined as the ratio of the drained area (triangle 1–4–5) to the total area (rectangle 1–2–3–4). This dimensionless ratio will be called the degree of drainage

$$U = \frac{\text{Drained Area}}{\text{Total Area}} \qquad (4)$$

or expressed as a percentage: $U\% = 100\,U$. In addition to U, it is expedient to introduce another dimensionless quantity

$$T = \frac{2\,t\,k\,H}{c\,n_e\,L^2} \qquad (5)$$

that will be called the time factor.

Introducing U and T in Eq. 3 gives the formula for the progress of drainage:

$$T = 2\,U^2 \qquad (6)$$

Eq. 6 is valid for $U \leqq 0.5$ (or $\leqq 50\%$). By a similar derivation one obtains the solution for the second half of the drainage process, for which the variable triangle 1–2–6 has a constant base length L and a variable height h (Fig. 2(*b*)).

The starting equations are:

$$dq = -\frac{L\,n_e}{2}\,dh \tag{7a}$$

$$dq = k\,\frac{h}{2}\,\frac{h}{x}\,dt = k\,\frac{h^2}{2\,x}\,dt \tag{7b}$$

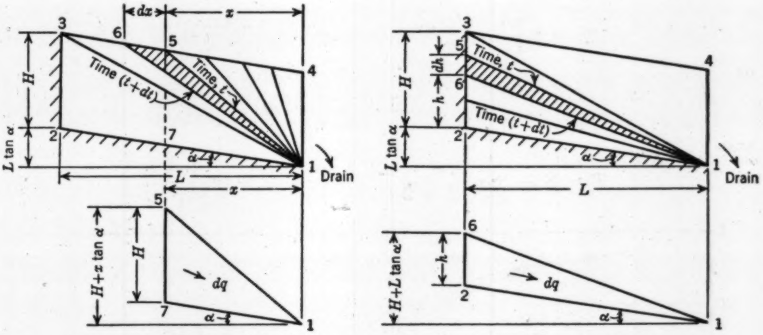
and

$$dq = k\,\frac{h^2}{c\,x}\,dt \tag{7c}$$

The final solution can be expressed by the following relation between the dimensionless quantities T and U

$$T = \frac{U}{2 - 2\,U} \tag{8}$$

Eq. 8 is valid for $U \geqq 0.5$ (or $\geqq 50\%$).


dx
x
3
6
5
H
Time (t+dt)
Time, t
4
L tan α
2
7
α
1
Drain
L
x
5
H+x tan α
H
dq
7
α
1
3
5
dh
H
6
h
Time, t
4
Time (t+dt)
L tan α
2
α
1
Drain
L
6
H+L tan α
h
dq
2
α
1

(*a*) U EQUAL TO OR GREATER THAN 50%

(*b*) U EQUAL TO OR LESS THAN 50%

FIG. 3.—ASSUMED PROGRESS OF FREE WATER SURFACE—SLOPING BASE

Analysis of Sloping Base.—The assumptions on which the following analysis is based are similar to those made for the analysis of the horizontal base and are illustrated in Fig. 3. The mathematical derivation, however, is not quite as simple. For the first half of the drainage process (Fig. 3(*a*)) the starting equations are

$$dq = \frac{H\,n_e}{2}\,dx \tag{9a}$$

$$dq = k\,\frac{H}{2}\,\frac{H + x\tan\alpha}{x}\,dt = k\,\frac{H^2 + H\,x\tan\alpha}{2\,x}\,dt \tag{9b}$$

and

$$dq = k\,\frac{H^2 + H\,x\tan\alpha}{c\,x}\,dt \tag{9c}$$

To simplify the presentation of the solution, it is desirable to introduce (in addition to T and U as previously defined) a third dimensionless quantity named the slope factor, which is defined by

$$S = \frac{H}{L \tan \alpha} \ldots\ldots\ldots\ldots\ldots\ldots \tag{10}$$

in which α is the angle of slope of the base layer.

The final solution can be expressed by the following relationship among the dimensionless quantities T, U, and S:

$$T = 2\,U\,S - S^2\, ln \frac{S + 2\,U}{S} \ldots\ldots\ldots\ldots\ldots\ldots \tag{11}$$

which is valid for $U \leqq 0.5$ (or $U \leqq 50\%$). By a similar derivation (Fig. 3(*b*)) the solution for the second half of the drainage process is obtained. The starting equations and the final solution are as follows:

$$dq = -\frac{L\, n_e}{2}\, dh \ldots\ldots\ldots\ldots\ldots\ldots \tag{12a}$$

$$dq = k \frac{h}{2} \frac{h + L \tan \alpha}{L}\, dt = k \frac{h^2 + h\, L \tan \alpha}{2\, L}\, dt \ldots\ldots\ldots \tag{12b}$$

or

$$dq = k \frac{h^2 + h\, L \tan \alpha}{c\, L}\, dt \ldots\ldots\ldots\ldots\ldots\ldots \tag{12c}$$

and

$$T = S + S\, ln \frac{2\,S - 2\,U\,S + 1}{(2 - 2\,U)\,(S + 1)} - S^2\, ln \frac{S + 1}{S} \ldots\ldots\ldots \tag{13}$$

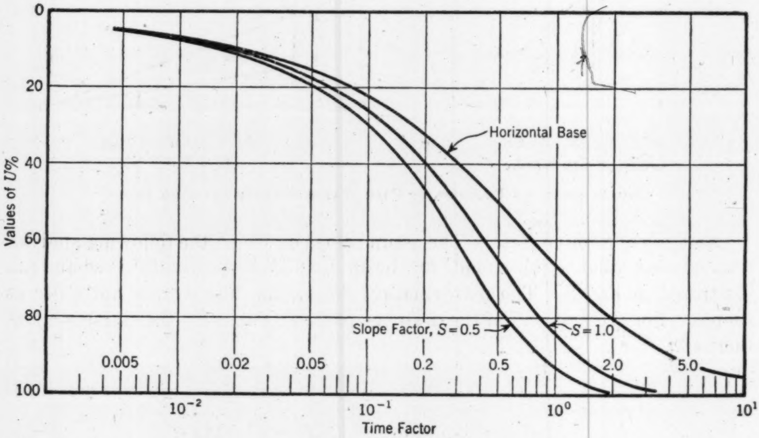


FIG. 4.—TYPICAL TIME CURVES

Discussion of Theoretical Results.—Eqs. 6 and 8 are shown graphically for the horizontal base in Fig. 4; and Eqs. 11 and 13 are shown for an inclined base having a slope factor of $S = 0.5$ and 1.0. For the most purposes it is convenient to plot such U versus T curves on semilogarithmic paper. Since the time factor

(T) is directly proportional to the time (t), the effect of a slope on the rate of drainage can be readily compared in Fig. 4. For example, a sloping base, with a slope factor of $S = 0.5$, requires about one half the time of a horizontal base to obtain 50% drainage, and about one quarter of the time to reach 90% drainage.

It should be noted that, for a horizontal base, one U versus T curve represents the solution for all possible combinations of k, n_e, H, L, and c, since all these variables, and also the time t, are contained in the time factor T. To use theoretical T versus U curve for an actual case, one need only transform the T-scale into a time scale using Eq. 5.

LABORATORY MODEL TESTS

In the past the flow of a viscous fluid between parallel plates has been used to study problems of steady seepage. At first the application of this principle to problems of nonsteady flow was found unsatisfactory because any viscous fluid will adhere to the glass plates. By extensive experimentation this obstacle was overcome in the following manner:

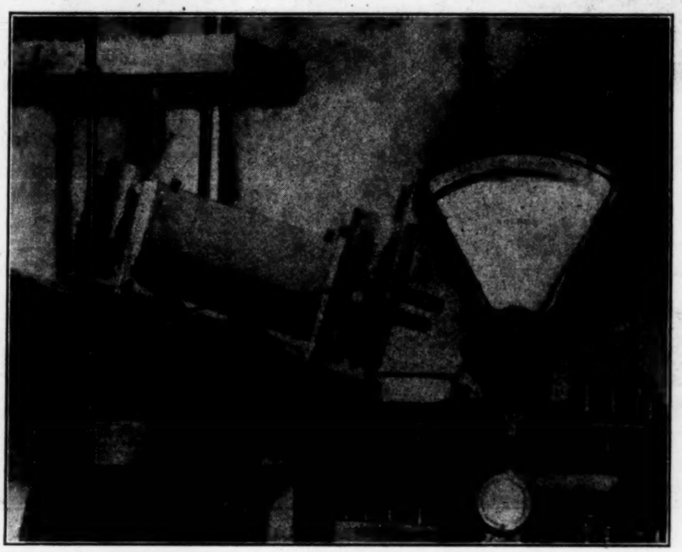


FIG. 5.—CONSTANT VELOCITY APPARATUS FOR PERMEABILITY TESTS

(*a*) The glass plates were covered with a very thin film of a soft grade of paraffin, by dipping the plates into melted paraffin and withdrawing them quickly after the plates had reached approximately the temperature of the paraffin. Then the paraffin was removed from the outside surface to increase visibility.

(*b*) Glycerine was used as viscous liquid, and about 10 gm of sodium hydroxide was added to each 100 cc of glycerine to increase the viscosity; a small amount of phenolphthalein was used to produce a light red color.

The models consisted essentially of two parallel glass plates with an inside space of ¼ in., mounted on a tilting frame. The bottom and the left end of these models were closed, and drainage was permitted through the right end into a vessel that rested on an accurate indicating scale. Model dimensions and fluid viscosity were so controlled that flow was laminar at all times. By proper lighting of these models, the surface of the colored glycerine could be made clearly visible through the coated plates. The glycerine was prevented from discharging during filling by a temporary stop at the open end. After the model was filled, it was inclined quickly to the desired slope, and the temporary stop was removed.

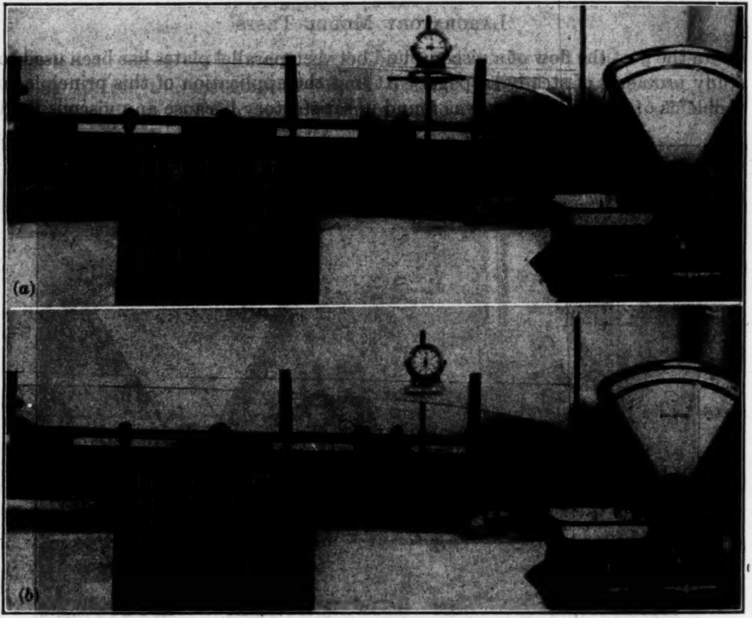


Fig. 6.—Viscous Fluid Model Test

(a) 15 Seconds After Start of Drainage

(b) 2 Minutes After Start of Drainage

During a test photographs were taken periodically, and measurements were made of the weight of the drained fluid, the position of the free surface, and the temperature of the fluid. In addition, samples of the drained fluid were taken and tested for viscosity. The coefficient of permeability that was effective in the model tests was determined by means of direct permeability tests using two different types of apparatus. Most of the tests were performed on the constant velocity type apparatus shown in Fig. 5, the plate treatment and spacing being the same as that used in the models. Several tests were performed on a free discharge apparatus similar to that shown in Fig. 6 with the left side connected to a reservoir in which a constant head was maintained. For each permeability test the Saybolt-Fural viscosity was determined on a sample of the fluid taken from the discharge. The relationship between velocity and hydraulic gradient, as obtained from these tests and reduced to a constant viscosity, is plotted in Fig. 7. The values given are for a plate spacing of 0.635 cm and have been reduced to a constant Saybolt-Fural viscosity of 400 sec per 60 cc. The practically linear relationship between hydraulic gradient (*i*) and velocity (*v*) shows that the coefficient of permeability is independent of the hydraulic gradient and that the velocity is defined by $v = k\,i$ in which k is a constant of proportionality. In other words, the law that governs the flow in these models is identical with Darcy's law.

Since it proved impossible to maintain a constant viscosity for the fluid used in all model tests, the viscosity had to be determined simultaneous to each model test on a sample taken from the discharge. The corresponding coefficient of permeability was then read from Fig. 8, which shows the relation between the viscosity and the reciprocal of the coefficient of permeability.

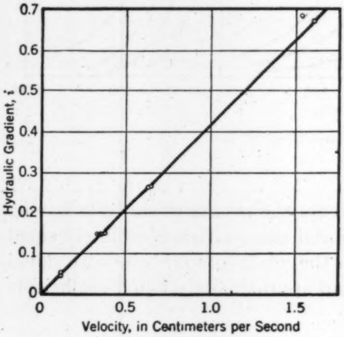


FIG. 7.—TEST RESULTS OBTAINED WITH CONSTANT-VELOCITY PERMEABILITY APPARATUS

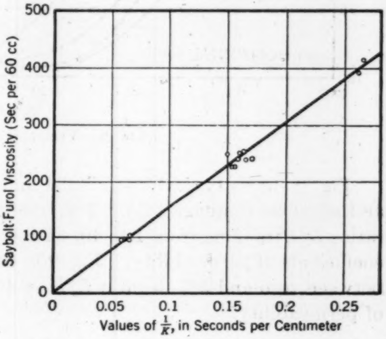


FIG. 8.—RELATION BETWEEN VISCOSITY AND PERMEABILITY

Although it would appear from Fig. 7 that the flow between the treated glass plates follows strictly the law of laminar flow with zero velocity along the walls, it is believed that the boundary velocity is not actually zero. This is concluded from the manner in which a film of the fluid will flow off the paraffin-covered surface, showing that there is no affinity between the fluid and the paraffin. The influence of this boundary velocity on the relationship between velocity and hydraulic gradient would merit an accurate investigation over a much larger range of hydraulic gradient.

To study the effect of base dimensions on drainage, models having a length-to-height ratio (L/H) varying from 1 to 40 were tested. Most of the models had a height of 6 in., and the largest model had a length of 20 ft. Longer models were impracticable to construct in the laboratory. The models were tested with slopes varying between zero and 5%.

Photographs of a typical test, taken at two different elapsed times from start of drainage, are shown in Fig. 6. It was found convenient to present the test results by plotting values of $U\%$ versus the logarithm of time. Corresponding to the definition used in the theoretical analysis, $U\%$ is taken as the ratio (in percentage) of the volume of fluid drained after a given elapsed time to the total volume of fluid in the model at the beginning of the test.

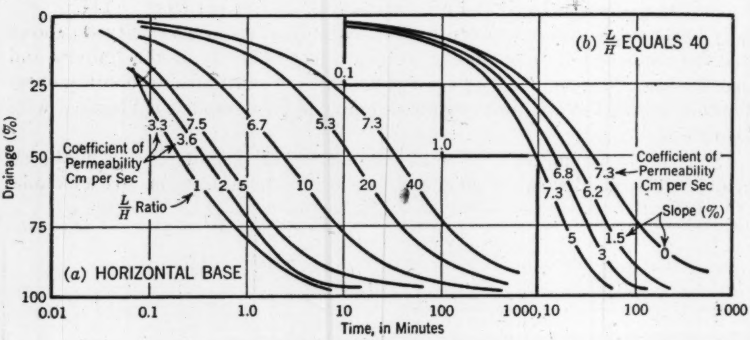


FIG. 9.—TYPICAL DRAINAGE CURVES

Fig. 9 shows typical test results and illustrates the effect of the ratio L/H and slope on drainage (%). Fig. 9(*a*) presents the results of six tests having ratios L/H ranging from 1 to 40, with horizontal base and approximately equal coefficients of permeability. Fig. 9(*b*) shows the results of four tests with slopes between zero and 5%, a ratio L/H of 40, and approximately equal coefficients of permeability.

FULL-SCALE TESTS

In addition to laboratory models, a full-scale field test installation was built in which base drainage under controlled conditions was studied. The test area consisted of four sections, numbered 1 to 4, each 10 ft wide, 75 ft long, and sloping 1.5%. The base materials and thicknesses used are listed in Table 1. Each test section was so constructed that its sides, top, bottom, and upper end were impervious. The impervious top was weighted with sand and gravel to resist the uplift forces during saturation of the base. At the lower end a drain was provided. Wells were provided to saturate the base and to observe the elevation of the water surface in the base during drainage tests.

The operation of a test section consisted of saturating the base with the drain closed, using water that was passed through a filter for de-airing, stopping the inflow of water, and simultaneously opening the drain. During drainage the quantity of water discharged, the elevation of the water surface at numerous locations in the base, and the temperature of the discharged water were measured. A typical profile for field test section 4, showing the water surface at various elapsed times, is shown in Fig. 10.

TABLE 1.—DATA FROM FIELD TESTS (SEE FIG. 15)

Test section No.	Base material	Slope (%)	COEFFICIENT OF PERMEABILITY, k, IN CENTIMETERS PER SECOND					EFFECTIVE POROSITY n_e		
			Free Discharge Tests			Constant Velocity Tests[a]		Range		
			Average	Range		Depth (in.)	k	Average	From:	To:
				From:	To:					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
(a) SIX-INCH BASE: $H = 0.5$ FT; AND $L = 75$ FT										
1	Washed pea gravel	1.5	13	10	16	....	....	0.40	0.31	0.62
2	Sandy gravel	1.5	0.13	0.11	0.15	....	....	0.03	0.02	0.04
(b) EIGHTEEN-INCH BASE: $H = 1.5$ FT; AND $L = 75$ FT										
3	Washed pea gravel	1.46	16	13	21	....	....	0.43	0.41	0.46
4	Sandy gravel	1.63	0.27	0.21	0.33	15 9 4	0.33 0.074 0.027	0.07	0.05	0.09

[a] Coefficient k, in centimeters per second, for the different depths of flow, in inches.

For a comparison with the theory, it was necessary to determine the coefficient of permeability of the base materials and their effective porosity. Determinations of coefficient of permeability were made by direct testing of the base. A trench was excavated the full width of the base at the upper end of each section, and de-aired water was introduced into the test section at this point until a steady state of flow was obtained. Two methods were used for performing the permeability test, these methods being the same in principle as the two methods used for the detemination of the coefficient of permeability for the laboratory models.

In the free discharge tests the coefficient of permeability was computed by means of two different procedures. One procedure was based on flow-net analysis, and the other method consisted of the use of a simplified formula in which an average hydraulic gradient and average cross section of flow was substituted in Darcy's law. Both procedures gave similar results, which are summarized in Table 1.

Constant velocity tests were performed on Section 4. The average values for the coefficient of permeability, as computed from these tests, are also summarized in Table 1. It should be noted that the coefficient of permeability shows a large decrease with a decreasing depth of flow. It is believed that this difference is due principally to downward segregation of fine particles during construction as well as during the drainage tests. In addition, the lower layers have received a greater degree of compaction.

The effective porosities of the base material in each test section, computed from the drainage tests, are listed in Table 1.

COMPARISON OF THEORY WITH RESULTS OF EXPERIMENTS AND FIELD OBSERVATIONS

Comparison with Laboratory Experiments.—In Fig. 11 are shown the semilogarithmic plots of the theoretical curves for zero slope and for a slope factor of 0.5. For comparison, the observations from fluid model tests No. 1 and No. 4 with a ratio of $L/H = 20$ are also given. For these laboratory tests a plate spacing of 0.635 cm was used, the height (H) was 15 cm, the base width (L) was 609 cm, and the value of k was 7.3 cm per sec. The theoretical curves are matched through the 50% point. From this and numerous other comparisons between the theoretical and experimental results, it is concluded that Eqs. 6, 8, 11, and 13 match closely the shape of the experimental curves. The deviations are small and are probably of the same order of magnitude as the experimental error.

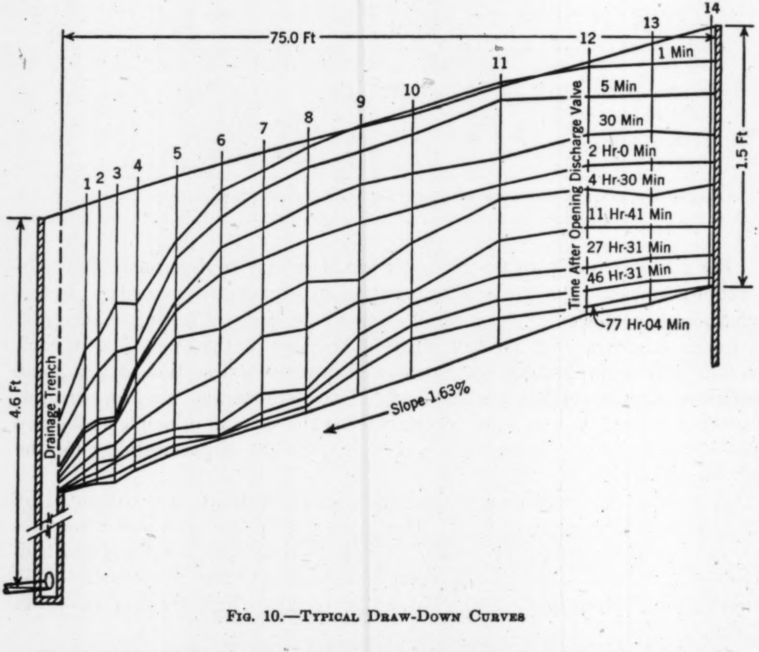


FIG. 10.—TYPICAL DRAW-DOWN CURVES

The position of the time curves on a semi-logarithmic plot is determined by the parameter c which was introduced in Eq. 2*b*. By comparing the time factor for 50% drainage with the actual time required for drainage, using Eq. 5, one can compute the parameter c that best represents given test results. For the two examples shown in Fig. 11, the parameters (c) of the observed curves are 1.5 and 2.4. A satisfactory average value for the conditions prevailing in airfield design is a value of c equal to 2, which happens to coincide with the value that had been recommended for design purposes before model tests were available.

In Fig. 12 the actual shape of the free surface corresponding to 50% drainage is compared with the assumed straight-line surface. As can be seen, the shape of the draw-down curve at any elapsed time differs radically from the straight line assumed in the theoretical analysis. By means of such plots of flow nets for various stages of this nonsteady flow problem, it became apparent that the shape of the actual draw-down curve could best be approximated by ellipses.

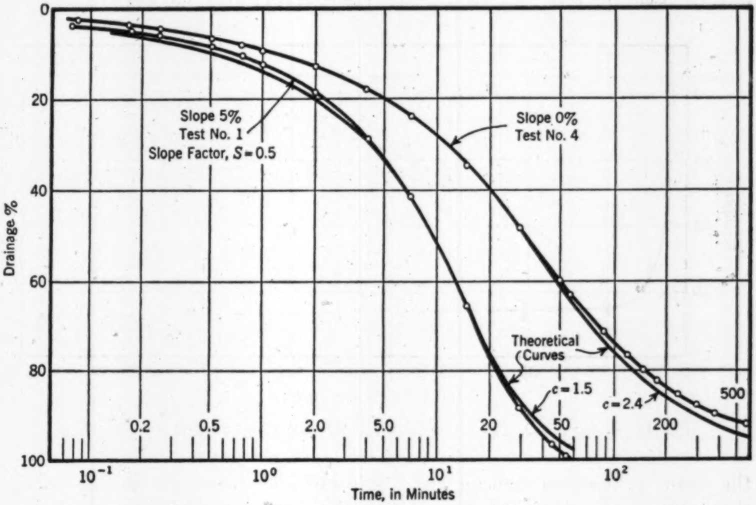


Fig. 11.—Comparison of Theoretical Drainage Curves with Laboratory Test Results

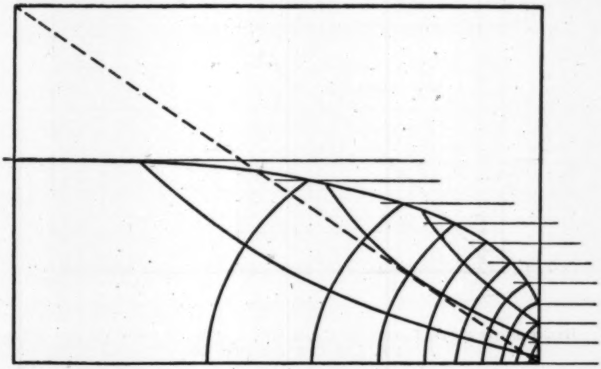


Fig. 12.—Comparison of Actual Free Surface with Assumed Free Surface for 50% Drainage

Louis A. Pipes solved this problem using an elliptical shape for the free surface. He found that the formulas resulting from such an assumption were essentially identical with the formulas derived on the basis of a straight-line assumption.

This paradoxical result can be understood by comparison with the simple Dupuit formula for the quantity of seepage through an earth dam. Although Dupuit's formula is based on a radical simplification of the free surface, quantitatively it checks very well with the few rigorous solutions that are available for the steady problem of seepage with a free surface. Generally, it may be said that whenever quantities of seepage are to be computed, even a rough approximation of the free surface will yield a good approximation of the quantity of seepage.

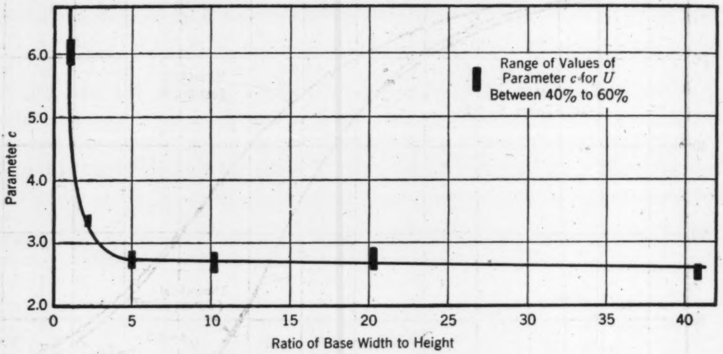


FIG. 13.—PARAMETER c COMPUTED FROM LABORATORY TESTS WITH HORIZONTAL BASE

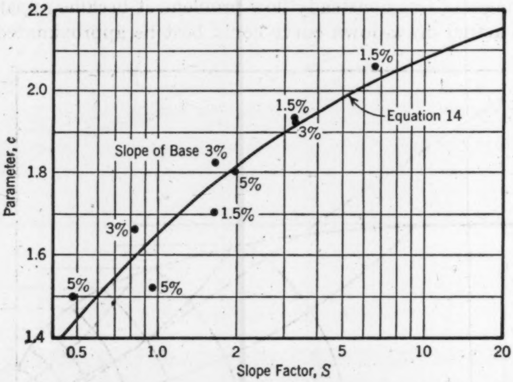


FIG. 14.—RELATION OF SLOPE FACTOR TO PARAMETER c DERIVED FROM VISCOUS MODEL TESTS, FOR L/H GREATER THAN 10

More detailed information on the relation between the ratio of base width to height, L/H, and the parameter c is shown in Fig. 13 as derived from horizontal viscous-fluid model tests, and in Fig. 14 from tests on models with different slopes. In these figures the values of c were determined in each case as the arithmetic average of the values of c for 40% drainage, 50% drainage, and 60% drainage. By using such an average, it is believed that a better approximation is obtained than by taking only one value that may be inordinately influenced by an individual error in reading.

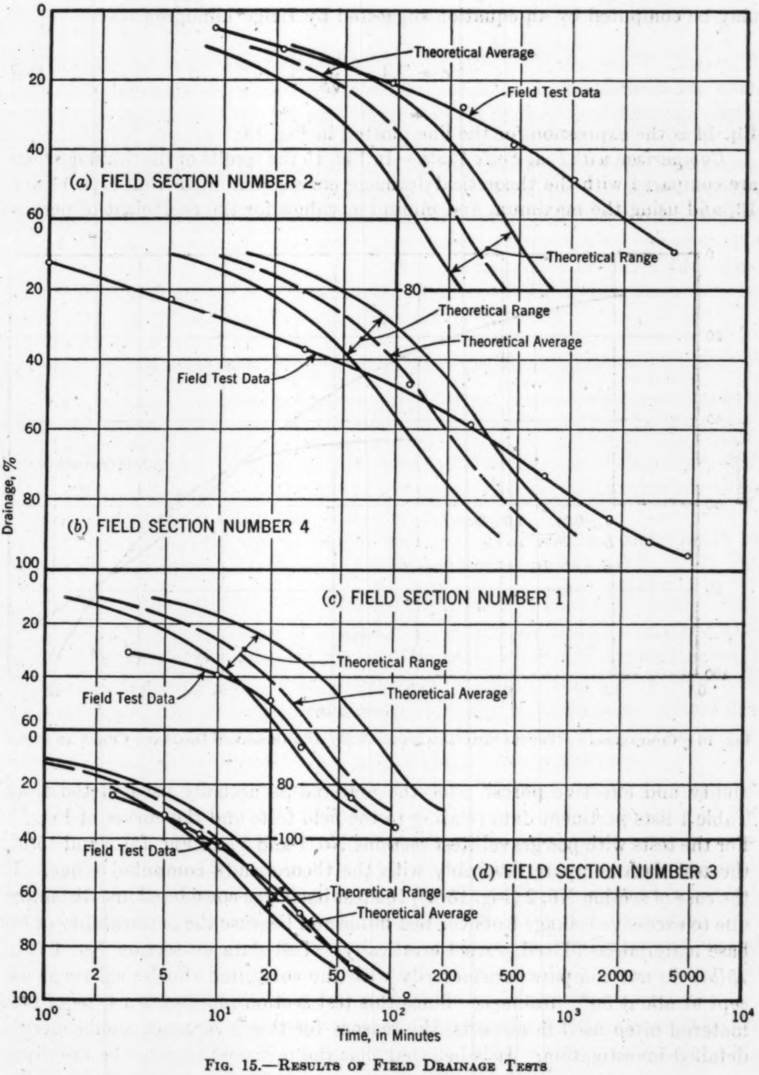


FIG. 15.—RESULTS OF FIELD DRAINAGE TESTS

In Fig. 13 it can be seen that for horizontal models and for ratios L/H greater than 5, the parameter c is essentially constant. For sloping models it is evident in Fig. 14 that the variable determining the parameter c is not the slope itself but the slope factor (S). Within the range of slope factors that are applicable in the design of airfields, the parameter c varies between 1.5 and 2.4, and it may be computed by an equation suggested by Helge Lundgren.

$$c = 2.4 - \frac{0.8}{\sqrt[3]{S}} \quad \text{......................} \quad (14)$$

Eq. 14 is the expression for the line plotted in Fig. 14.

Comparison with Full-Scale Tests.—In Fig. 15 the results of the drainage tests are compared with the theoretical drainage curves computed from Eqs. 11 and 13, and using the maximum and minimum values for the coefficient of permeability and effective porosity for the material as actually used in the tests. Table 1 lists pertinent data relating to the field tests and the curves of Fig. 15. For the tests with pea gravel (test sections No. 1 and 3 and Fig. 15(c) and 15(d)) the test data compare favorably with the theoretically computed range. In the case of section No. 2 (Fig. 15(a)) the test itself was considered unsatisfactory due to excessive leakage from the test flume and because the permeability of the base material, as placed, varied erratically. Test data on section No. 4 (Fig. 15(b)) do not compare satisfactorily with the computed theoretical range except at about 50% drainage. Since this test section represents a type of base material often used in airports, the reasons for this divergence would merit a detailed investigation. It is believed that the principal reasons for the divergence are the fact that the coefficient of permeability decreases with decreasing

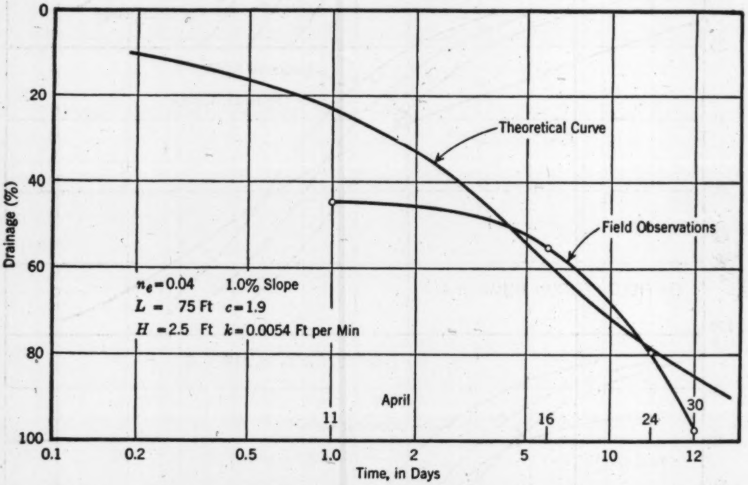


FIG. 16.—COMPARISON OF THEORETICAL DRAINAGE CURVE AND OBSERVED DRAINAGE CURVE AT SITE B

average depth of flow (see Table 1) and the possibility that placement and compaction has resulted in stratification of the base. To some extent the appreciable thickness of the zone of capillary saturation also contributes to the divergence.

It should be noted that the divergence between theoretical and experimental results was found only in the well-graded sand and gravel that also contained a slight quantity of silt. The tests on washed pea gravel showed excellent agreement.

Comparison with Field Observations.—At site B, described in section 2, sufficient field data were obtained to construct a time-drainage curve and compare it, as shown in Fig. 16, with computed theoretical time-drainage curves using Eqs. 11 and 13. At site A, the base was fully saturated on April 16 and during the following forty days, 35% drainage of the base occurred. Using Eq. 11 the computed time for 35% drainage is about 50 days. For these two sites the agreement between observed and computed values is considered reasonable. For sites D and E, comparisons between observed and computed values are not possible because of complicated subsurface drainage and probable additional water supply by infiltration through the pavement while drainage was in progress. At site C the base did not become saturated.

RECOMMENDATIONS FOR DESIGN OF BASE DRAINS

As a result of the investigations reported herein, the following recommendations are made, and base drainage should be required at:

1. Locations where considerable ice segregation in the subgrade is anticipated;
2. Locations where infiltration of surface water through a pavement may be expected to result in saturation of the base;
3. Locations where the pavement may become inundated occasionally or where the ground-water table may rise into the base course; and
4. At the low points of longitudinal grades sloping more than 2%.

Generally, a base course is not liable to become saturated where the subgrade is relatively pervious, except in those cases where ice segregation may develop in the subgrade. The requirement for base drains may be modified at locations where subsurface drains are provided to lower a ground-water table in the subgrade or to intercept water flowing in a pervious zone. Such drainage should be designed to provide also for drainage of the base course.

The principal factors that determine drainage design are the dimensions, the slope, the coefficient of permeability of the base, and the spacing of the drains. In the design, these factors may be varied to determine a practical solution. For example, it may be more economical to use a poorly draining base material combined with a close spacing of the drains than a more pervious but more expensive base material combined with much wider drain spacing.

For wide aprons and runways it may be necessary to provide several lines of base drains. However, for the usual runway and taxiway pavements, base drains at pavement edges, only, will usually prove adequate.

If base drains are to be installed, in the absence of special requirements it is recommended that these drains be so designed that the time required for 50%

drainage of the base course is not more than about 10 days. This recommendation is based primarily on the writers' investigations and experience. It is believed that base drainage designed in accordance with such a rule will be effective, yet not excessively costly. With this rule one of the following two procedures may be used:

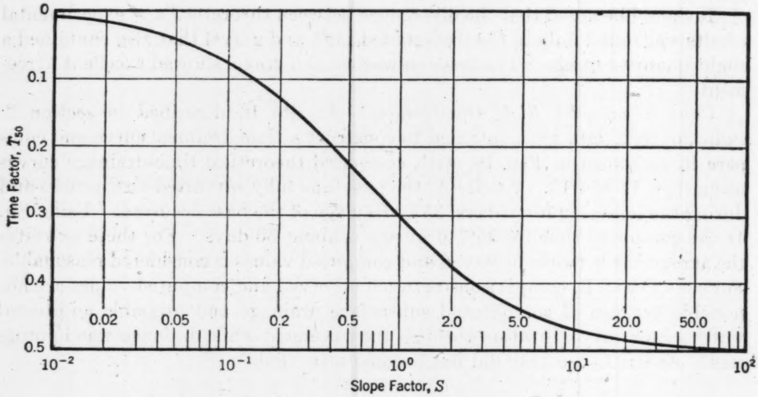


FIG. 17.—RELATION BETWEEN SLOPE FACTOR AND TIME FACTOR FOR 50% DRAINAGE

Procedure I.—Fig. 17 is a plot of the relation between the time factor T_{50} and the slope factor S obtained from Eq. 11. This plot permits direct reading of the value of T_{50} for any given slope factor S. Using this value of T_{50}, the corresponding time (t_{50}) is then computed from

$$t_{50} = \frac{c\, n_e\, T_{50}\, L^2}{2\, k\, H} \qquad \text{(15a)}$$

in which the parameter c is taken from Fig. 14.

Procedure II.—For most practical purposes a further simplification is permissible by use of the following equation:

$$t_{50} = \frac{n_e\, L^2\, S}{2\, k\, H\, (S + 1)} = \frac{n_e\, L^2}{2\, k\, (H + L \tan \alpha)} \qquad \text{(15b)}$$

For the working range of H, L, and slope generally used in airfield design Eq. 15*b* was found to give within about ± 10% the same result as Procedure I. Eq. 15*b* has the advantage that it does not contain the empirical parameter c and that no charts are needed. The degree of approximation of Eq. 15*b* is illustrated in the following:

Slope factor, S	Ratio, R
0.2	0.94
1.0	1.00
10.0	1.14

in which, for the working range of slope factors S, the corresponding ratio—

$$R = \frac{t_{50} \text{ from Eq. 13}}{t_{50} \text{ from Eq. } 15b} \quad \cdots\cdots\cdots\cdots \quad (16)$$

—is correlated.

Example

An application of the recommended design procedure is illustrated in the following example:

Runways 300 ft wide, for heavy airplanes, are to be built on a soft clay subgrade, as shown by the cross section in Fig. 18. Because of local conditions occasional saturation of the base is considered possible. The base course, 4.5 ft thick, consists of sand and gravel with an average coefficient of permeability of $k = 0.01$ ft per min and an effective porosity, $n_e = 0.1$.

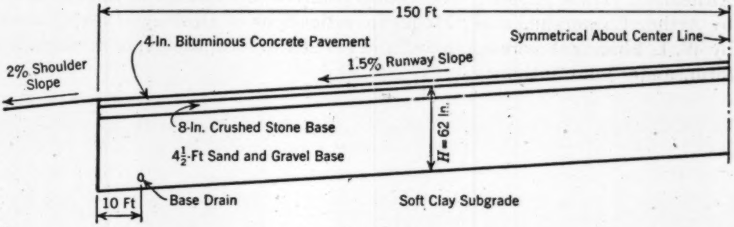


FIG. 18.—EXAMPLE OF DESIGN OF BASE DRAINS ALONG EDGE OF RUNWAY

The rule of 50% drainage in 10 days is assumed for the design of the base drainage. Because of the thickness of the base, the drains may be placed in the base, near the bottom, as shown in Fig. 18, thus making $L = 140$ ft. For the drainage analysis the depth of the base is assumed to be the thickness of the sand and gravel and crushed stone ($H = 62$ in.). The coefficient of permeability for the entire layer is assumed equal to that of the sand and gravel, an assumption which is on the conservative side.

To determine the suitability of the section, compute from Eq. 10, $S = 2.46$; from Fig. 17, $T_{50} = 0.395$; from Fig. 14, $c = 1.85$; and from Eq. 15a, $t_{50} = 9.6$ days. Hence, two lines of base drains, each 10 ft inside the pavement edge as shown in Fig. 18, are considered satisfactory.

During construction and before the pavement is placed, the base drains will drain water infiltrating into the base due to precipitation. Complete saturation of the base will probably result during each period in which the precipitation exceeds the storage in the base. Therefore, the question is pertinent as to how quickly construction operation can be resumed after a rain storm. It is considered reasonable to assume that construction can proceed when the base is only 10% drained. For instance, if the base is partly constructed to a depth of 2 ft, 10% drainage would require about 29 hr, as computed using Eqs. 11 and 5. Hence, the proposed drains are believed to provide satisfactory drainage of the base during construction.

ACKNOWLEDGMENT

The investigations described in this paper were started in 1945 by the New England Division Office, Corps of Engineers, for the Airfields Branch of the Office, Chief of Engineers, in connection with a comprehensive study of criteria for the design of base drainage and subsurface drainage of airport pavements. They are described in detail in several unpublished reports by the New England Division.

As a member of the Board of Consultants on Airfield Pavements for the Office, Chief of Engineers, the senior writer made the theoretical studies and suggested the use of the viscous fluid models. The junior writer, as chief of the Frost Effects Laboratory, New England Division Office, was in charge of the laboratory and field investigations during the period 1945 to 1947.

At the spring meeting, ASCE, in Philadelphia, Pa., on April 18, 1946, two papers were presented by the writers, as a symposium on "Subdrainage of Airfields." These papers ("Theoretical Analysis of Drainage of Base Courses" by Arthur Casagrande and "Model Investigations of Drainage Base Courses" by W. L. Shannon) were combined and revised to form the paper in its present form, under joint authority.

DISCUSSION

Edward S. Barber,[3] A. M. ASCE.—The field and laboratory data presented are very worthwhile. Some simplification of the theoretical development could be effected by considering the cross section and gradient at x in Fig. 3, assuming that the slope of the base is fully reflected in the hydraulic gradient. Thus, as an alternative to Eq. 9c

$$dq = k\,H\left(\frac{H}{c_3\,x} + \tan\alpha\right) dt \ldots\ldots\ldots\ldots (17)$$

For a steady state with $x = L$ and using $c_3 = 2$, from Dupuit's formula for a horizontal base, this gives

$$k = \frac{q\,L}{H\left(\frac{H}{2} + L\tan\alpha\right)} \ldots\ldots\ldots\ldots (18)$$

If Eq. 18 were used instead of Eq. 9b, Col. 6 in Table 1 would read 9, 0.09, 13, and 0.22, which compares better with the flow net analysis values in Col. 5 Table 1.

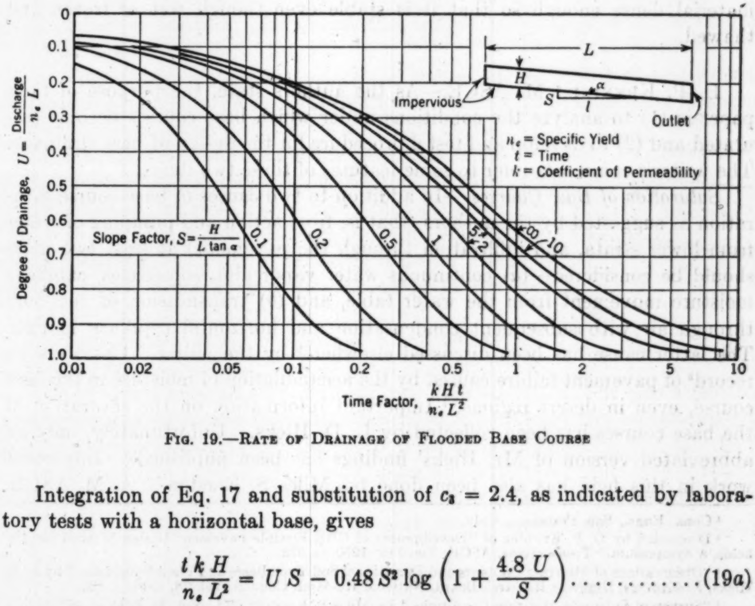


Fig. 19.—Rate of Drainage of Flooded Base Course

Integration of Eq. 17 and substitution of $c_3 = 2.4$, as indicated by laboratory tests with a horizontal base, gives

$$\frac{t\,k\,H}{n_e\,L^2} = U\,S - 0.48\,S^2 \log\left(1 + \frac{4.8\,U}{S}\right) \ldots\ldots\ldots\ldots (19a)$$

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[3] Associate Prof., Civ. Eng., Univ. of Maryland, College Park, Md.

for $0 \leqq U \leqq 0.5$. Similarly for $0.5 \leqq U \leqq 1$:

$$\frac{t\,k\,H}{n_e\,L^2} = 0.5\,S - 0.48\,S^2 \log\left(1 + \frac{2.4}{S}\right) + 1.15\,S \log \frac{S - U\,S + 1.2}{(1 - U)(S + 2.4)} \quad \text{..(19}b\text{)}$$

Eqs. 19 are plotted in Fig. 19. For $U = 0.5$ the following approximate formula is useful

$$t_{50} = \frac{n_e\,L^2}{k\,H}\,\frac{0.44}{0.74 + \dfrac{1}{S}} \quad \text{....................(20)}$$

in which t is time, n_e is specific yield, and k is the coefficient of permeability. Comparison with the recommended calculation indicates that $c_3 = 2.4$ for all values of S as illustrated in Table 2.

The fact that some dense-graded base course materials have been found satisfactory even though they are not drainable according to the criterion proposed in the paper suggests that it is possible to have a material dense enough so that it is stable even though wet or frozen and thawed.

TABLE 2.—COMPARATIVE VALUES OF $\dfrac{t_{50}\,k\,H}{n_e\,L^2}$

Slope factor S	Procedure I	Equation 19*a*	Equation 20
0.5	0.16	0.16	0.16
2	0.34	0.34	0.35
10	0.49	0.51	0.52

D. P. KRYNINE,[4] M. ASCE.—As the authors state, the purpose of their paper is (1) to analyze the conditions under which base courses become saturated and (2) to develop and test a procedure for the design of base drainage. The writer's comments refer to some features of these two items.

Saturation of Base Courses.—In addition to two causes of base course saturation as suggested by the authors (that is, frost action and pumping of water from lower strata, and infiltration through the pavement) at least two more should be considered—(*a*) continuous water vapor and, sometimes, capillary moisture movement from the water table, and (*b*) transmission of moisture through air with subsequent condensation and horizontal capillary motion. The latter cause has been discussed elsewhere[5] by the writer. Cases are on record[5] of pavement failure caused by the accumulation of moisture in the base course, even in desert regions. Important information on the saturation of the base courses has been collected by L. D. Hicks. Unfortunately, only an abbreviated version of Mr. Hicks' findings has been published.[6] Important work in this field has also been done by Miles S. Kersten,[7,8] A. M. ASCE.

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[4] Cons. Engr., San Francisco, Calif.

[5] Discussion by D. P. Krynine of "Development of CBR Flexible Pavement Design Method for Airfields, a Symposium," *Transactions*, ASCE, Vol. 115, 1950, p. 572.

[6] "Observations of Moisture Contents and Densities of Soil Type Bases and Their Subgrades," by L. D. Hicks, *Proceedings*, Highway Research Board, National Research Council, Vol. 28, 1948, p. 422.

[7] "Survey of Subgrade Moisture Conditions," by Miles S. Kersten, *ibid.*, Vol. 24, 1944, p. 497.

[8] "Subgrade Moisture Conditions Beneath Airport Pavements," by Miles S. Kersten, *ibid.*, Vol. 25, 1945, p. 450.

Theoretical Analysis of Base Drainage.—Ingenious simplifications of the actual conditions have led Mr. Casagrande to conclusions in many respects similar to those of the theory of consolidation. The writer has followed the derivations of the paper and has obtained the same results as given by Eqs. 6 and 8. As to the sloping base, the writer notices that the value of the "slope factor" S as given by Eq. 10 tends to become infinite when $\tan\alpha = 0$, that is, when the base becomes horizontal. In other words, Eqs. 11 and 13 should become Eqs. 6 and 8, respectively, for the case $S = \infty$. Apparently some more mathematics is needed to clarify the infinite solutions obtained by such an operation.

Discussion of Theoretical Results.—The action of a base drain as depicted in Fig. 18 is similar to the action that prevails in pumping test. At the beginning of the pumping test the water level falls towards the pump steeply; hence both the hydraulic gradient and what may be called the effective cross section of the stream are considerable but decrease rapidly. An analogous although schematized pattern is presented in Figs. 2 and 3. Hence, the action of a drain removing water from a saturated subgrade is more or less efficient in the beginning of the drainage process only and slows down rapidly. After a certain interval of time the drainage process takes so much time that, before the value of the "degree of drainage" reaches a high value (say, 90%), a new saturation is possible. Therefore, what is called the second part of the drainage process in the paper is not essential in practice. This point of view is implicitly shared by the authors themselves when, in their final recommendations for the design of base drains, they do not extend their study beyond half of the drainage process (Eq. 15).

The authors rightly state that the present development of mathematical tools does nor permit a rigorous solution of the problems of unsteady (transient) flow. Approximate solutions with a high degree of precision have been obtained in particular cases, however. As an example, an interesting solution of the spreading problem, in many respects similar to the base course drainage problem, should be mentioned.[9] It is noteworthy that in the base course drainage problem the shape of the actual drawdown (saturation) line is not important for computations of seepage quantities, as may be concluded from work of Louis A. Pipes,[10] who used not only an elliptic drawdown approximation, but also a parabolic approximation; an approximation based on a horizontal free surface, and the triangular approximation similar to that demonstrated by Fig. 2. In the opinion of the writer, what is really important is the lower boundary of the base course. If the base course is saturated by water pumped from the lower strata, the subgrade is also saturated, at least at its uppermost layers (Fig. 20). Hence, the drain must remove water not only from the base course but also

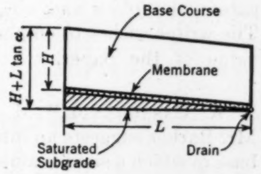

FIG. 20

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[9] "Ground-Water Movement Controlled Through Spreading," by Paul Baumann, *Proceedings-Separate No. 86*, ASCE, August, 1951.

[10] "Subsurface Drainage Investigations 1945–1946. Report on Theoretical Analysis of Drainage of Base Courses," by Louis A. Pipes, Corps of Engrs., U. S. Army, Boston, Mass., March, 1946.

from the subgrade. This is possibly one of the reasons why, in such cases as that shown in Fig. 15(*b*), the time required in the second part of the drainage process in the field is much longer than the time indicated by theory.

The writer agrees with the statement that the coefficient of permeability decreases with decreasing average depth of flow. As a matter of fact the assumption of a constant coefficient of permeability is a vulnerable point in all theories of this kind, beginning with the theory of consolidation. In the drainage process, moisture is not removed from the surface of a particle in a single blow, but the particle is gradually peeled from the moisture covering it, and the closer this cover adheres to the particle, the more difficult is that peeling. Hence, a retardation of the drainage process may be expected in the soils possessing a certain value of the specific attractive power, such as silty sands in this case. The amount of moisture attracted by pea gravel is exceedingly small, hence there is practically no retardation of the drainage process in a subgrade of this material.

Theory and Laboratory Experiments.—If the subgrade is isolated from the base course by an impervious asphaltic membrane and the base course is saturated by infiltration through the pavement, the field-test results would be closer to the theory than if there were no membrane. In addition, if the coefficient of permeability is kept constant, a perfect agreement of the tests with the theory can be expected. The laboratory experiments explained in the paper precisely duplicate this particular case. As may be concluded from the paper, these experiments were performed with the utmost care, as generally all laboratory work of the Corps of Engineers, United States Army, is done; but, nonetheless, in a general case they cannot be admitted as a proof of the applicability of the theory developed in the paper.

Summary.—The work presented in this paper is a definite step forward in a relatively unexplored field and a considerable help to engineering investigators who follow and who are interested in further research along these lines. The writer wishes to emphasize that his discussion does not disregard the high value of the paper.

A. CASAGRANDE,[11] M. ASCE, AND W. L. SHANNON,[12] A. M. ASCE.—Mr. Barber suggests an interesting modification of the equation for the sloping base in which a single value for the parameter c may be used for all slope factors. This permits the plotting of one set of curves in which the degree of drainage U is plotted against the time factor as shown in Fig. 19. Mr. Barber also suggests that Eq. 18 is a better solution of the steady problem of seepage with sloping base than the corresponding equation which follows from the assumption represented by Eq. 9*b*, and he bases his conclusion on a comparison with the results shown in Col. 6, Table 1. However, these results include k-values which were in part computed by Eq. 9*b* and in part derived from flow net solutions. Therefore, the values in Col. 6 cannot be used to decide which approach is more correct. Besides, the coefficient of permeability in these field tests varied greatly over the thickness of the layer, so that such tests would hardly be suitable for a comparison of these equations. However, viscous fluid model tests—particularly tests with steep base slopes to accentuate the differences—should be valuable for such a comparison. In the absence of such tests, Eq. 18 and the corresponding equation based on Eq. 9*b* have been compared on the basis of flow net solutions of several cases and it was found that Eq. 18 gave from 30% to more than 50% larger values for the ratio q/k than the flow net analysis, whereas Eq. 9*b* gave results for q/k which were from 5% to 20% too small. It can also be shown by simple theoretical considerations that the deviations from correct solutions should be of that order of magnitude. Therefore, the writers believe that, for the solution of steady-state flow problems with a sloping base, the assumptions in Eq. 9 are more accurate than those on which Eqs. 17 and 18 are based.

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[11] Prof. of Soil Mechanics and Foundation Eng., Harvard Univ., Cambridge, Mass.

[12] Cons. Engr., Seattle, Wash. (formerly Materials Engr., Corps of Engrs., U. S. Dept. of the Army, Portland, Ore.).

Since Eqs. 15*b* and 20 give practically identical results and since the available test data are not sufficient to prove conclusively which one is more accurate, the simpler Eq. 15*b* appears preferable for design purposes.

The writers are not in agreement with Mr. Barber on the value of dense-graded base courses, and question whether such base courses have indeed stood the test of saturation. In the writers' experience, also dense-graded bases have developed failures when they became saturated from one cause or another. Besides, dense-graded bases are likely to contain, locally, an objectionable amount of fines from the standpoint of frost action.

Mr. Krynine has called attention to other possible causes for base saturation. In his discussion of the theoretical results, he notes that subsurface drains must not only remove water from the base course but also from the subgrade and that water draining from the subgrade may result in slowing down the last portion of drainage from the base course. This phenomenon might be true in some cases. Mr. Krynine suggests that this effect may have been present in the field drainage tests as indicated by Fig. 15(*b*). In these particular tests the subgrade was simulated by an impervious membrane composed of asphalt impregnated fabric (prefabricated bituminous surfacing, known as PBS, surplus from World War II) which, so far as could be determined, did not contribute to water drainage. Some troubles were experienced in making this membrane leak proof, which may have influenced the results in the opposite direction.

Although the paper has emphasized only the drainage of base courses beneath pavements, the findings are applicable to pervious subgrades where the thickness of the pervious material is limited as, for example, a sand subgrade created by hydraulic filling over an impervious formation. Assuming that in a humid climate an island 10,000 ft in diameter is created by a 20-ft-thick layer of sand on an impervious clay, it is obvious that a ground-water table in the middle of this island will remain developed substantially above the surrounding free-water level. Depending on replenishment by precipitation and on the thickness and permeability of the sand fill, the ground-water table in the fill may remain so close to the surface of the fill that it would be undesirable to build pavements upon the sand without special provisions to increase the depth from the ground surface to the water table. To provide such an increase in depth to the water table (*a*) the thickness of the fill might be increased, (*b*) an extensive system of subdrainage could be provided, or (*c*) in the case where the sand fill is placed on a relatively thin impervious layer of clay or a silty or peaty meadow mat overlying a pervious deposit, this impervious layer could be entirely or partly removed prior to construction of the sand fill. This last method, where applicable, may achieve the desired results most economically.

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Paper No. 2517

MODEL TESTS USING LOW VELOCITY AIR

By James W. Ball,[1] A. M. ASCE

With Discussion by Messrs. David W. Appel and James W. Ball

Synopsis

The use of low velocity air in testing closed hydraulic systems, or testing in which air can be substituted for water without introducing any appreciable error due to compressibility, is discussed. The advantages, disadvantages, errors to be expected, and cautions to be observed when the hydraulic engineer employs this method of testing are also introduced.

Introduction

For the purpose of this paper, low velocity air tests will be considered, those that yield, for all practical purposes, the same results as hydraulic tests. Low air velocities usually range up to about 250 ft per sec or about 25% of sonic speed, depending upon the nature of the particular problem.

History of Air Testing.—Air testing, or aerodynamic testing as it is commonly termed, is not new, but the technique of using low velocity air as a fluid to obtain solutions to hydraulic problems is a new development. References[2,3,4] to aerodynamic testing are found in the early part of the twentieth century, but these references usually pertain directly to aircraft research or to fluid mechanics research and not to testing of the type under discussion in this paper. In this early research work the experimental data from tests using different fluids, including air and water, were combined to illustrate a fundamental law or principle rather than to prove that air could be substituted for water in determining the flow characteristics of hydraulic systems. Low velocity air testing is now being used in solving problems of this nature. Among the first to make use of this method, and a pioneer in this field, was C. Keller. Some of Mr. Keller's work was published as early as 1937, and in 1939 he was co-author of an

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Note.—Published in June, 1951, as *Proceedings-Separate No. 76*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Hydr. Engr., Bureau of Reclamation, U. S. Dept. of Interior, Denver, Colo.

[2] "Wind Tunnels," by Ludwig Prandtl and Oskar G. Tietjens, *Applied Hydro- and Aeromechanics*, McGraw-Hill Book Co., Inc., New York, N. Y., 1934, pp. 252–265.

[3] "Similarity of Motion in Relation to the Surface Friction of Fluids," by T. E. Stanton and J. R. Pannell, *Philosophical Transactions of the Royal Society of London*, Series A, Vol. 214, 1914, p. 199.

[4] "The Reynolds Number," by Boris A. Bakhmeteff, *Mechanical Engineering*, Vol. 58, 1936, pp. 625–630.

article on air testing of turbines.[5] In the same year another article,[6] which cited some of Mr. Keller's work and discussed low velocity air testing of hydraulic and steam operated machines, was published. Everything set forth in this article is just as applicable today when leading engineering organizations, universities, and private industries are using air testing as a low-cost, useful tool to solve some of their hydraulic problems.

Instruments, test equipment, and measuring techniques are beyond the scope of this paper, but they offer no problem in low velocity air testing because they have been highly developed through the extensive aerodynamic work pertaining to aircraft design. On the other hand, much work still can be done towards defining the limits to which aerodynamic testing can be substituted for hydraulic testing. Some of the limits already established will be discussed subsequently.

Increasing Use and Advantages of Low Velocity Air Testing.—There are numerous reasons why low-velocity air testing is gaining popularity with hydraulic engineers. The following advantages contribute to this popularity:

1. The low density of air minimizes the structural requirements of the test facilities.
2. Power requirements to circulate the test fluid are comparatively low.
3. Absolute fluid tightness is not essential but more easily attained. The wetting problem is nonexistent, permiting use of wide variety of materials.
4. The atmosphere serves both as a supply reservoir and a catch basin.
5. Test procedure is usually greatly simplified but requires certain types of more highly developed and sensitive instruments.
6. Reynolds numbers can be made about the same for either air or water in a given model. Use of a larger model to test at a higher Reynolds number might be advantageous in some cases.
7. The relationships for noncompressible fluids can be applied with negligible error if the air velocities are kept below certain values (less than $\frac{1}{4}$ sonic speed).

All except the last item are advantages that result directly when air is used instead of water for studying the flow characteristics of closed hydraulic systems. However, without proof questions might arise as to whether or not the simple relationships for an incompressible fluid are applicable when air is used in place of water. It will be endeavored to supply this proof without going into extensive mathematical details.

Indicated Disadvantages.—It seems appropriate at this point to mention some of the disadvantages of using air as a test medium and to emphasize that the discussion applies principally to systems of short length where friction is not a major factor. Air has a comparatively low bulk modulus and does not become discontinuous as in the case when the vapor pressures of liquids are reached; and since air is a gas, there is no free surface. From these properties it appears that air could not be used to replace a liquid where effects of compressibility, cavitation, and gravity effects are predominating factors. This is not always the case, but it is necessary to be sufficiently familiar with the dynamics of gaseous flow in order to recognize when such studies are applicable. The use of the equations for incompressible flow will indicate the limits where compressibility is of little or no consequence. A logical and valid conclusion without the aid of mathematics would be that the velocity of sound for the test medium should not be exceeded in any event, since beyond this point it is not valid to assume the condition of adiabatic changes. A more elaborate theoretical treatment of the problem is required in such cases.

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[5] "A Method for Determining the Cavitation Factor by Air Tests," by C. Keller and H. Bleuler, *Escher Wyss News*, Zurich, Vol. 1-2, 1939, pp. 19-24.

[6] "New Method of Aerodynamic Research," by C. Keller, *ibid.*, pp. 4-13.

A pertinent question at this point is "How near can the speed of the air approach sonic velocity without introducing errors of appreciable magnitude?"

LIMITATIONS ON USE OF COMPRESSIBLE FLOW

Application and Limitations of Formulas for Incompressible Flow.—It can be shown that large errors are introduced by using near-sonic velocities when the simpler hydraulic relationships are applied. This being the case, it would appear that the first problem of a hydraulic engineer wishing to use air instead of water for testing a hydraulic system would be to determine the maximum air velocity for his particular study.

In most tests, if the velocities are kept low and there are no large changes in pressure or velocity, it will not be necessary to make an evaluation of the compressibility effects. However, if there is some question as to whether or not the compressibility effects should be of concern, this evaluation can be made as will be outlined. The problem may or may not be easy depending upon the difficulty in establishing approximate expressions for the flow conditions in relation to the pressures and velocities for both incompressible and compressible flow. The maximum permissible velocity for any problem can be determined by setting up the expressions for the pressures in incompressible and compressible flow at two carefully selected points, computing the pressure change from one point to the other for both types of flow and plotting these pressure changes against the respective velocities at the reference point.

If the velocity of the air is kept within the range in which the pressure difference between the two selected points is essentially the same for both types of flow, then the simpler hydraulic relationships can be used in analyzing the test data from a hydraulic system using air instead of water as the test medium.

Theoretical Limiting Velocity.—The velocity of the air which will result in a given degree of accuracy in a system can be found by considering two general equations derived from the Bernoulli relationship for incompressible and compressible flow. The equation for incompressible flow is

$$\frac{h_1}{h_0} = 1 - \frac{(V^2_1 - V^2_0)}{2\, g\, h_0} \qquad (1)$$

and the equation for compressible flow is

$$\frac{h_1}{h_0} = \left[1 - \frac{(K-1)\,(V^2_1 - V^2_0)}{2\, C^2_0} \right]^{\frac{K}{K-1}} \qquad (2)$$

in which h is the pressure head, V is the velocity, g is the acceleration of gravity, K is the gas constant, C is the acoustic velocity, and the subscripts 0 and 1 refer to the reference station and selected station, respectively.

The type of testing under consideration requires all velocities to be much less than sonic and all pressures to be near atmospheric. Therefore, the term $\frac{(K-1)\,(V^2_1 - V^2_0)}{2\,C^2_0}$ will be less than unity, and the right side of the equation can be expanded into a convergent series by means of the binomial theorem and written:

$$\frac{h_1}{h_0} = 1 - \frac{K\,(V^2_1 - V^2_0)}{2\,C^2_0}\left[1 - \frac{(V^2_1 - V^2_0)}{4\,C^2_0} - \frac{(K-2)\,(V^2_1 - V^2_0)^2}{24\,C^4_0} \cdots\right] \quad (3)$$

V_1 can be expressed in terms of V_0 as

$$V_1 = \beta\,V_0 \qquad \ldots\ldots (4)$$

in which β is the ratio of flow areas $\frac{A_0}{A_1}$. Then

$$\frac{h_1}{h_0} = 1 - \frac{K\,V^2_0\,(\beta^2 - 1)}{2\,C^2_0}\left[1 - \frac{V^2_0\,(\beta^2 - 1)}{4\,C^2_0} - \frac{(K-2)\,V^4_0\,(\beta^2 - 1)^2}{24\,C^4_0} \cdots\right] \quad (5a)$$

or

$$h_1 - h_0 = -\frac{V^2_0\,(\beta^2 - 1)}{2\,g} \times \left[1 - \frac{V^2_0\,(\beta^2 - 1)}{4\,C^2_0} - \frac{(K-2)\,V^4_0\,(\beta^2 - 1)^2}{24\,C^4_0} \cdots\right] \quad ..(5b)$$

since

$$\frac{h_0\,K}{C^2_0} = \frac{1}{g} \qquad \ldots\ldots (6)$$

also, for incompressible flow,

$$h_1 - h_0 = -\frac{V^2_0\,(\beta^2 - 1)}{2\,g} \qquad \ldots\ldots (7)$$

Examination of Eqs. 1 to 7 shows that the relative magnitude of the compressibility effects will be reflected only in the second and subsequent terms within the brackets and that this magnitude depends on three factors: (1) The velocity of the air in the test system; (2) the sonic velocity at the pressure in the test system; and (3) the shape of the flow passage.

A quick estimate of the probable error introduced by using air instead of water may be made for a particular test setup by substituting approximate values of V_0, C_0, and β in the second term within the bracket. For example, let $V_0 = 250$ ft per sec, $C_0 = 1{,}000$ ft per sec, and $\beta = 2$. This error would be $\left[\frac{1}{4}\,\frac{(250)^2}{(1{,}000)}\,(4 - 1)\right] = 0.047$ or 4.7%. The effect of the third and subsequent terms is insignificant. Examination shows that error would be reduced to about 1% if V_0 was limited to 50 ft per sec. However, at the lower velocity, pressure intensity and pressure differentials might be more difficult to measure, and more precise instruments would be required.

Practical Limiting Velocity.—The use of hydraulic equations for air flow that is actually governed by aerodynamic relations introduces errors in the test results. The magnitude of these errors can be determined in some cases, while in others the limiting velocity can be determined at which the two regimes start to separate.

One of the least complicated examples of the determination of permissible test velocity is that of steady flow past a solid body, such as a pitot tube used in making a test, held stationary in a uniform stream. In this case the points selected for pressure measurement are in the undisturbed flow upstream from the body and on the leading edge of the body where stagnation pressure exists. The equation for the case of incompressible fluid is

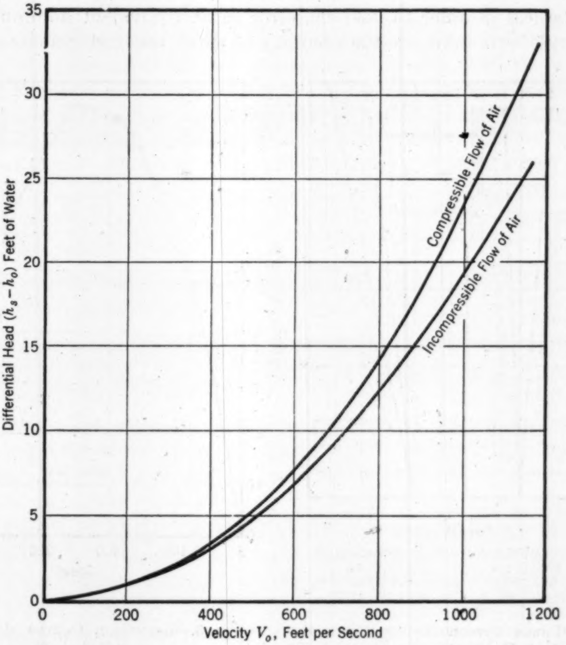


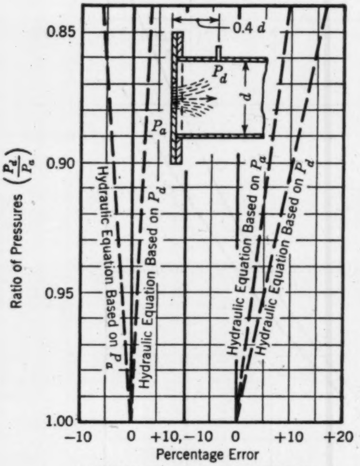
FIG. 1.—PRESSURE DIFFERENTIAL VERSUS VELOCITY CURVES FOR STAGNATION POINT

$$h_s - h_0 = \frac{V^2_0}{2\,g} \qquad (8)$$

and for compressible fluid at subsonic velocity:

$$h_s - h_0 = \frac{V^2_0}{2\,g}\left[1 + \frac{V^2_0}{4\,C^2_0} - \frac{(k-2)\,V^4_0}{24\,C^4_0} \cdots\right] \qquad (9)$$

The subscripts 0 and s refer to the station in the undisturbed flow and at the stagnation point, respectively. By assuming various values (less than sonic) for V_0, computing values of $h_s - h_0$ and plotting the $(h_s - h_0)$ versus V_0 terms for both cases, Fig. 1 is obtained. An examination of the graph discloses that the values for both conditions are essentially the same for velocities up to about 250 ft per sec. For this type of study then, one would conclude that results comparable to hydraulic tests could be obtained with air, providing the value of V_0 was kept below about 250 ft per sec. Reliable results could be expected from a pitot tube for this velocity range. However, if other points in such a system are selected for measurement, it is possible that the value of V_0 would have to be less in order to hold the error to an equivalent minimum. Compressibility effects influence the accuracy of many test instruments.



(a) VARIATION BASED ON SPECIFIC VOLUME AT P_d

(b) VARIATION BASED ON SPECIFIC VOLUME AT P_a

FIG. 2.—ERROR INTRODUCED BY USING HYDRAULIC INSTEAD OF AERODYNAMIC EQUATION FOR AN INTAKE ORIFICE

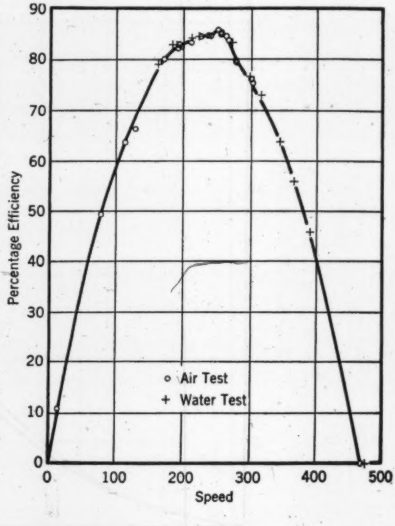


FIG. 3.—EFFICIENCY CURVES MEASURED ON A MODEL TURBINE

Flow through an orifice is an example of the determination of the magnitude of error introduced. The accuracy of the formula for incompressible flow through an intake orifice will vary depending on the pressures at which it operates and the reference pressure used in making the computation. The four lines in Fig. 2 show the percentage error introduced by using the hydraulic equation instead of thermodynamic relations for the four possible conditions based on the different reference points. The error is a minimum when the base pressure used is that in the pipe downstream, and results are compared with those obtained by the equation for compressible flow using the specific volume for the downstream pressure. With the low velocities suggested for the air tests, the pressure differentials are small and the ratio of downstream pressure to upstream pressure so near unity that the error is small in any case. The use of the mean pressure would result in even less error if the quantity were based on the specific volume at the downstream pressure. It may be concluded from this illustration that the equation for incompressible flow for an intake orifice may be used without introducing serious error if the pressure differential across the orifice is kept small (not more than 1 ft of water).

Examples for stream tubes in general and for flow in a venturi meter may be found in standard texts.[7] The evaluation of the compressibility effects on the accuracy of data taken from the test structure and on any instruments that are to be used in making the tests might be only the initial phase of a study in which air is substituted for water. In most cases the need for this initial work can be eliminated by maintaining the test velocities as low as possible while still providing accurately measurable quantities. With a little experience the hydraulic engineer is able to recognize whether or not the initial investigation of the compressibility effects is necessary.

There are other factors not apparent in the equations that might influence the accuracy of the results or make them qualitative only. Specific examples will be cited later to illustrate some of these factors which have been discovered mainly through experience.

HYDRAULIC PROBLEMS STUDIED

Air has been used successfully in hydraulic problems of the following nature: (*a*) Design of water passages, runners, and impellers of hydraulic turbines and pumps; (*b*) study of operating characteristics of hydraulic turbines and pumps including the efficiency and cavitation factor; (*c*) study and determination of losses and capacities of water distribution systems; (*d*) determination of losses and capacity of comparatively large water tunnels; (*e*) study of the mechanics of turbulent flow and its application to liquids (turbulence loss, development, and decay below screens); (*f*) study of the hydraulic characteristics of large outlet valves; (*g*) study of the hydraulic characteristics of complex passages such as those of hydraulic operating systems of large valves; (*h*) study of the pressure distribution and losses for various shapes of penstock inlets; (*i*) capacities and hydraulic characteristics of certain types of fluid meters (nozzles, orifices, etc.); and (*j*) study of the diffusion of air jets and its application to such structures as submerged sluice gates.

Turbine and Pump Design.—Air testing in connection with turbine design seems to have been done mainly by German and Swiss engineers in the late 1930's. A great deal of their work was published in Swiss technical papers,[8] that contained the comparison between aerodynamic and hydraulic tests shown in Fig. 3. The method has not been adopted by manufacturers in the United States, for these companies still use water for testing their models. The same is true of United States pump manufacturers.

Large Water Tunnel Design.—An unusual instance in which air has been used in place of water to determine the flow characteristics of a system of water tunnels was discussed in papers[9,10] given by two French engineers, G. Remenieras and P. Bourguignon at the 1949 meeting of the International Association for Hydraulic Structures Research in Grenoble, France. These tests were made before the tunnels were placed in operation and later checked by hydraulic tests. The test was performed by passing air through the tunnels while taking measurements of pressure changes, ascertaining the loss coefficients, and computing the tunnel capacities. Good agreement was found when the results were checked hydraulically after the system was placed in operation. The air tests in this case actually proved the adequacy of parts of the system before construction of the entire system was completed as is necessary for water tests. Such air tests permit an on-the-spot check of a system before the contractor has moved from the site and any deficiency could still be corrected at a minimum cost.

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[7] "Fluid Mechanics," by Russell A. Dodge and Milton J. Thompson, McGraw-Hill Book Co., Inc., New York, N. Y., 1st Ed., 1937.

[8] "Aerodynamische Versuchsanlagen für Hydraulische Maschinen," by C. Keller, *Schweizerische Bauzeitung*, Vol. 110, 1937, p. 203.

Turbulence Testing.—Work is being done at the State University of Iowa (at Iowa City) laboratory to study fundamental laws dealing with turbulence. Air is used instead of water because suitable instruments have been developed for measuring turbulence in air while instruments for measuring turbulence in water are still in the experimental stage. The diffusion of an air jet and its application to submerged sluice gates is another of this university's projects.

Characteristisc of Hydraulic Outlet Structures.—The Bureau of Reclamation Hydraulic Laboratory at the Denver (Colo.) Federal Center first employed this type of testing to study the flow characteristics of a diffuser cone. Later it was used to investigate the hydraulic properties of outlet valves to determine the deficiencies or advantages of the existing and proposed physical arrangement and dimensions of the water passages in regard to the capacity and possible occurrence of cavitation.

In one case the time required for the closing and opening cycle of an outlet needle valve when operated hydraulically was under study. The speed of the cycle was governed mainly by the capacity of the flow passages of the operating mechanism. A very simple representation of this mechanism was constructed of wood and sheet metal and tested with air. The results proved very satisfactory, and required less work and a fraction of the time required for hydraulic tests. Tests in the laboratory have included the study of the pressure distribution in the rectangular entrance to a conduit having a radial gate for regulation and the determination of loss coefficients for a complicated system of branching penstocks. Comparison of the results of these air studies with water studies have not been made available.

MISCELLANEOUS FACTORS AFFECTING LOW VELOCITY AIR TESTS

Turbulence.—Some very interesting facts pertinent to air testing were brought to light during the studies just discussed.

In the initial air tests on diffusers an unexpected discrepancy was noted in the results when compared with the results of water tests. Fig. 4 is a plot of this

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[9] "Prédétermination de la perte de charge dans une canalisation d'eau sous pression à partir de celle mesurée sur la même canalisation parcourue par de l'air," by G. Remenieras, *Paper III-2*, Third Meeting of International Assn. for Hydraulic Structures Research, Grenoble, France, 1949.

[10] "Prédétermination de la perte de charge dans une canalisation d'eau sous pression à partir de celle mesurée sur la même canalisation parcourue par de l'air; contrôle de la validité de la méthode sur la Galerie d'Amenée, R. G. de l'usine hydro-eléctrique de Pont-Escoffier," by P. Bourguignon, *Paper III-9* Third Meeting of International Assn. for Hydraulic Structures Research, Grenoble, France, 1949.

test. The air tests indicated considerably more loss in the diffusers than did the water tests. In an attempt to introduce an additional loss ahead of the diffuser to study the effect of such loss on the action of the diffuser, a wire mesh waste-basket was held over the inlet to the test apparatus shown as Case I in Fig. 4. A decrease in loss was indicated immediately. It was concluded that turbulence was almost nonexistent without the basket in the incoming air stream because of the arrangement that used the entire volume of the laboratory as a supply reservoir. Thus, flow conditions were not correctly represented. The apparatus was rearranged as shown for Case II, and the tests repeated.

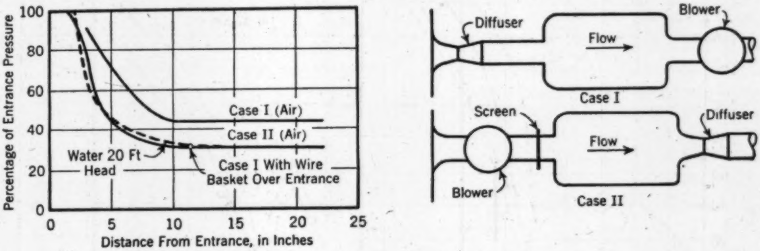
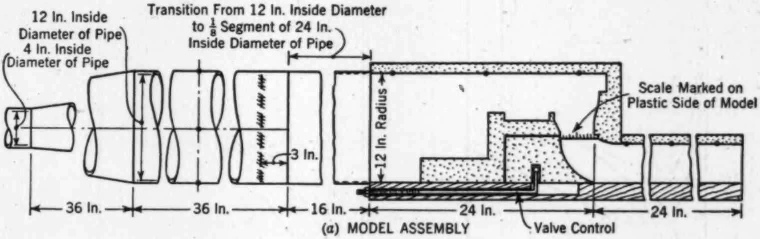
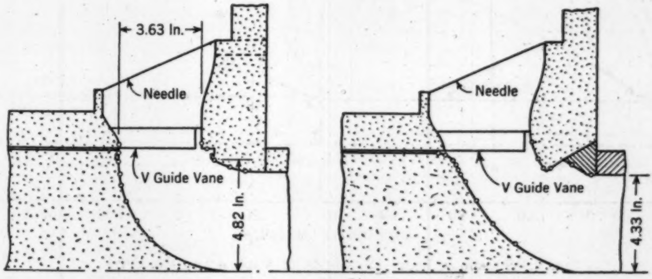


FIG. 4.—EFFECT OF TURBULENCE ON AIR TEST


12 In. Inside Diameter of Pipe
4 In. Inside Diameter of Pipe
Transition From 12 In. Inside Diameter to $\frac{1}{8}$ Segment of 24 In. Inside Diameter of Pipe
12 In. Radius
3 In.
Scale Marked on Plastic Side of Model
36 In.
36 In.
16 In.
24 In.
24 In.
Valve Control

(a) MODEL ASSEMBLY


3.63 In.
Needle
V Guide Vane
4.82 In.
Needle
V Guide Vane
4.33 In.

(b) EXISTING INSTALLATION (c) PROPOSED REDESIGN

FIG. 5.—SHOSHONE DAM AERODYNAMIC MODEL

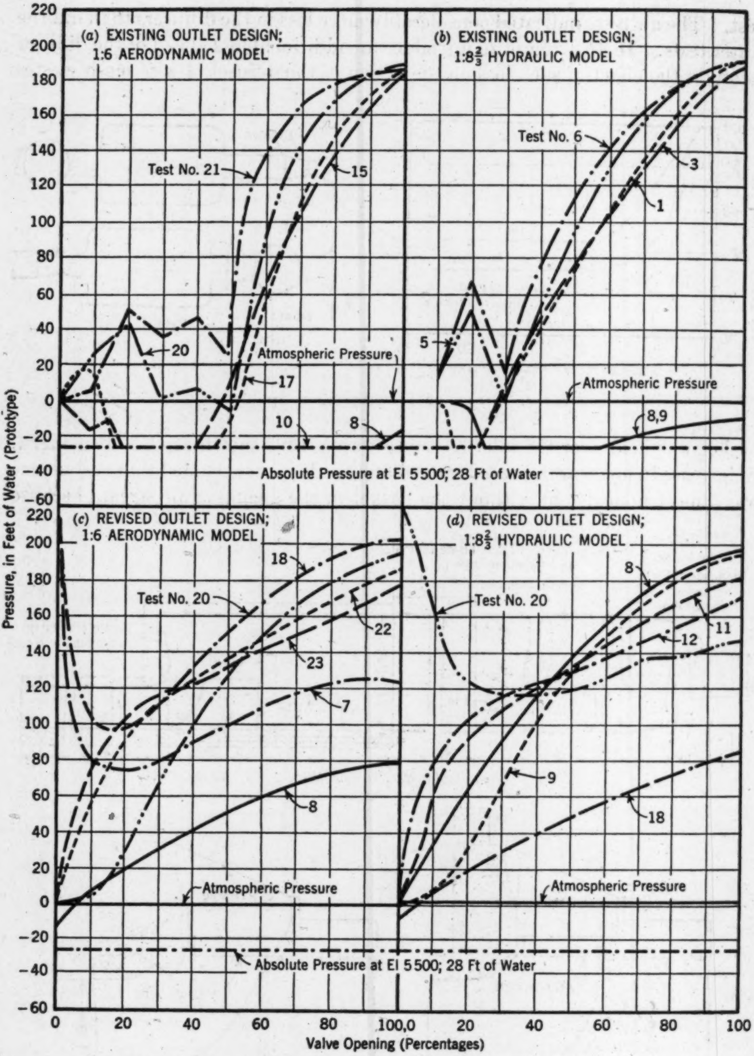


Fig. 6.—Shoshone Dam—Pressures in Valves and Conduits

With the new arrangement of the air supply passing through the screen before flowing through the diffusers, there was a marked change in the results, and good agreement between the aerodynamic and hydraulic tests ensued. The situation of a virtually turbulence-free fluid at the beginning of the model, which first occurred inadvertently in the air test, rarely, if ever, is encountered

in a hydraulic problem. This is the reverse of the case in aerodynamic works where the turbulence-free conditions of the atmosphere can best be approximated only in wind tunnels. The plot of Fig. 4 illustrates the error which might be introduced by improper turbulence.

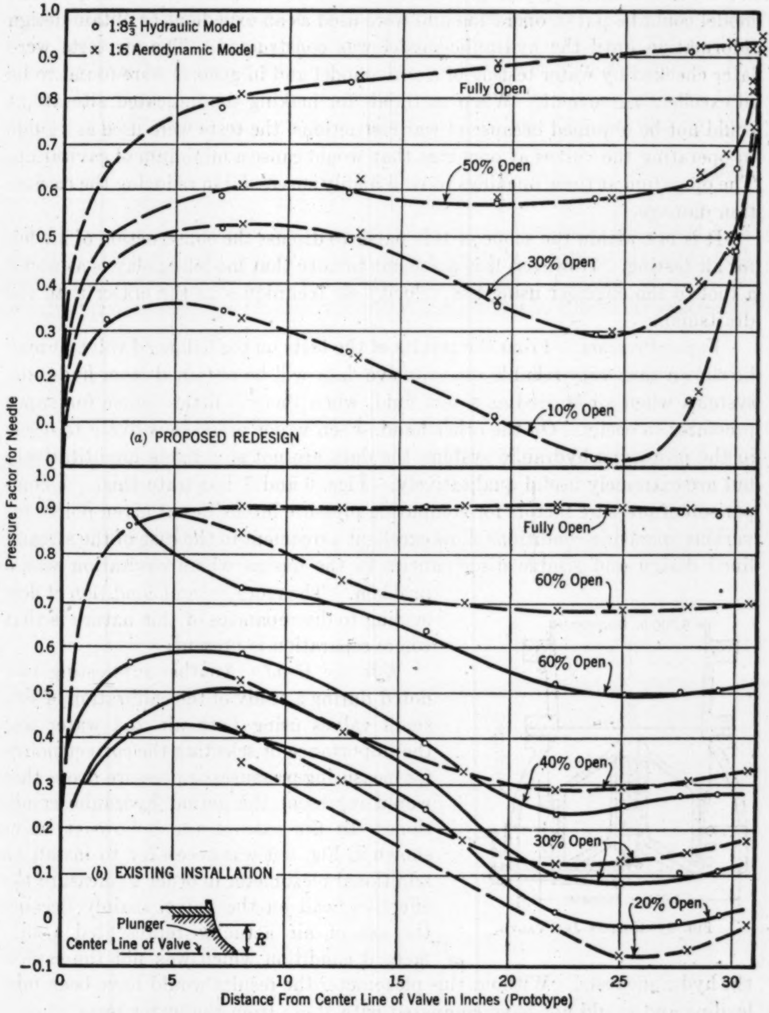


FIG. 7.—SHOSHONE DAM—COMPARISON OF PRESSURE FACTORS

***Cavitation.*—Several items of interest were discovered in the Bureau of Reclamation Laboratory during the air tests pertaining to the design of outlet valves. The tests were conducted on the Ensign balanced valve, needle**

valves, and the more recent development, the hollow-jet valve. In the case of the balanced valve extensive air tests were made on the existing design and proposed alterations to eliminate cavitation. Fig. 5 shows the general dimensions of the aerodynamic model for a 58 in.-balanced valve in the outlet chamber of Shoshone Dam in Wyoming. The air tests were completed before a hydraulic model could be put in operation and were used as an expedient to obtain design information until the hydraulic model was constructed. The air tests were later checked by water testing of a scale model and in general were found to be in excellent agreement. When materials for making the indicated alterations could not be obtained because of war restrictions, the tests were used as a guide in operating the valves at openings that would cause a minimum of cavitation. The operation at these openings proved highly successful in reducing the cavitation damage.

It is not within the scope of this paper to discuss the construction of models for air testing. However, it is pertinent to note that modeling clay is as useful a tool to the engineer using low velocity air techniques as the eraser is to the draftsman.

Vapor Pressure.—From the results of the tests on the balanced valve, it may be shown that very reliable quantitative data will be obtained from hydraulic systems when air is used as a test fluid, when there is little chance for vapor pressures to occur. On the other hand, when vapor pressure is likely to occur in the prototype hydraulic system, the data are not so reliable quantitatively but are extremely useful qualitatively. Figs. 6 and 7 illustrate this. A comparison of pressure factors for computing pressure intensities at given points for various operating conditions show excellent agreement in the case of the streamlined design and sizable discrepancies in the design where cavitation was a problem. The more general condition of flow leading to discrepancies of this nature is that where separation is present.

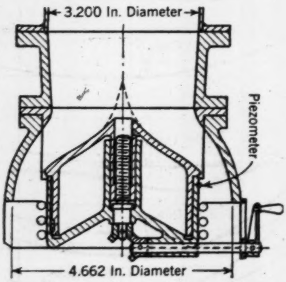


FIG. 8 —HOLLOW-JET VALVE

Effective Head.—Another interesting fact noted during a study of the calibration of two small valves using both air and water was the importance of selecting the correct points for measuring pressures or pressure drops that would represent the actual hydraulic conditions. In the case of the hollow-jet valve shown in Fig. 8 it was necessary to install an additional piezometer in order to measure the effective head on the valve, mainly because the use of air actually represented a submerged condition which was not the case in the hydraulic tests. Without this piezometer the results would have been misleading and would not have compared with those from the water tests.

In another instance it was found necessary to remove an apparently essential part of the test valve in order to make it represent the hydraulic conditions which would be present in the field structure.

CONCLUSION

No doubt there are other factors involved in the proper representation of boundary conditions influencing the application and accuracy of air test results that have not been discussed in this paper. The factors presented here are either readily apparent from an examination of the physical relationships involved or those encountered by the Bureau of Reclamation Laboratory or other engineers who have conducted low velocity air tests for solving problems of a hydraulic nature. It would be interesting to hear from others who have had experience with or have knowledge of hydraulic problems being solved by low velocity air testing.

Although low velocity air testing of hydraulic systems is a very useful, expeditious, and economical method of solving many hydraulic problems, it is doubtful if it will ever be used extensively or result in accomplishments of a sensational nature. It is used mainly as a substitute for hydraulic tests and serves as an expedient for urgent or preliminary studies. The final proof of the design of a hydraulic system will, therefore, in most cases still be its successful operation under hydraulic conditions.

DISCUSSION

David W. Appel,[11] J. M. ASCE.—Although the author emphasizes the use of air tests in problems of hydraulic design rather than in fundamental hydraulic research, it is still difficult to justify the indecisive nature of his conclusions. His discussion of the limitations on the use of air for hydraulic model tests, although incomplete, is more detailed than necessary, since the general principles of hydraulic similitude—which apply to any fluid—have long since been verified and are now generally accepted. In particular, the "miscellaneous factors" which he lists as affecting air tests are all embodied in the general requirements for similitude and in themselves introduce no uncertainties in the air-test results. In every case where discrepancies in test results were observed, one of these requirements was not met. After removing this uncertainty as to the existence of unaccountable factors which may affect air-test results, one can only conclude that air tests should play an ever-growing role in hydraulic design as well as research.

The general principles of hydraulic similitude were established, confirmed, and accepted by the profession a long time ago. The author's assertion that these do not prove that air can be substituted for water in determining the characteristics of specific hydraulic systems is untenable; the use of air in place of water is as fully warranted and fully prescribed as merely the change in boundary scale and rate of flow. To establish similtude, certainly it is necessary to consider those properties of air, as a gas, which differ from those of water, as a liquid. These differences were given in the first part of the paper: Air is compressible, cannot maintain a free surface, and cannot become discontinuous; whereas water is assumed to be incompressible, can maintain a free surface, and becomes discontinuous when the vapor pressure is reached. The limitations due to each of these differences were considered. The compressibility effect was then found to be negligible for low to moderate velocities; closed systems were assumed in order to avoid free-surface effects; and it was indicated that only qualitative results can be obtained when cavitation occurs in the prototype. In other words, for tests of closed systems, where air velocities in the model are not high and pressures in the prototype are above the vapor pressure, the well-known principles of hydraulic similitude should be sufficient to establish practically perfect similarity between model and prototype.

The discrepancies noted in the particular tests described can now be explained in terms of well-established limitations. For example, the hollow-jet valve tests represent a case in which a free surface exists in the prototype which cannot be reproduced in the model using air. It is significant that comparable results were obtained in spite of this by selecting suitable points (using the principles of fluid mechanics) for measuring the effective head. The difference in results in the tests on diffusors, although caused by a difference in turbulence in the flow, should be attributed to geometric dissimilarity between the model and prototype. The results, using nonturbulent flow of air in Case I, could be reproduced equally well using water, by having the diffuser supplied directly from a large tank, thus eliminating turbulence from the approaching flow. The selection of the proper arrangement of equipment depends on the arrangement in the prototype. Geometric similitude requires that the flow system on either side of the test section be similar, or at least equivalent, in the model and prototype. This will be sufficient to make sure of correct results in nearly all tests of hydraulic models. The exception will be the case in which viscous effects are not negligible, as was assumed by Mr. Ball, and similitude of the Reynolds type would be required.

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[11] Research Associate, Iowa Inst. of Hydr. Research, State Univ. of Iowa, Iowa City, Iowa.

It is evident from these examples that careful consideration must be given to all factors that may influence the flow in a particular model. Similitude in all models—whether tested in air or in water—is usually only approximate due to the practical necessity of ignoring various secondary requirements. A comparison of the precision desired in the results with estimates of the probable deviations from perfect similitude should indicate whether additional tests are necessary under conditions more closely approximating those in the prototype. The model turbine test cited by the author is an example of the excellent degree of similitude that often can be obtained.

The advantages of using air instead of water in model tests were cited in a paper by Hunter Rouse, M. ASCE, in 1947,[12] which emphasized in particular the simplification of many phases of instrumentation which the use of air permits. The availability of more highly developed and sensitive instruments is an advantage in favor of using air, rather than just a requirement to be met as the author suggests. This is especially true in basic research where instruments such as the hot-wire anemometer can be used to make measurements in air that cannot readily be obtained in water at the present time.

The list, given by Mr. Ball, of the types of hydraulic problems studied emphasizes the uses of air in model tests for the design of hydraulic structures. The importance of the uses of air in fundamental hydraulic research warrants more attention to this phase of air testing. To indicate the wide variety of research problems that have been studied in air, examples of investigations conducted at the Iowa Institute of Hydraulic Research, in Iowa City, will be given.

The early experiences of the staff of the Iowa Institute with air tests during World War II have been described.[12] The relative ease of conducting the air tests compared with similar water tests was so apparent that air-flow equipment has since been explained and applied to many other problems. Two of these were mentioned in the paper: The investigation of flow through screens and baffles,[13] and the investigation of the diffusion of submerged jets.[14] One significant fact about these investigations is that the results are applicable whereever these flow situations are encountered, in the flow of water just as much as in the flow of air. The results obtained give hydraulic engineers all the basic information they are likely to need for flow of these two types without recourse to supplementary tests with water.

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[12] "Use of the Low-Velocity Air Tunnel in Hydraulic Research," by Hunter Rouse, *Bulletin No. 31*, Studies in Engineering, State Univ. of Iowa, Iowa City, Iowa, 1947 (*Proceedings*, Third Hydraulic Conference).

[13] "An Investigation of Flow Through Screens," by W. D. Baines and E. G. Peterson, *Transactions* ASME, Vol. 73, July, 1951, p. 467.

[14] "Diffusion of Submerged Jets," by M. L. Albertson, Y. B. Dai, R. A. Jensen, and Hunter Rouse, *Transactions*, ASCE, Vol. 115, 1950, p. 639

Another extensive investigation has been made of the development of the boundary layer on both smooth and rough surfaces.[15,16] Here again the experimental results are being obtained from air tests. These are of immediate importance in the problem of predicting the underwater drag of ship hulls and will undoubtedly be applied in the design of hydraulic structures in the future. The results of the air tests have already been used by William J. Bauer,[17] J. M. ASCE, in an investigation of the growth of the boundary layer in the accelerated flow of water in steeply inclined channels.

In connection with another project at the Iowa Institute, it was necessary to determine, experimentally, the best head form for a cylindrical body which would make possible a simple indication of the magnitude and inclination of the velocity of flowing water by measurements of pressures at suitable locations. A large number of head forms, such as blunt, partly rounded, hemispherical, ellipsoidal, ogival, and conical, were tested in a small open-throat air tunnel to obtain the pressure distributions around the head forms at different angles to the flow. The work in running the tests in air was only a small fraction of what would have been required for tests in a water tunnel, and the results agreed with the few data that were available from earlier water-tunnel tests.

The usefulness of air tests for the prediction and elimination of cavatition is illustrated well by the tests on outlet valves described by the author. Often, advantage can be taken of larger models for this type of test. At the Iowa Institute, water-tunnel tests on the occurrence of cavitation at gate slots[18] were supplemented by air-tunnel tests at a greatly enlarged scale to permit not only much finer detail in the local pressure measurements but also a relatively convenient control of the velocity distribution in the boundary layer to determine its effect on the pressure distribution.

Air tests have also been used to obtain data about gravitational phenomena in which the Froude criterion for similitude is just as applicable as it is to flow problems involving a free liquid-gas interface. In several instances the free convection above sources of heat was studied, for convenience, in air,[19,20,21]

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[15] "An Exploratory Investigation of Boundary-Layer Development on Smooth and Rough Surfaces," by W. D. Baines, thesis presented to the State University of Iowa, at Iowa City, Iowa, in August, 1950, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

[16] "An Experimental Investigation of the Boundary-Layer Development Along a Rough Surface," by Walter L. Moore, thesis presented to the State University of Iowa, at Iowa City, Iowa, in August, 1951, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

[17] "The Development of the Turbulent Boundary Layer on Steep Slopes," by William J. Bauer, thesis presented to the State University of Iowa, at Iowa City, Iowa, in August, 1951, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

[18] "Cavitation and Pressure Distribution at Gate Slots," by Aristokli Spengo, thesis presented to the State University of Iowa, at Iowa City, Iowa, in June, 1949, in partial fulfilment of the requirements for the degree of Master of Science.

[19] "Gravitational Diffusion From a Boundary Scource in Two-Dimensional Flow," by Hunter Rouse, *Journal of Applied Mechanics*, Vol. 14, September, 1947, p. A225.

[20] "Free Convection Due to a Point Source of Heat," by Chia-Shuen Yih, thesis presented to the State University of Iowa, at Iowa City, Iowa, in August, 1948, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

[21] "Gravitational Convection from Line Sources," by Harold W. Humphreys, thesis presented to the State University of Iowa, at Iowa City, Iowa, in February, 1950, in partial fulfilment of the requirements for the degree of Master of Science.

the results being applicable as well to convection in water, whether produced by the addition of heat or (inversely) of sediment-laden liquid. In another instance, the fall velocity of spherical particles in a cylindrical container of liquid was being studied as the ratio of the particle diameter to the diameter of the container was varied. Practical difficulties were met in these fall-velocity experiments when relatively large spheres were used. Therefore, dynamically similar air tests were made with the spheres fixed in an air stream passing through a short cylindrical duct. Instead of measuring the fall velocity, pressure measurements were made to determine the drag of the spheres, which in turn could be related to their fall velocities.[22] A large scale was chosen for the air-test model for convenience in construction and to obtain high Reynolds numbers, an advantage in the use of air tests noted by Mr. Ball.

In one current thesis project air flow is actually used to simulate the nonuniform flow of water with a free surface. Impossible as it may seem at first, the distribution of turbulence in the hydraulic jump is being studied in this manner. This is done by shaping a closed-conduit transition according to the profile of the jump at each of a series of Froude numbers. The similarity of the flow pattern is checked by measuring the pressure distribution on the boundary representing the free surface and comparing this with the distribution corresponding to the assumed hydrostatic conditions within the jump. Pitot traverses will be used to verify the mean velocity distribution at various sections and, finally, hot-wire anemometer measurements will give the distribution of turbulence. The extra work involved in using air in this case is warranted only because a reliable instrument is not yet available for measuring the various components of turbulence in water.

Besides these examples of the use of air tests in research, it is obvious that air flow may be very convenient for demonstrations, student instruction, and the preparation of motion pictures, particularly when the flow pattern is to be observed by means of foreign agents. Disposing of air containing smoke, heat, or chemicals is no problem, whereas cleaning or disposing of contaminated water may be very difficult or uneconomical. The student laboratory equipment at the institute includes a small air tunnel and an air-pipe assembly for the purpose of familiarizing students with the advantages of air tests and their applications in hydraulic problems. Two other air tunnels and supplementary blower systems are available for more extensive applications.

It is evident from the tests cited in the paper, as well as from the investigations mentioned in this discussion, that the use of air as a model fluid provides a convenient and economical means of solving many problems in hydraulic design and in hydraulic research. These examples are in accord with the established fact that practically perfect similitude can be obtained in a large number of problems in which there are no free-surface or cavitation effects. Moreover, even when such close similarity is not a practical possibility, the use of air for qualitative or exploratory studies is advantageous. The availability of highly developed, sensitive instruments permits measurements to be made quite simply in air and, furthermore, makes possible studies of turbulence in air that cannot be made in water at the present time. In spite of the author's conclusions to the contrary, the writer predicts that the many advantages accruing from the use of air will result in its playing an increasingly important role in the solution of hydraulic problems.

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[22] "Drag of Spheres Within Cylindrical Boundaries," by John S. McNown and John T. Newlin, *Proceedings*, First U. S. National Cong. of Applied Mechanics (publication pending).

James W. Ball,[23] A. M. ASCE.—Many excellent examples of low velocity air testing have been presented by Mr. Appel. The work cited is extremely interesting and indicative of the many outstanding facts and principles in fluid mechanics that can be established by this type of testing. The introduction of similitude increases the scope of the paper to include the fluid mechanics viewpoint. The paper is not a dissertation on similitude; neither does it misuse or advocate the misuse of this most valuable tool. The proper representation of boundary conditions is stressed as these are very important in attaining similitude in low velocity air testing as in hydraulic testing. Some boundary conditions are negligible whereas others are absolutely essential, but even these can often be represented artificially as illustrated in the paper.

Although not stated explicitly, the paper was written from the hydraulic point of view for engineers who are fearful of using compressible fluid data for solving problems relating to incompressible flow. There is no intention of implying that the results of air tests on hydraulic systems must be regarded with suspicion; rather, it is intended to encourage the use of air tests, but to caution that boundary conditions must be selected carefully. One of the most important factors retarding the use of low velocity air testing of models or full-size hydraulic structures is the problem of persuading hydraulic and design engineers to apply compressible fluid test data, confidently, to problems involving incompressible fluids. This reticence results from unfamiliarity with fluid mechanics and compressible fluids. A definite effort must be made to offset this factor when introducing air testing in laboratories where most of the problems concern incompressible flow. The paper represents such an effort.

In such laboratories (including those confined to pumps or turbines alone) it is not expected that outstanding hydraulic designs will be developed by direct testing with a low velocity compressible fluid, although the designs might be influenced considerably by similar tests which have established certain fundamental principles. Until the hydraulic engineers and designers are convinced that tests with low velocity air are as reliable as tests with water, they will insist on a check in the field of work with which they are familiar.

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[23] Hydr. Engr., Bureau of Reclamation, U. S. Dept. of Interior, Denver, Colo.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2518

LONGITUDINAL MIXING MEASURED BY RADIOACTIVE TRACERS

By Harold A. Thomas, Jr.,[1] and Ralph S. Archibald,[2] J. M. ASCE

With Discussion by Messrs. Conrad P. Straub and Donald A. Pecsok; Alfred C. Ingersoll; and Harold A. Thomas, Jr., and Ralph S. Archibald

Synopsis

A method of determining the magnitude and effect of horizontal mixing in pipes and tanks is available to the engineer through the use of radioactive tracers. These materials have shown many advantages over the commonly accepted use of salts and dyes.

The foremost advantage of radioisotopes is the ability to detect them in minute concentrations, eliminating misleading density currents that may occur when brine is used. In addition, precise methods of detection are available, and, in some instances, it is not necessary to take samples of the fluid being tested.

The paper developes a longitudinal mixing parameter that characterizes a given system and gives examples of its use. The experimental results are reported and analyzed, and techniques are briefly discussed.

Introduction

Salts and dyes have been used as tracers in flow systems primarily for the measurement of discharge. In the Allen[3] salt-velocity method, a slug or cloud of salt solution is abruptly injected into the flow, and a measurement is made of the time taken by the salt slug to travel to a downstream station. In sanitary engineering research, this technique has another and more important application, namely, the measuring of longitudinal mixing.

Part I—Theory of Longitudinal Mixing

The phenomenon of "shortcircuiting" of flow in the passage of fluids through conduits, streams, tanks, and lakes is the result of three factors: (1) The non-

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Note.—Published in August, 1951, as *Proceedings-Separate No. 84.* Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Associate Prof. of San. Eng., Div. of Eng. Sciences, Harvard Univ., Cambridge, Mass.

[2] Engr., Pipe Founders Sale Corp., Boston, Mass.

[3] "The Salt Velocity Method of Water Measurement," by Charles M. Allen and Edwin A. Taylor, *Transactions*, A.S.M.E., Vol. 45, 1923, p. 285.

uniform distribution of velocity caused by the curvature of and the shear at the solid boundaries of the flow; (2) eddy diffusion engendered by the turbulence of the moving stream; and (3) irregular displacements due to wind, local density differences, sludge removal mechanisms, boats, and pumps. The mixing from these sources causes the tracer cloud to disperse as it moves downstream. This action may be depicted diagrammatically for a one-dimensional flow system in which the origin represents the point of injection of the tracer, as shown in Fig. 1.

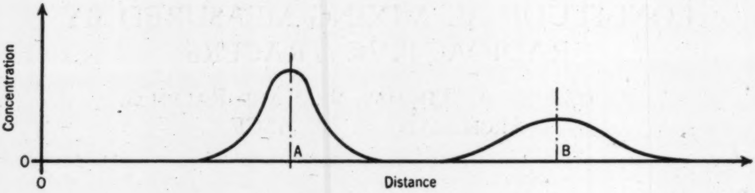
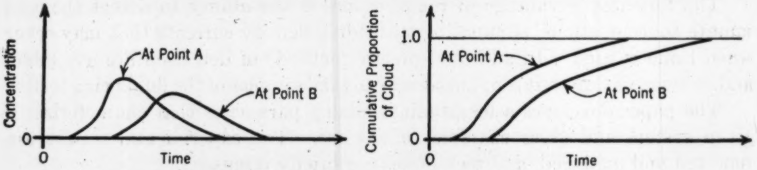


Fig. 1.—Space Distribution of Tracer Cloud

In some flows, such as those occurring in settling tanks and in ponds, the mixing action causes portions of the fluid to travel to a downstream station with velocities many times larger than the average velocity. The time required by different portions of the flow to travel to downstream stations may be measured by observing the rate of arrival of the dispersing cloud of tracer. This may be shown in two types of diagrams depicted in Fig. 2. The concentration-time distribution is generally asymmetrical because the slug continues to disperse during the period that it is passing the observation station.



(a) DIFFERENT PROPORTIONS OF FLOW (b) CUMULATIVE DISTRIBUTION

Fig. 2.—Time of Travel of Salt Cloud

The Effect of Longitudinal Mixing

In comparing the degree of longitudinal mixing in different flow systems it is convenient to refer the actual flow time to the average time of flow. Average flow time (T) is generally taken as the arithmetic mean time of flow that may be calculated for unsteady flow from the formula

$$T = \frac{\int_0^{t_1} t\, Q\, dt}{\int_0^{t_1} Q\, dt} \qquad (1)$$

in which Q is the variable discharge, and t_1 is the time for which the flow is averaged. For steady flow the equation becomes

$$T = \frac{V}{Q} \qquad (2)$$

in which V is the volume of the fluid between the two reference stations. The average flow time is the abscissa of the centroid of the time-concentration distribution (Fig. 2(a)).

The importance of longitudinal mixing is due to the fact that many dangerous and objectionable substances in water, such as pathogenic bacteria, human excrement, putrescible settleable solids, and radioactive wastes are destroyed (or removed) at rates approximately proportional to their concentrations. In still water the rate of disappearance (or decay or die-away) of such substances may be calculated approximately from the formula

$$u = u_0 \, e^{-kt} \qquad (3)$$

in which u is the concentration at time t (u_0 representing the initial concentration), k is a constant that may readily be determined experimentally, and e is the Napierian base of logarithms. The more complicated case of the die-away curve in turbulent flow is analyzed in the appendix.

When wastes of these types are faced in streams, pipes, or tanks, the downstream concentrations may be considerably higher than those given by Eq. 3 when t is taken as the average flow time. This is caused by short-circuiting in the flow. The efficiency of sewage treatment, water purification, and disinfection systems, and the capacity of streams and ponds for handling pollutional loads depends to a large extent upon the amount of longitudinal mixing occurring in the flow. The greater the amount of mixing, the higher the concentrations found downstream; laboratory tests performed in still water as a rule yield lower downstream concentrations than those attainable in moving water.

Some idea as to the possible magnitude of the short-circuiting effect may be had from an example. Assume a chlorine contact tank for the protection of a bathing beach has a detention period of 10 min and handles settled sewage containing 1,000,000 coliform organisms per milliliter. The tank is provided with 9 transverse baffles. Laboratory tests in still water determined that coliform organisms are destroyed by the chlorine in accordance with Eq. 3 at a rate corresponding to a k of 1.0 per min. The density of bacteria in the effluent may be calculated (a) by assuming that no mixing takes place, that is, the fluid moves as a unit, or (b) by assuming that violent mixing occurs and that the bacterial concentration in the tank in all regions is uniform.

In accordance with assumption (a) the coliform density in the effluent would be

$$u_1 = 10^6 \, e^{-10} = 50 \text{ coliform per ml}$$

In accordance with assumption (*b*) the density would be given approximately by the expression $u_2 = \frac{10^6}{10}\left[e^{-0.5} + e^{-1.5} + \cdots + e^{-9.5}\right]$ in which it is assumed that as a result of the violent mixing the effluent consists of ten portions having average detention times of 0.5, 1.5, $\cdots$ 9.5 min each. A more accurate expression is given by the equation $u_2 = \frac{10^6}{10}\int_0^{10} e^{-t}\,dt = -\frac{10^6}{10}\left[e^{-t}\right]_0^{10} = 10^5$ coliform per ml. The effect of longitudinal mixing in this case causes a bacterial density in the effluent of 2,000 times that occurring with no mixing. Actually the tank would give a performance between these limiting values. However, many existing tanks tend to perform more nearly in accordance with the second assumption than the first, and longitudinal mixing may seriously interfere with the efficiency of disinfection.

COEFFICIENT OF QUIESCENCE

The mixing performance of a tank or stream may be anticipated from the concentration-time distribution characteristics of the system obtained in tracer tests. The average concentration ($\bar{u}$) of a foreign substance at a downstream station in steady flow may be computed by the formula

$$\bar{u} = \int_0^\infty u\,p\,dt \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

in which $p\,dt$ represents the proportion of the flow (p is the ordinate of Fig. 2(*a*) or slope of Fig. 2(*b*)) passing the observation point during the time interval dt. The value of u can be found by Eq. 3. The average concentration ($\bar{u}$) may be computed by an arithmetic integration of Eq. 4, or it may be approximated by a simpler procedure. Many time-concentration distributions, it has been found, may be fitted satisfactorily by the equation

$$p\,dt = \frac{1\,N^N}{T\,(N-1)!}\left(\frac{t}{T}\right)^{N-1} e^{-\frac{Nt}{T}}\,dt \quad\ldots\ldots\ldots\ldots\ldots(5)$$

in which N is the coefficient of quiesence, a parameter that depends upon the degree of longitudinal mixing. For flow systems in which the mixing is very violent N approaches a limiting lower value of 1.0; for systems in which the mixing is zero (uniform, irrotational flow) N equals infinity. Test results for ordinary tanks and ponds show N-values range from 1.2 to about 6, depending upon the shape, baffling, inlet and outlet arrangements of the tank and the amount of stirring induced by wind, density differences, and sludge removal mechanisms.

A number of different procedures may be used to evaluate N from a particular tracer time-concentration distribution. The simplest method of fitting Eq. 5 consists of making the means and modes of the theoretical and observed curves equal. This leads to the relation

$$N = \frac{\text{mean time of flow}}{\text{mean time of flow—modal time of flow}} \quad\ldots\ldots\ldots\ldots(6)$$

The mean time of flow is the centroid of the distribution, T, and the modal time is the time corresponding to the peak concentration.

Integrating Eq. 4 by using the values of u and p as given by Eq. 3 and Eq. 5, the following relation is obtained:

$$\bar{u} = u_0 \left(1 + \frac{k\,T}{N} \right)^{-N} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Values of the relative proportion of the concentration $\left(\frac{\bar{u}}{u_0} \right)$ remaining in the flow as it passes the downstream section, for various values of $k\,T$ and N, are shown in Table I.

TABLE I.—PERCENTAGE OF INITIAL CONCENTRATION REMAINING IN FLOW AFTER PASSAGE TO DOWNSTREAM SECTION

$\frac{k\,T}{N}$	Values of coefficient of quiescence, N				
	1	2	4	6	10
0	100	100	100	100	100
1	50	25	6.25	1.56	0.98
2	33.3	11.1	1.23	0.14	1.7×10^{-3}
4	20	4	0.16	6.4×10^{-3}	1.0×10^{-5}
6	14.3	2.04	4.2×10^{-2}	8.5×10^{-4}	3.5×10^{-7}
10	9.09	0.83	6.8×10^{-3}	5.6×10^{-5}	3.9×10^{-9}

An application of the formulation embodied in Eq. 7 is given in the following example:

A mill pond of 6 acres, with an average depth of 12 ft, receives settled sewage from a village of 1,800 persons. During a period of low summer flow of 11 cfs the average ultimate BOD of the outlet is 2.42 ppm. The temperature is 68° F, k is 0.44 per day. Assume 0.25 lb per capita per day ultimate BOD.

1. Calculate the short-circuiting parameter N:V = volume of pond = 6(43,560) (12) = 3,130,000 cu ft; T = theoretical detention period = V/Q = (3,130,000)/(11) (86,400) = 3.30 days; L_a = BOD load = (0.25) (1,800) = 450 lb per day; and L_T = BOD in effluent = 2.42 (8.33) (11/1.547) = 144 lb per day.

Then, substituting in Eq. 7 $\frac{L_T}{L_a} = \frac{144}{450} = 0.32 = \left[1 + \frac{(0.44)\,(3.30)}{N} \right]^{-N}$. By cut-and-try solution, $N = 2.6$.

2. Calculate the BOD in parts per million at the mill pond outlet after construction of a reservoir upstream that is operated so as to increase the summer flow to 25 cfs. The increased dilution will reduce the BOD concentration; this effect, however, will be partly offset by the reduced time of flow. Assume that the value of N will remain the same. T = (3,130,000)/(25) (86,400) = 1.44 days; and $L_T = \dfrac{450}{\left(1 + \dfrac{0.44\,(1.44)}{2.6} \right)^{2.6}}$ = 255 lb per day. Therefore, BOD in pond effluent = $\dfrac{(255)\,(1.547)}{(8.33)\,(25)}$ = 1.9 ppm.

While Eqs. 5 and 6 have only an empirical basis, they have been found to formulate the effect of longitudinal mixing in a simple approximate way. A more rational approach is presented in a succeeding section.

PART II—EXPERIMENTAL TECHNIQUE

Prior to the production of radioisotopes in substantial quantities from atomic piles, tracer tests of flowing liquids were made with salts and dyes. These substances had disadvantages that under some conditions seriously impaired the value of test results. In testing a sedimentation tank, for example, the brine solution introduced at the inlet often would cause density currents that disrupted the normal patterns of flow, and dyes were apt to be partly adsorbed on solid surfaces or else react with other substances in the water to such an extent as to cause a color change. In general both salts and dyes may conveniently be detected in the flow if their concentration exceeds about 1/10 ppm (1/10 milligram per liter), but in many flow systems the mixing is so marked that a considerable proportion of the tracer clouds is dispersed to concentrations appreciably less than the threshold level of 0.1 ppm. The amount of tracer recovered at the downstream station was frequently found to be significantly smaller than that added to the flow. As a consequence, in early tests the so-called actual flow times, obtained by determining the centroid of observed time-concentration curves, were often much smaller than the theoretical average flow times as calculated by Eq. 1 or Eq. 2. From this it was inferred that dead spaces of stationary fluid existed in the flow. Recent tests with more sensitive methods of measuring concentration indicate that such regions do not ordinarily exist and that some mixing occurs in all parts of the flow. The so-called volumetric efficiency (actual flow time divided by theoretical average flow time), therefore, is really a measure of the sensitivity with which concentrations are measured rather than the effective volume of a tank or basin. When sensitive instruments are used, together with suitable tracers—ones that are not adsorbed or changed by reactions with other substances in the water—it has been found that actual flow times approximate the theoretical average time of flow.

Radioisotopes have advantages over the various salts and dyes that have been used as tracers. These may be listed as:

a. They may be detected in extremely small concentrations. A good count (100–1,000 disintegrations per min) may be obtained from samples taken from flows containing as little as 10^{-10} to 10^{-11} mg of radioactive substance per liter. In one test on a pond with a capacity of 18.3 acre-ft, only 5 ml of a solution containing 2×10^{-6} grams of radioiodine were used. These extremely low concentrations do not cause observable density currents, so the flow pattern remains undisturbed.

b. The presence of radioisotopes may be readily and positively identified by modern counters of many types. Radioisotopes from natural sources may occasionally be detected in water, but they are normally found in very small amounts and do not interfere with the identification of the tracer. The apparatus for making counts is simple to manipulate and gives precise results.

In small flow systems, hydraulic models, and the like, the passage of the tracer cloud may be detected by radiation (gamma rays) coming through conduit walls so that it is not necessary to sample the flow.

c. No chemical or biochemical reaction occurring in the flow interferes with the rate of disintegration of a radioisotope; temperature changes do not affect the rate of disintegration. Moreover, adsorption of the tracer on solid surfaces can be made a negligibly small factor with a suitable choice of isotope.

The tracer selected for the testing program was radioiodine, I-131. The cost of this radioisotope is not high. One millicurie of I-131 costs $0.75 (1951) and has sufficient activity for a tracer cloud having a volume of 250,000 gal. I-131 gives off both beta and gamma rays; the latter were found to be sufficiently penetrating so that detection was possible through the walls of small pipes. Iodide is not readily precipitated, adsorbed, or chemically changed by substances normally occurring in water and sewage. Another merit of I-131 was the fact that it yields a linear relationship between concentration and count rate over extensive range of concentrations.

Most important of all, I-131 has a half life of 8 days, which is about ideal for this form of tracer work. The half life of an isotope is the period required for one half of the initial energy to be dissipated. A shipment of 20 millicuries, for example, would at the end of 8 days have a strength of 10 millicuries and at the end of 16 days a strength of 5 millicuries. Elements with long half lives are undesirable because if accidentally spilled or ingested they may be expected to remain active for long periods, giving off radiation that interferes with testing or that may even be dangerous to the health of laboratory personnel. The short-lived elements, on the other hand, are apt to be expensive, since much of their energy may be wasted by disintegration during shipment or delays in testing programs.

One disadvantage pertaining to the use of radioactive tracers as a general research tool is the hazard to the personnel when overexposed to radiation. Excessive exposure may result in cancer, genetic effects, and other damaging results. The guard against excessive radiation must be continual. Precautions include blood tests, the wearing of instruments that record the amount of radiation received, and constant care in handling of radioactive solutions to prevent serious mistakes.

The Atomic Energy Commission[4] has published tolerance limits for various forms of radiation, and organizations using radioisotopes are required to comply with a number of regulations relating to personnel protection. The amounts of radioactive material necessary to perform flow tests of the type described in this paper were small relative to those common in other forms of research, and the safety measures observed were probably more stringent than really necessary.

EXPERIMENTAL RESULTS

To test the use of radioactive tracers in the salt-velocity method in conduits, both field and laboratory experiments were made. Both types of tests were entirely successful, but the laboratory results are perhaps the more interesting and informative. These laboratory tests were performed on a straight length of 1.25-in. pipe. The radioactive solution was injected quickly into the flow at an upstream point, and points on the curve of the concentration were detected by a Geiger tube placed against the pipe at points along the conduit downstream. Thus by varying the flow rate and the distance from the source, the variation in the cloud spread could be noted in relation to distance and flow rate.

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[4] "Interim Recommendations for the Deposit of Radioactive Wastes by Off-Commission Users," *Isotopes Division Circular B-6*, United States Atomic Energy Commission, Isotopes Division, Oak Ridge, Tenn.

TABLE 2.—RADIOACTIVE TRACER TEST DATA, PIPE LENGTH = 94 FEET

Flow (gallons per second) (1)	Reynolds number (2)	Theoretical time of flow, T (seconds) (3)	Mode (percentage of T) (4)	First trace (percentage of T) (5)	Average time (percentage of T) (6)	Relative height of mode (7)
0.0205	2,180	292.6	56.8	46.3	70.9	1.89
0.0841	9,410	71.7	82.0	73.6	88.6	....
0.1150	10,710	52.4	90.8	82.2	99.4	1.12
0.1367	14,210	44.3	98.4	89.4	100.0	0.89

Tables 2 and 3 illustrate these changes. In Table 2 the variation of the parameters with flow rate is shown. All of these values were obtained at a point a constant distance from the injection station. The results show that the mode (time of maximum intensity of radiation), that is given in terms of the theoretical time of passing, increases quite rapidly with the Reynolds number and is thus quite unreliable as a measure of discharge. The mode is often used

TABLE 3.—RADIOACTIVE TRACER DATA, CONSTANT FLOW TESTS

Distance (feet) (1)	Theoretical time of flow, T (seconds) (2)	Mode (percentage of T) (3)	First trace (percentage of T) (4)	Average time (percentage of T) (5)	Height of mode (6)
(*a*) REYNOLDS NUMBER = 14,210					
21.4	10.2	103.8	73.1	139.0	1.36
36.5	17.1	95.4	73.3	101.2	....
60.9	28.3	92.9	82.2	108.2	1.28
75.6	34.0	93.9	82.5	100.0	1.17
94.0	44.3	98.4	89.4	100.0	0.89
(*b*) REYNOLDS NUMBER = 7,490					
31.3	27.8	56.2	45.9	78.6	....
44.8	39.4	69.1	55.4	80.1	....
53.4	46.9	72.1	63.0	86.0	....
78.2	68.7	77.1	69.9	85.2	....
(*c*) REYNOLDS NUMBER = 218					
21.4	69.4	57.7	39.0	84.7	5.30
36.5	116.5	60.1	38.6	67.6	2.80
60.9	195.5	70.3	38.3	60.4	....
75.6	219.4	54.8	44.2	57.9	1.06
94.0	292.6	56.8	46.3	70.9	....

in dye studies for this purpose, but the results of these tests show that an error of 20% could be expected. The first trace of the cloud also shows the same unreliable characteristics. The average time of passage seems to be fairly reliable at the higher Reynolds numbers and would seem to be the best measure of discharge of the group.

In Table 3 the average time also stands out as the best measure, although at low Reynolds numbers it too is extremely inaccurate. The variation in the relative height of the mode is also shown, and it is interesting to note the rapid decrease of this quantity as the flow becomes more turbulent.

Results of a tracer test on a rectangular primary settling tank of a sewage treatment plant are shown in Fig. 3. The tank has an average depth of 8.75 ft

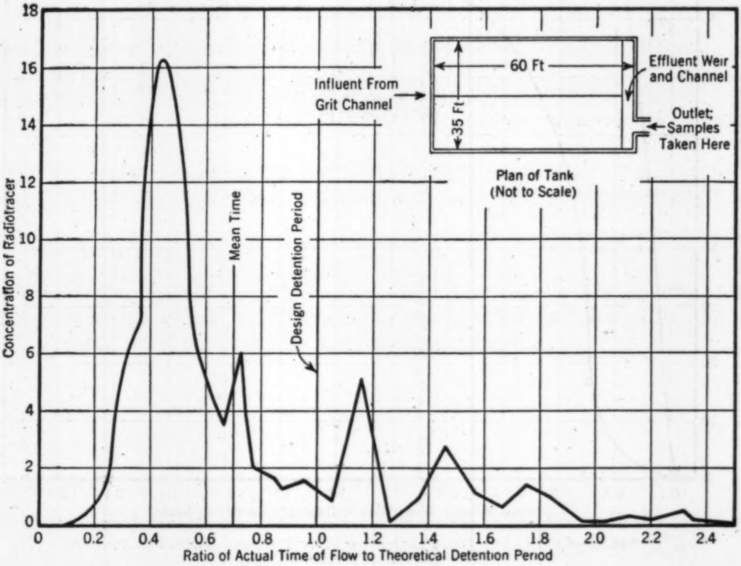

Fig. 3.—Concentration-Time Distribution for a Rectangular Primary Sewage Settling Tank

and at the time of the test the flow averaged 2.8 mgd. Sewage enters the tank through a series of pipes in parallel and leaves over a 32-ft long weir. A tracer dose of 5.4 millicuries of I-131 was used. The irregularities in the concentration time curve are to be attributed in part to small fluctuations in the rate of flow and in part to inherent irregularities in the tracer cloud. The peak shown at time ratio 1.2 was due to an increase in flow, but the other peaks are the result of normal mixing action. The average time of passage of the tracer (centroid of Fig. 3) was 41 min. The first sample (taken at 10 min) exhibited a significant count. It is of interest to note that dye that had been added at the same time at which the radioisotope was added had traveled only about 25% of the way across the tank toward the outlet at the time the first measurement was made.

The time for the first 10% of the cloud to pass through the tank was 22 min; the last 10% started through at 72 min, and the median flow time was 28.1 min. The N-value (Eq. 5) was found to be 3.0, indicating that a considerable amount of mixing took place.

In Fig. 4 the cumulative time distribution of the tracer cloud is indicated. A typical salt-tracer (sodium chloride) test on another settling tank reported by Charles H. Capen, Jr.,[5] M. ASCE, is presented. The very short flow times shown on this curve relative to the theoretical average detention time and also relative to the flow times of the other curve obtained with I-131 are noteworthy. This effect is to be attributed mainly to the density currents caused by the addition of brine.

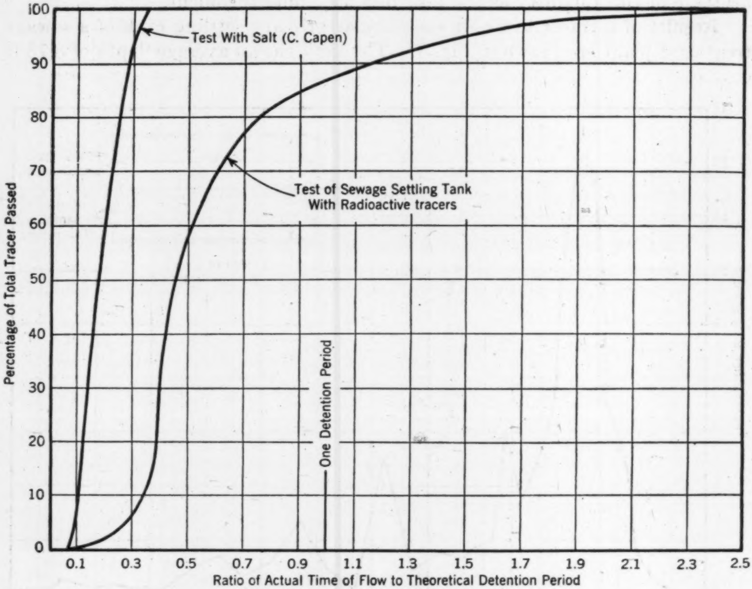

FIG. 4.—CUMULATIVE DISTRIBUTION OF FLOW TIME FOR TRACER CLOUD IN RECTANGULAR SETTLING TANK

ACKNOWLEDGMENT

The foregoing tests were made at Harvard University, and various places in eastern Massachusetts in connection with an experimental project to investigate the utility of radioactive tracers as a tool of the hydraulician. The project was financed mainly by funds provided by the Boston Society of Civil Engineers. The junior author was awarded a John R. Freeman Fellowship (1948) by this

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[5] "Study of Sewage Settling Tank Design," by C. H. Capen, Jr., *Engineering News-Record*, Vol. 99, 1927, p. 833.

organization. Additional aid was furnished by the Sanitary Engineering Division of the Massachusetts Department of Public Health.

Many other flow tests were made in other tanks, including model tanks, pipes, and sewers, and in streams and lakes; these are described in detail in a report by Mr. Archibald[6] to the Freeman Committee.

APPENDIX—ANALYSIS OF THE DIE-AWAY CURVE IN TURBULENT FLOW

If a waste, that in still water disappears in accordance with Eq. 3, is added at a constant rate at the head of a long stream, the concentration profile after a steady state has been attained will decrease monotonically to zero at infinity downstream. An exact mathematical analysis giving the shape of the concentration profile under general conditions is extremely complex. The following assumptions are made to simplify the formulation and do not greatly lessen the generality of the solution in so far as the mixing caused by eddy diffusion is concerned.

(1) The flow in the stream is steady, uniform, and two dimensional (wide rectangular channel).

(2) The waste does not settle out, nor is it adsorbed on boundaries.

(3) The velocity distribution at a cross section (x-constant) is parabolic,

$$v = v_s - c(h - y)^2 \qquad (8)$$

in which h is the depth of the stream and v_s the surface velocity (the velocity at the bottom of the stream is not necessarily zero). This assumption for the velocity distribution, together with the Prandtl equation

$$\epsilon = \frac{\tau}{\rho \dfrac{dv}{dy}} = \tau_0 \frac{\left(1 - \dfrac{y}{h}\right)}{\rho \dfrac{dv}{dy}} \qquad (9)$$

in which ϵ is the kinematic eddy viscosity and the term $\sqrt{\tau_0/\rho}$ is the shear velocity, leads to the predication of a constant ϵ in the y-direction (normal to the direction of flow):

$$\epsilon = \frac{\tau_0}{2\,c\,h\,\rho} \qquad (10)$$

The turbulence is taken to be isotropic.

(4) The diffusion of substances in suspension or solution in water proceeds exactly as water in water; that is, it is assumed that the rate of transport in a direction s will be $-\,\epsilon \dfrac{\partial u}{\partial s}$. These assumptions lead to the differential equation

$$\epsilon \frac{\partial^2 u}{\partial x^2} + \epsilon \frac{\partial^2 u}{\partial y^2} - v \frac{\partial u}{\partial x} - k\,u = 0 \qquad (11)$$

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[6] "Radioactive Tracers in Flow Tests," by Ralph S. Archibald, *Journal*, Boston Soc. of Civ. Engrs., Vol. 37, 1950, pp. 49–116.

in which ϵ is constant, v is given by Eq. 8, x is the direction of flow (positive downstream), and y is the direction perpendicular to the flow (positive upward).

An appropriate set of boundary conditions are the following:

(*a*) The waste uniformly mixed with the stream flow at a given initial concentration (u_0) enters the channel with a high velocity (as induced by a pump or weir) so that diffusion in the upstream direction against the current is impossible. Then for all y-values at $x = 0$, continuity of transport requires that $u'_0 v - \epsilon \dfrac{\partial u}{\partial x_{x=0}} = v\, u_0$ in which u'_0 is the concentration in the channel at $x = 0$ and u_0 is the concentration entering the channel (upstream from the pump)

(*b*) The concentration for all values of y at $x = \infty$ is zero.

(*c*) For all values of x, $\dfrac{\partial u}{\partial y_{y=0}} = 0$, $\dfrac{\partial u}{\partial y_{y=h}} = 0$.

Using a method of relaxation, arithmetic integration of Eq. 10 subject to the foregoing boundary conditions, with various values of the parameters c and ϵ has indicated that the former has the greater effect on the concentration profile, particularly upon the order of contact for higher values of x.

An exact solution of Eq. 10 for $c = 0$ (uniform velocity distribution) is given by the following expression:

$$u = \frac{2\, u_0}{1 + \sqrt{1 + \dfrac{4\, k\, \epsilon}{v^2}}}\, e^{-j} \quad \text{(12)}$$

in which (to simplify typography):

$$j = \frac{\sqrt{1 + 4\, k\, \epsilon/v^2} - 1}{2}\, \frac{vx}{\epsilon} \quad \text{(13)}$$

The solution for this case depends upon two parameters, k/v and ϵ/v; if these are great, then the profile will differ markedly from that obtained assuming $\epsilon = 0$, namely,

$$u = u_0\, e^{-\frac{kx}{v}} \quad \text{(14)}$$

This equation corresponds to Eq. 3.

DISCUSSION

Conrad P. Straub,[7] **A. M. ASCE, and Donald A. Pecsok**[8].—The determination of flow patterns and mixing in tanks has been a problem which has confronted design engineers for many years. As stated by the authors (see "Introduction"), salt and dye methods have been used, but the results obtained by these techniques have not always been satisfactory. The use of radioactive tracers offers the engineer a convenient and simple procedure for studying the complex phenomena of hydraulic flow and evaluating some of the parameters involved. Of course, special equipment must be available for measuring the radioactivity in the pipe channel itself, or in the tanks. Equipment has been developed which makes possible the continuous monitoring of activity in pipe lines or in tank effluents. With such equipment it should be unnecessary to withdraw and measure individual samples. With future development of immersion-type counting equipment, it may be possible to measure the activity throughout the liquid in a tank, conduit, or natural body of water.

Experience gained at Oak Ridge National Laboratory (ORNL), Oak Ridge, Tenn., in the use of carrier-free and carrier-added radioiodine-131 has indicated that when carrier-free I–131 is added to activated sludges or when passed through trickling filters, much of the I–131 will be adsorbed or combined with the organic matter. These removals, which have occurred to a lesser degree with raw sewage, are at variance with the statement by the authors (see fourth paragraph preceding "Experimental Results"): "Iodide is not readily precipitated, adsorbed, or chemically changed by substances normally occurring in water and sewage." It is assumed that the authors have reference to radioactive iodine in the iodide form, in weak basic sodium sulfite solution, which is the form in which it is shipped from the Operations Division, ORNL. The ORNL studies show that the amount of radioactive I–131 removed from solution is dependent upon several factors, most pertinent of which are: The amount of organic matter in the medium used, the time of contact, pH, and the amount of carrier KI added.

Before use is made of any radioactive tracer for flow tests in tanks, natural basins, or channels in which sewage or industrial wastes may be present, it would be desirable to run jar-test studies to determine the "take-up" of the radio element by the material contained in the waste. Such studies will also provide data on the concentrations of the radioisotope that must be used. For example, if it is found that 80% to 90% of the I–131 would be removed by the organic matter present in an aeration tank under investigation, it would be necessary to increase the initial dose by a factor of 10 to obtain desirable measurable concentrations in the effluent—that is, concentrations comparable to those that would have been obtained if there had been no removal of I–131 from solution by the organic matter. Tests with raw sewage have indicated that from 10% to 20% of the I–131 could be adsorbed on the organic matter. It is presumed that organic matter found in streams would react in a similar fashion. To reduce the uptake of activity from solution by organic matter the addition of carrier KI has been found quite effective. Dosages as large as 10 ppm of KI have been adequate for this purpose.

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[7] San. Engr., Public Health Service, Oak Ridge, Tenn.

[8] Senior Asst. San. Engr., Public Health Service, Oak Ridge, Tenn.

Since there is the possibility that I–131 may be volatilized and lost into the atmosphere, the use of this tracer is not recommended when longitudinal mixing is investigated, in tanks containing acid liquids or in acid streams.

The definition of half life given by the authors (in the third paragraph preceding "Experimental Results")—

> "The half life of an isotope is the period required for one half of the initial energy to be dissipated"—

may be confusing, although the example clarifies the statement somewhat. A preferable definition would be

> "The half life is the time required for the radioisotope to lose one half of its total activity; that is, the time required for one half of the original atoms to disintegrate."

If the authors' definition had been stated in terms of total energy, less confusion might have resulted. Actually, the energy released per disintegration is constant and has no relation to the age of a particular radioisotope.

Alfred C. Ingersoll,[9] A. M. ASCE.—The authors are to be commended for opening a new channel in the field of tracer technology in sanitary engineering and for finding another peacetime use of radioactive isotopes. Radioactive "markers" have been used in many industrial applications since 1946, as in locating pipe obstructions and serving as boundary markers for successive shipments of oil in pipe lines. The extension presented in this paper, to quantitative analysis of the mixing process, will provide a valuable research tool for attacking problems on the flow through mixing tanks, aeration chambers, and settling basins.

In the example of the effect of longitudinal mixing in which a chlorine contact tank is considered under the two extreme cases of no mixing and of violent mixing, it was assumed that, as a result of the violent mixing, the effluent consisted of ten portions having average detention times 0.5, 1.5, ··· 9.5 min each. Such an assumption would lead to an average detention time of only 5.0 min for the tank with a 10-min nominal detention period. Although this may not seem incompatible with reality, in view of the violent mixing which certainly causes much of the flow to be detained for less than half the nominal period, the fact is that the average detention time in the perfect mixing tank is equal to the displacement detention time, or twice that implied by the authors. The distribution curve of the individual detention times among the particles of an effluent sample can be shown to be essentially the same as the concentration-time distribution curve (Fig. 2 (*a*)), which represents the probability of detention time for any fluid element at the influent. The concentration-time distribution curve for the ideal mixing tank is given by the differential equation,

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[9] Asst. Prof. of Civ. Eng., California Inst. of Technology, Pasadena, Calif.

$$\frac{dc}{dt} = -\frac{c}{T} \qquad (15)$$

which integrates into

$$\frac{c}{c_0} = e^{-t/T} \qquad (16)$$

in which c is the concentration at time t (c_0 representing the initial concentration). Since the total area under this dimensionless curve is unity, the area between any two ordinates represents the proportion of the flow having detention time in this range. Next, the left side of Eq. 16 can be considered as a relative probability and the exponent (or abscissa) on the right side can be written as a function of the area, A, under the probability curve

$$\frac{t}{T} = \log_e \frac{1}{1 - A} \qquad (17)$$

Eq. 3 then becomes

$$u = u_0 e^{-kT_4/T} \qquad (18)$$

and for the final result, with violent mixing (introducing Eq. 17 into the exponent in Eq. 18):

$$u_2 = 10^6 \int_0^1 e^{10 \log_e (1-A)} dA = \frac{10^6}{11} \qquad (19)$$

which is seen to be not greatly different from the result obtained by the authors, the reason being that u_2 is determined principally by the extreme short-circuiting of the first portions of the flow to reach the effluent.

In their discussion of experimental technique (Part II) the authors state that "In general both salts and dyes may conveniently be detected in the flow if their concentration exceeds about 1/10 ppm * * *" In his experience with sodium phosphate as a tracer, in the study of model separating chambers, the writer has found concentrations of the phosphate ion of only 0.006 ppm to be clearly distinguishable with the use of a standard colorimetric technique.[10] Such concentrations were found to produce no discernible density current effects. Although sodium chloride is notorious for its density current effects, it does not seem fair to compare the Capen tests[5] with a salt tracer to the tests with radioactive tracers in Fig. 4 on the basis mainly of density currents. Of equal importance is the fact that the tests were conducted on different tanks with different inlet devices. When sodium chloride is used in the test of a settling tank the density effect often shows up as an excessive delay in the modal or mean time and the final scavenging of the salt from the tank. This occurred on tests by H. E. Babbitt, M. ASCE, and H. E. Schlenz,[11] A. M. ASCE, in which the peak of the distribution curve was delayed beyond the nominal detention time.

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[10] "Determination of Hydraulic Characteristics of Separating Chambers," by Alfred C. Ingersoll, *Proceedings*, Midwestern Conference on Fluid Dynamics, 1950, p. 393.

[11] "Results of Tests of Sewage Treatment," by H. E. Babbitt and H. E. Schlenz, *Bulletin 198*, Univ. of Illinois Eng. Experiment Station, Part 5, 1929, pp. 54–83.

The data given in Tables 2 and 3 provide some interesting information on the flow of fluids in pipes. The percentage values for the first trace, given in Col. 5, for example, can be interpreted as giving the ratio of the average to the maximum velocity in the pipe, if it may be assumed that the first trace arrives with the velocity of the fastest part of the flow (generally considered to be along the center line of the pipe). If it be assumed, more logically, that the first trace arrives with a combined velocity, made up of the maximum instantaneous velocities of the fastest moving particles in the turbulent flow, then the values in Col. 5 would be expected to be less than the values of V/V_{max}. With the foregoing in mind, it will be illuminating to compare values from Table 2 with the best obtainable values for V/V_{max}, computed from the Nikuradse formula,

$$\frac{V}{V_{max}} = \frac{1}{1 + 1.435\sqrt{f}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

with values of the friction factor, *f*, given by Lewis F. Moody,[12] for commercial steel or wrought-iron pipe.

Table 4 shows that the first trace behaves in accordance with the second of the foregoing assumptions for the first of the runs considered, but there is no ready explanation for the seemingly anomalous condition in the second and third runs. It may well be that the tracer test has provided a more accurate measure of V/V_{max} than can be obtained from the Nikuradse formula. The first of the four runs given in Table 2 cannot be used in this comparison since it lies in the critical range between laminar and turbulent flow.

TABLE 4.—COMPARISON OF FIRST TRACE TIME WITH V/V_{max} FOR PIPE FLOW

Reynolds number	First trace[a]	Friction factor, *f*	V/V_{max} (%)
9,140	73.6	0.0332	79.2
10,710	82.2	0.0324	79.5
14,210	89.4	0.0306	79.9

[a] Percentage of *T* from Table 2.

Given an accurate concentration-time distribution curve for the flow in a uniform reach of pipe, it should be possible to construct a reasonably valid profile of velocity distribution. This method might then be applied to such analyses in cases where it would be easier than the installation of a pitot tube.

Laminar flow in tubes affords an especially good testing ground for the radioactive tracer, for in this case the first trace should not appear in less than half the nominal time, except through molecular diffusion. The results of Table 3(*c*), at a Reynolds number of 2,180, show the percentage for the first trace to be significantly less than 50. This shows that the velocity distribution in the critical flow range is even more distorted from the uniform than for pure laminar flow. It will be valuable if the authors are able to present one run in the bona fide laminar range to prove the tracer diffusion rate. In such a run, if the first trace appears in appreciably less than 50% of the nominal time, the radioactive tracer will be open to question as an accurate indicator of the flow.

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[12] "Friction Factors for Pipe Flow," by Lewis F. Moody, *Transactions*, A. S. M. E., November, 1944.

The writer concurs with the authors in their conclusion that the modal time is not a reliable measure of discharge. It has been his experience, however, that the median time is a more reliable indicator of short-circuiting behavior than is the average time, which is always affected by the long tail of the distribution curve. This is not to contradict the authors' statement that the average time seems to be the best measure of discharge of the group.

Certainly there are many applications of tracer technology which may soon be expected to benefit from the multiple advantages of radioactive tracers. It is to be hoped that researchers in their use will be actively pursued so as to further the information now available.

Harold A. Thomas, Jr.,[13] and Ralph S. Archibald,[14] J. M. ASCE.—The discussion has been gratifying in that a number of relevant points pertaining to tracer technology have been elucidated.

Mr. Ingersoll perfers a method of calculating the coliform content in a chlorine contact basin under the assumption of violent mixing, which the writers agree is rational. Indeed, it was a generalization of his line of reasoning that underlies Eqs. 5 and 7. His numerical result may be obtained by direct substitution in Eq. 7.

His thoughts on the use of the tracer test to obtain a measure of the velocity distribution are perceptive. The writers concur with Mr. Ingersoll that in the turbulent regime the maximum velocities, calculated from the time of appearance of the first trace, should be larger than those given by Eq. 20. However, tests continue to yield the seemingly anomalous results noted by Mr. Ingersoll. For example, in a carefully conducted run in a 6,400-ft reach of an old tuberculated 20-in. pipe having a friction factor, f, of 0.074, and carrying a discharge of 6.75 cu ft per sec, the first trace was observed at a time corresponding to 78.2% of the mean flow time. According to Eq. 20 this value should be no larger than 71.8%.

Part of the discrepancy, at least, may be attributed to the difficulty in timing the arrival of the first trace precisely. The first portion of the slug is highly diluted and yields counts differing only slightly from the background count. Consequently, the net count may be evaluated only with a low degree of precision. It is hoped that this question will be resolved with special counting equipment in the Sanitary Engineering Laboratory at Harvard University. Apparatus entailing a heavy iron shield for the elimination of local gamma radiation, together with a bundle of anti-coincidence tubes for screening cosmic background, yielded very low background counts (3 to 8 counts per min). Part of this remaining background was found to be due to natural radioactivity (K^{40}) in the glass of commercial GM tubes. Therefore, special tubes have been developed. Preliminary tests indicate that a considerably higher order of sensitivity and precision is attainable for tracer studies in pipes, streams, and lakes.

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[13] Associate Prof. of San Eng., Div. of Eng. Sciences, Harvard Univ., Cambridge, Mass.

[14] Engr., Pipe Founders Sale Corp., Boston, Mass.

Messrs. Straub and Pecsok mention certain imperfections of I-131 as a tracer, and emphasize the necessity of using a carrier in many types of tests. The writers agree as to the importance of preliminary tests to ascertain "take-up" by various chemical, biochemical, and physical processes. Only by such investigation is it possible to ascertain the proper amount of radio tracer and carrier for a particular test.

Although I-131 undoubtedly is not the hypothetical "perfect tracer," under some conditions it has properties that are not far from ideal. In tests in the 6,400-ft pipe previously mentioned, a 90% recovery of I-131 was obtained in a run in which no carrier was added. With 10 ppm of NaI the recovery was virtually 100%. In runs with P-32, on the other hand, recoveries of less than 25% were obtained even in runs involving high isotopic dilution. This undesirable characteristic would appear to limit the utility of phosphorus as a tracer, whether measured by radioactivity or by chemical tests mentioned by Mr. Ingersoll.

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Paper No. 2519

APPLICATION OF HIGHWAY CAPACITY RESEARCH

By J. P. Buckley,[1] M. ASCE

With Discussion by Messrs. Donald S. Berry, Joseph Barnett, O. K. Normann, and J. P. Buckley

Synopsis

Practical application of the principles of highway capacity research are outlined in this paper. The development and use of comprehensive capacity charts are illustrated for the cases of two-lane rural roads, two-way city streets, and one-way city streets.

Introduction

The principles of highway capacity research as well as the practical applications of these principles are outlined in the paper by O. K. Normann. The study referred to in that paper[2] is recognized as one of the most thorough in highway research history and one of the most important advancements in highway engineering in many years.

Capacity, the number of vehicles a highway, road, or street can accommodate safely and efficiently, is one of the fundamental problems in motor transportation. The objective of highway engineers in this field is to provide sufficient capacity on highways and streets to carry safely and efficiently the traffic volumes at the minimum cost. Inaccurate estimations of required capacity can be costly, since the design and construction of facilities with more than required capacity is obviously a waste of public funds. However, under-design, or failure to provide adequate capacity, is equally wasteful. Lack of capacity means congestion, costly delays, and accidents, and often the cost of correction is far more than the cost of proper initial design and construction.

Although highway capacity has been extensively studied for many years, the work of the committee on Highway Capacity of the Highway Research Board covers the subject more extensively than any previous investigation and

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Note.—Published in November, 1951, as *Proceedings-Separate No. 101*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Chf. Engr., Highways Div., Automotive Safety Foundation, Washington, D. C.

[2] "Principles of Highway Capacity Research," by O. K. Normann, *Transactions*, ASCE, Vol. 117, 1952, p. 913.

gives the basic factors necessary to insure greater accuracy in capacity determinations and design.

PLANNING OF SURVEYS

Design engineers and traffic engineers are continuously faced with the problem of capacity determinations on specific sections of highways and streets. However, the problem is more complex in connection with state-wide highway planning studies.

The principal objectives of these studies are to determine the physical deficiencies and needs of all highways, roads, and streets in the state and to prepare a long-range plan to meet these needs on an efficient and economical basis.

In a highly urbanized state, such as Ohio, with many large cities and extensive mileage of roads and streets carrying high traffic volumes, the problem of capacity has unusual importance. Not only must the practical capacity of the existing roads and streets be determined, but in addition it is necessary to know the type of design required to provide adequate capacity at a minimum cost on new facilities. The extensive mileage of roads involved in this determination requires that the field studies be made by several hundred state, county, and city engineers. Since these engineers take on this work in addition to their regular activities and since there is a time factor, it is highly important that the task be simplified by every possible means without sacrificing accuracy. Further, since one of the requirements of the survey is to determine the comparative needs of the state highway, county, and city systems, the work must be done on a uniform basis with standards of appraisal. Finally, in view of the large amount of detail involved and the importance of producing a sound and accurate program, the central supervisory engineering staff of the study must have a method of checking the projects submitted by state and local engineers to insure their accuracy. So, far as capacity is concerned, the work of the Committee on Highway Capacity has provided the tools for meeting these conditions.

However, Mr. Normann has indicated the very large number of variables involved in the determination of capacity, lane width, sight distance, terrain, operating speed, and many others. The comprehensive report of the Committee shows the importance of sound engineering judgment in applying the data. The problem, then, is to determine a method of applying the basic factors on a mass production basis within reasonable limits of accuracy.

HIGHWAY CAPACITY CHARTS

As part of the organization of the Ohio Highway Study Committee, special engineering advisory committees, composed of the state's most able highway and traffic engineers, were established. Following several months of study of the report of the Highway Capacity Committee, a series of three basic circular charts were prepared for use in determining the capacity of: (*a*) two-lane rural roads; (*b*) two-way city streets; and (*c*) one-way city streets. These three charts actually are the final product of a long series of charts combined for more efficient handling. The circular scheme lends itself to a tabular arrangement of a number of variables in a limited space.

The normal method of applying the capacity factors for a specific problem would be to consider successively each of the factors that affect capacity and go through the mathematical steps of applying these factors to arrive at the final answer. In preparation of the charts the staff engineer has gone through this process for several hundred possible sets of conditions. To simplify the work, only the basic factors that have a material effect on capacity were considered in this process. Field checks in several parts of the country have indicated that the limits of accuracy are reasonable.

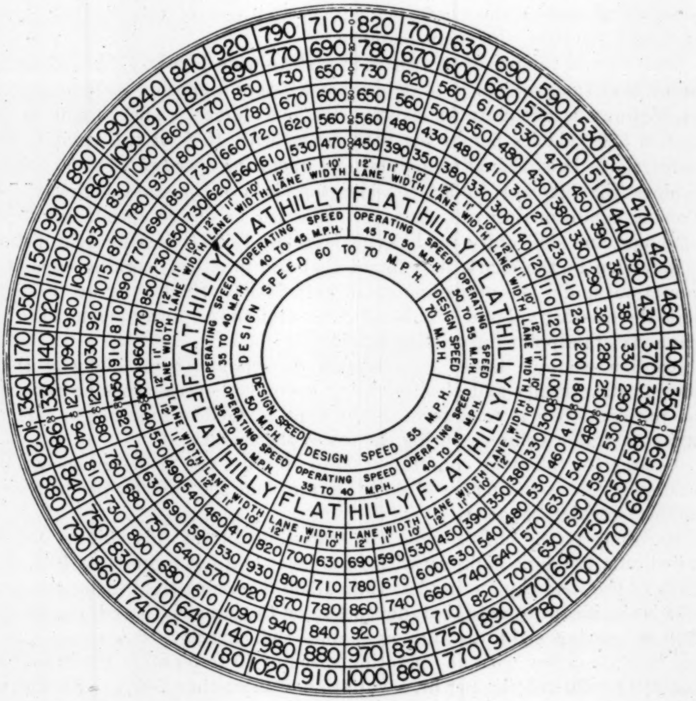

FIG. 1.—PRACTICAL WORKING CAPACITIES FOR TWO-LANE RURAL ROADS IN TERMS OF TOTAL VEHICLES PER HOUR IN TWO DIRECTIONS

In accordance with the definition established by the Highway Capacity Committee, the capacity figures in the charts are practical working capacities —the maximum number of vehicles that can pass a given point during 1 hr without producing a traffic density so great as to cause unreasonable delay, hazard, or restriction to the driver's freedom to maneuver under the prevailing roadway and traffic conditions.

Practical Working Capacities for Two-Lane Rural Highways.—The first chart (Fig. 1), covering capacities for two-lane rural roads, sets up practical capacities under 252 sets of conditions. These conditions include design speed, operating speed, terrain, lane width, and sight distance. Additional simple calculations that further refine the answers can be made for variations in the percentage of commercial vehicles. For the purposes of this chart, a commercial vehicle is considered to be any vehicle having dual tires. The values given in Fig. 1 include an assumed 10% of commercial vehicles. For each 1% that the actual number of commercial vehicles varies from the assumed figure, it is necessary to add or subtract 1% from the capacity figures in flat terrain or 2% from the capacity figures in rolling terrain. The outer six concentric circles of the chart indicate percentages of the roadway having less than 1,500 ft passing sight distance.

To illustrate the use of this chart in a state-wide highway planning study, assume that the problem is to determine whether a specific section of a two-lane highway is adequate for existing traffic volumes. The volume in peak hours is 580 vehicles per hr for both lanes, with 10% of the vehicles being commercial. It is desired to maintain an operating speed of 45 to 50 miles per hr, since the highway under consideration is a main state highway. The following conditions prevail: 60 to 70 miles per hr design speed; flat terrain; 11-ft lanes; and 60% of the road having less than 1,500 ft passing sight distance.

Referring to the chart, it is found that the practical working capacity of the section, under the prevailing conditions, is 560 vehicles per hr, or slightly under the existing volume. This indicates that the highway is adequate from a capacity standpoint for existing traffic.

Another example might be a section of two-lane rural highway that is deficient structurally and also operating beyond practical working capacities. The problem then is to determine the type of facility required to carry the maximum traffic volumes during the life of the facility.

Assume the following conditions: 60 to 70 miles per hr design speed; 40 to 45 miles per hr operating speed; hilly terrain; and 850 vehicles per hr required practical working capacity.

Referring to the chart, under the stated conditions this volume could be carried on a two-lane highway with 12-ft lanes and not more than 40% of the length of the roadway having less than 1,500 ft passing sight distance.

It is significant to note that, depending on the lane width and percentage of 1,500 ft passing sight distance available, the capacity of a two-lane rural road in flat country with an operating speed of 45 to 50 miles per hr can very from 350 to 820 vehicles per hr, a variation of more than 200%. Similar wide variations in capacity are found for other sets of conditions. The significance of the true economy of proper design has never been illustrated better.

Some rural roads are known to carry more vehicles per hour than are shown on this chart, but they do so only at the expense of reduced safe operating speeds.

Practical Working Capacities for Two-Way Streets.—City street capacities are affected by a wider variety of factors than are rural roads, and the second chart (Fig. 2.), covering capacities for two-way streets, includes more basic factors. The chart is first divided into the three accepted districts of a city—downtown, intermediate, and outlying. The factors of parking, percentage of commercial vehicles, existence or prohibition of left turns, and curb to curb widths of streets are also taken into consideration. The chart gives directly the capacity under 144 sets of conditions. Interpolations may be made quickly for odd-width streets.

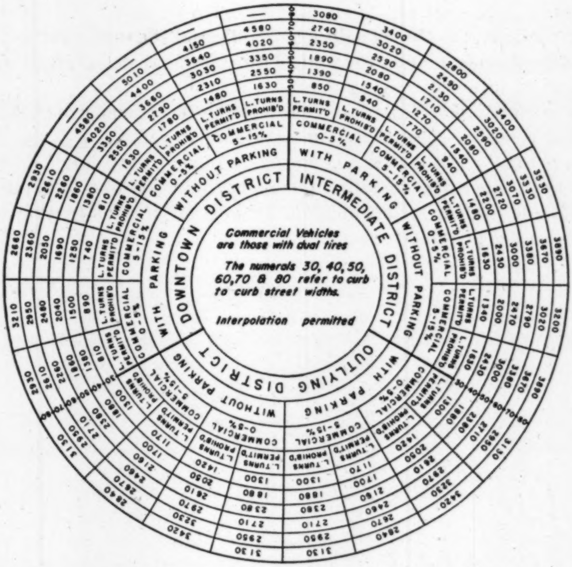

FIG. 2.—PRACTICAL WORKING CAPACITIES FOR TWO-WAY STREETS IN TERMS OF TOTAL VEHICLES PER HOUR

Additional computations for special conditions are as follows:

(1) If commercial vehicles exceed 15% of the total traffic, subtract 1% from the capacity value for each 1% of commercial vehicles over 15%;

(2) If the street being studied is a bus route, multiply final value by 0.88;

(3) If the street is a streetcar route, regardless of district, use values of downtown—with parking;

(4) If fixed time signal is used, multiply the capacity by the ratio of green interval to total cycle; and

(5) If streetcar turns, deduct the time from the seconds of green.

This chart is used in the same manner as the chart for rural capacities, by checking through the appropriate conditions to arrive at the capacity figure and by making such adjustments for additional factors as may be required.

The basic data of the highway capacity committee, as condensed in this chart, illustrate dramatically the amazing increases in practical capacity that can be secured by comparatively inexpensive and simple methods, instead of the alternative of expensive widening or other major construction.

For example, the practical capacity of a 40-ft street in the downtown district, carrying 10% commercial traffic when both parking and left turns are permitted, is 1,250 vehicles per hr. If parking were eliminated during the rush hours and left turns prohibited, the practical capacity could be doubled to 2,550 vehicles per hr.

It is recognized that public support for such measures is often a problem. However, such facts as these enable the public and the public officials to decide intelligently whether or not the added capacity and its resulting freedom of movement and lessened congestion are worthwhile.

In the application of these data in appraising the capacity of existing streets, engineers are shown specifically the results that can be obtained by taking these other steps. Without this evidence, in some cases it might be wrongfully assumed that expensive construction is the only solution. These data will prove valuable also in reconciling the submission of construction projects in the light of other possible treatments.

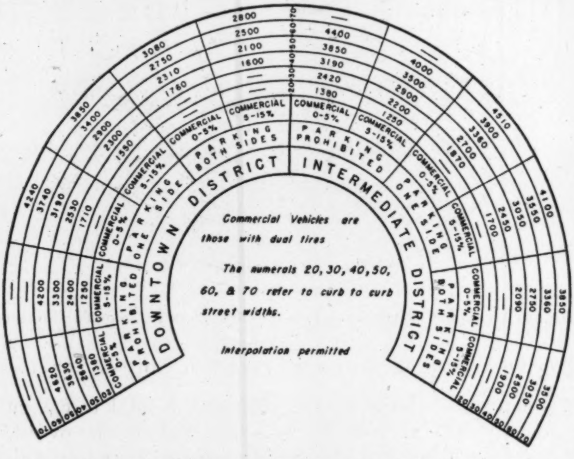

FIG. 3.—PRACTICAL WORKING CAPACITIES FOR ONE-WAY STREETS IN TERMS OF TOTAL VEHICLES PER HOUR

Although the major increases in capacity that are possible through these steps are important and substantial, obviously, it must be recognized that such steps will not solve all problems and, under some conditions, major construction of freeways or other high-volume facilities is the only answer. However, proper application of these data does insure that all practical steps will be considered and that the resulting program will be both conservative and necessary. It should be noted that there is a very narrow margin between the practical working capacity as shown in Fig. 2 and the existence of undesirable or almost unbearable congestion.

Practical Capacities for One-Way Streets.—The third chart (Fig. 3) covers practical capacities for one-way streets and is similar to and includes the same factors as the chart for two-way streets. In addition to its utility in determining the practical capacities of either existing or proposed one-way streets, the tabulation of basic material again quickly illustrates the efficiency of one-way streets when they can be established. For example, a 40-ft street in a downtown district, having a capacity of 2,550 vehicles per hr as a two-way street under the same basic conditions, could carry 3,630 vehicles per hr (or an increase of 42%) when operated as a one-way street.

In the application of these capacity data to long-range highway planning studies, similar determinations have been made for multi-laned divided rural highways and freeways, and for freeways in cities. Again, application of capacity determinants has helped to establish uniform standards for such facilities and results in a method for checking the need and the adequacy of proposed improvements.

ACKNOWLEDGMENT

The Bureau of Public Roads, the Highway Research Board Committee on Highway Capacity, and numerous state, county, and city engineers gathered the basic field data on capacity through countless observations over a long period of time. The Committee on Highway Capacity and, particularly, its chairman, Mr. Normann, and his assistants in the Bureau of Public Roads have worked many years in preparing the committee's report.

Glenn Logue and Harold Eckhardt, engineers of the Ohio Department of Highways, were principally responsible for devising the circular capacity charts used in this paper.

DISCUSSION

Donald S. Berry,[3] A.M. ASCE.—The capacity charts described by Mr. Buckley were designed for use in determining deficiencies in state and county highways and in city street systems, as part of a study of "Ohio Highway Needs." The method of determining capacity necessarily was to be simple for uniform application by several hundred state, county, and city engineers. As stated by Mr. Buckley (see under the heading, "Planning of Surveys"), "* * * it is highly important that the task be simplified by every possible means without sacrificing accuracy."

Use of these charts for other than their original purpose can lead to questionable results, especially when comparing the capacities of streets under one-way operation and two-way operation. Each chart sets forth capacity values for a single street under one-way or two-way operation, for a fixed distribution of traffic by direction. The charts do not have correction factors for variations in directional distribution of traffic or for variations in the operating conditions of the parallel street which normally is used under one-way operation to carry traffic in the opposite direction.

Comparisons of capacities for a single street under one-way versus two-way operation may be valid for studies on urban extensions of state highways, in which both directions of state highway traffic are routed over the two-way street, and in which the parallel street is not attractive to traffic under the two-way traffic plan. One-way operation of such streets "spreads the load," as revealed by capacity surveys in the State of Washington.[4,5]

In many downtown areas, however, both of the parallel two-way streets being considered for one-way operation are already through streets and are both attractive to traffic in the peak hours. Using Figs. 1, 2, and 3 for comparing capacities of such streets for one-way and two-way operation can lead to results which vary considerably from those obtained when using similar charts in *The Highway Capacity Manual*.[6]

Table 1 shows an example of a method of presenting comparative capacities of two parallel streets under one-way versus two-way operation. The capacity values shown were obtained by computing the practical capacities, by direction of movement, for each of two parallel streets, and combining the results to obtain directional volumes for the pair of streets. The charts[6] of *The Highway Capacity Manual* were used in making the computations.

When the two 50-ft streets of Table 1 are both attractive to through traffic, their computed capacity under one-way operation, with parking on both sides, is only 2% greater than with two-way operation (1,490 vehicles per hr versus 1,465 vehicles per hr). Figs. 2 and 3 show a 24% advantage for a one-way street of the same width and location (1,050 vehicles per hr versus 845 vehicles per hr).

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[3] Asst. Director, Inst. of Transportation and Traffic Eng., Univ. of California, Berkeley, Calif.

[4] "Spokane Urban Highway Capacity Survey," Washington Dept. of Highways, February, 1951.

[5] "Urban Highway Route Evaluation," by H. C. Higgins and R. E. Dunn, *Proceedings*, Highway Research Board, National Research Council, January, 1952.

[6] "Highway Capacity Manual," Bureau of Public Roads, U. S. Dept. of Commerce, U. S. Govt. Printing Office, Washington, D. C., 1950, pp. 79 and 84, Figs. 24 and 26.

TABLE 1.—COMPARATIVE CAPACITIES OF TWO-WAY AND ONE-WAY STREETS[a]

These data apply to two 50-ft east-west streets in a downtown area. In the peak hour, 60% of traffic is eastbound, with 10% commercial vehicles, 10% left turns, and 10% right turns. Signals are timed for 30-sec green in a 60-sec cycle. There is no bus stop.

Parking control and direction of movement	PRACTICAL CAPACITY OF TWO STREETS, IN VEHICLES PER HOUR			
	ONE STREET NOT ATTRACTIVE TO THROUGH TRAFFIC[b]		BOTH STREETS ATTRACTIVE TO THROUGH TRAFFIC	
	Two-way operation	One-way operation	Two-way operation	One-way operation
(*a*) PARKING BOTH SIDES				
Eastbound........	660	895	880	895
Westbound	440	595	585	595
Both directions....	1,100	1,490	1,465	1,490
(*b*) PARKING PROHIBITED ON SOUTH SIDE OF BOTH				
Eastbound........	990	1,380	1,320	1,380
Westbound	660	920	880	920
Both directions....	1,650	2,300	2,200[c]	2,300
(*c*) PARKING PROHIBITED ON NORTH SIDE OF ONE STREET AND SOUTH SIDE OF THE OTHER				
Eastbound........	825	1,380	1,100	1,380
Westbound	550	920	735	920
Both directions....	1,375	2,300	1,835	2,300
(*d*) NO PARKING BOTH SIDES				
Eastbound........	1,310	1,975	1,750	1,975
Westbound	880	1,320	1,170	1,320
Both directions....	2,190	3,295	2,920	3,295

[a] Computations of capacities in the limiting direction are based upon values shown in Figs. 24 and 26, *Highway Capacity Manual*, 1950. The capacities for the westbound direction (carrying only 40% of the traffic) are taken as two thirds of the limiting capacities in the eastbound direction, in Tables 1(*a*) and 1(*d*). In Tables 1(*b*) and 1(*c*), however, the limiting condition for two-way streets with a 60-40 directional distribution is the capacity of the side of the street on which parking is not prohibited.

[b] This assumes that the two-way street which is not attractive to through traffic will carry only 50% of the volume carried by the other street in each direction.

[c] If directional distribution of traffic is 67% to 33% instead of 60% to 40%, this two-way operation will carry 2,640 vehicles per hr versus 2,070 vehicles per hr for one-way operation.

When parking is prohibited on one side of the street (the peak-direction side for two-way streets), the comparative capacities depend upon the directional distribution of traffic. With a 60% to 40% directional distribution of traffic, the computed capacity for the two streets of the example in Table 1(*b*) is 2,300 vehicles per hr for the one-way operation as compared with 2,200 vehicles per hr for two-way operation of the streets. If the directional distribution of traffic is 67% to 33%, the practical capacity for two-way operation is 28% greater than for one-way operation.

Under conditions of no parking on both sides, Table 1(*d*) shows that the computed value of the practical capacity for the two one-way streets is 13% greater than under two-way operation (3,295 versus 2,920). Figs. 2 and 3 indicate a capacity advantage of 38% (2,100 versus 1,515) for a one-way street under the same conditions.

When one of the streets is not attractive to traffic under the two-way plan, the resulting capacity of the two two-way streets may be quite low, as shown in the example in Table 1. Under the one-way plan in such cases, traffic is forced to use the less attractive street and, as shown in Table 1, the capacity of the pair of one-way streets greatly exceeds the capacity under two-way operation.

When one-way streets can be used, they may have many other advantages over two-way streets in addition to their increased capacity. They simplify movements at complex intersections, reduce delays due to double parking, facilitate progressive timing of traffic signals, reduce travel times in nonrush hours, and spread traffic loads over little-used parallel streets. The actual increase in capacity obtained by changing two existing two-way downtown through streets with parking over to one-way operation may not be great, however, if the basic data in *The Highway Capacity Manual* on the capacity of one-way streets are accepted as correct for design and traffic control purposes.

The Department of Traffic and Operations, Highway Research Board, has voted to reactivate its Committee on Highway Capacity. This should afford an opportunity to collect data on operating capacities of many additional one-way streets, and to reappraise the chart values on the capacity of one-way streets.[7]

JOSEPH BARNETT,[8] M. ASCE.—One of the principal parts of this paper is a description of highway capacity charts which show practical working capacity for various conditions. As stated in the paper, these are based on data in *The Highway Capacity Manual* of the Highway Research Board[9] and eliminate the arithmetic necessary to account for certain variables when using the charts in *The Highway Capacity Manual.* The writer questions the advantage of the circular forms shown in Figs. 1, 2, and 3. He redrew one of the charts in the conventional rectangular form, with speeds increasing from left to right, and found it much easier to use.

Figs. 2 and 3, showing practical working capacity in terms of total vehicles per hour, should be used with caution as they can be greatly in error. The capacities of streets in downtown and intermediate districts are determined to such a great extent by the volume of traffic on cross streets that to generalize on the capacities may be out of order. Where the volumes of through and cross traffic justify control by traffic lights, the additional computation to correct for ratio of green interval to total cycle affects the capacity volume more than most of the factors included in the chart. It would have been well to state in the title of the chart that volumes are for the hour of green interval. For intersections with traffic light control an excellent set of charts was prepared[10] by Jack E. Leisch, A.M. ASCE, a member of the Highway Research Board Committee on Highway Capacity.

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[7] "Highway Capacity Manual," Bureau of Public Roads, U. S. Dept. of Commerce, U. S. Govt. Printing Office, Washington, D. C., 1950, p. 84, Fig. 26.

[8] Chf., Urban Highway Branch, Bureau of Public Roads, U. S. Dept. of Commerce, Washington, D. C.

[9] "Highway Capacity Manual," Bureau of Public Roads, U. S. Dept. of Commerce, U. S. Govt. Printing Office, Washington, D. C., 1950.

O. K. **Normann**[11].—The circular charts which Mr. Buckley has presented were prepared for estimating needed highway improvements in Ohio. They were used by state, county, and city engineers for a specific purpose in which the required accuracy could be attained by applying data for average conditions. It was realized at the time they were prepared that some degree of accuracy had to be sacrificed for simplicity. Many of the variable factors which ordinarily influence the capacity of a highway or street could not be included in the charts, and, therefore, average values which apply only to conditions in Ohio were used in preparing the charts. The charts served their purpose very well.

In other sections of the United States where other conditions are present, or even in Ohio, if the information contained in *The Highway Capacity Manual* is to be applied for other purposes, the information contained in Figs. 1, 2, and 3 should not be applied without first making certain that they fit the particular conditions under study. In fact, in most cases it will be necessary to revise the data contained in the charts to fit local conditions. The variation in traffic flow, its distribution by directions, the effect of commercial vehicles, and the traffic volumes, which may be considered "practical capacities" may be quite different in other sections of the United States than in Ohio.

In many cases where a greater accuracy is desired and where overestimates will not balance the underestimates, it may not be advisable to use charts of this type. This is especially true when considering various design improvements at a specific location or comparing increases in capacities that may be attained at a specific location by such traffic regulations as the elimination of parking, one-way street operation, or the elimination of certain turning movements at intersections.

J. P. **Buckley**,[12] M. ASCE.—Messrs. Normann and Berry are correct in cautioning against use of the Ohio charts for other than their original purpose. The charts were designed to meet the needs of a large-scale highway planning study in Ohio. The basic method is adaptable to similar studies in other states but the data would have to be revised to fit the conditions in each state.

The writer does not recommend the use of charts of this type for specific capacity problems. Detailed analysis taking into consideration the specific factors involved, as set forth in *The Highway Capacity Manual,* gives far more valid results in individual cases.

Mr. Berry correctly points out that variations in directional distribution of traffic and in attractiveness of a street to through traffic may materially affect the flow or volume. However, the capacity or the ability of the street to handle traffic remains the same. The capacity is there whether it is used or not.

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[10] *Public Roads*, Vol 26, No. 6, February, 1951, U. S. Govt. Printing Office, Washington, D. C.

[11] Chf., Traffic Operations Section, Highway Transport Research Branch, Bureau of Public Roads, Washington, D.C.

[12] Chf. Engr., Highways Div., Automotive Safety Foundation, Washington, D. C.

Since directional flow is usually unbalanced, certain flow assumptions were made in the preparation of Fig. 2 which are not stated in the paper. These assumptions are as follows:

Item	Areas	One direction	The other direction
1	Downtown	55%	45%
2	Intermediate	60%	40%
3	Outlying	66⅔%	33⅓%

Fig. 3 shows the maximum practical capacity of one-way streets on certain stated conditions. A pair of one-way streets under the same conditions would have twice the capacity of one street. However, because of the unbalanced flow the actual volumes would normally be considerably less, as indicated by Mr. Berry.

Mr. Berry's method, therefore, gives a more realistic analysis of the advantages of a pair of one-way streets against a pair of two-way streets from the standpoint of actual volumes carried.

Mr. Barnett points out that the titles of Figs. 2 and 3 should state that the volumes are per hour of green interval. This is covered by special condition (4) under Fig. 2. However, it would certainly be clear if both figures were labeled as Mr. Barnett suggests.

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DEFLECTIONS IN GRIDWORKS AND SLABS

By Walter W. Ewell,[1] J. M. ASCE, Shigeo Okubo,[2] and Joel I. Abrams,[3] J. M. ASCE

With Discussion by Messrs. Jack R. Benjamin, Alexander C. Scordelis, Paul J. Brennan, Serge Leliavsky Bey, W. Watters Pagon, Phil M. Ferguson, Robert V. Whitman, and Walter W. Ewell, Shigeo Okubo, and Joel I. Abrams

Synopsis

Because the exact determination of elastic deformations in plates and slabs by the mathematical theory of elasticity has proved to be impractical in all but the simple cases, engineers have been forced to devise approximate means of solution. A horizontal gridwork of beams intersecting at right angles, with deformation characteristics analogous to those of a slab, will develop an elastic surface under normal load similar to the surface of the slab under the same load. A technique is presented to determine the elastically deformed surface of a plate or slab that has been transformed into an equivalent grid system. The method employs an auxiliary force system for controlling vertical displacements of the joints and a moment and torque distribution process for transmission of the displacement effects.

Introduction

Approximate methods of determining the deflected surfaces of slabs are not new. A system of beams, intersecting at right angles to one another, forms a gridwork that yields a deflected surface similar to that of a slab when analyzed under normal loads. This statement appears obvious when one considers the similar elastic surfaces developed by a slab and by a simple window screen supported and loaded in like fashion. The analysis of such a gridwork is not simple, however.

One method of determining the deflection in a gridwork of beams is presented by Miklós Hetényi,[4] who assumes that the individual beams comprising the gridwork deflect without rotation at their intersections with other beams.

Note.—Published in September, 1951, as *Proceedings-Separate No. 89.* Positions and titles given are those in effect when the paper or discussion was received for publication.

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[4] "Beams on an Elastic Foundation," by Miklós Hetényi, The Univ. of Michigan Press, Ann Arbor, Mich., 1946, pp. 185–192.

Simultaneous differential equations are then solved in order to obtain joint displacements.

When it is assumed that no bending or twisting moment is transmitted at grid beam intersections, another method of grid analysis is possible. With such an assumption, Stephen S. Timoshenko employs trigonometric series to express the elastic curves developed by the individual grid beams.[5] A gridwork, unable to transmit bending and torsional moments, is not, however, analogous to a slab or plate in its action.

If a realistic rigid joint, capable of resisting bending and torsion, is assumed at each beam intersection, the analysis of the resulting grid is a tedious process. Linear equations can be formed involving three unknown elements at each joint—the loads on the beam, the moment change in the longitudinal beam, and the moment change in the transverse beam.[6] Both moment changes are developed by torsional effects in the mutually perpendicular beams. An inordinate amount of time is involved in the solution of the simultaneous equations particularly since there are three times as many equations as there are joints in the grid.

A technique is here presented for eliminating two of the three unknown quantities at each joint by a moment and torque distribution process. In some of its aspects, this procedure is similar to the method of finite differences developed by Henri Marcus[7] and extended to skew slabs by Vernon P. Jensen.[8]

Finite difference equations have been applied to rectangular and skew slabs divided into a network of points determined by arbitrarily ruling the surface with a series of intersecting lines. Moments and deflections are obtained at each point of the network by solving simultaneously two groups of linear equations: one group containing moment values as unknowns, the second group expressing deflection values as unknowns. The number of equations to be solved in one group is, in the general case, equal to the number of points in the arbitrary network. The solution of the two groups of simultaneous equations can be avoided if the original fourth-order differential equation of the plate is not divided into two parts. When finite difference equivalents are determined directly from the fourth-order equation, a solution is required for only one series of linear equations. Nevertheless, this procedure is not recommended by Mr. Marcus.

In comparison, the method used in this paper is applicable to a physically analogous grid rather than to the ruled surface of the slab itself. After the bending moments and torsional moments have been distributed over the grid, only one series of linear equations need be solved in order to define the deflection pattern. This is possible because the equations are written in terms of unknown deflections produced by auxiliary forces at the grid points. Bending moments and torsional moments can then be found without recourse to the second series of simultaneous equations that are required in the simplified finite difference solutions.

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[5] "Über die Biegung von Trägerrosten," by Stephen S. Timoshenko, *Zeitschrift für angewandte Mathematik und Mechanik*, Band 13, 1933, p. 153.

[6] "Trägerrost und Platte," by Béla Enyedi, *Beton und Eisen*, Vol. 34, Heft 3, 1935, p. 42.

[7] "Die Theorie elasticher Gewebe und ihre Anwendung auf die Berechnung biegsamer Platten," by Henri Marcus, Julius Springer, Berlin, 1924.

[8] "Analysis of Skew Slabs," by Vernon P. Jensen, *Bulletin Series No. 332*, Univ. of Illinois, Urbana, Ill., 1941.

PROCEDURE

The procedure for determining the elastic surface of a plate or slab subjected to normal loading consists of the following steps:

1. The arbitrary division of the slab into orthogonal strips that are considered the beams of the analogous gridwork.
2. The determination of factors, based on the individual beam's resistances to bending and to twist, for distributing the unbalanced moments at each grid beam joint.
3. The vertical displacement of each singular joint in the grid and consequent introduction of fixed-end moments on the beams. Joints where deflection produces a distinctive moment and torque pattern are known as singular joints.
4. The conversion of the fixed-end moments into bending and twisting moments by a moment and torque distribution process.
5. The computation of reactions at all joints in terms of the unknown displacements.
6. The computation of the moments and torques at each joint which were expressed by the distribution process in terms of the arbitrary displacements.

Transformation of the Slab into a Comparable Grid.—Any plate or slab, when divided into strips in two mutually perpendicular directions, can be transformed into a grid system provided that each strip be considered as a grid beam. In fact, an analogy between slab and gridwork is indicated by the close correspondence of slab deflection values found by the grid method and checked by the theory of elasticity. Obviously, a finer division of the slab will increase the number of grid beams and will lead to a more accurate definition of the elastic surface.

If the slab shown in Fig. 1 is divided into two series of parallel orthogonal strips, the resulting system simulates a group of rigidly connected continuous beams mutually supported along their center lines. For example, the east-west and north-south grid strips are considered continuous over supports 1–1, 2–2, 3–3, and over supports 1′–1′, 2′–2′, 3′–3′, located at the center lines of the two perpendicular strips. Furthermore, a beam segment such as 1–2–2–1 is assumed free to rotate and to twist over its supports. Along the edges 1–2 the resistances to bending and twist that are presumably neglected in the east-west beams are actually considered as vertical shear forces in the north-south beams. Because the cross section of any grid beam is identical to that of a comparable slab strip, the gridwork is actually a pseudogrid. The over-all general appearance of the system is no different from that of the slab because no openings exist between grid beams.

Since the equivalent gridwork is to be loaded and consequently subjected to moments and torques, relative stiffnesses and distribution factors of the individual grid beams must be calculated.

Relative Stiffnesses and Distribution Factors.—If an applied moment (M), in the plane of the beam A 1 C, is released on the rigidly connected system shown in Fig. 2, the tangent to the elastic curve will rotate through an angle θ. If torsional fixity is realized at points D and B, the beam D 1 B will twist through the same angle θ at point 1. Similarly, a moment applied in the plane D 1 B will produce rotation of the tangent to the elastic curve in that plane and consequent twist in the beam A 1 C.

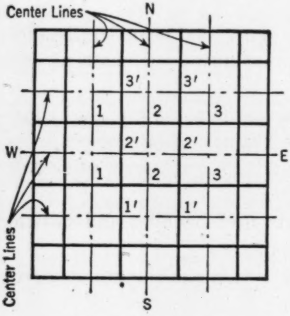

FIG. 1.—DIVISION OF SLAB

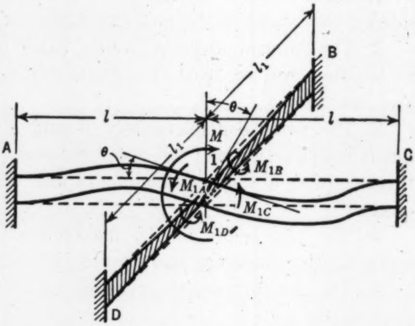

FIG. 2.—BENDING AND TWISTING OF A ROTATED RIGID JOINT

In order to resist the applied moment M, bending moments M_{1A} and M_{1C} will be developed in beam A 1 C, and torsional moments M_{1D} and M_1B will be introduced in beam D 1 B.

When the rotation through the angle θ has taken place,

$$M = M_{1A} + M_{1C} + M_{1D} + M_{1B} \ldots\ldots\ldots\ldots (1)$$

For a unit rotation θ in beam A 1 C, using slope deflection equations,

$$M_{1A} = \frac{4\,E\,I}{l} \quad \text{and} \quad M_{1C} = \frac{4\,E\,I}{l} \ldots\ldots\ldots\ldots (2)$$

For a unit twist θ in the beam D 1 B, with circular cross section,

$$M_{1D} = \frac{G\,I_p}{l_1} = M_{1B} \ldots\ldots\ldots\ldots (3a)$$

with rectangular cross section, or

$$M_{1D} = \frac{\beta\,c\,b^3\,G}{l_1} = M_{1B} \ldots\ldots\ldots\ldots (3b)$$

in which c is the width of grid beam; b is the depth of beam; G is the modulus of elasticity in shear; I_p is the polar moment of inertia of beam or grid strip; I is the moment of inertia with respect to an axis perpendicular to the plane of bending; β is the coefficient depending on cross-sectional properties[9]; and E is the modulus of elasticity. With the expressions for moment in the longitudinal and transverse beams known, distribution factors can be written for any joint. For example:

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[9] "Elements of Strength of Materials," by S. Timoshenko and Gleason H. MacCullough, D. Van Nostrand Co., Inc., New York, N. Y., 1949, pp. 265–266.

$$M_{1A} = \frac{4\,E\,I/l}{4\,E\,I/l + 4\,E\,I/l + G\,I_p/l_1 + G\,I_p/l_1} \qquad \text{(4)}$$

Distribution factors for the ordinary grid will be the same at every point when lengths between intersections are made constant in each direction and identical cross sections are selected.

Auxiliary Force Systems for Controlling Deflections.—The moment distribution process to be used cannot be applied directly to any loaded structure whose joints undergo translation. In order to control or prevent these joint translations, it is necessary to calculate a series of auxiliary force systems that arise from the arbitrary displacements of joints. These forces are applied at every joint, and the translations they cause or prevent must be such as to produce an equilibrium with the forces actually acting on the structure.

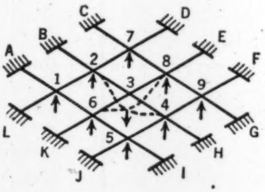


FIG. 3.—GRIDWORK WITH JOINT 3 DISPLACED

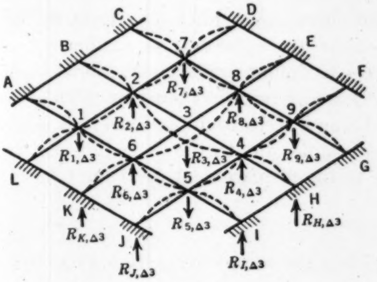


FIG. 4.—ASSUMED ELASTIC SURFACE AFTER DISTRIBUTION OF MOMENTS CAUSED BY DEFLECTION AT JOINT 3

After the distribution factors for the unbalanced moments and torques at every joint have been determined, an arbitrary displacement (Δ) is introduced, in turn, at each singular joint as shown in Fig. 3. The singular joints of the particular grid illustrated are at points 1, 2, and 3. Deflections at all other joints will induce similar moment and torque patterns. The auxiliary force system shown consists of reactions at all joints other than joint 3 at which point the Δ_3 displacement has been produced. These reactions prevent displacement from the horizontal datum plane but do not prohibit rotation and twist in the grid members. The lengths of the grid members being equal in both directions, the deflection Δ_3 induces fixed-end moments equal to $6\,E\,I\,\Delta_3/l^2$ at the initially fixed joints 6, 2, 8, 4, and at joint 3 where the deflection occurs.

Moments are placed on the grid first in the east-west direction and are assigned the customary moment distribution signs as viewed from the south. Ordinary fixed-end moments need only be introduced on the beams in one direction, for the symmetry of the gridwork yields identical distribution patterns in the mutually perpendicular directions.

Next, the joints are progressively released and balanced, introducing bending moments and torques at all beam intersections. After all the joints are balanced, a carry-over process is effected, torque moments carrying over to opposite ends of the beam at their full values but with opposite sign, and bending moments carrying over as half values with the same sign. The second cycle is completed by rebalancing each joint with the usual distribution factors. Three or four cycles generally furnish a thorough dispersion of the original fixed-end moments.

When the distribution process is complete at any one joint, bending moment values at the beam ends are used to determine the reactions in terms of the Δ_3 displacement. Although a distribution is performed in the east–west direction only, a similar group of bending moments must be placed on the north–south beam in order to define the complete reaction pattern. Fig. 4 shows an assumed deflection pattern due to a Δ_3 displacement at joint 3. The notation should be self evident. For instance, $R_{9,\Delta 3}$ is the reaction at joint 9 caused by a deflection Δ_3 at joint 3. It should be remembered at this point that all reactions are expressed as coefficients of $\Delta_3 EI/l^3$. In the particular grid selection, similar displacements must be introduced at the other singular points 1 and 2, and reactions must be calculated.

Using a reciprocal relationship it is noted that the reaction at any point 4, for example, due to the deflection Δ_3 ($R_{4,\Delta 3}$) will be equal to the reaction at 3 due to the displacement Δ_4 at joint 4.

$$R_{4,\Delta 3} = R_{4,\Delta 4} \dots\dots\dots\dots\dots\dots\dots\dots \text{(5)}$$

Thus the complete reaction at joint 3, with all points of the grid displaced through unknown Δ values, will be

$$R_{1,\Delta 3} + R_{2,\Delta 3} + R_{3,\Delta 3} + R_{4,\Delta 3} + R_{5,\Delta 3} + R_{6,\Delta 3} + R_{7,\Delta 3} + R_{8,\Delta 3} + R_{9,\Delta 3} \dots \text{(6}a\text{)}$$

or

$$R_{3,\Delta 3} + R_{3,\Delta 2} + R_{3,\Delta 3} + R_{3,\Delta 4} + R_{3,\Delta 5} + R_{3,\Delta 6} + R_{3,\Delta 7} + R_{3,\Delta 8} + R_{3,\Delta 9} \dots \text{(6}b\text{)}$$

Since only 3 singular points exist, at joints 1, 2, and 3,

$$R_{3,\Delta 1} = R_{3,\Delta 7} = R_{3,\Delta 9} = R_{3,\Delta 5} \dots\dots\dots\dots\dots\dots \text{(7}a\text{)}$$

$$R_{3,\Delta 2} = R_{3,\Delta 8} = R_{3,\Delta 4} = R_{3,\Delta 6} \dots\dots\dots\dots\dots\dots \text{(7}b\text{)}$$

Therefore, $R_{3,\Delta 3} + 4\,R_{3,\Delta 1} + 4\,R_{3,\Delta 2}$ is the total reaction influence at 3. Similar equations can be written at points on the grid and equated to a load P that might exist at the intersection, or else equated to zero if there is no load at the joint.

In the case of a uniform load across the plate, a load P exists at every intersection and the displacement of three points 1, 2, 3 will define the elastic surface. Consequently, in this simple case, only three simultaneous equations need be solved for the unknowns Δ_1, Δ_2, and Δ_3.

With the deflection values known at each point the moments and torques expressed in the distribution process in terms of Δ can be accumulated by simple multiplication and addition.

Example A—Deflections in a Simple Gridwork.—In this first application of the analysis no attempt has been made to draw an analogy between a gridwork and a slab. Fig. 5 illustrates a gridwork consisting of circular bars rigidly connected at each junction, with complete fixity against bending and twist realized at beam ends. Deflections will be determined in this structure when it is subjected to equal normal loads P at all joints and when a single load P is applied at joint 1. Since all lengths are equal and cross sections are the same, the distribution factors at each of the joints are identical. The distribution factors are computed, assuming $G = 0.36\,E$ and $I_p = 2\,I$.

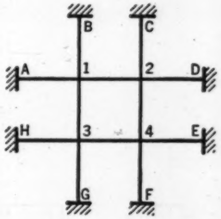


FIG. 5.—GRIDWORK OF RIGIDLY CONNECTED BEAMS

$$M_{1A} = \frac{4\,E\,I\,\theta}{l} = M_{12} \quad \text{........} (8)$$

Assuming joints A, B, 2, and 3 temporarily fixed

$$M_{1B} = \frac{G\,I_p\,\theta}{l} = \frac{0.72\,E\,I\,\theta}{l} = M_{13} \quad \text{...............} (9)$$

If $\frac{E\,I\,\theta}{l}$, known to be constant, is temporarily omitted,

$$M_{1A} = \frac{4}{4 + 4 + 0.72 + 0.72} = 0.4237\,M \quad \text{...........} (10)$$

in which M is an unbalanced moment released at the joint. Likewise,

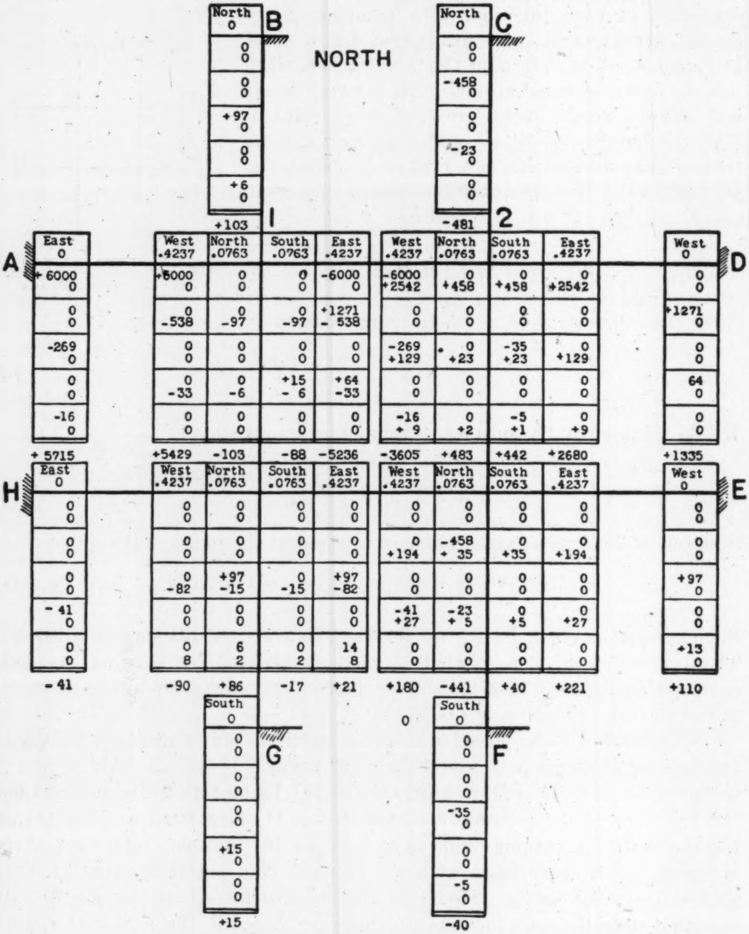
$$M_{IB} = \frac{0.72}{4 + 4 + 0.72 + 0.72} = 0.0763\,M \quad \text{...........} (11)$$

Similar equations can be set up for the remaining points on the gridwork. The percentage of the unbalanced moment resisted by bending moments developed in beams is 42.37%, and the percentage resisted by torque moments in the north-south beams is 7.63%.

When joint 1 is displaced a distance Δ_1, fixed-end moments of $6\,E\,I\Delta_1/l^2$ are induced at points A, 1, and 2 along the beam A D and also at 3, 1, and B along the beam C B. Fig. 6 shows the fixed-end moments multiplied by 1,000 but exclusive of the constant quantity $E\,I\,\Delta_1/l^2$ along the east-west beams, together with the ensuing moment and torque distribution. The signs of the moments are in accordance with the moment distribution convention when viewed from the south. Moments are not distributed in the north-south direction since they are identical to those developed in the east-west beams.

Equilibrating reactions are derived from the end moments in the east-west direction obtained by the distribution process. The equations of static equilibrium are used to determine these reactions after the beams are subdivided into their component lengths, as shown in Fig. 7. Because of the constant beam length l, the reaction values shown are multiples of the term $E\,I\,\Delta_1/l^3$. When an equal series of reactions derived from the north-south end moments are superimposed upon the grid and algebraically added to the first group of reactive forces, the equilibrated system shown in Fig. 8(a) results.

By the theorem of reciprocity, the reaction 13.188 $E\,I\,\Delta_1/l^3$ at junction 2, caused by the deflection Δ_1 at joint 1, is equal to the reaction at joint 1 caused by an equal deflection Δ_2 at joint 2. If a similar reasoning is employed at all grid points, the total reaction at joint 1 can be written as follows: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+6 0
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C
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-16 0
+5715
West .4237 | North .0763 | South .0763 | East .4237
+6000 0 | 0 0 | 0 0 | -6000 0
0 -538 | 0 -97 | 0 -97 | +1271 -538
0 0 | 0 0 | 0 0 | 0 0
0 -33 | 0 -6 | +15 -6 | +64 -33
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-41 +27 | -23 +5 | 0 +5 | 0 +27
0 0 | 0 0 | 0 0 | 0 0
+180 | -441 | +40 | +221
West 0
E
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+97 0
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+13 0
+110
South 0
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0 0
0 0
0 0
+15 0
0 0
+15
0
South 0
F
0 0
0 0
-35 0
0 0
-5 0
-40

FIG. 6.—DISTRIBUTION OF EAST-WEST, FIXED-END MOMENTS. DISPLACEMENT AT JOINT 1

$$R_1 = 39.970\,E\,I\,\Delta_1/l^3 - 13.188\,E\,I\,\Delta_2/l^3 - 0.260\,E\,I\,\Delta_4/l^3 - 13.188\,E\,I\,\Delta_3/l^3 \qquad (12)$$

If every grid joint supports a load of P, the following expression can be obtained by equating vertical forces to zero:

$$P = 39.970\, E\, I\, \Delta_1/l^3 - 13.188\, E\, I\, \Delta_2/l^3 \\ - 0.260\, E\, I\, \Delta_4/l^3 - 13.188\, E\, I\, \Delta_3/l^3 \ldots\ldots\ldots\ldots (13)$$

But with equal forces P located at all joints of the symmetrical grid,

$$\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 \ldots\ldots\ldots\ldots (14)$$

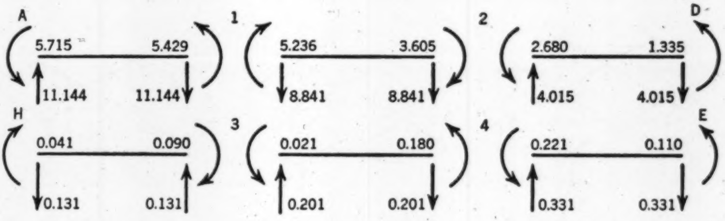


FIG. 7.—MOMENTS AND REACTIONS ON GRID BEAMS

Therefore,

$$13.332\, E\, I\, \Delta_1/l^3 = P \ldots\ldots\ldots\ldots (15a)$$

or transposing

$$\Delta_1 = \frac{P\, l^3}{13.332\, E\, I} = \frac{0.075\, P}{l^3\, E\, I} \ldots\ldots\ldots\ldots (15b)$$

Fig. 8(b) illustrates the deflected grid subjected to equal loads P at all joints. The deflections are expressed as coefficients of $\frac{P\, l^3}{E\, I}$.

When a single concentrated load P is applied at joint 1, the deflections developed at joints 1, 2, 3, and 4 are not equal. By equating vertical forces at all grid joints to zero the following linear equations are developed:

$$39.970\, E\, I\, \Delta_1/l^3 - 13.188\, E\, I\, \Delta_2/l^3 \\ - 0.260\, E\, I\, \Delta_4/l^3 - 13.188\, E\, I\, \Delta_3/l^3 = P \ldots\ldots\ldots\ldots (16a)$$

$$- 13.188\, E\, I\, \Delta_1/l^3 + 39.970\, E\, I\, \Delta_2/l^3 \\ - 13.188\, E\, I\, \Delta_4/l^3 - 0.260\, E\, I\, \Delta_3/l^3 = 0 \ldots\ldots\ldots\ldots (16b)$$

$$- 0.260\, E\, I\, \Delta_1/l^3 - 13.188\, E\, I\, \Delta_2/l^3 \\ + 39.970\, E\, I\, \Delta_4/l^3 - 13.188\, E\, I\, \Delta_3/l^3 = 0 \ldots\ldots\ldots\ldots (16c)$$

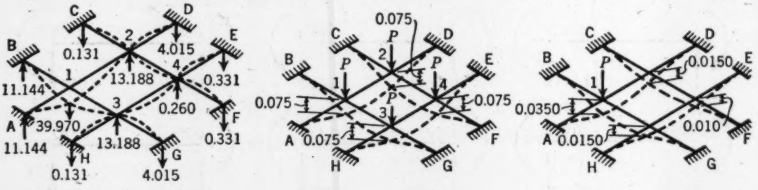
$$- 13.188\, E\, I\, \Delta_1/l^3 - 0.260\, E\, I\, \Delta_2/l^3 \\ - 13.188\, E\, I\, \Delta_4/l^3 + 39.970\, E\, I\, \Delta_3/l^3 = 0 \ldots\ldots\ldots\ldots (16d)$$

Solved simultaneously, the equations yield the following values for deflection:

$$\Delta_1 = 0.0350\, P\, l^3/E\, I \\ \Delta_2 = 0.0150\, P\, l^3/E\, I \\ \Delta_3 = 0.0101\, P\, l^3/E\, I \\ \Delta_4 = 0.0150\, P\, l^3/E\, I$$

Fig. 8(c) shows the deflections as coefficients of $P\, l^3/E\, I$ at each of the four joints.

Since the bending and the torsional moments have been determined in terms of the various displacements, it is a simple matter to calculate the moments after the deflections have been determined.



(*a*) ASSUMED ELASTIC SURFACE AFTER DISTRIBUTION OF MOMENTS DUE TO Δ_1

(*b*) DEFLECTION SURFACE LOAD *P* AT JOINTS 1,2,3,4

(*c*) DEFLECTION SURFACE LOAD *P* AT JOINT 1

FIG. 8.—DEFLECTIONS IN A SIMPLE GRIDWORK

From Fig. 6 the moment at A due to displacement at joint 1 is $+ 5{,}715\,E\,I\,\Delta_1/l^2$ and the torque along beam A 1 is $103\,E\,I\,\Delta_1/l^2$ units, as shown at B. The torque of $103\,E\,I\,\Delta_1/l^2$ could be calculated directly by distributing fixed-end moments in the north-south direction, but this procedure is unnecessary due to symmetry. Furthermore, with a displacement at joint 2, the moment $+ 1{,}335\,E\,I\,\Delta_1/l^2$ shown at point D in the figure would develop as $- 1{,}335\,E\,I\,\Delta_2/l^2$ at point A. The change in sign arises because original fixed-end moments should be introduced as $- 6{,}000\,E\,I\,\Delta_2/l^2$. With displacements at all joints the total moment at A is

$$M_{A\,1} = + 5{,}715\,E\,I\,\Delta_1/l^2 - 1{,}335\,E\,I\,\Delta_2/l^2 - 41\,E\,I\,\Delta_3/l^2 - 110\,E\,I\,\Delta_4/l^2 \quad (17a)$$

Substituting values for Δ as previously determined,

$$\mathrm{M}_{A\,1} = + 200.03\,P\,l - 17.03\,P\,l - 0.62\,P\,l - 1.11\,P\,l \ldots\ldots (17b)$$

$$M_{A\,1} = + 181.27\,P\,l \ldots\ldots\ldots\ldots\ldots\ldots (17c)$$

in which the positive sign is based on moment distribution convention. All other moments and torques can be determined in like fashion.

Example B—Analysis of a Slab, With and Without a Center Support.—As a second illustration of the applicability of the method, the 1/16-in.-thick slab shown in Fig. 9 has been selected for analysis. A combination of fixed and free boundaries has been chosen for the slab in order to prove that complexities arising from variable edge conditions are easily handled. The effects of continuity will be studied by determining slab deflections both with and without a center vertical support at joint 12.

If the slab shown is divided into a series of 3-in.-wide strips, each strip can be considered a beam of an equivalent grid system. The grid beams and their center lines, along which the deflections will be determined, are shown in Fig. 9. Because of the fixed- and free-end conditions, three sets of distribution factors are required for the distribution of moments and torques over the gridwork.

Interior joints require but one set of distribution constants, for when a displacement is introduced two equal length and equal cross section beams twist through an angle as two identical but perpendicular beams rotate through the same angle. The same stiffness factors are developed, and consequently the same distributive effects are required for the distribution of fixed-end moments in the perpendicular direction. Edge joints require different constants in the two directions, for when moments are distributed in the north-south direction, one east-west beam is subjected to torque; and when moments are distributed in the east-west direction two north-south beams are subjected to twist. The values of the distribution factors for interior and exterior joints are shown in Fig. 10. These constants can be computed by assuming all beams to be of constant length l, and also assuming that $G = 0.377\,E$, and that the ratio of beam depth to beam width = 1/16 in. to 3 in. = 1/48. The expression for moment required to produce a unit angle of twist in a rectangular beam has been taken from Mr. Timoshenko.[9]

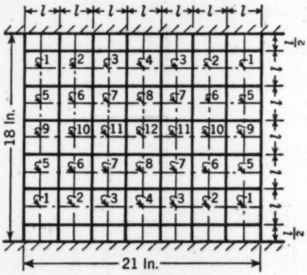
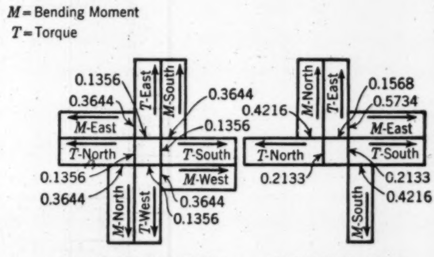


FIG. 9.—DESIGNATION OF GRID JOINTS ON A PLATE



(a) INTERIOR JOINT (b) EXTERIOR JOINT

FIG. 10.—DISTRIBUTION FACTORS AT TYPICAL JOINTS

When the slab is subjected to a uniform vertical load of w lb per sq in., the equivalent gridwork is loaded at each joint with a concentration $P = w\,l^2$. Twelve deflections, Δ_1 through Δ_{12} at the numbered joints shown in Fig. 9, will completely define the elastic surface. Consequently, equal but arbitrary displacements are introduced at each of the similarly numbered singular grid joints in turn, while the remaining joints are temporarily fixed on a horizontal datum plane by vertical reactions. When all similarly numbered joints are displaced simultaneously, a symmetrical distribution results and the computation can be confined to one quarter of the slab.

Inasmuch as the numerical process of distribution is similar at all joints, the procedure is illustrated for displacements at joint 9 only, as shown in the chart of Fig. 11. This chart, showing only the north-west quarter of the slab, facilitates the simultaneous distribution of the north-south and east-west, fixed-end moments of $6\,E\,I\,\Delta_9/l^2$ developed at joints 9, 5, and 10. The simultaneous distribution is necessitated by the different end conditions in the two directions. For convenience, moments are represented by the constant coefficient 6 multiplied by 1,000. As in example A, the carry-over factors for bending moment and torsional moment are 1/2 and -1, respectively. The singular

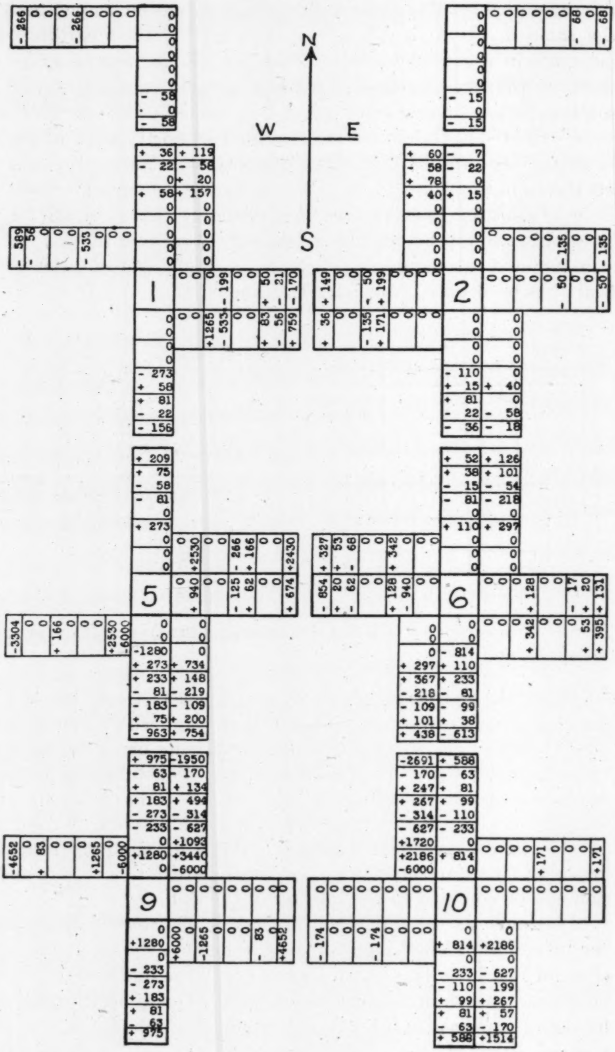

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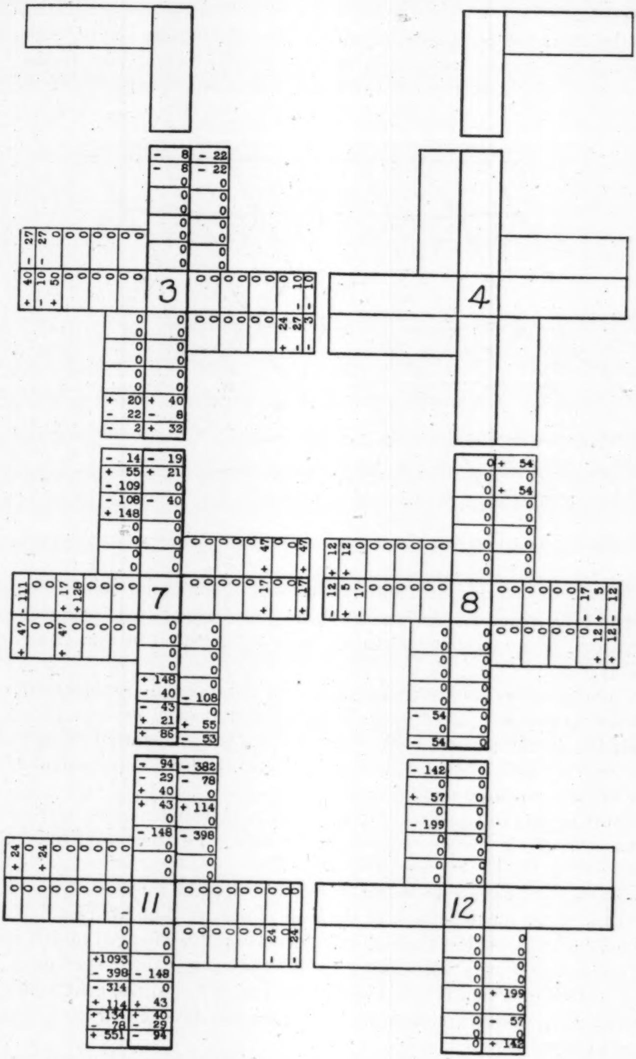
FIG. 11.—DISTRIBUTION OF FIXED-END



MOMENTS. DISPLACEMENT Δ_9 AT JOINT 9 joint reactions, exclusive of $E\,I\,\Delta_9/l^3$ caused by the Δ_9 displacements are illustrated in Fig. 12.

Similar distributions are performed at each of the 12 singular joints, each operation reducing by two the number of unknowns. The complete process reduces from 36 to 12 the number of linear equations to be solved simultaneously.

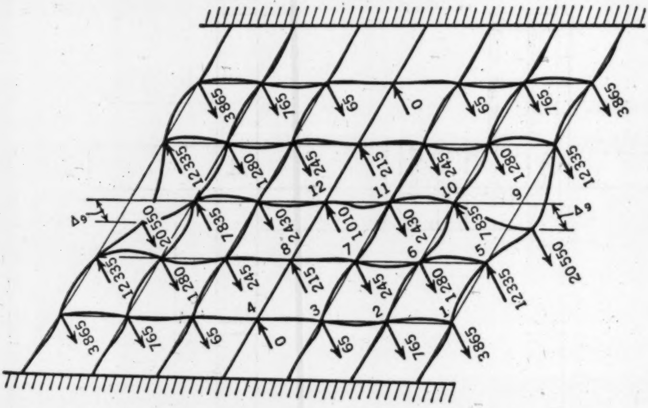


Fig. 12.—Assumed Elastic Surface After Distribution of Moments Caused by Deflection Δ_9

After the effects of the displacements at each of the singular joints have been determined, the reactions shown in Table 1 are calculated, using the methods of statics. The reactions caused by Δ_9 are illustrated in Fig. 12 and entered in Table 1(*a*).

When the reactions—expressed in terms of the unknown Δ displacements but still exclusive of the term $E\,I/l^3$—are accumulated at each joint and each reaction equated to a vertical load 1,000 P, the twelve equations used to determine the twelve Δ's result. Since the equations are merely a restatement of Table 1, they are not repeated at this point.

With a center support at joint 12, only the first eleven equations are required, inasmuch as Δ_{12} is known to be equal to zero. Furthermore, the Δ_{12} term is not present in the eleven equations solved.

The solution of these equations is facilitated by using the method of M. H. Doolittle.[10] The values for deflection at the grid joints are tabulated in Table 1(*b*). Since 1,000 P was used in the equation formation, the effect of multiplying $6\,E\,I\,\Delta/l^3$ by 1,000 was obliterated. All deflection values shown are multiples of $P\,l^3/E\,I$, in which the E, I, and l are properties of the grid beam.

As in example A, moments and torques may be determined after the deflections are known.

Example C—Analysis of a Uniformly Loaded Square with Clamped Edges.—The square slab shown in Fig. 13 has been selected for analysis to show the

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[10] "Application of the Theory of Least Squares to the Adjustment of Triangulation," by Oscar S. Adams. *Special Publication No. 28*, U. S. Coast and Geodetic Survey, Washington, D. C., 1915.

TABLE 1.—REACTIONS AND DEFLECTIONS AT SINGULAR JOINTS CAUSED BY Δ DISPLACEMENTS

Joint number	Displacement												
	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	Δ_7	Δ_8	Δ_9	Δ_{10}	Δ_{11}	Δ_{12}	P
(a) SUMMATION OF REACTIONS IN TERMS OF $E I \Delta/l^3$													
1	+24,790	− 7,470	+ 2,490	− 480	−14,030	+ 860	+ 180	− 105	+ 3,865	+ 765	+ 140	− 30	= +1,000
2	− 7,470	+35,610	−12,100	+ 3,205	+ 860	−14,350	0	+ 410	+ 765	+ 3,225	+ 585	+ 120	= +1,000
3	+ 2,490	−12,100	+40,800	−11,850	+ 190	0	−13,215	+ 15	+ 65	+ 530	+ 3,505	+ 515	= +1,000
4	− 960	+ 6,410	−23,700	+37,600	− 230	− 830	0	−13,625	0	+ 250	+ 1,030	+ 3,390	= +1,000
5	−14,030	+ 860	+ 190	− 115	+24,450	− 7,230	+ 2,530	− 525	−12,335	+ 1,280	+ 300	− 145	= +1,000
6	+ 860	−14,350	0	+ 415	− 7,230	+35,420	−12,050	+ 3,330	+ 1,280	−13,040	+ 340	+ 460	= +1,000
7	+ 180	0	−13,215	0	+ 2,530	−12,050	+40,690	−11,790	+ 245	+ 450	−11,850	+ 325	= +1,000
8	− 210	+ 820	+ 15	−13,625	− 1,050	+ 6,660	−23,580	+37,355	− 215	+ 830	+ 645	−12,330	= +1,000
9	+ 7,730	+ 1,530	+ 130	0	−24,670	+ 2,560	+ 490	− 215	+20,550	− 7,835	+ 2,430	− 505	= +1,000
10	+ 1,530	+ 6,450	+ 1,060	+ 250	+ 2,560	−26,080	+ 450	+ 830	− 7,835	+31,900	−12,575	+ 3,215	= +1,000
11	+ 140	+ 1,170	+ 1,030	+ 1,030	+ 600	+ 680	−23,700	+ 645	+ 2,430	−12,575	+37,240	−12,330	= +1,000
12	− 120	+ 480	+ 2,060	+ 6,780	− 580	+ 1,840	+ 1,300	−24,660	− 1,010	+ 6,430	−24,660	+33,970	= +1,000
(b) VALUES OF DEFLECTIONS IN TERMS OF $P l^3/E I$													
Without support	1.04	1.01	0.98	0.97	2.69	2.60	2.52	2.51	3.41	3.33	3.21	3.20	....
With support	0.78	0.58	0.33	0.21	1.86	1.36	0.64	0.29	2.27	1.59	0.68	0.00	....

accuracy of the grid method as used to define the entire elastic surface of a loaded slab. The characteristics of this slab are as follows: $G = 4{,}000{,}000$; $E = 10{,}600{,}000$; $\mu = 0.33$; $a = 2$ in.; $b = 1/16$ in. The slab has a uniform load of w lb per unit area. The dashed lines in Fig. 13 represent the grid beam center line. Deflection values obtained at ten different points on the slab will be compared with the exact values obtained by the mathematical theory of elasticity.[11]

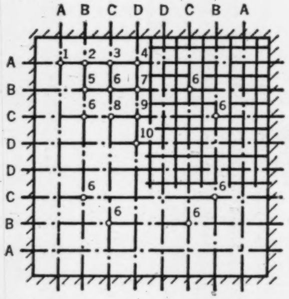

FIG. 13.—SELECTION OF GRIDWORK

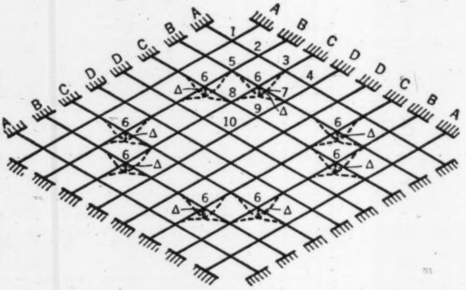

FIG. 14.—SIMULTANEOUS DISPLACEMENT OF JOINT 6

Nine grid beam center lines, each two inches apart, are selected in the two orthogonal directions, in order to convert the slab into a pseudogrid system. This slab division gives rise to the ten singular joints shown. Under the action of a uniform load, the square slab will deflect symmetrically around its north-south, east-west, and diagonal axes. Because of this symmetry, the 10 singular joints contained within any one octant of the grid will completely define the elastic surface.

Since the grid beams in both directions have identical lengths and cross sections, the resistances to bending and to twist will be constant at every joint. When the physical characteristics of the slab and the constant length and cross-sectional properties of the grid beams are involved in the proper expression,[10] the following distribution factors will result:

$$\text{Percentage moment} = 13.49 \text{ torque} \ldots\ldots\ldots\ldots \quad (18a)$$

$$\text{Percentage moment} = 36.51 \text{ bending} \ldots\ldots\ldots\ldots \quad (18b)$$

If an unbalanced moment M is applied on any joint of the gridwork in either direction, 36.51% of the applied moment is resisted by two beams in bending, and two beams in torsion resist 13.49% of the applied moment.

The multiple symmetry existent in this example permits the simultaneous displacement of all similarly numbered joints and the calculation of the auxiliary force systems developed by these displacements. As an example, the eight joints designated "6" in Fig. 13 will be deflected through equal distances when the gridwork is uniformly loaded. Furthermore, these eight

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[11] "The Calculation of Maximum Deflection, Moment, and Shear for Uniformly Loaded Rectangular Plate with Clamped Edges," by I. A. Wojtaszak, *Transactions*, A.S.M.E., Vol. 59, 1937, p. A-173.

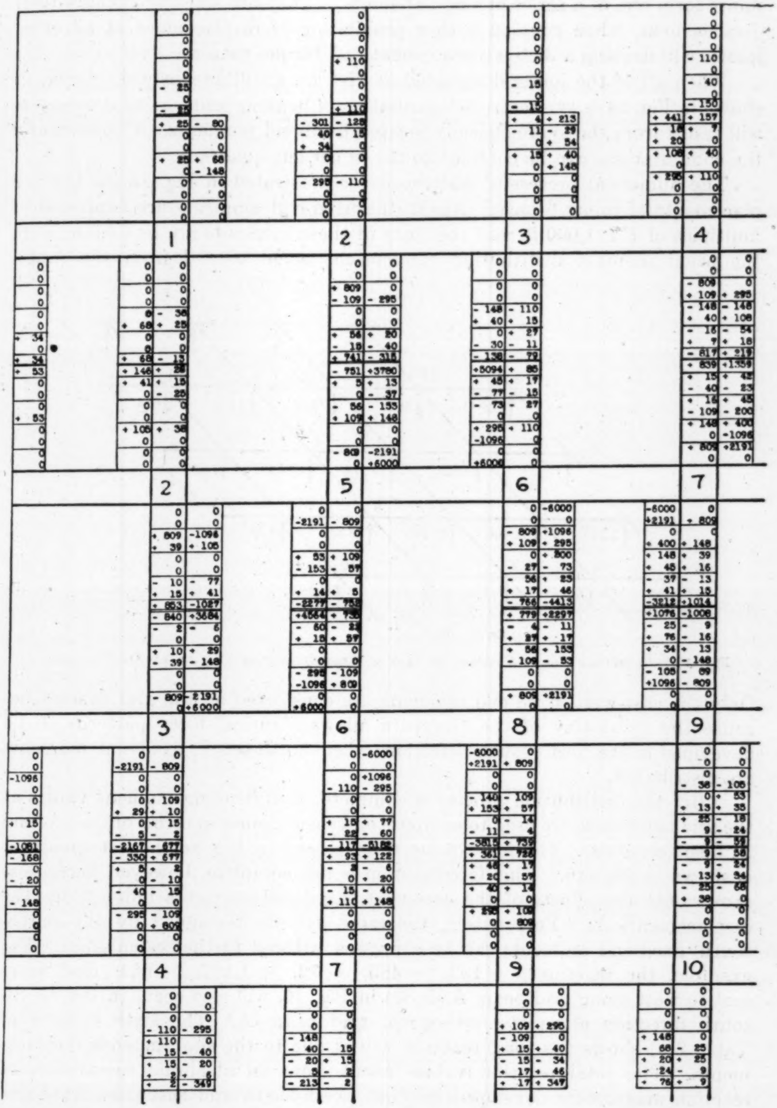
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FIG. 15.—DISTRIBUTION OF EAST-WEST, FIXED-END MOMENTS CAUSED BY DISPLACEMENT Δ_6 AT JOINT 6

joints form one of a series of singular joints. They are singular because their displacement, when coupled with a prohibition of displacement at all other joints, will develop a distinctive moment and torque pattern.

When all of the joints designated as "6" are simultaneously displaced, as shown in Fig. 14, a symmetrical distribution of bending and torsional moments will result from the symmetrically located fixed-end moments. Consequently the computations can be confined to the upper left quadrant.

The numerical process of distribution is illustrated in Fig. 15 for the displacements of joints 6 only. Again, the fixed-end moments are expressed as multiples of $EI/1{,}000\,l^2$, and the signs of these moments are in keeping with the usual moment distribution conventions when viewed from the south.

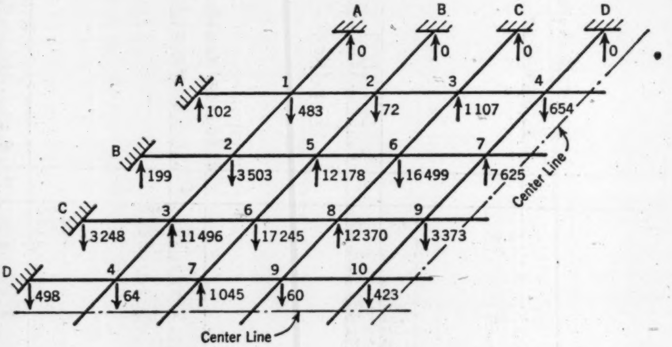

FIG. 16.—AUXILIARY FORCE PATTERN IN NORTH-WEST QUADRANT CAUSED BY DEFLECTION Δ_6

Only the east-west, fixed-end moments are considered on the grid system because the symmetry of the structure allows identical force patterns to be developed in the north-south direction, when north-south, fixed-end moments are distributed.

After the distribution process is complete, final bending moment values at the grid beam ends are used to compute end shear values in terms of the original Δ_6 displacements. The force pattern developed in the north-west quadrant is shown in Fig. 16. This force system is not complete, however, because it shows only the effects of the east-west, fixed-end moments induced by the displacements Δ_6. Fortunately, the force pattern developed by the north-south, fixed-end moments can be expressed without further calculation. For example, the reactions, + 102, − 483, − 72, + 1,107, − 654, read from east to west along grid beam A shown in Fig. 16, will also occur in the north-south direction along the orthogonal grid beam A. The sixth column of Table 2(*a*) shows the total reaction values due to the simultaneous displacements. The total reaction values result from an algebraic summation of reaction magnitudes developed by both north-south and east-west, fixed-end moments.

Similarly, the remaining 9 groups of singular joints are displaced, the fixed-end moments distributed, and the auxiliary forces calculated. These

TABLE 2.—SIMULTANEOUS EQUATIONS AND SOLUTIONS FOR DEFLECTION, UNIFORMLY LOADED SQUARE PLATE

At oint	Δ1	Δ2	Δ3	Δ4	Δ5	Δ6	Δ7	Δ8	Δ9	Δ10	External load
(*a*) TOTAL REACTION VALUES EXPRESSED IN TERMS OF $K I \Delta/l^3$ =											
1	−40,870	+25,000	− 6,764	+ 1,090	+ 2	− 966	+ 266	− 120	+ 32	0	−1,000*P*
2	+12,500	−37,559	+11,289	− 2,512	+12,924	− 3,575	+ 280	− 511	+ 29	+ 17	−1,000*P*
3	− 3,382	+11,290	−36,942	+ 8,597	− 191	+12,603	− 483	− 3,368	− 58	+ 148	−1,000*P*
4	+ 543	− 2,513	+ 8,599	−25,590	− 291	− 718	+12,808	− 631	− 3,745	+ 703	−1,000*P*
5	0	+25,850	− 380	− 586	−34,130	+24,356	− 5,238	− 384	− 638	− 88	−1,000*P*
6	− 484	− 3,575	+12,605	− 718	+12,180	−33,744	+ 8,672	+12,375	− 3,438	− 419	−1,000*P*
7	+ 134	+ 281	− 478	+12,806	− 2,619	+ 8,670	−21,834	− 835	+11,437	−2,937	−1,000*P*
8	− 120	− 1,022	− 6,734	− 1,260	− 382	+24,740	− 1,668	−32,598	+18,396	−1,410	−1,000*P*
9	+ 16	+ 28	− 59	− 3,743	− 319	− 3,433	+11,436	+ 9,197	−22,204	+8,215	−1,000*P*
10	0	+ 34	+ 300	+ 1,406	− 86	− 846	− 5,970	− 1,404	+16,428	−9,646	−1,000*P*
(*b*) SOLUTIONS											
...	0.272	0.766	1.177	1.395	2.135	3.304	3.935	5.135	6.134	7.341	....

operations are not included because of their repetitious character. All auxiliary force values, expressed as multiples of $E I \Delta/1{,}000\, l^3$, are shown on the left side of Table 2.

The ten linear equations defining the elastic surface of the slab under load are found by equating the 10 separate reaction magnitudes at each joint to the externally applied load *P* at the same joint. Consequently, Table 2 is not only an expression of auxiliary force systems but also a statement of the ten linear equations of equilibrium.

The force *P* is everywhere equal to $w\, l^2$. For convenience, the load *P* is multiplied by 1,000, thus nullifying the effect of the thousandfold increase in moment value. The solutions of these equations are shown in Table 2(*b*) in which the values of displacements are multiples of $P\, l^3/E\, I$.

TABLE 3.—COMPARISON OF DEFLECTIONS IN TERMS OF $w\, a^4/E\, h^3$

Method of calculation	1	2	3	4	5	6	7	8	9	10
Method of I. A. Wojtaszak	0.00046	0.00133	0.00204	0.00240	0.00371	0.00592	0.00691	0.00905	0.01087	0.01308
Grid analogy	0.00049	0.00140	0.00215	0.00255	0.00390	0.00604	0.00720	0.00939	0.01122	0.01343

In this classical slab-problem, the magnitudes of deflections over the loaded surface, as determined by the theory of elasticity and by the grid analysis procedure, are found to be in excellent agreement. A comparison of deflections at the ten selected positions is shown in Table 3. The results obtained by the theory of elasticity, as expressed by I. A. Wojtaszak,[11] are in terms of $w\,a^4/E\,h^3$, in which a is the side length of square slab; h is the thickness of slab; and the other terms are as previously defined.

Since the length of the grid beam (l) in this example is equal to $a/9$, the grid analysis values shown in Table 2 can be converted as follows:

$$\Delta_6 = 3.304\,\frac{P\,l^3}{E\,I} = \frac{3.304\,(w\,l^2)\,l^3}{E\,l\,h^3/12} = \frac{3.304\,(12)\,w\,l^4}{E\,h^3}$$

$$= \frac{3.304\,(12)\,w\,a^4}{9^4\,E\,h^3} = \frac{0.0604\,w\,a^4}{E\,h^3}.$$

Along the slab boundary there is less grid beam definition since the nearest grid beam is one half a grid beam length removed from the edges. As a result the greatest error in the grid analysis occurs near the slab boundary.

CONCLUSIONS

The analysis of gridworks and slabs subjected to normal load is a tedious process, for the calculation of deflections and the determination of their significance should be performed at many points in order to define clearly the resulting elastic action. Neither the exact mathematical methods nor the approximation schemes can circumvent this difficulty.

The procedure developed in this paper should compare favorably with any known method for determining slab and gridwork deflections. The method is fundamentally a numerical process that results in the solution of only one series of simultaneous equations. When any series of linear equations contains frequent coefficients of zero values for the unknowns, the solution of the equations is relatively simple. This condition may be realized in the slab deflection equations of this method by performing fewer cycles of distribution, thus confining the dispersed moments to fewer joints. Obviously, more errors are incurred when this expediency is employed.

When applied to a true gridwork, the outlined method results in an "exact" solution, for no degree of indeterminateness is avoided. The moment and torque distributions express the unknown moments and twists at each joint in terms of the unknown displacements that comprise the terms of the linear equations. It is believed that the initial threefold indeterminacy assumed at each joint represents a structural condition more realistic than ordinarily considered by other methods of grid analysis.

While the authors can offer no mathematical proof of the analogy between slab and gridwork, numerous results computed by the grid method and checked by the theory of elasticity seem to validate the grid analysis procedure. Obviously, the accuracy of any analysis is dependent on the number of grid beams used to approximate the slab. Using an analogous grid consisting of six north-south beams and six east-west beams and solving directly linear equations containing all three unknowns at each joint, Béla Enyedi[6] has shown that the computed center moment on a uniformly loaded square plate agrees remarkably well with Mr. Marcus' solution by the theory of elasticity. It is interesting to note that the deflections in example C, computed by the grid method, agree more closely with the exact values than those offered by Mr. Marcus' finite difference analysis[12] using the same slab division.

The method of moment and torque distribution for determining slab and gridwork deflections is relatively simple to apply to a wide variety of problems, heretofore difficult to solve.

The degree and the variation of fixity or freedom existing at the slab or gridwork boundary does not complicate the solutions for deflections and moments, since the distribution factors at grid beam terminals are so assigned as to reproduce the given boundary condition. When the fixed boundary conditions of the slab in example C were imposed on the grid beam ends, the elastic surface developed by the grid system was found to be strikingly similar to that computed by Mr. Wojtaszak. Fig. 17 illustrates a slab irregular in shape and variable in boundary conditions and the possible arrangement of grid beams defining the various conditions.

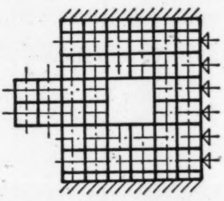

FIG. 17.—GRID BEAMS FOR VARIABLE BOUNDARY CONDITIONS

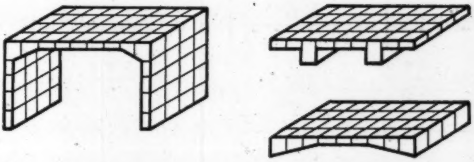

FIG. 18.—SUGGESTED GRID BEAM LAYOUTS

The analysis of flat slabs that are continuous over column or wall supports is possible by this method. As shown in example B, the center column support actually simplifies the analysis of the flat slab by prohibiting deflection at the support joint and consequently reducing the number of unknowns.

Slabs of variable cross sections, when divided into analogous grid beams, submit to solutions if the variation in distribution factors is considered. Tentative grid beam layouts for various cases are shown in Fig. 18.

Other extensions of the moment and torque distributions are feasible. Investigations are being made of slabs on elastic foundations in order to predict deflections and stresses in the slabs.

In general, the moment distribution method for determining deflections and moments in gridworks and slabs exhibits a versatility that is not characteristic of other analytical methods. There are some who will reject the simplicity of the numerical approximation for the elegance of the exact mathematical analysis. Where solutions by the theory of elasticity are known, this attitude seems justified, although a thorough evaluation of slab action by any scheme is a time-consuming process. The real merit of the grid analogy approximation used in this paper has yet to be explored on slabs heretofore defying solution.

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[12] "Die Theorie elasticher Gewebe und ihre Anwendung auf die Berechnung biegsamer Platten," by Henri Marcus, Julius Springer, Berlin, 1924, p. 152.

APPENDIX. COMPUTATION OF DEFLECTIONS

Mr. Wojtaszak gives the following expressions for the calculation of deflection of a uniformly loaded rectangular plate with clamped edges:

$$w = \frac{P}{8\,D}\left(\frac{a^2}{4} - x^2\right)\left(\frac{b^2}{4} - y^2\right) - \frac{P\,a^3}{16\,\pi^2\,D}\left\{\sum_{n=1,3}^{\infty}\frac{A_n}{n^2}\,Y_n\,\frac{\cos\dfrac{n\,\pi\,x}{a}}{\cosh^2\dfrac{n\,\pi\,b}{2\,a}} + \sum_{n=1,3}^{\infty}\frac{B_n}{n^2}\,\frac{b^3}{a^3}\,X_n\,\frac{\cos\dfrac{n\,\pi\,y}{b}}{\cosh^2\dfrac{n\,\pi\,a}{2\,b}}\right\} \qquad \text{(19)}$$

in which w is the deflection; P is the load per unit area; a $(= b)$ is the length of side; x and y are coordinates of plate, origin at center; D is the plate modulus $\left[= \dfrac{E\,H^3}{12\,(1 - \mu^2)}\right]$; μ is Poisson's ratio (= 0.33); $X_n = a \sinh \dfrac{n\,\pi\,a}{2\,b} \cosh \dfrac{n\,\pi\,x}{b} - 2\,x \cosh \dfrac{n\,\pi\,a}{2\,b} \sinh \dfrac{n\,\pi\,x}{b}$; and $Y = b \sinh \dfrac{n\,\pi\,b}{2\,a} \cosh \dfrac{n\,\pi\,y}{a} - 2\,y \cosh \dfrac{n\,\pi\,b}{2\,a} \sinh \dfrac{n\,\pi\,y}{a}$ for $b/a = 1.00$.

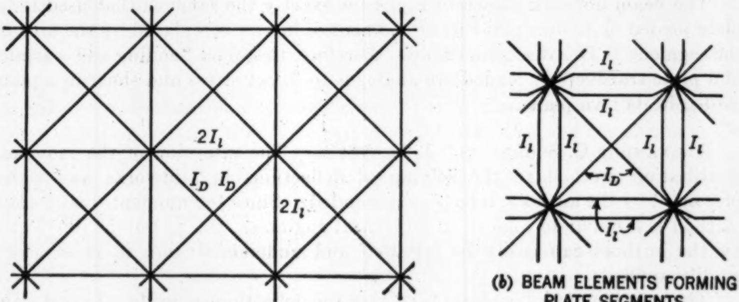
n	A_n	B_n
1	+ 1.4138	+ 1.4138
3	+ 0.0943	+ 0.0943
5	− 0.1096	− 0.1096
7	+ 0.0765	+ 0.0765
9	− 0.0523	− 0.0523
11	+ 0.0365	+ 0.0365

The deflection values shown in Table 3 were calculated using these expressions. Deflection influences can be determined for any point defined by the coordinates x and y by expanding the above series. Since the series converges very rapidly, only three or four terms were deemed necessary. Throughout the calculation an accuracy of five decimal places was used.

DISCUSSION

Jack R. Benjamin,[13] A. M. ASCE.—For a considerable time the writer has been working with the problem of a plate loaded in its own plane rather than transverse to that plane. The two problems are very similar and can be solved by the same general procedures. The authors replace the slab or plate by intersecting beams having both bending and torsional resistance. Some of the fundamental plate characteristics become lost when this is done and the evaluation of the elastic properties of the network members is then only approximate. The solution obtained by the authors shows the errors to be minor in character for the particular problem solved.

The more exact network of beams can be determined readily from the fundamental properties of a finite piece of plate, thus yielding the equivalent beam network shown in Fig. 19. This network differs from that in the paper in two respects—(1) only bending stiffness is considered, and (2) diagonal intersecting beams are added. The elastic properties of the beam network members are as follows:



(a) BEAM NETWORK

(b) BEAM ELEMENTS FORMING PLATE SEGMENTS

Fig. 19

$$I_l = \frac{I}{2(1-\mu)} \left(\text{in which } I = \frac{l\,h^3}{12}\right) \dots\dots\dots\dots\dots (20a)$$

$$I_D = \frac{I\,\mu\,\sqrt{2}}{1-\mu^2} \text{ (for pure bending)} \dots\dots\dots\dots\dots (20b)$$

and

$$I_D = \frac{I\,\sqrt{2}}{2(1+\mu)} \text{ (for warping)} \dots\dots\dots\dots\dots (20c)$$

If μ is 0.333, Eqs. 20 reduce to

$$I_l = \frac{3}{8}\,I \dots\dots\dots\dots\dots\dots\dots\dots (21a)$$

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[13] Associate Prof. of Structural Eng., Stanford Univ., Stanford, Calif.

and

$$I_D = \frac{3}{8}\sqrt{2}\,I \text{ (for both bending and warping)} \dots\dots\dots\dots (21b)$$

The entire plate is then formed by the consideration of all the beam elements (Fig. 19). The side elements add stiffnesses to become 0.75 *I*. The entire plate then has been replaced by a finite number of segments, each with sides of length *l*. The plate deflections will then agree exactly with the beam network deflections at all beam intersections if a large number of segments are used. Errors have been found to be very small when only a very few segments are used.

The solution of the beam network requires considerable work and if extreme care is not taken in the selection and adaptation of procedures a very difficult convergence problem arises. Generally a numerical iterative solution is used and, although errors at any given point may appear small, they may be cumulative at one or more points, resulting in large local errors.

The beam network shown in Fig. 19 is exactly the same as that used for a plate loaded in its own plane if the moment of inertia is replaced by the area of the segment. This discussion shows, therefore, that pure bending and warping of a plate transversely loaded are analogus to direct stress and shear in a plate loaded in its own plane.

ALEXANDER C. SCORDELIS,[14] J. M. ASCE.—The extension of the moment-distribution method to the solving of deflections in gridworks and slabs, presented by the authors, is to be commended. Since the moment-distribution method is familiar to almost all structural engineers, the technique developed by the authors can easily be followed and understood and, in most cases, readily applied.

The solution of any problem involving the deflections in gridworks and slabs is a tedious procedure. Using the method presented in this paper, it can be seen that for gridworks that lack symmetry and have many joints the work involved becomes appreciable. A separate solution must be made for each joint displacement, then for each of these solutions reactions or holding forces must be calculated at each joint, and, finally, to obtain the deflections a set of simultaneous equations which satisfy shear relations must be solved.

In comparing this method with the moment-distribution method used in sidesway problems, an alternate procedure suggests itself. Sidesway problems—or better yet, problems involving joint dispacements for frames loaded in the plane of the structure—may be solved in one of two ways: The first is the so-called superposition method, which is analogous in many respects to that used by the authors in solving their problem; and the second is that of successive shear corrections. This latter procedure is considered by many to be superior to the superposition method since, when the number of possible joint displacements becomes great, it eliminates the necessity for making separate solutions and solving simultaneous equations.

The method of successive shear corrections can also be used in solving for deflection in gridworks and slabs. As a general case, consider the gridwork shown in Fig. 20, with a load of 1,000 P, at joint 1. This gridwork is the same as that shown in Fig. 5. For the solution of deflections by successive shear corrections, the general procedure can be described in five steps.

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[14] Asst. Prof. of Civ. Eng., Univ. of California, Berkeley, Calif.

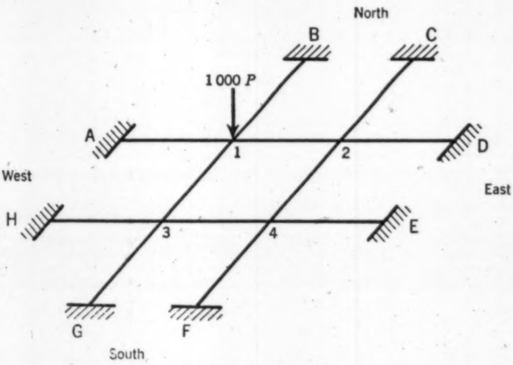


FIG. 20.—GRIDWORK

Step 1.—One shear equation is written for each possible joint displacement. These equations will involve the external load at the joint and the shears at the far ends of the members framing into the joint. These shears can be expressed in terms of the moments at each end of the four members. For example, at joint 1:

$$\left(\frac{M_{A1} + M_{1A}}{L}\right) - \left(\frac{M_{12} + M_{21}}{L}\right) + \left(\frac{M_{B1} + M_{1B}}{L}\right) - \left(\frac{M_{13} + M_{31}}{L}\right) - 1{,}000\ P = 0 \ldots (22)$$

or simplifying, at joint 1:

$$M_{A1} + M_{1A} - M_{12} - M_{21} + \mathrm{M}_{B1} + M_{1B} - M_{13} - M_{31} - 1{,}000\ P\ L = 0 \ldots (23a)$$

Similarly at joint 2—

$$M_{12} + M_{21} - M_{2D} - M_{D2} + M_{C2} + M_{2C} - M_{24} - M_{42} = 0 \ldots (23b)$$

at joint 3—

$$M_{H3} + M_{3H} - M_{34} - M_{43} + M_{13} + M_{31} - M_{3G} - M_{G3} = 0 \ldots (23c)$$

and, at joint 4—

$$M_{34} + M_{43} - M_{4E} - M_{E4} + M_{24} + M_{42} - M_{4F} - M_{F4} = 0 \ldots (23d)$$

The signs of the moments in Eqs. 23 are in accordance with the moment-distribution convention when viewed from the south and the west.

Step 2.—With all joints locked each joint is displaced. These displacements produce fixed-end moments in the members framing into each joint.

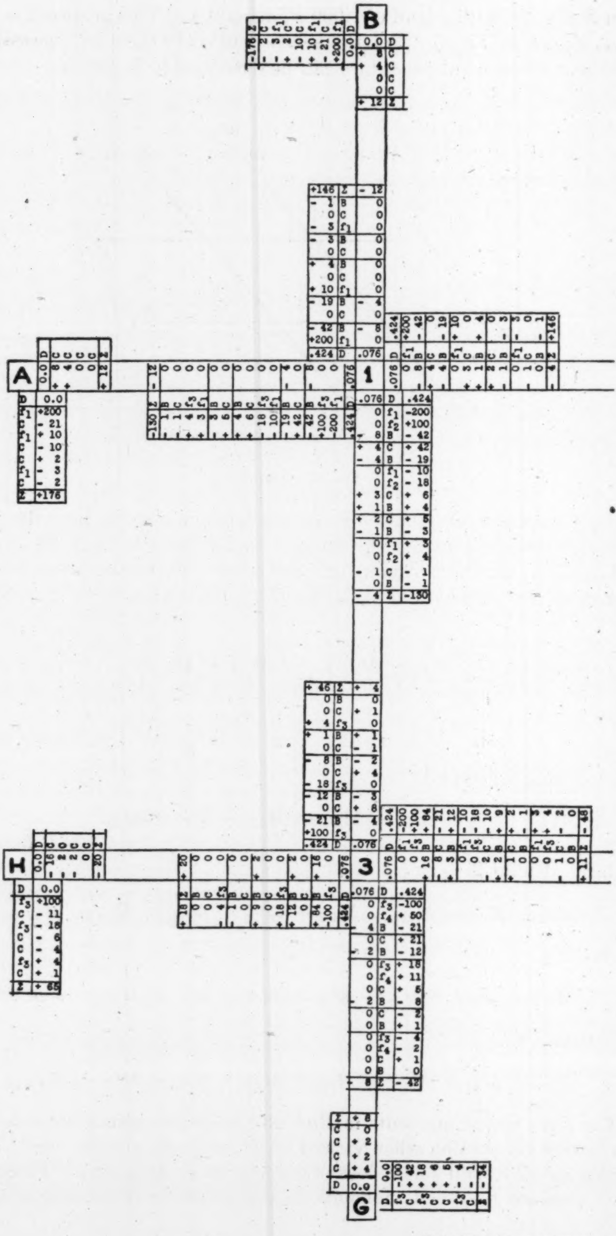
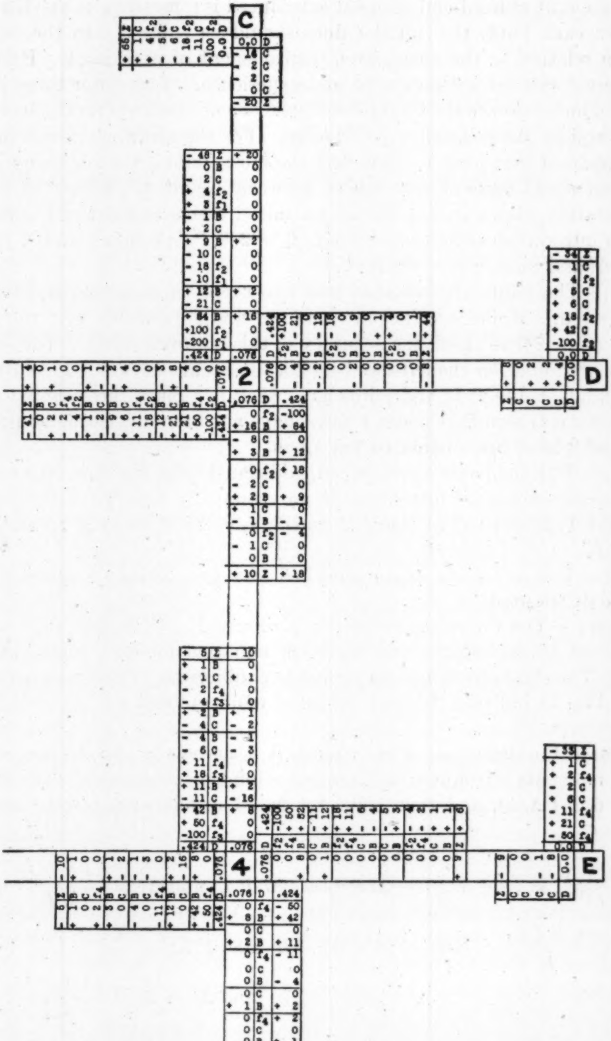


FIG. 21.—DISTRIBUTION OF



MOMENTS AND TORQUES

Since the amount of fixed-end moment introduced is a measure of the displacement given each joint, the initial values should be entered into the tabular solution in relation to the anticipated displacement of each joint. Here the analyzer must exercise his judgment and experience. The closer these initial fixed-end moments are to the total joint-displacement moments for the structure the more rapidly the solution will converge. For the gridwork shown in Fig. 20, it is apparent that joint 1 will deflect the most, joint 4 the least, and joints 2 and 3 an equal amount somewhere between 1 and 4. Therefore in the tabular solution shown in Fig. 21, as an initial estimate fixed-end moments have been introduced as follows: Joint 1, $f_1 = 200\ P\ L$; joints 2 and 3, $f_2 = f_3 = 100\ P\ L$; and joint 4, $f_4 = 50\ P\ L$.

Step 3.—The joints are unlocked and two cycles of moments and torques are distributed. Moment values are summed and substituted into the shear equations (Eqs. 23) to check the accuracy of this approximation. The following results are found for the left-hand side of the equations: Joint 1, $-\ 76\ P\ L$; joints 2 and 3, $+\ 144\ P\ L$; and joints 4, $-\ 88\ P\ L$. Since these should all be equal to zero, it is seen that joints 1 and 4 have not been displaced enough and joints 2 and 3 have been displaced too much.

Step 4.—With the joints again locked, additional fixed-end moments created by joint displacement are introduced in an attempt to satisfy the shear equations: Joint 1, $f_1 = 10\ P\ L$; joints 2 and 3, $f_2 = F_3 = -\ 18\ P\ L$; and joint $4 = 11\ P\ L$.

Step 5.—The joints are again unlocked and two cycles of moments and torque are distributed.

Summary.—The foregoing procedure is repeated until finally all moments about a joint are in balance and the shear equation for each joint has been satisfied. The structure is now in complete equilibrium. The moment totals shown in Fig. 21 indicate the true values of moments and torque at each end of the members.

To obtain the deflection of each joint, it is necessary only to sum up the fixed-end moments introduced by successive joint displacements, equate these totals to $6\ E\ I\ \Delta/L^2$, and then solve for Δ. The following results are then obtained:

$$\frac{6\ E\ I\ \Delta_1}{L^2} = (200 + 10 - 3)\ P\ L = 207\ P\ L;\ \text{and}\ \Delta_1 = \frac{34.5\ P\ L^3}{E\ I} \quad \text{. . (24}a\text{)}$$

$$\frac{6\ E\ I\ \Delta_2}{L^2} = \frac{6\ E\ I\ \Delta_3}{L^2} = (100 - 18 + 4)\ P\ L = 86\ P\ L;$$

$$\text{and}\ \Delta_2 = \Delta_3 = \frac{14.4\ P\ L^3}{E\ I} \quad \text{. . (24}b\text{)}$$

and

$$\frac{6\ E\ I\ \Delta_4}{L^2} = (50 + 11 - 2)\ P\ L = 59\ P\ L;\ \text{and}\ \Delta_4 = \frac{9.9\ P\ L^3}{E\ I} \quad \text{. . . (24}c\text{)}$$

The values of Δ in Eqs. 24, when divided by 1,000, compare favorably with the results obtained by solving Eqs. 16. More accuracy could be obtained by increasing the cycles of distribution and by using more significant figures.

The foregoing method of successive shear corrections has some advantages and some disadvantages. The advantages are as follows:

1. A single solution is required no matter how many joints are involved.
2. To find the deflections no simultaneous equations are needed at the end of the solution. This becomes important when the number of joints involved is great.
3. Final answers for torques and moments are obtained at the end of a single solution.
4. The analyzer can use his judgment and experience in estimating the relationships between the various joint displacements, thus speeding the solution.
5. The method of first using a coarse net, and then with the displacements obtained therefrom proceeding to a finer net, can be pursued.
6. In cases of symmetry of loading it is possible to use short cuts similar to those used by the authors.

The disadvantages are as follows:

a. In certain cases the single solution mentioned as advantage 1 is somewhat cumbersome; and

b. Each new loading condition on the structure involves a separate solution.

In any particular problem the analyzer must decide whether to use the superposition method or the successive shear corrections method. Each has a definite advantage in certain types of problems, and the analyzer should be familiar with both.

Paul J. Brennan,[15] J. M. ASCE.—Evidence on the action of reinforced concrete slabs is obtained from analysis, testing, experience, and judgment. Information from one source may substantiate evidence from other sources, may disagree with evidence from other sources, or may introduce evidence that is not apparent in other sources. Nonetheless, all are necessary for an understanding of the structural action of reinforced concrete slabs.

Mathematical analysis is an important and relatively inexpensive source of evidence and a necessity in the interpretation of data obtained from tests. The absence of a satisfactory method of analysis is one of the major impediments in the study of slabs. At present, slab analysis is restricted to the mathematical theory of elasticity, methods of static control, approximate formulas, and systems that assume the slab to be a grid. Each approach is limited.

The mathematical theory of elasticity is limited to homogeneous and isotropic plates. The expression defining the deflections must be known, and must satisfy the boundary conditions and the Lagrange equation. The Lagrange expression, which equates the statics to the slab deformations through the properties of the material, is not general. It is only applicable when the moduli of elasticity are the same in all directions and the moments of inertia about horizontal axes are equal in all directions. Reinforced concrete slab construction does not lend itself to these conditions.

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[15] Asst. Prof. of Civ. Eng., Yale Univ., New Haven, Conn.

The static control approach is powerful and exact for some conditions. Although the total moment across any section or sections of a slab may be known, the manner in which these moments are distributed is not known. This may not be serious, however, because, if provision is made to resist this total moment, the slab will probably adjust automatically to resist it and there is reason to believe that it does.

The general grid approach has the advantage of showing a physical picture. It is somewhat straightforward and uses tools that have become familar to many engineers. However, it does have disadvantages. When a distribution procedure is used in some methods convergence is not automatic. The number of strips to be considered is not obvious. Dependence is placed on values of torsional resistance in concrete which are nebulous, but the lack of such values is probably not serious.

The grid analysis system explained in the paper is one of a number of systems that may be developed. For example, a system using a grid of beams dependent upon linear simultaneous equations for the solution of deflections was presented in 1933 by John N. Pirok.[16] The method presented by Messrs. Ewell, et al., has the disadvantage that it is dependent upon the solution of linear simultaneous equations. The grid system that is dependent, in whole or in part, on linear simultaneous equations is open to discussion. It is possible to solve a few equations containing a few unknowns if the equations are of the right form. Symmetrical slabs with symmetrical loadings do not necessarily involve a prohibitive number of equations. The unsymmetrical slab with an unsymmetrical load is common in structures and by the system described involves one equation and one unknown for each grid intersection. The satisfactory solution of these equations for a sufficient number of grid beams to represent this slab is questionable.

The writer believes that a grid system approach to the analysis of reinforced concrete slabs is valuable and may be an important source of evidence in the study of slabs. However, it must be applicable to any slab, with any variation in thickness, any support conditions, and any loading. The mathematical approach is limited by the assumptions on which it is based. If the reinforced concrete slab does not conform to the assumptions, the mathematical analysis may be an unsatisfactory source of evidence. Even with reasonable conformation between the assumed action and the structural action, mathematical analysis is not sufficient by itself. Testing, experience, and judgment must be considered and evaluated with analysis.

Serge Leliavsky Bey,[17] M. ASCE.—Not only does this paper have outstanding merit, but also much timeliness, for the problem it deals with—namely, the bearing capacity of a thin plate supported (or fixed) on four or two sides—calls for further and possibly deeper study, which, the writer believes, may result (if successful) in a material saving in the expenditure on steel gates and various other works. It should, nevertheless, be realized that the principle of substituting a grid for a slab is not as obvious, as it may possibly appear at first glance, for from the standpoint of elastic deformation analysis, a set of parallel beams does not deflect in the same manner as a plate —even if the mean second moments, per unit width, are the same in both cases —because of the lateral elastic constraint in the plate.

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[16] "Studies in the Analysis of Slabs," by John N. Pirok, thesis presented to the University of Illinois, at Urbana, Ill., in 1933, in partial fulfilment of the requirements for the degree of Master of Science in Civil Engineering.

[17] Director, Designing Service, Reservoirs & Nile Barrages Dept., Ministry of Public Works, Cairo, Egypt.

In Fig. 22 the elastic deformation is intentionally exaggerated, and the plate is assumed to be supported on two sides only, that is, AB and CD. As the plate bends, compression is developed in the upper part of the central section, abcd, and tension is developed in the corresponding lower part. Considering element 1 as a free body, its top width *ab* becomes a_1b_1 because of the lateral effect of the main stress while *cd* deforms to the length, c_1d_1.

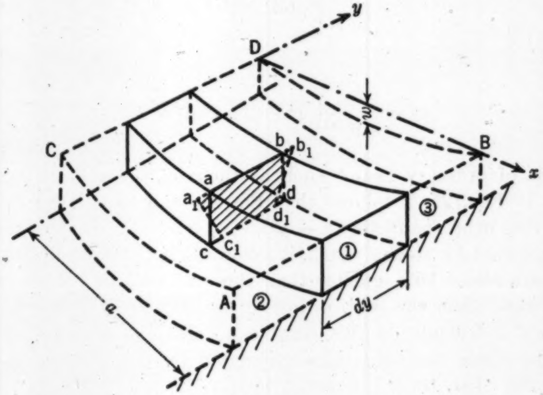

FIG. 22

However, since elements 2 and 3, adjacent to element 1, prevent these lateral deformations from taking place, a secondary compressive stress must develop, at right angles to the main, primary bending stress in the upper part of the plate; also, a secondary tensile stress must be found to occur in the lower part of the same section. These additional stresses can be visualized as being caused by a constraint moment, acting at right angles to the main bending moment—very much as if one side of the plate had been heated while, at the same time, deformations were prevented.

To estimate quantitatively the importance of this lateral effect, the general equation of the elastic plate

$$\frac{\partial^4 w}{\partial x^4} + 2\,\frac{\partial^4 w}{\partial x^2\,\partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p}{N} \quad \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

may be employed. The symbol *w* represents the vertical deflection, whereas *N* is the so-called "rigidity factor"

$$N = \frac{Eh^3}{12\,(1 - \gamma^2)}$$

in which E and γ are, respectively, the Young modulus and the Poisson ratio, and h is the thickness of the plate. Since in Fig. 22 w is constant for given values of x, Eq. 25 becomes[18]

$$\frac{\partial^4 w}{\partial x^4} = \frac{p\, f(x)}{N} \quad \text{(26)}$$

Assuming that p is constant, integrating, and substituting $w = 0$ and $M_x = 0$, for either $x = 0$ or $x = a$,

$$w = \frac{p}{24\, N}\left(x^4 - 2\, a\, x^3 + a^3\, x \right) \quad \text{(27a)}$$

as against

$$w = \frac{p}{2\, E\, h^3}\left(x^4 - 2\, a\, x^3 + a^3\, x \right) \quad \text{(27b)}$$

for the case of ordinary beams such as those of the grid. Comparing the coefficients in Eqs. 27, it follows that Eq. 27*a* can be derived from Eq. 27*b* by multiplying it by the factor $(1 - \mu^2)$.

Assuming that for steel $\mu = 0.3$, it follows that the deflections of an equivalent grid are about 10% greater than those of a plate. Thus, the plate is 10% more rigid than the grid, owing to the effect of the lateral constraint. The authors' comments on this point can enhance the value of the paper. In particular since the deflections calculated by the authors' method were always greater than those computed by Eq. 19 (see Table 3).

In most engineering structures such as steel gates or reinforced concrete decks, the designer is not concerned with a single unit, but with a group of contiguous plates. This fact introduces a new element in the mechanical analysis of the problem—namely, the horizontal component, S, of the reaction. Had the plate been ideally rigid or, alternatively, had its edges been provided with sliding or roller bearings, this force could not have developed. However, since the plate is usually even more flexible than a beam, and its edges are either cast together, riveted, or welded to the supporting beams (which possess a certain degree of lateral rigidity, or are prevented from moving transversly by the plates spanning the adjacent panels), it follows that each individual panel will work as a chain or rope, somewhat in the same manner as a suspension bridge.

To the usual equation of the elastic axis of the plate—

$$E\, I\, (1 + \mu^2)\, \frac{d^2 w}{dx^2} = \frac{p\, a^2}{4} - \frac{p\, x^2}{2} \quad \text{(28)}$$

—another term, $-S\, w$, must, therefore, be added; and, thus,

$$E\, I\, (1 + \mu^2)\, \frac{d^2 w}{dx^2} = \frac{p\, a^2}{4} - \frac{p\, x^2}{2} - S\, w \quad \text{(29)}$$

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[18] "Elastische Platten," by A. Nadai, J. Springer, Berlin, Germany, 1925, p. 70.

Integrating Eq. 29, Ph. Forchheimer[19] obtains

$$w = \frac{P}{S\,r^2}\left[1 - \frac{\cosh\,(r\,x)}{\cosh\left(\frac{r\,a}{2}\right)}\right] - \frac{p\left(\frac{a^2}{4} - x^2\right)}{2\,S} \qquad (30)$$

in which

$$r = \sqrt{\frac{S}{E\,I\,(1 + \mu^2)}} \qquad (31)$$

For the maximum deflection, in the middle of the span, Eq. 30 yields

$$w_0 = \frac{p}{S\,r^2}\left[1 - \operatorname{sech}\left(\frac{r\,a}{2}\right)\right] \qquad (32)$$

and the corresponding moment, obviously, is equal to

$$M_0 = \frac{p\,a^2}{8} - S\,w_0 = \frac{p}{r^2}\left[1 - \operatorname{sech}\left(\frac{r\,a}{2}\right)\right] \qquad (33)$$

The problem is thus solved, provided S is known. In order to calculate this force, the length, β, of the elastic axis must be known; then

$$S = \frac{\beta - a}{a}\,E\,(1 + \mu^2)\,h \qquad (34)$$

It is well known that analytical determination of the lengths of arcs is frequently a difficult problem. In the present case Forchheimer introduced certain simplifications, and thus obtained a solution that can be presented in the following form:

$$\frac{16\,h^4\,E\,(1 + \mu^2)}{p\,a^4} = f\left(\frac{r\,a}{2}\right) = \phi \qquad (35)$$

For the function $f(r\,a/2)$ he produced a table which, for practical convenience, the writer has converted into a graphical form as shown in Figs. 23 and 24, depending upon whether the plate is freely supported or fixed. In the latter case, that is, for a fixed plate, the formula for the bending moment will include an additional term, the fixation moment M_f, and the symbol ψ will be used to denote the relevant function (in place of ϕ).

For practical applications of the grid analogy, forming the subject matter of the paper under discussion, it is rather important to consider the force, S, for in this manner a solution may possibly be obtained, explaining—at least partly—the fact that, in nature, plates are capable of withstanding far greater loads than those calculated from the various elastic bending theories.

To afford an idea of the exceedingly high capacity of elastic plates in nature it will suffice to quote some of the classical experiments of C. Bach,

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[19] "Die Berechnung ebener und geecrümmter Behälterboden," by Ph. Forchheimer, W. Ernst, Berlin, Germany, 1909, p. 3.

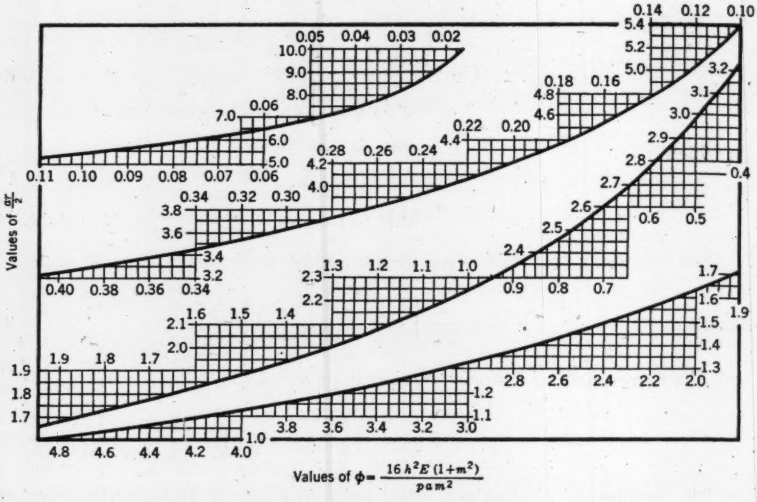

FIG. 23.—FORCHHEIMER'S FUNCTION ψ, FOR PLATES FREELY SUPPORTED

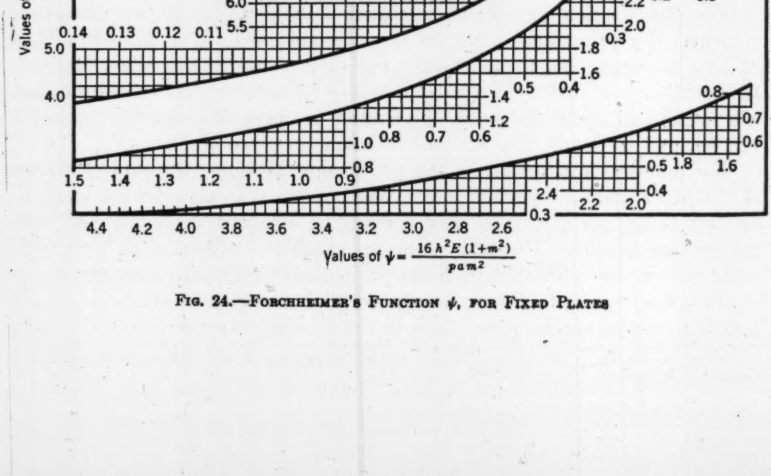

FIG. 24.—FORCHHEIMER'S FUNCTION ψ, FOR FIXED PLATES

which are mentioned in so many reference books (without, however, giving the actual test data):

a. Square Plate.—Sides $a = b = 80$ cm, thickness $h = 0.8$ cm; computed permissible unit load $p = 0.48$ atmosphere. The pressure was raised to 24 atmospheres without failure, or any sign of failure.

b. Square Plate, Same Size As Item a But with $h = 1.6$ Cm.—Calculated $p = 1.92$ atmospheres. The pressure was raised to 20 atmospheres. There was no sign of failure, but the test was discontinued because of leakage in the joints.

c. Rectangular Plate.—$a = 40$ cm and $b = 80$ cm, $h = 0.8$ cm; computed load $p = 1.2$ atmospheres; pressure raised to 28 atmospheres. No sign of failure.

d. Rectangular Plate, Same Size As Item c But with $h = 1.6$ cm.—Calculated $p = 4.8$ atmospheres; actual $p = 34$ atmospheres, after which the test was discontinued, because of leakage in the joints.

It appears from these data that here are phenomena belonging to the plastic range of conditions. It may be of interest to mention in this connection that, according to R. N. Southwell, the relaxation method, employed by the authors, is particularly suited for dealing with such cases, and it would therefore be of considerable interest to develop the solution in this direction.

To sum up this discussion, there are three points that require further explanation or development—(*a*) the effect of the $(1 + \mu^2)$-factor, (*b*) the effect of the *S*-force, and (*c*) the effect of plasticity.

W. Watters Pagon,[20] M. ASCE.—There have been solutions of this problem for slabs of circular and square shape, but when the shape becomes irregular exact analysis has heretofore been impossible. In general, it remains so. Of course, following the authors' method, the work involved becomes stupendous, unless the computations can be so arranged as to be handled by a mechanical computer.

A trapezoid, such as that in a skew span, has been treated fairly extensively by others, but in modern concrete bridges there often arises the problem of stress in an irregularly shaped quadrilateral such as the type occurring at the entrance or exit of a ramp approach. Such a slab at times also requires a central "edge beam" extending from a salient corner, which makes the problem essentially one of two adjacent, continuous spans straddling the central edge beam with negative moment over it, and all complicated by the irregular shape. A very flat dome roof, with reinforcing ribs and local concentrations, in addition to the customary uniform load causing radial and circumferential stress, might well be analyzed by the authors' method.

Specifications of the American Association of State Highway Officials (AASHO), for the design of an edge beam, are a compromise, and permit the design of a deep curb beam or the addition of extra reinforcement to the outer edge of the slab. This is a problem which in the writer's opinion deserves re-analysis, and might perhaps be done by the authors' method. Of course, the writer fully realizes that these suggestions contemplate extensive work, and the use of mechanical or electrical computers; however, it does not seem that there is any other recourse to some of the problems other than by using expensive and elaborate model tests.

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[20] Cons. Engr., Baltimore Life Bldg., Baltimore, Md.

The authors are to be congratulated on their efforts.

Phil M. Ferguson,[21] M. ASCE.—The possibility of analyzing slabs by procedures with which many structural engineers are already familiar is introduced in this well written paper. It is true that other methods of slab analysis, involving difficult mathematics, have made great progress and have been the basis for productive numerical methods. Nevertheless, the field of such methods thus far has been limited to relatively simple boundary conditions. This new approach appears to be on a very flexible basis; even variable sections and slab openings are minor rather than major difficulties.

This is an entirely new approach in the sense that related earlier work appears not to have found its way into English publications. Since foreign language reports are not actually available to most American engineers, those who report promising developments from abroad deserve recognition. In this case the authors have gone further and introduced American procedures that are a distinct improvement.

The possibilities for this method are many, but several factors need further appraisal. The first factor is the accuracy of the results. Deflections of slabs are of secondary importance structurally compared to bending moments in slabs. Methods that are adequate for deflections may not be entirely adequate for establishing satisfactory moments. The authors have checked the accuracy of their methods and the results shown in Table 3 are quite good. Nevertheless, other comparisons would add much to the paper. It is hoped that their closing discussion will show such comparisons, which they report as available, and will also include a brief restatement of Béla Enyedi's numerical comparison.[6] The writer would like to have this additional basis for evaluating the general accuracy and usefulness of the method.

In addition to the approximations involved in using finite widths of strips (and relatively large finite widths) instead of differential quantities, it appears that this procedure now neglects the Poisson ratio effect. Possibly this is one reason the authors' values in Table 3 are slightly too large. (Another factor is discussed later that should make these calculated values too small; the two effects partly offset each other.) It appears that it might be necessary to incorporate Poisson's ratio in some way before really good values of moment would be obtained. The writer does not quite visualize the manner in which this can be done, but believes that it is possible, certainly as an approximation if not more accurately.

The method as presented is limited by the fundamental difficulty of handling a large number of simultaneous equations. Before much further progress can be made, moment distribution methods using successive approximations must be made available for this type of problem. At present such methods for extended grids are themselves difficult. The writer has used successive approximations to solve grids slightly more complex and more extended than the twleve-joint quadrant used in Example B; but such solutions can scarcely be classified as practical. If grid methods can also be used for slab analysis, as this paper suggests, further study of such procedures is quite important.

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[21] Chairman, Dept. of Civ. Eng., Univ. of Texas, Austin, Tex.

The writer would like to recommend the tabulation form used in Fig. 6 as being far superior to that used in Fig. 11. It is realized that Fig. 11 covers rotations about both axis *xx* and axis *yy*. These calcuations would be easier to follow if separate tabulations were set up for these two axes. For members intersecting at right angles there is no transfer of data between the calculations for the two axes; hence, tabulations need not be adjacent.

In one respect the writer is critical of the authors' detailed procedure. Figs. 9 and 13 show slabs with fixed edges. The gridwork substituted for these slabs ignores a half strip width adjacent to each fixed edge. Since these half strips also deform, their neglect seems equivalent to assuming a fixed edge one half strip inside the ones originally shown in the statement of the problem. All the authors' calculated deflections should thus be too small. Table 3 indicates otherwise, which might seem to refute this argument. However, the writer believes that this is simply evidence that some other important factor giving an opposite effect has been omitted, possibly Poisson's ratio.

Further study of this method is definitely needed before it can be fully accepted. However, the method has such promise (if it can be entirely validated) that such further study is more than justified.

The writer knows of no other method that approaches this close to the solution of practical problems: (*a*) Of flat slabs that are not entirely regular, (*b*) of slab band construction, or (*c*) of slabs with holes cut through them—in other words, the design of slabs as the structural engineer encounters them, day in and day out. Even if the method cannot be fully validated, it may yield relative values that will be significant and illuminating. Hence, he predicts that much more will be heard of this method in the future. The authors are to be commended for this pioneering step.

ROBERT V. WHITMAN,[22] J. M. ASCE.—This discussion is concerned with the authors' treatment of the so-called grid analogy for obtaining solutions to problems in the bending of flat plates. The fundamental concept behind this analogy is certainly sound. However, the paper leaves in doubt several points concerning the stiffness factors to be applied to the beams of the grid.

First, consider the choice of the torsional stiffness factor. In both of the plate examples treated by the authors, the aspect ratio of the grid beams was calculated and the corresponding value of the factor β in Eq. 3*b* was then selected. Suppose that the number of the beams in the grid were to be increased many times, the plate thickness remaining unchanged. The factor β would decrease rapidly, and correspondingly the relative importance of the torsional stiffness factor would increase as compared to the bending stiffness factor. In the limiting case of infinitely many very thin beams, β would vanish and the calculation would indicate no twisting moments in the plate.

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This consideration indicates that the use of a torsional stiffness factor coupled to the aspect ratio of the grid beams is not correct. Rather, the value $\beta = \frac{1}{3}$ should be used at all times. The results of the calculations can then be used for a plate of any thickness, assuming of course that the limitation on the magnitude of the transverse shear forces is observed. Following the authors' line of reasoning, the deflection values listed in Tables 1 and 3 would hold accurately only for one value of plate thickness, and small changes in the distribution factors would lead to slightly different deflections for other thicknesses. The values of β used by the authors were very nearly one third, and thus these comments affect the general theory rather than their numerical results.

The choice of the bending stiffness factor is also open to question. The authors use a factor that is independent of Poisson's ratio, although it is well known that the bending stiffness of the beam elements of a plate is increased in the ratio $\frac{1}{1 - \mu^2}$ since these elements are restrained against lateral change in thickness. It would seem that the grid analogy would be most accurate if the beams of the grid were to conform as nearly as possible to elements cut from a plate, and that the use of the increased stiffness would hence be more logical. Unfortunately, no simple, concise argument is at hand to aid in a definite choice of the proper factor.

If the suggestions in the preceding paragraphs are followed, the distribution factors appearing in Eqs. 18*a* and 18*b* will become

$$\text{Percentage moment} = 12.62 \text{ torque}$$
$$\text{Percentage moment} = 37.38 \text{ bending.}$$

A similar change would be made for the values shown in Fig. 10. These changes are not large, and the auxiliary forces listed in Table 2, for example, would not be greatly altered. Since small changes in the coefficients of simultaneous equations sometimes cause considerable change in the solutions of these equations, it is difficult to estimate how much the values in the bottom line of Table 2 will change. However, the formula used to convert the values in this line into the deflection values in Table 3 will contain the factor $\frac{1}{1 - \mu^2}$, and here the effect will definitely be noticeable.

Positive answers to these questions can be obtained only from a mathematical demonstration of the validity of the grid analogy. Such a task was undertaken by Milton Platt[23] who wrote the equilibrium and continuity equations for the joints and beams of the grid system, and then applied difference and summation procedures to combinations of these equations. In the limiting case of infinitely many beams these operations yielded the differential equation for the bending of plates, thus establishing the analogy. Mr. Platt's work confirms the use of the same torsional stiffness factor for all grid beam proportions. Since he applied the demonstration only for the case $\mu = 0$, further study is necessary to establish definitely the proper bending stiffness factor.

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[23] "A Network Procedure for the Analysis of Flat Plates Subjected to Transverse Distributed Loads," by Milton M. Platt, thesis presented in 1946, to the Massachusetts Inst. of Technology, Cambridge, Mass., in partial fulfilment of the requirements for the degree of Doctor of Science.

As stated by the authors, the labor required to solve plate problems in this fashion is still considerable. As a result of some experience with the procedure, it is estimated that an experienced person using this procedure on a plate such as the one in Example C would consume about 40 hr for the determination of the auxiliary force system, and another 10 hr to solve and check the simultaneous equations and to assemble the desired results. By far the major part of the work is in the moment distribution steps. The authors suggest one possibility for reducing this numerical effort, that is, using a limited number of cycles of distribution. Another approximation can be obtained by neglecting the torsion of the grid beams. Then beams intersecting at a joint will have the same deflection but not necessarily the same slopes. It is instructive to see the magnitude of the errors that may result from such an assumption.

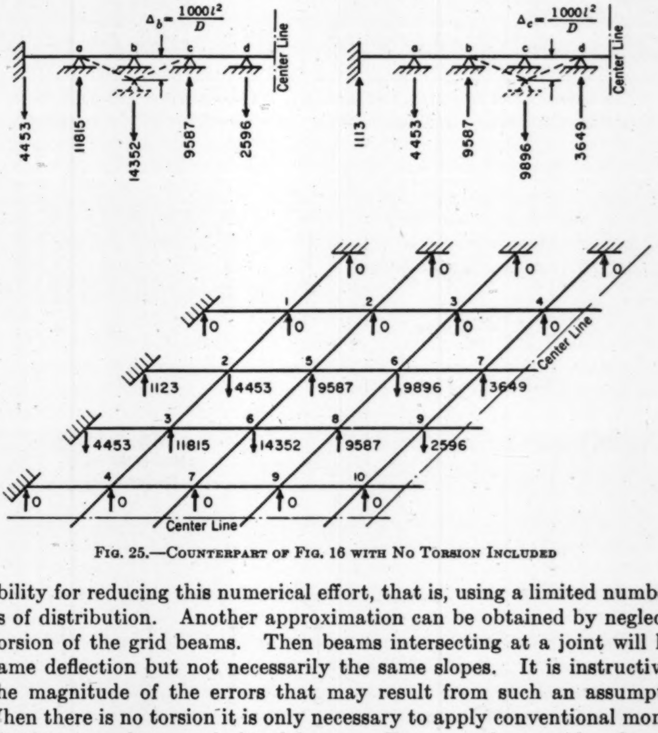
$\Delta_b = \frac{1000l^2}{D}$
a
b
c
d
Center Line
4453
11815
14352
9587
2596
$\Delta_c = \frac{1000l^2}{D}$
a
b
c
d
Center Line
1113
4453
9587
9896
3649
0
0
0
0
1
2
3
4
0
0
0
0
0
2
5
6
7
1123
4453
9587
9896
3649
Center Line
3
6
8
9
4453
11815
14352
9587
2596
4
7
9
10
0
0
0
0
0
Center Line

FIG. 25.—COUNTERPART OF FIG. 16 WITH NO TORSION INCLUDED

When there is no torsion it is only necessary to apply conventional moment distribution procedures to isolated beams. For example, consider the plate in Example C. Fig. 25 indicates how the auxiliary forces caused by deflection Δ_6 can be obtained by superimposing the results of distributions on only two beams. With but two additional distributions a table corresponding to Table 2 can be prepared. The work of forming the table has been reduced, and it will contain a good number of zeros. Such a "no-torsion" solution has been made assuming the stiffness of the grid beams to be proportional to $\frac{E\,I}{1-\mu^2}$, and

TABLE 4.—COMPARISON OF MAXIMUM DEFLECTIONS AND MOMENTS FOR UNIFORMLY LOADED, SQUARE PLATES (See Fig. 13)

Method of Computation	(*a*) FOUR EDGES CLAMPED (EXAMPLE C)			(*b*) TWO ADJACENT EDGES	
	Deflection	Positive moment	Negative moment	Deflection, point 10	Moment, line D
(1)	(2)	(3)	(4)	(5)	(6)
I. A. Wojtaszak[a]	$0.1308 \frac{w a^4}{E h^3}$	$0.0233\ w\ a^2$	$0.0513\ w\ a^2$	....	....
Grid Analogy—Authors'	....	....	....	$0.292 \frac{w a^4}{E h^3}$	$0.241\ w\ a^2$
No torsion	$0.01647 \frac{w a^4}{E h^3}$	$0.0233\ w\ a^2$	$0.0542\ w\ a^2$	$0.705 \frac{w a^4}{E h^3}$	$0.334\ w\ a^2$
Percentage error	26	0	6	142[b]	39[b]

[a] *Transactions*, A.S.M.E., Vol. 59, 1937, p. A. 173. [b] Percentage error with no torsion.

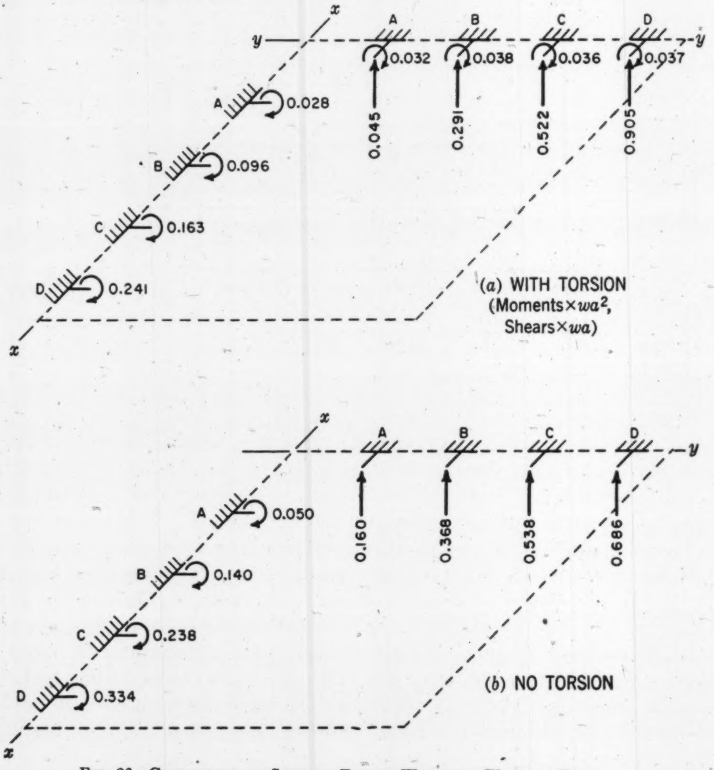
x
y
A
B
C
D
y
0.032
0.038
0.036
0.037
0.045
0.291
0.522
0.905
A 0.028
B 0.096
C 0.163
D 0.241
x
(a) WITH TORSION
(Moments × wa^2,
Shears × wa)
x
A
B
C
D
y
0.160
0.368
0.538
0.686
A 0.050
B 0.140
C 0.238
D 0.334
x
(b) NO TORSION

FIG. 26—COMPARISON OF SUPPORT FORCES WITH AND WITHOUT TORSION

the solution has been summarized in Table 4(*a*). Although the error in the maximum deflection is considerable, the agreement in the moments is remarkable.

As a second example of the importance of torsional forces, consider the case of a square plate with sides of length *a*, with two adjacent edges clamped and the other two edges completely free, and subject to a uniformly distributed load. All other properties are the same as for the plate of Example C. The manner of choosing grid beams is the same as for one quadrant of that plate. Except for the distribution factors used at joints 4, 7, 9, and 10 (Fig. 13) all numerical work follows exactly the same form. Table 4(*b*) and Fig. 26 show the results of applying the grid analogy to this problem, first with torsion included and then with no torsion. The edge shears shown do not include the effect of the loads on the grid beams of width $\frac{1}{2}\,l$ which lie immediately adjacent to the fixed supports, and which do not enter into the distributions.

In this case the assumption of no torsion gives deflections which are very much in error, and the discrepancies in the moments and shears are also considerable. These larger errors are not surprising since the case is one in which torsional couples are certain to play an important part.

It is clear that the approximation of no torsional forces must be used cautiously, and only after the investigation of numerous other cases. For the engineer who is primarily concerned with obtaining estimates of the maximum stresses it may prove to be a useful tool.

Finally it is of interest to observe the accuracy of the authors' method when applied to this last example (Table 4(*b*)). When moment equilibrium about the side x–x is investigated, it is found that the various edge forces contribute to resisting the moment of the applied load in the following amounts: Bending moments along x–x, 26%; shear forces along y–y, 67%; and twisting moments along y–y, 7%. Since, theoretically, there should be no twisting moments along the clamped edges, 7% is a fair estimate of the over-all accuracy of the external forces.

WALTER W. EWELL,[24] J.M. ASCE, SHIGEO OKUBO,[25] A.M. ASCE, AND JOEL I. ABRAMS,[26] J.M. ASCE.—The writers are gratified by the interest shown in their paper and would like to thank the discussers for their worthwhile contributions.

Mr. Brennan emphasizes the importance of correlating experience and judgment with mathematical analyses of reinforced concrete slabs; with this point, the writers are in complete agreement. This is particularly true when the mathematical analysis is to be predicated on a series of ideal conditions rather than on those which actually exist. However, analysis must be considered a guide to engineering judgment and in many cases more than one type of analysis can and should be carried out. It is with this fact in mind that the grid method is presented, as an analysis to augment rather than to supplant other well-known methods.

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[26] Instr., Civ. Eng., The Johns Hopkins Univ., Baltimore, Md.

It is believed that the grid method of analysis can be readily applied to slabs characterized by nonisotropic materials and varying thickness. Important examples of this type of irregularity have been discussed by Mr. Pagon, who emphasizes the need for re-examination of the stress problem in an irregularly-shaped quadrilateral slab complicated by a central "edge beam," and in a flat roof dome reinforced by ribs. A grid analysis of a geometrically irregular slab is naturally more difficult because of the increase in the number of linear simultaneous equations required for a solution. Still, an approximate solution of a complicated problem is valuable. This can be effected by using a smaller number of grid beams and thereby reducing the number of equations to be solved simultaneously. Accuracy would be sacrificed for the sake of obtaining an answer that would at least indicate the structural behavior of the system.

The distribution procedures used in this paper, although not necessarily characterized by rapid convergence, can be extended to any degree of finality that is desired. This characteristic is common to most of the numerical procedures used in engineering practice. The analysis of a slab that is unsymmetrical in loading and geometry requires the solution of a large number of equations, as has been stated by both Mr. Ferguson and Mr. Brennan. However, if a problem warrants a solution at all, and if a solution cannot be obtained in another manner, this should not be considered a consequential deterrent. Mr. Pagon wisely suggests the use of electronic computing devices when such problems are encountered.

The accuracy desired in the solution determines the number of grid strips to be selected. In the limiting case of infinitely many grid strips, Mr. Platt[23] has established the fact that a grid system acts as a slab. The writers reluctantly admit that no attempt has yet been made to correlate the variation in accuracy with the variation in the number of grid strips, but future studies should provide this information. Of course, unorthodox problems must always be given individual consideration.

The writers are indebted to Mr. Scordelis for his discussion of the shear correction method as applied to the moment distribution process and his subsequent application of this method to a grid system. The procedure obviates the simultaneous solution of the shear equations. However, if the process is to be most effective, the analyst must judiciously choose a group of joint displacements that will accelerate the convergence. In a complicated slab problem, the analyst might find it difficult to select the best displacement values. Moreover, the shear correction method also becomes tedious when a large number of grid joints must be considered. In spite of these disadvantages, Mr. Scordelis' application represents an important refinement to the basic idea.

As indicated by Mr. Whitman, the effect of restraint that actually exists in any beam cut from a plate was not considered in the grid analogy. In order to make the plate and gridwork systems comparable, the restraint effect should be considered when choosing the stiffness factors. In this sense, the use of the term "analogy" is somewhat misleading.

Inasmuch as the inclusion of the restraint factors causes no new difficulty in the fundamental process, it should be done in order to make the grid conform more closely to the plate. This important fact was emphasized by Leliavsky Bey, Mr. Ferguson, and Mr. Whitman.

The choice of restraint factors, for all cases, is still not completely clear. Mr. Whitman suggests the use of $\beta = \frac{1}{3}$ as the torsional stiffness factor for all values of the aspect ratio of the grid beam. Whether this is a valid assumption is open to discussion and further study. Mr. Whitman explains his choice by citing the case of infinitely many grid beams with slab depth held constant where, he claims, the β value vanishes. The contrary is true, however. With infinitely many grid beams,[27] the aspect ratio (depth divided by width) approaches infinity and β approaches $\frac{1}{3}$. Since all grid strips cut from the plate are restrained by adjacent material, the torsional stiffness factor should be modified. When the cross-sectional dimensions are small as compared to the length of the member, the constraint has a negligible effect upon the torsional stiffness.[28] This is true when several of the cross sections are constrained, but its effect on the case under discussion is questionable.

Leliavsky Bey and Mr. Whitman indicate that the bending stiffness should be modified by the factor $1/1\text{-}\mu^2$. The writers are especially indebted to Leliavsky Bey for his demonstration of this fact.

Leliavsky Bey attributes a plate's capacity to carry more load than that calculated by existing theory to the force in the plane of the plate, and, furthermore, presents a very practical method of calculating this force. The Forchheimer graphs (Figs. 23 and 24) greatly simplify the lateral force determination. On a gridwork of beams it is conceivable that the magnitude of axial force within any beam can be calculated by some of the better-known methods of structural analysis. In fact, Mr. Benjamin has shown how a gridwork can be used to determine the forces developed in the plane of a plate subjected to edge loading. Perhaps Mr. Benjamin's grid study can be coupled with the writers' study in order to determine axial, bending, and torsional effects simultaneously.

Mr. Ferguson has questioned the advisability of ignoring a half grid strip adjacent to the plate boundary. Still, the writers are convinced that they have used the most rational plate division. Such a plate division is not tantamount to assuming the boundary one-half grid strip away from the actual plate boundary since the perpendicular grid beams have their terminal reactions on the plate's edge and are consequently free to deflect from the edge inward. Furthermore, the slab division into equal-length grid beams reduces the computational procedure that must follow—a simplification that appears to be justified when the number of grid beams is selected so as to minimize the error.

Mr. Ferguson has requested a statement of Mr. Enyedi's comparison of moment value with that obtained by Mr. Marcus. On a simply-supported plate subjected to uniform loading with $\mu = 0.25$, Mr. Enyedi found the moment at the center to be $0.0372w\,a^2$, whereas Mr. Marcus gave a moment value equal to $0.0368w\,a^2$. For the uniformly-loaded fixed-edge plate illustrated in Fig. 13, the maximum moment occurring at the midpoint of the fixed edge[29] was 15.90 w (in.-lb) whereas Mr. Timoshenko's value[30] for the same moment was 16.62 w ($\mu = 0.3$).

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[27] "Theory of Elasticity," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1934, pp. 243 and 248.

[28] "Strength of Materials," by S. Timoshenko, D. Van Nostrand Co., Inc., New York, N. Y., 1951, Part II, p. 282.

The writers appreciate the wealth of discussion that has been presented and recognize the fact that it has greatly enhanced the value of the original paper.

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[29] "A Grid Analogy for Analyses of Flat Plates," by Joel I. Abrams, thesis presented to The Johns Hopkins University, in Baltimore, Md., in 1950, in partial fulfilment of the requirements for the degree of Master of Science.

[30] "Theory of Plates and Shells," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1940, p. 228.

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PRINCIPLES OF HIGHWAY CAPACITY RESEARCH

By O. K. Normann[1]

With Discussion by Messrs. E. L. Fieldhammer and O. K. Normann

Synopsis

The problem of highway capacity design has relied mainly on empirical factors developed by the road builder. Very few factual data have been available and the literature contains few helpful references.

This paper summarizes the results of research in highway capacity. The types of investigations that were conducted and the conclusions derived are listed and discussed. These include investigations of overtaking and passing practices, motor vehicle performance, driver behavior, and traffic flow at intersections.

The factors of desired speed and practical capacity of roads are related to the effects of various limitations. Suggestions are made as to the practical application of the research data in design and construction.

Introduction

Few engineering problems are of more interest to so many people and have as large effect on their everyday lives as those problems relating to traffic conditions. Almost everyone is aware of the overloaded streets and highways that exist at many places during certain periods. Many are also aware of the fact that some highways will accommodate much higher traffic volumes before becoming congested than others, and that it is more comfortable to travel on certain highways than on others when the density of traffic is a factor. Few drivers, however, have any conception of the roadway width required to accommodate a given volume of traffic or the details of design that are necessary to obtain maximum safety for travel at reasonable speeds. These features are the responsibility of the engineer.

In 1937, however, when a review was made of all literature relating to the effect that various highway design features have on the volume of traffic

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Note.—Published in November, 1951, as *Proceedings-Separate No. 105.* Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Chf., Section of Traffic Operations, Bureau of Public Roads, U. S. Dept. of Commerce, Washington, D. C.

that can be accommodated by a highway and the speed and safety of movement, it was astonishing to find that there was little information available that had been based on facts gathered through research. Apparently, engineers engaged in the planning and construction of highways had directed their efforts in the research field almost entirely to improving the strength and life of the structure, and to more economical construction in order to utilize the funds available for highway improvement over the greatest mileage. Their knowledge of the required geometric features was based largely on their own experiences as drivers. There had been little concentrated effort to discover the technical facts relating to the dynamics of flow that are of prime importance if a highway is to serve traffic effectively. These factors involve the effect that various highway design features such as surface and shoulder width, alinement, and gradient have on the safety and economy of operation of individual vehicles, and how the movement of each vehicle is affected by, and in turn affects, the movement of other vehicles in the traffic stream.

HISTORY OF CAPACITY RESEARCH

The intensive research projects in traffic operations carried out since 1930 by many organizations have resulted in the development of several entirely new principles that provide the basis for highway design standards required to accommodate the needs of traffic. Substituting facts obtained through research for personal opinion has resulted in major changes in the approach to many traffic problems.

It is fortunate that one of the most intensive research projects carried out in the field of traffic operations deals with highway capacity.[2] Basically, this subject concerns the effectiveness of various facilities in the service of traffic and involves the effect on traffic flow of the many elements of highway design, vehicle and driver performance characteristics, and traffic control measures. It is also fortunate that conditions that tend to increase the capacity of a highway also result in improved traffic safety whether they apply to the highway, the vehicle, or the driver.

Considering the scope of the many individual yet interrelated studies that have been conducted to obtain fundamental data, and the comprehensive analyses to which these data have been subjected, it is apparent that the technical facts now available (1951) will permit new highways to be designed or old ones to be revamped with the assurance that highway operating conditions will be consistent with the justifiable needs of traffic. These studies have included the development of new electro-mechanical instruments that have made feasible the collection of data in larger volume and with greater accuracy than was previously possible, and the recording of some types of information that could not otherwise have been obtained. The following are but a few of the more important investigations in traffic operations conducted by the Bureau of Public Roads, generally in cooperation with various state highway departments and other private and public agencies:

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[2] "Highway Capacity Manual," Bureau of Public Roads, U. S. Dept. of Commerce, Washington, D. C.

1. Investigations were conducted of overtaking and passing practices on two-lane roads. During these studies, the speed and position of each vehicle in relation to other vehicles on the highway was recorded continuously over a 0.5–mile section. The data was obtained in nine states geographically distributed to include any major differences in driving habits and included actual practices of drivers while performing some 20,000 passing maneuvers. The results showed the physical measurements of time and space involved in performing passing maneuvers with safety, the frequency of maneuvers at various speeds in relation to traffic volumes, and the portion of any section of highway that must be designed to provide for passing. Conversely, the effect of restrictive sight distances in limiting the capacity of a highway was determined.

2. Investigations were made of motor vehicle performance. These included such studies as grade ability, braking ability, and acceleration and turning characteristics of new and used vehicles. The results of these investigations, in combination with other studies, provide information of particular significance in determining the effect that truck loading practices and truck movements have on highway capacity. They also provide means for determining the most economical combination of alinement, profile, and surface width that will effectively overcome the problem of slow moving vehicles on grades.

3. Driver behavior studies were made. During these studies the speed and transverse position of each vehicle and its spacing in relation to all other vehicles traveling in the same or opposing direction that might have influenced its speed or transverse position were recorded. At each location, data were obtained during periods when traffic increased from relatively light flows up to those at or approaching capacity conditions. The data that have been analyzed in detail include information for well over 1,000,000 vehicles at about 350 locations having various highway and roadside conditions. Additional data are also provided by the periodic speed studies conducted by the state highway departments at nearly 800 locations. To date, some 5,500 such studies have been made, including a total of nearly 2,000,000 vehicles.

The results of these studies have provided information regarding normal driving practices under various roadway and traffic conditions and have made possible the determination of the most appropriate combination of lane and shoulder width, the effect of curvature of varying degrees, the effect of curbs of various types, the effect of the presence of roadside objects such as narrow culverts or bridge rails that encroach on the normal shoulder width, and many other design details that have a considerable psychological effect on the driver.

4. Studies of traffic flow at intersections were conducted. From an analysis of data recorded at hundreds of the most heavily traveled intersections throughout the country, it has been possible to determine the effect of street width, parking regulations, turning movements, and many other factors on the possible and practical capacity of rural and urban intersections.

Application of Studies

These are only typical examples of the scope and type of studies that, in combination with evidence of traffic growth and other information obtained by the planning survey organizations, form the basis of available geometric standards for highways. The results also provide information for adapting the many miles of existing roads and streets that must continue in use for extended periods of time.

The results of these studies, as they apply to highway capacity, have recently been published under one cover as a manual.[2] Many of the results, however, have been available in preliminary form and have been applied since 1947 by the more progressive highway organizations in their technical direction of the long-range highway needs studies that have been completed in several states.

No attempt will be made to present the details of this report, but a few of the most fundamental criteria will be pointed out as pertinent when selecting design standards for new construction or when revamping existing highways to accommodate increased traffic.

Desired Speed.—In open country, all but a few drivers at one time or another desire to travel between 50 and 70 miles per hr on highways of adequate design. Few exceed 70 miles per hr and most of the time the large majority travel between 45 and 55 miles per hr. On expressways in urban areas, most drivers desire to travel between 35 and 50 miles per hr during off-peak periods when traffic volumes are low. For similar conditions in rough or mountainous terrain, and on local roads and streets, desired speeds are 5 to 10 miles per hr slower.

Under the less favorable conditions of traffic density that occur during peak periods of flow, most drivers will voluntarily, and without feeling unduly restricted, lower their speeds to 45 miles per hr on main highways in rural areas and to 30 miles per hr on expressways in urban areas. Freedom to travel at these speeds with safety is the quality of service that highway and street systems must provide to meet the demands of traffic. There is no apparent justification for an added expenditure of highway funds to provide for higher speeds in the foreseeable future. Similarly, when drivers are forced to travel at unreasonably low speeds, the highway is overloaded or the design is inadequate.

Practical Capacity.—The highest hourly traffic volumes that a highway will accommodate safely at reasonable speeds is termed the practical capacity. The following list gives practical capacities of different types of facilities having 12-ft traffic lanes, excellent alinement, adequate shoulders, and no intersections at grade with major cross movements:

Type of highway	Capacity—in passenger cars per hour
Two-lane rural	900 total
Three-lane rural	1,500 total
Multilane rural	1,000 for each lane in the direction of heavier travel
Multilane urban expressway	1,500 for each lane in the direction of heavier travel

When present, the average commercial vehicle with dual rear tires can, for purposes of estimating capacity, be considered as 2 passanger cars in level terrain and as 4 passenger cars in rolling terrain.

Although some highways are now being constructed with 12-ft traffic lanes, excellent alinement, wide shoulders, and grade separations for cross traffic, most of our existing highways do not have these features, and it will be uneconomical and unnecessary to provide all of these features on many highways as they are constructed or rebuilt.

Effect of Limiting Factors.—The effect of these less favorable conditions on practical capacities has been obtained from the research studies. Nine-foot traffic lanes on two-lane roads, for example, have only 70% of the capacity of 12-ft lanes. A road with narrow shoulders and frequent obstructions within 2 ft of the pavement edge has only 80% of the capacity of a similar road with wide, clear shoulders. If a vehicle break down should occur, the capacity of the road with inadequate shoulders might be reduced as much as 60%. A vehicle break down occurs, on an average, at least once every 10,000 vehicle-miles of travel. To eliminate the obstruction to free flow and the resulting hazard of vehicles parked on the surface of rural highways while making emergency repairs or for other purposes, a shoulder width of 10 ft is required.

The effect of the alinement on the capacity of two-lane roads can now be determined from entirely new criteria developed from driver behavior and motor vehicle performance investigations. These criteria involve the percentage of the length of the highway on which the driver can see 1,500 ft ahead. A highway on which a driver can see 1,500 ft ahead during only 50% of the time as he travels at a uniform speed, for example, has only about 80% of the capacity of a highway on which the driver can always see 1,500 ft ahead. This comparison is applicable even though the designs of both highways are adequate for safe travel at 60 miles per hr during light traffic.

INTERRELATION OF DESIGN ELEMENTS

Most existing roads have been designed by considering various elements such as curvature, gradient, and sight distance independently, and applying to each some standard maximum or minimum. It is now known that the operating efficiency of a road can be predetermined only by considering these various elements in combination with one another, not independently. Adherence in the past to some maximum gradient to permit reasonable truck speeds on grades, for example, has resulted in the introduction of so much curvature that sometimes passing-sight distance has been nearly eliminated. Consequently, these roads now become badly congested at exceedingly low traffic volumes. The alinement and profile, together with other details of design that are selected for an important rural two-lane highway, can result in practical capacities ranging from 200 to 900 vehicles per hr and a corresponding (if not larger) variation in construction costs. Standards for various sections of highway must necessarily differ, therefore, with the volume and type of traffic for which the highway is being constructed or reconstructed. Any other approach to the problem would either be uneconomical or result in unsatisfactory operating conditions.

One of the more important elements limiting the capacity of any highway facility, especially city streets, is the intersection at grade. In the past far too little was known to relate effectively the roadway conditions and traffic control measures with the type and number of vehicles that an intersection would accommodate. It has been known for many years that a wide street will handle more traffic than a narrow street, that the elimination of curb parking and left turns will increase capacity, and that more vehicles can be accommodated on a one-way street than on a two-way street when other conditions are the same. What has not been known, however, is the quantitative effect of each of these and many other variables that affect the flow of traffic at intersections. Some are still unknown quantities, but the effect of many of the more important variables has been determined from research carried on over a sufficient period and with adequate repetition to justify confidence in the results. By application of this knowledge, it is possible to compare the capacity of an entire street system, as it is being operated, with the present or estimated future traffic demand. If the demand exceeds the capacity under the prevailing conditions, traffic control methods should first be appraised to determine the possibility of improvement. Next, the extent to which relief can be provided by recognized measures such as the elimination of curb parking, or adoption of one-way street systems, should be determined. Finally, if these measures will not provide the desired relief from congested traffic conditions, the benefits gained in traffic capacity by widening existing streets or constructing some new facilities should be considered.

The principles relating to highway capacity have been developed through years of research by many individuals and organizations. Their laboratory has been the highways and streets of the nation; their subjects have been millions of drivers operating vehicles under normal conditions. It is now the responsibility of engineers to apply the principles developed so that the safety and economy of highway transportation will be improved.

DISCUSSION

E. L. FIELDHAMMER,[3] J. M. ASCE.—Extensive use of *The Highway Capacity Manual*,[2] to which Mr. Normann refers, during the design of an expressway facility, proved it to be an invaluable aid in determining the geometric requirements of the expressway itself, as well as the capacity of the city street system into which the expressway traffiic emerged.

However, in attempting to "***compare the capacity of an entire street system, as it is being operated, with the present or estimated future traffic demand" (see under the heading, "Interrelation of Design Elements"), the writer frequently encountered two interrelated problems for which no solution was readily available, either in *the Manual* or elsewhere, namely: (1) The determination of the capacity of an unsignalized intersection and (2) the lack of a signal control warrant based on intersection capacity. The first of these problems would seem to offer opportunities for further research. It will, therefore, be presented here in the hope that some large organization with adequate facilities will deem it a worthwhile project since the massive volume of data required before a conclusion can be reached on a matter such as highway capacity prevents an individual from undertaking the research himself. The second problem will be discussed also since the two are not readily separable.

These problems usually occurred in the following manner: An intersection would be encountered which did not satisfy any of the warrants for signal control given in *The Manual on Uniform Traffic Control Devices for Streets and Highways*.[4] However, if such an intersection were analyzed under the assumption that it was signalized, the result would indicate that the intersection was operating at, or near, its practical (and sometimes, its possible) capacity. That is, the total time required for all movements would approach 100%.

In an attempt to determine, rationally, the reasons for the foregoing result, it appeared that the signal-control warrants bore little relationship to the capacity of the intersection, and that the unsignalized capacity was larger than the signalized capacity of a given intersection. Although, in the absence of any data, there were at least three distinct possibilities for the relationship of the capacities of signalized and unsignalized intersections:

1. The unsignalized capacity of an intersection was larger than the signalized capacity (a very limited number of observations tend to favor this theory);
2. The signalized and unsignalized capacity of an intersection were the same, within the limits of accuracy of traffic calculations; and
3. The signalized capacity of an intersection was greater than the unsignalized capacity.

Since a measure of the capacity of unsignalized intersections was not available, the following method of analysis, utilizing possibility 2, was applied whenever unsignalized intersections were encountered:

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[3] D. B. Steinman, New York, N. Y.

[4] "Manual on Uniform Traffic Control Devices for Streets and Highways," by the Bureau of Public Roads, U. S. Dept. of Commerce, Washington, D. C., 1948, Sections 159, 207-213, 234, 253.

a. The capacity of the intersection approaches was determined on the basis of the capacity charts and procedures for signalized intersections, as given in *The Highway Capacity Manual*, and elsewhere[5];

b. The actual traffic on each approach was compared with the capacity to determine the time required for each movement;

c. If the time required totalled, say, 85% to 100%, signal control was warranted; and

d. If the time required by any one approach was less than 20%, but the total time was within the 85% to 100% range, traffic actuated equipment was considered.

If possibility 1 were true, and a measure of unsignalized intersection capacity were available and had been substituted in step *a*, the procedure outlined would have resulted in a reduction of intersection capacity below actual flow upon the installation of signal control. This hardly seems desirable.

If possibility 3 were true, on the other hand, and a measure of unsignalized capacity had been used in step *a*, the resultant warrant would be most satisfactory since it would establish signal control and thus increase capacity as the unsignalized intersection approached its maximum flow.

Were a measure of the capacity of unsignalized intersections available, comparison of it with the capacity of signalized intersections would decide whether possibility 1, 2, or 3 corresponded most closely to actuality. Once this was established it would be possible to analyze an unsignalized intersection and determine its influence upon the traffic flow of a city street system. It would also enable the traffic engineer to apply a realistic warrant for signal control, based on intersection capacity, as well as to have some foreknowledge of the result of its use. Thus it appears that research leading to a measure of the capacity of unsignalized intersections would be fruitful.

O. K. NORMANN[6].—The remarks of Mr. Fieldhammer enlarge upon the subject matter of the paper beyond its originally intended scope; nevertheless, they are very pertinent. Of the three assumptions listed by him, only one can be valid for any particular intersection, but that one might be any of the three, depending upon the conditions at the intersections. Thus there are three classes of intersections, the first of which has an unsignalized capacity greater than if signalized. An example of this class would be a multi-lane arterial way on which relatively high speeds prevail (where unsignalized), intersecting a minor cross street on which traffic is comparatively light. Such intersections occur most frequently in surburban areas. The free-flowing volume on the arterial way may be greatly in excess of the volume that could be accommodated if the movement were interrupted for even a very short interval of time by a traffic signal. If a traffic signal should be installed for any one or more of several reasons (such as failure of drivers on the arterial highway to yield the right of way to traffic on the minor street), the capacity of the major highway would be diminished. Should there be an insufficient increase in the volume on the cross street to offset the loss to the arterial way, the capacity of the intersection would be decreased as a result of the installation. Furthermore, the average delay per vehicle would be increased. Despite such economic losses, however, installations of this type are commonplace, and traffic departments throughout the United States are besieged with requests for still more.

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[5] *Public Roads: A Journal of Highway Research*, Vol. 26, No. 6, February, 1951.

[6] Chf., Section of Traffic Operations, Bureau of Public Roads, U. S. Dept. of Commerce, Washington, D. C.

In many instances, however, the capacity of the major facility is controlled by major intersections at other locations. In such a case, the installation of a traffic signal at intermediate locations at minor roads might actually improve the traffic flow on the major facility if the signals are properly sychronized.

The second assumption is believed to be correct for most intersections where streets are of moderate or narrow width and where vehicular speeds are relatively low, so that a driver wishing to enter the intersection from the cross street may feel reasonably sure that through traffic will yield the right of way and that the possibility of serious accident is slight. Another requirement would be that the traffic demand be less than the possible capacity of the intersection. If not, the intersection would most likely become completely blocked and traffic on all the approaches would come to a standstill.

Should the volume of traffic approaching any intersection exceed that which is able to clear the intersection—that is, if it exceeds the possible capacity of the intersection—the third assumption would apply. It is a fundamental principle of highway capacity that, when the number of vehicles attempting to use a facility exceeds the capacity of that facility under the prevailing conditions, traffic at some point will be at a standstill. The unregulated entry of vehicles into an intersection, may cause a demand exceeding the possible capacity of the intersection, with the result that the volume of traffic will reduce to zero and remain there until orderly movement has been restored. In most instances, a traffic signal will prevent the occurrence of complete congestion within the intersection and thereby improve its capacity when the volume is at or near its possible signalized capacity. Such was doubtless the objective in the initial development of traffic signals, which performed this mission so well that the signal was soon regarded as a panacea for all traffic ills. In retrospect, this may have been unfortunate in the sense that if signals had not become so commonplace it is conceivable that more attention might have been devoted to the development of proper speed-zoning regulations, more realistic right-of-way rules, and the invention of other regulatory measures or devices that would have aided the flow of traffic at less hazard and without so seriously impairing street capacities.

Accepting the theory that signals may either increase, decrease, or fail to alter the capacity of an intersection—depending upon the prevailing conditions—the question remains as to what application can be made of this knowledge. Except in so far as it may be of value in basing a signal warrant on intersection capacities, this question is perhaps of academic interest only. As a practical matter most intersections for which the traffic volume is sufficiently heavy to justify an investigation of their capacity are either signalized or will become signalized. As soon as the volume of traffic at an unsignalized intersection approaches its capacity as a signalized intersection, signalization is almost certain to take place. There is little likelihood that the signal will be removed from any intersection that is already signalized. In short, if an intersection is unsignalized at present it probably will become signalized before its capacity is reached, and, if it is already signalized, it will probably remain that way. Hence, the engineer, in computing capacities, should regard intersections as being signalized, whether or not the signal actually has been installed. Aside from capacities his primary concern will be in signal warrants so that he can estimate the date at which a signal will be required.

In the itemized steps *a* and *b* of his procedure, Mr. Fieldhammer has outlined a method for allocating the green time for an imaginary signal installed at an unsignalized intersection. Once this has been accomplished, computation of the capacity of the intersection as if it were signalized is a relatively simple operation. As to the volume for which signal control is warranted, there appears to have been far too little factual research, but the proposal of Mr. Fieldhammer in item *c* seems realistic and plausible. For some time, it has been the writer's contention that the traffic volume warrants for signal installations should be based on the relation between the traffic demand and the possible capacity of the intersection. The 85 percentile that Mr. Fieldhammer suggests may be desirable at intersections involving a major and a minor facility. With the traffic flow more evenly divided on the two facilities, however, it would seem that a signal would be warranted at the intersection of two two-lane roads when the demand is less—possibly approximating 60% of the intersection's capacity—and at a considerably lower percentage for the higher-capacity facilities with two or more traffic lanes for each direction of travel.

In most instances where actuated control is at all feasible, better allocation of signal time can be obtained by using this device than is possible with fixed-time control. Mr. Fieldhammer proposes that, if the time required by any one approach is less than 20%, traffic-actuated control should be considered. It is doubtful that he intends to restrict consideration of traffic actuation within this limit, however, but rather to insure against the use of short green intervals of the order of 10 sec or 15 sec. Such practice is commendable, as green intervals shorter than 20 sec are undoubtedly less efficient than the longer intervals, and it is questionable whether the curves of Fig. 24 of *The Highway Capacity Manual*[2] can be applied properly where the green interval is shorter than 20 sec. At fixed-time installations it is sometimes practicable to adhere to the 20-sec minimum while retaining a low percentage of the total time by lengthening the total cycle. This is particularly true at isolated intersections. Instances are known of 100-sec cycles that have had little or no objection voiced against them.

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TRANSACTIONS

Paper No. 2522

UTILIZATION OF GROUND WATER IN CALIFORNIA

By T. Russel Simpson,[1] M. ASCE

With Discussion by Messrs. Dean C. Muckel and T. Russel Simpson

Synopsis

Great quantities of ground water are used for irrigation in the State of California. The paper traces the development of this ground-water utilization and gives detailed descriptions of the use of ground water in the major hydrographic areas of the state. Ultimate development is also treated in terms of unused ground-water storage areas and more efficient use of areas already developed.

Ground-Water Utilization

Utilization of ground water for irrigation was unimportant in California prior to 1880, except in Los Angeles and San Bernardino counties, where nearly 1,000 flowing artesian wells had been developed prior to that time. In 1880 the total area under irrigation in various sections of the state from both surface and ground-water supplies was estimated to be as follows:

Location	Irrigated acreage
Southern California	82,500
San Joaquin Valley	188,000
Sacramento Valley	13,400
Sierra foothill valleys	9,000

The total irrigated area in the state in 1950 was almost 6,000,000 acres and more than 90% of the total water utilization in the state at that date was for irrigation purposes.

The total gross pumpage of ground water in California in 1950 was estimated to be about 10,000,000 acre-ft or about 40% of the total in the United States. The total diversion of surface waters, including the Colorado River, for irrigation, domestic, and industrial uses in California approximated ground-water extractions. A discussion of ground water utilization in various areas of the State of California is most conveniently broken down into the seven major hydrographic areas shown in Fig. 1.

Note.—Published in November, 1951, as *Proceedings-Separate No. 102*. Positions and titles given are those in effect when the paper or the discussion was received for publication.

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1. *South Coastal Area.*—The south coastal area embraces the drainage basins tributary to the Pacific Ocean from Ventura County to San Diego County, inclusive. The average annual combined runoff in these drainage basins for the 53-yr period, 1894–1895 to 1946–1947, was slightly more than 1,000,000 acre-ft, or about 1.7% of the total runoff for the state. It is estimated that additional importations, over and above existing rights in the Colorado River, Owens River Basin, and Mono Basin will be required ultimately for development in the south coastal area.

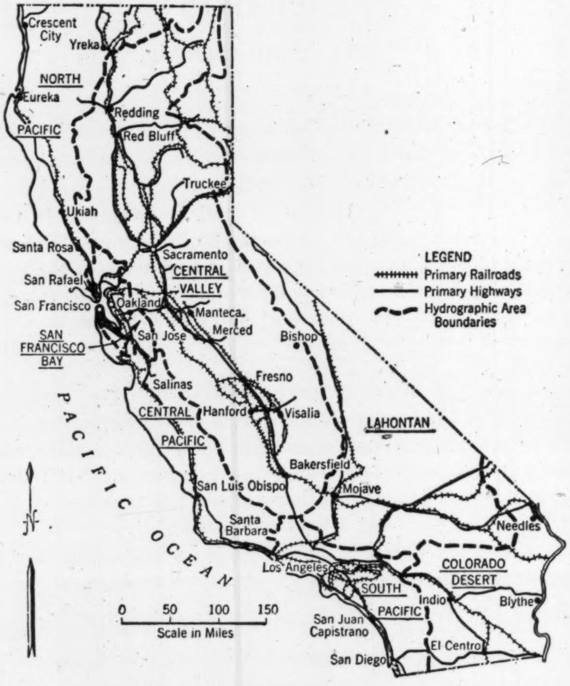
Crescent City
Yreka
NORTH
Eureka
PACIFIC
Redding
Red Bluff
Truckee
Ukiah
Santa Rosa
Sacramento
CENTRAL
VALLEY
San Rafael
San Francisco
Oakland
Manteca
Merced
Bishop
SAN FRANCISCO BAY
San Jose
Fresno
Salinas
CENTRAL
PACIFIC
Hanford
Visalia
LAHONTAN
PACIFIC OCEAN
Bakersfield
San Luis Obispo
Mojave
Santa Barbara
Needles
Los Angeles
COLORADO
DESERT
SOUTH
PACIFIC
Indio
Blythe
San Juan Capistrano
El Centro
San Diego
0 50 100 150
Scale in Miles
N
LEGEND
Primary Railroads
Primary Highways
Hydrographic Area Boundaries

FIG. 1.—MAJOR HYDROGRAPHIC AREAS OF CALIFORNIA

Annual ground-water uses in the south coastal area amounted to about 800,000 acre-ft in 1950 and are nearly double those from surface supplies, including importations from the Colorado River, Owens Valley, and Mono Basin. Los Angeles County has for many years been the leading county in the United States in income from agricultural production. This production is supported largely by irrigation from ground water. Some 120,000 acres in Ventura County are devoted to high value crops and urban development. Water service for this area is largely dependent on ground water. There is some ground-water overdraft and evidence of sea water intrusion in wells near the ocean in which water levels fall below sea level in a series of dry years.

The habitable area in this county is about 70% developed. Additional water supplies will be required to eliminate overdraft and provide water to meet probable ultimate requirements in Ventura County.

The West Coast Basin lies between the Inglewood fault and the ocean, and extends from the Santa Monica Mountains to Long Beach in western Los Angeles County. Marine intrusion resulting from long-standing overdraft threatens to destroy the usability of this ground-water basin.

About 600,000 acre-ft of water are being pumped annually from the ground water of the Los Angeles, San Gabriel, and Santa Ana river systems. In 1950 there was an annual overdraft of about 45,000 acre-ft on ground water in the San Gabriel and Santa Ana river basin systems. Roughly 2,000,000 acre-ft of water is stored in a 50-ft zone below the water levels that existed at that time in the upstream basins, and, while this is not a part of the permanent supply, it is available for emergency use pending a permanent solution of the problem. Maintenance of salt balance requires that some water be wasted, but the balance is a function of the water passing through the basin rather than the amount in storage. Industrial wastes and oil well brines have contaminated portions of ground-water basins in Los Angeles and Orange counties.

About 100,000 acres in the coastal drainage of San Diego County are devoted to high-value irrigated crops and urban developments. There is a large additional area suitable for irrigation and domestic occupancy in the county, and the ground-water basins comprise only a small part of the area. They are narrow and shallow in the upper reaches, and near the ocean the usable storage capacity is limited by the threat of sea water intrusion. Their effectiveness as regulators of the widely varying runoff is small. Roughly one third of the present water supply is derived from ground water in San Diego County. The San Diego County Water Authority imported about 70,000 acre-ft of water from the Colorado River in 1948, of which the City of San Diego received about 50,000 acre-ft.

2. *Colorado Desert.*—The principal ground-water development in California within the Colorado River Basin is in Coachella Valley, northwest of Salton Sea. The gross pumpage of ground water in Coachella Valley has been approximately 100,000 acre-ft annually. Service by a supplemental surface water from the Colorado River was extended to this area in 1949. The valley fill in Imperial Valley is chiefly silt and compacted sands of low permeability, and there is essentially no irrigation from wells. The ground-water problems south of the Salton Sea are those of drainage due to high water table.

3. *Lahontan Basin.*—Antelope Valley is situated in the southwest portion of the Lahontan Basin. It embraces about 500,000 acres of valley land, half of which is irrigable land of good quality. With about 50,000 acres under irrigation from ground water in 1947, the annual overdraft of water was estimated to be 50,000 acre-ft. This will doubtless increase materially, but there is no local supply available for development to meet present deficiencies and provide water for anticipated expansion. However, the empty underground storage capacity is large.

Mojave Valley ground-water basin is situated east of Antelope Valley. Mojave River inflow to the valley averages about 80,000 acre-ft annually, about 80% of which percolates and is retained in the ground-water basin. It is estimated that about 40,000 acre-ft per year is wastefully consumed by native vegetation on the low lands along the river. There is ample irrigable land within easy reach to utilize the available supply, but only a small part of it is irrigated.

The City of Los Angeles, during the three dry years, 1929 to 1931, inclusive, pumped about 340,000 acre-ft from ground-water basins in Owens Valley for export through the Owens Valley Aqueduct. There have been no extractions for export since 1931, and the ground-water basins are now fully recharged. The glacial moraine and lava sand deposits south of Mono Lake in Mono Basin appear to contain substantial undeveloped ground waters.

Ground-water draft is light in the remainder of the Lahontan Basin, although there are a number of domestic and stock-watering wells in shallow valley fills between Mono Lake and Honey Lake. There are also about a dozen irrigation wells in the Susan River delta and a few flowing wells on the west side of Suprise Valley that supply a limited amount of irrigation water.

4. *Central Coastal Area.*—The central coastal area embraces the drainage basins tributary to the ocean from San Francisco County to Santa Barbara County, inclusive. The combined mean annual runoff from these drainage areas approximates 2,000,000 acre-ft. A large part of the domestic, industrial, and irrigation development is dependent on underlying ground waters for its water supply. The draft on several of the major ground-water basins in the area has been so heavy that serious problems have arisen in respect to their adequacy, not only for future developments but also for maintaining the existing economy. Water levels at wells close to the coast have fallen to elevations below sea level in Pajaro, Salinas, and Arroyo Grande valleys and on the south coastal plain of Santa Barbara County. Progressive lowering of water levels is occurring in Santa Ynez, Santa Maria, San Benito, and south Santa Clara valleys.

The annual gross pumpage of ground water in the central coastal area approximated 600,000 acre-ft in 1950, about 60% of which is in Salinas Valley. The annual overdraft in Salinas Valley at that time was approximately 30,000 acre-ft, and the ultimate may approach 76,000 acre-ft. Small overdrafts also exist in Pajaro and south Santa Clara valleys. Safe yield and overdraft estimates have not been made in the other ground-water basins in the central coastal area. Unused underground storage and surface reservoir sites are generally available for salvage of such surpluses on the various stream systems as may be needed under ultimate development to supplement existing water supplies.

5. *San Francisco Bay Area.*—The San Francisco Bay area embraces portions of nine counties that drain into San Francisco, San Pablo, and Suisun bays. The total average annual unimpaired runoff from the drainage basins in the area in 1950 approximated 1,240,000 acre-ft. Twenty-two surface reservoirs with a combined capacity of about 400,000 acre-ft have been constructed in the area. Five of these in Santa Clara County, with a combined capacity of 43,100 acre-ft, are operated in conjunction with spreading works to

recharge ground waters. Additional reservoirs will provide an aggregate capacity of 106,000 acre-ft, all in Santa Clara County.

Ground waters are utilized to the extent of about 300,000 acre-ft annually in the San Francisco Bay area. Ground-water uses in the area surrounding the south bay in San Mateo, Santa Clara, and Alameda counties exceed the safe yield. The annual deficiency in Santa Clara Valley in 1950 approximated 38,000 acre-ft and ultimately may approach 100,000 acre-ft.

The ground-water resources underlying about 60,000 acres along the eastern bay shore in southern Alameda County are threatened with destruction through depletion of underground storage and deterioration in quality. Proximity to the bay, water levels at wells prevailing materially below sea level, and decreasing amounts of chlorides easterly from the bay strongly suggest sea water intrusion into the ground-water resources of the Niles Cone area. The Alameda County Water District provides some artificial recharge by ponding for absorption.

Delivery of water through the Contra Costa Canal, a unit of the Central Valley Project, will provide adequate supplemental water to the Contra Costa Water District to meet probable ultimate water requirements in that area. The canal could, with storage constructed en route, be extended to furnish additional water supplies to the Richmon-Pinole area.

Small overdrafts existed in 1950 in the limited ground-water basins in the vicinity of Suisun and Fairfield in Solano County. The ground-water problems in Napa County have been relieved by construction of the Conn Valley and Rector Creek reservoirs with an aggregate capacity of 34,400 acre-ft.

Water is imported to the San Francisco Bay area from three sources: (1) Hetch Hetchy system on the Tuolumne River by the City of San Francisco; (2) Pardee Reservoir on the Mokelumne River by the East Bay Municipal Utility District; and (3) The Sacramento-San Joaquin delta, through the Contra Costa Canal, by the Bureau of Reclamation (USBR), United States Department of the Interior. These three systems, completely developed, could provide a total importation of about 900,000 acre-ft annually.

6. *North Coastal Area.*—The north coastal area embraces about 16,000 sq miles and this area yields about three eighths of the total runoff of the state. The average runoff for the 53-yr period, 1894–1895 to 1946–1947, approximated 29,000,000 acre-ft. The Mattole River, tributary to the ocean about 37 miles south of Eureka, has a water supply about equal to the combined runoff from all drainage basins in the previously mentioned south coastal area.

The combined ground-water uses in the north coastal area were on the order of 50,000 acre-ft annually in 1950. Use of ground water for irrigation was restricted to comparatively small areas in Santa Rosa Valley, Eel River delta, and Butte Valley in Siskiyou County. A limited amount of ground water is also used for domestic and municipal purposes.

7. *Central Valley.*—Utilization of ground water for irrigation in the Central Valley did not become significant until after 1900. More or less complete direct diversion of surface water supplies during the summer season in south San Joaquin Valley prior to 1910 gave impetus to development of ground water.

The combined capacity of wells in the San Joaquin Valley south of Chowchilla was about 7,300 cu ft per sec by 1919 and about 20,600 cu ft per sec by 1929. Overdrafts on ground water occurred in much of the area prior to 1929 and have prevailed since that time. Estimated usable underground storage capacity of approximately 20,000,000 acre-ft has made the long-continued overdraft possible. Depletion of underground storage generally has been severe on the west side of south San Joaquin Valley, in the Edison-Arvin area, and on the east side between Dinuba and McFarland. Water levels are generally high in the Kings River and Kern River service areas. Average water levels between 1939 and 1947 underlying 250,000 acres in the Kings River service area were less than 10 ft below ground surface, causing losses by evapo-transpiration to occur directly from the water table.

The combined gross pumpage of ground water from about 35,000 wells in the San Joaquin Valley (south of Merced River) during the seasonal year ending April 1, 1948, is estimated as being close to 6,000,000 acre-ft, or about 60% of the total in the state. The combined pumpage of ground water in that year in the lower San Joaquin Valley approximated 1,000,000 acre-ft. General extension of electric energy to the ground-water basins and development of deep-well turbine pumps were important factors enabling ground-water utilization since 1925.

The annual pumpage of ground water in the Sacramento Valley, that embraces a gross area of about 6,000 sq miles or one-third of the Central Valley was about 1,000,000 acre-ft annually in 1950. Heavy pumping in the Peach Bowl area in Sutter County and in portions of the counties of Yuba and Placer has induced problems of localized overdrafts. Problems caused by the quality of water have been noted in the Coche Creek and Sutter Basin areas. During the 9-yr period from 1940 to 1948 the water table was perennially less than 10 ft below ground surface over an area of about 780,000 acres in the Sacramento Valley. The Sacremento River, and its tributaries throughout most of the ground-water basin, drain surplus ground waters into the Sacramento-San Joaquin delta. The irrigable land in the Sacramento Valley is only about one-third developed so use of ground water there can be expected to increase as long as its development can compete favorably with the cost of supplemental surface storage.

Expected Ground-Water Developments

Possible ultimate ground-water developments in California include utilization of (*a*) unused ground-water storage to the extent of safe yield in ground-water basins not fully developed, and (*b*) increments in safe yield of ground-water basins as additional surface water supplies are made available. A major portion of the water utilization, both surface and ground waters, is within the ground-water basins in the state. The maximum amount of firm water (water that is available on demand) under ultimate development of the state's water resources could be obtained through operation of surface reservoirs, in so far as possible, on an average yield basis in conjunction with cylic underground storage. An adequate supply of energy would be necessary in a series of dry years to pump ground water.

Unused Ground-Water Storage.—The largest body of unused ground-water storage in the state is in the Sacramento Valley. Results of appraisal of geologic features of Sacramento Valley ground-water basins show over 28,000,000 acre-ft of underground storage capacity within the 20-ft to 200-ft zone below ground surface underlying 2,066,000 acres in this area. This area will generally support heavy draft pumping. An additional area of about 630,000 acres of basin deposits has an estimated ground-water storage capacity of more than 5,000,000 acre-ft within the same depth zone. The basin deposits are less permeable, and low well yields are generally obtained therein, but determination has not been made of the usable portion of the storage capacity. The Sacramento Valley ground-water basins were fully charged in 1950, except for limited areas in Sutter, Yuba, Placer, and Solano counties. The total pumpage in the Sacramento Valley in that year was about 1,000,000 acre-ft, and the net draft on ground water for irrigated crops was probably less than half this amount under the prevailing irrigation practices at that time. The safe yield of this extensive ground-water reservoir has not been determined. If average ground-water levels were drawn down about 10 ft below those prevailing in 1950, there would be a great salvage of uneconomic consumption on some 780,000 acres of high water table land and drainage from the ground-water basin through the stream system.

The unused underground storage capacity in the Camphora-Greenfield area in Salinas Valley, within the 125-ft zone below ground surface, was estimated to be 575,000 acre-ft in 1946.[2] It was further estimated that utilization of about 130,000 acre-ft of unused storage within the 60-ft zone below ground surface in that area would provide an adequate solution of the ground-water problem in Salinas Valley. Studies have been undertaken of methods of using such unused ground-water storage in conjunction with surface storage to provide a complete solution of flood control and water conservation problems in Salinas Valley under probable ultimate development.

As previously mentioned there was fully recharged underground storage in 1950 in Owens Valley now being held as stand-by for the City of Los Angeles. The city also has substantial stand-by underground storage in San Fernando Valley. Mention is made again of the uneconomic consumption of ground water in Mojave Valley. This water could be salvaged for beneficial uses through full utilization of the unused ground-water storage in that basin.

In addition, similar uneconomic consumption is evident in the ground water underlying about 250,000 acres of high water table land in the Kings River service area. Greater utilization of ground-water supplies would result in salvaging a substantial portion of such natural disposal.

Intensive ground-water investigations in California prior to 1948 have largely been confined to the areas where overdraft conditions existed.

Increments in Safe Yield.—An essential feature of the Central Valley Project in the San Joaquin Valley is the utilization of ground-water reservoirs for the storage and subsequent extraction of water supplies by pumping. Underground reservoir utilization is particularly important in the southern

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[2] "Salinas Basin Investigation," by T. Russel Simpson, *Bulletin No. 52*, Div. of Water Resources, Dept. of Public Works, State of California, Sacramento, Calif., 1946.

San Joaquin Valley where experience has already demonstrated its practicability and value, and where wells and pumping plants with an aggregate capacity of over 25,000 cu ft per sec are in operation.

The usable underground storage capacity in the southern San Joaquin Valley south of Chowchilla totals over 20,000,000 acre-ft, and in the northern San Joaquin Valley about 3,000,000 acre-ft. The Central Valley Project provides full utilization of the underground capacity in the southern San Joaquin Valley with operation thereof coordinated with surface storage regulation. The chief cost involved in the utilization of the underground storage would be for the operation of pumping installations, on which it is estimated a capital outlay of approximately $200,000,000 had been made up to 1950. Augmentation of surface water supplies on the east side of the southern San Joaquin Valley by an average of about 1,500,000 acre-ft annually will increase the safe yield of ground-water basins by the amount of deep percolation from unconsumed irrigation water and from spreading of water for direct replenishment. Net draft of ground water should keep pace with increment in safe ground-water yield to prevent occurrence of drainage problems.

Studies have been made to determine the feasibility of introducing surplus water into empty underground space in wet years. If this procedure is feasible, it is believed the amount of loss in subsurface outflow would be small in a limited period of wet years compared with retention for rediversion. If depth of storage is held to more than 12 ft from ground surface, then uneconomic losses through evapo-transpiration direct from the water table would be insignificant.

As additional surface supplies of water are made available in free (unconfined) ground-water zones (as planned for various ground-water basins in the state), the safe yield of the basins will be increased. This will necessitate increased ground-water utilization, in so far as enhancement of supplies exceeds overdrafts, in order to maintain hydrologic balance between supplies and disposal.

DISCUSSION

Dean C. Muckel[3].—The reader of this paper will be impressed by the fact that information and data exist for a complete analysis of the ground-water situation in the State of California. Credit is due to the agencies and individuals responsible for collecting basic data and making pertinent investigations over periods of many years. The author has made a brief, concise accounting of the ground-water supply, and suggests the ultimate development possible.

Each of the seven major hydrographic areas used by the author contain numerous surface drainage areas, ground-water basins, and subbasins. Some of these basins are interconnected in such a manner that extractions or replenishments in one will eventually affect adjacent basins within the same hydrographic area. In many cases, however, there is no connection between basins, and the hydrographic areas as delineated will function not as a unit but as a group of individual units each with its own characteristics.

Several ground-water reservoirs are discussed with relation to the expected ground-water development. Since most of the development has come about by the extraction of water through wells located without any basin-wide consideration or control, future development will probably follow the same lines. Under this system of development, a ground-water basin by concentrations of wells, by unequal replenishment throughout the basin, or by a combination of both, may have conditions of overdraft in one part of the basin and have surpluses and wastes at the same time in other parts of the basin. To obtain the ultimate development, then, each ground-water basin must be given detailed study, and the proper location of the extractions planned as well as the total quantities of water to be removed by pumping.

In the southern San Joaquin Valley the full utilization of the underground storage depends partly on the success or failure of spreading imported water for artificial replenishment. Although spreading has been carried on successfully in certain areas (notably the South Coastal Basin of California), conditions in the southern San Joaquin Valley do not generally lend themselves to spreading. The prevailing soils are fine-textured with low rates of percolation. In some places hardpans, clay pans, and other impervious or nearly impervious strata retard the deep percolation of water applied to the surface. There are now (1952) plans in the San Joaquin Valley to spread water on soils classified as loams and fine sandy loams underlain in places by lenses of hardpan. The natural rates of percolation are low—less than 1 acre-ft per acre per day and decreasing with the time of submergence. The writer has been participating in attempts to improve the natural rates of percolation, which have been successful on small test ponds. (Joint investigation with the Soil Conservation Service (Research) U. S. Department of Agriculture; U. S. Bureau of Reclamation; North Kern Water Storage District; California State Division of Water Resources; and California Agricultural Experiment Station, participating.) Although no opportunity has presented itself as yet to carry the trials to large-scale spreading, it is obvious that successful spreading on these soils will require careful planning and study and the expenditure of large sums of money for land, construction of spreading systems, and possibly for soil treatments and management.

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Although the author has treated the southern San Joaquin Valley as a single underground reservoir, it functions more or less as a series of subbasins because of heavy pumping concentrations and unequal natural replenishment throughout the area. Artificial replenishment by spreading, therefore, requires that spreading systems be located in the intake areas of that part of the ground-water basin or reservoir needing replenishment and having underground storage space. The location of spreading systems requires careful investigation, as misplaced systems might feasibly cause more damage than good by creating perched water table resulting in unfavorable drainage conditions.

The author obviously did not intend to enter into a detailed analysis of the various ground-water basins and their individual complexities. The foregoing comments are offered to emphasize the fact that ultimate development of any underground reservoir requires considerably more study and analysis than is involved in an inflow-outflow equation on a basin-wide basis.

T. RUSSEL SIMPSON,[4] M. ASCE.—Several separate ground-water units that exist in each of the seven major hydrographic areas in California are mentioned by Mr. Muckel. Comprehensive ground-water investigations have been made under the writer's direction in four of the hydrographic areas covering about 2,000,000 acres in twenty-seven units during the 6-yr period, 1945–1950. Ground-water overdrafts either have occurred in these units or threaten to occur in the near future. Mr. Muckel has been doing excellent laboratory and field research with the objective of stimulating additional ground-water recharge through artificial percolation by spreading. Results of the research will be helpful in solving ground-water problems.

It was not intended to enter into a detailed analysis of the various ground-water units and their individual complexities because such a discussion would be quite extensive. It may be of interest to summarize the types of basic data collected in the aforementioned comprehensive ground-water investigations in order to provide information for making a detailed analysis:

(*a*) Establish supplemental precipitation stations to obtain precipitation records well-distributed over the ground-water basin for evaluation of weighted average annual precipitation from isohyets or by the Thiessen polygon method.

(*b*) Install supplemental stream-gaging stations to obtain complete data on surface inflow to, and outflow from, the ground-water basin.

(*c*) Collect all available well-driller logs, and drill such test wells as are needed where logs are not available, to give good coverage on valley fill.

(*d*) Make a general geologic study of valley fill from well-log groups to obtain probable specific yields at various depth ranges from ground surface down to desirable maximum pumping lifts; make a peg model and delimit probable zones of confined or partly confined aquifers from the geologic study and from ground-water behavior during recharge and discharge.

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[4] Cons. Engr.; Prof. of Irrig. Eng., Univ of California, Berkeley, Calif.

(*e*) Collect all antecedent information on ground-water levels, pumping records, water analyses, and cultural surveys; obtain annual records of agricultural power sales back through the base-period, and all pump test records during the past 3 years; obtain such other information as may be useful in estimating the historic use of water and the crop pattern.

(*f*) Select test tracts of all crops generally grown in the area on various soil textures using usual methods of irrigation, and obtain detailed records of water use under average practices throughout the period of investigation. Have pump tests made and determine the irrigation efficiency of all test tracts; also collect pump test records made during the investigation elsewhere in the ground-water basin.

(*g*) Develop information from which consumptive use of all types of water-consuming areas can be estimated by the available heat method or the irrigation efficiency method.

(*h*) Select a grid of measuring wells, spaced about ¾ mile apart over the ground-water basin, from existing wells supplemented with jettied test wells where required. Water levels should be measured at all wells under as nearly static conditions as possible after the close of the season of heavy draft, and again after the season of recharge. Use about 10% of the measuring wells as control wells for monthly water level measurements (oftener if indicated) during the investigation.

(*i*) Make an annual comprehensive cultural survey during the period of investigation, from aerial photos transferred to the United States Geological Survey base. Classify culture into equal water-consuming groups and determine acreage by map cutting-and-weighing method.

(*j*) Collect water samples from all operating wells and make field analyses for total mineral solubles and chlorides. Make complete analyses on about 10% of the samples. If unusal concentrations of mineral solubles are found from field tests, make such complete analyses in the zone of pollution as to identify extent, type, and source of pollution.

(*k*) Make a land classification and soil survey, if not included in antecedent information.

(*l*) Use one of the direct methods developed to determine subsurface inflow, subsurface outflow, and safe ground-water yield, where applicable.[5,6,2,7,8]

(*m*) Locate areas of overdraft and near-by sources of surplus water. Develop feasible plans for development of supplemental water to meet present and probable ultimate needs—in successive steps or phases if possible—and determine the cost of such supplemental water.

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[5] "Utilization of Ground-Water Storage in Stream System Development," by Harold Conkling, *Transactions*, ASCE, Vol. 111, 1946, p. 275.

[6] Discussion by Raymond A. Hill of "Utilization of Ground-Water Storage in Stream System Development," by Harold Conkling, *ibid.*, p. 306.

[7] "Ground Water," by C. F. Tolman, McGraw-Hill Book Co., Inc., New York, N. Y., 1937, pp. 490-491.

[8] "Water Resources of Southern San Joaquin Valley," by S. T. Harding, *Bulletin, No. 11*, California Div. of Eng. and Irrig. and Water Rights, Documents Section Printing Div., State of California, Sacramento, Calif., 1937.

The supply of water to a ground-water unit must balance its disposal during any specified time. Separate evaluation should be made of each item included in total water supply and its disposal during the base-period selected for analysis. After inserting all items in supply and disposal, if the totals are out of balance within the limit of accuracy of the total basic data involved, individual items must be adjusted in accordance with relative probable error to effect closure. Further investigation is indicated if the discrepancy exceeds the limit of accuracy of total basic data.

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Paper No. 2523

PILE FOUNDATIONS FOR LARGE TOWERS ON PERMAFROST

By L. A. Nees,[1] A. M. ASCE

With Discussion by Messrs. Harry Carlson, and L. A. Nees

Synopsis

The strategic importance of the Arctic region and the lack of knowledge of engineering conditions in this area make this paper of general interest to the profession. Confronted with the design of tower bases on permafrost soil, it was found necessary to conduct a series of tests to derive values for the factors of design. These tests are described and compared with previous research.

The design finally selected for the tower base involved a system of steel beams and piles that spread the load from the tower leg through annular rings and spider beams to a series of 8-in. piles. The design of this system is described, and steps are given for the design of the heat diverting pad necessary to prevent melting of the permafrost and resultant loss of pile friction. The question of pile imbedment is also discussed briefly.

Introduction

The vast reservoir of natural resources in the Arctic and the militarily strategic importance of these regions make it inevitable that they will attract the attention of the engineer with ever-increasing interest. It is safe to say that the background of engineering knowledge on Arctic conditions, especially in the field of permafrost, is not sufficiently advanced to provide well-established theories and criteria for design—especially on important structures. Great strides are being made in the preparation of design standards by a number of agencies in the United States, Canada, and Great Britain. Until these are available, the designer must depend largely upon empirical methods and isolated experiences. It is hoped that this paper, by outlining a rational method of design for an important structure and by describing a few model pile tests, will help to plug the gap in engineering knowledge.

Note.—Published in November, 1951, as *Proceedings-Separate No. 103*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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Experiences with large tower construction in the Arctic in 1947 made it seem desirable to reduce the use of concrete in the foundations to a minimum. The idea of using piling seemed excellent, provided a reasonable method of design could be assured and a reliable estimate could be made of the adhesion of the piles to the permafrost under uplift stresses. The question of heat transmission was recognized as being a very important variable in the general problem.

TEST PROGRAM

Research and Development.—Work was started on the preparation and analysis of a typical design, and at the same time a study was made of available data on the adhesive force of freezing. This study indicated the need for additional information. Therefore, a series of small-scale tests was run to determine the envelope of the values of bond stress developed between steel piles and permafrost.

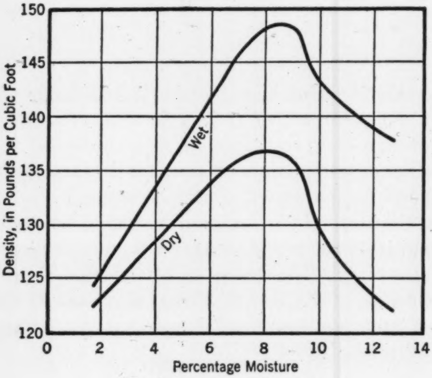

FIG. 1.—PROCTOR ANALYSIS OF TEST SOIL

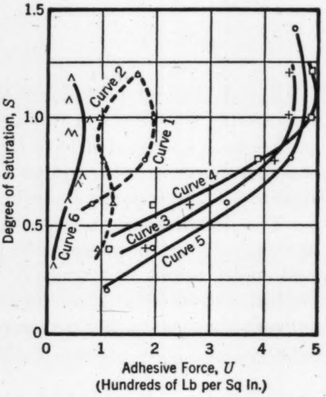

FIG. 2.—ADHESIVE FORCE OF VARIOUS SOILS

The tests were conducted with $1\frac{1}{4}$-in. standard steel pipe, ranging in length from 6 in. to 24 in., imbedded in a local sand-silt having characteristics as shown in Fig. 1. The sieve analysis by weight was as follows:

Sieve number	Percentage finer
1 in.	98.3
$\frac{3}{4}$ in.	95.6
$\frac{3}{8}$ in.	89.3
No. 4	85.4
No. 10	80.0
No. 40	65.5
No. 100	56.1
No. 200	53.1

The test apparatus consisted of a 1-cu-yd wood box containing the test pile and soil. A hydraulic jack with dial gauge was used to apply uplift loads and was mounted on a cross girder that transferred the jack reactions into the box frame. Temperatures of the soil and of points along the test pile were measured by recording thermocouples. A total of 15 tests were run, covering various combinations of density, moisture content, and pile length.

Test Results.—Fig. 2 gives the adhesive force of various soils to wet wood for varying degrees of saturation. It compares the results of experiments conducted in Russia[2] (curves 1 to 5) with experimental results of the tests described in this paper (curve 6). The characteristics of the soils tested are tabulated as follows:

Curve number (Fig. 1)	Type of soil	Temperature of test (degrees Fahrenheit)
1	Clayey	29–30
2	Silty	29–30
3	Clayey	11–15
4	Silty	11–15
5	Sandy Loam	11–15
6	Sand–Silt	18–25

Comparison of the Russian data with the test results indicates that, although the general trends of behavior appear similar, the test results appear to yield much lower values than would be expected as a result of the differences between wood, concrete, and steel. Unfortunately, insufficient data were available on the Russian tests to permit comparison of soil types, optimum moistures and densities, and other important test factors.

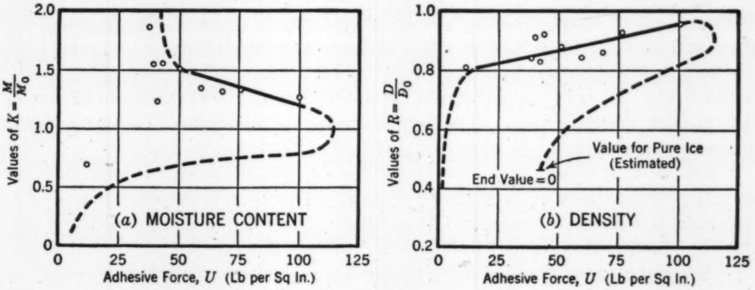

FIG. 3.—EFFECT OF VARIOUS FACTORS ON THE ADHESIVE FORCE

It is interesting to note that for all soils there appears to be a reversal of curvature at a saturation amount of approximately 1.0. It can be expected that all curves will become asymptotic to the value of pure ice as the saturation increases. On the dry side it can be anticipated that all soils will approach a minimum value at zero moisture content that will result from the pure mechanical grain friction of the soil in its loosest state. This point is probably very close to zero for all soil types.

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[2] "The Force of Congelation of Frozen Grounds with Timber and Concrete," by I. S. Vologdina, *SOPS and KIVM*, Academy of Sciences, Vol. II, 1936.

The variation of the force of adhesion with soil density and moisture content is given in Fig. 3. The curve of values of u versus the ratio of observed moisture content M, to optimum moisture content M_0, is shown in Fig. 3(*a*). The variation of values of u with the ratio of observed density, D, to density at optimum moisture content is plotted in Fig. 3(*b*).

An interesting feature of the tests was the high refreezing strengths developed. After the test had been carried to failure, the load was removed for a period of 1 to 2 min. The pile was then rapidly loaded. It was found that values of up to $\frac{1}{2}$ to $\frac{2}{3}$ u were developed before failure again took place.

Working Value of Adhesive Force.—After considerable study of all available data it was decided that a value of $u = 20$ lb per sq in. could be used as a safe equivalent of the adhesive force u for steel piles in all soils except those with a natural density less than 85% of optimum or with a moisture content of less than 80% of optimum. This excludes highly organic soils and very dry soils such as well-drained sand or gravel terraces.

The value of u was further qualified for the design under consideration by the following criteria: (1) The minimum imbedment of a pile below the permafrost table will be 20 ft; and (2) no joint or coupling will be permitted less than 5 ft below the permafrost table.

DESIGN OF TOWER FOUNDATION

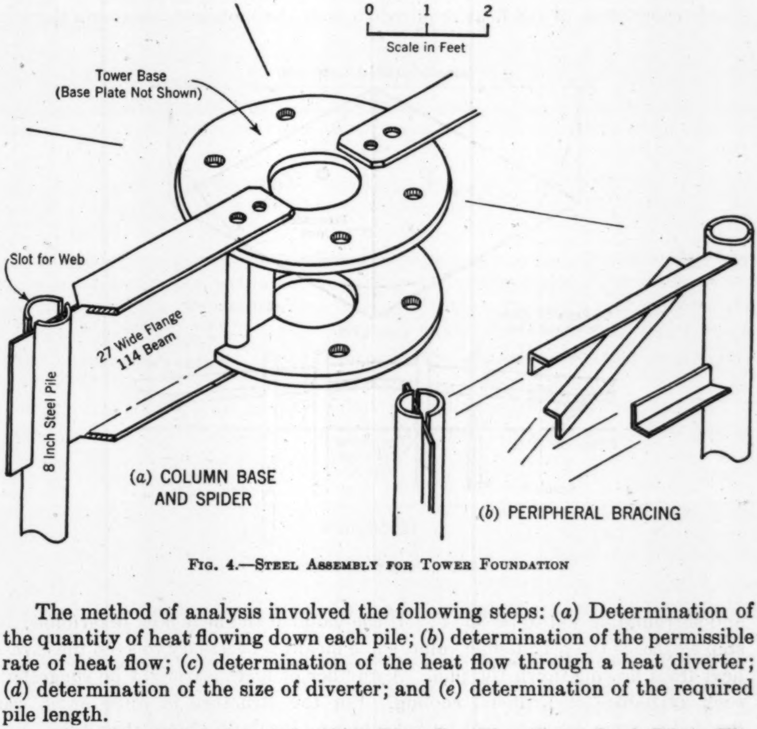
Structural Arrangements.—Fig. 4 shows the type of design finally adopted. The tower loadings are transferred through the bolt system and annular rings into a set of 6 radial spider beams that are supported on 8-in. double extra strength, steel pipe piles. Peripheral horizontal bracing was designed to distribute the horizontal shear to all piles and to provide added lateral rigidity to the structure. The principle of the design was to eliminate as much field welding as possible and to provide a structure that could be assembled easily in the field. The design is intended for use with piles either drilled in or driven into locally thawed zones. The former method, of course, is to be preferred since it ensures greater accuracy in positioning the piles.

Reduction of Heat Transmission.—The structural analysis presented no unusual applications of established theory. The heat transmission problem, however, is of interest because of its complexity and the method used to resolve it. The factors that influence the amount of heat flowing down into the permafrost are the mass of the tower, the ambient temperature, the intensity of radiation, the emissivity of the tower, the effect of winds, the temperature and thermal conductivity of the permafrost, and the specific resistance of the path of flow.

It is obvious that the critical period in the design of the structure occurs during the months of July and August when the temperatures are highest and maximum sunlight prevails. During this season, fortunately, the probability of occurrence of winds on the order of design magnitude (100–150 miles per hr) is very low.

An attempt was first made to establish standard daily curves, or reliable average values, for factors such as temperature, radiation, and emissivity. Available data were insufficient to permit establishment of a reliable daily curve. This problem in variable flow was reduced, therefore, to one of steady flow by assuming the tower above the insulators as a mass of constant temperature. For the purposes intended, a temperature of 80°F was chosen as being amply conservative.

The other end condition to be fixed was the temperature of the permafrost. Observations by the Corps of Engineers, United States Army, indicated that for depths of permafrost up to about 25 ft the average summer temperature could be expected to be about 1° to 2° below freezing. Therefore the temperature for the bottom of the pile was chosen as 30° F.



(*a*) COLUMN BASE AND SPIDER

(*b*) PERIPHERAL BRACING

FIG. 4.—STEEL ASSEMBLY FOR TOWER FOUNDATION

The method of analysis involved the following steps: (*a*) Determination of the quantity of heat flowing down each pile; (*b*) determination of the permissible rate of heat flow; (*c*) determination of the heat flow through a heat diverter; (*d*) determination of the size of diverter; and (*e*) determination of the required pile length.

a. The Total Quantity of the Heat That Can Flow Down Each Pile.—The equation for steady heat flow was used for this determination.

$$Q_T = \frac{\Delta_T}{\frac{x_1}{k_1A_1} + \frac{x_2}{k_2A_2} + \frac{x_3}{K_3A_3} \cdots \frac{x_n}{k_nA_n}} \qquad \text{.....(1)}$$

in which Q_T is the total heat flow in British Thermal Units (BTU) per hour; Δ_T is the difference in temperature between ends of system in degrees Fahrenheit; X is the length of particular section of path in feet; k is the thermal conductivity of a section in BTU per square foot per hour, per foot per degree Fahrenheit; and A is the mean cross-sectional area of a section in square feet. The subscripts 1, 2, 3 . . . n identify the number of sections in the total path.

b. The Maximum Permissible Rate of Heat Flow Down the Pile.—This flow was assumed to be a rate that would melt out an annular layer around the pile of 0.05-in. thickness at a rate not to exceed 1 ft vertically in 24 hr. The value of 0.05 in. was based upon the soil used in the tests; and for any soil, the thickness of the annular layer was fixed as the size at which 85% was finer. Making an assumption as to the moisture content and density, this item resolved into a simple calculation of the heat required to melt the moisture, assuming the pile and surrounding soil to be 32° F. If the value of the heat flow determined in step a exceeds the permissible value, some means is necessary to keep the excess heat from flowing down the pile. A number of methods might be suggested, such as baffles or artificial cooling. For the structure in question it was decided to try a concrete pad buried under an insulating blanket and designed to spread heat in a lateral direction and then down into the permafrost. This pad is illustrated in Fig. 5.

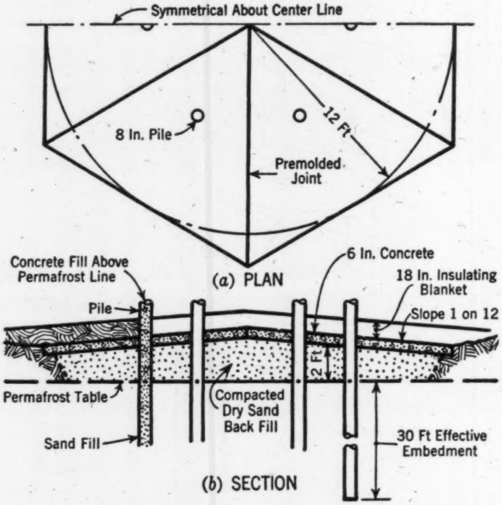

Symmetrical About Center Line
8 In. Pile
12 Ft
Premolded Joint
(a) PLAN
Concrete Fill Above Permafrost Line
Pile
6 In. Concrete
18 In. Insulating Blanket
Slope 1 on 12
2 Ft
Permafrost Table
Compacted Dry Sand Back Fill
Sand Fill
30 Ft Effective Embedment
(b) SECTION

Fig. 5.—Heat Diverter Pad

c. Heat Flow Through Diverter.—The point of diversion is first picked. In this case the point was on the pile 2.75 ft above the assumed permafrost table. Knowing the permisssible rate of flow and the temperature of the bottom of the pile, the temperature at the point of diversion is calculated by a simple application of the basic equation for heat flow. If the subscript p is used to denote permissible flow through the pile:

$$\Delta T_p = \frac{Q_p X_p}{k_p A_p} \qquad (2a)$$

or

$$\Delta T_p = Q R_p \qquad (2b)$$

in which $R_p = \frac{X_p}{k_p A_p}$ may be defined as the total resistance of the system or path. Since the temperature at the point of diversion, T_p, cannot exceed $30° + \Delta T_p$ the quantity of heat to be handled by the diverter can be determined. The value of R of the pile, R_p, is deducted from the total system value, R, that is computed from step a. The total heat flow in the system, Q_T, is then

$$Q_T = \frac{T_t - T_D}{R - R_p} \qquad (3)$$

in which T_t is the temperature of the tower; then

$$Q_T - Q_p = Q_D \qquad (4)$$

in which Q_D is the quantity of heat to be handled by the diverter. For the particular example analyzed, this quantity amounted to 1.90 − 0.45 = 1.45 BTU per hr.

d. Diverter Size.—It was assumed that only one half of the circumferential area around each pile would be effective in dissipating heat. The action of the diverter may be likened to flow through a pipe having an ever-increasing cross-sectional area (the concrete pad) and having an infinite number of infinitesimal side outlet pipes (the soil above the permafrost table) discharging into a reservoir of fixed elevation (the permafrost).

The first step is to adopt a trial size. A radius is assumed, and the effect on melting the permafrost is checked by determining the time required to melt an inch of depth using the formula:

$$\frac{t = A\ d\ M\ D\ 144}{12\ Q_D} \qquad (5)$$

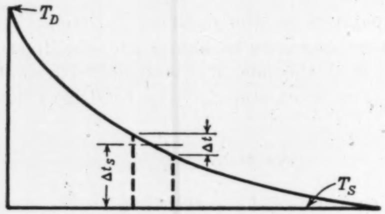
in which A is the horizontal area of permafrost under the diverter; d is the depth of layer (in this case 1 in.); M is the moisture content of soil by weight as a decimal; D is the density; t is the time in hours; and Q_D is the flow through the diverter in BTU per hour. In practice it will be found that almost any reasonable size of diverter will keep the rate of melting within safe limits. For the case in question a radius of 6 ft resulted in a rate of melting of 1 in. in about 55 days. It should be noted that the limit of 1 in. is arbitrary and could be established at other values depending upon the designer's judgment. The value of t should be equal to the length of the thawing (summer) season.

The thickness of concrete must be selected next. In this case a thickness of 6 in. was selected for economy. The relationship of heat flow in the soil and concrete warrant keeping this thickness as small as practicable.

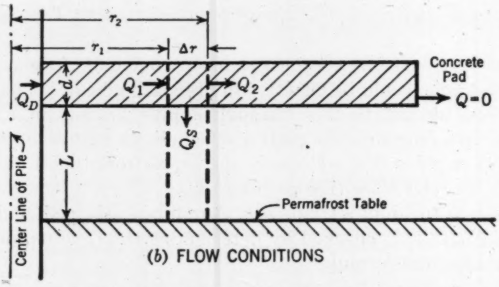
It is evident from an inspection of the problem that, for any small finite interval of radius

$$Q_2 = Q_1 - Q_s \qquad (6)$$

in which Q_1 and Q_2 are flow in the concrete pad, and Q_s is the flow through an incremental soil cylinder. The relationships are indicated in Fig. 6. It should be noted that the conditions indicated are not completely correct physically because the elevation of the permafrost table should rise a certain amount. The assumptions implied, however, are considered to be on the side of safety.



(a) TEMPERATURE DIAGRAM



(b) FLOW CONDITIONS

FIG. 6.—ASSUMED FLOW CONDITIONS IN DIVERTER

This relationship is solved by an arithmetic process as follows:

1. Assume Δt;

2. Compute $\Delta t_s = T_D - T_s - \left(\Delta t_1 + \Delta t_2 \ldots \Delta \frac{t_n}{2}\right)$ in which T_s is the temperature of the permafrost;

3. Compute $Q_s = \frac{k_s A_s \Delta t_s}{L}$ in which $A_s = \frac{\pi}{2}(r^2_2 - r^2_1)$ and the Q_s is the average heat flow in the annular soil ring;

4. Then $Q_1 - Q_s = Q_2$;

5. Compute Q_{ave} as a check against $\frac{Q_1 + Q_2}{2}$ from step 4 above: Q_{ave} $\frac{k_c A_s \Delta t}{\Delta r}$ in which A_c = log mean area; and

6. Revise the assumption for Δt as indicated by the result of step 5.

At the end of the process the final flow in the pad should be zero and the allowable total temperature drop ($\Sigma \Delta t$) completely consumed. If residual flow still exists in the concrete, the thickness or the radius must be increased or both. The computation, once set up, can be solved fairly rapidly for a number of combinations of r and d.

It was noted for the case in question that the critical region for the diverter lay in radii of less than about 2 ft. In this zone the soil cylinder is capable of diverting very little heat flow, and excessive thickness of concrete would be required. To eliminate this an annular sheet copper element was employed, extending to an average radius of 3 ft. The step calculations outlined are readily adapted to include provision for the copper element, assuming (for simplicity) that all the flow in the pad is taken by the copper. In general it would appear economical to keep the concrete thickness to a minimum consistent with other requirements, and in the critical region close to the pile to attain the required heat transmission with a high conductivity metal. The calculations indicated that less than ⅛-in. thickness of copper was required. This was arbitrarily increased to 5/16 in. for structural reasons.

Fig. 5 shows the type of element and pad adopted for the design in question. Particular pains have been taken to prevent infiltration of moisture for two reasons:

1. To reduce the possibility of galvanic corrosion; and
2. To prevent feeding of heat to the pile due to subsurface flow.

e. Required Length of Pile.—The following criteria were set up:

For winter conditions with the pile completely frozen in, the allowable value of skin frictional resistance (u) was taken as 20 lb per sq in. of pile surface. The minimum length of effective imbedment in permafrost is 20 ft, and the highest pile joint was taken to be at least 5 ft in permafrost.

For summer conditions when the pile may be all or partly frozen in, the minimum pile imbedment was assumed to be 30 ft based on an allowable rate of melting of 1 ft per day for 30 consecutive days. The pile, when completely unfrozen, should develop sufficient skin friction to resist forces due to a 60-mile per hr wind. Effective imbedment specifies that all layers of highly organic materials, ice lenses, and other impurities, according to the judgment of the designer, should be excluded from contributing to the pile resistance. The criterion for minimum imbedment under summer conditions is based upon the assumption that 30 consecutive days is the maximum period under which the design temperature conditions can be expected to prevail. This was based on examination of temperature records for a number of locations in the Arctic and may be adjusted for any specific locality in which reliable information is available.

SUMMARY

The method of design presented above is admittedly no more than a crude approach to a method that will become more accurate commensurate with an increase of knowledge in the basic factors, such as temperature regimens, radiation, and soon, an increasing knowledge of permafrost mechanics. However, it is recommended to the readers' attention because it indicates that the question of heat transmission may be important even on unheated structures; and it provides a method of attack that can be employed for any structure founded on piles. It also recognizes certain important factors that affect the success of the foundation and provides the framework within which they can be bracketed.

Acknowledgment

The tests and studies described in this paper are in connection with work performed for the Air Materiel Command, under the direction of Col. J. G. Griggs, USAF, Chief of the Air Installations Division. The author wishes to express his appreciation for the assistance of John W. Bierhorst, Jr., who ably criticized much of the work and assisted in the later phases of the design analyses, and Chief Warrant Officer Philip K. Head, USAF, who ably and untiringly assisted in the construction of the equipment and in the accomplishment of the tests.

DISCUSSION

Harry Carlson,[3] A. M. ASCE.—The author has described tests and studies which are of great interest and importance to engineers designing pile foundations for sites in arctic and subarctic regions where permafrost composed of frost-susceptible soil is present.

It is assumed from the statements under the heading, "Test Program; Test Results," describing Fig. 2, that curves 1 to 5, inclusive, indicate the results of tests on wet wood and curve 6 indicates the results of tests on 1½-in. steel pipe. It has been observed that the adhesive force between wood and various soils increases as the temperature decreases.[4] This would account for the behavior of the clayey soils in curves 1 and 3 as well as the silty soils in curves 2 and 4, respectively. The sandy loam soil of curve 5 also falls in the same area of the graph as the clayey and silty soils at the 11°F to 15°F temperature range. The adhesive force developed between soils and steel or wood depends, among other things, on the surface characteristics of the pile materials. Taper in the proper direction or surface roughness on wood piles could be more effective in developing adhesive force than the relatively smooth surface of steel pipe. It is not indicated in the paper what type of failure occurred in the tests made on the steel pipe; that is, whether the pipe slipped in the frozen soil or whether a diagonal tension failure occurred in the soil. It would be interesting to know what results would be obtained from tests on full-size pile specimens and whether the indicated unit adhesive force is affected by the diameter and depth of imbedment of the pile specimen in frozen ground.

Although the structural loads and other factors may have prevented the use of wood piles in the case described the heat transferred into the ground by wood piles would be much less than that transferred by steel piles. This is evident from the coefficients of thermal conductivity of these materials—0.8 for wood and 312.0 for steel. The quantity of heat conveyed into the ground by a pile is a function of its thermal conductivity and cross-sectional area. A 12-in. wood pile would give a $(k\,A)$-value of $0.8 \times 113 = 91$. An 8-in. XXH steel pipe pile would have a $(k\,A)$-value of $312 \times 21.4 = 6{,}677$. Thus, for a given temperature differential and length of pile, the steel pipe would transmit about seventy-five times as much heat as the wood pile.

In the design of the heat diverter pad shown in Fig. 5, it is indicated that a compacted dry sand backfill is used between the concrete pad and the permafrost table. As it would be undesirable to transmit heat from the pad to the permafrost in the vicinity of the piles, a less conductive material than a compacted sand should be used if possible. Generally, the coefficient of thermal conductivity of materials varies in direct proportion to the density. Thus, for the least heat transfer, fill materials with low density should be selected.

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[3] Chf., Permafrost Div., St. Paul District, Corps of Engrs., U. S. Dept. of the Army, St. Paul, Minn.

[4] "Permafrost or Permanently Frozen Ground and Related Engineering Problems," by S. W. Muller, J. W. Edwards, Inc., Ann Arbor, Mich., 1947.

A fill, containing material with a high void content which is saturated and frozen, will have a high volumetric latent heat of fusion and will be very effective in resisting heat transfer until the ice is entirely melted.

Filling of the steel pipe piles with relatively high-density materials such as sand and concrete will tend to increase the total quantity of heat flowing down the pile. It appears that the central part of the concrete pad bounded by the piles is unnecessary since only the part outside the pile circle is effective in dissipating heat laterally to areas where it will not affect the permafrost adjacent to the piles. Since the coefficient of thermal conductivity of the concrete in the pad would generally be nearly the same as the soil on which it rests, it would be desirable to use a material in the pad having a much larger coefficient of thermal conductivity than the supporting soil, or lateral heat transfer would not be effected.

Under the heading, "Reduction of Heat Transmission; *e* Required Length of Pile,"—"* * * the minimum pile imbedment was assumed to be 30 ft based on an allowable rate of melting of 1 ft per day for 30 consecutive days." Thus, the adfreezing strength of the pile could be zero at the end of the thawing season. As refreezing normally would occur from the ground surface downward, uplift forces could develop with ice segregation in frost-susceptible soils near the surface before refreezing would occur at lower levels. It would be desirable, therefore, to provide sufficient anchorage to resist uplift in the annual frost zone. Normally piles cannot be driven into permafrost but must be placed in holes formed by drilling or steaming operations. By attaching or forming anchors at the bottom of the pile it is possible to increase the resistance to withdrawal very substantially over that obtained by the bond developed from the adfreezing strength.

L. A., NEES,[6] A. M. ASCE.—The increase in adhesive force between wood and soil, as temperature decreases, accounts for the difference between curves 1 and 3 and curves 2 and 4 of Fig. 2, as stated by Mr. Carlson. The curve relationships further imply that, except for soils that differ widely in classification, the soil type may have only a minor effect on adhesive force—at least at lower temperature ranges. These factors do not invalidate the opinion expressed by the writer that the test values ranged lower than was expected. In no case—even on tests discarded because of doubt as to the results—did any of the values of *u* reach 125 lb per sq in. Only two tests exceeded 100 lb per sq in.—one at 102 lb per sq in. and the other at 121 lb per sq in. The remainder were observed well below 100 lb per sq in.

During and immediately after each test run, a careful examination of the soil mass was made to obtain evidence of shear failure. In no case except that of very dry soil conditions (*M* less than 6%) did the soil fail in shear. All other failures consisted of slippage of the pipe in the soil. The writer recognizes that the dividing line for the two types of failure in terms of moisture content will depend on the type of pile material used, all other variables being held constant.

The writer has not as yet been able to make the necessary series of full-scale tests which would answer questions on scaling.

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[6] Chf., Installations Eng. Section, Air Installations Div., Air Materiel Command, Wright-Patterson Air Force Base, Dayton, Ohio.

Mr. Carlson's comments regarding the thermal efficiency of wood piles are pertinent. The use of wood piles was considered in the design; however, the tower loadings and site conditions dictated the use of steel piles.

The sand-blanket backfill was used to reduce the volume of gravel required, gravel being scarce. Actually, a coarse gravel might be more desirable. The 18-in. insulating layer was counted on to provide the major portion of the thermal resistance.

Whether to fill the pile or to cap it, leaving it hollow, is open to question and would probably reflect the individual judgment of the designer. The thermal disadvantages and structural advantages of filled piles are not of major importance.

The central part of the pad is unnecessary and the diverter was designed on that basis. However, the pad was made continuous over the area to decrease the danger of surface-water infiltration.

As described in the paper, the diverter employs copper for an average radius of 3 ft. It thus provides against the contingencies that Mr. Carlson describes. In the basic design, values of $k = 1.0$ for concrete and 0.33 for soil were used. The analysis on this basis indicated that concrete cannot deliver sufficient heat laterally until a radius of the order of 3 ft to 5 ft is established. This necessitated the use of the copper sheet.

It is true that refreezing normally occurs from the ground surface downward. However, when air temperatures go below freezing the heat flow is reversed. The tower is now a structure capable of dissipating tremendous quantities of heat energy. For the design conditions, the frictional resistance of the unfrozen length of pile is approximately 25 kips, which is sufficient to hold forces developed by surface freezing to a depth of approximately 1 ft. No analysis was made, but it was intuitively concluded that, before this could be exceeded, the pile bottom would be "refrozen" into the permafrost for a sufficient depth to overcome forces developed by surface freezing. However, in any specific design where serious doubts exist regarding this condition, the designer can either adopt Mr. Carlson's suggestion of anchors, or increase the length of pile. Anchors may prove most desirable for "steamed-in" piles. However, such anchors should have a diameter of at least three pile diameters to insure success. For "drilled-in" piles, it should prove more economical to increase the pile length.

The writer wishes to thank Mr. Carlson for his interesting discussion. It revealed the weak points in the paper and illustrated the need for more and extensive investigations into problems of this nature.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 2524

CONSUMPTIVE USE OF WATER

A SYMPOSIUM

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AMERICAN SOCIETY OF CIVIL ENGINEERS

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TRANSACTIONS

DEFINITIONS, METHODS, AND RESEARCH DATA

BY HARRY F. BLANEY,[1] M. ASCE

WITH DISCUSSION BY MESSRS. GEORGE H. HARGREAVES, WILLIAM W. DONNAN, AND HARRY F. BLANEY

SYNOPSIS

Consumptive use (or evapo-transpiration) includes loss of water by evaporation of moisture from the surface of the soil and loss from interception by vegetative cover and plant transpiration. This paper discusses the subject of consumptive use with special reference to definitions, methods, and results of research. The procedure for determining consumptive use of water in valley areas from climatology, irrigation, and other data is outlined. The results of experimental measurements of evapo-transpiration by agricultural crops and natural vegetation are presented briefly. The paper provides the practicing engineer the information required to estimate consumptive use in water utilization investigations.

INTRODUCTION

Knowledge concerning the various phases of the hydrologic cycle is incomplete. Consumptive use of water (or evapo-transpiration) is one of the important elements in this cycle of water movement from the time water falls on the land as rain or snow until it reaches the ocean. The subject of consumptive use includes evaporation of moisture from land and transpiration by vegetation and is increasingly significant, particularly in the irrigated areas of the western part of the United States. It involves problems of water supply both surface and underground, as well as those of the management of, and general economics of, irrigation and multiple-purpose projects. Data on the use of water are essential in planning federal, state, and private irrigation projects. The consumptive-use requirement for water has become an important factor in the arbitration of controversies regarding major stream systems, such as the Rio Grande and the Colorado River, in which the welfare

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of the people of valleys, cities, states, and even nations is involved.[2,3] Before the available water resources and an equitable division of the use of the waters of a drainage basin in arid and semiarid regions can be satisfactorily ascertained, careful consideration must be given to the consumptive-use requirements for water in various subbasins.[3,4]

In water-supply and irrigation investigations, engineers are called upon to make, within a limited time, estimates of probable past, present, and future evaporation and evapo-transpiration losses from areas in river valleys. Sometimes few long-period hydrologic records, except climatological data, are available.[3] The results of research on evapo-transpiration provide the practicing engineer with the basic data and methods required to estimate consumptive use in utilization of water investigations.[4] A progressive improvement of methods and rapid increase in available literature on the subject sometimes make it difficult and tedious for the engineer who does not specialize in this field to evaluate the data properly and select the methods best suited for solving his particular problem.

DEFINITIONS AND SOME INTERPRETATIONS

Definition of Consumptive Use.—In some instances there has been confusion as to the meaning of terms employed in reports on the use of water. Such terms as consumptive use, evapo-transpiration, transpiration, total evaporation, stream-flow depletion, water requirements, irrigation requirements, and duty of water have been interchanged. In this paper the terms "consumptive use" and "evapo-transpiration" are considered synonymous. They may be defined as:

> "The sum of the volumes of water used by the vegetative growth of a given area in transpiration and building of plant tissue and that evaporated from adjacent soil, snow, or intercepted precipitation on the area in any specified time, divided by the given area. If the unit of time is small, the consumptive use is expressed in acre-inches per acre or depth in inches, whereas, if the unit of time is large, such as a crop-growing season or a 12-month period, the consumptive use is expressed as acre-feet per acre or depth in feet or inches."

This definition has been found satisfactory for use in many reports since 1935.[2,3,4] In 1939 it was adopted with minor changes by the Committee on Irrigation of the American Society of Agricultural Engineers and is now in general use by engineers.

Other Interpretations.—Although evaporation and transpiration have been studied for more than 200 years, the term consumptive use probably was not applied to water consumption prior to 1900. Many of the writings concerning the consumptive use of water are in the form of unpublished engineering reports. The term consumptive use, as originally applied to irrigation, was defined

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[2] "Water Utilization, Upper Rio Grande Basin," by Harry F. Blaney, Paul A. Ewing, O. W. Israelsen, Carl Rohwer, and F. C. Scobey, National Resources Committee, Washington, D. C. Part III. February, 1938.

[3] "Consumptive Use of Water in the Irrigated Areas of Upper Colorado River Basin," by Harry F. Blaney and Wayne D. Criddle, Div. of Irrig. and Water Conservation, SCS., U. S. Dept. of Agri., Logan, Utah, April, 1949.

[4] "Consumptive Water Use and Requirements: Report of the Participating Agencies, Pecos River," by Harry F. Blaney, Paul A. Ewing, Karl V. Morin, and Wayne D. Criddle, Joint Investigation of the National Resources Planning Board, Washington, D. C., June, 1942.

as a seasonal loss of water in acre-feet per acre irrigated. Among the first published writings dealing directly with the consumptive use, a report of an ASCE committee in 1930 is noteworthy.[5] The committee proposed certain definitions for consumptive use of water in a basic sense, and also for the farm, the project, and the valley. It also reviewed previous estimates of consumptive use for large river systems and made reference to 24 articles, published and unpublished, dealing with net water requirements and consumptive use. One of the definitions suggested by this committee follows:

> "Consumptive Use in a Basic Sense: The consumptive use, 'U', is here defined as the quantity of water, in acre-feet per cropped acre per year, absorbed by a crop and transpired or used directly in the building of plant tissue, together with that evaporated from the crop-producing land."

In a report[6] in 1930 on rainfall penetration and consumptive use of water, consumptive use was defined as "the sum of water used by the vegetative growth of a given area in transpiration or building of plant tissue and that evaporated from the area."

In 1934, the Committee on Absorption and Transpiration, Hydrology Section, American Geophysical Union, proposed the following definition[7]:

> "Consumptive Use: The quantity of water per annum used by either cropped or natural vegetation in transpiration or in the building of plant-tissue, together with water evaporated from the adjacent soil, snow, or from intercepted precipitation. It is sometimes termed 'evapo-transpiration'."

In 1935 the ASCE Special Committee of the Board of Direction, on Irrigation Hydraulics, adopted the following definitions[8]:

"Consumptive Use: The quantity of water transpired and evaporated from a cropped area."

"Evapo-transpiration: Combined loss of water from soils by evaporation and plant transpiration."

In some reports, the term "total evaporation" is applied to the combined evaporation and transpiration loss in drainage basins.[9]

The handbook[10] of the ASCE Committee on Hydrology of the Hydraulics Division of the ASCE, published in 1949, states:

> "* * * the terms 'evapo-transpiration' and 'consumptive use' have received general acceptance, these terms denote the quantity of water

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[5] "Consumptive Use of Water in Irrigation," Progress Report of the Duty of Water Committee of the Irrig. Div., *Transactions*, ASCE, Vol. 94, 1930. p. 1349.

[6] "Rainfall Penetration and Consumptive Use of Water In Santa Ana River Valley and Coastal Plain," by Harry F. Blaney, Colin A. Taylor, and Arthur A. Young, *Bulletin No. 33*, Div. of Water Resources, California State Dept. of Public Works, Pomona, Calif., 1930.

[7] "Report of the Committee on Absorption and Transpiration, 1933–34," *Transactions*, Am. Geophysical Union, Washington, D. C., June, 1934, Part II, pp. 295–296.

[8] "Letter Symbols and Glossary for Hydraulics," *Manual of Engineering Practice No. 11*, ASCE, 1935.

[9] "Hydrology," *Physics of the Earth Series*, Vol. IX, ed. by O. E. Meinzer, McGraw-Hill Book Co., Inc., New York, N. Y., 1942, p. 314.

[10] "Evaporation and Transpiration," in Hydrology Handbook, *Manual of Engineering Practice No. 28*, ASCE, 1949, Chapter 4.

transpired by plants during their growth or retained in the plant tissue, plus the moisture evaporated from the surface of the soil and the vegetation, expressed in feet or inches depth of water lost or used in a specified time."

The definitions of other terms which are sometimes confused with consumptive use are as follows:

1. Duty of water: The quantity of irrigation water applied to a given area for the purpose of maturing its crop;
2. Irrigation requirement: The quantity of water, exclusive of precipitation, that is required for crop production. It includes surface evaporation and other economically unavoidable wastes;
3. Stream-flow depletion: The amount of water that flows into a valley, or onto a particular land area, minus the amount that flows out of the valley or off from the particular land area[2];
4. Transpiration: The quantity of water absorbed by the crop and transpired and used directly in the building of plant tissue, in a specified time. It does not include soil evaporation;
5. Water requirement: The quantity of water, regardless of its source, required by a crop in a given period of time, for its normal growth under field conditions. It includes surface evaporation and other economically unavoidable wastes.

RESEARCH STUDIES

Factors Affecting Consumptive Use.—Many factors operate singly or in combination to influence the amount of water consumed by plants. The effects of these factors are not necessarily constant but may fluctuate from year to year as well as from place to place. Some involve the human factor, but others are related to natural influences and the environment.

The rate of evapo-transpiration depends on climate, soil-moisture supply, vegetative cover, soils, and land management. The factors included in climate that particularly affect consumptive use are precipitation, temperature, humidity, wind movement, and length of growing season. The quantity of water transpired by plants depends upon the amount of water at their disposal, as well as on temperature and dryness of the air, wind movement, the intensity of sunlight, the stage of the development of the plant, the amount of its foliage, and the nature of its leaf.

The effect of sunshine and heat in stimulating transpiration was studied as early as 1691, according to a review of the literature by Cleveland Abbe.[11] Measurements of transpiration of various kinds of plants by L. J. Briggs and H. L. Shantz indicate a close correlation between transpiration and evaporation from free water surfaces, air temperature, solar radiation, and wet-bulb depression readings.[12] Scientists have made many other studies of the effects of temperature moisture and light on plant growth.[13]

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[11] "A First Report of the Relations Between Climates and Crops," by Cleveland Abbe, *Bulletin No. 36*, Weather Bureau, U. S. Dept. of Agri., Washington, D. C., 1905.

[12] "Daily Transpiration During the Normal Growth Period and its Correlation with the Weather," by L. J. Briggs and H. L. Shantz, *Journal of Agricultural Research*, Vol. 7, 1916.

[13] "The Water Requirements of Plants," by Lyman J. Briggs and H. L. Shantz, *Bulletin No. 284*, Bureau of Plant Industry, U. S. Dept. of Agri., Washington, D. C., 1913.

Studies and Results.—Research studies by federal, state, and other type agencies have been made on evaporation from soil and evapo-transpiration at various times during the past 50 years in the United States.

One of the first studies[14] of evapo-transpiration losses of irrigated crops in California was made in 1903 by Irrigation Investigations, Office of Experiment Stations, United States Department of Agriculture (USDA). At various times since that date this agency, now known as the Division of Irrigation and Water Conservation of the Soil Conservation Service (SCS), has studied and measured consumptive use of water by different agricultural crops and natural vegetation in many sections of the west, in cooperation with state engineers as well as state agricultural experiment stations, and other agencies. Usually evaporation, temperature, humidity, precipitation, and wind movement were recorded at the same time. Thus, data are available in many areas for correlating consumptive use measurements with temperature and other climatological observations.

Extensive studies in the correlation of evaporation, temperature, humidity, wind movement, and evapo-transpiration were conducted by the author in 1919, at the Irrigation Field Laboratory of the USDA, located at Denver, Colo. These studies included meteorological observations as well as measurements of evaporation from various sized pans, evaporation from soil tanks, and consumptive use of water by irrigated crops grown in tanks.[15]

The results of these studies and other investigations in California, New Mexico, and other areas indicate the observed evaporation data may be used as a means of estimating evapo-transpiration by phreatophytes (water-loving vegetation) having access to an ample water supply when the relation of the two is known for a particular area.[2,4] As an example, for two locations in California, for tules growing in large tanks within the confines of a swamp area, the consumptive use, with reference to evaporation from a near-by exposed United States Weather Bureau pan, was 95%. The percentage varied from month to month, increasing during the summer and becoming smaller in the cooler months, but 95% was the average obtained from a two-year record.

From long-period records of evaporation, temperature, and humidity in New Mexico and Texas, together with consumptive-use measurements at Carlsbad, N. Mex., empirical formulas were developed for computing evaporation and consumptive use when temperature and humidity data are available.[4] Consideration of these results and the factors

TABLE 1.—COEFFICIENTS FOR CONSUMPTIVE USE OF WATER, CARLSBAD, NEW MEXICO—1940

Type of vegetation	Depth of water table (feet)	EMPIRICAL COEFFICIENTS	
		k_w	k_s
Sacaton	4	0.0044	0.0139
Sacaton	2	0.0063	0.0154
Salt cedar (tamarisk)	2	0.0075	0.0216

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[14] "Evaporation Losses in Irrigation and Water Requirements of Crops," by Samuel Fortier, *Bulletin No. 177*, Office of Experiment Stations, U. S. Dept. of Agri., Washington, D. C., 1907.

[15] "Progress Report on Evaporation and Use of Water Studies at Denver Irrigation Field Laboratory, Colorado," by Harry F. Blaney, Div. of Irrig., U. S. Dept. of Agri., Los Angeles, Calif., 1920 (unpublished).

involved is shown in the expression:

$$u = k_m\, t\, p\, (114 - h) = k_m\, c \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which u is the monthly consumptive use (or evaporation) in inches; k is the monthly empirical coefficients; t is the mean monthly temperature, °F; p is the monthly percentage of daytime hours of the year; h is the average monthly humidity; and $c = t\, p\, (114 - h)$ is the monthly use index (climatic factor). The formula for annual consumptive use (or evaporation) in inches is

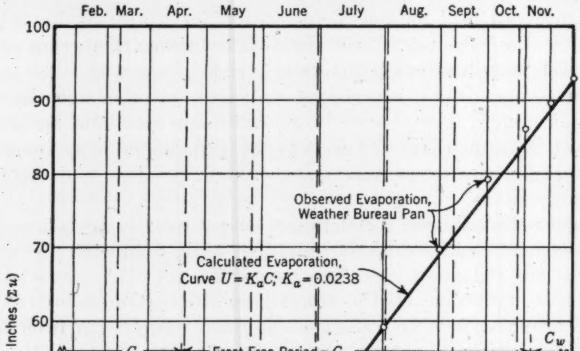

FIG. 1.—OBSERVED AND CALCULATED CURVES FOR USE OF WATER, CARLSBAD, NEW MEXICO, 1940

$$U = K_a\, C = K_w\, c_w + k_s\, c_s \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which K_a is the empirical coefficient for the entire year; C is the use index for the entire year; k_w is the empirical coefficient for the winter period; k_s is the empirical coefficient for the growing season or frost-free period; c_w is the use index for the winter season; and c_s is the use index for the growing season or frost-free period.

The values of k_w and k_s may be computed from observed values of consumptive use, temperature, and humidity by the relation $k = u/c$.

Computed coefficients for winter and summer consumptive use of water for natural vegetation (phreatophtes) growing along the Pecos River, based on evapo-transpiration, temperature, and humidity measurements at Carlsbad for the year 1940, are shown in Table 1. The computed annual coefficient K for lake-surface evaporation is 0.0164. The summer coefficient for alfalfa, growing in an area of high water table at Albuquerque, N. Mex., in 1936, is computed to be $k_s = 0.0170$.

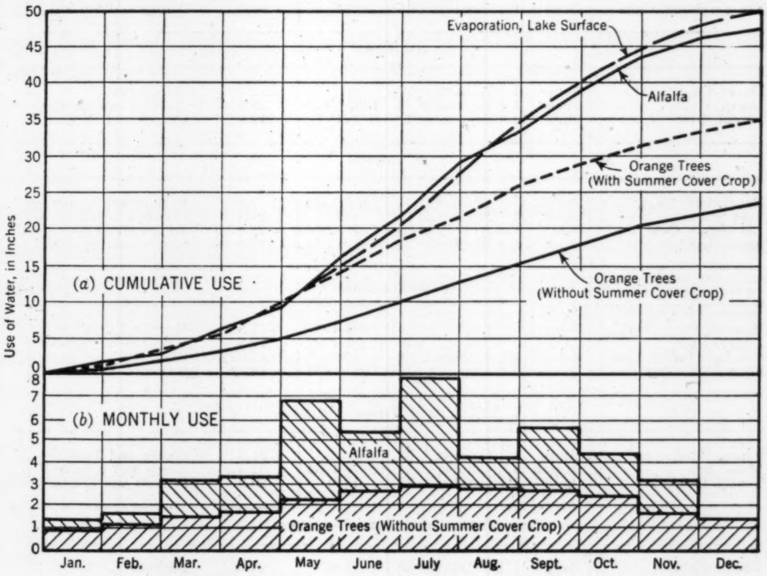

FIG. 2.—USE OF WATER, SAN FERNANDO VALLEY, CALIFORNIA

Fig. 1 shows the observed and calculated curves for use of water for evaporation from a Weather Bureau pan, a tank of sacaton growth, and bare soil tank at Carlsbad in 1940. An example of the results of research studies of the transpiration use of water is shown in Fig. 2 which illustrates the difference between evaporation from a lake surface and the use of water by alfalfa and citrus trees in the San Fernando Valley, in California.[16]

The results of some measurements by various investigators of consumptive use by agricultural crops and natural vegetation are shown in Tables 2 and 3, respectively.

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[16] "Progress Report on Cooperative Research Studies on Water Utilization, San Fernando Valley, California, Irrigation Season of 1950," by Harry F. Blaney and Homer J. Stockwell, Jr., Div. of Irrig., SCS, U. S. Dept. of Agri., June, 1941 (unpublished).

TABLE 2.—Seasonal Consumptive Use of Water by Various Irrigated Crops as Determined by Field Experiments

Location	Growing Season or Period		Year	Consumptive use of water (inches)	Authority
	From	To			
(1)	(2)	(3)	(4)	(5)	(6)
(a) Alfalfa					
Bonners Ferry, Idaho..	May 5	September 25	1940–1947	24.0	Marr and Criddle[a]
Carlsbad, N. Mex......	April 18	November 10	1940	38.6	Blaney[b]
Davis, Calif...........	April 1	September 30	1939	30.4	Veihmeyer[c]
Fort Stockton, Tex.....	April 13	November 11	1940	40.5	Blaney and Bloodgood[b]
Logan, Utah..........	May 7	October 11	1902–1929	25.0	Pittman and Stewart[d]
San Fernando, Calif....	April 1	October 31	1940	37.4	Blaney and Stockwell[e]
(b) Corn					
Davis, Calif...........	June 1	September 30	...	12.0	Veihmeyer[c]
Logan, Utah..........	June 1	September 30	1902–1929	25.0	Pittman and Stewart[d]
Vernal, Utah..........	June 10	September 20	1948	19.4	Criddle and Peterson[f]
(c) Cotton					
Bakersfield, Calif......	April 1	October 31	1927–1930	29.2	Beckett and Dunshee[g]
Fort Stockton, Tex.....	April 13	November 11	1940	28.9	Blaney and Bloodgood[b]
Los Banos, Calif.......	May 1	November 30	1932	25.5	Adams and Veihmeyer[h]
Mesa, Ariz............	April 1	October 31	1935	30.9	Harris and Hawkins[i]
(d) Small Grains					
Bonners Ferry, Idaho..	May 5	August 5	1930–1947	17.5	Marr and Criddle[a]
Logan, Utah..........	May 10	August 10	1902–1929	17.5	Pittman and Stewart[d]
San Luis Valley, Colo..	June 1	August 31	1936	14.05	Blaney[b]
Scottsbluff, Nebr.......	April 20	July 25	1932–1933	14.72	Bowen[j]
(e) Orchard Crops—Oranges					
Azusa, Calif...........	April 1	October 31	1929	21.8	Blaney and Taylor[k]
Mesa, Ariz............	March 1	October 31	1931–1934	32.4	Harris and Kinnison[l]
Tustin, Calif..........	April 1	October 31	1929	20.9	Beckett and Pillsbury[m]
(f) Orchard Crops—Walnuts					
Tustin, Calif..........	April 1	September 30	1928	26.30	Beckett[n]
Tustin, Calif..........	April 1	September 30	1929	27.43	Beckett[n]
(g) Orchard Crops—Deciduous Trees					
Albuquerque, N. Mex..	May 1	September 31	1936	19.5	Blaney[b]
Davis, Calif...........	March 1	November 30	...	26.4	Veihmeyer[c]
Wenatchee, Wash......	April 15	October 22	1908	23.0	Fortier[o]
Ontario, Calif.........	April 1	September 30	1928	28.4	Blaney and Taylor[k]
(h) Potatoes					
Bonners Ferry, Idaho..	May 8	September 27	1947	22.95	Marr and Criddle[a]
Logan, Utah..........	May 20	September 15	1902–1929	15.0	Pittman and Stewart[d]
Ontario, Ore..........	April 20	August 31	1941–1942	17.9	Sanford and Criddle[p]
San Luis Valley, Colo...	June 1	September 30	1936	19.89	Blaney[b]

TABLE 2.—*Continued*

Location	Growing Season or Period		Year	Consumptive use of water (inches)	Authority
	From	To			
(1)	(2)	(3)	(4)	(5)	(6)
(*i*) Sugar Beets					
Davis, Calif.	April 1	September 30	...	25.20	Veihmeyer[c]
Logan, Utah	April 15	October 15	1902–1929	25.0	Pittman and Stewart[d]
Scottsbluff, Nebr.	April 20	October 15	1932–1936	24.0	Bowen[j]
(*j*) Tomatoes					
Davis, Calif.	June 1	October 31	1933–1935	22.8	Veihmeyer[c]
Mercedes, Tex.	March 25	June 30	1918–1920	17.0	Rockwell[q]
(*k*) Truck Crops					
Stockton, Calif.	May 1	September 30	1925–1928	21.4	Stout[r]
Stockton, Calif.	April 1	October 31	1925–1928	24.6	Stout[r]

[a] "Consumptive Use of Water Studies in Idaho," by Wayne D. Criddle and James C. Marr, Div. of Irrigation, Soil Conservation Service, U. S. Dept. of Agriculture, Boise, Idaho, 1945.

[b] "Consumptive Water Use and Requirements," by Harry F. Blaney, Karl V. Morin, and Wayne D. Criddle, The Pecos River Joint Investigation Reports of the Participating Agencies, National Resources Planning Board, 1942, pp. 170–230.

[c] "Irrigation Studies," by Frank J. Veihmeyer, Univ. of California, Berkeley, Calif., 1939 (unpublished).

[d] "Twenty-Eight Years of Irrigation Experiments near Logan, Utah," by D. W. Pittman and George Stewart, *Bulletin 219*, Utah Agricultural Experiment Station, 1930, pp. 1–15.

[e] "Progress Reports on Cooperative Research Studies on Water Utilization, San Fernando Valley, California," by Harry F. Blaney and Homer J. Stockwell, 1940–41 (unpublished).

[f] "Consumptive Water Use and Requirements: A Progress Report on Colorado River Area Investigations in Utah," by Wayne D. Criddle and Dean F. Peterson, Jr., Div. of Irrigation and Utah Agricultural Experiment Station, Soil Conservation Service, U. S. Dept. of Agriculture, 1949.

[g] "Water Requirements of Cotton on Sandy Loam Soils in Southern San Joaquin Valley," by S. H. Beckett and Carroll F. Dunshee, *Bulletin 537*, California Agricultural Experiment Station, 1932, pp. 1–48.

[h] "Cotton Irrigation Investigations in San Joaquin Valley, California, 1926–1935," by Frank Adams, F. J. Veihmeyer, and Lloyd N. Brown, *Bulletin 668*, California Agricultural Experiment Station, 1942.

[i] "Irrigation Requirements of Cotton on Clay Loam Soils in the Salt River Valley," by Karl Harris and R. S. Hawkins, *Bulletin 181*, Arizona Agricultural Experiment Station, 1942, pp. 421–459.

[j] "Uses and Efficiencies of Water by Some Farm Crops under Irrigation in Western Nebraska," by Leslie Bowen, Div. of Irrigation, Soil Conservation Service, U. S. Dept. of Agriculture, Scottsbluff, Nebr., 1937.

[k] "Rainfall Penetration and Consumptive Use of Water in Santa Ana River Valley and Coastal Plain," by Harry F. Blaney, C. A. Taylor, and A. A. Young, *Bulletin 33*, Div. of Water Resources, California State Dept. of Public Works, 1930, pp. 1–162.

[l] "Use of Water by Washington Navel Oranges and Marsh Grapefruit Trees in Salt River Valley, Arizona," by Karl Harris, A. F. Kinnison, and O. W. Albertt, *Bulletin 153*, Arizona Agricultural Experiment Station, 1936, pp. 441–496.

[m] "Irrigation Requirements of California Crops," by Arthur A. Young, *Bulletin 51*, Div. of Water Resources, California State Dept. of Public Works, 1945, pp. 1–132.

[n] "Irrigation Requirements in Southern California," by S. H. Beckett, *Bulletin 32*, Div. of Water Resources, California State Dept. of Public Works, 1930, Chapter IX, pp. 57–60.

[o] "Orchard Irrigation," by Samuel Fortier, *Farmers' Bulletin 1518*, U. S. Dept. of Agriculture, 1927, pp. 1–27.

[p] Studies (unpublished) of Hollis Sanford, and Wayne D. Criddle, 1941–1943.

[q] "Duty of Water in Irrigation," by William L. Rockwell, Div. of Irrigation and Texas Board of Water Engineers, Bureau of Public Roads, U. S. Dept. of Agriculture, 1914–1920.

[r] "Consumptive Use of Water in the Delta of Sacramento and San Joaquin Rivers," by O. V. P. Stout, *Bulletin 27*, Div. of Water Resources, California State Dept. of Public Works, 1931, pp. 68–75.

METHODS OF DETERMINING WATER USE

Scientists have tried various means to determine the amount of water transpired by plants.[9] One of the earliest methods consisted of weighing freshly cut leaves, twigs, or stalks of growing plants immediately after cutting and at short intervals thereafter, during a period of hours or days, or until wilting commenced. Experimenters have also grown thousands of individual plants in small pots, weighing them periodically to determine the transpiration

TABLE 3.—ANNUAL OR SEASONAL CONSUMPTIVE USE OF WATER BY NATIVE VEGETATION AS DETERMINED BY TANKS AND SOIL-MOISTURE STUDIES IN VALLEY AREAS

Location (1)	Vegetation (2)	Depth to water table (inches) (3)	Period: From (4)	Period: To (5)	Period: Months (6)	Consumptive use (inches) (7)	Authority (8)
(*a*) NATIVE GRASS							
San Bernardino, Calif...	....	....[a]	Oct. 1928	Sept. 1929	12	10.0	Blaney and Taylor[b]
Ontario, Calif..........	....	....[a]	Oct. 1927	Sept. 1928	12	13.4	Blaney and Taylor[b]
Anaheim, Calif.........	....	....[a]	Oct. 1927	Sept. 1928	12	12.6	Blaney and Taylor[b]
Cucamonga, Calif......	....	....[a]	Oct. 1929	Sept. 1930	12	15.0	Blaney and Taylor[b]
(*b*) NATIVE BRUSH							
San Bernardino, Calif...	....	....[a]	Oct. 1927	Sept. 1930	12	21.6	Blaney and Taylor[b]
Muscoy, Calif.........	....	....[a]	Oct. 1927	Sept. 1928	12	17.7	Blaney and Taylor[b]
Palmer Canyon, Calif...	....	....[a]	Oct. 1929	Sept. 1930	12	19.6	Blaney and Taylor[b]
(*c*) SALT GRASS							
Owens Valley, Calif.....	....	18	Jan. 1911	Dec. 1911	12	48.8	Lee[c]
Owens Valley, Calif.....	....	35	Jan. 1911	Dec. 1911	12	40.2	Lee[c]
Owens Valley, Calif.....	....	59	Jan. 1911	Dec. 1911	12	13.4	Lee[c]
Santa Ana, Calif.......	....	12	May 1931	Apr. 1932	12	42.8	Blaney and Taylor[d]
Santa Ana, Calif.......	....	24	May 1931	Apr. 1932	12	35.3	Blaney and Taylor[d]
Santa Ana, Calif.......	....	36	June 1931	Apr. 1932	11	23.8	Blaney and Taylor[d]
Santa Ana, Calif.......	....	48	May 1931	Apr. 1932	12	13.4	Blaney and Taylor[d]
San Luis Valley, Colo...	....	14	Apr. 1928	Oct. 1928	7	17.8	Tipton and Hart[e]
San Luis Valley, Colo...	....	24	Apr. 1928	Oct. 1928	7	15.6	Tipton and Hart[e]
Los Griegos, N. Mex....	....	14	Oct. 1926	Sept. 1927	12	33.2	Elder[e]
Los Griegos, N. Mex....	....	26	Oct. 1927	Sept. 1928	12	22.7	Elder[e]
Isleta, N. Mex.........	....	8	June 1936	May 1937	12	31.5	Blaney and Morin[e]
Mesilla Valley, N. Mex.	....	14	July 1936	June 1937	12	39.8	Blaney[e]
Escalante Valley, Utah.	....	23	May 1927	Oct. 1927	6	22.6	White[f]
Escalante Valley, Utah.	....	26	May 1927	Oct. 1927	6	17.9	White[f]
(*d*) TREES							
San Luis Rey, Calif.....	Cottonwood, willows, grass	48	Apr. 1941	Mar. 1943	12	62.5	Muckel and Blaney[g]
Safford Valley, Ariz.....	Cottonwood, willows, grass	84	Oct. 1943	Sept. 1944	12	91.7	Gatewood and Robinson[h]
Safford Valley, Ariz.....	Tamarisk	83.6	Oct. 1943	Sept. 1944	12	87.8	Gatewood and Robinson[h]
Safford Valley, Ariz.....	Tamarisk	24	May 1940	Dec. 1940	8	61.1	Turner and Halpenny[i]
Safford Valley, Ariz.....	Tamarisk	48	May 1940	Dec. 1940	8	47.9	Turner and Halpenny[i]
Carlsbad, N. Mex......	Tamarisk	36	Jan. 1940	Dec. 1940	12	57.3	Blaney and Morin[j]
Safford Valley, Ariz.....	Mesquite	120	Oct. 1943	Sept. 1944	12	32.5	Gatewood and Robinson[h]

TABLE 3.—*Continued*

Location	Vegetation	Depth to water table (inches)	Period			Consumptive use (inches)	Authority
			From	To	Months		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
			(*e*) MISCELLANEOUS VEGETATION				
Safford Valley, Ariz.....	Baccharis	24	May 1940	Dec. 1940	8	52.0	Turner and Halpenny[i]
Safford Valley, Ariz.....	Baccharis	48	May 1940	Dec. 1940	8	39.7	Turner and Halpenny[i]
Safford Valley, Ariz.....	Baccharis	72	Oct. 1943	Sept. 1944	12	54.2	Gatewood and Robinson[h]
Escalante Valley, Utah.	Grease wood	26	May 1927	Oct. 1927	6	25.2	White[f]
San Luis Valley, Colo...	Meadow grass	0	June 1936	Nov. 1936	6	36.3	Blaney and Morin[j]
Carlsbad, N. Mex......	Sacaton	36	Jan. 1940	Dec. 1940	12	44.8	Blaney and Morin[j]
San Bernardino, Calif...	Bermuda grass	24	May 1929	Apr. 1931	12	34.4	Blaney and Young[d]
Fort Collins, Colo.:....	Sedge grass	6	May 1930	Oct. 1930	6	60.2	Parshall[k]
Fort Collins, Colo......	Sedge grass	18	May 1930	Oct. 1930	6	53.6	Parshall[k]
Isleta, N. Mex.........	Sedge grass	3	June 1936	May 1937	12	76.9	Blaney and Morin[e]
Santa Ana, Calif.......	Wire rush	24	Aug. 1930	July 1931	12	78.9	Blaney and Young[d]
Fort Collins, Colo......	Rushes	....	July 1931	Oct. 1931	4	52.6	Parshall[k]
Victorville, Calif.......	Tules	0	Jan. 1931	Dec. 1932	12	78.4	Blaney and Taylor[d]
San Luis Valley, Colo...	Tules	0	June 1936	Nov. 1936	6	38.8	Blaney[e]

[a] Determined by Soil Sampling. No high water table. [b] "Rainfall Penetration and Consumptive Use of Water in Santa Ana River Valley and Coastal Plain," by H. F. Blaney, Colin A. Taylor, and Arthur A. Young, *Bulletin No. 33*, Div. of Water Resources, California State Dept. of Public Works, Pomona, Calif., 1930. [c] "Hydrology," *Physics of the Earth Series*, ed. by O. E. Meinzer, McGraw-Hill Book Co., Inc., New York, N. Y., 1942, p. 295. [d] "Water Losses Under Natural Conditions from Wet Areas in Southern California," by Harry F. Blaney, C. A. Taylor, M. G. Nickle, and A. A. Young, *Bulletin No. 44*, Div. of Water Resources, California State Dept. of Public Works, Pomona, Calif., 1933. [e] "Water Utilization, Upper Rio Grande Basin," by Harry F. Blaney Paul A. Ewing, O. W. Israelsen, Carl Rohwer, and F. C. Scobey, National Resources Committee, Washington, D. C., Part III, February, 1938. [f] "A Method of Estimating Ground-Water Supplies Based on Discharge by Plants and Evaporation from Soil," Results of Investigations in Escalante Valley, Utah, by W. N. White, *Water-Supply Paper 659-A*, Geological Survey, U. S Dept. of the Interior, Washington, D. C., 1932. [g] "Utilization of the Waters of Lower San Luis Rey Valley, San Diego County, California," by Dean C. Muckel and Harry F. Blaney, Div. of Irrigation and Water Conservation, Soil Conservation Service, U. S. Dept. of Agriculture, Los Angeles, Calif., April, 1945. [h] "Use of Water by Bottom-Land Vegetation in Lower Safford Valley, Arizona," by J. S. Gatewood, T. W. Robinson, B. R. Colby, J. D. Hem, and L. C. Halpenny, *Water-Supply Paper No. 1103*, Geological Survey, U. S. Dept. of the Interior, Washington, D. C., 1950. [i] "Ground-Water Inventory in the Upper Gila River Valley, New Mexico and Arizona: Scope of Investigations and Methods Used," by S. F. Turner and L. C. Halpenny, *Transactions*, Am. Geophysical Union, 1941, Part III, Washington, D. C., August, 1941, p. 738. [j] "Consumptive Water Use and Requirements: Report of the Participating Agencies, Pecos River," by Harry F. Blaney, Paul A. Ewing, Karl V. Morin, and Wayne D. Criddle, Joint Investigation of the National Resources Planning Board, Washington, D. C., June, 1942. [k] "Use of Water by Native Vegetation," by Arthur A. Young and Harry F. Blaney, *Bulletin No. 50*, California State Div. of Water Resources, Sacramento, Calif., 1942.

or evapo-transpiration losses. Most of the methods of this type are artificial, and it is questionable whether the results can be extended to field conditions.

Various methods have been used by engineers to determine consumptive use of water by agricultural crops and natural vegetation under field conditions.[17] Regardless of the method, the problems encountered are numerous. The source of water used by plant life, whether from precipitation alone or irrigation or ground water plus precipitation, is a factor in selecting a method. The methods most widely used in engineering investigations are:

(*a*) soil-moisture studies on plots;
(*b*) tank or lysimeter experiments;
(*c*) ground-water fluctuations;
(*d*) inflow-outflow measurements;
(*e*) integration;

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[17] "Field Methods of Determining Consumptive Use of Water," by Harry F. Blaney, Div. of Irrig., U. S. Dept. of Agri., Los Angeles, Calif., 1938.

(*f*) effective heat;
(*g*) correlation of water use, climatological data, and other data; and
(*h*) analysis of irrigation and precipitation records.

Soil-Moisture Studies on Plots.—This method is usually employed to determine the consumptive use of irrigated field plots in which the soil is fairly uniform and the depth to ground water is such that it will not influence the soil-moisture fluctuations within the root zone. Soil samples are taken by means of a standard soil tube before and after each irrigation, with some samples between irrigations. The samples are in 6-in. sections for the first foot and thereafter in 1-ft sections in the major root zone. Usually a great number of soil samples must be taken.[6,18]

Standard laboratory practices are used in determining the moisture content of the soil samples. The samples are weighed and dried in an electric oven at 110° C and the dry weights determined. The water content of a sample is expressed as a percentage of the oven-dry weight of the soil. From the moisture percentage thus obtained, the quantity of water in acre-inches per acre (inches) removed by evaporation and transpiration from each foot of soil is computed by using the formula

$$D = \frac{P\,V\,d}{100} \qquad (3)$$

in which P represents the moisture percentage by weight; V, the apparent specific gravity (or volume weight) of the soil; d, the depth of soil in inches; and D, the equivalent depth of water in inches lost by the soil (acre-inches per acre).

The total number of acre-inches of water extracted from the soil by evapotranspiration is computed for each period and later reduced to equivalent losses in acre-inches or inches for a 30-day period. The 30-day period losses may be plotted and a use-of-water curve for the season obtained. The average use of water for each month is taken directly from the curve (see Fig. 1). Examples of monthly consumptive-use-of-water determination made by this method are shown in Table 4 for several crops.

Tank and Lysimeter Experiments.—One of the more common methods of determining use of water by individual agricultural crops and natural vegetation is to grow the plants in tanks or lysimeters and measure the quantity of water necessary to maintain the growth satisfactorily. Tanks as large as 10 ft in diameter and 10 ft deep have been used. However, in most consumptive-use studies, the tanks are about 2 to 3 ft in diameter and 6 ft deep.[6,19]

The practicability of determining consumptive use by means of tanks or lysimeters is dependent on the accuracy of reproduction of natural conditions. In addition to environment, artificial conditions are the result of the limitations of soil, size of tank, or regulation of water supply. Weighing is the precise means of determining the consumptive use from tanks and this method was

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[18] "Irrigation Water Requirement Studies of Citrus and Avocado Trees in San Diego County, California, 1926 and 1927," by S. H. Beckett, Harry F. Blaney, and C. A. Taylor, *Bulletin No. 489*, College of Agri., Agri. Experiment Station, Univ. of California, Berkeley, Calif., 1930.

[19] "Use of Water by Native Vegetation," by Arthur A. Young and Harry F. Blaney, *Bulletin No. 50*, California State Div. of Water Resources, Sacramento, Calif., 1942.

used as early as 1907. However, conditions and facilities will not always permit the weighing of tanks. It has been found that all tank vegetation must be protected from the elements by a surrounding growth of the same species. Soil tanks equipped with suitable supply tanks have been used successfully in evapo-transpiration measurements from water tables at various depths. This equipment was recommended by the National Resources Committee. The tanks best adapted for this use are the double-type tank with an annular space between the inner and outer shells.[6,19]

Charles Hamilton Lee, M. ASCE, in tests conducted in Owens Valley, California, 1911, employed tanks to measure the use of water by natural vegetation.[9] Since 1919 this method has been employed in California, Colorado, and New Mexico.[2,4,6]

TABLE 4.—MONTHLY CONSUMPTIVE USE OF WATER AS DETERMINED BY SOIL-MOISTURE STUDIES

Month	CONSUMPTIVE USE OF WATER (INCHES)							
	Oranges[a]	Peaches[b]	Alfalfa	Alfalfa	Cotton	Cotton	Beets	Potatoes
	(Los Angeles, Calif.)	(Ontario, Calif.)	(Los Angeles, Calif.)[a]	(Scottsbluff, Nebr.)	(Bakersfield, Calif.)	(Mesa, Ariz.)[c]	(Scottsbluff, Nebr.)	(Scottsbluff, Nebr.)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
April	2.2	1.0	3.3	....	0.8	1.1	....	....
May	2.2	3.5	6.7	4.0	1.1	2.2	1.9	....
June	3.1	6.7	5.4	7.0	3.3	3.6	3.3	....
July	3.4	8.0	7.8	7.1	5.6	6.0	5.2	3.4
August	3.7	6.5	4.2	6.4	7.7	8.8	6.9	5.8
September	3.1	2.7	5.6	....	6.4	7.2	5.8	4.4
October	2.9	1.4	4.4	....	3.6	2.9	1.1	....
Total	20.6	29.8	37.4	....	28.5	31.8	24.2	....

[a] "Progress Report on Cooperative Research Studies on Water Utilization, San Fernando Valley, California, Irrigation Season of 1940," by Harry F. Blaney and Homer J. Stockwell, Jr., Div. of Irrigation, Soil Conservation Service, U. S. Dept. of Agriculture, June, 1941 (unpublished). [b] "Rainfall Penetration and Consumptive Use of Water in Santa Ana River Valley and Coastal Plain," by Harry F. Blaney, Colin A. Taylor, and Arthur A. Young, *Bulletin No. 33*, Div. of Water Resources, California State Dept. of Public Works, Pomona, Calif., 1930. [c] "Irrigation Studies in Arizona," by Karl Harris, Div. of Irrigation and Water Conservation, Soil Conservation Service, U. S. Dept. of Agriculture (unpublished).

Ground-Water Fluctuations.—Evapo-transpiration losses are indicated by the daily rise and fall of the water table. Diurnal measurements of the ground-water table fluctuations provide a basis for computing consumptive use of water by overlying vegetation. This method has been used successfully by the Geological Survey (USGS), U. S. Department of the Interior, in Escalante Valley, Utah; Santa Ana Valley, California; Gila Valley, Arizona; and other areas.[20,21,22] The procedure used is to install observation wells, equipped with

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[20] "A Method of Estimating Ground-Water Supplies Based on Discharge by Plants and Evaporation from Soil," Results of Investigations in Escalante Valley, Utah, by W. N. White, *Water-Supply Paper 659-A*, Geological Survey, U. S. Dept. of the Interior, Washington, D. C., 1932.

[21] "Water Losses under Natural Conditions from Wet Areas in Southern California," Part I, by Harry F. Blaney, C. A. Taylor, M. G. Nickle, and A. A. Young; Part II, by Harold Troxell, *Bulletin No. 44*, Div. of Water Resources, California State Dept. of Public Works, Pomona, Calif., 1933.

[22] "Use of Water by Bottom-Land Vegetation in Lower Safford Valley, Arizona," by J. S. Gatewood T. W. Robinson, B. R. Colby, J. D. Hem, and L. C. Halpenny, *Water-Supply Paper 1103*, Geological Survey, U. S. Dept. of the Interior, Washington, D. C., 1950.

water-stage registers, at representative locations in the area under consideration and to obtain graphic records of the ground-water fluctuations. The specific yield of the soil is determined by standard methods. The diurnal cycle, as indicated by the ground-water table, represents a curve showing the relation of consumptive use to ground-water discharge or recharge.

Inflow-Outflow Measurements.—Consumptive use of water by cottonwoods, willows, tules, and other riparian vegetation common to small streams may be determined by measuring the inflow to and the outflow from a selected area by means of flow recorders located on bed-rock controls.[6,21] In investigations conducted in the Upper Rio Grande,[2] the Pecos River[4] and other river valleys,[23] engineers have used the inflow-outflow method to determine the consumptive use in irrigated valleys containing land areas up to 400,000 acres.

In the Upper Rio Grande Valley investigation the inflow-outflow method was considered to include the measurement of the amount of water flowing into and out of a given area, including precipitation and change in ground-water storage. Thus, valley consumptive use (U) is equal to the amount of water that flows into the valley during a 12-month year (I), plus the yearly precipitation on the valley floor (R), plus the water in ground storage at the beginning of the year (G_s), minus the amount of water in ground storage at the end of the year (G_e), minus the yearly outflow (O). All amounts are measured in acre-feet. This relation may be expressed by the equation

$$U = (I + R) + (G_s - G_e) - O \qquad (4)$$

The difference between the storage of capillary water at the beginning and end of the year is considered negligible. It is also assumed that stream measurements are made on bed-rock controls and there is little or no subsurface flow. Example of inflow-outflow measurement in several typical areas are given in Table 5.

Integration Method.—Briefly stated, the integration method determines consumptive use by the summation of the products of consumptive use for each crop, times its area, plus the consumptive use of natural vegetation times its area, plus water surface evaporation times water surface area, plus evaporation from bare land times its area. This method was used in the Upper Rio Grande Basin[2] and the Pecos River[4] investigation to determine valley consumptive use.

Before the integration method can be used successfully, it is necessary to know unit evapo-transpiration, or consumptive use of water, and the areas of various classes of agricultural crops, natural vegetation, bare land, and water surfaces. Unit values of the evapo-transpiration can be obtained by soil-moisture, tank, or some of the other methods previously described. By using aerial maps and field surveys, the areas of the various types or vegetative cover and the land and water surface areas can be determined, and the valley consumptive use computed.[2,4]

Effective Heat.—Many formulas have been developed for determining evaporation from meteorological observations, but formulas for estimating consumptive use are not so numerous. A few methods for determining consumptive use, based on climatic factors, have been suggested and have been found to give reasonable results. For many years irrigation engineers have used temperature data in estimating valley consumptive use of water in areas of the west.[5] C. R. Hedke developed the effective heat method on the basis of studies of the Rio Grande. By this method consumptive use is estimated from a study of the heat units available to the crops of a particular valley. It assumes that there is a linear relation between the amount of water consumed and the quantity of available heat. Studies of the Bureau of Reclamation (USBR), United States Department of the Interior,[23] in 1941 developed a somewhat similar method that has been employed by the USBR in making its estimates of consumptive use. This method assumes a linear relation between consumptive use and accumulated daily maximum temperatures above 32° F during the growing season.

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[23] "Consumptive Use of Water for Agriculture," by Robert L. Lowry, Jr., and Arthur F. Johnson, *Transactions*, ASCE, Vol. 107, 1942, p. 1243.

TABLE 5.—EXAMPLES OF VALLEY CONSUMPTIVE USE OF WATER DETERMINATIONS BY INFLOW-OUTFLOW METHOD

Location	Year	Area (acres)	ANNUAL CONSUMPTIVE USE		Authority
			Total (acre-feet)	Average (feet)	
San Luis Valley, Colo.	1925–1935	400,000	664,900	1.66	Blaney-Rohwer[a]
San Luis Valley, Colo.	1936	400,000	685,423	1.71	Blaney-Rohwer[a]
San Luis Valley, Colo.	1930–1932	17,300	26,215	1.52	Tipton-Hart[a]
Isleta-Belen, N. Mex.	1936	17,500	38,700	2.28	Blaney-Morin[a]
Mesilla Valley, N. Mex.	1919–1935	109,000	297,756	2.73	Blaney-Israelsen[a]
Mesilla Valley, N. Mex.	1936	110,418	303,683	2.75	Blaney-Israelsen[a]
Carlsbad, N. Mex.	1921–1939	51,700	129,752	2.51	Blaney-Morin-Criddle[b]
Carlsbad, N. Mex.	1940	51,700	119,898	2.33	Blaney-Morin-Criddle[b]
New Fork, Wyo.	1939–1940	25,000	....	1.59	Lowry-Johnson[c]
Michigan-Illinois-Colo.	1938–1940	43,000	....	1.50	Lowry-Johnson[c]
Uncompahgre, Colo.	1938–1940	137,700	....	2.28	Lowry-Johnson[c]

[a] "Water Utilization, Upper Rio Grande Basin," by Harry F. Blaney, Paul A. Ewing, O. W. Israelsen, Carl Rohwer, and F. C. Scobey, National Resources Committee, Washington, D. C., Part III, February, 1938. [b] "Consumptive Water Use and Requirements; Report of the Participating Agencies, Pecos River," by Harry F. Blaney, Paul A. Ewing, Karl V. Morin, and Wayne D. Criddle, Joint Investigation of the National Resources Planning Board, Washington, D. C., June, 1942. [c] "Consumptive Use of Water for Agriculture," by Robert L. Lowry, Jr., and Arthur F. Johnson, *Transactions*, ASCE, Vol. 107, 1942, p. 1243.

In 1940, in connection with the Pecos River Joint Investigation,[4] a method of estimating consumptive use by natural ground-water vegetation and by irrigated crops having access to an ample water supply was developed. Correlation of the relationship between evapo-transpiration, evaporation, temperature, percent of daytime hours, length of growing season, and humidity was used in the estimate.[24] In 1945 this method was modified to make possible estimates of consumptive use for areas in which humidity records are not available.[25] This method has been used by the SCS and other agencies in estimating rates of water consumption in many areas.

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[24] "Evaporation and Consumptive Use of Water Empirical Formulas," by Harry F. Blaney and Karl V. Morin, *Transactions*, Part I, Am. Geophysical Union, August, 1942, p. 76.

[25] "A Method of Estimating Water Requirements in Irrigated Areas from Climatological Data," by Harry F. Blaney and Wayne D. Criddle, Div. of Irrig. and Water Conservation, SCS, U. S. Dept. of Agri., Logan, Utah, 1945.

CORRELATION OF WATER-USE, CLIMATOLOGICAL DATA AND OTHER DATA

Actual measurements of consumptive use under each of the physical and climatical conditions of any large area are expensive and time consuming. Therefore, some rapid method of transferring the results of careful measurements, made in several areas, to other areas of similar conditions is needed. The Division of Irrigation and Water Conservation, SCS, has developed such a method to estimate rates of water consumption.[26,27] Briefly, the procedure is to correlate existing consumptive-use data with monthly temperature, percentage of daytime hours, precipitation, frost-free (growing) period, or irrigation season. The coefficients so developed for different crops are used to translocate or transpose consumptive-use data from one section to other areas in which climatological data alone are available.

TABLE 6.—TENTATIVE NORMAL CONSUMPTIVE-USE COEFFICIENTS FOR THE MORE IMPORTANT IRRIGATED CROPS AND NATURAL VEGETATION OF THE WEST

Item	Length of growing season or period	Consumptive-use coefficient (K)
(a) IRRIGATED LAND		
Alfalfa	frost-free	0.85
Beans	3 months	.65
Corn	4 months	.75
Cotton	7 months	.62
Citrus orchard	7 months	.55
Deciduous orchard	frost-free	.65
Pasture, grass, hay, annuals	frost-free	.75
Potatoes	3 months	.70
Rice	3 to 4 months	1.00
Small grains	3 months	.75
Sorghum	5 months	.70
Sugar beets	5½ months	.70
(b) NATURAL VEGETATION[a]		
Very dense (large cottonwoods, willows)	frost-free	1.30
Dense (tamarisk, willows)	frost-free	1.20
Medium (small willows, tamarisk)	frost-free	1.00[b]
Light (saltgrass, sacaton)	frost-free	.80

[a] Ample moisture available from ground water or water table. [b] $K = 1.00$ for lake evaporation in arid areas and 0.90 for lake evaporation in coastal areas.

Although it is recognized that consumptive use of water is affected by numerous independent and related variables, records of temperatures and precipitation are by far the most universally available data throughout the western states. Research studies also indicate that, of all the climatic factors, these elements, together with daylight, undoubtedly have the greatest influence on plant growth. Disregarding the unmeasured factors, consumptive use varies with the temperature and the extent of daytime hours, and with the available moisture (precipitation, irrigation, and ground water). By multiply-

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[26] "Irrigation Practice and Consumptive Use of Water in Salinas Valley, California," by Harry F. Blaney and Paul A. Ewing, Div. of Irrig. and Water Conservation, SCS, U. S. Dept. of Agri., Los Angeles, Calif., June, 1946.

[27] "Determining Water Requirements in Irrigated Areas from Climatological and Irrigation Data," by Harry F. Blaney and Wayne D. Criddle, Div. of Irrig. and Water Conservation, SCS, U. S. Dept. of Agri., SCS-TP-96, Washington, D. C., 1950.

ing the mean monthly temperature (t) by the monthly percentage of daytime hours of the year (p), a monthly consumptive-use factor (f) is obtained. Then it is assumed that the consumptive use varies directly as this factor when an ample water supply is available. Expressed mathematically

$$U = K F = \Sigma k f \tag{5}$$

in which U is the consumptive use of crop (or evaporation) in inches for any period; F is the sum of the monthly consumptive-use factors for the period (sum of the products of mean monthly temperature and monthly percentage of daytime hours of the year); K is the empirical consumptive-use coefficient (irrigation season or growing period); and the other terms are as previously defined.

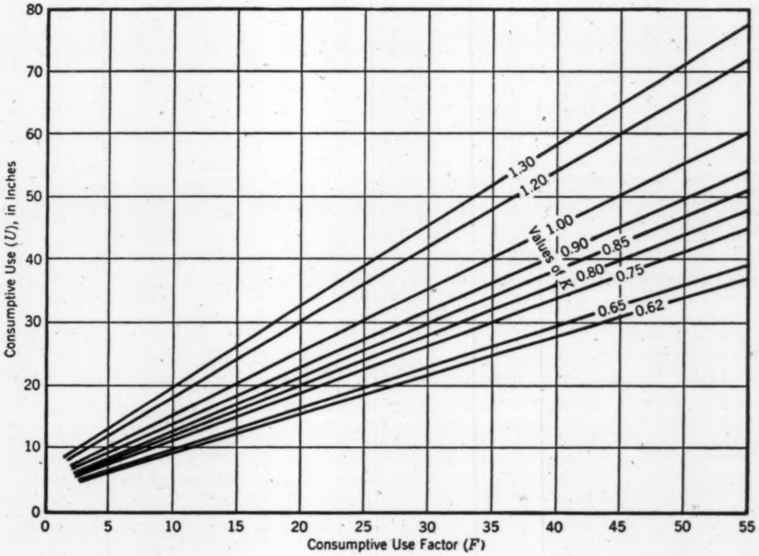

FIG. 3.—RELATION OF CONSUMPTIVE USE TO TEMPERATURE AND PERCENTAGE OF DAYLIGHT HOURS

The consumptive-use factor (F) for any period may be computed for areas in which monthly temperature records are available. Then, by knowing the consumptive-use coefficient (k) for a particular crop in some locality, an estimate of the water use by the same crop in some other area may be made by application of Eq. 5.

The consumptive-use coefficients (K) for the more important irrigated crops (grown under normal conditions in the West) and natural vegetation are shown in Table 6. These coefficients were developed from actual measurements of consumptive use in tank and soil-moisture field studies; and inflow-outflow measurements made throughout the West over a period of years by the Division of Irrigation and Water Conservation and other agencies. These coefficients are based on the assumption that the plants or trees receive a full water supply throughout the growing season or frost-free period. Fig. 3 shows the relation of consumptive use (U); consumptive-use factor (F); and coefficient (K). Monthly percentage of annual daytime hours computed from sunshine tables[28] are shown in Table 7.

TABLE 7.—MONTHLY PERCENTAGE OF DAYTIME HOURS OF THE YEAR FOR LATITUDES 24° TO 50° NORTH OF THE EQUATOR

Month	LATITUDES IN DEGREES NORTH OF EQUATOR													
	24	26	28	30	32	34	36	38	40	42	44	46	48	50
January	7.58	7.49	7.40	7.30	7.20	7.10	6.99	6.87	6.76	6.62	6.49	6.33	6.17	5.98
February	7.17	7.12	7.07	7.03	6.97	6.91	6.86	6.79	6.73	6.65	6.58	6.50	6.42	6.32
March	8.40	8.40	8.39	8.38	8.37	8.36	8.35	8.34	8.33	8.31	8.30	8.29	8.27	8.25
April	8.60	8.64	8.68	8.72	8.75	8.80	8.85	8.90	8.95	9.00	9.05	9.12	9.18	9.25
May	9.30	9.38	9.46	9.53	9.63	9.72	9.81	9.92	10.02	10.14	10.26	10.39	10.53	10.69
June	9.20	9.30	9.38	9.49	9.60	9.70	9.83	9.95	10.08	10.21	10.38	10.54	10.71	10.93
July	9.41	9.49	9.58	9.67	9.77	9.88	9.99	10.10	10.22	10.35	10.49	10.64	10.80	10.99
August	9.05	9.10	9.16	9.22	9.28	9.33	9.40	9.47	9.54	9.62	9.70	9.79	9.89	10.00
September	8.31	8.31	8.32	8.34	8.34	8.36	8.36	8.38	8.38	8.40	8.41	8.42	8.44	8.44
October	8.09	8.06	8.02	7.99	7.93	7.90	7.85	7.80	7.75	7.70	7.63	7.58	7.51	7.43
November	7.43	7.36	7.27	7.19	7.11	7.02	6.92	6.82	6.72	6.62	6.49	6.36	6.22	6.07
December	7.46	7.35	7.27	7.14	7.05	6.92	6.79	6.66	6.52	6.38	6.22	6.04	5.86	5.65
Total	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

Examples of consumptive-use rates in the Upper Colorado River Basin,[3] computed by the above method, are shown in Table 8.

The consumptive-use coefficients shown in Table 6 are suitable for average conditions in the semiarid areas of the West. Studies to develop correction factors for humid conditions in the eastern states have been undertaken. It is anticipated that evaporation, humidity, or solar radiation records can be

TABLE 8.—ESTIMATES OF NORMAL UNIT CONSUMPTIVE-USE RATES DURING THE IRRIGATION PERIOD FOR IRRIGATED CROPS FOR SEVERAL LOCALITIES IN THE UPPER COLORADO RIVER BASIN

Location	NORMAL RATE OF CONSUMPTIVE USE, IN INCHES[a]			
	Alfalfa	Pasture	Grains	Corn
Chinle, Arizona	28.3	24.2	15.4	20.1
Eagle, Colorado	18.5	16.2	11.6	11.6
Cortez, Colorado	24.5	21.5	14.6	18.9
Durango, Colorado	22.9	20.1	14.0	17.1
Farmington, New Mexico	32.1	28.4	15.3	20.4
Green River, Utah	32.5	28.6	16.2	21.7
Price, Utah	26.7	23.5	15.4	19.9
Eden, Wyoming	17.2	15.1	13.4	....
Pinedale, Wyoming	14.8	13.0	....	....

[a] Values used in the formula $U = KF$ are: Alfalfa, 0.85; pasture, 0.75; grain, 0.75; and corn, 0.75.

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[28] "Sunshine Tables," *Bulletin No. 805*, Weather Bureau, U. S. Dept. of Agri., Washington, D. C., 1905.

used for this purpose until measured consumptive-use data are available. Basic solar radiation data are now available at many stations throughout the United States.

ANALYSIS OF IRRIGATION AND PRECIPITATION RECORDS

In the consumptive-use studies conducted on the Pecos River in 1940, a method of determining unit consumptive use for various agricultural crops from irrigation and precipitation was developed.[4] This method has been successfully employed in several areas in California.[26] It consists of estimating evaporation and transpiration losses during the irrigation season and winter period based on data of the number of irrigations and depth of water applied, farm duty of water, and efficiency of irrigation water application for various crops and soils; and precipitation and evaporation from free water surface. Precipitation may be a major factor in winter consumptive use.

A field survey of conditions through the Pecos Valley area was made. Water superintendents of irrigation district and water companies, engineers, county farm advisors, and farmers were interviewed to ascertain the quantity of irrigation water supplied to various crops, both for the growing season and the winter period. Precipitation, irrigation, and other records were collected and estimates made of the efficiency of irrigation for various crops in the different areas. The latter data were supplemented by water-application efficiency studies, made on typical farms by measuring the quantity of water applied and taking soil samples before and after irrigation. By this means it was possible to determine the proportion of soil moisture available for crop transpiration use within the root zone. These data were analyzed and the unit rate of water consumption for each crop computed. The total consumptive use in acre-feet may be computed by multiplying unit values of consumptive use for each crop by its area.

ACKNOWLEDGMENT

This paper serves as an introduction to several others on this same general subject[29,30,31,32] and should provide the background necessary for complete understanding of the specialized phases of the field.

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[29] "Consumptive Use in the Rio Grande Basin," By Robert L. Lowry, *Transactions*, ASCE, Vol. 117, 1952, p. 1014.

[30] "Consumptive Use of Water by Forest and Range Vegetation," by L. R. Rich, *ibid.*, p. 974.

[31] "Consumptive Use of Water on Irrigated Land," by Wayne D. Criddle, *ibid.*, p. 991.

[32] "Consumptive Use in Municipal and Industrial Areas," by George B. Gleason, *ibid.*, p. 1004.

DISCUSSION

George H. Hargreaves,[33] A. M. ASCE.—A long needed stimulus for additional research in irrigation requirements has been provided by Mr. Blaney. Not only is this a matter of importance in the United States but also in the irrigated areas of the world. Engineers everywhere need a better understanding of this problem in order that the benefits of irrigation may be extended by the prevention of waste. In Latin America a sound technical basis must be provided for substituting a beneficial use criterion for the rule "one liter per second per hectare." An example of the need for irrigation requirement data can be had from the irrigated areas near Lima and Piura in Peru. Both areas depend almost entirely on irrigation, the mean annual rainfall being 1.4 in. and 3.5 in., respectively. During a 12-month growing season evaporation at Piura averages 4.4 times that at Lima.

Irrigation in the tropics is becoming increasingly important. Monthly irrigation requirement data based on monthly consumption use factors, as suggested by Mr. Blaney, or on the transfer of monthly consumptive-use data making use of evaporation data, are needed to determine the needs for irrigation and probable crop benefits. At Aux Cayes, Haiti, with a mean annual rainfall of 83 in., the need for irrigation is not generally recognized. However, this area has a definite 4-month to 5-month dry season and, based on estimated consumptive-use requirements, moisture deficiencies occur one third of the time or oftener except during May, August, and October in the rainy season.

Use of Evaporation Data.—Very briefly Mr. Blaney mentions the use of evaporation in estimating evapo-transpiration. More space in the discussion should be given this approach and its use should be more widely encouraged. Assuming comparable conditions of stage of growth, leaf surface area, and irrigated practice, the water requirement of a crop is closely proportional to evaporation from a water surface during the growing season. This relationship is not constant month by month throughout the growing season as transpiration increases proportionally with increasing leaf surface area. However, if irrigation requirements at comparable stages of growth are related to evaporation, this fact forms the most practical means of transferring irrigation requirements and consumptive-use data from one area to another.

Use of Emperical Formulas.—By using Eq. 1 and substituting evaporation, e, in place of consumptive use, u, values of k_m were computed based on temperature, humidity, and Piche evaporometer records in Peru. Average values of $k_m = e/c$ were computed for the growing season at several stations. At Lima the computed annual K_a for the Piche evaporometer is 0.015 whereas at Piura, the computed K_a is 0.036. In each case values of k_m were fairly constant throughout the 12-month growing season. From this it would appear that use of Eq. 1 is limited both by the lack of available relative humidity data and the difficulties involved in choosing suitable k_m-values. Evaporation records from the Weather Bureau type pans are available for many stations, particularly in California.[34] These can be used for evaluating the formula $u = kf$ by substituting evaporation, e, in place of the consumptive use. Then $k = e/f$. Values of k were computed for the growing season at forty-six evaporation stations. It was found that these values fell into natural classifications—coastal plain, uplands and mountains, Central Valley, and desert. These values of k are given in Table 9. For the climatic zones in California k-values increase with temperature averaging 38% to 43% higher in July than for the first month of the growing season. If evaporation is interchangeable with consumptive use in Eq. 5 then evaporation must vary directly as the monthly consumptive use of factor, f. The data presented in Table 9 show that this assumption, although useful as an index of irrigation requirements, is subject to considerable error.

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[33] Civ. Engr., Inst. of Inter-American Affairs, Port-au-Prince, Haiti.

TABLE 9.—VALUES OF k FROM THE FORMULA $k = e/f$ OR RATIO OF EVAPORATION OF CONSUMPTIVE-USE FACTOR

Location or area	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
California:												
Entire state	...	...	1.03	1.15	1.28	1.36	1.43	1.40	1.30	1.08	0.92	...
Coastal Plain	...	...	0.86	0.99	1.10	1.15	1.19	1.12	1.06	0.89	0.66	...
Uplands and mountains	...	...	0.99	1.04	1.17	1.26	1.37	1.36	1.27	1.08	0.95	...
Desert	...	1.41	1.65	1.86	1.90	1.94	1.97	1.87	1.74	1.53	1.48	...
Central Valley	...	...	...	1.09	1.33	1.51	1.56	1.52	1.35	0.97	...	...
Davis	...	...	...	1.08	1.38	1.37	1.43	1.42	1.34	1.03	...	...
Haiti:												
Port-au-Prince	0.94	0.93	0.97	1.04	0.95	1.01	1.02	0.93	1.00	0.98	0.81	0.76
Peru[a]:												
Piura	1.51	1.63	1.65	1.90	1.34	1.71	1.52	1.58	1.55	1.28	1.42	1.61
Paramonga	0.81	0.82	0.80	0.75	0.53	0.59	0.50	0.46	0.50	0.51	0.75	0.64
Lima	0.56	0.56	0.52	0.46	0.36	0.27	0.29	0.27	0.28	0.35	0.41	0.46
Tacna	0.80	0.82	0.79	0.60	0.54	0.43	0.47	0.52	0.52	0.60	0.65	0.77
Cusco	1.54	1.47	1.65	...	...	...	...	...	...	1.79	1.78	1.60

[a] Coefficients for Peru are based on Piche evaporometer measurements, which are believed to give about 80% of Weather Bureau type pan evaporation. Evaporation data for California and Haiti are from the Weather Bureau type pan.

Conclusions.—Based on the Piche evaporometer records, the monthly use index (c being the climatic factor) was found to correlate well with evaporation at a given location. This relationship varies with the location of the station probably because of other unmeasured climatic factors, indicating that the application of Eq. 1 to irrigation problems will be difficult. A study of available evaporation records from the Weather Bureau type pans indicates that the monthly consumptive-use factor, f, does not (as assumed by Mr. Blaney) vary directly with evaporation and consumptive use. However, from the California data, average month-by-month coefficients based on the ratio of evaporation to consumptive-use factors can be computed for a given climatic zone such as the coastal plain or the desert. These coefficients should provide a useful method for transferring irrigation requirement and consumptive-use data when evaporation data are not available. Month-by-month coefficients will provide data for canal capacity design and the analysis of irrigation requirements and benefits in tropical areas where, under conditions of high annual rainfall, the month-by-month distribution of available moisture is one of the principal factors limiting agriculture production.

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[34] "Evaporation from Water Surfaces in California, Basic Data," by Arthur A. Young, *Bulletin No. 54 A*, California State Div. of Water Resources, Sacramento, Calif., 1948.

William W. Donnan,[35] A. M. ASCE.—Throughout the United States engineers are faced with the paradox of expanding water use and dwindling supplies. The conservation, preservation, and beneficial use of these water supplies is a challenging problem that can be solved only by careful analysis. Of particular interest to the writer is the section dealing with the evaluation of consumptive use by "Correlation of Water Use, Climatological Data and Other Data." During the course of an investigation (reported in 1952[36]), the Division of Irrigation Engineering and Water Conservation was called upon to determine the annual consumptive use of water in the Los Angeles West Coast Basin for a 20-year previous period. These data were needed so that an adjudication could be made of the present available supply and long-term mean supply. By securing published Weather Bureau data on temperatures, precipitation, and evaporation in the West Coast Basin of Los Angeles County, it was possible to make valid estimates of water consumption using the Blaney-Criddle method.[27] In making these estimates, the long-term mean unit consumptive use was computed for each land-use classification. These unit values were then modified by the temperature and precipitation occurring in each individual year. The resultant consumptive-use values were applied to land-use acreages for the individual years of the study. When applied to the inflow-outflow equations of the hydrologic balance for the basin, these data made it possible to determine the degree of overdraft on the underground water supplies.

The question may arise in the minds of some readers as to whether a determination of unit consumptive use by the correlation method gives valid estimates. In an investigation[37] of irrigation and drainage problems in the San Fernando Valley in cooperation with the city engineer of Los Angeles, the writer and others had a chance to check on the validity of these estimates.

San Fernando Valley comprises about two fifths the area of the City of Los Angeles. Since roughly one fourth of its 130,000 acres is still used for agriculture, it presents a unique combination of urban and rural land use. The valley is an enclosed basin with a 400-sq-mile watershed draining to the valley from surrounding foothills and mountains and with a confined outlet at the Dayton Avenue narrows. In making an analysis of input-outflow in this basin over a 20-year period, measured data on each factor were available. The precipitation on the basin is measured at about forty different locations.

All importations to the basin are carefully metered. The exported water is sold through domestic meters. Sewage out of the basin is metered. The surface outflow at the Dayton Avenue gaging station is measured accurately. A small amount of subsurface outflow was accounted for. Because there are about one hundred and fifty observation wells in the valley floor, an accurate ground-water contour map could be drawn for each year and yearly changes in underground storage determined. The largest item in the hydrologic equation, comprising approximately 65% of the aggregate outflow, was the consumptive use. This was computed for each individual land-use classification and modified by yearly temperatures and precipitation. The hydrologic balance, equating input (consisting of precipitation and imports) to outflow (consisting of surface and subsurface runoff, exports, sewage, consumptive use, and plus or minus change in underground storage) came within 1.6% of checking each other.

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[35] Drainage Engr., SCS Research, U. S. Dept. of Agri., Los Angeles, Calif.

[36] "West Coast Basin Reference, Draft of Report by Referee," by Dept. of Public Works, State of California, Referee, acting through the State Engr., Sacramento, Calif., February, 1952.

[37] "Ground Water and Drainage Investigations in San Fernando Valley, Los Angeles, California," by William W. Donnan, G. Marvin Litz, and V. S. Aronovici, Div. of Irrig. Eng. and Water Conservation, U. S. Dept. of Agri., Los Angeles, Calif., 1950 (mimeographed publication).

HARRY F. BLANEY,[38] M. ASCE.—The discussion by Mr. Hargreaves calls attention to the world-wide expansion of irrigated areas resulting from the foreign program of the United States. The writer has received requests for more details on procedure in computing consumptive use from Austria, Brazil, Egypt, Greece, India, and Mexico. Eq. 5 is being widely used. For humid areas the seasonal coefficient K shown in Table 6 should be reduced by about 10%. Table 10 illustrates the computations necessary to determine monthly consumptive use.

TABLE 10.—COMPUTED NORMAL CONSUMPTIVE USE OF WATER BY GRASS PASTURE, VICTORIA, TAMAULIPAS, MEXICO

TABLE 11.—MONTHLY CONSUMPTIVE-USE COEFFICIENTS k FOR CROPS AND NATURAL VEGETATION[a]

Month (Tables 10 and 11)	Mean monthly temperature (t)	Day-time hours (p%)	CONSUMPTIVE USE			SALINAS	CORONA	DAVIS[c]		OCEANSIDE[d]	
			Factor[a] (f)	Coefficient[b] (k)	Product[c] u, in inches	Alfalfa[b]	Citrus[c]	Prunes	Tomatoes	Cottonwood trees	Tules
(1)	(2)	(3)	(4)	(5)	(6)	(2)	(3)	(4)	(5)	(6)	(7)
April	77.0	8.60	6.62	0.60	4.0	0.60	0.46	0.42	...	1.11	0.89
May	80.2	9.29	7.45	0.65	4.8	0.70	0.52	0.55	...	1.24	1.14
June	82.0	9.18	7.53	0.70	5.3	0.80	0.52	0.94	0.44	1.27	1.16
July	83.3	9.39	7.82	0.75	5.9	0.85	0.57	0.90	0.55	1.49	1.26
August	83.8	9.04	7.58	0.75	5.7	0.85	0.61	0.65	0.94	1.48	1.16
September	79.9	8,31	6.64	0.65	4.3	0.85	0.61	0.48	0.94	1.45	1.16
October	74.8	8.10	6.06	0.60	3.6	0.70	0.60	0.43	0.77	...	1.01

Table 10: [a] $f = \frac{t \times p}{100}$ = monthly consumptive-use factors. [b] k = monthly coefficient. [c] $u = k f$ = monthly consumptive use.

Table 11: [a] Developed from data observed in the State of California. [b] Harry F. Blaney and Orson W. Israelsen. [c] Harry F. Blaney and G. Marvin Litz. [d] Harry F. Blaney and Paul A. Ewing.

In this paper the writer did not assume (as indicated by Mr. Hargreaves) that the monthly consumptive-use factor f in Eq. 5 varies directly with evaporation and consumptive use. Coefficient K in Table 6 are seasonal values. When monthly values of k are available, the writer has recommended in other publications[26, 39] that they be used. Also, Fig. 2 and Table 4 indicate that the monthly value of k may vary. Monthly coefficients developed by the writer, based on measured consumptive u'-values and temperatures t (as reported by Orson W. Israelsen,[39] by Harry F. Blaney and G. Marvin Litz,[40] and by Harry

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[38] Prin. Irrig. Engr., Div. of Irrig. Eng. and Water Conservation, SCS, Research, U. S. Dept. of Agri., Los Angeles, Calif.

[39] "Irrigation Principles and Practices," by Orson W. Israelsen, John Wiley & Sons, Inc., New York, N. Y., 1950, p. 311.

[40] "Consumptive Use of Water in California," by Harry F. Blaney and G. Marvin Litz, Div. of Irrig. Eng., SCS, U. S. Dept. of Agri., Los Angeles, Calif., 1952 (unpublished).

F. Blaney and Paul A. Ewing[26]), are shown in Table 11. Monthly evaporation coefficients *k*, similar to those given for Weather Bureau pans by Mr. Hargreaves in Table 9, have been presented by the writer in reports on Salinas Valley,[26] California, and other areas.

The values of *k* for evaporation from pans presented by Mr. Hargreaves in Table 9 are useful in estimating monthly consumptive use, particularly phreatophytes such as alfalfa, willows, tules, and salt cedar. However, the writer has observed that monthly evaporation may not indicate the relative monthly evapo-transpiration for some agricultural crops such as citrus. The coefficients shown in Table 9 for evaporation from Weather Bureau pans are useful in estimating evaporation from reservoirs when reduced to evaporation from large water surfaces by a correction factor varying from 0.65 to 0.80. However, some physicists have questioned their value as well as the reliability of Piche evaporomenter measurements for estimating evaporation from reservoir or lake surfaces.

Mr. Donnan's reference to the Los Angeles West Coast Basin study illustrates the value of consumptive-use data in the adjudication of water rights.

TABLE 12.—EXAMPLE OF COMPUTATIONS OF SEASONAL CONSUMPTIVE-USE AND IRRIGATION REQUIREMENTS FOR CROPS IN THE MONTROSE AREA, COLORADO

Explanation of Column Headings—

Col. 3. F = Sum of monthly consumptive-use factors (f) for the growing or irrigation season.
Col. 4. K = Empirical consumptive-use coefficient determined experimentally.
Col. 5. $U = K F$ = Consumptive use for growing or irrigation season in inches.
Col. 6. R = Sum of monthly precipitation for growing or irrigation season, in inches.
Col. 7. E = Field irrigation efficiency (percentage).
Col. 8. $I = \frac{U - R}{E}$ = Irrigation requirement at head of the field, in inches.

Culture (1)	Growing season (2)	CONSUMPTIVE USE			U Minus R (6)	E (%) (7)	I (in.) (8)
		Factor F (3)	Coef. K (4)	Product U (5)			
Alfalfa	May 6 to October 6	31.12	0.85	26.45	21.73	70	31.0
Grass hay	May 6 to October 6	31.12	0.75	23.34	18.62	60	31.0
Corn	May 6 to September 6	26.21	0.75	19.66	16.02	65	24.6
Small grain	May 6 to August 6	19.83	0.75	14.87	12.55	65	19.3
Orchards	May 6 to October 6	31.12	0.65	20.23	15.51	70	22.2
Seeped land	May 6 to October 6	31.12	0.80	24.90	20.18	...	...
Dense natural vegetation	May 6 to October 6	31.12	1.20	37.34	32.62	...	...

Another example is the estimate of rates of water consumption made by the Division of Irrigation and Water Conservation in 1947-1948 in each of the states of Arizona, Colorado, New Mexico, Utah, and Wyoming at the request of the Upper Colorado River Basin Compact Commission.[3] Some of the results are shown in Table 8. A report was completed in 1948 and several months later a compact allocating a percentage of water to each state, based primarily on consumptive use, was agreed upon by the states and approved by Congress early in 1949.

Summary.—One of the major uses of basic consumptive-use data shown in Tables 2, 4, and 8 is in estimating the water requirement of existing or proposed irrigation projects and in determining the quantity of water required for crop production on individual farms. The water required for irrigation is dependent not only on consumptive use but also on the irrigation efficiency and usable summer precipitation. In some farm-planning programs it is necessary to estimate irrigation requirements of each crop at the point of water delivery to the field. This may be accomplished by dividing consumptive use minus precipitation by field irrigation efficiency, as shown in Table 12. By making an allowance for farm-lateral loss from the farm headgate to the field, the amount of water that should be delivered to the farm headgate may be estimated.

The writer acknowledges the assistance rendered by George Dewey Clyde, M. ASCE, chief of the Division of Irrigation Engineering and Water Conservation, SCS, USDA, in the preparation of this paper. His helpful suggestions and advice are appreciated.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

FOREST AND RANGE VEGETATION

By L. R. Rich[1]

With Discussion by Messrs. M. J. Youhotsky, and L. R. Rich

Synopsis

Consumptive use of water by forest and range vegetation is important to the irrigation economy of the West, because the water available for irrigation is the residual precipitation or that left over after watershed requirements have been met. Consumptive use of water by forest and range vegetation depends on the distribution and occurrence of precipitation, the amount of water held by the soil, and the character and type of the vegetation.

Under natural rainfall conditions, consumptive use of water can be determined by dividing the year into four periods: (1) Soil moisture recharge; (2) water surplus; (3) soil moisture utilization; and (4) water deficit. Each period must be considered separately. Measured field data are presented to show the workability of this method. Data are also presented that show the consumptive use of vegetation grown under natural rainfall conditions in open-top metal containers at El. 2600 and El. 5100. Consumptive-use data obtained from watersheds varying from desert grassland-chaparral vegetation at El. 3800 (16.22 in. precipitation) to pine-fir forest vegetation from El. 6500 to El. 8000 (34.19 in. precipitation) have followed a similar pattern.

Data from measured areas have shown small differences in consumptive use between those kept bare of vegetation and those in various types of vegetation, and no significant differences in consumptive use between a watershed in good and poor grass cover. The major problem is not whether watersheds kept bare of vegetation would yield most water, but to determine the type of vegetation that interferes least with water yield and still controls erosion and sediment. In the Southwest, grasses appear to interfere least with water yields, while half-shrubs, winter annuals, and evergreen shrubs (all characteristic of deteriorated vegetation) tend to reduce water yields.

Note.—Published in October, 1951, as *Proceedings-Separate No. 90.* Positions and titles given are those in effect when the paper or discussion was received for publication.

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INTRODUCTION

Consumptive use of water by forest and range vegetation is important to the irrigation economy of the West. The water available as stream flow for irrigated and riparian areas is the residual precipitation, or that remaining after watershed requirements have been met. As existing streams approach full development and increasing populations demand more and more water, the major hope for maintaining and increasing water supplies lies in better management and use of watersheds.

CONDITIONS THAT INFLUENCE CONSUMPTIVE USE

In contrast to irrigated areas and other areas that receive unlimited or supplemental water, the only water available to forest and range vegetation on the watersheds is the natural precipitation. Consumptive use is therefore limited by the amount and distribution of precipitation and the moisture held in the soil after periods of abundant precipitation. An unlimited water supply is seldom available to forest and range vegetation during the entire year. The exceptions to this condition may be riparian vegetation along perennial streams and wet meadows, or other limited areas at higher elevations, such as some areas in eastern New Mexico, where precipitation is high, where evapo-transpiration losses are low, and where rainfall distribution approaches potential evapo-transpiration. The wide variations in the consumptive-use data at hand show the need for consideration of climatic and watershed conditions as well as type of vegetation.

Distribution of Precipitation.—Climatic variations influence consumptive use of watershed vegetation. Winter precipitation decreases rather uniformly from almost 100% of the yearly total on the Pacific Coast to less than 20% in eastern New Mexico. In the northern part of the United States winter precipitation occurs as snow, whereas in the Southwest it is a combination of rain and snow, depending on winter temperatures that may vary widely from year to year.

At a particular location there are variations in both the amount and the distribution of annual precipitation that affect consumptive use and require data from long periods of measurement to determine averages. In the Southwest particularly, hydrologic data and the complex relationship between climate and the physical environment unfold slowly because of the low precipitation, infrequent periods of heavy precipitation, long and frequent droughts, and wide variations in these factors from year to year. There is danger in using averages in any study relating to water use, water yields, and climate. For example, during the 15-year period of record at the large undisturbed-soil lysimeters on the Sierra Ancha Experimental Forest, Tonto National Forest, in central Arizona, the highest consumptive use and more than half of the total water yield were recorded in one winter (1940–1941). On the other hand, the highest summer consumptive use and more than half of the total summer water yield resulted from one storm (September 17 to 19, 1946).

Amount of Precipitation.—Consumptive use does not vary directly with annual precipitation. An area seldom uses all the rainfall that falls on it, and there is some stream flow even in the driest years. Increasingly higher consumptive use and stream flow take place during wet years. This is exemplified by rainfall, runoff, and consumptive-use data from three watersheds at Sierra Ancha Experimental Forest, plotted in Fig. 1. Consumptive use accounts for most of the first 10 in. to 15 in. of precipitation, leaving only a relatively small amount available for stream flow. With higher precipitation, there is an increasingly lower percentage of consumptive use and a resulting higher water yield.

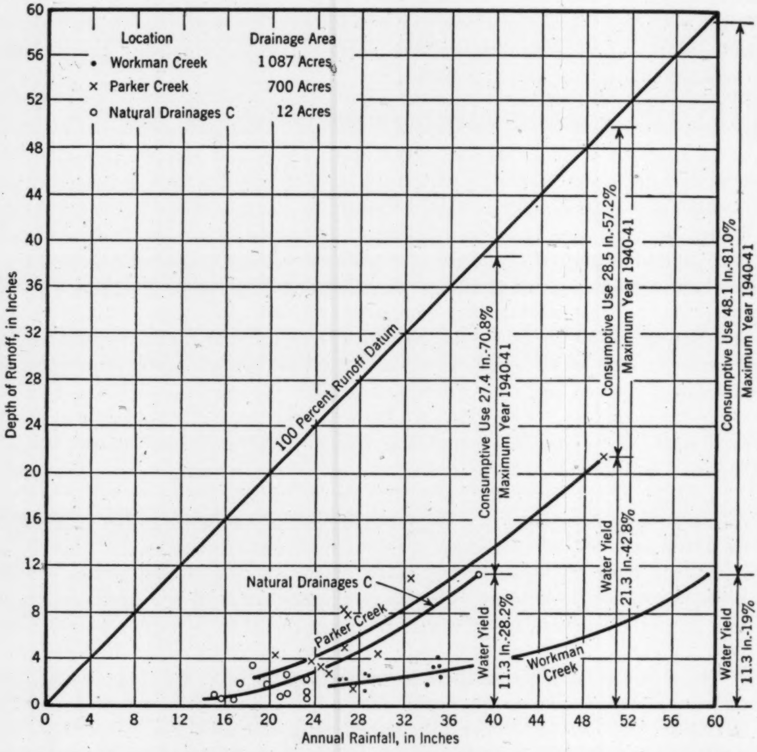

FIG. 1.—CONSUMPTIVE USE AND WATER YIELDS FOR SIERRA ANCHA, ARIZONA, WATERSHEDS

Potential consumptive use (the amount of water an area would use if an adequate supply were available during the entire year) was not satisfied even in 1940–1941 (the wettest year since 1905) when precipitation was over 200% of average. During the period of record there has not been a year of sufficiently high annual rainfall to locate the point where the yield line parallels the 100% datum line in Fig. 1. This is the point at which the consumptive use becomes constant and the difference is yielded as stream flow. Consumptive use has been 27 in. on "Natural Drainages" (El. 4555 to El. 4900), 28 in. on Parker Creek (El. 5500 to El. 7300), and 48 in. on Workman Creek (El. 6500 to El.

8000), all in central Arizona. All these totals are higher than average annual rainfall, thus pointing out the limitations of averages.

Variations in Watershed Conditions.—Watershed conditions that influence consumptive use are also subject to wide variation. The term watershed condition as used in this paper refers to topography, depth of soil, vegetation, and slope of the area. Some watersheds are flat, with deep soils that retain water from periods of excess precipitation for use during periods of deficient precipitation. Other watersheds are steep, the soils are shallow, there are large areas of exposed bedrock, the drain-out is rapid, and comparatively little moisture is held in the soil. Different types and densities of vegetation grow on watersheds, and the use of moisture varies with the species of plants and their relative abundance. For example, winter annuals grow and use water during the winter, evergreen shrubs use some water throughout the year, but water use by grasses is largely limited to the summer growing season. With different densities of vegetation there are different ratios of transpiration from plants and evaporation from bare soil.

POTENTIAL CONSUMPTIVE USE

Many studies have been made of water use by cultivated crops and associated native vegetation[2,3,4] and by trees.[5,6] The results all confirm the great amount of water required to produce one unit of dry vegetation matter.

Actual consumptive use is less than potential consumptive use over most forest and range watersheds in the Southwest because of deficient rainfall and rainfall distribution that differs from use requirements of the plants. This difference can be shown by a comparison of annual rainfall with transpiration losses from plants with an unlimited water supply. To determine transpiration losses at Sierra Ancha (El. 5100) with an unlimited water supply, three evergreen chaparral species, point-leaf manzanita (*Arctostaphylos pungens*), shrub live oak (*Quercus turbinella*), and desert ceanothus (*Ceanothus greggi*) were

TABLE 1.—WATER USE IN INCHES BY PLANTS WITH UNLIMITED WATER SUPPLY

Item	PERIOD					Total for year
	November–March	April–May	June	July–September	October	
Precipitation, inches...	17.55	1.00	0.11	7.55	0.59	26.80
Evaporation, inches...	13.46	18.03	10.94	25.80	5.68	73.91
Use by shrubs, inches..	4.85	10.28	8.54	24.76	5.46	53.89

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[2] "The Water Requirement of Plants. II," by L. J. Briggs and H. L. Shantz, *Bulletin No. 285*, Bureau of Plant Industry, U. S. Dept. of Agri., 1913.

[3] "Relative Water Requirement of Plants," by Lyman J. Briggs and H. L. Shantz, *Journal of Agricultural Research*, U. S. Dept. of Agri., Vol. 3, 1914, pp. 1–64.

[4] "The Water Requirement of Plants at Akron, Colo.," by H. L. Shantz and Lydia N. Piemeisel, *Journal of Agricultural Research*, U. S. Dept. of Agri., Vol. 34, 1927, pp. 1093–1190.

[5] "Water Utilization by Trees, with Special Reference to the Economic Forest Species of the North Temperate Zone," by Oran Raber, *Miscellaneous Publication No. 257*, U. S. Dept. of Agri., 1937.

[6] "Transpiration by Forest Trees," by Robert E. Horton, *Monthly Weather Review*, U. S. Weather Bureau, Vol. 51, 1923, pp. 571–581.

grown in sealed-top metal containers. Water was added at weekly or biweekly intervals to maintain the supply at optimum field-moisture capacity. The results are summarized in Table 1, and monthly use figures with average precipitation for the area are shown in Fig. 2.

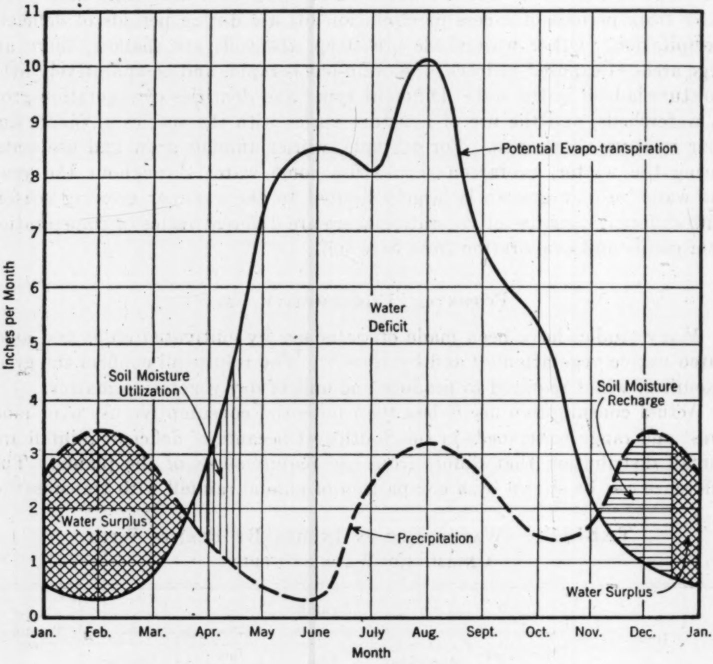

FIG. 2.—WATER USE AND WATER SURPLUS PERIODS FOR SIERRA ANCHA EXPERIMENTAL FOREST, ARIZONA

Precipitation for the year of measurements totaled 26.80 in., and evaporation for the same period was 73.91 in., as measured by a United States Weather Bureau evaporimeter. The average consumptive use of 53.89 in. was almost double the precipitation. It amounted to nearly 73% of the evaporation. Results of this study show that enormous quantities of water will be transpired by plants in the chaparral zone when unlimited supplies are made available.

These data may be applicable to riparian vegetation along perennial streams that have unlimited amounts of water available for use. However, except for relative comparisons, these data point out the limitations of sealed-top containers in studying consumptive use of native vegetation on the well-drained slopes that make up the greater portion of watershed areas in the West.

ACTUAL CONSUMPTIVE USE

Transpiration, unless the attendant soil evaporation is considered, is of very little use in determining consumptive use.[7] A. F. Meyer, M. ASCE,[8] has developed curves for evaporation from land areas and also for transpiration. These curves are based on temperature and monthly rainfall for evaporation and temperature for transpiration. Mr. Meyer points out, however, that availability of water can be a factor in both evaporation and transpiration. Evaporation from soil areas[9] is usually less than from a free water surface, and the difference in the quantity of evaporation increases as the soil becomes drier. Vegetation, in that it decreases wind velocity, also decreases evaporation. Transpiration is independent of the soil moisture content as long as the latter is above the wilting points of the vegetation. If rainless periods during the growing season are sufficiently prolonged, available moisture may be used by evapo-transpiration regardless of the size, kind, or density of the vegetation.

METHOD OF DETERMINING CONSUMPTIVE USE

Since forest and range vegetation does not have an unlimited water supply, a method[10] that appears suited to determining consumptive use under natural conditions consists of dividing the year into four periods and then considering the amount of water available and the amount used during each period. These periods are: (*a*) Soil moisture recharge; (*b*) water surplus; (*c*) soil moisture utilization; and (*d*) water deficit. A generalized graph showing these periods is given in Fig. 2. The cycle starts at the point where the potential evapo-transpiration curve crosses the precipitation curve (in the fall of the year) and becomes less than the rainfall. For a period that depends on both the amount of precipitation and depth and storage capacity of the soil, the surplus precipitation is satisfying the soil moisture deficit. The second period begins when the soil moisture reaches field capacity and the surplus water is yielded as stream flow. The third period begins when the potential evapo-transpiration crosses the rainfall line and continues until the available soil moisture is used. The fourth period is the summer water-deficit period in which there is less water available than is needed for consumptive use.

Fig. 2 illustrates the influence of climatic and watershed conditions. The proportion of the rainfall diagram that falls under the potential use curve reflects the influence of climate. In general, when the precipitation pattern is similar to the use curve, consumptive use is highest. In winter, use is low, and high precipitation results in water available for stream flow. The influence of watershed conditions is reflected in the amount of water held in the soil during the period of soil moisture recharge. In general, deep soils on gently sloping watersheds hold more water than shallow soils on steep areas.

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[7] "Hydrology," *Physics of the Earth Series*, Vol. IX, ed. by Oscar E. Meinzer, McGraw-Hill Book Co., Inc., New York, N. Y., 1942, pp. 259–330.

[8] "The Elements of Hydrology," by Adolph F. Meyer, 2d Ed. Rev., John Wiley & Sons, Inc., New York, N. Y., 1928.

[9] "Forest Influences," by Joseph Kittredge, McGraw-Hill Book Co., Inc., New York, N. Y., 1948.

[10] "An Approach Toward a Rational Classification of Climate," by C. W. Thornthwaite, *The Geographical Review*, Vol. XXXVIII, 1948, pp. 55–94.

TEST OF METHOD

Water-use data from plants grown in open-top metal containers and the water-yield data from large natural lysimeters were combined to test the workability of this method.

The water-use study in the chaparral zone was conducted in the Sierra Ancha Experimental Forest at El. 5100. Annual precipitation in this zone averages about 25 in., of which two thirds falls in the winter (October through May) and one third in the summer (June through September). Evaporation from a United States Weather Bureau evaporimeter was about 72 in. annually, and the soil was a residual silty clay loam derived from quartzite. Plants were transplanted in their original soil cores without disturbance to galvanized iron containers. The plants used were of about the same size and density as the plants under natural conditions. The open-top metal containers were buried in the ground so that approximately normal soil temperatures would prevail, and percolation water was drained at the bottom of the cans. Consumptive use was determined by weekly weighings. Containers of bare soil without plants served as checks for measurement of consumptive use (evaporation) by bare soil. During long dry periods it was necessary to add water to keep the plants functioning normally in their confined space, where they could not draw on moisture from deeper soil layers. The data derived from this test are shown in Fig. 3 and Table 2. Consumptive use varied during the year. Summer use depended on the amount of water available through rainfall, and winter use depended on growing conditions. Evaporation was the factor that most nearly described use conditions during the winter period. Estimating equations were developed from the actual amounts of water used by plants by relating the summer use to rainfall and by relating the winter use to evaporation. Other variables, such as temperature, humidity, and wind movement, were added to the estimating equations without serving any appreciable

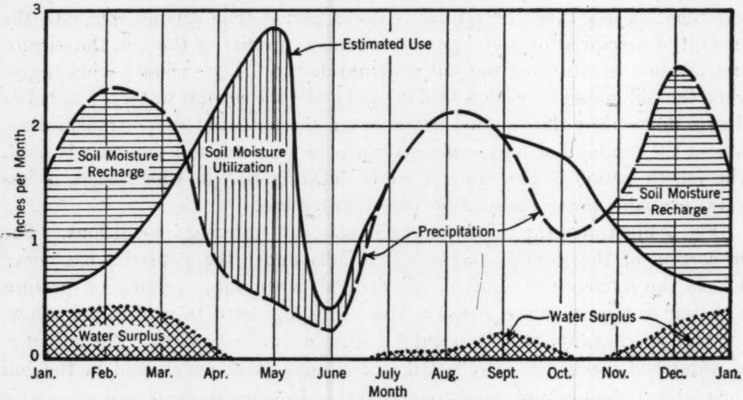

FIG. 3.—CONSUMPTIVE USE AND WATER YIELD FROM AN UNTREATED NATURAL LYSIMETER AT SIERRA ANCHA FOREST, ARIZONA

improvement in accuracy. Using average monthly rainfall for the summer period and evaporation for the winter period, the estimated use was derived from these equations and is shown in Col. 4, Table 2.

Water-yield data (Cols. 6 and 7, Table 2) are from the untreated Base Rock lysimeter located at Sierra Ancha Experimental Forest. This lysimeter is unique in that an undisturbed block of soil 18 ft by 50 ft and from 3 to 6 ft deep, overlying impervious quartzite bedrock, is used for measurements of rates and quantities of surface and subsurface runoff. The soils fall within the reddish chestnut soil grouping. Except for the surface soil, the profile shows no lines of demarcation or leaching between soil horizons. The entire profile contains partially disintegrated quartzite rock fragments from which the soils were derived, is noncalcareous, has a pH of about 6.3, and contains a high percentage of fine silt and clay in the subsoil. Vegetation cover is composed of the species characteristic of the semidesert grassland. The present ocular density is about 30%. The elevation of the test site is 4,500 ft.

TABLE 2.—ESTIMATED DISTRIBUTION OF PRECIPITATION AS DETERMINED BY THE UNTREATED BASE ROCK LYSIMETER AT SIERRA ANCHA EXPERIMENTAL FOREST, ARIZONA, 1935–1948 (IN INCHES)

Month (1)	Average precipitation (2)	Evaporation (3)	Estimated use (4)	Surplus (5)	RUNOFF		Accumulated surplus (8)	CONSUMPTIVE USE	
					Surface (6)	Subsurface[a] (7)		Actual (9)	Potential[b] (10)
(a) WINTER PERIOD									
October	1.04	5.54	1.77	0	0.01	0	0	1.03	5.46
November	1.38	3.33	1.04	0.34	0.03	0	0.31	1.04	2.27
December	2.50	2.21	0.67	1.83	0.01	0.25	1.88	0.67	0.80
January	1.58	1.89	0.57	1.01	0	0.39	2.50	0.57	0.62
February	2.30	2.45	0.75	1.55	0.01	0.42	3.62	0.75	0.32
March	2.13	4.33	1.37	0.76	0.01	0.37	4.00	1.37	0.84
April	0.76	6.71	2.15	0	0	0.01	2.60	2.15	2.80
May	0.50	8.99	2.90	0	0.01	0	0.19	2.90	7.48
Subtotal	12.19	35.45	11.22	5.49	0.08	1.44	....	10.48	20.59
(b) SUMMER PERIOD									
June	0.24	10.64	0.40	0	0	0	0	0.40	8.54
July	1.45	10.27	1.50	0	0.03	0	0	1.45	7.98
August	2.11	8.61	2.05	0	0.03	0.01	0	2.07	10.06
September	1.93	7.64	1.90	0	0.06	0.14	0	1.73	6.72
Subtotal	5.73	37.16	5.85	0	0.12	0.15	....	5.65	33.30
Annual Total	17.92	72.61	17.07	5.49	0.20	1.59	....	16.13	53.89

[a] Percolation yield. [b] Determined from sealed-top cans.

Annual precipitation at the Base Rock lysimeters has averaged 17.92 in. (12.19 in. during the winter and 5.73 in. in summer). Evaporation, as determined by a United States Weather Bureau evaporimeter, was 72.61 in. annually, and the water yield has averaged 1.79 in. or 10% of the precipitation. The water yield was distributed 89% in winter and 11% in summer. Nearly

91% of the winter yield and 56% of the summer yield has occurred as subsurface flow. Consumptive use (Col. 9, Table 2) has averaged 16.13 in. annually or 90% of the precipitation (10.48 in. of the winter precipitation and 5.65 in. of the summer moisture). The estimated use (Col. 4) is 17.07 in. or 0.94 in. higher than actual use. In this tabulation, October was considered a winter month, and the water use was dependent on growing conditions. Average rainfall is less than estimated use for the first part of the month, and 0.74 in. of the deficit is incurred in the first part of October, before the precipitation exceeds the consumptive use. The remaining 0.20 in. is the surface runoff that results from storm intensity and surface soil conditions rather than total water supply. The estimated potential consumptive use by vegetation grown in sealed-top containers is 53.89 in.

Potential consumptive use is much higher than precipitation; yet there is a period of surplus water available for stream flow. Between November and March precipitation is high, and evaporation from the soil and transpiration from plants are low. During this period precipitation exceeds consumptive use, the soil fills with water, and surplus water is yielded as stream flow. On the average, there is 9.9 in. of precipitation during the 5-month period, of which 4.4 in. goes to consumptive use (evapo-transpiration), 4.0 in. is held in the soil, and 1.5 in. is yielded as stream flow, mainly through percolation. This period is the only consistent water-yielding period.

The moisture held in the soil reservoir is used during the spring dry period. The effect of depth and type of soil on consumptive use is illustrated by the data in Table 2. A deeper soil or a soil with a higher moisture-holding capacity could easily retain, for use during the spring and early summer drought, the rainfall that was yielded as stream flow. A shallower soil or one with less total soil moisture-holding capacity should retain less water for use during the spring drought and allow a higher water yield. The water yield from summer precipitation was largely due to surface runoff from thunderstorms, when rainfall intensities exceeded the infiltration capacity of the soil, and also from a rare type late summer storm of large total quantity and long duration. One storm of this type (September 17 to 19, 1946) is included in the data shown in Table 2, and, therefore, summer water yields are higher than the average to be normally expected.

Data on Consumptive Use

Consumptive-use data (the difference between precipitation and water yield) from the Sierra Ancha Experimental Forest are available from transpiration studies at El. 2500 and El. 5100 (Table 3), and from watersheds that are typical of the semidesert grassland, chaparral, ponderosa pine, and pine-fir forest vegetation types from El. 3800 to nearly El. 8000 (Table 4). Rainfall at these stations is greater than that at corresponding elevations in most of the arid regions of the West, and the concentration of precipitation in winter probably results in lower consumptive use and higher water yield. All watersheds studied except those at Summit, Ariz., are underlain by nearly level beds of impervious quartzite. Measuring weirs are tied into this bedrock, and since there is little, if any, water diverted to other watersheds or to deep seepage, the runoff measured is virtually the total surface and subsurface yield. Summit watersheds are located in the granitic type soil, and measuring weirs are not tied into bedrock. The wide variations in consumptive use in different watersheds and between average and maximum and minimum years suggest that little credence should be given to figures on consumptive use by forest and range vegetation unless climatic and watershed conditions are duly considered. El. 2500 when there was 26.33 in. annual rainfall consumptive use was 89% of the rainfall as determined by cans of bare soil, 92% as determined by grasses and 98% of the rainfall as determined by winter annuals. In all cases, consumptive use in summer was 100% of the rainfall. At El. 5100, with 30.32 in. of precipitation annually, consumptive-use values were 78% of the rainfall for bare soil, 81% for grasses, and 84% of the rainfall for shrubs. At Base Rock lysimeters, El. 4500, in semidesert grassland cover (annual rainfall of 17.92 in.), consumptive use averaged 90% of the rainfall, but varies from 74% in 1940–1941 (26.7 in. of rainfall) to 94% of the rainfall in 1938–39 (11.07 in. of rainfall). The high consumptive use from bare soil is noteworthy. Relatively little additional water was used by vegetation.

Data from watersheds have followed a similar pattern. Consumptive use at Summit watersheds (El. 3800) with semidesert grassland-chaparral vegetation has averaged 96.8% of the rainfall for nine small watersheds, but varied from 94 to 98% on these adjacent watersheds. Precipitation for the period averaged 16.78 in. The four Natural Drainages are adjacent watersheds varying from 9 to 18 acres, with grassland-chaparral vegetation (El. 4500 to

TABLE 3.—TESTS OF WATER USE FROM OPEN–TOP METAL CONTAINERS, SIERRA ANCHA EXPERIMENTAL FOREST, ARIZONA

Condition measured	Year	WINTER			SUMMER			TOTAL YEAR		
		Rain (inches)	Consumptive use (inches)	Use percentage of rain	Rain (inches)	Consumptive use (inches)	Use percentage of rain	Rain (inches)	Consumptive use (inches)	Use percentage of rain
(a) ELEVATION 2,500 FEET										
Bare soil	1936	14.41	11.49	80	11.92	11.98	101	26.33	23.47	89
Grasses	1936	14.41	12.10	84	11.92	12.17	102	26.33	24.27	92
Half-shrubs	1936	14.41	13.40	93	11.92	11.96	100	26.33	25.36	96
Shrubs	1936	14.41	13.26	92	11.92	12.08	101	26.33	25.34	96
Woody legumes	1936	14.41	12.83	89	11.92	11.99	101	26.33	24.82	94
Winter annuals	1936	14.41	13.77	96	11.92	11.98	101	26.33	25.75	98
(b) ELEVATION 5,100 FEET										
Bare soil	1936	18.16	9.36	52	12.16	11.13	92	30.32	20.49	68
	1938	14.18	8.63	61	7.96	10.36	130	22.14	18.99	86
	1939	17.36	12.55	72	10.84	9.93	92	28.20	22.48	80
Grasses	1936	18.16	12.79	70	12.16	11.50	95	30.32	24.29	80
	1938	14.18	10.58	75	7.96	7.88	99	22.14	18.46	83
	1939	17.36	13.98	81	10.84	9.42	87	28.20	23.40	83
Shrubs	1936	18.16	13.89	76	12.16	11.73	96	30.32	25.62	84
	1938	14.18	10.52	74	7.96	7.82	98	22.14	18.34	83
	1939	17.36	14.86	86	10.84	9.24	85	28.20	24.10	85

El. 4900), and consumptive use has varied from 90% to 95% of the rainfall. The difference is probably related to soil type. The upper parts of all watersheds (varying from 26 to 55% of their area), are composed of diabase-derived soil, whereas the soil on the lower parts is derived from quartzite. The areas with the highest percentage of diabase-derived soil showed the highest consumptive use, those with the largest areas of quartzite-derived soil the lowest.

TABLE 4.—CONSUMPTIVE USE OF WATER BY FOREST AND RANGE VEGETATION, SIERRA ANCHA EXPERIMENTAL FOREST

Item studied	WINTER			SUMMER			TOTAL YEAR		
	Rain (inches)	Consumptive use (inches)	Use percentage of rain	Rain (inches)	Consumptive use (inches)	Use percentage of rain	Rain (inches)	Consumptive use (inches)	Use percentage of rain
(*a*) SUMMIT WATERSHEDS[a]									
Watershed Number 9 (low use)	11.16	10.71	96	5.54	4.92	89	16.70	15.63	94
Watershed Number 6 (high use)	11.21	11.03	98	5.65	5.55	98	16.86	16.58	98
(*b*) BASE ROCK[b] LYSIMETER									
Average of 13 years	12.19	10.48	86	5.73	5.65	99	17.92	16.13	90
Maximum year 1940–41	29.68	20.64	70	5.67	5.63	99	35.35	26.27	74
Minimum year 1938–39	7.92	7.19	91	3.88	3.88	100	11.80	11.07	94
(*c*) NATURAL DRAINAGE WATERSHEDS[c]									
Watershed A (open range)	14.37	12.82	89	7.49	7.27	97	21.86	20.09	92
Watershed B (protected)	14.37	13.40	93	7.49	7.33	98	21.86	20.73	95
Watershed C (protected)	14.87	12.88	87	7.07	6.87	97	21.94	19.75	90
Watershed D (moderately grazed)	14.87	12.82	86	7.07	6.88	97	21.94	19.71	90
(*d*) PARKER CREEK WATERSHED[d]									
Average of 15 years	18.69	12.55	67	9.36	9.09	97	28.05	21.64	77
Maximum year 1940–41	43.03	21.89	51	6.78	6.63	98	49.81	28.52	57
Minimum year 1947–48	12.37	10.68	86	4.90	4.90	100	17.27	15.58	90
(*e*) WORKMAN CREEK WATERSHED[e]									
Average of 10 years	23.47	20.40	87	10.72	10.37	97	34.19	30.77	90
Maximum year 1940–41	52.01	41.32	79	7.41	6.79	92	59.42	48.11	81
Minimum year 1947–48	15.78	14.69	93	9.13	8.97	98	24.91	23.66	95

[a] 0.80 and 1.05 acres of grassland-chaparral at El. 3800–3900. [b] Semidesert grassland at El. 4500. [c] 9 to 18 acres of mixed grassland-chapparral at El. 4500–4900. [d] 700 acres of ponderosa pine-chaparral at El. 5500–7300. [e] 1,087 acres of pine-fir forest at El. 6500–8000.

Consumptive use on Parker Creek, a 700-acre watershed with ponderosa pine-chaparral vegetation (El. 5500 to El. 7300), has averaged 77% or 21.64 in. consumptive use from 28.05 in. precipitation. The consumptive use has varied from 57% to 90% of the rainfall. The soils are shallow, and there are areas of exposed bedrock, the slopes are steep, and the drain-out of excess precipitation is rapid. Ninety-six per cent of the water yield occurs during the winter and results from winter precipitation. Workman Creek and Parker Creek also illustrate the wide variations between watersheds less than a mile apart. Consumptive use on Workman Creek, in pine-fir forest type vegetation (El. 6500 to El. 8000), has averaged 90% of the precipitation (30.77 in. consumptive use from 34.19 in. of precipitation) and has varied from 81% to 95%. The high use is probably caused by storage of relatively large amounts of water in the deep soils during the winter surplus period, and later used by evapotranspiration during drought periods.

EFFECT OF GRASS DENSITY

The importance of watershed and climatic factors is illustrated by preliminary results from a 12-acre watershed (Natural Drainage C) with semidesert grassland-chaparral vegetation. The area was fenced and livestock use eliminated in 1934. The terrain is steep, the drain-out of excess precipitation is rapid, and generally there is a period of no runoff between storms. Between 1934 and 1947 there were 163 rainfall-runoff periods varing in duration from a few hours to 6 months. A highly significant correlation exists between precipitation and runoff in the equation:

$$Y = 1242\,X^{1.83} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which Y is the runoff in cubic feet, and X is the rainfall in inches.

During the period from 1934 to 1947, vegetation density more than doubled (to 20% ocular density). If vegetation has a significant influence on runoff, the plotting of residuals from the above equation should show a definite trend through the years—upward if increased vegetation favored water yield and downward if vegetation favored consumptive use. However, the residuals by years show no trend. There is a scatter of points with approximately the same number above and below the curve for any one year, and consumptive use in this particular area appears to be independent of vegetation changes that range between about 10% and 20% ocular density. However, from the standpoint of usability of water and sediment content, there is a marked difference. As vegetation densities increase, both erosion and the amount of soil carried in streams decrease.

RELATION OF CONSUMPTIVE USE TO WATER YIELD

Since the water available for consumptive use on irrigated areas is the residual precipitation from watersheds, some comment on the effect of vegetation on stream flow is in order. Data from studies at the Sierra Ancha Experimental Forest have shown small differences in consumptive use between areas kept bare of vegetation and areas grown in various types of vegetation, and no significant differences in consumptive use between a watershed with good and poor grass cover. Decreased use on bare watersheds is largely hypothetical since no watershed can be kept bare of all vegetation; some plants will grow under extremely adverse conditions. If perennial grasses are depleted through abuse, inferior species such as annuals, weeds, and half-shrubs usually replace the original vegetation. If chaparral species are burned out, the sprouting varieties readily recover and may even increase.

From a water-yield standpoint, the major question is not whether watersheds bare of vegetation would yield most water, but the type of vegetation that interferes least with water yield and still controls sediment. In the Southwest, with very few exceptions, grasses are dormant and do not use water during the winter water-yielding period. Water use is largely confined to the summer period when there is no surplus water, and surface evaporation alone could consume all the precipitation. During the winter, high transpiration by half-shrubs, winter annuals, and evergreen shrubs, all characteristic of a deteriorated vegetation, tends to reduce water yield. Total annual use by grasses is slightly higher than evaporation from bare ground but lower than losses from a deteriorated vegetation. The benefit of well-developed and well-maintained plant cover in checking soil erosion and sedimentation of reservoirs outweighs by many times the value of the slight amount of water it uses.

SUMMARY

Consumptive use of water by forest and range vegetation depends primarily on climatic and watershed conditions and on water availability. There are few areas in the West in which unlimited water is available for full potential consumptive use. The ability of plants to grow and use water is greatest in summer and least in winter. Actual water use is dependent on growing conditions when moisture is available. The availability of moisture depends on the distribution of precipitation and the moisture held in the soil for use during drought periods. Water use in the semidesert grassland zone was 92% of the precipitation for perennial grasses, 98% for winter annuals, and 89% of the precipitation lost from bare soil by evaporation. Consumptive use of water in the chaparral zone was 81% of the precipitation for grasses, 84% of the precipitation for shrubs, and 78% of the precipitation lost from bare soil by evaporation. The use on watersheds in the mixed grassland-chaparral zone ranged from 90% to 95% of the precipitation on 4 adjacent watersheds, depending on soil type and depth of soil. Water use on forested watersheds has varied from 77% to 90% of the rainfall, depending on depth of soil and slope.

A method that appears suited to determining consumptive use of water by forest and range vegetation is to divide the water year into four periods: (1) Soil moisture recharge; (2) water surplus; (3) soil moisture utilization; and (4) water deficit; and to consider each period separately. Vegetation or evaporation from bare soil can always use more water than falls as precipitation during the summer period in the Southwest, and consumptive use is dependent on total precipitation less the small amount of surface runoff that results from summer thunderstorms. Part of this surface runoff is dissipated from dry stream channels by riparian vegetation and evaporation. Water use during the winter period depends on growing conditions. The water that is surplus first satisfies the soil-moisture deficit, and the balance is yielded as stream flow. Water use during the spring and early summer depends on the precipitation and the amount of moisture held in the soil from winter precipitation. Thus, consumptive use by forest and range vegetation depends on the amount and distribution of the rainfall, the climate, topography, and the storage capacity of the soils of the watershed as well as on the type of vegetation and the degree to which the vegetation use overlaps with the period of water surplus. The principal way in which man can influence water yield is by sound management and use of vegetation cover, through proper livestock grazing and timber harvesting, so that consumptive use overlaps and interferes least with the water-yielding period.

DISCUSSION

M. J. Youhotsky[11].—The data on the consumptive use of water by forest and range vegetation, as presented by Mr. Rich, were obtained by observing transplanted plants, and from the untreated Base Rock lysimeter. The task of obtaining direct evapo-transpiration data for some stands of timber, in their natural condition, and correlating such data to the volume of vegetation in the plant community, appears to be interesting but difficult. Under certain conditions, when the stand is fed by a shallow ground-water level, the proposed method of observing the fluctuations of the water-table elevations may prove useful.

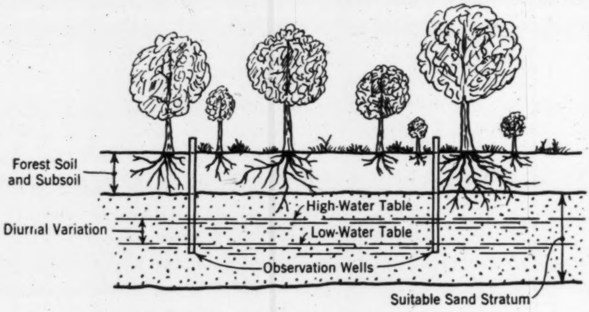

Fig. 4

In connection with a proposed hydrological survey of the Red Gum (Eucalyptus Rostrata) forests,[12] in Australia, the writer has suggested that this method be given a trial. These forests grow on flat alluvial land in the central part of the basin of the Murray River and thrive in the nearly arid climate of southeastern Australia with 15 in. of annual rainfall. Conditions most favorable for this type of observation are demonstrated diagrammatically in Fig. 4. The forest soil is shown overlying a stratum of uniformly-grain alluvial sand for which the porosity coefficient is easily obtainable. Tubular wells with accurate hook gages are arranged within each test plot as shown in Fig. 5. The diurnal variation of the water table, multiplied by the area of the test plot and by the porosity coefficient, will give the evapo-transpiration use of water by the given plant community during a given day.

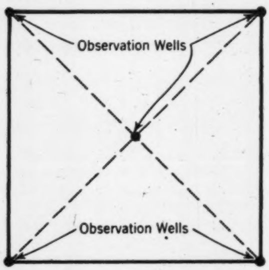

Fig. 5.—Test Plot

Consider a fairly extensive forest, situated at a low elevation, with flat terrain. If the gradation of the underlying sand is sufficiently small, any inflow of water from outside the bounds of the test plot may be disregarded, because the fluctuations of the water table are a function of evapo-transpiration.

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[11] Box 2967, G. P. O., Sydney, New South Wales, Australia.

[12] *New South Wales Forestry Recorder*, No. 1, 1948.

Correction may be needed for the possible tidal effect of the moon. If so, this phase could be investigated on a sufficiently large, cleared, land area of the same type and geology as the test plot.

L. R. RICH[13].—As stated by Mr. Youhotsky, the task of obtaining direct evapo-transpiration data for stands of timber in their natural condition and correlating such data to the volume of vegetation in the plant community is both interesting and difficult. The method suggested—diurnal fluctations of the water table where the vegetation is fed by a shallow ground-water table—should prove satisfactory under certain conditions. Previous work by G. E. P. Smith, M. ASCE, and Walter N. White[14] has shown that diurnal fluctuations of the water table can result from growing vegetation, drawing its supply from a shallow water table. Mr. White suggested a method for computing consumptive use of water by the vegetation. C. H. Lee, M. ASCE, discussed this method as follows[15]:

> "The method has application in areas of shallow water table where vegetation depends entirely on ground water. * * * Its greatest value is in areas where inflow and outflow are at scattered or inaccessible points and not susceptible of accurate measurements."

The experiment Mr. Youhotsky suggests should add worth-while data to the subject of consumptive use of water by forest vegetation under natural conditions. However, the evapo-transpiration rates from this type of experiment would probably be much higher than those from the forests at Sierra Ancha Experimental Forest, and higher than most of the forest areas in the arid West, because most of these areas do not have an unlimited water supply such as a shallow water table.

Forests grow under different conditions. Some areas have an unlimited water supply. In these areas consumptive use depends on growing conditions and would probably exceed average percipitation over much of the West. Other areas, which make up most of the forest and range lands of the West, do not have an unlimited water supply. The writer illustrated this point in Fig. 1, in which it was demonstrated that during 1940–1941, when percipitation was almost double the average, consumptive use in the forested watershed of Workman Creek exceeded the average annual precipitation; yet, during years that approached average conditions as well as years of below average precipitation, the trees made some growth, and stream flow, although below that of 1940–1941, was maintained. In this area most of the forest grows on well-drained slopes; an unlimited water supply is confined to a period in the winter and spring when use is limited by growing conditions. This period is followed by a water-deficit period during late spring, summer, and early fall months when water use would be highest if there were an unlimited supply. Consequently, consumptive use by forest trees on these well drained slopes depends on water available rather than on the capability of the forest vegetation to use water.

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[13] Hydrologist, Southwestern Forest and Range Experiment Station, Tuscon, Ariz.

[14] "A Method of Estimating Ground-Water Supplies Based on Discharge by Plants and Evaporation from Soil, Results of Investigations in Escalante Valley, Utah," by Walter N. White, *Water-Supply Paper No. 659-A*, 1932, Geological Survey, U. S. Dept. of the Interior, Washington, D. C.

[15] "Hydrology," *Physics of the Earth Series*, Vol. IX, ed. by Oscar E. Meinzer, McGraw-Hill Book Co., Inc., New York, N. Y., 1942, pp. 259–330.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

TRANSACTIONS

IRRIGATED CROPS

By Wayne D. Criddle[1]

With Discussion by Messrs. Clyde E. Houston, and Wayne D. Criddle

Synopsis

Generally not all irrigation water consumed in an area is used for crop production. Frequently great amounts of water can be recovered or saved. This paper gives a range in the amount of water consumed by varying types of vegetation under different climatic conditions in the United States. Alfalfa may consume 4.5 acre-ft of water per acre in the Salt River Valley of Arizona and only 1.5 acre-ft per acre in areas of Colorado. However, spring wheat, if it matures, will use about 1.25 to 1.50 acre-ft per acre regardless of where it is grown.

Introduction

Importance of Consumptive-Use Information.—The importance of a knowledge of consumptive use of water is realized by all people living in the irrigated areas of the world. Just as it is important to know, before starting on a long trip across some of our sparsely settled West, whether the old car will give 10 or 20 miles per gal of gasoline, so it is important to know how much irrigation water will be required and consumed by the crops on an irrigated tract. Likewise, a knowledge of consumptive use of water is vital in arriving at an equitable division of the waters of a river system, particularly if interstate and international rights are involved.

After carefully estimating present and possible future consumptive use and stream-flow depletion, planning for future hydroelectric power plants and their power production becomes possible. Engineers and others working on water supply, drainage, flood control, and many other fields are finding it increasingly important to have a knowledge of the amount of water consumed by agricultural crops and other uses incidental to irrigated agriculture.

Note.—Published in November, 1951, as *Proceedings-Separate No. 98.* Positions and titles given are those in effect when the paper or discussion was received for publication.

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Although information on consumptive use is important in basin or large area studies, it is important too for the individual farm. In the design and layout of a farm irrigation system one of the first questions that must be answered is "How much water will be required by the crops and when must it be applied?" To answer this question satisfactorily requires a knowledge of the soil including water-intake rate, water-holding capacity, and other soil characteristics. It also requires a knowledge of the crops to be grown, the total seasonal water requirement of each crop, and the peak daily, weekly, or sometimes peak monthly use of water. A knowledge of how much of the summer precipitation will be effective in satisfying the consumptive requirement is necessary, and a determination must be made of how much of the winter precipitation will be carried over as soil moisture and the contribution of ground water to consumptive use, if any.

Incidental Use of Irrigation Water.—In all irrigated areas, there is always a certain percentage of the water used within the area that is not actually consumed by the crops. Along with the crop use there is always some incidental use that cannot be economically avoided. This consumptive use that is incidental to irrigation varies widely between areas. It frequently offers an excellent opportunity for obtaining a better water supply for the crops or for expanding the irrigated acreage on the project. As the demand for water in a valley increases, the percentage consumed by argicultural crops usually becomes greater.

The total amount of water consumed by agricultural crops varies widely throughout the United States, but the consumption of irrigation water is considerably more variable. For some purposes a knowledge of the total amount of water consumed by the crops may be of prime importance. For other purposes an understanding of the amount of irrigation water required to satisfy the consumptive needs of the plants is considered most important.

FACTORS AFFECTING AGRICULTURAL USE

Consumptive use of water in an area is influenced by many factors including (1) climate; (2) cropping pattern; (3) available water supply; (4) soils and topography; (5) irrigation practices, and others.

1. *Climate.*—Undoubtedly climate is the factor having the greatest influence on the amount of water consumed by agricultural crops of an area. Investigators in the field generally agree that consumptive use of water will vary with temperature, length of growing season, precipitation, solar radiation, and humidity.

However, it is believed that the greatest effect of these factors is on the perennial crops, such as alfalfa and grass hay. The use of water by some annual crops, providing the crop matures, does not seem to be greatly affected by wide ranges of climate. As an example, the longest growing season and hottest temperatures found for any of the irrigated areas in Idaho is Lewiston, where the Snake River leaves the state. Here wheat is usually planted in late February or early March and harvested by mid-June. In contrast, spring wheat is usually not planted until late May on the Upper Snake River above Idaho Falls, Idaho, and is harvested the latter part of August or first part of September. Consumption of water by the wheat crop appears to be about the same at the two places, even though the crop requires only the first half of the growing season to mature at Lewiston but nearly all of the growing season at the location above Idaho Falls. Of course double cropping might be practiced at Lewiston, and the total seasonal water requirement for each acre of land might be twice as great as on the Upper Snake River. Fig. 1 shows consumptive water requirements of wheat crops grown at Lewiston and above Idaho Falls.

With perennial crops water use and crop yields vary more directly with the temperature and length of the growing season. Alfalfa may be expected to produce more hay in the Salt River Valley of Arizona than on the Upper Snake River in Idaho. With these greater seasonal yields will come a higher use of water.

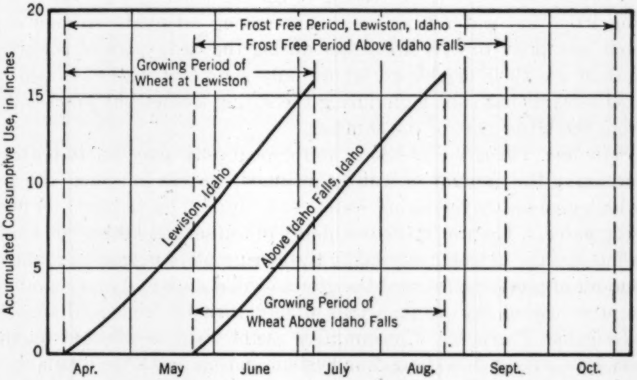

Fig. 1.—Accumulated Consumptive Water Requirement of Wheat at Locations in Idaho

2. *Cropping Pattern.*—Another important factor that must be considered with consumptive use of water is the cropping practice in the area. Crops such as alfalfa will generally use considerably more water each season than small grain crops. If an irrigated area has adequate water and is planted entirely to wheat, it may be expected to use less water consumptively than if planted entirely to alfalfa. However, the peak daily or weekly use for these crops may be quite similar.

3. *Available Water Supply.*—Probably the factor having the greatest influence on consumptive use, other than climate, is the water supply. Quantity, quality, cost, and availability are important considerations. Without an adequate supply, the consumption certainly will be less than when water is plentiful, all other factors remaining constant. With no irrigation water available, seasonal consumptive use cannot exceed the effective summer precipitation plus any carry-over of winter precipitation in the form of soil moisture.

Likewise, the quality of the water available may have an influence on the amount of water consumed. If the water is highly saline, good crop production may require frequent heavy applications of water to maintain a downward movement of the salts and prevent a harmful concentration near the surface of the land. Under such practices evaporation from the wet or damp soil surface may materially increase the amount of water consumed inasmuch as consumptive use includes, by definition, the evaporation from the soil and water surfaces adjacent to the plants.

Cost may have a decided effect on the amount of water consumed by crops. Where water is cheap and plentiful, copious irrigations are frequently applied, causing high evaporation and incidental losses. As water becomes more expensive in relation to crop values and to other costs, it is usually applied more efficiently; sometimes less than the amount necessary for optimum crop production is applied.

Availability of water as needed has some effect on consumptive use but may be most important in increasing irrigation efficiencies. Those farmers receiving irrigation water from reservoirs having adequate storage capacity are much less likely to over-use water during the early part of the irrigation season than are their neighbors, whose only source of water is from an unregulated stream that runs high during the spring freshet but practically dries up during the latter part of the summer.

4. *Soils and Topography.*—Soils and topography play an important role in determining the amount of water consumed. Areas in which the fertility level is high give heavy yields, but water consumption, particularly for perennial crops, is greater. Texture, structure, depth of top soil, and other soil characteristics affect the use of water, especially the consumptive irrigation requirement. The amount of precipitation that the ground will absorb and make available for consumptive use of the plants varies widely with the soils and topography.

5. *Irrigation Practices.*—Consumptive use of water is affected by the way a farmer lays out his farm irrigation system and manages the available water. The wise irrigator will apply water when and as needed to keep the crop growing at or near a maximum rate. Such practices will tend to give high crop yields with a relatively great use of water per unit area. However, bad practices may also result in fairly great use of water because of the resulting incidental uses. If the land was not properly prepared for irrigation, ponding and drowning out of crops may result in some areas. Evaporation from such areas is great, and consumption of water in the area may be similarly high.

Consumptive Use Incidental to Irrigated Agriculture

Factors Responsible For Consumptive-Use Variations.—As has been already stated, water is consumed in most irrigated areas that are not directly necessary for the production of agricultural crops. This use includes evaporation from free water surfaces and seeped lands and the transpiration of the natural vegetation that is inevitable when water is available. In areas, such as those along the Pecos River above Lake McMillan in New Mexico or the Gila River above Safford, Ariz., these incidental uses become extremely important. In each of these areas vast acreages of salt cedars consume great quantities of water. In other areas, willows, cattails, and sedges consume water. A portion of this water will undoubtedly be salvaged and used beneficially for agricultural production when the need for water requires action.

The amount of water consumed by natural vegetation varies widely from one irrigation project to the next because of many factors. The length, slope, and shape of irrigation canals and ditches and the soils through which they pass have great effect on the amount of water lost in transit and consumed by natural vegetation. If the canals and ditches are lined and the amount of seepage is controlled, areas in which seepage appears and large patches of plants that consume great quantities of water will not develop. However, if the canals are unlined and traverse areas of flat, highly permeable soils, there are apt to be great quantities of water escaping from the canal. Unless the underdrainage is good and the seepage water escapes underground from the project, wet areas will develop. Soon natural water-loving vegetation will start growing. This growth, together with evaporation from any existing water surface and from the moist soil surface, can account for great water use.

Adaptability of Natural Vegetation.—The amount of water consumed by natural vegetation is somewhat dependent upon the type of vegetation that will grow in the area under natural conditions. In general, the larger the vegetation, the greater the amount of water that will be consumed. Usually tall cottonwoods or other trees will have a dense undergrowth of smaller plants such as willows and grass. However, some types of vegetation grow better under some climatic conditions than they do under others. For instance, salt cedars or tamarisk, commonly found in many of our southwestern areas, do not grow and thrive naturally under northern or more rigid temperature conditions. In its natural habitat, tamarisk appears to be an important user of water. Further north and in the higher elevations willows replace the salt cedar, but the willows are likewise great users of water, although they may not have as high a peak use rate as the tamarisk.

In high mountain meadow country, such as on the headwaters of the Colorado River, large areas of dense willows are found. In fact, many ranchers must exert a continual effort to keep the willows out of the pasture or wild hay lands. Such vegetation begins using water early in the spring and continues to use water, if available, until late in the fall.

Likewise, sacaton, a plant that loves a high ground-water table, grows rank and dense under southwestern conditions, but will not live in the northern areas.

Frequently, on irrigated farms low spots are found in or at the end of the fields into which large amounts of surface waste water and silt collect. Such conditions are conducive to high farm consumptive use and give a particularly high use if based only on irrigated cropped lands.

High ground water, whether natural or caused by leaks from canals or irrigation applications, is conducive to great use of water. Many plants, including alfalfa, use much more water if it is readily available. To this greater use by the plants themselves must be added the increase in evaporation from the land and water surfaces.

Variability of Use Throughout the West

Regional Variations in Consumptive Use.—It is quite generally understood that the consumptive requirement of water varies throughout the West. The greatest use is, of course, in those areas having the longest growing seasons, highest temperatures, and lowest humidity. Some of the valleys in the United States where consumptive use is greatest are the Salt River and Yuma areas in Arizona, and the Central and Imperial valleys of California.

Mimimum use is found in the higher, cool valleys. In high mountain valleys where frost may occur every month of the year, only a light crop of wild hay may be harvested some years. Under such climatic conditions little consumptive use occurs each season because of the short period during which crop growth takes place. The New Fork area above Pinedale, Wyo., is typical of such an area.

Use may vary widely because of the difference in crops grown in the various areas. There are a number of small valleys in southern California where one would expect a rather great use of water. Based on temperature and growing season records, the use in these valleys might be quite similar to that in the Imperial or Coachella valleys of California. However, because of lack of adequate water for irrigation, only relatively low yielding grain or beans is normally grown. Furthermore, after the grain crop is planted, there is an attempt made to utilize all the precipitation possible, both winter and early spring. Precipitation alone will not produce a crop in these valleys, so one or two small applications of water may be made.

Such practices result in an extremely low average consumptive use when compared to that in an area such as the Mojave River area near Victorville, Calif., where climatic conditions are not too much different but where a major irrigated acreage is planted with high producing alfalfa. Table 1 gives some examples showing the variation in use between different areas:

TABLE 1.—Result of Studies on Consumptive Use of Water by Alfalfa and Small Grains

Location	State	Consumptive Use of Water by Crops, in Inches	
		Alfalfa	Small grains
Bonners Ferry	Idaho	24.0	17.5
Logan	Utah	25.0	17.5
Prosser	Washington	36.0	18.0
Mesa	Arizona	52.5	...

Consumptive Irrigation Requirement.—Rather frequently it becomes necessary to know the total amount of water consumed by a crop and also the net amount that must be supplied by means of irrigation and other sources. In areas in which ground water does not contribute to the requirement the only other sources of water are summer precipitation and soil moisture carry-over from the winter precipitation. Determining the proportional contribution from each source is extremely difficult since it varies widely from place to place. One of the most controversial points in any consumptive-use-of-water study is determining these contributions.

In the hotter climates, such as the Salt River Valley in which total annual precipitation is small, and a major portion of this occurs in storms of less than 0.5 in., there is always a question as to whether any of the precipitation is really effective. Under these Arizona conditions a characteristic summer storm develops rapidly.' Rainfall occurs at a rather high intensity for a short duration, and then the storm clouds pass. The high rate of fall is frequently accompanied by heavy surface runoff from the fields. Precipitation that remains after surface runoff is soon evaporated from the wet foliage and ground surface without any significant quantity penetrating into the root zone of the crop. Under such conditions, the consumptive irrigation requirement is probably reduced little, if at all, because of the summer storm. However, in such areas, the total amount of precipitation that falls during the summer is relatively small in comparison with the total requirement. Therefore, whether this precipitation is considered as being effective or not may make little difference to the total irrigation requirement.

Under conditions found in the northwestern United States a rainstorm, regardless of its size, is frequently accompanied by cloudiness and cooler weather that may last for several days. This lowering of temperatures and increased humidity may reduce consumptive use so that a large part of the precipitation might be considered effective. Also, in this area it is characteristic for the rain to fall more gently and over a longer period of time. This allows greater time for the precipitation to percolate into the root zone of the crops and become effective in reducing the consumptive irrigation requirement. This is particularly true under coastal conditions in Washington and Oregon and in northern Idaho. In the more arid portions of the Northwest, precipitation that normally falls during the growing season may be so small that it is practically insignificant in relation to the annual consumptive use.

East of the 100th meridian normal rainfall more nearly satisfies the water requirements of the crop. In fact, irrigation in the Middlewest and East has been considered practical on only a few high-priced truck crops. With the development of cheaper and more dependable portable sprinkler equipment, farmers in many areas of the East are finding it economically sound to provide for one or more irrigations each season to keep their crops growing at a maximum rate throughout the season. This supplemental irrigation may not be necessary each year, but, at least during the drier years, it may mean the difference between good yields or seriously reduced yields.

In order to understand the quantity of irrigation water that must be applied and when it should be applied, the amount and distribution of the effective precipitation should be known. Applying water in excess of crop needs is certainly not economical and may be harmful. By far the most economical way is to supply through natural processes the moisture needed by the crop. However, it is sometimes necessary to help nature supply water for plant requirements just as New York, N. Y., finds it necessary to seed the clouds with dry ice or silver iodide in hopes of increasing its water supply.

The use of irrigation water to supplement rainfall is believed to be more clearly presented by chart, as in Fig. 2. This figure shows the cumulative consumptive use and cumulative precipitation during the growing season at four locations having widely different climatic conditions.

The chart for Phoenix, Ariz., shows a possible total use for alfalfa of approximately 56 in. during the average frost-free period from February 10 to December 3. Normally, less than 6 in. of precipitation occurs during this period. Whether all of this precipitation is considered effective or whether it is disregarded entirely will not make a significant difference in the irrigation water requirement. Probably 75 to 80 in. of water would normally be applied to the field to provide the 56 in. required for consumptive use.

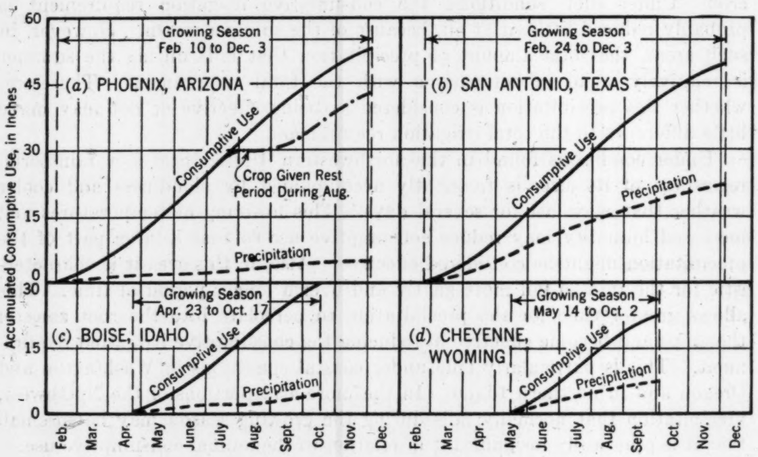

FIG. 2.—POTENTIAL CONSUMPTIVE WATER REQUIREMENT OF ALFALFA

When the chart for San Antonio, Tex., with a similar frost-free period is studied, it can be seen that, if precipitation is neglected, it would make a considerable difference in the irrigation requirement. Here the growing season precipitation amounts to about 23 in., or nearly half the total possible requirement of alfalfa at that location.

At Boise, with a much shorter growing season, the summer precipitation amounts to only 4 in., and the consumptive use of alfalfa is believed to be about 30 in. Here, as at Phoenix, neglecting the summer precipitation will not make too much difference in the irrigation water requirement.

At Cheyenne, Wyo., with only a short growing season, the summer precipitation of 8 in. is a good proportion of the 23 in. that is the estimated consumptive requirement of alfalfa at this location.

PEAK USE OF WATER BY CROPS

If the combined weights of all vehicles passing a point on a highway during a month were determined and this total divided by the number of vehicles, the average weight of each vehicle would result. However, if an engineer were to design a new bridge (using this average weight as the live loading), the engineer would be looking for a new job as soon as the first oil tanker attempted to cross this bridge.

So it is with the design of an irrigation system. The system, particularly if it is of the sprinkler type, must be designed to handle the peak water requirement of the crop. The frequency of irrigation that must be designed will depend on the length of time the water added to the soil and stored for plant use will last when consumptive use is at a maximum rate.

Frequently peak use for a project as a whole has been determined from records of past water demands on some similar operating project. This method works well where all of the factors affecting water requirement, including cropping pattern, are similar. Another method that has been used occasionally is to estimate the average monthly irrigation water requirements of each crop[2] at the location under study, choose the peak monthly use, and divide by the number of days in the month. This average daily rate for the peak month is then multiplied by a factor which may be as large as 1.5.

To illustrate this method, assume that a design for a sprinkler system for an alfalfa field having light shallow soils near Grand Junction, Colo., is desired. The normal seasonal consumptive use[3] is estimated to be 32.4 in. Maximum use normally occurs during the month of July and is estimated to be 6.78 in. However, normal July precipitation is 0.75 in., leaving a deficit of 6.03 in. that must be supplied by some other means. The sprinkler system, therefore, would be designed to supply 1.5 × 6.03, or about 9 in. of water for consumption of the crop plus various losses. Assuming an irrigation efficiency of 75%, the sprinkler system must be designed to convey 12 acre-in. of water per acre if optimum crop-growing conditions are to be maintained during the extremely hot dry periods. Daily peak consumptive-use rates being used by technicians of the Pacific Coast Region of the Soil Conservation Service for three broad climatic zones of the western states for the more common crops are given in Table 2.

TABLE 2.—VALUES OF PEAK DAILY USE OF SOIL MOISTURE FOR COMMON IRRIGATED CROPS PRODUCING OPTIMUM YIELDS

Crop	USE OF WATER IN INCHES DEPTH PER DAY		
	Cool climate	Moderate climate	Hot climate
Alfalfa	0.15	0.20	0.25–0.30
Pasture	0.12	0.16	0.20–0.25
Small grains	0.15	0.20	0.22
Potatoes	0.10	0.12	0.14
Sugar beets	0.12	0.15	0.20
Deciduous orchard	0.15	0.20	0.25
Orchard with cover crop	0.20	0.25	0.30

SUMMARY

In general, consumptive use of water in irrigated areas might be broken into two categories:

a. Consumption by agricultural crops; and
b. Consumption or use incidental to irrigation.

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[2] "Determining Water Requirements in Irrigated Areas from Climatological and Irrigation Data," by Harry F. Blaney and Wayne D. Criddle, Div. of Irrig. and Water Conservation, SCS, U. S. Dept. of Agri., Washington, D. C., September, 1949.

[3] "Consumptive Use of Water in the Irrigated Areas of the Upper Colorado River Basin," by Harry F. Blaney and Wayne D. Criddle, Div. of Irrig. and Water Conservation, SCS, U. S. Dept. of Agri., Washington, D. C., April, 1949.

The amount of water consumed by agricultural crops is affected by many factors, of which climate is probably the most important. Other influencing factors are soils, topography, water supply, irrigation practices, and the crops grown. The total depth of irrigation water required for consumptive use during the growing season is dependent on the effective precipitation that falls during the summer and the carry-over soil moisture from winter precipitation.

Perennial crops show wide variations in use of water at different locations of the West. Nearly 4½ ft of water per yr are normally used by alfalfa in the Phoenix and Yuma areas of Arizona. A minimum use where alfalfa is grown successfully occurs in the higher northern valleys and is probably about 1½ ft of water per year. Annual crops usually do not show such a wide variation in use. Approximately 15 to 18 in. of water will be used consumptively in producing a good irrigated wheat crop regardless of where it is grown.

Water consumed by natural vegetation incidental to irrigation may vary widely. Probably the greatest factor influencing the proportion of an irrigation supply thus consumed is the relative cost of the water in relation to other farm costs and to the value of the crops produced. However, there are other factors involved that also affect this proportion.

In many arid valleys, potentially short of water for agricultural purposes, reduction in the amount of water used by nonagricultural plants may present the greatest opportunity for expanding the agriculture or improving present supplies. Canal linings, better irrigation practices, better land leveling for irrigation, and other irrigation improvements might be used to salvage water. Consumptive use of water by crops cannot be decreased without decreasing yields, and it even appears that, because of increasing yields, the consumption of water per acre might also be increasing. The efficiency with which irrigation water is conveyed and applied thus offers the greatest opportunity for saving water.

Farm and field irrigation systems should be designed with capacity for supplying sufficient water to the crops during periods of unusually high demand. For some conditions a capacity of 1½ times the irrigation requirement during the normal peak month has been considered desirable. However a factor smaller than 1½ may be entirely satisfactory in areas of deep soils with high water-holding capacities.

DISCUSSION

Clyde E. Houston,[4] A.M. ASCE.—Mr. Criddle has given an excellent presentation of a subject which should receive primary consideration by present-day irrigation engineers and planners. Past experience has shown that there is more to building a successful irrigation project than the construction of great dams, control works, and canals. There is need for concentrated study of the actual use of the water by crops on the farm. If, as a result of inaccurate calculations or inefficient water use, the cost of dams and canals cannot be repaid, what good are they but as monuments to poor planning?

To Mr. Criddle's list of "Factors Affecting Agricultural Use" might be added a sixth factor—improved crop varieties. With the advent of hybrid corn and its increased yield probably comes an increase in consumptive use. Improved alfalfa varieties on irrigation experimental plots at Reno, Nev., indicate an increased consumptive use of water with an increased yield. As improved irrigation practices take place to influence crop yield materially there may be an increase in the consumptive use of water.

The period of small, simple, and relatively inexpensive water projects has closed and a period of large, complex, and relatively costly undertakings has opened. It should be recognized that projects of various types will increase still further in size, complexity, and cost. Furthermore, the supply of water for the use of these projects is limited. Under these circumstances it is clear that effective control and efficient use of water must be obtained for maximum utilization and the greatest total benefit. Consumptive-use determinations are fundamental to a sound, permanent, and profitable irrigation agriculture.

Wayne D. Criddle[5].—In suggesting that the introduction of improved crop varieties will increase consumptive use, Mr. Houston is entirely correct. Any increase in crop yield—whether it is caused by the use of improved varieties, higher fertility levels, or better irrigation and management practices—will tend to increase the consumptive use of water by the crop, although not in direct proportion to yield increases. Also, as shown by the work of the Soil Conservation Service at Prosser, increasing the frequency of irrigation and maintaining the soil moisture at a higher level will greatly increase the consumption of water, although yields may not be greatly influenced.

The data on the consumptive use of water were based on what the writer believes to be approximately average conditions for the areas, both as to crop yields and irrigation practices. Any general increase in yield or change in irrigation practice for any large area usually comes slowly. Therefore, the data presented in the paper should be valid for a number of years.

Since the paper was written in 1950, much additional information has been gathered on the peak use rate. Current studies at various locations in the West indicate the approximate range of the daily rate at which water is used by crops. The consumptive-use rate for any one day, or the average over a short series of days, can be considerably higher than the average daily rate for an entire month. Likewise, use during the peak month is much higher than the average monthly rate during the entire growing period. Frequency of irrigation depends on the length of time that the water stored in the soil will meet the plant needs, or on how much water the soil will hold and how fast it is used. Therefore, the less water that can be stored in the root zone at each irrigation, the more frequent the irrigation must be, assuming that the use rate remains the same. Also, the less water that can be stored at each irrigation and the more frequent the irrigation, the larger the irrigation system must be to meet the crop needs during periods of high use.

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[4] Irrig. Engr., Div. of Irrig. Eng. and Water Conservation, SCS, U. S. Dept. of Agri., Reno, Nev.

[5] Senior Irrig. Engr., Div. of Irrig. and Water Conservation, SCS, U. S. Dept. of Agri., Boise, Idaho.

Studies have shown the relationship between the daily use of water by crops during short peak periods and the average during a peak month. This relationship was used in constructing Table 3. By considering the characteristics of the soil in the crop root zone and by knowing, or computing, the peak monthly consumptive-use rates of the crops, Table 3 might be used to design an irrigation system with adequate capacity to meet the varying crop needs.

TABLE 3.—PEAK CONSUMPTIVE-USE RATES FOR THE DESIGN OF IRRIGATION SYSTEMS

Computed peak monthly use, in inches	PEAK DAILY CONSUMPTIVE-USE DESIGN RATES, IN INCHES PER DAY							
	Depth of water to be stored[a]							
	1 in.	2 in.	3 in.	4 in.	5 in.	6 in.	7 in.	8 in.
3.0	0.16	0.14	0.12	0.11	0.11	0.10	0.10	0.10
4.0	0.21	0.18	0.17	0.15	0.15	0.14	0.13	0.13
5.0	0.27	0.23	0.21	0.19	0.18	0.17	0.16	0.16
6.0	0.32	0.27	0.25	0.23	0.22	0.21	0.19	0.19
7.0	0.37	0.32	0.29	0.27	0.25	0.24	0.23	0.23
8.0	0.43	0.36	0.33	0.31	0.29	0.27	0.26	0.26

[a] Depth of soil moisture to be replaced in the crop root zone by each irrigation.

An example of the use of Table 3 follows: The design capacity of an irrigation system might be desired, in which the peak monthly use of the crop is 6 in. The soil, which is light-textured and well-drained, will hold only 1 in. of water between wilting point and field capacity for each foot of soil depth. Under the existing conditions, the rooting depth of the crop is 3 ft and the irrigation water must be applied by the time two thirds of the available soil moisture has been extracted. Thus, a 2-in. application would be required at each irrigation. From Table 3 it may be seen that with a peak monthly consumptive-use requirement of 6 in. and where only 2 in. can be stored at each irrigation, the system must be able to apply 0.27 acre-in. per acre each day. The irrigation interval could not exceed about eight days. However, irrigation water cannot be applied on a field basis without some loss. A good irrigation efficiency is generally considered to be about 75%. Thus, the system in this example must have a capacity of 0.27/0.75, or 0.36 acre-in. per acre per day, which is about 6.75 gal per min of continuous flow for each acre of land.

If the soil in the example had been deep, with a high water-holding capacity from which 7 in. of water could safely be withdrawn between irrigations, the average daily peak use would have been 0.19 acre-in. per acre per day. If applied at 75% efficiency, this would have required a system capacity of only 0.25 acre-in. per acre per day, or 4.7 gal per min of continuous flow for each acre of land.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

MUNICIPAL AND INDUSTRIAL AREAS

By George B. Gleason,[1] A. M. ASCE

With Discussion by Messrs Robert H. Born and Meyer Kramsky

Synopsis

A typical survey to determine the consumptive use of water in municipal and industrial areas is described in this paper. In order to determine a fair distribution of water rights in the Raymond Basin Area, Pasadena, Calif., an evaluation was made of the factors involved. The results of this study are presented, and the data are analyzed to emphasize points that may be of assistance in the planning and execution of similar surveys.

Introduction

Consumptive use of water in municipal and industrial areas includes transpiration and evaporation from lawns, shrubs, and trees; evaporation from bare surfaces; and the relatively very small increments evaporated in household and industrial uses or retained in industrial products. It does not include sewage, surface runoff, or water percolating beyond the root zone of the covering vegetation, all of which are or can be made available for re-use.

A municipality can be defined as that area devoted to residences, commercial and industrial establishments, churches, schools, parks, and cemeteries. Residential areas vary in their water consuming characteristics, the amount consumed depending primarily upon what part of the area is devoted to each of the several types of vegetation and what part to buildings, paved surfaces, and so on. Commercial areas also show a consumptive use, ranging from closely built-up centralized districts in which water-consuming vegetation is virtually nonexistent to scattered establishments in the suburbs around which there may still be considerable areas of lawns, shrubs, and trees.

Survey of Raymond Basin Area

In connection with a suit for the adjudication of water rights,[2] brought by the City of Pasadena against other producers of water from the Raymond

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Note.—Published in November, 1951, as *Proceedings-Separate No. 99.* Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Supervising Hydr. Engr., State of California, Div. of Water Resources, Los Angeles, Calif.

[2] "City of Pasadena vs. City of Alhambra," No. Pasadena C-1323 in the Superior Court of the State of California, in and for the County of Los Angeles, February, 1939.

Basin Area in Los Angeles County, it was deemed necessary that the consumptive use within the basin be evaluated. The Division of Water Resources, Department of Public Works of the State of California made an investigation and report on the physical facts.

Cultural Classification of Area.—During 1938 and 1939 the City of Pasadena, as the first step in this investigation, completed a detailed survey of the culture of the area involved. Of the approximately 23,000 acres in the western unit of the area, nearly 17,000 acres (or about three fourths of the total) was devoted to urban types of culture. For purposes of the survey this urban culture was classified as follows: (1) Estates; (2) class A residential; (3) class B residential; (4) rural residential; (5) commercial; (6) semicommercial; (7) reservoir sites; (8) parks; (9) schools; and (10) streets. Although it is probable that there are no two lots that are identical in the relative proportions of buildings, driveways, lawns, shrubs, and trees, there is materially less variation between lots within each of the above classes than between those in different classes. The results disclosed by the survey are presented in the following tabulation:

Classification	Acreage	Classification	Acreage
Estates	2,604	Shrubs	4
Class A residential	2,167	Lawns and ornamental trees	63
Class B residential	3,170	Ornamental trees	1,481
Rural residential	765	Avocado and citrus	904
Commercial	193	Deciduous	294
Semicommercial	532	Vineyard	1,024
School	272	Nursery and truck	296
Park and cemetery	453	Vacant	2,252
Reservoir site	63	Brush	2,275
Streets	3,722	River wash	354
Lawns and shrubs	5		
		Total	22,893

The first ten classifications listed in this tabulation constitute urban culture. With the exception of commercial and streets, which were assumed to include no water-consuming vegetation, each of these classifications includes, in various proportions, vacant areas, buildings, driveways, lawns, shrubs, trees, and possibly other water-consuming vegetation. It was assumed that an accurate determination of the relative proportion of the area that was devoted to each type of culture on a few representative areas in each classification would provide a reliable basis for evaluating the acreage in each type on the whole urban area. The same assumption was applied to river wash areas that included areas of brush and bare sand and gravel. Table 1 presents a composite of the results obtained on the typical areas. Applying these percentages to the area of each urban classification as determined by the survey and adding the areas not included in those classes, the total acreage in each type of culture was obtained and is also listed in Table 1.

Unit Consumptive-Use Values.—The next step in the evaluation of consumptive use was the assignment of unit values of consumptive use for each of the several types of culture covering the area. Prior to this considerable experimental work had been carried on, having as its objective the evaluation of unit consumptive-use values for agricultural crops and native vegetation. So far as could be determined, similar work on the types of vegetation that are peculiar to municipal development had been negligible. Because of this lack of information the City of Pasadena, in cooperation with the United States Department of Agriculture, carried out an extensive program of soil sampling for moisture determination throughout the period October, 1937, to October, 1939, and the State Division of Water Resources installed and operated an

TABLE 1.—CONSUMPTIVE-USE SURVEY OF THE WESTERN UNIT OF RAYMOND BASIN AREA

Type of culture	Area in acres	Classification									Estimated consumptive use in feet
		Estates	Residential			Semicommercial	Reservoir site	Parks	Schools	River wash	
			Class A	Class B	Rural						
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
(a) NUMBER OF TYPICAL AREAS STUDIED											
		5	5	5	3	5	5	3	3	1	
(b) PERCENTAGE OF CLASSIFICATION DEVOTED TO SPECIFIC CULTURE											
Impervious	6,371	19	33	23	14	43	43	17	28	...	0.50
Lawn and shrubs	1,894	19	23	18	12	7	4	28	25	...	3.00
Lawn and trees	1,533	20	16	13	5	3	0	30	0	...	3.33
Shrubs	831	7	10	8	9	6	24	11	4	...	...
Vacant	3,617	4	6	19	33	23	18	9	38	...	1.50
Ornamental trees	2,607	20	5	10	12	8	11	5	5	...	2.33
Deciduous	977	8	4	8	12	8	...	...	...	...	1.75
Citrus	1,098	3	3	1	1	2	...	...	...	...	2.00
Nursery and truck	312	...	...	...	2	...	...	...	...	...	3.00
Sand and gravel	206	...	...	...	...	...	...	...	...	58	0.50
Brush	2,423	...	...	...	...	...	...	...	...	42	1.67
Vineyard	1,024	...	...	...	...	...	...	...	...	...	1.67
TOTAL	22,893	100	100	100	100	100	100	100	100	100	

experimental plot consisting of eight tanks planted with lawn grass and surrounded by lawn. Water, including rainfall, was applied as required by the grass in the tanks, and the amount of water that was withdrawn as it accumulated in the bottom of the tanks was measured during the period from June, 1939, to July, 1945. Based on the experimental data so obtained, the unit values shown in Table 1 were assigned.

Combining the values presented in Table 1, unit values of annual consumptive use for each cultural classification were obtained and are listed as follows:

Cultural classification	Consumptive use in feet	Cultural classification	Consumptive use in feet
Estates	2.07	Reservoir sites	1.34
Class A residential	1.92	Park	2.40
Class B residential	1.88	Schools	1.63
Rural residential	1.78	River wash	0.99
Semicommercial	1.32		

The commercial classification was assumed to consume 6 in. Applying these values to the acreages as listed in Table 1 resulted in an estimated total consumptive use for the western unit of Raymond Basin Area equal to 36,166 acre-ft.

Subsurface Outflow.—In the Raymond Basin study the recognized principle that the hydrologic equation must balance was utilized to evaluate the subsurface outflow across Raymond fault, the lower boundary of the ground-water basin. Each of the items of supply and disposal other than the subsurface outflow was independently estimated in the manner of the following derivation:

Item	Quantity (acre-feet)	
Water entering area:		
Precipitation	40,810	
Inflow	11,650	
Import	3,480	
Water coming from storage in basin	5,810	
Subtotal		61,750
Water leaving area on surface:		
Surface outflow	8,090	
Export	14,630	
Consumptive use	36,170	
Subtotal		58,890
Subsurface outflow		2,860

Of the items appearing in the above derivation, inflow, outflow, import, and export were for the most part measured. The estimate of water coming from storage in the basin was based on measured changes in water table elevation at numerous and fairly well-distributed wells and also on specific yield values established by earlier detailed studies of the Division of Water Resources.[3] The number and distribution of precipitation records was adequate.

No detailed independent estimate of the subsurface outflow appeared to be feasible at the time of the study, but general geological considerations indicated that the derived value was reasonable. Assuming that all values used in its derivation other than consumptive use were substantially correct, it follows that the value of over-all consumptive use was also substantially correct. It does not necessarily follow that the unit values presented in Table 1 are exact.

ANALYSIS OF RESULTS

Analyses of evaporation from soils indicate that the unit consumptive-use value of 0.50 ft, arbitrarily assigned to impervious areas, may be high. Assuming that the true unit value is 0.30 ft, the consumptive use from the impervious area would be reduced by 1,274 acre-ft. Evidence available indicates that deciduous culture consumes more water than citrus culture. Assuming a unit value of 2.25 ft for deciduous culture results in an increase of 488 acre-ft., leaving 786 acre-ft to be accounted for if the over-all consumption from the area is to remain unchanged. An increase of a little more than $\frac{1}{10}$ ft in the unit values assigned to the three types of water-consuming culture that are peculiar to urban development (lawns and shrubs, lawns and trees, and ornamental trees) would accomplish the purpose. It is doubtful that unit values that are applicable to large areas can be established experimentally within such close limits. However, each of the values used in the hydrologic equation is subject to some error, so the desirability of independently evaluating, with the greatest possible accuracy, consumptive use, as well as each of the other factors, is apparent. Experimental data relating to unit consumptive-use values for the types of culture that are peculiar to urban development are evidently limited to that obtained in connection with the Raymond Basin study. More work along this line is justified.

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[3] "South Coastal Basin Investigation, Geology and Ground Water Storage Capacity of Valley Fill," *Bulletin No. 45*, State of California, Dept. of Public Works, Div. of Water Resources, 1934.

The effect of changes in culture on the supply of water required is of especial interest in areas that overlie ground-water basins of large capacity and in which rapid and extensive urban development is under way or anticipated. The paving of streets and the installation of sewers tend to increase the outflow. On the other hand the consumptive use may decrease. The weighted average unit consumptive use by a combination of the first four items of culture in Table 1 is 1.61 ft. This value is less than any of the unit values for the agricultural crops shown and also less than that for brush. It is greater than the values for vacant land and bare waste land.

To arrive at an approximation of the effect of future development on runoff, sewage production, and consumptive use in the western unit of the Raymond Basin Area, it was arbitrarily assumed that there would be no change in waste land along streams; but that ultimately 21% of the area would be devoted to streets, 5%' would remain vacant, and the remainder would be devoted to municipal development, with each classification proportional to its 1939 acreage, as shown in Table 1. Under these conditions it is estimated that runoff would increase by about 2,800 acre-ft over the 1939 values and that sewage production would increase by about 4,600 acre-ft but that consumptive use would decrease by about 300 acre-ft. The study further indicated that percolation to the ground water from precipitation and irrigation would increase under these conditions by about 2,200 acre-ft.

It was further estimated that the change from natural culture to that of 1939 resulted in an increase of about 3,400 acre-ft in runoff, an increase of about 8,100 acre-ft in sewage production, a decrease of about 2,300 acre-ft in consumptive use, and an increase of about 8,200 acre-ft, in percolation to the ground water from precipitation and irrigation.

The foregoing estimates indicate that existing urban culture in the Raymond Basin Area has decreased rather than increased consumptive use and has increased deep percolation to the ground water. On the other hand, it has materially increased surface runoff and the production of sewage. Whether or not all the effects would be comparable elsewhere depends not only upon the type of development but also upon the similarity of climate. Consumptive use by native vegetation is directly related to the amount of precipitation available, and consumptive use by all vegetation is related to temperature. Average annual precipitation on the western unit of Raymond Basin Area ranges from 18 to 25 in. Where precipitation averages much greater than this, it is probable that the reduction in consumptive use resulting from urban development would be greater than the value estimated. Where precipitation is much less, it is possible that urban development might increase rather than decrease the consumptive use. During the period from June, 1939, to September, 1940, in which the lawn grass tanks were operated, the temperature at the plot ranged from 54.8° F in February to 75.2° F in July, 1940, and averaged 64.1° F during the year from July 1, 1939, to June 30, 1940. Had the temperatures been higher, the unit values presented in Table 1 would have been greater, and vice versa.

CONCLUSIONS

When considering consumptive use by urban development, each area presents its own problem. Whether or not in a particular case a culture survey as detailed as that outlined in this paper would be justified would depend upon the nature of the study. However, the value of more such surveys and further experimental work, with a view to more definitely establishing unit values for the various types of culture involved, cannot be questioned.

DISCUSSION

Robert H. Born,[4] J. M. ASCE.—Information obtained in the Raymond Basin court reference has been widely accepted as an important contribution to the fund of knowledge concerning municipal water use. The writer concurs in Mr. Gleason's conclusion that further experimental work is necessary to establish, more definitely, the units of water use for various types of urban culture.

As a research project in connection with the current California State-Wide Water Resources Investigation under the direction of the State Water Resources Board, the Division of Water Resources has amassed additional water-use data in the Los Angeles and Vernon industrial areas. Conducted in the fall of 1949 and spring of 1950, the study had as its objective, the determination of units of water usage applicable to large industrial sectors in the Los Angeles metropolitan area. The area surveyed covered approximately 2,500 acres of land devoted entirely to industrial purposes wherein were located representatives of the following industries: Steel mills, breweries and wineries, soft drink manufacturers, canneries (processed food), chemical plants, paint and varnish manufacturers, rubber products manufacturers, paper mills, meat packers, tanneries, wool pullers, rendering plants, ceramic industries (pottery, china, and glass), ice and cold storage plants, fabricated metal products manufacturers, warehouses, and railroad marshalling yards.

The total water delivered to the area was determined from metered records of the cities of Los Angeles and Vernon and from records and estimates of water pumped from private wells. It was determined that approximately 27,000 acre-ft of water was applied during the fiscal year, 1949–1950, to the 2,500 acres of industrial lands, giving an applied water-use factor of approximately 10.8 acre-ft per acre per year.

TABLE 2.—Selected Industrial Water-Use Factors

Industry	Number of plants surveyed	Water-Use Factor, in Acre-Feet per Acre per Year	
		Average	Range
(1)	(2)	(3)	(4)
Paper mills	four	162.8	11.9 to 247.5
Meat packing	eight	54.5	16.2 to 97.3
Ice and cold storage	two	16.1	14.9 to 17.2
Chemical	two	12.9	2.2 to 109.1

Among the interesting facts revealed in this survey, it was discerned that adequate information concerning quantities of water consumptively used (that is, the water lost through evaporation from steam lines, boiler blow-off, and cooling towers) is not generally available due, for the most part, to the lack of sufficient and reliable waste measurements.

It was also noted that a tremendous variation exists in the water use between the several types of industries as well as between industries of the same general category. An indication of the quantities of water required by a few of the heavy water users in the area surveyed is given in Table 2.

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[4] Asst. Hydr. Engr., Div. of Water Resources, California Dept. of Public Works, Los Angeles, Calif.

The lack of correlation between industrial water use and plant area is readily apparent in Table 2. The effect of this variation is even more pronounced when these water-use data are compared on a percentage basis. Within the four paper mills surveyed, for example, the applied water-use factors varied from 7% to 150% of the average of the four plants. Within the total industrial area surveyed, applied water-use factors varied from 12% to 2,300% of the average factor for the entire area.

The significant variation in industrial water use leads to the conclusion that water consumption expressed as a quantity of use per unit of area may not be a valid indicator of industrial water requirements except where the factor has been obtained from a survey of a large area comprising many types of industries. With so wide a variation in unit values of water applied, there must also be significant variation in the quantities consumed per acre.

The results of the aforementioned survey have stimulated the writer to investigate further a more logical basis on which to determine water requirements of individual industries. Preliminary information obtained thus far indicates that, in the meat packing industry for instance, water-use factors expressed as a quantity of water per unit of plant production (gallons per 1,000 lb of kill) are the most consistent of all.

MEYER KRAMSKY,[5] A.M. ASCE.—The author has presented a comprehensive analysis of consumptive use in the western unit of the Raymond Basin Area of Southern California. That area is part of metropolitan Los Angeles, and is predominantly suburban residential in character, with heavy industry practically nonexistent. The City of Pasadena occupies most of the area.

It is assumed that consumptive use in the Raymond Basin Area is confined to natural evapo-transpiration losses from the various surfaces included therein. Weighted average unit consumptive-use factors are determined for the several land-use classifications based upon relative proportions of the various types of vegetative cover and bare surface included within each classification. Total consumptive use is obtained by applying the appropriate unit factor to the total acreage devoted to that particular land use.

This procedure is entirely valid for regions where native vegetation, agriculture, and municipal residential use predominate, but is not necessarily applicable where an appreciable part of the area under consideration is devoted to heavy industry. There is no particular relationship between area occupied and water consumed in manufacturing processes, such as exists for the other land-use categories, and the infinite variety of industrial water usage renders fallacious any generalized conclusions drawn from limited studies.

Industrial Process Consumptive Use in West Coast Basin.—As part of its investigation as referee in an adjudication of water rights,[6] the Division of Water Resources, Department of Public Works of the State of California, conducted an extensive study of water use and disposal in the West Coast Basin of Los Angeles County. This area encompasses 101,000 acres, in which extensive industrial installations are located with oil refineries predominating. More than 40% of all applied waters (excluding rainfall) in the West Coast Basin are used for industrial purposes.

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[5] Senior Hydr. Engr., California Div. of Water Resources, Los Angeles, Calif.

[6] California Water Service Co., et al, vs. City of Compton, et al, No. 506806 in the Superior Court of the State of California, in and for the County of Los Angeles, October 24, 1945.

The large proportion of water used by industry, and the preponderance of water evaporation processes involved in the refining of oil, indicate that industrial processing accounts for a major part of the consumptive use of water in the West Coast Basin. This use is in addition to, and entirely unrelated to, the normal evapo-transpiration losses from the ground surface.

Surface losses in the West Coast Basin were analyzed and evaluated by the conventional methods demonstrated by the author. Consumptive use by industrial processing, however, was measured by comparison of industrial water usage with resulting sewage effluent, based on the assumption that industrial waters are disposed of by either consumptive use or discharge as industrial wastes. It was assumed that the industrial water used for purposes other than industrial processing and incidental sanitary use was negligible. Under these assumptions, the difference between applied industrial water and industrial waste is a measure of consumptive use by industrial processing.

TABLE 2.—INDUSTRIAL PROCESS CONSUMPTIVE USE IN ACRE-FEET, WEST COAST BASIN

Year	SEWERED AREA								Consumptive use-ratio (%)	ENTIRE BASIN	
	Number of dwelling units	Unit water requirements	Domestic water delivered	Domestic waste	Total sewage	Industrial waste	Applied industrial water	Industrial process consumptive use		Applied industrial water	Industrial process consumptive use
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1932-33	70,400	0.237	16,700	10,000	15,800	5,800	14,300	8,500	59	22,700	13,500
1933-34	71,200	0.237	16,800	10,100	14,600	4,500	14,300	9,800	68	22,600	15,500
1934-35	75,400	0.237	17,900	10,700	14,700	4,000	12,900	8,900	69	20,700	14,300
1935-36	83,100	0.237	19,100	11,500	16,200	4,700	18,100	13,400	74	26,700	19,800
1936-37	85,000	0.237	20,100	12,100	14,600	2,500	18,400	15,900	86	27,400	23,700
1937-38	89,100	0.237	21,000	12,600	16,100	3,500	18,200	14,700	81	25,800	20,800
1938-39	92,600	0.237	21,900	13,100	16,800	3,700	19,700	16,000	81	29,200	23,700
1939-40	98,000	0.237	23,200	13,900	18,600	4,700	18,500	13,800	75	28,900	21,600
1940-41	110,000	0.224	22,900	13,700	19,800	6,100	21,200	15,100	71	32,200	22,900
1941-42	116,200	0.241	27,900	16,700	22,900	6,200	21,800	15,600	72	33,500	24,000
1942-43	122,300	0.268	32,600	19,600	25,200	5,600	24,000	18,400	77	42,400	32,500
1943-44	135,100	0.260	34,900	20,900	31,000	10,100	31,200	21,100	68	53,900	36,400
1944-45	139,700	0.277	38,500	23,100	33,400	10,300	39,200	28,900	74	67,300	49,600
1945-46	145,800	0.320	46,400	27,800	36,200	8,400	32,900	24,500	74	56,400	42,000
1946-47	151,900	0.296	44,800	26,900	36,200	9,300	28,900	19,600	68	53,400	36,200
1947-48	161,700	0.299	48,100	28,900	38,400	9,500	38,900	29,400	76	57,500	43,500
1948-49	169,600	0.298	50,300	30,200	40,900	10,700	44,600	33,900	76	62,900	47,800
1949-50	182,400	0.283	51,300	30,800	48,100	17,300	48,600	31,300	64	67,900	43,700

Applied waters in the basin were segregated into three broad classifications: Municipal, industrial, and irrigational. Municipal use was estimated by applying the average water demand per dwelling unit, as determined from detailed studies of representative water distribution systems, to the total number of dwelling units in the basin, taken from census reports and studies of the Regional Planning Commission of Los Angeles County. The water used for irrigation was ascertained by examination of the records of the various water producers and distributors. The remaining applied water was assumed to represent industrial use (Col. 11, Table 2).

The volume of industrial waste was evaluated as the difference between total sewage and domestic waste, as presented in Table 2. A count of the total number of sewage connections in the basin was converted to an equivalent number of dwelling units in the sewered area. Domestic waste was estimated as 60% of the water supply to these dwelling units, based on the relationship between water and sewage in representative residential areas. The remaining sewage was considered to be industrial waste from the sewered area.

Industrial waste in the basin is disposed of through the several sewerage systems or by discharge into Dominguez Channel, a surface water course. Since industrial waste determination has been limited to that portion discharged into the sewerage system, it was necessary to determine the corresponding volume of industrial water served to the sewered area. This was accomplished by deducting the water supply of industrial installations tributary to Dominguez Channel from the total applied industrial water.

In accordance with the basic assumption employed herein, industrial process consumptive use in the sewered area was computed as the difference between applied industrial water and industrial waste as previously evaluated. The large volumes of water evaporated and the extensive recirculation of water effected by the industries are reflected in the extremely high consumptive-use ratio, averaging more than 70% of the applied water. This ratio, as determined for the sewered area, was considered representative of the entire basin and was used to determine the total annual consumptive use by industrial processing, as shown in Table 2.

Since consumptive use by industrial processing is not amenable to analysis and determination on an acreage basis, the techniques presented by the author are not particularly adaptable to this problem. A reasonable estimate of industrial process consumptive use can be obtained through comparison of the volume of applied industrial water with the resulting industrial waste. Evaluation of these items, however, is usually laborious and involved, and should not be undertaken unless the cultural development of the area under consideration includes large industrial establishments whose consumptive use is an appreciable item of water disposal.

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TRANSACTIONS

SPECIAL CASE IN RIO GRANDE BASIN

By Robert L. Lowry,[1] M. ASCE

With Discussion by Messrs. Isom H. Hale and George A. Whetstone; H. R. McDonald; and Robert L. Lowry

Synopsis

The necessity of controlling the consumptive use of water in the irrigated sections of the Rio Grande is a problem of national interest. The beneficial use of water in this river basin has increased in recent years only through control of nonbeneficial uses and careful husbanding of all available supplies.

The case of the El Paso (Tex.)-Juárez (Mexico) section of the Rio Grande Valley is presented to illustrate the proportion of available water that was lost because of the natural channel conditions and to indicate the method used in reducing these losses. The manner in which the consumptive use was determined is shown in detail. The effect of river control is analyzed, and recommendations are made for the planning of similar projects.

Introduction

With the waters of southwestern streams of the United States almost wholly appropriated for irrigation development, it is urgent that users be made to realize that the major opportunity for further expansion in irrigation rests in the possibility of converting nonbeneficial consumptive uses to beneficial purposes. The nonbeneficial uses in some areas are exacting a heavy toll of water and consist largely of the water demands of native vegetation plus losses incident to evaporation from swamps and badly deteriorated stream channels. In many places the areas covered by native vegetation that are using water directly from the streams are increasing, and natural channels are rapidly deteriorating as the result of sedimentation. The Rio Grande is no exception to the general rule. Any further expansion in irrigation in the Rio Grande Basin upstream from Fort Quitman, Tex., will probably have to come through what now seems the one remaining opportunity to provide

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additional water—a reduction in the natural losses. Most of the available water supply for the Rio Grande in this upper reach is now regulated by reservoirs that furnish water for practically full irrigation development. The problem of water use and control in the Rio Grande Basin has become a matter of national concern.[2]

CONTROL OF CONSUMPTIVE USE

Reservoir Development.—Many reservoirs have been constructed throughout the Rio Grande Basin to provide irrigation water. These reservoirs have a combined storage capacity of more than 8,100,000 acre-ft. International reservoirs on the main river, downstream from Fort Quitman, now under construction or actively contemplated[3] (1950), will add another 8,500,000 acre-ft of storage capacity. These reservoirs provide a controlled water supply for the irrigation of more than 2,500,000 acres of land, of which about 1,780,000 acres are in the United States, with the remaining 720,000 acres in Mexico.

Losses Through Nonbeneficial Consumptive Use.—In the Rio Grande Basin the most notable example of stream depletion by nonbeneficial consumptive use, for which there is some hope of relief through remedial measures, is that of the area of more than 15,000 acres at the head of Lake McMillan, on the Pecos River in New Mexico, now covered by a dense growth of salt cedars. A plan of the Bureau of Reclamation (USBR), United States Department of the Interior, for by-passing this area with a side channel has been developed with the expectation that an annual saving of from 30,000 to 40,000 acre-ft of water may be realized.

Another area in which nonbeneficial uses are large is the Middle Rio Grande Basin in New Mexico. A project to bring about improvement of this condition by channel rectification has been jointly submitted to Congress by the USBR and the Corps of Engineers, United States Department of the Army. When consummated, it is expected that a substantial saving in water in that area will result.

EL PASO-JUÁREZ VALLEY STUDY

Basis of Studies.—An example of what can be accomplished by the conversion of nonbeneficial uses to beneficial uses is afforded by the records for the El Paso-Juárez Valley of the Rio Grande. Soon after 1900 there began a steady expansion of the irrigated area of this valley without any appreciable change in the water supply. This continued expansion has been made possible by a marked decrease in the natural losses between El Paso and Fort Quitman.

Data on inflow and outflow for this reach of the river have been assembled and are shown in Table 1. The determination of these natural losses was made as the result of an inflow-outflow study covering the 49-yr period from 1900 to 1948. Most of the inflow to the area (Col. 9) is measured in the river at the El Paso gaging station, which has been in operation since 1889. A minor quantity, contributed by the normally dry arroyos between El Paso and Fort Quitman, was estimated from the records by the Hydrographic Office of the United States Section of the International Boundary and Water Commission (IBWC). The outflow from the area (Col. 10) has been measured since 1924 at the Fort Quitman gage. Estimates for the period 1900 through 1923 were based largely on the records at El Paso and the Upper Presidio (Tex.) gage. The difference between the inflow and outflow represents the total

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[2] "Ten Rivers in America's Future," The Report of the President's Water Resources Policy Commission, Vol. 2, U. S. Govt. Printing Office, Washington, D. C., 1950.

[3] "Falcon Dam Stimulates Irrigation in Mexico," by J. C. Bustamante, *Civil Engineering*, May, 1951, p. 273.

TABLE 1.—STREAM DEPLETION IN THE EL PASO-JUÁREZ VALLEY OF THE RIO GRANDE

Year	Irrigated area (acres)[a]	Unit consumptive use[b] (acre-feet per acre)	Effective rainfall[c] (feet)	Unit stream depletion[d] (feet)	STREAM DEPLETION IN THOUSANDS OF ACRE-FEET			RIVER FLOW IN THOUSANDS OF ACRE-FEET	
					Irrigation only[e]	Total[f]	Channel[g] loss	Inflow[h]	Outflow[a] at Ft. Quitman
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1900	10,000	3.27	0.80	2.47	24.7	105.6	80.9	188.1	82.5
1901	12,000	3.26	0.66	2.60	31.2	201.4	170.2	373.1	171.7
1902	6,000	3.27	0.87	2.40	14.4	41.5	27.1	72.2	30.7
1903	8,000	3.16	0.83	2.33	18.6	398.4	379.8	1,048.8	650.4
1904	500	3.19	0.93	2.26	1.1	180.9	179.8	492.8	311.9
1905	9,000	3.15	1.13	2.02	18.2	491.6	473.4	2,041.8	1,550.2
1906	12,000	3.20	1.02	2.18	26.1	299.6	273.5	1,131.7	832.1
1907	14,500	3.33	0.73	2.60	37.7	325.0	287.3	1,855.9	1,530.9
1908	26,000	3.17	0.65	2.52	65.5	184.9	119.4	479.0	294.1
1909	40,000	3.20	0.53	2.67	106.8	303.9	197.1	880.9	577.0
1910	44,000	3.40[i]	0.29[i]	3.11[i]	116.8[i]	127.9	11.1	583.8	455.9
1911	45,000	3.26	1.10	2.16	97.2	494.1	396.9	1,575.6	1,081.5
1912	47,000	3.07	0.79	2.28	107.2	307.4	200.2	1,377.8	1,070.4
1913	33,000	3.07	0.69	2.38	78.5	155.7	77.2	309.7	154.0
1914	35,000	3.18	1.42	1.76	61.6	340.8	279.2	1,090.3	749.5
1915	38,000	3.13	0.87	2.26	85.9	351.3	265.4	727.2	375.9
1916	42,000	3.30	0.59	2.71	113.8	303.6	189.8	555.4	251.8
1917	45,000	3.22	0.51	2.71	122.0	404.7	282.7	899.2	494.5
1918	49,000	3.14	0.80	2.34	114.6	181.1	66.5	363.3	182.2
1919	52,700	3.17	0.90	2.27	119.6	275.4	155.8	537.7	262.3
1920	57,500	3.23	0.55	2.68	154.1	385.1	231.0	705.7	320.6
1921	62,300	3.30	0.67	2.63	163.8	423.6	259.8	875.7	452.1
1922	67,000	3.25	0.36	2.89	193.6	423.2	229.6	780.6	357.4
1923	71,800	3.18	0.76	2.42	173.8	401.5	227.7	733.9	332.4
1924	76,584	3.24	0.56	2.68	205.2	445.5	240.3	819.3	373.8
1925	90,411	3.27	0.67	2.60	235.1	377.8	142.7	645.7	267.9
1926	94,021	3.22	0.99	2.23	209.7	304.0	94.3	580.3	276.3
1927	95,137	3.36	0.56	2.80	266.4	382.2	115.8	623.1	240.9
1928	105,746	3.26	0.73	2.53	267.5	374.4	106.9	638.2	263.8
1929	108,877	3.20	0.84	2.36	256.9	351.0	94.1	562.8	211.8
1930	108,750	3.27	0.47	2.80	304.5	347.7	43.2	535.7	188.0
1931	107,300	3.29	0.86	2.43	260.7	321.1	60.4	532.7	211.6
1932	99,077	3.23	0.73	2.50	247.7	372.0	124.3	583.1	211.1
1933	101,662	3.32	0.51	2.81	285.7	395.5	109.8	609.3	213.8
1934	98,558	3.48	0.31	3.17	312.4	407.5	95.1	509.9	102.4
1935	98,991	3.30	0.53	2.77	274.2	319.3	45.1	464.7	145.4
1936	99,227	3.29	0.77	2.52	250.1	326.8	76.7	476.4	149.6
1937	108,334	3.34	0.62	2.72	294.6	368.9	74.3	547.2	178.3
1938	110,430	3.31	0.77	2.54	280.5	303.4	22.9	579.6	276.2
1939	111,054	3.31	0.51	2.80	311.0	366.0	55.0	518.2	152.2
1940	114,989	3.29	0.59	2.70	310.5	337.2	26.7	461.5	124.3
1941	118,104	3.25	1.44[e]	1.81	213.7	222.1	8.4	552.6	330.5
1942	119,514	3.31[i]	0.76[i]	2.55[i]	271.6[i]	291.6[i]	20.0[i]	1,562.0	1,270.4[i]
1943	121,102	3.25	0.56	2.69	325.8	401.1	75.3	634.4	233.3
1944	120,346	3.16	0.87	2.29	275.6	353.5	77.9	626.4	272.9
1945	123,203	3.25	0.58	2.67	329.0	368.5	39.5	576.2	207.7
1946	126,290	3.26	0.68	2.58	325.8	374.6	48.8	506.0	131.4
1947	122,891	3.32	0.59	2.73	335.5	368.9	33.4	459.7	90.8
1948	123,650	3.36	0.52	2.84	351.2	358.4	7.2	435.8	77.4

[a] From the records of the International Boundary and Water Commission (IBWC). [b] Based on the average annual temperature in the valley. [c] 85% of the average annual rainfall in the valley. [d] Col. 3 minus Col. 4. [e] Col. 2 times Col. 5. [f] Col. 9 minus Col. 10. [g] Col. 7 minus Col. 6. [h] Inflow at El Paso gaging station plus arroyo inflow as estimated by IBWC. [i] Arbitrarily adjusted to avoid negative figures.

stream depletion or that quantity of water lost from the river within this area (Col. 7). Obviously, this total stream depletion includes the uses of water by irrigated crops, as well as the natural losses. Losses of water by evaporation from the surface of the stream or from open water in swamped areas have been included with the natural losses. In order to reduce this total depletion to its component parts (use by irrigation and natural losses) it was necessary to make estimates of the consumptive use on the irrigated area. Each year since 1900 the area listed as irrigated (Col. 2), including the lands in both the United States and Mexico, was based on information obtained from the El Paso Office of the USBR and the Mexican Section of the IBWC. Unit consumptive use was determined from the relation established by a number of inflow-outflow studies[4] as indicated in Table 2. Temperatures were taken from the records of the Weather Bureau at El Paso. These temperatures have been modified for the period since the Weather Bureau station was moved from downtown El Paso to the municipal airport, 7 miles northeast of the city. Rainfall records used include both Weather Bureau and IBWC data. Only 85% of the total rainfall each year was considered as effective in order to prevent a duplication in the calculations of runoff. Then, by calculating the total water used by irrigation and deducting that quantity from the total depletion, the depletion by natural causes (called nonbeneficial uses) was left as the residual. The

TABLE 2.—Relation Between Consumptive Use and Mean Annual Temperature

Station or area	State	Consumptive use (feet)	Mean annual temperature (degrees Farenheit)	Source
		(*a*) Observed Data		
Wagon Wheel Gap	Colorado	1.30	33.0	United States Weather Bureau
Uncompahgre	Colorado	2.28	50.4	Colorado Water Conservation Board
San Jacinto River	Texas	3.19	68.3	Bureau of Reclamation[a]
Carlsbad Project	New Mexico	2.94	63.6	Bureau of Reclamation[a]
Messilla Valley	New Mexico-Texas	2.83	59.7	Bureau of Reclamation[a]
Lower Rio Grande Valley	Texas-Mexico	3.80	73.8	J. L. Burkholder[b]
Cypress Creek	Texas	3.02	65.6	Bureau of Reclamation[a]
East Fork of Trinity River	Texas	2.82	65.0	Bureau of Reclamation[a]
		(*b*) Determined from Effective Heat Units[c]		
Presidio	Texas	3.42	68.3	....
Eagle Pass	Texas	3.50	71.3	....
Del Rio	Texas	3.30	69.4	....
El Paso	Texas	2.91	63.7	....
Laredo	Texas	3.76	73.0	....
Substation 14	Texas	2.82	65.4	....
Fort Stockton	Texas	2.94	66.2	....
Falfurrias	Texas	3.63	73.2	....

[a] "Consumptive Use of Water for Agriculture," by R. L. Lowry, Jr., and A. F. Johnson, *Transactions*, ASCE, Vol. 107, 1942, p. 1243. [b] *Ibid.*, p. 1287. [c] *Ibid.*, p. 1252.

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[4] "Consumptive Use of Water for Agriculture," by R. L. Lowry, Jr., and A. F. Johnson, *Transactions*, ASCE, Vol. 107, 1942, p. 1243.

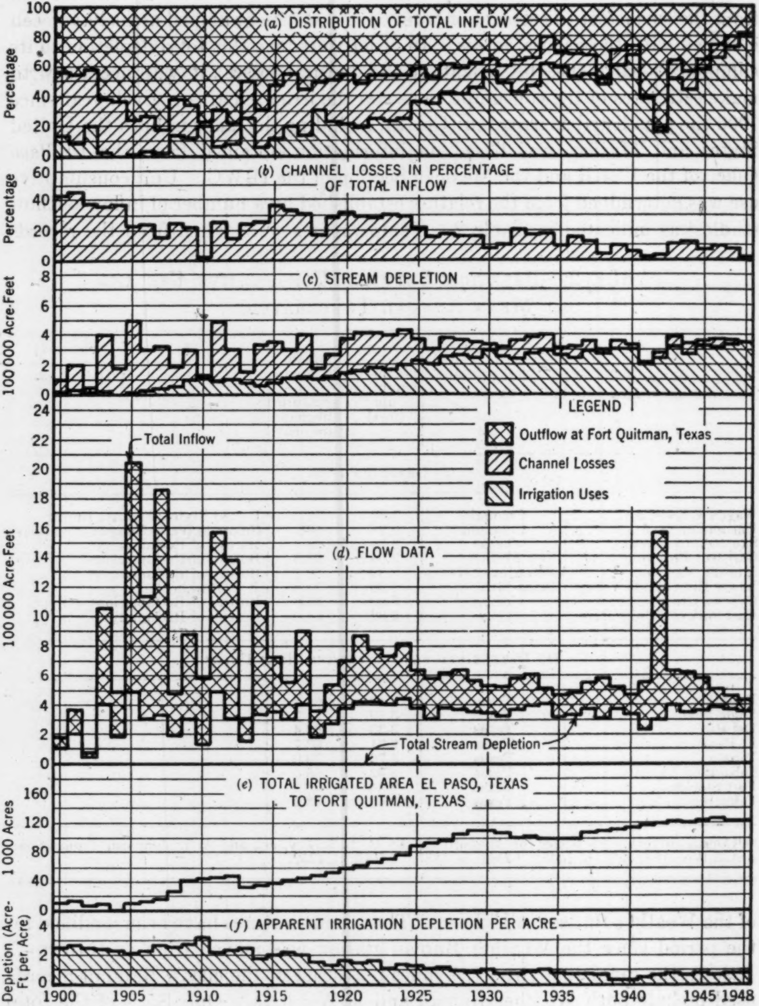

FIG. 1.—STREAM DEPLETION DATA IN THE EL PASO-JUÁREZ VALLEY OF THE RIO GRANDE

different elements of the calculation have been plotted in Fig. 1, which is self-explanatory. The lowest section of the figure shows the depletion caused by irrigation, assuming natural channel losses remained as they were prior to placing control on the river.

Analysis of Results.—Fig. 1 is significant in that it emphasizes the following:

1. The steady expansion in irrigation from 10,000 acres in 1900 to more than 120,000 acres in 1948. The drop in acreage in 1902–1904 was caused by severe drought conditions on the river;
2. The more-or-less constant inflow to the area since Elephant Butte Reservoir (New Mexico) started storing water in 1915; and
3. The constantly decreasing channel losses or nonbeneficial uses.

Irrigation uses that amounted to 25,000 acre-ft in 1900 have increased to as high as 350,000 acre-ft in 1948. Within the period the average annual inflow has been decreasing, as indicated by Table 3. Channel losses, which were practically the same during the early half of the period, have declined rapidly since 1924. Although less and less water is being released for irrigation, the acreage irrigated continues to increase. This expansion is being accomplished by the conversion of what were previously natural losses to beneficial use.

Several factors that have resulted in a saving of water have contributed to this conversion from nonbeneficial to beneficial uses. Prior to 1915 the river was uncontrolled. A major portion of the total runoff in the Rio Grande at El Paso came as floods in the form of snow-melt from the headwaters area. This situation resulted in the usual spring high water, followed by long periods of low flow. In the years when high spring floods occurred, the banks were overtopped, and wide areas of swampland were submerged, with consequent high natural losses. Conversely, in years of small spring floods and no overflow, natural losses were relatively low. Since two drought periods (1902–1904 and 1910–1913) occurred between 1900 and 1914, the average channel losses were somewhat lower than might be expected for the 14 years.

TABLE 3.—INFLOW AND CHANNEL LOSS—EL PASO-JUÁREZ VALLEY

Period	Length of period	AVERAGE ANNUAL QUANTITY IN ACRE-FEET	
		Inflow	Natural channel losses
1900–1914	15	900,000	210,000
1915–1924	10	700,000	215,000
1925–1937	13	562,000	91,000
1938–1948	10	535,000[a]	39,500

[a] Exclusive of 1942 when approximately 1,000,000 acre-ft were spilled from Elephant Butte Reservoir.

Since the beginning of storage in Elephant Butte Reservoir (in 1915) the river control through this section has regulated the annual snow-melt floods. Lands that had previously been swamped, however, continued as heavy water users, and the need for drainage at an early stage in the project became of paramount importance. A drainage program initiated in 1917 was virtually completed by the end of 1924, and the irrigated area practically doubled within this period.

The next 13-yr period (1925 through 1937) was marked by further irrigation expansion accompanied by a great decline in the natural losses. Irrigation increased rapidly until an average of more than 100,000 acres of irrigated land was reached. Natural losses averaged only 42% of the average that prevailed during the preceding period, and as the area became more fully developed the natural losses were more nearly confined to the limits of the channel itself.

By the end of 1937, the channel rectification through the El Paso-Juárez Valley was practically completed by the IBWC and resulted in a further reduction in the natural losses. The rectification project produced a straightening and shortening of the river channel between El Paso and Fort Quitman, reducing the channel length from 155 miles to approximately 88 miles. Data for the 11-yr period (1937–1948) show a still further expansion in irrigated area, to average about 119,000 acres during the period. This increase of 19,000 acres over the previous period was made possible by a reduction in the natural losses from an average of 91,000 acre-ft per yr to 39,500 acre-ft per yr.

TABLE 4

Period	Average irrigated area (acres)	AVERAGE ANNUAL QUANTITIES IN ACRE-FEET		
		Total depletions	Depletions due to irrigation	Outflow at Fort Quitman
1900–1914	22,400	263,600	53,700	637,000
1915–1924	56,200	359,500	144,600	340,300
1925–1937	101,000	357,500	266,500	204,600
1938–1948	119,200	345,400[a]	305,900	189,700[a]

[a] Exclusive of 1942.

The averages of the remaining elements in the calculations have been summarized in Table 4.

A feature that is noteworthy in Table 4 is that the total depletions of river water have remained practically constant since the river was controlled by Elephant Butte Dam. Within that same interval (1915–1948) the irrigated area has increased from 38,000 acres to more than 120,000 acres.

The procedure that has been so successfully employed in this area can be equally well applied to other areas. It involves one or more of the following three items of construction:

a. Regulation of the river by storage;
b. Channel rectification and improvement; and
c. Drainage and canal lining.

Full development of natural resources in the Southwest cannot be reached, and any further expansion in irrigation is limited until the natural losses have been reduced to an economic minimum.

DISCUSSION

Isom H. Hale,[5] and George A. Whetstone,[6] Associate Members, ASCE.—It has long been known that the three-point program defined at the end of this paper is effective. The data in this paper—applying as they do to 15 years of unregulated flow followed by nearly 40 years of progressively increasing regulation—fill a gap in hydrologic literature by demonstrating, quantitatively, a measure of this effectiveness.

Another phase of nonbeneficial use, upstream from El Paso deserves to be recorded. As Mr. Lowry has mentioned, Elephant Butte Dam was finished in 1915, and the reservoir functioned for many years thereafter as a single-purpose seasonal storage facility. From 1938 to 1940 a power plant was added at this dam, and a second dam was built 20 miles downstream, creating Caballo Reservoir with a volume of 346,000 acre-ft of which 246,000 acre-ft were reserved for regulating power releases from Elephant Butte Dam.

Although Caballo Dam and Reservoir have undoutedly been a means of permitting the operation of the power plant at Elephant Butte throughout the year rather than only in irrigation season, they have also had the adverse effect of creating a large lake in an area with a very high evaporation rate. The average annual storage in Caballo Reservoir is 122,400 acre-ft.[7] The surface area at this capacity is about 6,350 acres. The average annual reservoir loss due to evaporation, minus rainfall, from 1943 to 1950, inclusive, was about 42,600 acre-ft, not including losses during winter months to waters in transit from Elephant Butte to Caballo.

It would seem to the writers that a better solution (and one which could still be employed to advantage) would be to substitute for Caballo Reservoir, with its seasonal storage, a new daily or weekly reregulating reservoir with a volume of perhaps 5,000 acre-ft (the maximum daily power releases having been from 2,400 to 3,000 acre-ft) and then to operate Elephant Butte as a pumped-storage installation. A further advantage of this procedure would be that waters stored in Caballo Reservoir following flood periods would not necessarily have to be released to provide storage for power releases during the following off-irrigation season. This may be illustrated by the period following the 1942 season when Caballo Reservoir was almost full and facing a period of power releases from Elephant Butte Dam with no place to store the water. In 1951, there was an extreme need for such water.

It is instructive to compare the past operation of Elephant Butte since 1942 with that outlined in this discussion—that is, using pumped storage facilities at Elephant Butte, in lieu of Caballo Dam, for regulating power releases. The release lost during the period following the 1941–1942 floods, due to power production at Elephant Butte, was at least 75,000 acre-ft. The average annual prevented losses, due to avoiding the annual use of Caballo storage for power releases would have been approximately 20,000 acre-ft during the 8-yr period, 1943–1950, or a total of 160,000 acre-ft. This is very conservative since 20,000 acre-ft are not quite half as much as the average annual loss computed for Caballo Reservoir for the period since 1942. This means that approximately 235,000 acre-ft of additional water would have been in Elephant Butte Reservoir at the beginning of 1951 while at the same time serving project lands with the same volume of water during the period. In addition, the water that was in Caballo at the beginning of the year would have been in Elephant Butte. Due to the increase in stored waters in Elephant Butte, and resulting increase in power heads, the net effect on electrical energy potentials would have been an increase throughout the period with the same quantity of water released. Pumping operations on a day after day basis would have been necessary only during the off-irrigation season, October to February, or on an average of five months of each year.

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[5] Hydr. Engr., International Boundary and Water Comm., El Paso, Tex.

[6] Associate Prof., Civ. Eng., Texas Technological College, Lubbock, Tex.

[7] *Water Bulletin No. 19*, International Boundary and Water Comm., El Paso, Tex.

H. R. McDonald,[8] A. M. ASCE.—The data in this paper demonstrate the magnitude of nonbeneficial depletions in a stream channel under natural or virgin conditions. The methods that have been used so successfully in the El Paso-Juarez Valley undoubtedly could be applied to many other western streams with at least some degree of success.

Mr. Lowry has shown that there has been no significant change in total stream depletions in the El Paso-Juárez Valley since 1915 following the regulation of stream flow by Elephant Butte Reservoir, although the irrigated acreage has increased more than threefold. There is probably no important area not yet developed where such large reductions in nonbeneficial uses could be effected. However, sound planning for the development of water resources will require that wherever salvage can be made economically, such measures must be considered carefully.

TABLE 5.—Total Dissolved Solids (Mean Values, 1931 to 1936)

Gaging station	Tons per acre-ft	Total tons
Below Elephant Butte Dam (No large contiguous irrigated areas in this reach)	0.81	619,909
Leasburg Dam, New Mexico (Large contiguous areas irrigated in this reach. No important tributary inflow)	0.87	647,406
El Paso (Courchesne) (Large contiguous irrigated areas in this reach)	1.22	637,968
Fabens-Tornillo (Large contiguous irrigated areas in this reach)	1.93	496,014
Fort Quitman	2.75	473,129

Col. 10, Table 1, records the outflows at Fort Quitman, which are noted to have been decreasing since about 1905. The average outflow from 1937 to 1948 (189,700 acre-ft) may have equaled or exceeded the "economic minimum" mentioned by Mr. Lowry in his closing paragraph. It is generally recognized that, in order to maintain crop production and to prevent the development of saline or alkali soils, an approximate balance must occur between the average inflow and outflow of the total soluble salts. This balance may be only approximate because of the base exchange complex between chemical constituents contained in the soil and in the water that is applied. Normally the return flow from an irrigation project is much more highly mineralized than the irrigation water applied and contains relatively high percentages of the more soluble salts. However, there is a point beyond which the salt concentration cannot be tolerated permanently. In effect, this fact places a limit on the extent of beneficial uses and thereby the irrigated acreage that can be developed, unless further reductions in nonbeneficial consumptive uses can be made.

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[8] Engr., Hydrology Branch, Project Planning Div., Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo.

Table 5 shows the changes in total dissolved solids at gaging stations along the Rio Grande from Elephant Butte Dam to Fort Quitman.[9]

Robert L. Lowry,[10] M. ASCE.—In general, the discussions of this paper have indicated general agreement with the main theme of the paper, which is that—future expansion in irrigation throughout the Southwest is mostly limited to such savings in present nonbeneficial uses of water as may be possible. As stated by Messrs. Hale and Whetstone, there is ample opportunity to effect a saving in some instances by a more efficient operation of the existing reservoir systems. The method outlined for a possible saving of water on the Rio Grande, which would greatly benefit the irrigated area that is the subject of this paper, shows promise of something worth while. It is to be hoped that the authorities of the facilities involved will give the suggestion proper consideration.

The question raised by Mr. McDonald concerning the salt balance in the irrigated lands in this reach of the Rio Grande applies to all irrigation projects. The data on total dissolved solids cited in Table 5 were for the period prior to 1936. Later comparable data[11] show some improvement in the salt balance in this area. Table 6 shows that change in total dissolved solids for some of these same stations for the period from 1931 through 1949.

TABLE 6.—TOTAL DISSOLVED SOLIDS (MEAN VALUES, 1931 TO 1949, INCLUSIVE)

Gaging station	Tons per acre-ft	Total tons
Below Caballo Dam	0.71	589,000
Leasburg Dam	....	610,000
El Paso	1.11	634,000
Fabens-Tornillo	....	....
Fort Quitman	2.31	543,000

Of greater interest is the fact that operation of the project during 15 years following publication of the data for 1936 showed the situation to be improving.

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[9] "The Rio Grande Joint Investigation," National Resources Planning Committee, Part VI, Vol. 1, pp. 458–463.

[10] Consulting Eng., Austin, Tex.

[11] *Water Bulletin No. 19*, 1949 International Boundary and Water Comm., El Paso, Tex., pp. 63–64.

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GROUND-WATER MOVEMENT CONTROLLED THROUGH SPREADING

By Paul Baumann,[1] M. ASCE

With Discussion by Messrs. David K. Todd, Max Suter,
D. P. Krynine, and Paul Baumann

Synopsis

Over a period of years the Los Angeles County Flood Control District in California, under the writer's direction, has been engaged in water conservation by the diversion of clear stream flow to off-channel areas, causing the water to percolate into the soil from shallow basins and thereby adding it to the ground-water supply. This operation is known as spreading, and the areas upon which it is performed are called spreading grounds.[2]

In the course of spreading a mound will be formed above the initial ground-water table. The mound will be two dimensional or three dimensional, depending on the shape of the spreading ground. The recharge of ground water through wells is a special, three-dimensional case. The control of sea water intrusion into ground-water basins is of particular significance. The configuration of the mound, its limiting magnitude, its effective, temporary water retention both as to volume and time, its flow net for potential flow, and the drainage of the water stored in the mound are analyzed in this paper.

Finally, the theoretical results are compared with results derived from model tests on two-dimensional flow.

Introduction

The mound is two dimensional if spreading is done from strip areas and three dimensional if the area approximates a circle. The height H to which a mound may rise during a spreading operation depends on: (1) Rate (q_o) of spreading—that is, the rate of infiltration; (2) duration (T) of spreading; (3) permeability (K) of the soil; (4) porosity (μ) of the soil; (5) vertical distance (D) between the initial ground-water surface and the ground surface;

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[2] "Correlating Flood Control and Water Supply, Los Angeles Coastal Plain," by Finley B. Laverty, *Transactions*, ASCE, Vol. 111, 1946, p. 1127.

(6) depth (a_o) between initial ground-water surface and impervious stratum; (7) slope (i) of the impervious stratum; and (8) horizontal distance (L_d) to a control through which the downstream boundary conditions are uniquely established.

The mound phenomenon must be divided into four distinct phases. The first three occur during its growth and the fourth phase during its disappearance after discontinuation of spreading.

The First Phase.—The first phase consists of a mound wave created when the inflowing water is being put into storage, necessarily above the initial ground-water surface. For constant inflow q_o this wave must grow in magnitude and thereby represents a case of unsteady flow. The growth of this wave continues until a boundary is reached. The boundary in regard to x is called the "lateral control," as represented by a river, a lake, or a trench. The boundary in regard to y is called the "potential control." It is the surface of the spreading basin.

The Second Phase.—The second phase is transitory. It begins when: (a) The wave front reaches the lateral control, or (b) the wave crest reaches the potential control. In case (a), the wave length L has reached its maximum value. Discharge now begins and increases until it is equal to the inflow q_o. In case (b), the height H of the wave remains constant while the length L increases and, in the absence of a lateral control, theoretically becomes infinite. Hence, all the flow is put into storage at a steadily decreasing rate.

The Third Phase.—In the presence of a lateral control, stationary flow will occur during the third phase. Under these conditions potential flow will develop, and the storage in the mound will reach its maximum for a constant inflow q_o. It may greatly exceed available surface storage.

The Fourth Phase.—If the mound has reached equilibrium in the presence of a lateral control and if spreading is discontinued suddenly, it will drain itself in the course of time. If the mound has reached the potential control in the absence of a lateral control, and if spreading has subsequently been continued until, theoretically, the inflow q_o has become zero and the wave length L has become infinity, then this mound will never drain itself as long as there is no extraction from it. This serves to emphasize the fact that the drainage phenomenon is primarily related to the existence of a lateral control.

General Conditions.—The mathematical analysis of the mound phenomenon is based on the Darcy law[3,4] of percolation of water through sand. This law is valid as long as the flow is laminar. Laminar flow is governed by the Reynolds[5,6] number **R**. M. Muskat[7] and others have found that, for sand, laminar flow takes place if $R \leqq 1$.

Furthermore, the idealized soil is assumed to be homogeneous as well as isotropic. Hence, the permeability remains constant irrespective of the direc-

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[3] "Les fontaines publiques de la ville de Dijon," by Henri Darcy, Dalmont, Paris, France, 1856.

[4] "Recherches physica-mathematiques sur la théorie du mouvement des eaux courantes," by G. C. F. M. R. de Prony, *Mémoires de l'Institute National*, Paris, France, 1804.

[5] "Papers on Mechanical and Physical Subjects," by Osborne Reynolds, C. J. Clay and Sons, Cambridge, England, 1903.

[6] "The Two Manners of Motion of Water," by Osborne Reynolds, *Proceedings*, Royal Inst. of Great Britain, Vol. XI, 1884–1886, p. 44.

[7] "The Flow of Homogeneous Fluids Through Porous Media," by M. Muskat, McGraw-Hill Book Co., Inc. New York, N. Y., 1st Ed., 1937.

tion of flow which, admittedly, is not in keeping with natural soils. However, spreading grounds are located in soils that approach idealized conditions extending over great distances. Within these distances permeability and porosity of the soil and slope of the impervious stratum may be expressed by average values with satisfactory results.

Generally, the storage in the part of the mound directly beneath the spreading ground is small as compared to the total storage. Therefore, it may be neglected except in special cases.

The slope (i) of the impervious stratum is generally small, namely, of the order $\leqq 0.01$.

PART I.—THE TWO-DIMENSIONAL FLOW OF WATER THROUGH GRANULAR SOIL

1.—Analysis of Unsteady Ground-Water Flow

Outstanding among the classical presentations on unsteady ground-water flow is the one due to M. J. Boussinesq.[8] He derived the differential equation—

$$\frac{\partial y}{\partial t} = \frac{K}{\mu}\left[y\,\frac{\partial^2 y}{\partial x^2} + \left(\frac{\partial y}{\partial x}\right)^2 - i\,\frac{\partial y}{\partial x}\right] \ldots\ldots\ldots\ldots\ldots (1)$$

—which expresses this phenomenon and in which x and y are the coordinates of the free water surface. If the flow becomes steady or stationary, Eq. 1 reduces to

$$q_o = K\,y\left(i - \frac{\partial y}{\partial x}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

which is known as the Dupuit[9] equation or theorem. The validity of Eq. 2 requires that the vertical components of the velocity vectors are small as compared to the horizontal ones. Where such is the case, calculations based on the Dupuit equation lead to satisfactory solutions.

Eq. 1 is not integrable by exact calculus methods. Considering the fact that, normally in ground-water flow, the slope of the water surface and the height of the mound relative to the initial ground-water depth a_o are small, Boussinesq proceeds in neglecting $\left(\frac{\partial y}{\partial x}\right)^2$ and in making $y = a_o$. Hence, Eq. 1 reduces to

$$\frac{\partial y}{\partial t} = \frac{K}{\mu}\left(a_o\,\frac{\partial^2 y}{\partial x^2} - i\,\frac{\partial y}{\partial x}\right) \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Considering the further fact, however, that in connection with ground-water phenomena related to spreading, the term $i\,\frac{\partial y}{\partial x}$ is normally of the same order of magnitude as $\left(\frac{\partial y}{\partial x}\right)^2$ if, indeed, it is not generally smaller; $i\,\frac{\partial y}{\partial x}$ appears negligible with equal justification. Actually these terms are of opposite sign so long as i and $\frac{\partial y}{\partial x}$ are of equal sign. Their difference, then, is of higher order of smallness. Hence the neglect of both terms is, probably, a closer approximation than the neglect of $\left(\frac{\partial y}{\partial x}\right)^2$ only. The exception to this rule and its significance will be referred to in its proper place. With this neglect, or for $i = 0$, in Eq. 3 there results

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[8] "Essai sur la théorie des eaux courantes," by M. J. Boussinesq, *Memoires à l'Académie des Sciences de l'Institut de France*, Vol. 23, 1877, p. 257.

[9] "Etudes théoriques et pratiques sur le mouvement des eaux courantes," by Jules Dupuit Carilian-Goeury, Paris ,France, 1948.

$$\frac{\partial y}{\partial t} = K \frac{a_o}{\mu} \frac{\partial^2 y}{\partial x^2} \qquad (4)$$

which is identical in form with the well-known differential equation for the flow of heat through a nonradiating prismatic bar. Boussinesq referred to this analogue three quarters of a century ago.

Heat Analogue Solution; Bar of Finite Length[10] (*First, Second, and Fourth Phase*).—Based on the analogue of the flow of heat through a prismatic, nonradiating bar of length L_d with a continuous and constant heat source at the origin $x = 0$ (spreading ground) and constant temperature at the opposite end $x = L_d$ (control), the following solution for the first, second, and fourth phase of the mound phenomenon was developed for $i \neq 0$.

The Growing Mound Between the Spreading Ground and the Control.—To simplify typography, let:

$$j = \frac{K\, a_o\, t}{\mu} \qquad (5)$$

It may be verified through differentiation that

$$y = C_1\, e^{-\beta^2 j} \cos \beta\, x + C_2\, x + C_3 \qquad (6)$$

is a particular solution of Eq. 4 in which C_1, C_2, C_3, and β are arbitrary constants. It does not satisfy the boundary conditions for the growing mound phenomenon, however. Based on Fig. 1, these conditions are

1. $y' = 0$ at $t = 0$ for all values of x;
2. $y' = 0$ at $x = L_d$ for all values of t in which $0 < L_d < \infty$; and
3. $y' = f(x)$ at $t = \infty$.

Based on the fundamental concept that $y' \ll a_o$ upon which Eq. 4 is predicated, the linear equation for $f(x)$

$$y' = \left(\frac{q}{K\, a_o} - i\right)(L_d - x) \qquad (7)$$

is the approximate equation for the stable mound at $t = \infty$. The relations—

$$\beta = \frac{(2\,n - 1)\,\pi}{2\,L_d} \qquad (8a)$$

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[10] "Conduction of Heat in Solids," by H. S. Carslaw and J. C. Yaeger, Oxford Univ. Press, Oxford, England, 1947.

and

$$C_2 \, L_d = - \, C_3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8b)$$

—satisfy condition 2; and

$$C_2 = - \left(\frac{q}{K \, a_o} - i \right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8c)$$

satisfies condition 3. Hence Eq. 6 becomes:

$$y' = C_1 \, e^{-\beta^2 i} \cos \frac{(2 \, n - 1) \, \pi}{2 \, L_d} x + \left(\frac{q}{K \, a_o} - i \right) (L_d - x) \ldots\ldots (9)$$

Eq. 9 still does not satisfy condition 1, however.

A Fourier series expansion for the equation of the stable mound is

$$y' = \left(\frac{q}{K \, a_o} - i \right) (L_d - x) = \frac{8 \left(\frac{q}{K \, a_o} - i \right) L_d}{\pi^2} \sum_{n=1}^{n=\infty} \frac{1}{(2 \, n - 1)^2} \times \cos \frac{(2 \, n - 1) \, \pi \, x}{2 \, L_d} \ldots (10a)$$

and the final equation then becomes

$$y' = \left(\frac{q}{K \, a_o} - i \right) (L_d - x) - \frac{8 \left(\frac{q}{K \, a_o} - i \right) L_d}{\pi^2} \sum_{n=1}^{n=\infty} \frac{1}{(2 \, n - 1)^2} \times \left(\cos \frac{(2 \, n - 1) \, \pi \, x}{2 \, L_d} \right) e^{-\beta^2 i} \ldots (10b)$$

which satisfies all of the boundary conditions.

The Disappearing Mound Between the Spreading Ground and the Control.—The boundary conditions now become:

(*a*) $y' = \left(\frac{q}{K \, a_o} - i \right) (L_d - x)$ at $t = 0$;
(*b*) $y' = 0$ at $x = L_d$ for all values of t; and
(*c*) $y' = 0$ at $t = \infty$ for all values of x.

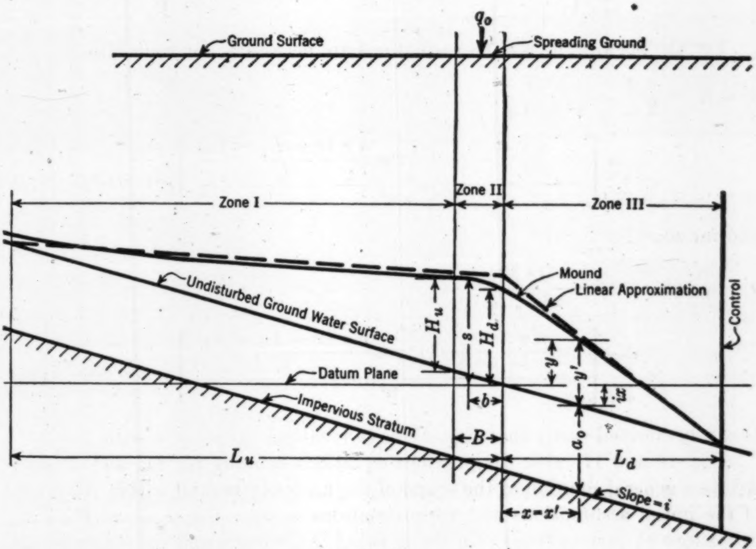
Eq. 10*b* indicates that condition (*a*) is satisfied if:

$$y' = \frac{8 \, (q - K \, a_o \, i) \, L_d}{K \, a_o \, \pi^2} \sum_{n=1}^{n=\infty} \frac{1}{(2 \, n - 1)^2} \left(\cos \frac{(2 \, n - 1) \, \pi \, x}{2 \, L_d} \right) e^{-\beta^2 j} \ldots (11)$$

and that conditions (*b*) and (*c*) are likewise satisfied.

The General Case of the Unsteady Mound for $i \neq 0$.—The foregoing was predicated on the presence of discontinuity immediately upstream from the spreading ground, for example, in the form of a basin barrier. In the absence of such a discontinuity the growth of the mound downstream from the spreading ground gives rise to a backwater condition upstream from it. Hence, there are three distinct zones to be considered as indicated in Fig. 1. The true configuration of the stable mound will be analyzed in a subsequent section. In the following, the mound surface in zone II is considered parallel to, and the distance H_d above, the initial ground-water surface. For a constant ground-water flow $q = K\,a_o\,i$:

$$K\,a_o\,i = K\,(a_o + y')\left(i - \frac{d\,(a_o + y')}{dx}\right) \qquad (12a)$$



q_o
Ground Surface
Spreading Ground
Zone I
Zone II
Zone III
Mound
Linear Approximation
Control
Undisturbed Ground Water Surface
Datum Plane
Impervious Stratum
H_u
s
H_d
y
y'
ix
b
B
a_o
L_u
L_d
Slope = i
$x = x'$

FIG. 1

Upon integration, Eq. 12a results in:

$$-\,i\,(x - B) = (H_d - y') - a_o \log_e \frac{y'}{H_d} \qquad (12b)$$

in which B = width of spreading ground. Since $2 > \frac{y'}{H_d} > 0$:

$$\log_e \frac{y'}{H_d} = \frac{y' - H_d}{H_d} - \frac{1}{2}\left(\frac{y' - H_d}{H_d}\right)^2 + \frac{1}{3}\left(\frac{y' - H_d}{H_d}\right)^3 \qquad (13)$$

After neglecting B and all but the first term, an approximate, linear equation is:

$$y' = \frac{H\,i}{a_o + H}\,(L_u + x) \qquad (14)$$

in which $0 < i < 0.01$.

The stable mound will now be considered approximated by Eqs. 7 and 14, as indicated by dash lines in Fig. 1. A Fourier series, to describe this stable mound in one equation, with the origin at the spreading ground, is

$$y' = \left(\frac{L}{\pi}\right)^2 \frac{H}{L_u\,L_d} \sum_{n=1}^{n=\infty} \frac{\cos\frac{n\,\pi\,x}{L} - (-1)^n \cos\frac{n\,\pi\,(x + L_u - L_d)}{L}}{n^2} \qquad (15)$$

in which $-L_u < x < L_d$ and $L_u + L_d = L$. An equation which satisfies Eq. 4 can be written for the disappearing mound.

$$y' = \left(\frac{L}{\pi}\right)^2 \frac{H}{L_u\, L_d} \times \sum_{n=1}^{n=\infty} \left[\frac{\cos \frac{n\,\pi\,x}{L} - (-1)^n \cos \frac{n\,\pi\,(x + L_u - L_d)}{L}}{n^2} \right] e^{-(n\pi/L)^2 j} \quad \text{..(16)}$$

For the growing mound two equations are necessary, for zone III—

$$y' = H - \frac{q_o\, x}{K\, a_o} - \frac{L^2\, H}{\pi^2\, L_u\, L_d} \times \sum_{n=1}^{n=\infty} \left[\frac{\cos \frac{n\,\pi\,x}{L} - (-1)^n \cos \frac{n\,\pi\,(x + L_u - L_d)}{L}}{n^2} \right] e^{-(n\pi/L)^2 j} \quad \text{..(17a)}$$

and for zone I—

$$y' = \frac{H}{L_u}\,(L_u + x) - \frac{L^2\, H}{\pi^2\, L_u\, L_d} \times \sum_{n=1}^{n=\infty} \left[\frac{\cos \frac{n\,\pi\,x}{L} - (-1)^n \cos \frac{n\,\pi\,(x + L_u - L_d)}{L}}{n^2} \right] e^{-(n\pi/L)^2 j} \quad \text{..(17b)}$$

It can be verified easily that Eqs. 17 have identical values at $x = 0$.

Eqs. 16 and 17, with the previous equations defining L_d, L_u, and H, constitute a general solution of the shape of the unsteady mound within the limits of the initial assumptions and approximations made. Fig. 2 shows the configurations at various times t for the growing (dash lines) and the disappearing (solid lines) mound for the following numerical values: $L_d = 4$ ft; $q_o = 0.82$ cu ft per hr; $K = 25$ ft per hr; $a_o = 0.5$ ft; and $\mu = 0.15$ for a growing mound (dash line in Fig. 2) and 0.06 for a draining mound (solid line in Fig. 2). These values correspond to those in the comprehensive model tests, except that $i \neq 0$. It is interesting to note a division of flow from the spreading ground between zones I and III during part of the growth. This conforms to the requirement of synchronous growth of the two zones at $x = 0$. The initial ground-water flow is checked during this period and goes entirely into storage in zone I. This feature is of particular significance in the control of sea water intrusion. It shows that the latter is checked when the infiltrated fresh water reaches the saline ground-water table. To maintain control indefinitely the rate of infiltration must be such as to cause rise of the mound to at least mean sea level. The use of recharge wells for this purpose will be discussed in the chapter on three-dimensional flow.

Eq. 4, which forms the basis for this solution, holds true strictly for $i = 0$ only, although it should lead to satisfactory results for $i \neq 0$ so long as $\left(\frac{\partial y}{\partial x}\right)^2$ and $i\,\frac{\partial y}{\partial x}$ are small, of the same order of magnitude and opposite in sign. The latter condition is not satisfied during just the aforementioned initial period of infiltrated flow into zone 1. However, the influence on the mound configuration of this discrepancy is not important in the light of the mathematical simplification thus achieved. Hence, mathematically, the base of the triangular mound is horizontal. This gives rise to a fictitious outflow at the upstream end and a discontinuity in slope because of the approximation of the upstream curve by a straight line. To conform to the physical system, the mathematical system must be superposed on the sloping base. The accumulated sum of the mathematical outflows at the two ends corresponds to the effective pore volume in the triangular mound, drained during time *t*.

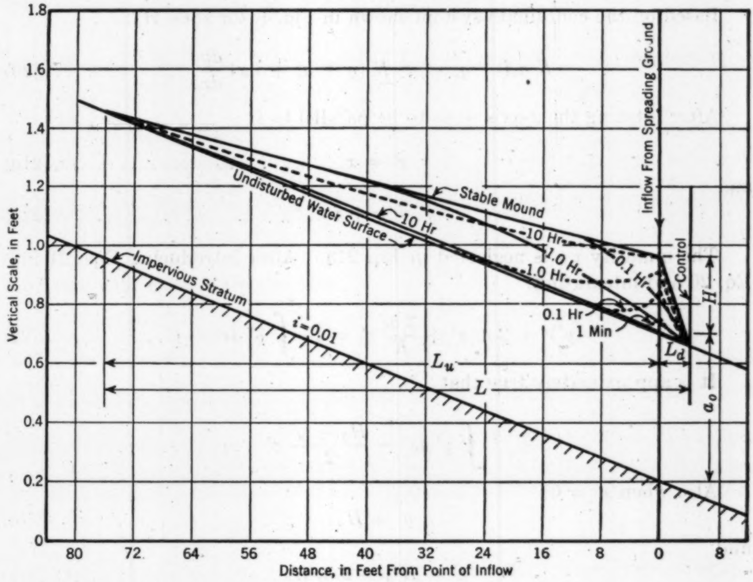


FIG. 2

The flow at any section is:

$$q = K\,a_o\left(i - \frac{\partial y'}{\partial x}\right) - K\,a_o i = -\,K\,a_o\,\frac{\partial y'}{\partial x} \quad \text{..........(18)}$$

Hence, the outflow at the control during the disappearance is

$$q_d = \frac{2\,K\,a_o\,L\,H}{\pi\,L_u\,L_d}\sum_{n=1}^{n=\infty}\frac{\left(\sin\frac{n\,\pi\,L_d}{L}\right)(1-(-1)^n)}{n}\,e^{-(n\pi/L^2)\,i} \quad \text{....(19a)}$$

and during the growth

$$q_g = q_o - \frac{K\,a_o\,L\,H}{\pi\,L_u\,L_d}\sum_{n=1}^{n=\infty}\left[\frac{2\sin\frac{n\,\pi\,L_d}{L}}{n}\right]e^{-(n\pi/L)^2 j} \quad \text{.......(19b)}$$

For $L_u = L_d$ Eqs. 16, 17, and 19 express the conditions for the symmetrical mound ($i = 0$).

2.—THE TRUE CONFIGURATION OF THE STABLE MOUND FOR $i \neq 0$

Based on the coordinate system shown in Fig. 1, for zone III:

$$K\,a_o i + q_o = -K\,(y + a_o + i\,x)\,\frac{dy}{dx} \qquad (20)$$

After rotating the x-axis so as to be parallel to i

$$x' = x \qquad (21a)$$

and

$$y' = y + i\,x \qquad (21b)$$

The quantity $i\,y$ is neglected in Eq. 21a. After introducing Eqs. 21 into Eq. 20 and integrating:

$$(y')^2 + 2\,a_o\,y' + \frac{2\,q_o}{K}\,x' = 2\,i \int y'\,dx' + C \qquad (22)$$

It is approximately true that

$$\int y'\,dx' = \frac{H_d + y'}{2}\,x' \qquad (23)$$

Also when $x' = 0$:

$$y' = H_d \qquad (24a)$$

and

$$C = H^2_d + 2\,a_o\,H_d \qquad (24b)$$

Substituting Eqs. 23 and 24 in Eq. 22 and solving for y':

$$y' = -\left(\frac{2\,a_o - i\,x'}{2}\right) + \sqrt{\left(\frac{2\,a_o - i\,x'}{2}\right)^2 - \left[\frac{2\,q_o\,x'}{K} - ((H_d + a_o)^2 - a^2_o) - i\,x'\,H_o\right]} \qquad (25)$$

For $x' = L'$, $y' = 0$. Hence:

$$H_d = -\left(\frac{2\,a_o + i\,L'}{2}\right) + \sqrt{\left(\frac{2\,a_o + i\,L'}{2}\right)^2 + \frac{2\,q_o\,L'}{K}} \qquad (26)$$

For zone II:

$$K\,a_o i + \frac{q_o}{B}\,(B + b) = -K\,(s + a_o + i\,b)\,\frac{ds}{db} \qquad (27)$$

Hence

$$\frac{ds}{db} = -\left[\frac{\dfrac{q_o\,b}{B\,K} + \dfrac{q_o}{K} + a_o i}{i\,b + s + a_o}\right] \qquad (28)$$

which is a linear, fractional, nonhomogeneous, differential equation. Substituting $s = s_1 + M$ and $b = b_1 + N$ in Eq. 28 and solving for M and N result in the homogeneous equation:

$$\frac{ds_1}{db_1} = - \frac{q_o\, b_1}{B\, K\, (i\, b_1 + s_1)} \qquad \text{(29)}$$

Introducing $U = \frac{b_1}{s_1}$ and $db_1 = s_1\, dU + U\, ds_1$; integrating and neglecting i^2 and i in $\frac{2\, q_o\, b_1}{B\, K\, s_1} + i$:

$$0 = \log_e \left[\frac{\frac{q_o\, b^2_1}{B\, K} + i\, b_1\, s_1 + s^2_1}{C^2} \right] - \frac{i}{\sqrt{\frac{q_o}{B\, K}}} \tan^{-1} \frac{\sqrt{\frac{q_o}{B\, K}}}{s_1}\, b_1 \qquad \text{(30)}$$

in which, with close approximation:

$$C^2 = \frac{B \left(\frac{q_o}{K} + 2\, a_o\, i \right) + \left((H_d + a_o) - \frac{B\, i}{2} \right)^2}{e^u} \qquad \text{(31)}$$

in which (to simplify typography):

$$u = \frac{i}{\sqrt{\frac{q_o}{B\, K}}} \tan^{-1} \frac{\sqrt{\frac{q_o\, B}{K}} \left(1 + \frac{K\, a_o\, i}{q_o} \right)}{(H_d + a_o) - i\, B}$$

The backup curve in zone I is determined by Eq. 12*b*. Hence the configuration of the stable mound for $i \neq 0$ is a steady curve, expressed by Eqs. 25, 30, and 12 and shown in Fig. 1.

3.—APPROXIMATE EQUATIONS FOR UNSTEADY GROUND-WATER FLOW

The Unique Flow Function—First Phase (*Zone III*).—In view of the limitations imposed upon Eq. 3, it is axiomatic that the respective solutions are valid only within these limitations, namely, $y \ll a_o$ and $\frac{dy}{dx}$ and i is very small. Although the latter condition is reasonably satisfied in connection with these phenomena, the former quite often is not. Hence, it was desirable to find a unique function $q = f(x)$ for unsteady flow which is in close agreement with those based on Eq. 3 within the imposed limitations but which permits extrapolation beyond these limits. This function must satisfy the following conditions:

1. $q = q'$ at $x = 0$, in which q' is that portion of q_o, flowing into zone III;
2. $q = 0$ at $x = L$;
3. $\frac{dy}{dx} = 0$ at $x = L$; and
4. q must fade away as $x \rightarrow L$.

The first three conditions are satisfied by a multitude of functions, whereas an exponential function is required to satisfy condition 4. All four conditions are satisfied by:

$$q = q' \left(1 - \frac{e\,x}{L}\, e^{-x/L}\right) \text{.................} (32)$$

In accordance with Fig. 3:

$$K\, a_o\, i + q = -\,(y + a_o + i\,x)\, K\, \frac{dy}{dx} \text{.............} (33)$$

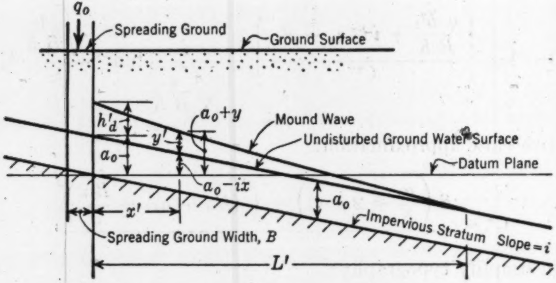

q_o
Spreading Ground
Ground Surface
h'_d
a_o+y
Mound Wave
y'
Undisturbed Ground Water Surface
a_o
Datum Plane
a_o-ix
a_o
x'
Impervious Stratum
Slope = i
Spreading Ground Width, B
L'

FIG. 3

After rotating the axes so as to make the x-axis parallel to i:

$$x' = x \text{..........................} (34a)$$

since $i\,y$ may be neglected again and

$$y' = y + i\,x \text{......................} (34b)$$

Hence:

$$\frac{dy}{dx} = \frac{dy'}{dx'} - i \text{.........................} (35)$$

and

$$\frac{q}{K}\, dx' = -\,(y' + a_o)\, dy' + y'\, i\, dx' \text{..............} (36)$$

Substituting for q and integrating

$$\frac{q'}{K}\left(x' + (x' + L')\, e^{(L'-x')/L'}\right) + \frac{(y')^2}{2} + y'\, a_o - i \int_o^{x'} y'\, dx' = C \text{..} (37)$$

The last member on the left side (which represents a small fraction of the area of the mound wave) may be simplified; thus

$$\int_o^{x'} y'\, dx' = \left(\frac{h'_d + y'}{2}\right) x' \text{................} (38)$$

and

$$(y')^2 + 2\,a_o\,y' + \frac{2\,q'}{K}\,(x' + (x' + L')\,e^{(L'-x')/L'}) - i\,x'\,(h'_d + y') = C \ldots (39)$$

For $x' = 0$, $y' = h'_d$ and $C = h'^2_d + 2\,a_o\,h'_d + \frac{2\,q'}{K}\,L'\,e$; therefore:

$$y' = -\left(\frac{2\,a_o - i\,x'}{2}\right) + \sqrt{\left(\frac{2\,a_o - i\,x'}{2}\right)^2 - \left\{\frac{2\,q'}{K}\left[x' + e\,L'\left(\left(\frac{x'}{L'} + 1\right)e^{-x'/L'} - 1\right)\right] - ((h'_d + a_o)^2 - a^2_o) - i\,x'\,h'_d\right\}} \,..(40)$$

For $x' = L'$, $y' = 0$, and

$$h'_d = -\left(\frac{2\,a_o + i\,L'}{2}\right) + \sqrt{\left(\frac{2\,a_o + i\,L'}{2}\right)^2 + \frac{2\,q'\,L'}{K}\,(1 + (2 - e))} \ldots (41)$$

The quantity q' flows into zone III. The remainder of q_o flows into storage in zones II and I. Except for a short initial period, the storage in zone II may be neglected. Hence q' follows from the condition of synchronized rise at $x' = 0$. For $i = 0$, and neglecting zone II, $q' = q_o$.

The Unique Time Function—Second Phase.—

Case (a).—At the instant the mound wave reaches the lateral control at $x' = L'$, the discharge there is zero, and the configuration of the wave is expressed by Eq. 40. On the other hand the configuration of the stable mound is expressed by Eq. 25. If the approximate time T_2 of the transition is to be finite (for example, the time during which 95% of the transition takes place) then within this period the term $e\,L'\left(\left(\frac{x'}{L'} + 1\right)e^{-x'/L'} - 1\right)$ in Eq. 40 must fade away; h'_d must change to H_d (which is accomplished if $(2 - e)$ in Eq. 41 fades away); and q' must change to q_o. Furthermore, at any time t the total inflow must equal the total storage plus the total outflow.

The unique time function that satisfies these conditions is:

$$f(t) = 1 - \frac{e\,t}{T_2}\,e^{-t/T_2} \ldots\ldots\ldots\ldots (42)$$

and the mound equation for the second phase is

$$y' = -\left(\frac{2\,a_o - i\,x'}{2}\right) + \sqrt{\left(\frac{2\,a_o - i\,x'}{2}\right)^2 - \left\{\frac{2\,x'}{K}\left[q_o + q'\left[\left(1 + \frac{1}{x'}\right)\left(e\,L'\left(\frac{x'}{L'} + 1\right)e^{-x'/L'} - 1\right) - q_o\right]\left(1 - \frac{e\,t}{T_2}\,e^{-t/T_2}\right)\right] - \left((h'_d - a_o)^2 - a^2_o\right) - i\,x'\,h'_d\right\}} \,..(43)$$

in which:

$$h'_d = -\left(\frac{2\,a_o - i\,L'}{2}\right) + \sqrt{\left(\frac{2\,a_o - i\,L'}{2}\right)^2 + \frac{2\,L'}{K}\left[q_o + \left(q'\,(3 - e) - q_o\right)\left(1 - \frac{e\,t}{T_2}\,e^{-t/T_2}\right)\right]} \,..(44)$$

Case (b).—If the mound wave rises to the potential control in the absence of a lateral control, then from Eq. 41, it becomes evident that for the wave height h'_d to remain constant, the product $q'\,L' = q_o\,L'$ must remain constant. Hence, as the wave continues to grow in length, the inflow q_o must diminish proportionally. This is strictly true for $i = 0$ and nearly so for $i\,L' \ll 2\,a_o$.

From a practical point of view this significance of the wave reaching the surface is that "waterlogging" of the ground may become critical, not only upstream from the spreading ground but also downstream from it.

The Approximate Time Requirement.—

First Phase.—Since the initial ground-water flow ($K\,a_o\,i$) remains unchanged throughout the wave configuration, it is evident that the water stored in zone III must be caused entirely by the flow q' and that, in order to satisfy continuity:

$$V'_1 = \mu \int_o^{L'} y'\,dx' = \int_o^{T'_1} q'\,dt \tag{45}$$

in which: V'_1 = pore volume; and T'_1 = time required to create wave length L' and height h'. The exact integration of Eq. 45 would be a formidable task so long as i is a factor in Eq. 40 and q' is variable. A graphical solution would be called for whereby the relation between t, q', h', and L', for arbitrary slopes i, could be established with close approximation.

Actually the influence on the final result of $0 < i < 0.01$ is such that it does not differ significantly from that due to $i = 0$.

For $i = 0$, Eq. 40 reduces to:

$$y = -\,a_o + \sqrt{(h + a_o)^2 - \frac{2\,q_o}{K}\left[x + e\,L\left(\left(\frac{x}{L} + 1\right)e^{-x/L} - 1\right)\right]} \tag{46}$$

and Eq. 41 to

$$h = -\,a_o + \sqrt{a^2_o + \frac{2\,q_o\,L}{K}\,(3 - e)} \tag{47}$$

The determination of the pore volume of the mound wave as represented by Eq. 46 is facilitated by expanding it into a binomial which converges for all values $x < L$, $(h + a_o) > \dfrac{2\,q_o}{K}\left[x + e\,L\left(\left(\dfrac{x}{L} + 1\right)e^{-x/L} - 1\right)\right]$. Hence,

$$y = h - \frac{q_o}{K\,(a_o + h)}\left[x + eL\left(\left(\frac{x}{L} + 1\right)e^{-x/L} - 1\right)\right] \tag{48}$$

For $y = 0$ and $x = L$, from Eq. 48:

$$h^2 + a_o\,h = \frac{q_o\,L}{K}\,(3 - e) \tag{49a}$$

whereas, from Eq. 47:

$$h^2 + 2\,a_o\,h = \frac{2\,q_o\,L}{K}\,(3 - e) \tag{49b}$$

It is obvious that, for $a_o = 0$, h^2 from Eq. 47 is just twice that from Eq. 48. Therefore Eq. 47 is closely approximated by

$$h = \frac{\breve{c}\,q_o\,L}{K\,(a_o + h)}\,(3 - e) \tag{50}$$

if the values shown in Fig. 4 are used for $\bar{c}$. This coefficient is not applicable to Eq. 48 if $x \neq L$, because $\bar{c}$ would have to be a function of x for $a_o \leqq h$. However, for $a_o > h$, Eq. 48 is a satisfactory approximation if $\bar{c} = 1$. The

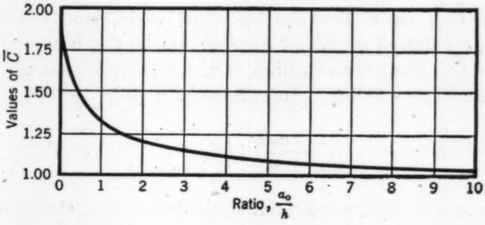


FIG. 4

volume V_1, based on Eq. 48 and $a_o > h$, is:

$$V_1 = \mu \int_o^L y\,dx = \mu \int_o^L \times \left\{ h - \frac{q_o}{K\,(h + a_o)} \left[x + e\,L \left(\left(\frac{x}{L} + 1 \right) e^{-x/L} - 1 \right) \right] \right\} dx = q_o\,T_1 \quad (51)$$

and upon integration

$$T_1 = \mu \left[\frac{L\,h}{q_o} - \frac{L^2}{K\,(h + a_o)} \left(e - \frac{5}{2} \right) \right] \quad (52)$$

The Second Phase, Case (a).—The binomial expansion of Eq. 43 for $a_o > h$ and $i = 0$

$$y = h - \frac{q_o}{K\,(h + a_o)} \left[x + e\,L \left(\left(\frac{x}{L} + 1 \right) e^{-x/L} - 1 \right) \left(1 - \frac{e\,t}{T_2} e^{-t/T_2} \right) \right] \quad (53)$$

The outflow q at $x = L$, at any time, t, is:

$$q = -\,K\,a_o \left(\frac{dy}{dx} \right)_L \quad (54)$$

and the volume ΔV_d of water discharged during t is:

$$\Delta V_d = \int_o^t q\,dt - K\,a_o \int_o^t \left(\frac{dy}{dx} \right)_L dt \quad (55)$$

Since t is not affected in regard to x

$$\left(\frac{dy}{dx} \right)_L = -\,\frac{\bar{c}\,q_o}{K\,(h + a_o)} \left(\frac{e\,t}{T_2} e^{-t/T_2} \right) \quad (56)$$

and from Eq. 55

$$\Delta V_d = +\,\frac{\bar{c}_m\,q_o\,a_o\,e}{(h_m + a_o)} \int_o^t \frac{t}{T_2} e^{-t/T_2}\,dt \quad (57)$$

in which $h_m = \frac{h + H}{2}$ is constant in order to make integration possible, and $\bar{c}_m$ is determined from Fig. 4 for $\frac{a_o}{h_m}$. The use of this mean value is permissible since the ratio h/H varies between the order of 1/2 and 1/3 for $a_o = 0$ and for a very large value of a_o; and $\bar{c}$ also applies to the binomial of Eq. 25 for $i = 0$ and $x = L$. For normal values of a_o the ratio h/H is nearly constant—namely, 1/3; and $h_m \sim 2/3\, H$. Integrating Eq. 57:

$$\Delta V_d = - \frac{\bar{c}_m\, q_o\, a_o\, e}{(h_m + a_o)} \left[(T_2 + t)\, e^{-t/T_2} - T_2\right] \ldots\ldots\ldots\ldots (58)$$

and the approximate, total accumulated outflow during the time T_2

$$V_d = \frac{\bar{c}_m\, q_o\, a_o\, T_2}{(h_m + a_o)} (e - 2) \ldots\ldots\ldots\ldots (59)$$

The volume V_m of water stored in the mound during the time T_2 is

$$V_m = \mu \int_0^L \Delta y\, dx \ldots\ldots\ldots\ldots (60)$$

in which Δy is the difference between y from Eq. 25 and from Eq. 43 for $i = 0$. Using the binomial of both equations:

$$V_m = \mu \left[(H - h)\, L + \frac{q_o\, L^2}{K\,(h + a_o)} \left(\frac{H - h}{2\,(H + a_o)}\right) - (3 - e)\right] \ldots (61)$$

Because of continuity

$$q_o\, T_2 - \Delta V_d = V_m \ldots\ldots\ldots\ldots (62)$$

and

$$T_2 = \frac{\mu}{1 - \frac{\bar{c}_m\,(e - 2)}{h_m + a_o}} \left[\frac{(H - h)\, L}{q_o} + \frac{L^2}{K\,(h + a_o)} \left(\frac{H - h}{2\,(H + a_o)} - (3 - e)\right)\right] \ldots (63)$$

The Unique Time Function—Fourth Phase.—The mathematical analysis, based on Eq. 4, of the drainage of the mound upon sudden discontinuation of spreading after it had reached a stationary condition has been treated in Sect. 1 for the general case, $i \neq 0$. The following approximate solution leads to results which are a fair representation of the drainage phenomenon for $i = 0$.

During the drainage period the flow through the downstream control is

$$q = - K\, a_o \left(\frac{dy}{dx}\right)_L \ldots\ldots\ldots\ldots (64)$$

in which $\left(\frac{dy}{dx}\right)_L$ is the slope at $x = L$. At the upstream end ($x = 0$), the flow through that section will be zero. Hence the slope at that point must be zero. Making the assumption that the slope varies linearly with x:

$$\left(\frac{dy}{dx}\right)_x = \frac{x}{L}\left(\frac{dy}{dx}\right)_L \qquad (65)$$

and integrating

$$y = \left(\frac{-L^2 + x^2}{2\,L}\right)\left(\frac{dy}{dx}\right)_L \qquad (66)$$

which is a parabola, symmetrical about the y-axis.

For one and the same slope $\left(\frac{dy}{dx}\right)_L = -\frac{q_o}{K\,a_o}$, the mound for stationary flow will have a greater volume than will the mound defined by Eq. 66. Therefore, there is a transition period during which the outflow remains constant. From Eq. 25 and $i = 0$ the pore volume is:

$$V_3 = \mu\left\{\left[-a_o\,L + \frac{K}{3\,q_o}\left(a^2_o + \frac{2\,q_o\,L}{K}\right)^{3/2} - a^3_o\right]\right\} \qquad (67)$$

whereas, for the mound shape defined by Eq. 66, the pore volume

$$V_4 = \mu\,\frac{q_o\,L^2}{3\,K\,a_o} \qquad (68)$$

The change in volume is

$$\Delta V_4 = V_3 - V_4 \qquad (69)$$

and since during this period the outflow is assumed to be q_o, the time is

$$\Delta T_4 = \frac{\Delta V_4}{q_o} = \frac{\mu}{q_o}\left(-a_o\,L + \frac{K}{3\,q_o}\left[\left(a^2_o + \frac{2\,q_o\,L}{K}\right)^{3/2} - a^3_o\right] - \frac{q_o\,L^2}{3\,K\,a_o}\right) \qquad (70)$$

Following this time interval the rate of outflow varies according to the unique time function:

$$q = q_o\,e^{-3\,j/L^2} \qquad (71)$$

which satisfies the condition that its integration, with respect to t between 0 and ∞, is equal to V_4.

4.—STATIONARY GROUND-WATER FLOW

The configuration of the stable mound for $i \neq 0$ was developed in Sect. 2. In the following. potential flow will be treated for $i = 0$.

Maximum Potential Flow[11,12].—To satisfy a physically possible system of the flow net between the spreading ground and the lateral control, motion must be irrotational, and there must exist streamlines and a velocity potential, ϕ. Furthermore, the condition of continuity must be satisfied, which is expressed by the Laplace equation

$$\nabla^2\,\phi = 0 \qquad (72)$$

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[11] "Hydraulik," by Ph. Forchheimer, B. G. Teubner, Leipzig und Berlin, Germany, 1930, pp. 82–83.

[12] "Theoretical Soil Mechanics," by Karl Terzaghi, John Wiley & Sons, Inc., New York, N. Y., 1943, pp. 239–246.

In order that this system shall represent flow under the action of gravity, the velocity potential is expressed as:

$$\phi = K\left(\frac{P}{\gamma} + y\right) \quad \text{.......................(73)}$$

in which P = pressure and γ = unit weight of water. Along the phreatic line, P is constant or zero, so

$$\phi = K\,y \quad \text{.......................(74a)}$$

and

$$\frac{d\phi}{dy} = K \quad \text{.......................(74b)}$$

An analytic complex function,[13] with the added condition that $\frac{d\phi}{dy} = K$ along the phreatic line, satisfies the mathematical requirements of potential flow. To satisfy the theory strictly, the surface of the spreading ground and of the control should be at a constant potential. This condition is normally not satisfied, a situation which leads to distortions.

Changing the constants in the equation by J. A. Kozeny,[14] to satisfy present requirements,

$$z = \frac{1}{2\,q\,K}\,w^2 \quad \text{.......................(75)}$$

in which:

$$z = x + iy \quad \text{.......................(76a)}$$

and

$$w = \phi + i\psi \quad \text{.......................(76b)}$$

in which: ϕ = a potential function; ψ = a stream function; and

$$i = \sqrt{-1}.$$

Introducing into Eq. 75 and equating real and imaginary parts,

$$x = \frac{1}{2\,q\,K}\left(\phi^2 - \psi^2\right) \quad \text{.......................(77)}$$

and

$$y = \frac{1}{q\,K}\left(\phi\psi\right) \quad \text{.......................(78)}$$

Constant values for ϕ and ψ determine the equipotentiality and streamlines as shown in Fig. 5. The phreatic line is defined by $\psi = q$. It is the basic parabola. The equation for the phreatic line is

$$\frac{2\,q}{K}\,x + \left(\frac{q}{K}\right)^2 = y^2 \quad \text{.......................(79)}$$

---

[13] "Fluid Mechanics for Hydraulic Engineers," by Hunter Rouse, McGraw-Hill Book Co., Inc., New York, N. Y., 1938, pp. 96–98.

[14] *Wasserkraft und Wasserwirtschaft*, Vol. 26, 1931, p. 28.

Along the floor, $y = 0$, $\psi = 0$; so, from Eqs. 77 and 73:

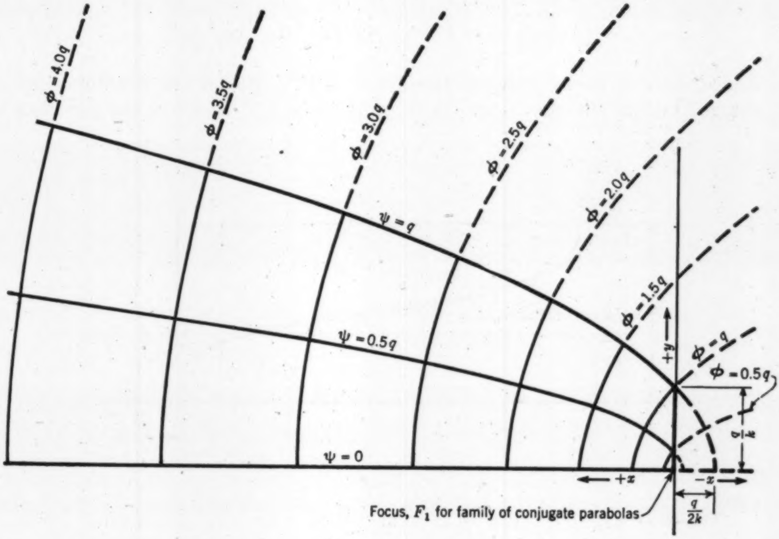
$$\frac{P}{\gamma} = \sqrt{\frac{2\,q\,x}{K}} \qquad (80)$$

which is the pressure head on the floor. Generally, $y = \frac{1}{q\,K}\,\phi\,\psi$ and $\psi = \frac{q}{n}$ in which n can be any number between 1 and ∞. Hence:

$$y = \frac{1}{q\,K}\frac{q}{n}\,\phi = \frac{\phi}{n\,K} \qquad (81)$$

and

$$x = \frac{1}{2\,q\,K}\left(k^2\,n^2\,y^2 - \frac{q^2}{n^2}\right) \qquad (82)$$



φ = 4.0q
φ = 3.5q
φ = 3.0q
φ = 2.5q
φ = 2.0q
φ = 1.5q
φ = q
φ = 0.5q
ψ = q
ψ = 0.5q
ψ = 0
+y
+x
−x
q/k
q/2k
Focus, F_1 for family of conjugate parabolas

FIG. 5

Therefore, the velocity for any streamline at any point is expressed as:

$$-\,n\,K\,\frac{\partial y}{\partial s} = -\,\frac{\partial \phi}{\partial s} = v \qquad (83)$$

Also, at any point on streamlines

$$\frac{dy}{dx} = \frac{q}{K\,n^2\,y} \qquad (84)$$

Hence, at $x = 0$; and $y = \frac{q}{K\,n^2}$

$$\frac{dy}{dx} = \frac{q}{K\,n^2}\left(\frac{K\,n^2}{q}\right) = 1 \qquad (85)$$

which means that at the control, the slope of every streamline is 45°. For $\frac{\partial y}{\partial s} = \sin\alpha$

$$v = -\,n\,K \sin\alpha \qquad (86)$$

in which α is the angle that any streamline makes with the horizontal at any point. The horizontal transport of the streamline is—

$$v \cos\alpha = -\,n\,K \sin\alpha \cos\alpha \qquad (87)$$

which is a maximum when

$$\frac{\partial}{\partial x}(v \cos\alpha) = 0 = -\,n\,K\,(\sin^2\alpha - \cos^2\alpha) \qquad (88)$$

Hence, $\sin\alpha = \cos\alpha$, which is true for $\alpha = 45°$. So for the maximum discharge through the least area the slope must be 45°, which is the condition existing at $x = 0$.

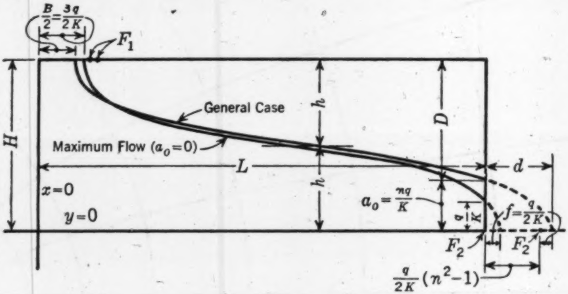


FIG. 6

This flow net represents, in general, the flow far from a spreading ground and in particular the case of a narrow spreading strip from which the water merely "rains" down upon the mound below.

The case of streamline flow from the spreading ground to the control is much more involved. Apparently no rigorous mathematical solution to the problem has so far been found. Therefore, the problem was approached as follows: As indicated in Fig. 5, the complete flow net for the basic parabola calls for the discharge q through the horizontal seepage surface between $x = 0$ and $x = -\frac{q}{2\,K}$. Hence, reversing and inverting the parabola, as shown in Fig. 6, results in a double parabola. If a flow q were infiltrated between F_1 and the vertex of the inverted parabola and an additional flow q were infiltrated between the vertex and the axis of symmetry, then $\psi = q$ at the former and $\psi = 2\,q$ at the latter. From Eq. 77, $\phi = 0$ and $\psi = n\,q$; therefore

$$x = \frac{n^2\,q}{2\,K} \qquad (89)$$

Hence, the distance between $\psi = q$ and $\psi = 2\,q$ along $y = \bar{H}$ is

$$\frac{B}{2} = \frac{3}{2}\,\frac{q}{K} \qquad (90)$$

and the double parabola is a streamline, common to both flow nets. Elimination of $\psi = q$ transforms this streamline into the phreatic line.

Thus far, each section of the curve has been referred to its own natural, coordinate system. The equations of the two parabolas can be referred to a common system of coordinates in which the plane of symmetry will be $x = 0$ and the impervious stratum $y = 0$ (Fig. 6).

The curve of the phreatic line between $x = \frac{3}{2}\,\frac{q}{K}$ and $x = \frac{L}{2} + \frac{q}{K}$ is

$$y = \bar{H} - \sqrt{\frac{2\,q\,x}{K} - \frac{3\,q^2}{K^2}} \qquad (91a)$$

and between $x = \frac{L}{2} + \frac{q}{K}$ and $x = L$:

$$y = +\sqrt{\frac{2\,q}{K}\,(L - x) + \frac{q^2}{K^2}} \qquad (91b)$$

Values of the equipotential ϕ will be based on the value of $\phi = 0$ at $x = L$, $y = 0$, the point of lowest potential within the system. So, at $y = \bar{H} = 2\,\bar{h}$:

$$\phi = 2\,\sqrt{q\,k\,L - q^2} = \Phi' \qquad (92a)$$

At any point on the phreatic line $\phi = K\,y$ or:

$$\phi = \left(\frac{y}{\bar{H}}\right) K\,\bar{H} = \frac{y}{\bar{H}}\,\Phi' \qquad (92b)$$

The flow net between $x = 0$ and $x = \frac{L}{2} + \frac{q}{K}$ was determined by the "relaxation method,"[15] that is, by progressive approximation.

The completed flow net is shown in Fig. 7 using the following values: $L = 46$ in. $= 3.833$ ft; and $\bar{H} = 12$ in. $= 1.00$ ft. Hence from Eq. 91a $\frac{q}{K} = \frac{3.833 - \sqrt{3.833^2 - 1}}{2} = 0.0667$ ft. Therefore, $\frac{B}{2} = \frac{3}{2}\,\frac{q}{K} = 0.10$ ft $= 1.20$ in.; and $\Phi' = K\,\bar{H} = K = 15\,q$.

---

[15] "Numerical Solutions for Tile Drainage of Layered Soils," by James N. Luthin and R. E. Gaskell *Transactions*, Am. Geophysical Union, Vol. 31, 1950, p. 595.

The numerical values correspond to those of the model tests on potential flow which will be discussed subsequently. The figures shown on the grid system correspond to the final approximation of the relaxation method. Partial stagnation is indicated below a boundary approximated by a quarter ellipse (Fig. 7). The parabolic boundary indicated corresponds to the one used in the model tests.

General Potential Flow.—The conditions for the general case of potential flow—that is, for any value of a_o between q/K and $\bar{H}$—are indicated in Fig. 6.

Three distinct stages of a_o must be considered, namely: (1) $0 \leqq a_o \leqq q/K$; (2) $q/K < a_o < D$; and (3) $D < a_o \leqq \bar{H}$, of which the first case has already been treated. For this case from Eq. 91*b* and from the condition that $x = L/2 + q/K$, $y = \dfrac{D + a_o}{2}$:

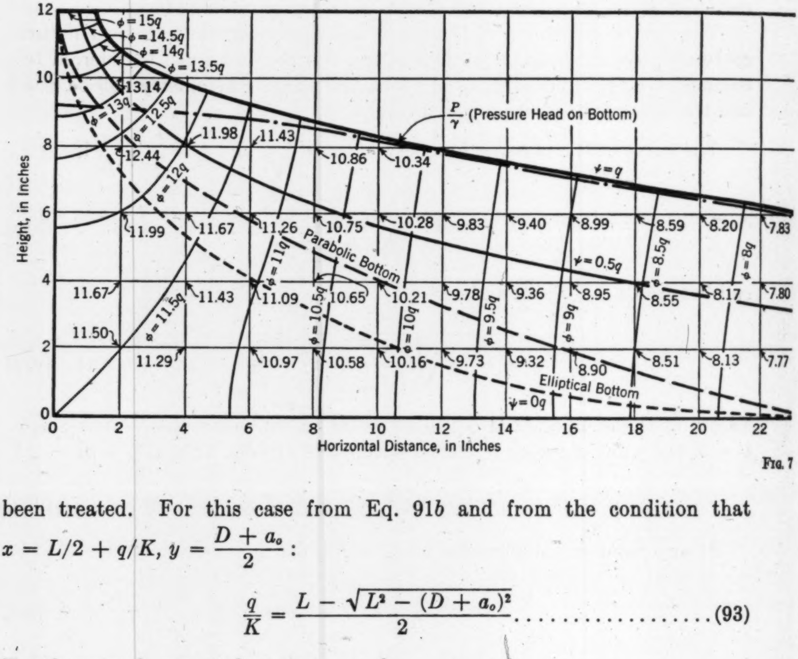


FIG. 7

$$\frac{q}{K} = \frac{L - \sqrt{L^2 - (D + a_o)^2}}{2} \ldots\ldots\ldots\ldots\ldots\ldots (93)$$

For the second case, at downstream end:

$$y = \sqrt{\frac{2\,q}{K}\,(L + d - x)} \ldots\ldots\ldots\ldots\ldots\ldots (94)$$

At $x = L$, $y = a_o$; therefore:

$$d = \frac{a^2_o\,K}{2\,q} \ldots\ldots\ldots\ldots\ldots\ldots (95)$$

and

$$y = \sqrt{\frac{2\,q}{K}\,(L - x) + a^2_o} \ldots\ldots\ldots\ldots\ldots\ldots (96)$$

The common point for both parabolas is at $y = \frac{D + a_o}{2}$ which lies at a distance $\frac{L + d - \frac{3\,q}{2\,K}}{2}$ from either vertex; hence at $x = \frac{L + d}{2} + \frac{3\,q}{4\,K}$, Eq. 91*a* yields:

$$\frac{q}{K} = \frac{L - \sqrt{L^2 - \frac{3}{2}\,(D^2 + 2\,D\,a_o - a^2_o)}}{3} \qquad \text{(97)}$$

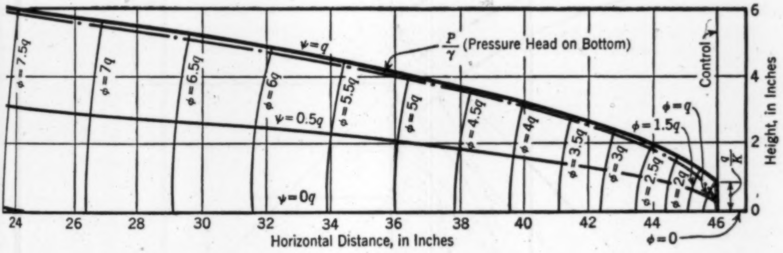
For the third case, at $x = L$, $y = a_o$ in Eq. 91*a* and

$$\frac{q}{K} = \frac{L - \sqrt{L^2 - 3\,D^2}}{3} \qquad \text{(98)}$$

which is zero when $D = 0$.

The flow net for the downstream parabola for $q/K \leqq a_o$ still follows from Eqs. 77 and 78, the focus, however, shifting the distance—

$$x = \frac{q}{2\,K}\,(n^2 - 1) \qquad \text{(99)}$$



beyond the control. At the control, along $\psi = q$:

$$y = a_o = \frac{n\,q}{K} \qquad \text{(100)}$$

and

$$\phi = n\,q \qquad \text{(101)}$$

Condition at the Control.—Fundamentally the problem arises because of the change in pressure along the seepage surface OA, Fig. 8, relative to that of the basic parabola. The latter pressure is shown as a cross-hatched area A_1. Under this pressure condition the flow between the equipotential surface AB and the vertical surface OA $= q_o/K = h_b$ is exactly q_o.

If the sand downstream from OA is removed, the pressure condition is changed. If $a_o = 0$, then the pressure at Sect. OA is atmospheric throughout and the pressure gradient between AB and OA is increased. Based on horizontal flow paths as a simple yet fair approximation, the flow q through surface OA $\sim 1.50\ q_o$ to satisfy the Darcy law. But since the flow q_o is stationary, the seepage surface OA must adjust itself to the new pressure condition and, therefore, must reduce its height. The critical height is

$$h_c \sim \sqrt{\frac{1}{1.50}}\, h_b \sim 0.82\, \frac{q_o}{K} \qquad (102)$$

With the same degree of approximation as before it may be shown that a tailwater depth $\bar{a}_o = 0.60\, h_b$ produces pressure on Sect. OA which is similar to the pressure, represented by area A_1 in Fig. 8; that generally

$$\bar{a}_o \sim \sqrt{2\, A_1} \qquad (103)$$

and that for this condition OA $\sim q_o/K$. Finally, it is clear that for a groundwater depth $a_o > \bar{a}_o$, Sect. OA must be greater than q_o/K so that a seepage surface will always maintain itself between the raised, phreatic line and the tailwater surface. For this condition the pressure gradient between A″B and OA (Fig. 8) is constant. Therefore

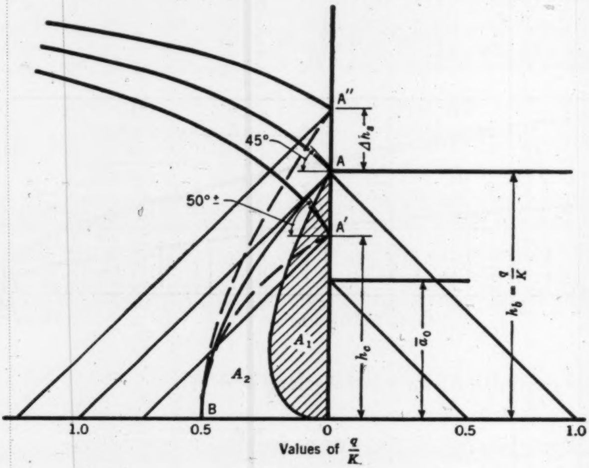


Fig. 8

$$q_s \sim \frac{\Delta h_s}{\Delta L_m}\left(\frac{q_o}{K} + \Delta h_s\right) K \qquad (104)$$

in which ΔL_m = mean horizontal length of flow path which does not differ significantly from that for area ABO = A_2 of the basic parabola. Hence,

$$\Delta h_s \sim \frac{q_o}{K}\left(-\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{A_2\, K^2}{q^2_o}}\right) \sim 0.26\, \frac{q_o}{K} \qquad (105)$$

For $a_o > q_o/K$ the height of the seepage surface diminishes rapidly with increasing a_o and may be expressed by

$$\Delta h_s \sim 0.26 \frac{q_o}{K} \left(\frac{q_o}{a_o K} \right)^3 \ldots\ldots\ldots\ldots\ldots\ldots (106)$$

PART II.—THE THREE-DIMENSIONAL FLOW THROUGH GRANULAR SOIL

1.—Horizontal, Impervious Stratum

Circular Spreading Ground—First Phase.—If spreading is performed on an area represented by a circle, the problem must be treated as three dimensional, as shown in Fig. 9. The unsteady flow radiates uniformly from the center of the circle.

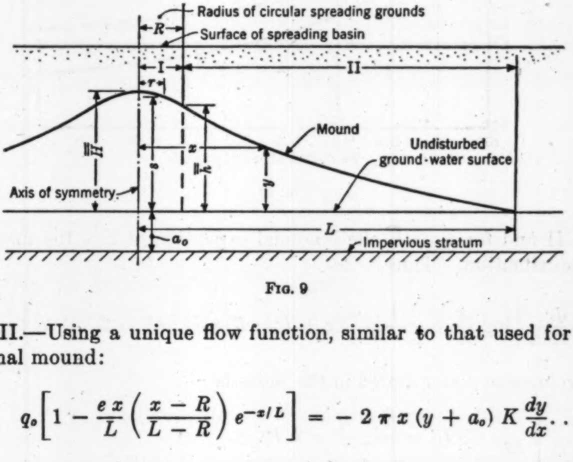

Radius of circular spreading grounds
R
Surface of spreading basin
I
II
r
Mound
Undisturbed ground-water surface
$\bar{H}$
s
$\bar{h}$
x
y
Axis of symmetry
L
a_o
Impervious stratum

Fig. 9

Zone II.—Using a unique flow function, similar to that used for the two-dimensional mound:

$$q_o \left[1 - \frac{e\,x}{L} \left(\frac{x - R}{L - R} \right) e^{-x/L} \right] = -\,2\,\pi\,x\,(y + a_o)\,K\,\frac{dy}{dx} \ldots\ldots (107)$$

which, on integration, leads to

$$y = -\,a_o + \sqrt{a^2_o - \frac{q_o}{\pi K} \left[\log_e \frac{x}{L} + \frac{1}{L - R} \left((x + L - R)\,e^{(L-x)/L} - 2\,L + R \right) \right]} \ldots (108)$$

For $x = R$, $y = \bar{h}$; hence:

$$\bar{h} = -\,a_o + \sqrt{a^2_o - \frac{q_o}{\pi K} \left[\log_e \frac{R}{L} + \frac{1}{L - R} \left(L\,e^{(L-R)/L} - 2\,L + R \right) \right]} \ldots (109)$$

Zone I.—In Zone I, Fig. 10:

$$\Delta q\,\pi\,r^2 = -\,2\,\pi\,r\,(s + a_o)\,K\,\frac{ds}{dr} = \frac{q_o\,r^2}{R^2} \ldots\ldots\ldots\ldots (110)$$

in which Δq is the spreading rate per unit area. Integrating:

$$s = -a_o + \sqrt{(\overline{\overline{H}} + a_o)^2 - \frac{q_o\, r^2}{2\,\pi\, K\, R^2}} \qquad \ldots\ldots\ldots\ldots(111)$$

For $r = R$, $s = \overline{\overline{h}}$ and, therefore, from Eqs. 109 and 111:

$$\overline{\overline{H}} = -a_o + \sqrt{a^2_o - \frac{q_o}{\pi\, K}\left[\log_e \frac{R}{L} + \frac{1}{L - R}\left((L - R)\, e^{(L-R)/L} - 2\,L + R\right) - \frac{1}{2}\right]} \ .(112)$$

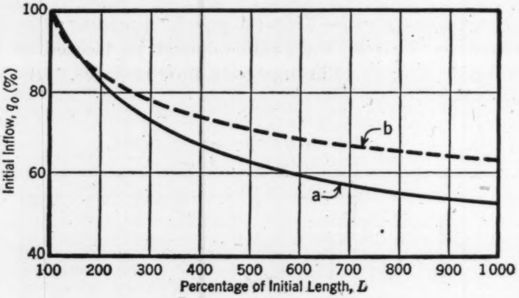


FIG. 10

For zone II and for $a_o \gg \overline{\overline{H}}$ the binomial expansion of Eq. 108 may be used as an approximation. Thus

$$y = -\frac{q_o}{2\,K\,a_o}\left[\log_e \frac{x}{L} + \frac{1}{L - R}\left((x + L - R)\, e^{(L-x)/L} - 2\,L + R\right)\right] \ldots.(113)$$

and the volume of water stored in the pores is

$$V''' = \frac{\mu\, q_o}{K\, a_o}\left[\frac{L^2}{4} + \frac{8\,L^3 - \dfrac{5\,L^2\,R}{2} - L\,R^2 + \dfrac{R^3}{2}}{L - R} \right.$$

$$\left. - \frac{e^{(L-R)/L}}{L - R}\,(3\,L^3 + 2\,R\,L^2) + \frac{R^2}{2}\left(\log_e \frac{R}{L} - \frac{1}{2}\right)\right] \ldots.(114)$$

Recharge Well—First Phase.—If, instead of a circular spreading ground, a well is used in recharge of ground water, then R relative to L, and the storage in the well, may be neglected. Then approximately:

$$V''' = \frac{\mu\, q_o}{10\, K\, a_o}\, L^2 = q_o\, T''' \qquad \ldots\ldots\ldots\ldots(115)$$

in which T''' = total spreading time. Hence:

$$T''' = \frac{\mu}{10\, K\, a_o}\, L^2 \qquad \ldots\ldots\ldots\ldots(116)$$

Circular Spreading Ground—Second Phase.—It would not be reasonable to consider the analogue of case (a) as previously analyzed, in connection with the three-dimensional mound because of the improbability of the existence of a circular control. However, the analogue of case (b) is all the more important.

When the three-dimensional mound has risen to the potential control, it follows from Eq. 109 that for $\bar{h}$ to remain constant L and q_o must be related at all times as shown in Fig. 10, curve a. It is based on typical numerical values, namely, $R = 500$ and initial length $L = 5{,}000$ ft.

Recharge Well—Second Phase.—As R is small as compared to L, the second term under the radical in Eq. 109 reduces to $\frac{q_o}{\pi K} (\log_e R/L + 0.72)$ which for a typical, initial ratio $R/L = 0.01$ leads to the relation shown in Fig. 10 as curve b.

2.—INCLINED, IMPERVIOUS STRATUM

It is possible to gain an insight into this complex case by assuming[16] the spreading flow as originating from a source of strength m, discharging vertically onto an inclined plane of infinite width and length. If the source is assumed at the origin and if U = initial uniform ground-water flow per linear ft, parallel to the x-axis, then:

$$U = K\, a_o i \quad \text{(117a)}$$

and if

$$2\,\pi\, m = q_o \quad \text{(117b)}$$

then the combined flow of the initial uniform flow, U, and the spreading flow q_o, can be represented by the complex function:

$$w = -\,U\,z - m \log_e z \quad \text{(118)}$$

In view of the definition of w and z, previously shown

$$\phi = -\,U\,x - m \log_e (x^2 + y^2)^{\frac{1}{2}} \quad \text{(119)}$$

and

$$\psi = -\,U\,y - m \tan^{-1} y/x \quad \text{(120)}$$

The flow net based on these equations may be drawn by Rankin's graphical method[17] and is shown in Fig. 11. The boundary of the mound is given by the streamline $\psi = 0$; so, from Eq. 120

$$y = -\,x \tan \frac{U\,y}{m} \quad \text{(121)}$$

from which it follows that the boundary asymptotically approaches—

$$y = \pm \frac{m\,\pi}{U} = \pm\, h_o \quad \text{(122)}$$

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[16] "Theoretical Hydrodynamics," by L. M. Milne-Thomson, Macmillan Co., Inc., New York, N. Y., 1950.

[17] "Elementary Demonstrations of Principles Relating to Stream-lines," by W. J. Macquorn Rankine, *Miscellaneous Scientific Papers*, Charles Griffin and Co., London, England, 1881, p. 522.

as $x \rightarrow \infty$ and that the mound extends upstream to the stagnation point

$$x_o = -\frac{h_o}{\pi} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (123)$$

Substituting for U and m in Eq. 122 gives the mound width at $x = \infty$:

$$W = 2\,h_o = \frac{q_o}{K\,a_o\,i} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (124)$$

Thus, the mound boundary has the effect of a training wall and relative to the initial ground-water stream of infinite width, the mound acts like an island of finite width and semi-infinite length.

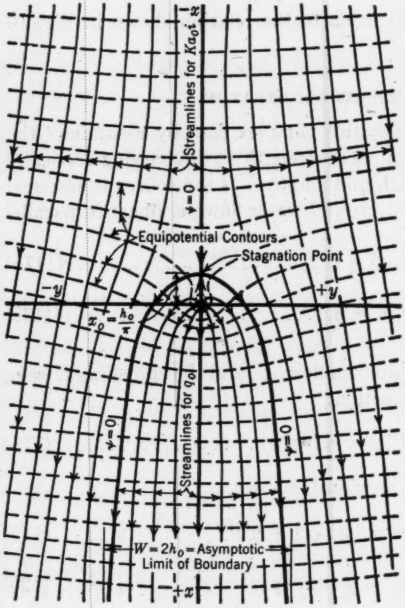


Fig. 11

Numerical Evaluation of Equations.—The following quantities are typical for one of the existing spreading grounds, the area of which may be approximated by that of a relatively small circle, namely: $R = 500$ ft; $a_o = 100$ ft; $K = 100$ ft per hr; and $q_o = 1 \times 10^6$ ft³ per hr. Hence for $i = 0.01$ from Eq. 124, $W = \frac{q_o}{K\,a_o\,i} = 10{,}000$ ft; and from Eq. 123 $x_o = -\frac{5{,}000}{\pi} = -1{,}592$ ft. In this case, based on the potential function the rise of the mound above the normal ground-water surface is of the order of 35 ft. This is its ultimate height.

For $i = 0$ and $L = 5{,}000$ ft, from Eq. 112, $\overline{H} = 33.47$ ft and from Eq. 114 $V''' = 34.23 \times 10^6$ ft³ = 786 acre = ft and $T''' = 34.23$ hr of spreading.

This clearly brings out the magnitude of the mound for $i = 0$ even during short spreading operations. They often extend over much longer periods.

PART III.—THE MODEL TESTS ON TWO-DIMENSIONAL GROUND-WATER FLOW

1.—Description of the Model Setup

Two distinct model arrangements were used to test the mechanics of the four phases for two-dimensional flow. The model sand was clean, hard, stream-worn material which had passed a No. 30 screen and had been retained on a No. 40 screen. The grain size, therefore, varied little from 0.5 mm.

First Model Arrangement.—The first model arrangement served to ascertain the wave phenomenon and unsteady flow characteristics during the first and second phases, the outflow hydrograph during the second phase, and the drainage of the mound during the fourth phase. A general view of the model setup in connection with this arrangement is shown in Fig. 12, and details of the model proper are shown in Fig. 13. The latter consisted of:

a. A watertight box approximately 12 in. wide, 12 in. deep, and 46 in. long, filled with sand with a grid-lined glass front;

b. A metal frame, 4 in. wide at the upstream end, simulating the spreading ground and extending clear across the box;

c. Twenty-five piezometers arranged in horizontal sets of five, each at five elevations above the bottom and each set extending 2 in., 4 in., 6 in., 8 in., and 10 in., respectively, into the sand from the end wall;

d. Eight deep wells, each equipped with threaded brass rod and steel contact point, adjustable as to vertical position within 0.005 in., and connected to a direct-current low voltage source;

e. Nine shallow wells behind and parallel to the deep wells, and equipped with one movable contact point of the same design as previously described;

f. Nine argon bulbs to indicate visually the instant of contact between the ground-water surface and the point gages;

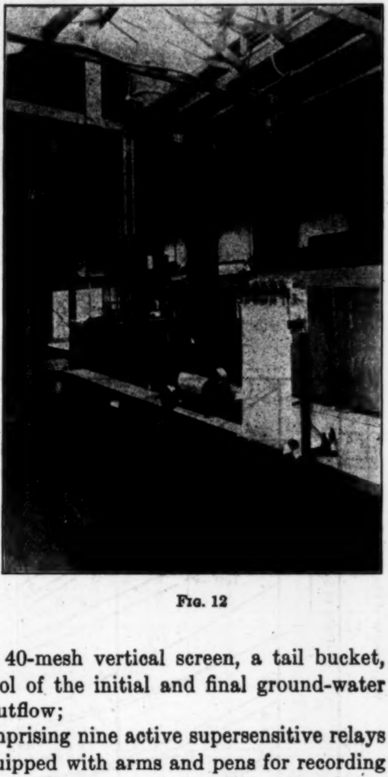


FIG. 12

g. A lateral control comprising a 40-mesh vertical screen, a tail bucket, and an adjustable weir for the control of the initial and final ground-water depths as well as for a check on the outflow;

h. An automatic chronograph, comprising nine active supersensitive relays and one spare supersensitive relay equipped with arms and pens for recording of the aforementioned well-point contacts in terms of time (the latter being controlled by a separate clock, recording 1-sec time intervals);

i. An automatic stage recorder to ascertain rate of outflow in terms of time;

j. A constant-head tank with float valve control, fed from a large primary tank and connected to a municipal source of water supply; and

k. An orifice-equipped manometer to determine the rate of inflow to the model. The rate of inflow was controlled by a needle valve, and the water was delivered to the spreading ground by a swing spout. The latter served to take volumetric measurements when desired.

Minimum aeration and uniform distribution of inflow into the spreading ground were accomplished by a trough, connected to a manifold with twenty-four small spouts placed at the surface of a $\frac{1}{2}$-in. layer of coarse diffuser sand overlying the test sand. A small U-tube connected to the bottom of the spreading-ground frame served to detect air lock, if present.

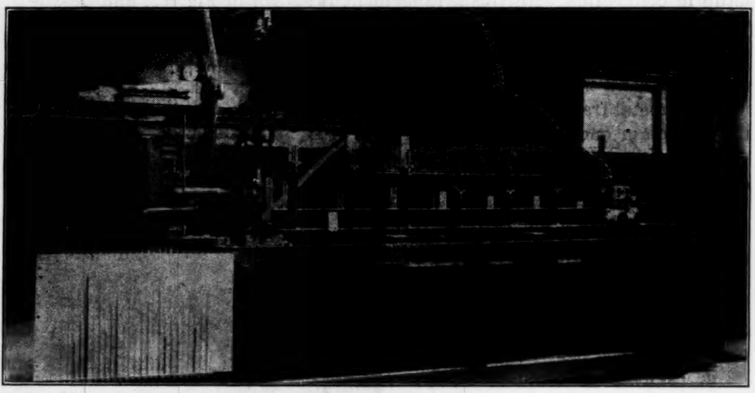


FIG. 13

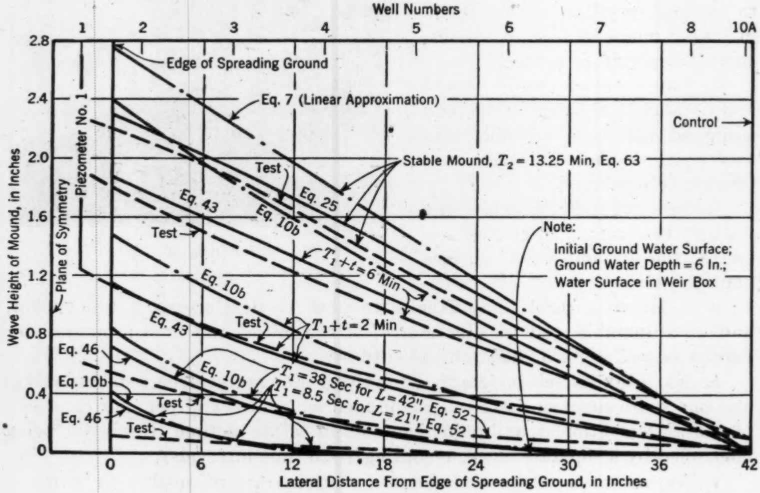


FIG. 14

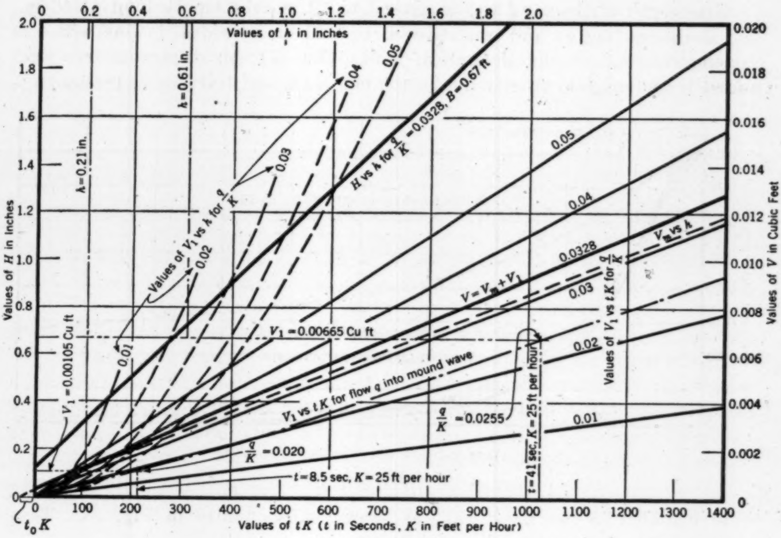


FIG. 15

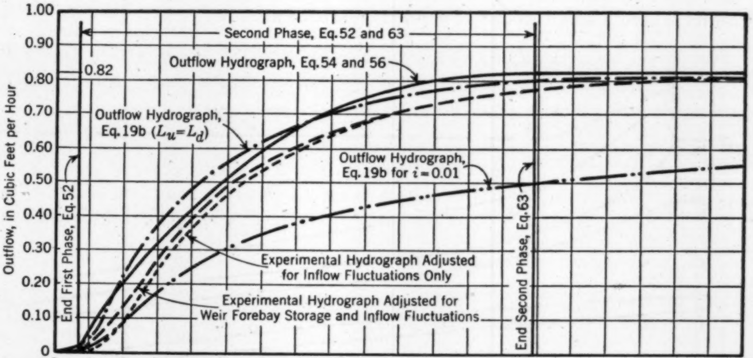


FIG. 16

Second Model Arrangement.—The second model arrangement served to ascertain the validity of the theory on potential flow. For this purpose both deep and shallow wells were removed, together with the electrical recording apparatus and the water-stage recorder at the discharge end.

The depth of the sand was increased to 12 in., the length kept at 46 in., and the spreading ground was reduced to 1.20 in. in width, in line with the dimensions used in the theoretical study (Fig. 7); eight piezometers were added to the original twenty five in the end wall, and thirty-nine piezometers were installed in the back wall, as shown subsequently in Fig. 18. They extended 2 in. into the sand.

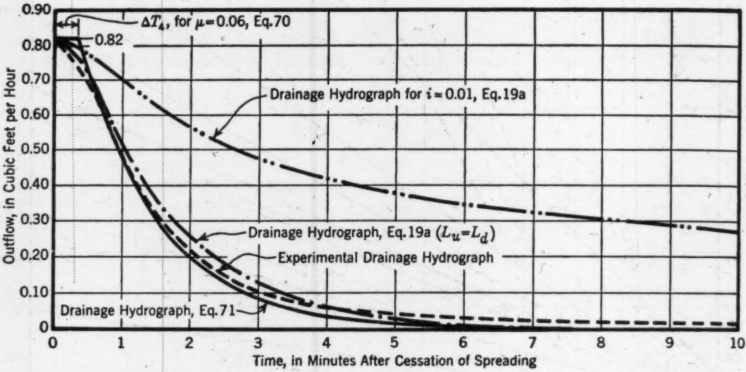


FIG. 17

Next, the sand was removed from the surface down to within 0.5 in. of the theoretical phreatic line in accordance with Fig. 7, except near the spreading ground where a small wedge of sand was left for stability.

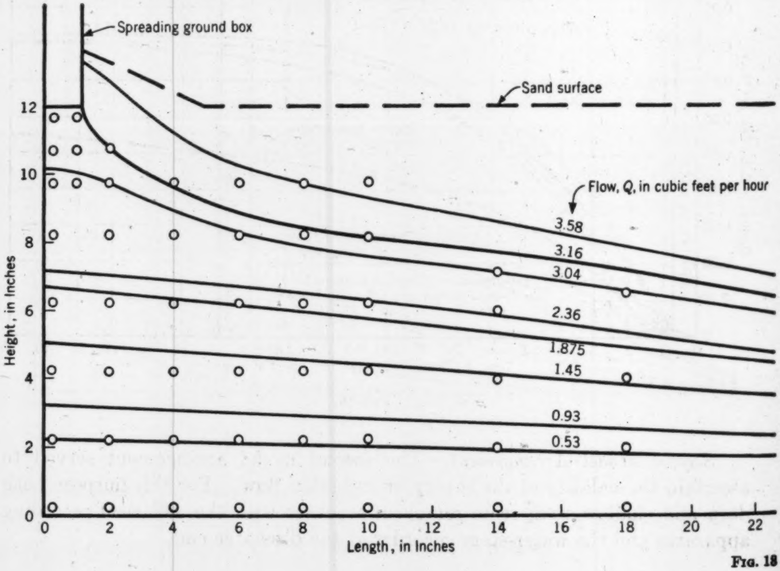


FIG. 18

Finally all the sand was removed. A parabolic boundary was constructed with the focus of the phreatic line and the vertical end wall as a tangent and with a transition to the horizontal floor near the midpoint. The sand was replaced to 0.5 in. over the theoretical phreatic line to conform to the previous condition.

2.—The Model Test Runs

All four phases of the mound phenomenon were included in the model tests. The rates of flow, actually observed in the model (which has the neat width of 11.25 in.) are distinguished from the rate of flow per linear foot, by *Q*. Rates of flow corrected for a width of 12 in. are shown by *q*. The latter are based, therefore, on the same premise as the theoretical rates of flow.

For the purpose of recording the first and second phases by the chronograph, the point gages in the wells were connected to the positive pole, and the ground water was connected to the negative pole of a direct-current circuit.

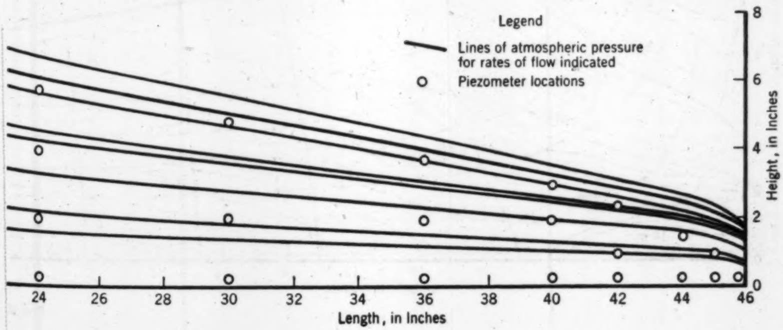
A comprehensive test of the first two phases was conducted for a ground-water stage $a_o = 6$ in., consisting of many consecutive runs at constant $q_o = 0.82$ cu ft per hr. During this time all the eight contact points were set at progressively higher levels to obtain the configuration of the unstable mound above the undisturbed ground-water surface in terms of time.

The stationary flow of the third phase was investigated by test runs on various initial ground-water stages a_o with the prime objective of establishing q_o (max) for each stage under this condition. It was not possible to maintain a continuous pressure system between the spreading ground ($B/2 = 4.00$ in.) and the control.

Tests for the fourth phase immediately followed those for the third phase. The rate of drainage was recorded automatically by the water-stage recorder. The recordings were supplemented by direct, volumetric measurements.

After rearranging the model as previously outlined, the test runs on potential flow were conducted:

1. With the sand surface 12 in. above the bottom;
2. With the sand surface shaved down to within 0.5 in. of the theoretical phreatic line as shown in Fig. 7; and



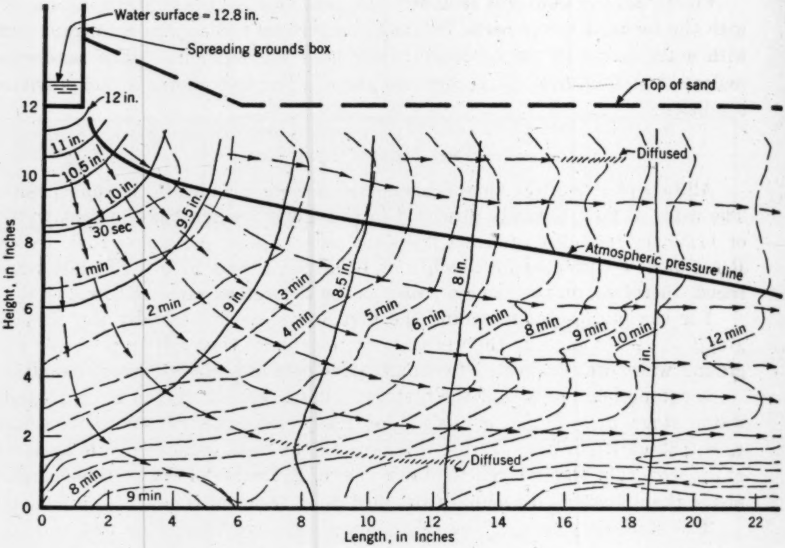

Water surface = 12.8 in.
Spreading grounds box
Top of sand
Diffused
Atmospheric pressure line
Diffused
12 in.
11 in.
10.5 in.
10 in.
9.5 in.
9 in.
8.5 in.
8 in.
7 in.
30 sec
1 min
2 min
3 min
4 min
5 min
6 min
7 min
8 min
9 min
10 min
12 min
8 min
9 min
Height, in Inches
0
2
4
6
8
10
12
Length, in Inches
0
2
4
6
8
10
12
14
16
18
20
22

FIG. 19

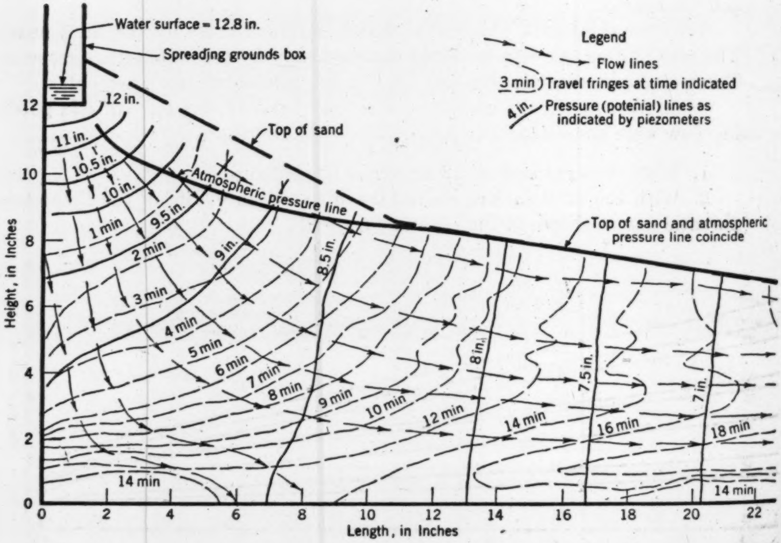
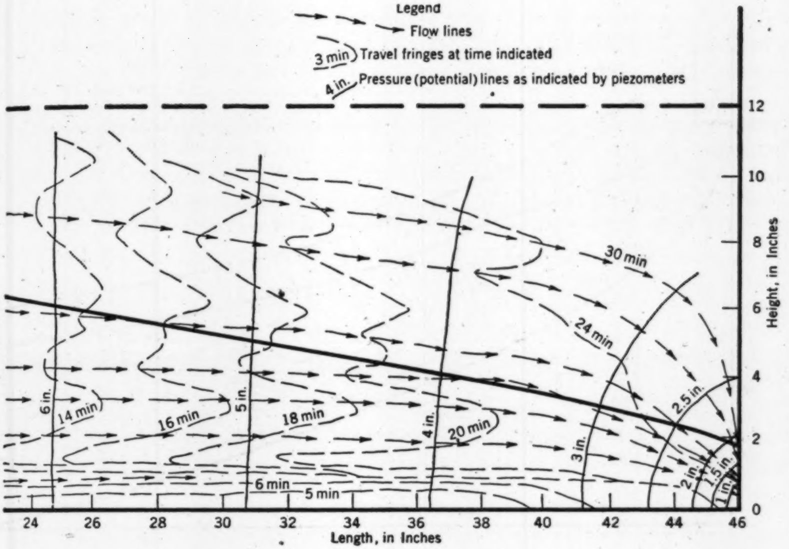
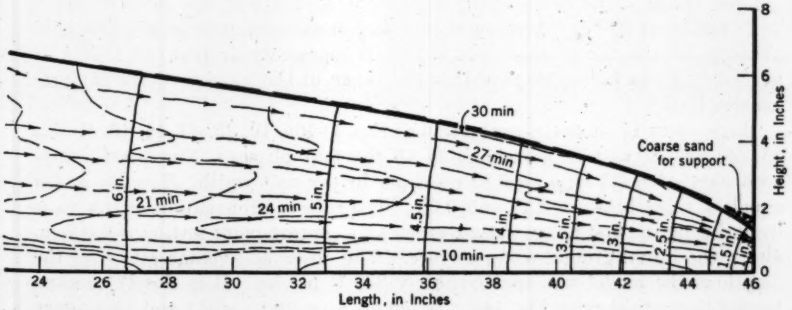

Water surface = 12.8 in.
Spreading grounds box
Legend
Flow lines
3 min
Travel fringes at time indicated
4 in.
Pressure (potenial) lines as indicated by piezometers
Top of sand
Atmospheric pressure line
Top of sand and atmospheric pressure line coincide
12 in.
11 in.
10.5 in.
10 in.
9.5 in.
9 in.
8.5 in.
8 in.
7.5 in.
7 in.
1 min
2 min
3 min
4 min
5 min
6 min
7 min
8 min
9 min
10 min
12 min
14 min
16 min
18 min
14 min
14 min
Height, in Inches
0
2
4
6
8
10
12
Length, in Inches
0
2
4
6
8
10
12
14
16
18
20
22

FIG. 20


Legend
Flow lines
3 min Travel fringes at time indicated
4 in. Pressure (potential) lines as indicated by piezometers
30 min
24 min
20 min
18 min
16 min
14 min
6 min
5 min
6 in.
5 in.
4 in.
3 in.
2.5 in.
2 in.
1.5 in.
1 in.
Height, in Inches
0
2
4
6
8
10
12
Length, in Inches
24
26
28
30
32
34
36
38
40
42
44
46

3. With the sand surface the same as in condition 2 and in addition with a parabolic lower boundary between $x = 0$ and $x \sim L/2$.

Fluorescein was injected into the spreading basin, and at intervals of 1 min, 2 min, or 5 min the progress of this fluorescein was traced on the glass that formed the face of the model, giving what may be called "travel fringes." In addition, dye was inserted adjacent to the glass at various elevations of the test length, and streamlines were drawn from the movement of this dye. Under this condition there was no difficulty in maintaining a continuous pressure system between the spreading ground ($B/2 = 1.20$ in.) and the control.


30 min
27 min
24 min
21 min
10 min
Coarse sand for support
6 in.
5 in.
4.5 in.
4 in.
3.5 in.
3 in.
2.5 in.
1.5 in.
1 in.
Height, in Inches
0
2
4
6
8
Length, in Inches
24
26
28
30
32
34
36
38
40
42
44
46

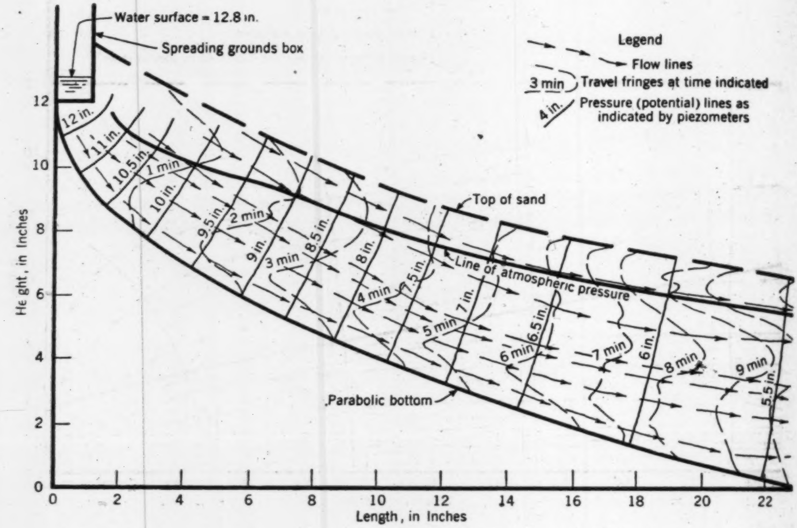

Water surface = 12.8 in.
Spreading grounds box
Legend
Flow lines
3 min Travel fringes at time indicated
4 in. Pressure (potential) lines as indicated by piezometers
Top of sand
Line of atmospheric pressure
Parabolic bottom
12 in.
11 in.
10.5 in.
10 in.
9.5 in.
9 in.
8.5 in.
8 in.
7.5 in.
7 in.
6.5 in.
6 in.
5.5 in.
1 min
2 min
3 min
4 min
5 min
6 min
7 min
8 min
9 min
Height, in Inches
0
2
4
6
8
10
12
Length, in Inches
0
2
4
6
8
10
12
14
16
18
20
22

Fig. 21

3.—Comparison Between Theory and Test Results

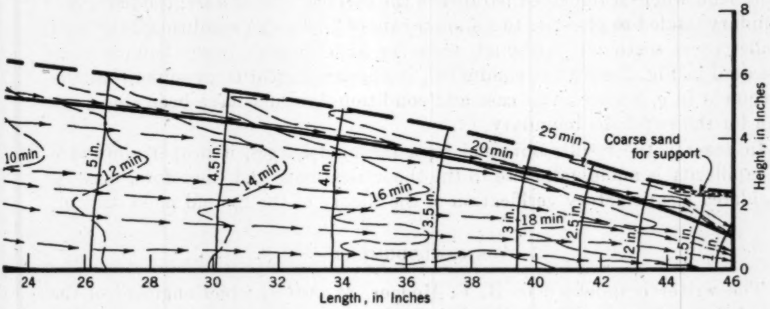
Comprehensive Tests.—Numerical values corresponding to the model tests were introduced into the equations. Since the floor of the model was horizontal, only those equations based on $i = 0$ were considered, except for the outflow from the mound during its growth and disappearance which was also computed for $i = 0.01$ for the sake of comparison.

The Reynolds number for the test sand, based on a unit gradient and a water temperature of 60° F, was 0.4. Hence, the flow was well within the laminar range. The permeability coefficient K of the model sand averaged 25 ft per hr at 60° F. A value of $\mu = 0.15$ represents average conditions of porosity for the rising stage; and $\mu = 0.06$ represents average conditions of porosity for the falling stage within the range of the mound of the comprehensive tests.

Figs. 14 to 17 show the evaluation of Eqs. 7, 10*b*, 19, 25, 43, 44, 46, 47, 52, 55, 56, 63, 70, and 71 pertaining to all the four phases. Numerical values used were $a_o = 0.5$ ft; $q_o = 0.82$ cu ft per hr per unit width; $K = 25$; and μ equals a value between 0.15 and 0.06. The rate of propagation of the wave front during the first phase was governed by a contact-point setting of 0.005 in. above the initial ground-water surface. The observed average rate over the length of the model was approximately 330 ft per hr. It is closely checked by the theoretical rate (Fig. 14). Based on $K = 25$ ft per hr and an average slope of 0.015, the corresponding average velocity of steady flow would be

0.375 ft per hr or nearly one nine hundredth of the average wave velocity. This explains the surprisingly short time intervals, often observed in the field, between beginning of spreading operations and the rise of the water surface in remote observation wells.

As previously stated, the effect of storage ΔV_m in the mound directly beneath the spreading ground was neglected in the derivation of the equations for the unsteady mound wave. Therefore, the respective correction was accomplished graphically as shown in Fig. 15. For the observed ratio $q_o/K = 0.0328$ and $\mu = 0.15$ this correction of the theoretical wave height h eliminates the major part of the discrepancy between experimental and theoretical values. Since the flow into the mound wave was affected by capillarity in addition to the storage ΔV_m, a flat check could not be expected. Both influences diminish with an increase in h and, therefore, with V_1. In a strict sense, Fig. 15 is applicable only to the first phase.



Potential Flow Tests.—Fig. 18 shows the test result for various rates of observed stationary flow Q for condition (1), which was necessarily affected by capillary flow. Hence the rates of flow shown at corresponding atmospheric pressure lines included the latter.

Fig. 19 shows the result for condition 1 for $q = 3.52$ ft per hr, $K = 24$ ft per hr, and $\bar{H} = 12.80$ in. Streamlines, equipotential lines, and travel fringes are indicated. Flow through the entire prism, except near the control, is evident. Fig. 20 shows the result for condition 2 for $q = 2.20$ cu ft per hr and $K = 22.6$ ft per hr. Hence, $q/K = 0.097$ ft^2 as compared to 0.067 ft^2, the dimension per linear foot being understood in this discussion of the tests for $\bar{H} = 12$ in. on which Fig. 7 is based. Capillary flow is evident in the small wedge of sand next to the spreading ground which could not be removed for reasons of stability. Partial stagnation in the lower left-hand corner is conspicuous, and so is the concentration of flow immediately below the phreatic line in the upstream half of the model. The effective head $\bar{H}$ averaged 12.80 in. and, therefore, was in excess of the theoretical one as shown in Fig. 7. The extra head was necessary to assure positive pressure on the floor of the spreading ground. The phreatic line maintained itself above the theoretical one as would be expected, except within the aforementioned wedge. Removal of even the smallest amount of sand from the parabolic surface below the wedge caused free water to appear. There was no difficulty in maintaining reasonably constant q and $\bar{H}$.

Fig. 21 shows the result for condition 3. Replacing the sand after installation of the parabolic floor resulted in a permeability which was greater than in the preceding test—namely $K = 24$ ft per hr. Hence, for the corrected observed $q = 1.95$ ft^3 per hr q/K was 0.081 ft^2. It is interesting to note that the phreatic line, contrary to the preceding test, dropped below the surface of the sand where it remained parallel to the surface. Therefore, the parabolic boundary tended to give rise to a similar rate of flow as for condition 2 through smaller cross sections. Although tests for an elliptical, lower boundary as indicated in Fig. 7 were not conducted, it appears logical to presume that the agreement in q, between this case and condition 2, would have been closer yet than for the parabolic boundary.

In general, the results obtained from the model tests, if proper allowance for capillarity is made, fall close to the theoretical ones and, therefore, may be considered a satisfactory verification of the theory of the mound phenomenon.

ACKNOWLEDGMENT

The writer is indebted to H. E. Hedger, M. ASCE, chief engineer of the Los Angeles County Flood Control District, for reviewing the paper and offering practical suggestions; to Maxwell F. Burke, A. M. ASCE, who supervised the model tests; to Karl F. Smallin and Milan W. Ransom, A. M. ASCE, whose mechanical skill and perseverance were instrumental in the achievement of the comprehensive test results; and to Richard W. Rutowski for conducting the tests on potential flow, for checking the mathematical work, and for his assistance and initiative in the evaluation of equations.

DISCUSSION

David K. Todd[18] J. M. ASCE.—In connection with the application of spreading to control sea-water intrusion, the author states: (paragraph following Eqs. 17) "To maintain control indefinitely the rate of infiltration must be such as to cause rise of the mound to at least mean sea level." For an unconfined homogeneous isotropic aquifer the mound elevation must exceed that of mean sea level to develop an effective equilibrium condition in repelling sea-water intrusion. According to the Ghyben-Herzberg principle,[19,20] the greater specific gravity of salt water will balance a correspondingly greater head of fresh water in saturated ground-water zones. Following the author's notation, this may be expressed as

$$\alpha_F (a_o + H_d + \Delta H) = \alpha_s (a_o + H_d) \ldots\ldots\ldots\ldots\ldots (125)$$

in which α_F equals the specific gravity of fresh water; α_s the specific gravity of salt water; and ΔH, the incremental head of fresh water required on the mound to maintain equilibrium. Solving for ΔH,

$$\Delta H = \frac{\alpha_s - \alpha_F}{\alpha_F} (a_o + H_d) \ldots\ldots\ldots\ldots\ldots\ldots (126)$$

To illustrate the magnitude of the increased fresh-water head required, the author's typical prototype values of $a_o = 100$ ft and $H_d = 35$ ft may be inserted in Eq. 126, together with values[21] of $\alpha_s = 1.026$ and $\alpha_F = 1.000$. A value of $\Delta H = 3.5$ ft is found. Thus, assuming that undiluted sea water is in direct contact with the fresh ground water, the fresh-water mound must be maintained more than 3 ft above mean sea level to control sea-water intrusion.

The author has limited his analytical treatment and model studies to cases of unconfined ground-water conditions only, and has not investigated the problem of pressure aquifers. However, the impression is given that an important application of spreading is that of control of sea-water intrusion. This application is important in coastal areas where unconfined aquifers do exist; but in California where the problem of a sea-water intrusion is becoming increasingly serious, most of the contaminated basins are located in pressure areas. In Table 1 are listed[22] the known localities of sea-water intrusion in California as of 1950; the areas in acres underlain by salt water; and the type of aquifer—confined or unconfined. Seven of the known areas of sea-water intrusion are situated in pressure aquifers, three in nonpressure aquifers, one in a semi-confined aquifer, and two in undetermined aquifers. Of the 90,525 acres that have been found to be underlain by sea water, 87,965 acres occur in pressure areas. To control intrusion in pressure areas, methods other than spreading must be employed. Current studies in California[23] include the technical and economic feasibility of constructing artificial subsurface barriers, and also of creating feash-water pressure ridges above mean sea level by means of injection wells.

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[18] Lecturer in Civ. Eng., Univ. of California, Berkeley, Calif.

[19] "Nota in verband met de voorgenomen put boring nabij Amsterdam" (Notes on the probable results of the proposed well drilling near Amsterdam), by W. Badon Ghyden, *Tijdschrift van het Koninkhijk*, Institut van Ingenieurs, The Hague, Holland, 1888–1889, p. 21.

[20] "Die Wasserversorgung einiger Nordseebader" (The water supply on parts of the North Sea coast), by B. Herzberg, *Journal Gasbeleuchtung und Wasserversorgung*, Vol. 44, Munich, Germany, 1901.

[21] "Specific Gravity of Sea-Water and the Ghyben-Herzberg Ratio in Hawaii," by C. K. Wentworth, *Transactions*, Am. Geophysical Union, Part IV, 1939, pp. 690–692.

[22] "Sea-Water Intrusion into Ground Water Basins Bordering the California Coast and Inland Bays," by H. O. Banks, G. B. Gleason, and R. C. Richter, *Report No. 1*, Water Pollution Investigations, Div. of Water Resources, State of California, 1950.

The evaluation of the mound width developed by a circular spreading ground above an inclined impervious stratum (Eq. 124) showed that a circle of 500-ft radius provided a mound width approaching 10,000 ft. The same equation may be applied to determine the mound width resulting from a recharge well. This would have application if consideration were being given to the maintenance of a fresh-water barrier along a coast in an unconfined aquifer using a line of recharge wells parallel to the coast. The mound width would determine the spacing of the wells. Since this width is independent of the spreading area size and depends only on the rate of aquifer recharge, q_o, and the physical parameters K, a_o, and i; an estimate can be made based on a representative rate of well recharge. Two published accounts giving quantitative measurements of recharge wells in Southern California[24,25] show average recharge rates varying from 0.52 to 2.92 acre-ft per day. Taking a value of $q_o = 3$ acre-ft per day as an example and using the author's values of $a_o = 100$ ft, $K = 100$ ft per hr, and

TABLE 1.—KNOWN AREAS AND TYPES OF AQUIFERS INVADED BY SEA WATER IN CALIFORNIA, AS OF 1950

Ground-water basin or valley	Area[a] (acres)	Type of aquifer	Authority
San Diego	Not known	Unconfined	A. J. Ellis and C. H. Lee[b]
San Luis Rey	Not known	Unconfined	A. J. Ellis and C. H. Lee[b]
Santa Margarita	Not known	Unconfined	A. J. Ellis and C. H. Lee[b]
Central Coastal Plain	3,500	Confined	G. Zander and G. B. Gleason[c]
West Coastal Plain	16,000	Confined	G. Zander and G. B. Gleason[c]
Malibu Creek	160	Not known	....
Trancas Creek	Not known	Not known	....
Oxnard Plain	500[d]	Confined	H. Conkling[e]
Salinas	7,000	Confined	T. R. Simpson[f]
Coastal strip[g]	530	Confined	J. M. Haley[h]
Pajaro	435	Confined	J. M. Haley[h]
Santa Clara	60,000	Confined	W. O. Clark[i]
Sacramento-San Joaquin[j]	2,400	Semi-confined	C. F. Tolman[k]
Total known area	90,525	....	....

[a] Area underlain by salt water, in acres. [b] *Water Supply Paper No. 446*, USGS, 1919. [c] *Bulletin No. 53*, Div. of Water Resources, State of California, 1947. [d] Approximately 500 acres. [e] *Bulletin No. 16*, Div. of Water Resources, 1933. [f] *Bulletin No. 52*, Div. of Water Resources, State of California, 1946. [g] Between Salinas Valley and Pajaro Valley. [h] "Santa Cruz-Monterez Counties Investigation," *Bulletin No. 5*, State Water Resources Board, State of California (publication pending). [i] *Water Supply Paper No. 519*, USGS, 1924. [j] Sacramento-San Joaquin Valley between Pittsburgh and Antioch. [k] "Ground Water," by C. F. Tolman, McGraw-Hill Book Co., Inc., New York, N. Y., 1937, pp. 378–379.

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[23] "Proposed Investigational Work for Control and Prevention of Sea-Water Intrusion into Ground Water Basins," by H. O. Banks and M. Bookman, Report to State Water Resources Board, Div. of Water Resources, State of California, 1951.

[24] "Spreading Water for Storage Underground," by A. T. Mitchelson and D. C. Muckel, *Technical Bulletin 578*, U. S. Dept. of Agri., Washington, D. C., 1937.

[25] "Surface Spreading-Operations by the Basin-Method and Tests on Underground Spreading by Means of Wells," by D. A. Lane, *Transactions*, Am. Geophysical Union, Vol. 15, 1934, pp. 523–527.

$i = 0.01$, Eq. 124 yields a mound width, W, of 54.5 ft. Actual wells would need to be spaced even closer than this distance to develop a suitable mound height and to leave no intervals between wells without fresh-water flow. In this example, the mound width would have reached only 54.0 ft at a distance of 1,740 ft downstream from each well. The spacing suggests that economic considerations alone would govern in applying this method to control sea-water intrusion. Having a line of recharge wells with only some 50 ft between each well indicates that recharge by spreading would be a better method in an actual situation approximating these conditions.

MAX SUTER,[26] M. ASCE.—This paper is an excellent mathematical treatment of the problem of water recharge, a problem that is becoming more and more important and on which the present literature is relatively scarce. The author shows how exceedingly complicated the mathematical part of the purely hydraulic phases can become in skilled hands unafraid of hard work. In practical applications of ground-water recharge many additional factors appear, such as nonuniformity of the soil and basin boundaries, variations in the chemical and physical character of the recharge water, silting of the soil (either by silt in the recharge water or as an effect on the soil particles), pollution of ground water, influences on the temperature and chemical composition of the ground water, predictions of rate of inflow, duration of inflow and of periods between recharges, pumpage in the vicinity, and backwater effects and general variations in ground-water levels due to rainfall and floods. Many of these factors cannot be represented by mathematical formulas as they depend too much on local conditions. It is good, therefore, to have some calculating methods for those phases that are amenable to mathematical treatment and in this respect this paper marks a great step forward.

Much of the mathematics used in the paper is probably unfamiliar to many practical engineers, as few of them had or remember differential equations. It may be asked, therefore, if all this higher mathematics is necessary for the solution of the problem, and if it is justified in view of the simplified assumptions that are made. In the following, only a few examples will be discussed to show the general trend of an answer to the foregoing question. To treat every development in the paper in the same manner would fill a book.

Eq. 10*a* checks the writer's Fourier series expansion provided the member $\frac{1}{2} a_o$ is dropped in the general formula for the Fourier series—

$$f(x) = \tfrac{1}{2} a_o + \sum_{n=1}^{\infty} [a_n \cos (n x) + b_n \sin (n x)] \dots\dots\dots (127)$$

The use of the Fourier series expansion makes Eq. 10*b* look formidable when it actually is nothing else than

$$y = \left(\frac{q}{K a_o} - i \right) (L_d - x) (1 - e^{-\beta^2 j}) \dots\dots\dots\dots (128)$$

To obtain this result it is not necessary to go through the Fourier series expansion.

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[26] Head, Eng. Research, State Water Survey, Urbana, Ill.

Applying condition 1 to Eq. 9,

$$C_1 \cos \frac{(2\,n - 1)\,\pi}{2\,L_d}\,x = -\left(\frac{q}{K\,a_o} - i\right)(L_d - x) \quad \text{(129)}$$

and, if this special value is introduced into Eq. 9, the result is a derivation of Eq. 128 without the detour of the Fourier series expansion. The development of Eq. 128 is based on Eq. 7 as the approximate equation of the stable mound at $t = \infty$. Referring to Fig. 1, Eq. 7 is the actual formula for y', not y. A somewhat simpler expression than Eq. 128 is obtained by using y. Eq. 7 is replaced by

$$y = H_d - \left(\frac{H_d}{L_d} - i\right) x \quad \text{(130)}$$

Similarly, in the place of Eq. 14,

$$y = \left(i - \frac{H_d}{L_u}\right) x - H_d \quad \text{(131)}$$

If the same procedure used in the derivation of Eq. 128 is applied to Eq. 130, a replacement of Eq. 128 becomes

$$y = \frac{H_d}{L_d}\,(L_d - x)\,(1 - e^{-\beta^2 i}) \quad \text{(132)}$$

The principles here developed can be expanded to other formulas in the paper, and it can be seen that, without recourse to the Fourier series expansions, formulas are obtained that are simpler in form and easier for numerical calculations than those in the paper. This is important as the Fourier series, although beautiful in theory, are notoriously cumbersome for numerical calculations.

It may also be questioned if the straight-line slope is the correct approximation for the steady mound. Eq. 1 is based on the assumption of Jules Dupuit that the velocity in a vertical is constant and horizontal and in an amount equal to that defined by the Darcy law—namely, $v = K\,(dy/ds)$ which is replaced by $v = K\,(dy/dx)$ for flat slopes for the sake of integrality of the equations.

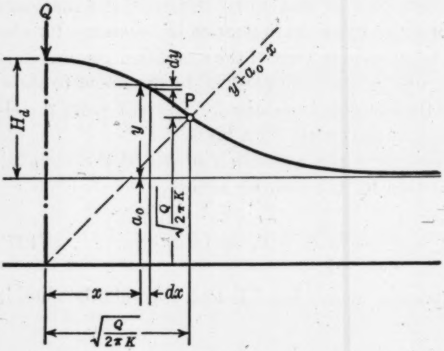

FIG. 22

The Dupuit assumptions are nonpotential but give close approximation in flat slopes, where the sine of the slope can be replaced by the tangent. The following developments are based on these assumptions and show that even these simplifications furnish curves of the general character obtained by the potential theory and by Mr. Baumann's experiments. Assuming a point source with circular runoff around it, the areas are as follows:

At distance	Area
x	$2\pi x (y + a_o)$
$x + dx$	$2\pi (x + dx)(y + a_o - dy)$

Therefore, the change in area is

$$dA = 2\pi [(a_o + y)\, dx - x\, dy] \qquad (133)$$

—neglecting the higher differential $dx\,dy$ (see Fig. 22).

Inasmuch as

$$v = \frac{Q}{A} = K \frac{dy}{dx} \qquad (134)$$

TABLE 2.—CHARACTERISTICS OF THREE CASES OF WATER SURFACE CURVES

Case	Value of dA	Area	Velocity	Slope	Water surface
1	0	Constant	Constant	Constant	Straight line
2	Positive	Increasing	Decreasing	Flattening	Concave line
3	Negative	Decreasing	Increasing	Steepening	Concave line

there are three cases for the water surface curve (see Table 2). When do these cases occur? Since

$$dA = 0 = (a_o + y)\, dx - x\, dy \qquad (135)$$

$$\frac{dx}{x} = \frac{dy}{a_o + y} \qquad (136)$$

Integrated,

$$\log x + \log C = \log (a_o + y) \qquad (137)$$

and

$$a_o + y = C\, x \qquad (138)$$

in which, at point P:

$$C = \frac{dy}{dx} \qquad (139)$$

For flat slopes $C < 1$ but the actual value is not yet known. Under the conditions assumed in the paper

$$C = \frac{H_d}{L_d} \qquad (140)$$

Since—

$$Q = 2\pi K x (a_o + y) \frac{dy}{dx} \qquad (141)$$

—$dA = 0$ at a point x,y defined as

$$y = \sqrt{\frac{Q}{2\pi K}} - a_o \qquad (142a)$$

and

$$x = C \sqrt{\frac{Q}{2\pi K}} \qquad (142b)$$

The condition dA exists therefore only in the immediate vicinity of an elevation y fixed by natural conditions. The value of C at that point can be determined by a study of the curve of the water surface. Integrating Eq. 141, an equation of the steady mound is derived:

$$y^2 + 2\,a_o\,y - \frac{Q}{\pi K}\log x + C = 0 \qquad (143)$$

For an infinitely large spreading ground, $C = \infty$ and for a limited spreading ground $C = \frac{Q}{\pi K}\log L_d$. In these two cases, Eq. 143 yields

$$y = -\,a_o \pm \sqrt{a^2_o + \frac{Q}{\pi K}\log x - C} \qquad (144a)$$

and

$$y = -\,a_o \pm \sqrt{a^2_o + \frac{Q}{\pi K}\log\frac{x}{L}} \qquad (144b)$$

—both of which lead to

$$\frac{dy}{dx} = -\,\frac{Q}{2\,\pi\,K\,(a_o + y)\,x} \qquad (145)$$

Eq. 145 becomes zero only for $x = \infty$, since $y_{\max} = H_d$. On second derivation,

$$\frac{d^2y}{dx^2} = \frac{Q}{2\,\pi\,K} \times \frac{1}{x^2} \times \frac{1}{a_o + y}\left(1 - \frac{Q}{2\,\pi\,K} \times \frac{1}{(a_o + y)^2}\right) \qquad (146)$$

Eq. 146 becomes zero for $x = \infty$ and for $y = \sqrt{\frac{Q}{2\,\pi\,K}} - a_o$.

The point with $dA = 0$ is therefore also a point of inflection and its x-coordinate, from the condition defined by Eq. 133, is computed by

$$x = \sqrt{\frac{Q}{2\,\pi\,K}} \qquad (147)$$

Therefore, this point lies on the line $a_o + y = x$, and in Eq. 138

$$C = \frac{dy}{dx} = -\,1 \qquad (148)$$

This steep slope explains why, in actual field and laboratory observations, there is a sharp drop near the recharge area.

A slope of 45° is never observed because, in the field and in the model, there are influences that flatten the slope—namely:

(*a*) For steep slopes Dupuit's assumptions do not yield a good approximation; and

(*b*) There is an area of inflow instead of a point of recharge. Eq. 146 shows two additional properties of the surface configuration:

(1) $\frac{d^2y}{dx^2}$ is positive if $y > \sqrt{\frac{Q}{2\,\pi\,K}} - a_o$

such a curve is convex between the source of recharge and point P. On the other hand,

$$(2)\ \frac{d^2y}{dx^2} \text{ is negative for } y < \sqrt{\frac{Q}{2\,\pi\,K}} - a_o$$

and such a curve is concave from point P outward. Considering these properties of the curve of the steady mound it does not seem appropriate to apply a refined computation involving an assumption that the water surface can be represented by a straight line.

The author presents curves of the type here developed, but also inverse curves. However, it is the writer's belief that the drop shown near the outflow is due to a backwater of less height than the critical depth[27]:

$$h_c = \frac{2}{\pi} \times \frac{Q_{\max}}{K} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (149)$$

D. P. Krynine,[28] M. ASCE.—The history of engineering reveals a great number of cases when successful structures were built and engineering operations performed from experience and partly by intuition, but without adequate theoretical background on the part of engineers. Theoretical clarifications based on physics and mechanics were given generally much later by scientists and scientifically inclined engineers other than the original ones. Such, for instance, was the case of ancient earth dams that were built since times immemorial without their builders having any notion of soil mechanics or subsurface flow net theories. Conversely, in the case of the present paper we have an extraordinary example of a modern engineer who for many years was engaged in the technical direction of spreading operations on a large scale in Southern California and then himself produced a complete theory of spreading tending to explain the minute details of the phenomenon. Under given circumstances it is exceedingly difficult to discuss this interesting paper, not only because of the abundant and rather complicated mathematical analysis presented, but chiefy because of the fact that the author, by virtue of his practice, is strongly prepared to defend his views. The latter, in addition, are backed by well-performed experiments which the writer had an opportunity to examine personally in 1948. The writer, therefore, wishes to limit his discussion mostly to the comparison of the author's philosophy of approach to the given problem with his own views.

After having read Mr. Baumann's paper the writer became convinced that at least three problems of the underground flow belong to the same class. These are—the theory of consolidation; drainage of the pavement base course;[29] and spreading. From these three the spreading problem is the most complicated because in this case the flow is confused especially at the experimental stage, by capillarity, mostly above the saturation line, and also by the boundary conditions, as at the bottom of the experimental box. In fact, the latter conditions are indicated by the travel fringes marked 5, 6, and so on, minutes in

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[27] "Technische Hydraulik," by Charles Jaeger, Birkhäuser, Basel, Switzerland, 1949, p. 366.

[28] Cons. Engr., San Francisco, Calif.

[29] "Base Course Drainage for Airport Pavements," by A. Casagrande and W. L. Shannon, *Transactions*, ASCE, Vol. 117, 1952, p. 792.

Figs. 19 and 20. In the opinion of the writer these fringes are produced by the colored liquid that falls to the bottom of the box through water and spreads horizontally due to the combined action of hydraulic head and momentum. The writer observed a somewhat similar phenomenon in his experiments on horizontal capillarity in cylindrical tubes filled with dry sand. In the latter case water just under a small head first very quickly spreads horizontally along the bottom of the tube and afterwards invades the tube in a capillary way moving upward.[30] The writer believes that the use of a parabolic bottom in the experimental box (Fig. 21) was a fruitful idea since in such a way the true character of flow in nature is better duplicated by the experiment than in the case of a horizontal bottom of the experimental box.

The solution of the three aforementioned problems is based in one way or another on the well-known differential equation of heat transfer through a prismatic, nonradiating bar with a constant coefficient proportional to the coefficient of permeability *k*. In the case of subsurface flow the coefficient *k* is not constant, however, and gradually decreases as either the process of drainage or spreading advances. This is a pronounced phenomenon in the case of soils with fine-grained admixtures. The situation is better in the case of flow through coarse pervious materials since in this case a relative constancy of the coefficient of permeability during drainage or spreading may be expected. Hence, the author's statement that "* * * spreading grounds are located in soils that approach idealized conditions extending over great distances" is encouraging. It is to be noticed that the same favorable condition prevails in the case of runway bases made of gravel or similar materials.

The hydraulics of the open-channel flow had existed before the advent of the theories of the subsurface flow; and undoubtedly the latter have been influenced by the former. Thus, in the theories of quasi-horizontal subsurface flow the existence of a horizontal or slightly inclined impervious bottom (like those occurring in some rivers) is predicated. In reality, however, the earth mass may be pervious to an indefinite depth and saturated with ground water. The most realistic explanation why an impervious bottom is introduced in the theories of quasi-horizontal subsurface flow is the necessity to have a plane from which the ordinates of the subsurface stream could be measured; otherwise the formulas cannot be derived. Such is the case in the Dupuit formula, the theory of the pumping test, the theory of the drainage of a pavement base,[29] and, finally, the theory of spreading as presented in this paper.

The writer feels, however, that, in the case of a very deep ground-water stream with large values of a_o, spreading would affect the lower part of this stream very little, if at all. It is stated sometimes in soil mechanics that water has no shearing strength, but this statement is of a relative value only. The very concept of viscosity is based on the possibility that shearing stresses exist in a viscous fluid such as water. The hydraulic gradient pushing the spreading stream down is larger than that of the underlying ground water; hence, the spreading stream moves with larger velocities than the underlying ground water. This causes shearing stresses and a corresponding increase of velocities in the ground water, which fact permits gradual incorporation of the mound wave into the original ground-water stream. The shearing stresses have to decrease rapidly from the surface of the original ground water downward and at a certain depth become negligible for all practical purposes. This pattern may be easily visualized by analogy from the diagram of shearing stresses in a body with a horizontal or slightly inclined upper surface over which a heavy object with a plane bottom surface is sliding. The writer's reasoning is valid, in his opinion, at least for the first and second phases of the spreading process.

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[30] "Some Capillary Phenomena in Sandy Materials," by D. P. Krynine, *Proceedings*, 29th Annual Meeting, Highway Research Board, National Research Council, Washington, D. C., 1950, pp. 520–530.

In conclusion the writer wishes to congratulate Mr. Baumann on his assiduous and conscientious work which represents a valuable contribution to the mathematical theory of spreading.

Paul Baumann,[31] M. ASCE.—In preparing this paper over a period of several years, the writer had two principal objectives, namely—to find sound, theoretical interpretation of observed ground-water phenomena related to spreading, and to clarify attendant, fundamental concepts which appeared to be afflicted, to some degree at least, by confusion. Above all, it was felt that the wave phenomenon and the significance of controls had not been fully evaluated and recognized, respectively.

Mr. Todd seems to interpret the writer's expression, "to at least mean sea level," as falling short of possibly 3.5 ft above mean sea level. Such was not the intention. Since 1950 the writer has been in charge of an investigation, by the Los Angeles County Flood Control District, of economic means for the stemming of sea-water intrusion in the West Coastal Basin. This basin (which Mr. Todd refers to in Table 1 as "West Coastal Plain") has a shore line about 12 miles in length along which about two thirds of the principal aquifer, known as Silverado Zone, are confined and the balance unconfined. The latter is predominantly located in the southern part of the area. In 1950 spreading tests were conducted to confirm the unconfined character of the aquifer there, to determine the permeability of the soil, and to ascertain the salinity of the ground water. The investigation led to the conclusion that, in this area, sea-water intrusion could best be controlled by means of a liquid barrier, created through spreading from a strip area or from shallow gravel-filled injection pits, 50 ft apart. This barrier would have to be located well inland from the seashore so as to have the assurance of control. Therein lies a distinct difference between the mechanics of a liquid barrier in an open and a confined aquifer. The latter, as Mr. Todd correctly states, is not treated in the paper, although a recharge test was conducted that same year on an abandoned well in the confined zone of the aquifer. Both tests plus a third one in the northern quasi-unconfined area were described in a report by the Los Angeles County Flood Control District, prepared in the spring of 1951, together with a study on the location and components of a complete liquid barrier over the entire 12-mile length. Finally, it included a recommendation for a pilot test on recharge wells, extending over a 1-mile front in the confined area, some 2,000 ft inland from the seashore. Although the chloride concentration at this location may correspond to a coefficient $\alpha_s = 1.01$, and may call for a rise of the barrier above mean sea level of 2 ft ±, the chloride concentration at the proposed injection pits of the open aquifer is very much smaller, so as to require a rise there of only a few tenths of a foot above mean sea level. Hence, the wording of the paper was meant to imply that, whereas in some locations the rise of the mound above mean sea level would be negligible to stem intrusion, it could be appreciable in other locations. Where the latter happens to be the case, each foot of extra rise may indeed represent a considerable part of the total effort. Hence, Mr. Todd's point is well taken.

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Fortunately, the conditions in the open area of the West Coastal Basin are not as difficult as they would be if the physical parameters used in the writer's prototype example applied. These values actually apply to a spreading ground in a debris cone area and not to the open aquifer of the West Coastal Basin. The respective values would be more nearly: $q_o = 0.25$ cu ft per sec $= 900$ cu ft per hr; $a_o = 300$ ft; $K = 10$ ft per hr; and $i = 0.004$. Hence, the corresponding mound width, $W = 75$ ft, would assure complete closure based on the aforementioned pit spacing of 50 ft.

In reading Mr. Suter's able discussion, one should bear in mind two fundamental premises: (1) The writer confined linearization of the stable or stationary mound to the two-dimensional case only and (2) "the improbability of the existence of a circular control" was emphasized in connection with the three-dimensional case. The linearization of the two-dimensional mound is a permissible approximation as long as $y \ll a_o$ in accordance with the basic premise. The discrepancy between the linearized and the true, stable mound shown in Fig. 14 is exaggerated because in the comprehensive model tests the aforementioned, basic premise was not satisfied. In fact, the height of the stable mound was nearly one half of the initial ground-water depth. This exaggeration was intentional to facilitate observation on the one hand and to ascertain the effect of this deviation from the basic premise on the theoretical results. It is a well-known fact that mathematics often furnishes valid results well beyond the imposed limitations.

Mr. Suter makes a commendable effort at simplification of Eq. 10*b* by introducing boundary condition 1 in Eq. 9. Apparently the statement, "Eq. 9 still does not satisfy condition 1 * * *" was thereby overlooked. Naturally, if Eq. 9 does not satisfy condition 1, then condition 1 does not satisfy Eq. 9. Hence, it is necessary, first of all, to transform Eq. 9 into an equation which does satisfy condition 1. This was accomplished through Eq. 10*b*. It is regrettable, of course, that condition 1 is singularly difficult to satisfy and that the elegant operation suggested by Mr. Suter does not answer the purpose. Unfortunately, it leads to the result expressed through Eq. 128, which describes a family of straight lines, radiating from the control toward the spreading ground. Hence, in accordance therewith, not only are the undisturbed ground-water surface and the surface of the stable mound linear but also all the intermediate surface configurations of the growing mound. Again, referring to Fig. 14, it is obvious that such is not the case. All equations derived from Eq. 4 must necessarily satisfy the latter. This is true of all of the writer's equations for the growing and the disappearing mound, but it is not true of Eq. 128.

Eqs. 16 and 17 which express the entire, two-dimensional phenomenon are not as formidable as they may appear at first glance. For $i = 0$, $L_u = L_d$ and Eqs. 17 check Eq. 10*b*. Length, L_d, may be any finite distance. Hence, in the case of L_d considerably in excess of the one represented in Fig. 2, it is possible to determine the progress and shape of the wave in both the upstream and downstream direction for such arbitrary time elements as correspond to the first phase. This is shown in Fig. 23 which is based on the same numerical values as the dash lines in Fig. 2 except for $L_d = 15$ ft instead of 4 ft. It shows that in this case the time required for the wave to reach the control, and for discharge in addition to the initial ground-water flow to commence, is of the order of 13 min as compared to 0.5 min in the first case. It also shows the flexibility of equations of this type. For $L_u = L_d$ ($i = 0$) Eqs. 17 also check the analogue of the equation[32] for heat flow in a nonradiating, prismatic bar of semi-infinite length which reads (in terms of ground-water parameters):

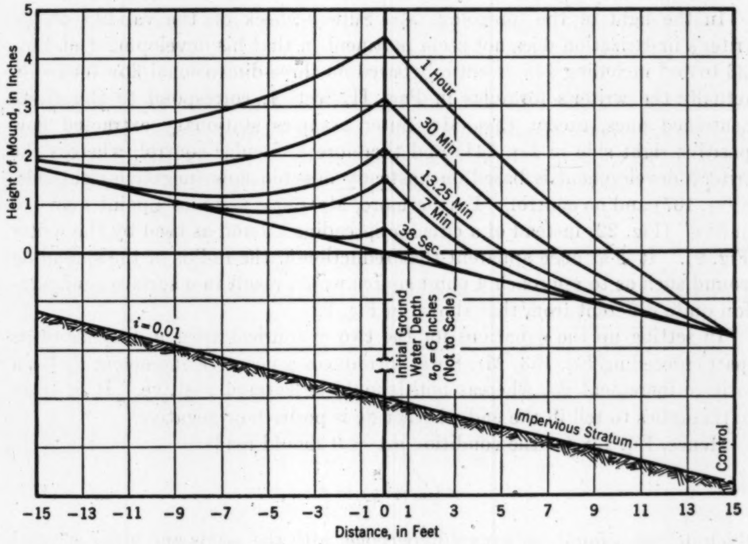

FIG. 23

$$y = \frac{q_0 \, x}{K \, a_0 \, \sqrt{\pi}} \left\{ \frac{e^{-x^2/(4j)}}{\sqrt{\dfrac{x^2}{4j}}} - \sqrt{\pi} \left[1 - \Phi_{(x)} \sqrt{\frac{x^2}{4j}} \right] \right\} \ldots\ldots (150)$$

in which (see Eq. 5) $j = \dfrac{K \, a_o \, t}{\mu}$; and $\Phi_{(x)} = \dfrac{2}{\sqrt{\pi}} \displaystyle\int_o^x e^{-t^2} \, dt$, the probability integral which is readily determined from tables.[33] For $x = 0$, Eq. 150 reduces to

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[32] "Heat Conduction," by Ingersoll, Zobel, and Ingersoll, McGraw-Hill Book Co., Inc., New York, N. Y., 1948, p. 110.

$$y = q_o \sqrt{\frac{4\,t}{K\,a_o\,\pi\,\mu}} \qquad (151)$$

which also checks the corresponding term derived from Eqs. 17.

Mr. Suter states correctly that the writer's equations for y are actually those for y' according to Fig. 1, and that the relations as shown in Eqs. 130 and 131 should be used for the respective transformation. Unfortunately, however, this transformation does not free Eq. 132 of the discrepancy attending Eq. 128.

In the light of the foregoing, Mr. Suter's check on the validity of the writer's linearization does not seem pertinent in that his development of Eqs. 133 to and including 148, is entirely based on three-dimensional flow for $i = 0$. Actually the writer's formulas in Part II, Sect. 1, correspond to the aforementioned ones, except that Mr. Suter assumes stationary extracted flow (positive right side in Eq. 141) and therefore a circular control, whereas the writer's development is based on unsteady injected flow (negative right side, in Eq. 107) and no control. Furthermore, Mr. Suter assumes a point source of "inflow" (Fig. 22) instead of a circular spreading ground as used by the writer (Fig. 9). It is at once apparent that reduction of the radius of the spreading ground such as to approach a point source would result in a surface configuration quite different from that shown in Fig. 22.

In setting up the equations for the two cylindrical areas a distance of dx apart, preceding Eq. 133, Mr. Suter introduces a negative increment dy for a positive increment dx, whereas both should be entered positive. It is up to mathematics to tell in the end whether dy is positive or negative.

Hence, Eq. 138 for the condition $dA = 0$ should read

$$x\,(a_o + y) = C \qquad (152)$$

which defines a family of square hyperbolas with the y-axis and a line parallel to, and a distance a_o below, the x-axis as asymptotes. The intersection of any of these hyperbolas with the undisturbed ground-water surface corresponds to the distance L_d at which the circular control is assumed. This solution satisfies the fundamental law that it is impossible to maintain all three—area, velocity, and slope constant—for increasing x. These hyperbolas are a special case of the so-called "cone of expression." It is important to emphasize, however, that a recharge well cannot be treated as a water well "in reverse"—that is, by assuming stationary flow as Mr. Suter did. The reason is that a water well is in itself a control whereas the recharge well is not. Hence, in the latter case the flow is unsteady and radiates from the well uniformly to any distance in form of a wave without ever becoming stationary within finite limitations. In the course of this performance the rate of recharge must diminish as illustrated in Fig. 10 (dash line). This diminution in the rate of recharge is, purely and simply, a hydromechanical necessity; yet, it has been attributed to causes other than the real one, by recent investigators. The writer is by no means oblivious to the fact that causes of an entirely different nature exist; but he believes in taking the first step first and in taking the probable into consideration ahead of the possible.

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[33] "Heat Conduction," by Ingersoll, Zobel, and Ingersoll, McGraw-Hill Book Co., Inc., New York, N. Y., 1948, see appendix.

In Mr. Suter's final paragraph, reference is made to the critical depth at the control in accordance with Charles Jaeger's determination,[27] which compares with Eq. 102. Both equations are approximations. Contrary to the writer's development in Sect. 4, Part I, Mr. Jaeger arrives at his value through substitution of circular arcs for the parabolic equipotential lines. This leads to a sharp drawdown of the phreatic line and a vertical tangent at the control. The point of contact of this tangent determines the critical depth. In either case the critical depth is defined as the minimum height of the phreatic line at the control regardless of how far below it the backwater surface may be. The true value of h_c probably lies between that defined by Eqs. 149 and 102.

Mr. Krynine's reference to a very deep ground-water stream reveals a keen concept of the mathematical premises necessary to make solution possible. The depth to the impervious stratum and its slope may never have been explored. In fact, a clear-cut, impervious stratum may not even exist. Instead, the aquifer may terminate in a merged zone which, under the influence of the critical surcharge, has broken down into a quasi-impervious soil. An effective, impervious stratum must then be assumed, preferably in the form of a horizontal plane. It must not be overlooked, however, that the pressure conditions, even in a deep ground-water stream, in an open aquifer, may be decisively affected through the creation of a spreading mound of commensurate height. As shown in Figs. 2 and 23, such a stream, regardless of how mighty, may be checked by strip spreading. The original ground-water flow downstream from the spreading ground is then replaced by infiltrated water from the latter during the first, and part of the second, phase. It is axiomatic that to accomplish this replacement the infiltrated flow per unit length of spreading ground must be greater than the corresponding initial ground-water flow, which reveals the fact that the spreading ground itself is a control even if the mound never rises to the surface. Hence, spreading grounds are powerful hydraulic "weapons" and must be operated with proper appreciation of that fact.

The progressive transmission of motion of the spreading wave from its surface to the initial ground-water stream at any depth through the medium of shearing stresses is a telling example of Mr. Krynine's insight into these phenomena. His reference to the fruitfulness of the parabolic bottom used in the model is no doubt closely related to the significance of shearing stresses. In this connection it is well to realize, however, that the mathematical treatment, based on the Boussinesq equation in general and on Eq. 4 (heat flow analogue) in particular is afflicted with a physical inconsistency. This is due to the neglect of inertia forces. It implies instantaneous instead of progressive acceleration through the spreading mound of initial ground-water bodies and instantaneous transmission of pressure to the control or infinity, during the first phase. The introduction of the intuitive, unique flow function in Part I, Sect. 3, tends to mitigate the aforementioned inconsistency in that a finite wave length is associated with a finite time element. It is interesting to note from Fig. 14 that equations based on the latter concept led to closer agreement with the test results than those based on Eq. 4.

In conclusion, the writer wishes to express his sincere appreciation to the discussers of the time and effort which were necessary to compose their valuable contributions to this paper. In the light of the combined efforts it is hoped that ground-water phenomena related to spreading will be more readily understood in the future than they have been in the past.

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FORCED VIBRATIONS OF CONTINUOUS BEAMS

BY EDWARD SAIBEL[1] AND ELIO D'APPOLONIA,[2] A. M. ASCE

WITH DISCUSSION BY MESSRS. W. H. HOPPMANN, AND EDWARD SAIBEL AND ELIO D'APPOLONIA

SYNOPSIS

The determination of the forced oscillations of a continuous beam due to an exciting force is presented in this paper. The procedure requires only a knowledge of the eigenfunctions and eigenvalues of the continuous beam from which all intermediate points of support have been removed (a simple beam). For the simple beam, these functions are known or can be readily evaluated.

The deflected shape of the continuous beam is developed in terms of the eigenfunctions of the simple beam.

The equations of motion are set up using the Lagrangian equations of motion, the intermediate supports being introduced through undetermined multipliers. The solution is found for any number of supports and a numerical example is given.

INTRODUCTION

The problem of the forced vibrations of beams has been a subject of discussion for over a century. The studies, in practically all cases, have been restricted to simple span structures. In the extension to continuous beams, the classical approach has been to cut the beam at the intermediate supports, destroying the continuity, and leaving a series of individual beams for each of which the solution can be found. Re-establishing the continuity of the beam by equating deflections, slopes, and moments at the sections that had been previously cut leads to a system of linear equations. In the case of the free oscillations of a continuous beam, the system of equations is homogeneous. The necessary condition for a nontrivial solution leads to the frequency equation from which the natural frequencies may be found. When the motion of the continuous beam is forced, the system of equations is nonhomogeneous

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and may be solved, in theory, for the deflection of the beam as a function of time. In practice, however, the system of equations that arises is intractable, and as a result, if the solution is to be of practical use, some other method of analysis must be devised.

A solution for the special case of a beam of two equal spans and simply supported at its extremities has been developed,[3] but this solution is apparently not capable of complete generalization. It is the purpose of this paper to present a solution for the forced vibrations of all types of continuous beams subjected to a fixed pulsating load. The method used is an extension of the one developed by the senior author[4] for the solution of the natural frequencies of continuous beams. This method had been developed earlier, in connection with other problems, by A. Weinstein.[5]

Basically the steps taken in the solution are: (*a*) the intermediate supports are removed leaving an ordinary simple beam for which the vibration frequencies and modes (eigenvalues and eigenfunctions) are known. The term, simple beam, as used herein refers to a beam without intermediate supports, but one whose end conditions can be any combination of simple, fixed, or free; (*b*) the deflection of the beam at any point along its length is represented as a generalized Fourier series in terms of the eigenfunctions of the simple beam; (*c*) the constraints at the intermediate supports are introduced through undetermined multipliers; and (*d*) from the Lagrange equations of motion and the conditions of constraint at intermediate supports (such as no deflection at a support if that happens to be the case) equations are developed that yield the solution of the problem.

This procedure may be applied to beams of any number of spans of arbitrary length and of varying flexural rigidity, and with any type of conditions at the extreme ends of the beam.

The important thing is that the eigenfunctions for the simple beam (the beam that is left when the intermediate supports are removed) must be known or be obtainable. In many practical cases these eigenfunctions are known.

In this paper the theory is applied to the case of a two-span uniform beam of unequal lengths subjected to a fixed pulsating force.

THEORY

Steady Vibrations.—The usual assumptions made in the classical theory of beam vibrations are assumed in this paper. Damping effects are neglected. The differential equation of the vibrating beam is

$$\frac{\partial^2}{\partial x^2}\left[E\,I\,(x)\,\frac{\partial^2 y}{\partial x^2}\right] + m\,(x)\,\frac{\partial^2 y}{\partial t^2} = P\,(x, t) \qquad \text{.............}(1)$$

in which E is the modulus of elasticity, I is the moment of inertia, x is the distance, $E\,I\,(x)$ is the flexural rigidity, $n(x)$ is the mass per unit length,

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[3] "Transverse Vibration of a Two-Span Beam Under Action of a Moving Constant Force," by R. S. Ayre, George Ford, and L. S. Jacobsen, *Journal of Applied Mechanics*, A.S.M.E., Vol. 17, No. 1, 1950, p. 1.

[4] "Vibration Frequencies of Continuous Beams," by Edward Saibel, *Journal of the Aeronautical Sciences*, Vol. 11, 1944, pp. 88–90.

[5] "Étude des spectres des equations aux derivées partielles de la théorie des plaques élastiques," by A. Weinstein, *Fascicule 88*, *Mémorial des Sciences Mathématiques*, Gauthier-Villars, Paris, France, 1937.

$y\ (x,\ t)$ is the deflection, and $P(x, t)$ is the externally applied force that, in this treatment, is exclusive of the intermediate or end supporting forces or moments.

As pointed out, the solution will be developed in the form:

$$y = \sum_{r=1}^{\infty} q_r(t)\ \phi_r(x) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which the values of q_r are the generalized coordinates of the vibrating system and the ϕ_r-values are the eigenfunctions of the simple beam. The ϕ_r-values constitute a complete orthonormal set—that is,

$$\int_o^L m(x)\ \phi_r(x)\ \phi_s(x)\ dx = \left.\begin{matrix} 0, & \text{when } r \neq s \\ 1, & \text{when } r = s \end{matrix}\right.$$

in which L is the total span length. Furthermore, associated with each ϕ_r, it is assumed that there is only one eigenvalue $p_r = \sqrt{\lambda_r}$ that is the circular frequency in the rth mode of vibration.

To establish the amplitudes of the forced vibrations of the continuous beam, it is first necessary to determine the equations expressing the generalized coordinates as a function of the eigenfunctions and eigenvalues of the simple beam and the frequency of the applied load.

This is accomplished by the use of the Lagrange equations of motion, in which are introduced as many undetermined multipliers μ_i as there are intermediate supports. It is through these undetermined multipliers that the constraints on the beam at the intermediate supports are taken care of. In the case in which the beam is allowed no deflection at the point, $x = c_i$, this constraint (f_i) takes the form:

$$f_i \equiv \sum_{r=1}^{r=\infty} q_r\ \phi_r\ (c_i) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

and assuming n such points along the beam, that is, n intermediate supports or a continuous beam of $(m + 1)$ spans, the Lagrange equation takes the form:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial q'_r}\right) - \frac{\partial T}{\partial q_r} + \frac{\partial V}{\partial q_r} - Q_r - \sum_{i=1}^{i=m} \mu_i(t)\ \frac{\partial f_i}{\partial q_r} = 0 \ldots\ldots\ldots (4)$$

In this equation, V is the potential energy due to bending, T is the kinetic energy, Q_r is the generalized force, and q'_r is $\frac{dq_r}{dt}$.

It can be shown that the potential energy due to bending is

$$V = \frac{1}{2}\int_o^L E\ I(x)\ \left(\frac{\partial^2 y}{\partial x^2}\right)^2 dx \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

It follows, on the substitution of Eq. 2 into Eq. 5, that

$$V = \frac{1}{2}\sum_{r=1}^{r=\infty}\int_o^L q^2{}_r \left[\frac{d^2}{dx^2}\left(E\ I(x)\ \frac{d^2\phi_r}{dx^2}\right)\right]\phi_r\ dx \ldots\ldots\ldots\ldots (6)$$

Furthermore, the eigenfunctions of the simple beam satisfy the equation,

$$\frac{d^2}{dx^2}\left(E\,I(x)\,\frac{d^2\,\phi_r}{dx^2}\right) - m(x)\,p^2{}_r\,\phi_r = 0 \ldots\ldots\ldots\ldots (7)$$

and, since the values of ϕ_r satisfy Eq. 3, the potential energy of the system becomes

$$V = \frac{1}{2}\sum_{r=1}^{r=\infty} p_r{}^2\,q^2{}_r \ldots\ldots\ldots\ldots (8)$$

in which p_r is the rth eigenvalue of the simple beam $(= \lambda_r)$. The kinetic energy of a vibrating beam is given by

$$T = \frac{1}{2}\int_0^L m(x)\left(\frac{\partial y}{\partial t}\right)^2 dx \ldots\ldots\ldots\ldots (9)$$

Using Eq. 2 and Eq. 3 it can be shown that Eq. 9 takes the form:

$$T = \frac{1}{2}\sum_{r=1}^{r=\infty} (q'_r)^2 \ldots\ldots\ldots\ldots (10)$$

In this work, the deflection of the beam at each intermediate support is zero, thus the constraint condition at the ith support is

$$f_i = \sum_{r=1}^{r=\infty} q_r\,\phi_r\,(c_i) = 0 \ldots\ldots\ldots\ldots (11)$$

defining $i = 1, 2, \ldots n$.

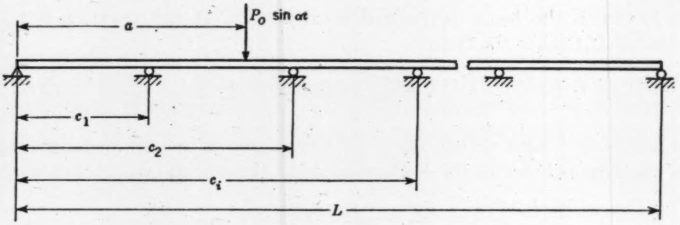

Fig. 1.—Continuous Beam Subject to Pulsating Load

Substituting Eqs. 8, 10, and 11 into Lagrange's equations of motion (Eq. 4), the resulting differential equation to be satisfied by each generalized coordinate becomes

$$q''_r + p^2{}_r\,q_r = Q_r + \sum_{i=1}^{i=m} \mu_i\,\phi_r\,(c_i) \ldots\ldots\ldots\ldots (12)$$

with $r = 1, 2, \cdots \infty$ in which $q''_r = \dfrac{d^2q_r}{dt^2}$. The solution of Eq. 12 may be obtained in the usual manner, that is, as the sum of the particular and of the homogeneous solutions. The form of the particular solution will depend on the right-hand side of the equation.

Consider the case of a continuous beam on which a load fixed in position $x = a$ but pulsating with circular frequency α is acting (Fig. 1). This load may be represented by $P_o \sin \alpha t$. The generalized force is

$$Q_r = (P_o \sin \alpha t)\, \phi_r(a) \ldots\ldots\ldots (13)$$

The particular solution may be formed by assuming

$$\mu_i = A_i \sin \alpha t \ldots\ldots\ldots (14a)$$

and

$$q_r = B_r \sin \alpha t \ldots\ldots\ldots (14b)$$

in which A_i and B_r are coefficients to be determined. After dividing out the factor $\sin \alpha t$, Eq. 12 takes the form:

$$-B_r \alpha^2 + B_r p^2_r = P_o \phi_r(a) + \sum_{i=1}^{i=m} A_i \phi_r(c_i) \ldots\ldots\ldots (15)$$

from which

$$B_r = \frac{P_o \phi_r(a) + \sum_{i=1}^{i=m} A_i \phi_r(c_i)}{p^2_r - \alpha^2} \ldots\ldots\ldots (16)$$

The coefficient A_i may be determined from the constraint conditions

$$\sum_{r=1}^{r=\infty} B_r \phi_r(c_i) = 0 \ldots\ldots\ldots (17)$$

in which $i = 1, 2, \ldots m$.

On substituting Eq. 16 into Eq. 17,

$$\sum_{r=1}^{\infty} \frac{P_o \phi_r(a)\, \phi_r(c_i) + \sum_{j=1}^{j=m} A_j \phi_r(c_j)\, \phi_r(c_i)}{p^2_r - \alpha^2} = 0 \ldots\ldots\ldots (18)$$

In expanded form, Eq. 18 becomes

$$A_1 \sum_{r=1}^{r=\infty} \frac{\phi^2_r(c_1)}{p^2_r - \alpha^2} + A_2 \sum_{r=1}^{r=\infty} \frac{\phi_r(c_1)\, \phi_r(c_2)}{p^2_r - \alpha^2} + \cdots = -P_o \sum_{r=1}^{r=\infty} \frac{\phi_r(a)\, \phi_r(c_1)}{p^2_r - \alpha^2} \ldots (19a)$$

$$A_1 \sum_{r=1}^{r=\infty} \frac{\phi_r(c_1)\, \phi_r(c_2)}{p^2_r - \alpha^2} + A_2 \sum_{r=1}^{r=\infty} \frac{\phi^2_r(c_2)}{p^2_r - \alpha^2} + \cdots = -P_o \sum_{r=1}^{r=\infty} \frac{\phi_r(a)\, \phi_r(c_2)}{p^2_r - \alpha^2} \ldots (19b)$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

The solution of this set of simultaneous linear equations for the values of A can be obtained for any number of supports. Only the first few terms of the series need be considered as the series converges rapidly. Hence the steady state part of the forced vibration solution can be expressed by

$$(y)_p = \sum_{r=1}^{r=\infty} \frac{P_o \phi_r(a) + \sum_{i=1}^{i=m} A_i \phi_r(c_i)}{p^2_r - \alpha^2} \sin \alpha t\, \phi_r(x) \ldots\ldots\ldots (20)$$

Eq. 20 is valid provided $p_r \neq \alpha$.

When the forcing frequency is equal to one of the natural frequencies of the simple beam a slight modification must be made. It is apparent from Eq. 15 that, when $p_r = \alpha$,

$$P_o\,\phi_r(a) + \sum_{i=1}^{i=m} A_i\,\phi_r(c_i) = 0 \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

consequently Eq. 16 becomes indeterminate, and B_r cannot be determined using it. However, the other values of B may still be found from this equation. The constraint Eq. 10 may be written

$$y(c, t) = \sum_{k=1}^{k=\infty *} q_k\,\phi_k(c) + q_r\,\phi_r(c) = 0 \ldots\ldots\ldots\ldots\ldots (22)$$

in which $\sum_{k=1}^{k=\infty *}$ indicates the sum over k with the $(k = r)$ term omitted. Since this must hold for all values of the time, it may be expressed as

$$\sum_{k=1}^{k=\infty *} \left[\frac{P_o\,\phi_k(a) + A\,\phi_k(c)}{p^2_k - \alpha^2} \right] \phi_k(c) + B_r\,\phi_r(c) = 0 \ldots\ldots\ldots (23)$$

in which, for simplicity, only one intermediate support is assumed and p_k is the kth eigenvalue of the continuous beam. The result is easily extended to any number of supports. From Eq. 21, for one inner support, there is only a single μ and consequently only one coefficient,

$$A = -\,\frac{P_o\,\phi_r(a)}{\phi_r(c)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

Substituting this value for A into Eq. 23,

$$B_r = -\,\frac{P_o}{\phi_r(c)} \sum_{k=1}^{k=\infty *} \frac{\phi_k(a)\;\phi_k(c) - \left[\dfrac{\phi_r(a)}{\phi_r(c)}\right] \phi^2\,(c)}{p^2_k - \alpha^2} \ldots\ldots\ldots (25)$$

The solution may now be written

$$(y)_p = \sum_{k=1}^{k=\infty *} B_k\,\phi_k(x) \sin \alpha\, t + B_r\,\phi_r(x) \sin \alpha\, t \ldots\ldots\ldots (26)$$

Consider the simple case of the steady state vibrations of a continuous two-span beam of uniform section and subjected to the fixed pulsating load $P_o \sin \alpha\, t$ as shown in Fig. 2. The eigenfunctions for the simple beam, for example, for the beam without the intermediate support $x = c$, are

$$\phi_r(x) = \sqrt{\frac{2}{L\,m}} \sin \frac{r\,\pi\,x}{L} \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

and the corresponding eigenvalues are

$$p^2_r = r^4\,p^2_1 = \frac{r^4\,\pi^4\,E\,I}{L^4\,m} \ldots\ldots\ldots\ldots\ldots\ldots (28)$$

From Eq. 18 the unknown coefficient,

$$A = -P_o \frac{\sum_{s=1}^{s=\infty} \frac{\phi_s(a)\,\phi_s(c)}{p^2_s - \alpha^2}}{\sum_{s=1}^{s=\infty} \frac{\phi^2_s(c)}{p^2_s - \alpha^2}} = -P_o K \quad \text{.............} (29)$$

For this particular case,

$$K = \frac{\sum_{s=1}^{s=\infty} \frac{\sin \frac{s\,\pi\,a}{L} \sin \frac{s\,\pi\,c}{L}}{s^4\,(1 - \beta^2_s)}}{\sum_{s=1}^{s=\infty} \frac{\sin^2 \frac{s\,\pi\,c}{L}}{s^4\,(1 - \beta^2_s)}} \quad \text{..................} (30)$$

in which $\beta^2_s = \frac{\alpha^2}{s^4\,p^2_1}$. Since the integer s occurs as the fourth power in the denominators of the summations, convergence of the series is rapid, and only a few terms of the series need to be considered to obtain excellent results.

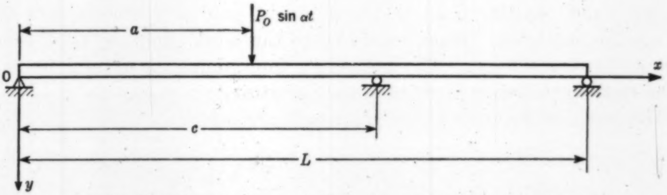

FIG. 2.—TWO-SPAN CONTINUOUS BEAM SUBJECT TO PULSATING LOAD

From Eq. 20, the steady state solution for the two-span continuous beam becomes

$$(y)_p = y_o \sum_{r=1}^{r=\infty} \frac{\sin \alpha\,t \sin \frac{r\,\pi\,x}{L}}{r^4\,(1 - \beta^2_r)} \left(\sin \frac{r\,\pi\,a}{L} - K \sin \frac{r\,\pi\,c}{L} \right) \quad \text{.......} (31)$$

in which $\beta^2_r = \frac{\alpha^2}{r^4\,p^2_1}$. The quantity,

$$y_o = \frac{2\,P_o\,L^3}{\pi^4\,E\,I} \quad \text{........................} (32)$$

is the approximate static deflection at the center of a simply supported uniform beam, of length L subjected to a concentrated load P_o at its center. It is apparent that, for the simply supported beam of span length L, the coefficient K is zero, and Eq. 31 reduces to the solution given by S. Timoshenko[6] and C. E. Inglis.[7]

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[6] "Vibration Problems in Engineering," by S. Timoshenko, D. Van Nostrand Co., Inc., New York, N. Y., 2d Ed., 1937.

[7] "A Mathematical Treatise on Vibrations in Railway Bridges," by C. E. Inglis, Cambridge Univ. Press, Cambridge, England, 1934.

Recognizing that the Fourier expansion for the concentrated pulsating load is

$$\sum_{r=1}^{r=\infty}\left(\frac{2\,P_o}{L}\times\sin\alpha\,t\times\sin\frac{r\,\pi\,a}{L}\times\sin\frac{r\,\pi\,x}{L}\right)\ldots\ldots\ldots\ldots(33)$$

and for the reaction,

$$\sum_{r=1}^{r=\infty}\left(\frac{2\,K}{L}\,P_o\times\sin\alpha\,t\times\sin\frac{r\,\pi\,c}{L}\times\sin\frac{r\,\pi\,x}{L}\right)\ldots\ldots\ldots\ldots(34)$$

it can be readily shown that solution of Eq. 31 satisfies the differential, Eq. 1, when $P(x,t)$ is replaced by the forces acting on the beam as well as the boundary conditions.

It is interesting to note that, if the frequency of the load is small in comparison with the frequencies of the simple beam, the term β_r can be neglected, and the equation of the curve of static deflection will be obtained. The physical interpretation that can be given to K is that it is a magnitude of the reaction at $x = c$ due to a unit load at $x = a$.

Complete Solution.—The complete solution of the forced vibration problem includes both the free and steady state oscillations. For most engineering problems, the free vibrations, due to damping, gradually become insignificant and can be neglected. However, to make this work complete, the free vibrations are now considered. The method for determining the natural frequencies of the restrained system is given in the literature.[4]

The complete solution of the problem is given by

$$y = (y)_p + (y)_F = \sum_{r=1}^{r=\infty}\frac{P_o\,\phi_r(a) + \sum\limits_{i=1}^{i=m} A_i\,\phi_r(c_i)}{p^2{}_r - \alpha^2}\sin\alpha\,t\,\phi_r(x)$$

$$+\sum_{k=1}^{k=\infty}\sum_{s=1}^{s=\infty}\frac{\sum\limits_{i=1}^{i=m}\bar{\mu}^{(k)}{}_i\,\phi_s(c_i)}{p^2{}_s - p^2{}_k}\,\phi_s(x)\sin\left(p_k\,t + e_k\right)\ldots\ldots\ldots\ldots(35)$$

in which (y_p) is the particular solution given by Eq. 20. As shown by one of the writers,[4] the components of the free oscillations of the continuous beam may be developed in terms of the eigenfunctions of the simple beam. Designate by $(y_k)_F$, the kth eigenfunction of the continuous beam. When expanded in terms of the eigenfunctions of the simple beam,

$$(y_k)_F = \sum_{s=1}^{s=\infty}\frac{\sum\limits_{i=1}^{i=m}\bar{\mu}^{(k)}{}_i\,\phi_s(c_i)}{p^2{}_s - p^2{}_k}\,\phi_s(x)\sin\left(p_k\,t + e_k\right)\ldots\ldots\ldots\ldots(36)$$

the barred symbols being used with reference to quantities associated with the free vibration of the continuous beam. The quantity $\bar{\mu}^{(k)}{}_i$ is the undetermined multiplier belonging to the ith support, when the kth mode of free vibration is considered. To determine the coefficients $\bar{\mu}^{(k)}{}_i$ and e_k the initial conditions are considered. Assume these conditions to be $y' = y = o$ at

$t = o$. From the condition $y = o$ when $t = o$, it is evident that $e_k = o$ and from $y' = o$ when $t = o$, it follows that

$$\sum_{r=1}^{r=\infty} \frac{\alpha \left[P_o \, \phi_r(a) + \sum_{i=1}^{i=m} A_i \, \phi_r(c_i) \right]}{p^2{}_r - \alpha^2} \phi_r(x)$$

$$+ \sum_{k=1}^{k=\infty} \sum_{s=1}^{s=\infty} \frac{\bar{p}_k \sum_{i=1}^{i=m} \bar{\mu}^{(k)}{}_i \, \phi_s(c_i)}{p^2{}_s - \bar{p}^2{}_k} \phi_s(x) = 0 \ldots\ldots\ldots\ldots\ldots (37)$$

Since this must be true for all values of x, the coefficients of each $\phi_r(x)$-term in Eq. 37 must be zero. This, in turn, leads to a system of linear equations for the values of $\bar{\mu}^{(k)}$. The solution of this system of equations is required. From the form of the equations, that comprise an infinite system of equations in an infinite number of unknowns, it may be seen that the conditions are satisfied for a solution to exist. Furthermore, only a few terms need be considered because of the rapidity of the convergence. Physically this means that only a few modes of the restrained system need be considered.

For the simple two-span continuous beam under discussion Eq. 37 becomes, when $r = s$,

$$\frac{\alpha \left[P_o \, \phi_r(a) - P_o \, K \, \phi_r(c) \right]}{p^2{}_r - \alpha^2} + \sum_{k=1}^{k=\infty} \frac{\bar{p}_k \, \bar{\mu}^{(k)} \, \phi_r(c)}{p^2{}_r - \bar{p}^2{}_k} = 0 \ldots\ldots\ldots (38)$$

Or, in expanded form, Eq. 38 becomes

$$\frac{P_o \left[\sin \frac{r \, \pi \, a}{L} - K \sin \frac{r \, \pi \, c}{L} \right]}{r^4 \, (1 - \beta^2{}_r)} + \frac{\bar{p}_1}{\alpha} \frac{\sin \frac{r \, \pi \, c}{L}}{r^4 \, (1 - \overline{\omega}^2{}_{1r})} \bar{\mu}^{(1)}$$

$$+ \frac{\bar{p}_2}{\alpha} \frac{\sin \frac{r \, \pi \, c}{L}}{r^4 \, (1 - \overline{\omega}^2{}_{2r})} \bar{\mu}^{(2)} + \frac{p_3}{\alpha} \frac{\sin \frac{r \, \pi \, c}{L}}{r^4 \, (1 - \overline{\omega}^{23}{}_r)} \bar{\mu}^{(3)} + \cdots = 0 \ldots\ldots (39)$$

in which $\overline{\omega}^2{}_{1r} = \dfrac{\bar{p}^2{}_1}{r^4 \, p^2{}_1}$; $\overline{\omega}^2{}_{2r} = \dfrac{\bar{p}^2{}_2}{r^4 \, p^2{}_1}$; $\cdots$. To solve for the undetermined multipliers $\bar{\mu}^{(1)}$, $\bar{\mu}^{(2)}$, $\bar{\mu}^{(3)}$, $\cdots$, that is, the multiplier associated with each mode of free vibration, Eq. 39 is expanded for the values of $r = 1, 2, 3, \cdots$.

It can be seen that in general the solution of Eq. 37 becomes cumbersome. However, for most problems only the first few modes of free vibration need be considered, and consequently the labor for the free vibration solution is reduced considerably.

The complete solution for the forced vibrations of a two-span continuous beam subjected to a fixed pulsating load $P_o \sin \alpha \, t$ is then given by

$$y = y_o \sum_{r=1}^{r=\infty} \frac{\sin \alpha \, t \sin \frac{r \, \pi \, x}{L}}{r^4 \, (1 - \beta^2{}_r)} \left(\sin \frac{r \, \pi \, a}{L} - K \sin \frac{r \, \pi \, c}{L} \right)$$

$$+ \frac{y_o}{P_o} \sum_{k=1}^{k=\infty} \sum_{s=1}^{s=\infty} \frac{\bar{\mu}^{(k)}}{S^4 \, (1 - \overline{\omega}^2{}_{ks})} \sin \frac{s \, \pi \, c}{L} \sin \frac{s \, \pi \, x}{L} \sin \bar{p}_k \, t \ldots\ldots (40)$$

in which as before, $\beta^2{}_r = \frac{\alpha^2}{r^4\, p^2{}_1}$; $\overline{\omega}^2{}_{ks} = \frac{p^2{}_k}{s^4\, p^2{}_1}$; $y_o = \frac{2\, P_o\, L^3}{\pi^4\, E\, I}$. Recognizing that

$$\frac{2}{L}\, \bar{\mu}^{(k)} \sin p_k\, t \sum_{s=1}^{s=\infty} \sin \frac{s\, \pi\, c}{L} \sin \frac{s\, \pi\, x}{L} \qquad (41)$$

is the Fourier expansion of the support reaction for *k*th mode of free vibration, it can be readily verified that the components of free oscillation given by the second term of Eq. 40 satisfy the differential equation of motion for a continuous uniform two-span beam.

NUMERICAL EXAMPLE

As a numerical example, consider a uniform beam simply supported at the ends and supported at an interior point so that the spans are of lengths 6 and 4.5, respectively, that is, $c = 6$, and $L = 10.5$ in Fig. 2. A driving force ($P_o \sin \alpha\, t$) with a frequency one fourth that of the lowest natural frequency of the simple beam is applied at the center of the longer span ($a = 3$).

For this beam, the values[4,6] of p are

$$p_1 = 0.0894 \sqrt{\frac{E\, I}{m}} \qquad (42a)$$

$$p_1 = 0.324 \sqrt{\frac{E\, I}{m}} \qquad (42b)$$

$$p_2 = 0.636 \sqrt{\frac{E\, I}{m}} \qquad (42c)$$

$$p_3 = 1.242 \sqrt{\frac{E\, I}{m}} \qquad (42d)$$

For $\frac{\alpha}{p_1} = \frac{1}{4}$, K is found to be 0.755. Using the first three terms of the series for the determination of the unknown multipliers, Eq. 39 becomes

$$1.165\, \bar{\mu}^{(1)} + 0.557\, \bar{\mu}^{(2)} + 0.281\, \bar{\mu}^{(3)} = 0.0509\, P_o \qquad (43a)$$

$$2.29\, \bar{\mu}^{(1)} - 0.358\, \bar{\mu}^{(2)} - 0.137\, \bar{\mu}^{(3)} = 0.0819\, P_o \qquad (43b)$$

$$0.1660\, \bar{\mu}^{(1)} + 0.734\, \bar{\mu}^{(2)} - 0.384\, \bar{\mu}^{(3)} = 0.01257\, P_o \qquad (43c)$$

Solving these equations,

$$\bar{\mu}^{(1)} = 0.0398\, P_o$$

$$\bar{\mu}^{(2)} = 0.0836\, P_o$$

$$\bar{\mu}^{(3)} = -\, 0.1500\, P_o$$

The deflection of the beam as a function of x and t is given by

$$y = y_o \left[\sin \alpha t \left(0.0509 \sin \frac{\pi x}{10.5} + 0.0819 \sin \frac{2 \pi x}{10.5} + 0.01257 \sin \frac{3 \pi x}{10.5} \right) \right.$$
$$+ \sin p_1 t \left(- 0.00320 \sin \frac{\pi x}{10.5} - 0.00602 \sin \frac{2 \pi x}{10.5} - 0.000455 \sin \frac{3 \pi x}{10.5} \right)$$
$$+ \sin p_2 t \left(- 0.001631 \sin \frac{\pi x}{10.5} + 0.001050 \sin \frac{2 \pi x}{10.5} - 0.00215 \sin \frac{3 \pi x}{10.5} \right)$$
$$\left. + \sin p_3 t \left(0.000758 \sin \frac{\pi x}{10.5} - 0.000370 \sin \frac{2 \pi x}{10.5} - 0.0001035 \sin \frac{3 \pi x}{10.5} \right) \right] . \quad (44)$$

The deflection at the point $x = 3$ as a function of time is given by

$$y_{x=3} = y_o \, [0.1248 \sin \alpha t - 0.00855 \sin p_1 t$$
$$- 0.001187 \sin p_2 t + 0.000189 \sin p_3 t] \ldots\ldots\ldots\ldots (45)$$

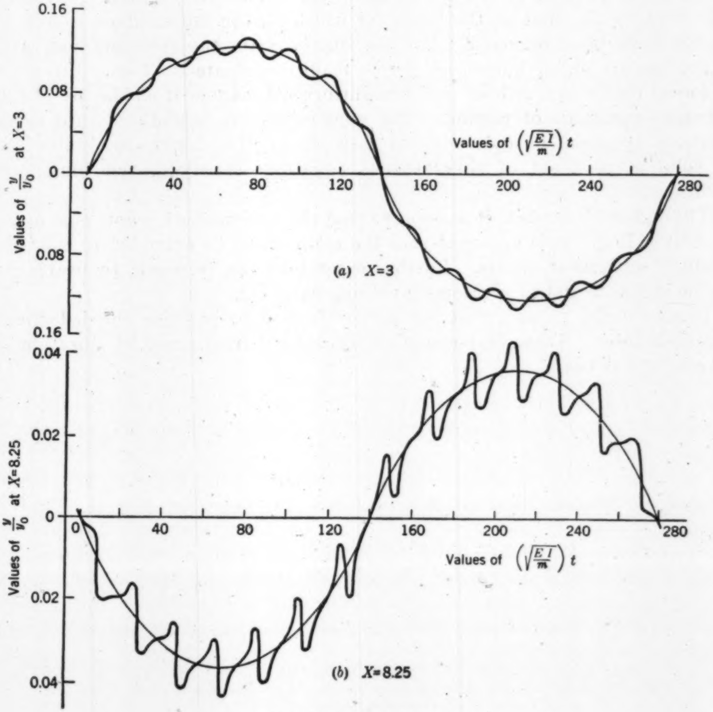

FIG. 3.—RATIO OF y/y

The result is shown graphically in Fig. 3(*a*). Similarly, for the point $x = 8.25$ at the center of the shorter span,

$$\begin{aligned} y_{x=8.25} = y_o \, [&- 0.0366 \sin \alpha t + 0.00345 \sin p_1 t \\ &- 0.000307 \sin p_2 t + 0.000924 \sin p_3 t] \ldots\ldots\ldots\ldots (46) \end{aligned}$$

Fig. 3(*b*) shows a plot of the ratio of the deflection at any time to the deflection y_o. It can be seen from Fig. 3 that the deflection (shown as a wavy line) due to the free vibrations is small in comparison with the forced vibration deflection shown as a light solid line. If damping were considered, the free vibrations would disappear with increase of time. By summing the absolute values of the coefficients in Eq. 45 and Eq. 46 it is estimated that the maximum deflections at $x = 3$ and $x = 8.25$ would be $0.134 \, y_o$ and $0.044 \, y_o$, respectively.

SUMMARY AND CONCLUSION

A method has been presented for the determination of the forced oscillations of a continuous beam due to an exciting force. It has been shown that the procedure requires only a knowledge of the eigenfunctions and eigenvalues of a simple beam, that is, the beam for which all the intermediate points of support have been removed. For the simple beam the eigenfunctions and eigenvalues are either known or can be readily evaluated. The solution for the forced oscillations follows in a straightforward manner from the use of the Lagrange equations of motion. The constraints are introduced into these equations by means of undetermined multipliers. The latter are obtained as the solution of a set of simultaneous equations satisfying the constraint conditions.

Throughout this work, it is assumed that the intermediate points of support are nonyielding. It is apparent that the method can be extended to the case in which settlement occurs. Further extensions can be made to beams on flexible supports and to problems involving damping.

It may easily be seen that, for certain forcing frequencies, the deflections become infinite. These correspond to the natural frequencies of vibration of the continuous beam.

DISCUSSION

W. H. **Hoppmann**[8].—The increasing importance of the knowledge of dynamical resistance of structures makes analytical devices that assist in the determination of deflections and strains highly desirable. The authors are to be commended for their contribution of such a device for use in analyzing the motion of continuous beams. Essentially it consists in treating the intermediate supports (all but the two end ones) by means of undetermined multipliers in setting up the usual Lagrangian equations of motion in generalized co-ordinates. The authors develop the analysis symbolically in some detail. However, they analyze one problem completely. This is a uniform two-span beam subjected to a single concentrated load of sinusoidal time variation. Their formulation of the problem implies a generality of treatment which would really require considerable additional analysis to work out in detail. In this connection the differential equation describing the motion of the beam contains the stiffness and mass as functions of the space variable defining distance along the beam. The numerical example provided as an illustration does not supply the vaguest hint of the additional labor involved if these physical quantities are allowed to vary along the beam.

In the introduction a reference is made to a paper by Mr. Weinstein,[9] a long study in which approximate frequencies of clamped plates and bars are determined. In the usual fashion these frequencies satisfy certain inequalities. The analysis is made through use of the calculus of variations and on the basis of the observation that there exists a formal mathematical relation of the Lagrangian plate equation to the equation of an elastic membrane, and of the Bernouilli-Euler beam equation to the classical string equation. There are no multiple supports and no undetermined multipliers. A specific remark by the authors as to why they refer to Mr. Weinstein's paper would be interesting. Possibly the writer has missed the point intended.

The applied force assumed by the authors is of the simple sinusoidal type concentrated at a point along the beam. It would be of interest to know what physical loading they had in mind. The writer is aware that it might represent the force produced by a simple vibration generator of the unbalanced inertia type. Also, it might be the force developed by a colliding body as shown by the writer.[10] In this particular case the time that the force was applied is short compared to the fundamental period. The phenomenon is then a transient one; however, the solution is such that it can be applied for the sinusoidal force assumed by the authors. The writer has, also, made an extensive experimental investigation on collisions of solid spheres with one-span, two-span, and three-span beams and compared the strains and deflections with those given by the theoretical solutions.[11] This problem requires consideration of both the motion of the colliding body as well as of the beam. However, it is shown that a sinusoidal force at the point of collision can be used to describe the real force by properly determining the amplitude and duration while satisfying the general laws of mechanics. The writer is also aware that the method of generalized co-ordinates and the Lagrangian equations of motion permit ready analysis of cases involving very general forces given as functions of space and time.

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[8] Associate Prof. of Mech. Eng., The Johns Hopkins Univ., Baltimore, Md.

[9] "Étude des spectres des équations aux dérivées partielles de la théorie des plaques élastiques," by A. Weinstein, *Mémorial des Sciences Mathématiques*, Gauthier-Villars, Paris, France, 1937, pp. 1–58.

[10] "Impact on a Multispan Beam," by W. H. Hoppmann, *Journal of Applied Mechanics*, A.S.M.E., Vol. 17, No. 4, December, 1950, p. 409.

In their summary the authors state that further extensions can be made to problems involving damping. It would be interesting to know what law of damping they have in mind. If they mean Stokes' viscous damping then it is not difficult to include it in the general analysis as shown by the writer[12] and others.[13] However, such a law of damping might not be the best to describe the actual physical phenomenon as discussed by R. D. Mindlin,[14] A. M. ASCE. In this event energy expressions for use in determining the Lagrangian equations of motion become rather delicate objects of analysis. In other words, one is faced with a need for determining a proper form of the dissipation function.[15] It is generally conceded that much more attention should be given to damping in connection with beam vibrations. This is a problem demanding theoretical and experimental investigations for possible application in connection with the dynamics of structures.

The authors appear to be concerned solely with deflections rather than strains. This seems to be dictated partly by expediency because of the complexity of the problem. They must solve, simultaneously, a finite number of linear equations each containing an infinite number of terms. They emphasize the rapid convergence of the series. This is not necessarily so for the corresponding strain equations. In fact for transient loading the series are rather slowly convergent. This has been encountered experimentally by the writer.[10,11]

Finally, the authors have not demonstrated a greater economy in numerical calculation using their method rather than solving the differential equation subject to given boundary, continuity, and initial conditions in the classical manner. It would appear that only experience with many actual solutions will answer the question, but the writer suspects on the basis of comparison of what already exists in the way of solutions that the latter method is at least as effective as the one proposed by the authors and possibly more effective.

EDWARD SAIBEL[16] AND ELIO D'APPOLONIA,[17] A.M. ASCE.—As far as the writers could determine at the time of the writing of this paper, the only case of a continuous beam under the action of a sinusoidal force was that of uniform beam of two equal spans, a solution of which was published[3] in 1949. If the classical solution were tractable this solution would have been unnecessary. From the fact that there were no such solutions extant, it is obvious that the classical method left much to be desired. In fact, a method that does not permit an actual solution is no solution at all. The solution which the writers present is more general than that cited[3] and can actually be extended to an accuracy as great as desired, in a reasonable length of time. A uniform beam of two unequal spans was selected to demonstrate the technique of applying the method and nothing would have been gained by the reader in studying the solution of a beam with more spans. As for the fact that the writers chose a sinusoidal load, it is evident that using the principle of superposition (applicable because the problem is linear) one can apply the method to any load that can be developed in a Fourier series.

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[11] "Impulsive Loading of Beams," by W. H. Hoppmann, *Proceedings of the Society for Experimental Stress Analysis* (publication pending), 1952.

[12] "Impact of a Mass on a Damped Elastically Supported Beam," by W. H. Hoppmann, *Transactions* A.S.M.E., Vol. 70, 1948, p. 125.

[13] "On the Decay of Waves in Visco-elastic Solid Bodies," by K. Sezawam, *Bulletin of the Earthquake Research Institute*, Toyko Imperial Univ., Vol. III, September, 1927, p. 50.

[14] "Response of Damped Elastic Systems to Transient Disturbances," by R. D. Mindin, F. W. Stubner, and H. L. Cooper, *Proceedings of the Society for Experimental Stress Analysis*, Vol. V, No. 14, 1948, p. 69.

[15] "The Theory of Sound," by Lord Rayleigh, Dover Publishing Co., New York, N. Y., Vol. I, p. 103.

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Mr. Hoppmann has indeed missed the point of the method described by Mr. Weinstein.[5] The basic philosophy underlying the treatment by Mr. Weinstein is the development of the solution of a difficult "eigenvalue" problem in terms of the known or easily worked out solution of a different "eigenvalue" problem. However, in the problems treated by Mr. Weinstein the boundary was exterior to the body in all cases. In the case under discussion however, the difficult problem is one in which part of the boundary conditions are given in the interior of the body, and this problem needs to be related to one which has only exterior boundary conditions—a much easier problem. Even in the case of a beam of nonuniform section the writers believe that their method is superior to the classical approach, but they do not wish to minimize the difficulties that may be encountered in this case.

In a solution published elsewhere[18] the stresses created by a moving force on a continuous beam are calculated. Although not as good as that for deflections, the convergence is shown to be quite rapid even without the necessity of using available methods for improving the rapidity of the convergence.[19] The convergence in the case of strains, of course, is of the same order as that of stress.

The writers are aware of the work done by Mr. Hoppmann on the impact of a solid sphere symmetrically applied to a continuous beam of equal span lengths. The fact that the real force between the two can be described by a sinusoidal force makes it indeed a much easier problem by the present method. However, if this were not the case, and if it were necessary to take into account the mass of the impacting body, this can be done by the method presented.[20]

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[18] "Vibration of a Continuous Beam under a Constant Moving Force," by Winston F. Z. Lee and Edward Saibel, *Journal of the Franklin Institute* (publication pending).

[19] "Partial Differential Equations," by J. D. Tamarkin and W. Feller, "Brown University Notes," Math. Dept., Brown Univ., Providence, R. I., 1941, Chapter 3, Section 3.

[20] "Free and Forced Vibrations of Constrained Beams and Plates," by Winston F. Z. Lee, thesis presented to the Carnegie Institute of Technology in Pittsburgh, Pa., in 1952, in partial fulfilment of the requirements for the degree of Doctor of Science.

The type of damping one wishes to consider is immaterial as far as the solution by the present method is concerned. The basic procedure will remain the same. However, what the writers had in mind was the inclusion of damping of a viscous nature—that is, dash-pot damping. Since the theory of the mechanism of internal friction is still in the incomplete stage and undoubtedly is so complicated as to be unmanageable in any differential equation treatment of vibration problems, the writers feel that the method of equivalent viscous damping is sufficient.

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TRANSACTIONS

Paper No. 2527

SURFACE CURVES FOR STEADY NONUNIFORM FLOW

BY ROBERT B. JANSEN,[1] J. M. ASCE

WITH DISCUSSION BY MESSRS. IVAN M. NELIDOV; J. C. STEVENS; MAURICE BISHAY AND H. E. BABBITT; K. C. WU; J. PIETRKOWSKI; AND ROBERT B. JANSEN

SYNOPSIS

Few problems in the field of hydraulic research have been more challenging or baffling than that concerned with plotting the surface curve assumed by a liquid in varied or nonuniform flow. Several investigators have sought the solution by attempting to integrate the varied flow equation. This paper, however, makes use of the differential equation by direct application of the surface slope that it defines. The paper is confined to the study of steady flow in prismatic trapezoidal channels, with the rectangular section considered to be a limiting case. Surface profiles plotted by this method are compared with those obtained by integration.

INTRODUCTION

A quick and simple method of determining the water surface profile in open channels carrying nonuniform flow has been the object of a search by engineers and mathematicians for a number of years. Although many exhaustive efforts have been made to solve this problem by integration, no entirely satisfactory answer has been found. The method advocated by Boris A. Bakhmeteff,[2] Hon. M. ASCE, has been applied with more confidence than has been accorded the works of other researchers, but even this approach has its deficiencies that are generally recognized.

The procedure outlined herein is, regrettably, not intended to be a final and complete solution. It is certainly not presented as a substitute for the scholarly achievements of Mr. Bakhmeteff, nor to detract from the works of other earnest investigators. The paper is essentially an enlargement upon the obvious relationship defined by the differential equation of varied flow, and is

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NOTE.—Published in October, 1951, as *Proceedings-Separate No. 96*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Civ. Eng. Associate, Bureau of Eng., City of Los Angeles, Calif.

[2] "Hydraulics of Open Channels," by B. A. Bakhmeteff, McGraw-Hill Book Co., Inc., New York, N. Y., 1932.

offered as an illustration of the results obtainable by direct plotting of the water surface slope.

Equation of Varied Flow

The flow of a liquid in an open channel is said to be uniform when factors such as bottom slope, depth, velocity, and cross-sectional area remain constant from point to point. Uniform motion can be considered to be a limit that is approached, for example, in a long reach of channel whose slope is just equal to the rate of loss in energy. In general, therefore, the case that must be considered is the one in which the contributing factors vary in such a manner that the energy gradient, the bottom slope, and consequently the water surface are not parallel.

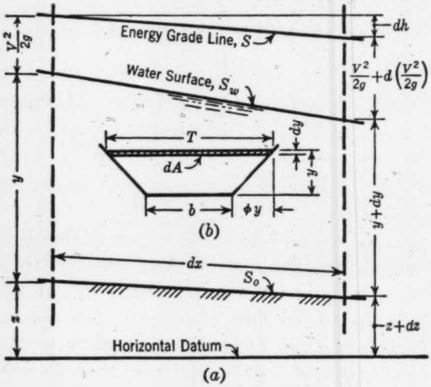

Fig. 1.—Flow in a Trapezoidal Channel

The following treatment is devoted to steady nonuniform (or varied) flow in prismatic channels. The slope of streamlines is assumed to be either constant or varying so gradually that the effect of centrifugal action can be neglected. That is, the pressure distribution is taken as hydrostatic. In the calculation of energy losses, application is made of the Manning version of the Chezy formula, that entails the assumption that the energy gradient in varied flow is parallel to that corresponding to uniform flow under the same conditions of depth, roughness, and velocity.

If x is the distance along the channel parallel to the bottom for an elementary length of channel (dx), as illustrated in Fig. 1(a), the total head (h) at any point in the stream is

$$h = z + y + \frac{V^2}{2\,g} \qquad (1)$$

assuming the velocity (V) to be constant with respect to depth. In Eq. 1, z is the elevation of the channel bottom above a horizontal datum, y is the depth of water in the channel, and g is the acceleration of gravity.

The derivative of h with respect to x is

$$\frac{dh}{dx} = \frac{dz}{dx} + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{V^2}{2\,g}\right) \qquad (2)$$

and $\frac{dh}{dx} = -S = -\frac{V^2}{C^2 R}$ from the Chezy formula, wherein $V = C\sqrt{R\,S}$, in which C is a coefficient; S is the slope of the energy gradient; and R is the hydraulic radius. Also, from Fig. 1(*a*) it is apparent that $\frac{dz}{dx} = -S_o$, in which S_o is the slope of the channel bottom. Referring to Fig. 1(*b*) if infinitesimals higher than the first order are neglected, $dA = T\,dy$, in which T is the surface width and A is the cross-sectional area of the channel.

Then, $\frac{d}{dx}\left(\frac{V^2}{2\,g}\right) = \frac{V}{g}\left(\frac{dV}{dx}\right) = \frac{Q}{A\,g}\left(\frac{dV}{dx}\right)$, and since $\frac{dV}{dx} = \frac{d}{dx}\left(\frac{Q}{A}\right) = -\frac{Q}{A^2}\left(\frac{dA}{dx}\right)$, it follows that

$$\frac{d}{dx}\left(\frac{V^2}{2\,g}\right) = \frac{-Q^2}{A^3\,g}\frac{dA}{dx} = \frac{-Q^2\,T}{A^3\,g}\frac{dy}{dx} \qquad (3)$$

Substituting those derived terms into Eq. 2 gives

$$\frac{-V^2}{C^2\,R} = -S_o + \frac{dy}{dx} - \frac{Q^2\,T}{A^3\,g}\frac{dy}{dx} \qquad (4)$$

By transposing terms, the following final expression for the slope of the water surface is obtained:

$$\frac{dy}{dx} = \frac{S_o - \dfrac{Q^2}{A^2\,C^2\,R}}{1 - \dfrac{Q^2\,T}{A^3\,g}} \qquad (5)$$

Now, if, as indicated in Fig. 1(*b*), the tangent of the channel side slope is designated as ϕ, then the area (A) is $A = y\,(b + \phi\,y) = b\,y + \phi\,y^2$, in which b is the bottom width of the channel, and the surface width is $T = b + 2\,\phi\,y$. Also, the wetted perimeter is $p = b + 2\sqrt{\phi^2\,y^2 + y^2} = b + 2\,y\sqrt{1 + \phi^2}$. The expression for the hydraulic radius is therefore

$$R = \frac{b\,y + \phi\,y^2}{b + 2\,y\sqrt{1 + \phi^2}} \qquad (6)$$

For a prismatic channel, S_o is constant. By the Manning formula, $C = \frac{1.486}{n}R^{\frac{1}{6}}$, squaring, $C^2 = \frac{2.208\,R^{\frac{1}{3}}}{n^2}$.

By inserting the foregoing factors, Eq. 5 can now be written

$$\frac{dy}{dx} = \frac{S_o - \dfrac{Q^2}{(b\,y + \phi\,y^2)^2\left(\dfrac{2.208\,R^{\frac{1}{3}}}{n^2}\right)(R)}}{1 - \dfrac{Q^2}{g}\left[\dfrac{b + 2\,\phi\,y}{(b\,y + \phi\,y^2)^3}\right]} \qquad (7a)$$

or,

$$\frac{dy}{dx} = \frac{S_o - \left(\frac{n^2\,Q^2}{2.208}\right)\left[\frac{(b + 2\,y\,\sqrt{1 + \phi^2})^{\frac{4}{3}}}{(b\,y + \phi\,y^2)^{\frac{10}{3}}}\right]}{1 - \frac{Q^2}{g}\left[\frac{b + 2\,\phi\,y}{(b\,y + \phi\,y^2)^3}\right]} \quad \text{(7b)}$$

The constant terms in Eq. 7*b* can be denoted as

$$N = \frac{n^2\,Q^2}{2.208}; \qquad P = 2\sqrt{1 + \phi^2}; \qquad \text{and} \qquad G = \frac{Q^2}{g}.$$

By the indicated substitutions, the differential equation can be changed to the form

$$\frac{dy}{dx} = \frac{S_o - N\left[\frac{(b + P\,y)^{\frac{4}{3}}}{(b\,y + \phi\,y^2)^{\frac{10}{3}}}\right]}{1 - G\left[\frac{b + 2\,\phi\,y}{(b\,y + \phi\,y^2)^3}\right]} \quad \text{(8)}$$

For the special case of a rectangular cross section, $\phi = 0$, and $P = 2$ so that, for this shape,

$$\frac{dy}{dx} = \frac{S_o - N\left[\frac{(b + 2\,y)^{\frac{4}{3}}}{(b\,y)^{\frac{10}{3}}}\right]}{1 - G\left[\frac{b}{(b\,y)^3}\right]} \quad \text{(9)}$$

Since for a prismatic channel the terms b, ϕ, and P do not vary, the expressions within brackets of Eq. 8 are functions of the depth alone. These variables in the numerator and denominator are designated as u and j, respectively.

$$u = \frac{(b + P\,y)^{\frac{4}{3}}}{(b\,y + \phi\,y^2)^{\frac{10}{3}}} \quad \text{(10)}$$

and

$$j = \frac{b + 2\,\phi\,y}{(b\,y + \phi\,y^2)^3} \quad \text{(11)}$$

The relation for the trapezoidal section can now be restated as

$$\frac{dy}{dx} = \frac{S_o - N\,u}{1 - G\,j} \quad \text{(12)}$$

The depth functions, u and j, have been evaluated and tabulated for common cases of side walls.

COMPUTATION PROCEDURE

To illustrate the direct use of the surface slope in solving problems of varied flow, the following elementary examples are given:

Example 1.—Water is flowing from under a sluice into a rectangular canal whose bottom width is 20 ft. The channel grade is 0.0010 and the value of Manning's n is 0.013. The sluice is regulated to discharge 1,080 cu ft per sec with a depth at the vena contracta of 1.5 ft. The surface curve is required.

The critical depth is found to be 4.49 ft. Flow under the conditions stated will usually lead to an hydraulic jump, starting between $y = 1.5$ and a critical depth $y_c = 4.49$. For the purpose of illustration, however, the curve will be computed on the assumption that the tailwater is low enough to prevent occurrence of the jump.

TABLE 1.—DETERMINATION OF SURFACE SLOPE, EXAMPLE 1

Depth y	Factor u	Factor j	Slope $\frac{dy}{dx}$
2.0	316.3×10^{-6}	312.5×10^{-6}	+0.00264
2.5	158.7	160.0	+0.00275
3.0	91.09	92.59	+0.00303
3.5	57.30	58.31	+0.00371
4.0	38.54	39.06	+0.00591

Evaluating the terms in Eq. 12, $S_o = 0.0010$, $N = \frac{n^2 Q^2}{2.208} = \frac{(0.013)^2(1080)^2}{2.208} = 89.3$, $G = \frac{Q^2}{g} = \frac{(1080)^2}{32.2} = 36{,}200$. From tables or by substitution in Eqs. 10 and 11, the values of u and j can be found. Solution of Eq. 12 is then completed.

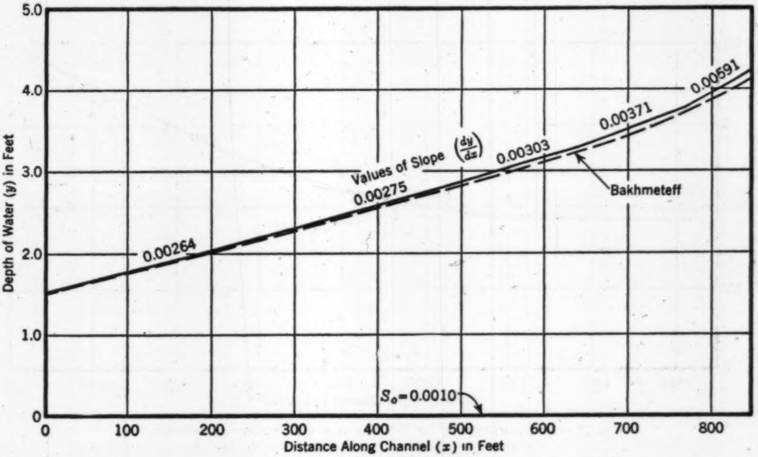

FIG. 2.—DETERMINATION OF WATER-SURFACE SLOPE—EXAMPLE 1

It should be remembered that $\frac{dy}{dx}$ is the water slope with respect to the channel bottom. By conventions of the calculus, its sign is positive with increasing depth in the direction of flow.

In the manner described above, values of $\frac{dy}{dx}$ for intermediate depths have been determined and are listed in Table 1.

The water-surface curve described by these slopes is plotted in Fig. 2. In order to attain uniform results in the sketching of curves in this paper the tangents have been made to intersect at the mid-points of the depth increments, that is, at $y = 2.25$, $y = 2.75$, and so on. In practice, this procedure can be modified to give more accuracy in that part of the curve where $\frac{dy}{dx}$ is varying rapidly.

TABLE 2.—DETERMINATION OF SURFACE SLOPE, EXAMPLE 2

Depth y	Factor u	Factor j	Slope $\frac{dy}{dx}$
2.0	8689×10^{-6}	5000×10^{-6}	+0.0009
2.5	4753	2560	+0.0020
3.0	2939	1481	+0.0024
3.5	1974	932.9	+0.0026
4.0	1408	625.0	+0.0027

For comparison, the surface plot obtained by the Bakhmeteff method is shown by the dashed curve in Fig. 2.

Example 2.—In an experiment conducted by the University of Illinois Engineering Experiment Station,[3] Urbana, Ill., 40.39 cu ft per sec of water was caused to flow through a rectangular channel 5 ft wide. The flume was laid on a slope of $S_o = 0.0030$ and was artificially roughened to obtain a value of $n = 0.019$. A backwater gate installed at the lower end of the reach was

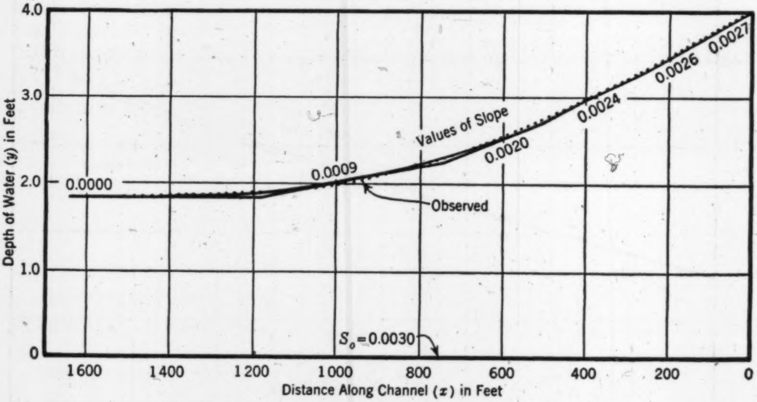

FIG. 3.—DETERMINATION OF WATER-SURFACE SLOPE—EXAMPLE 2

regulated in this particular test so that the depth at the downstream gage was $y = 4.0$ ft. The normal depth (y_o) for the given conditions is found to equal 1.84 ft. Also, using procedures already described, $N = 0.267$ and $G = 50.7$.

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[3] "An Investigation of the Backwater Profile for Steady Flow in Prismatic Channels," by W. M. Lansford and W. D. Mitchell, *Bulletin Series No. 381*, Univ. of Illinois, Urbana, Ill., 1949.

The slope calculations are summarized in Table 2. In Fig. 3 the surface plotted by means of these tangents is shown as compared with the curve observed in the laboratory.

CONCLUSIONS

Direct application of the water surface slope as defined by the varied flow equation may be expected to yield results in close agreement with those obtained by accepted integration procedures and by trial-and-error methods. Any of these approaches involves an uncertain amount of inaccuracy due to the assumptions of hydrostatic pressure and of nondivergent motion. The error introduced through determination of energy loss by the uniform-flow formula is probably not appreciable except in relatively steep or sharply nonparallel movement.

Profiles plotted by any procedure are inaccurate in the vicinity of the control section for most cases, since in the vicinity of the critical depth the effect of centrifugal forces is usually of considerable importance. The water slope at this point, when channel conditions permit, can be approximated by a curve sketched between adjacent sections upstream and downstream.

DISCUSSION

Ivan M. Nelidov,[4] M. ASCE.—In sketching the water-surface curves obtained with the aid of the slopes of water, computed by means of Eq. 12, the author has made the water-slope tangents intersect at the midpoints of the depth increments. This equation may be rewritten as follows:

$$\frac{dy}{dx} = \frac{S_o - S}{1 - Gj} = \frac{\frac{dE}{dx}}{\frac{dE}{dy}} \qquad (13)$$

in which (conforming to the notation of the paper), S is the friction slope and E is the specific energy, or energy referred to the invert. Eq. 2 can be represented as the Bernouilli equation if the depth y_1 is added to both sides, if the friction slope S is introduced, and if both sides are multiplied by dx; thus,

$$- S\,dx = - S_o\,dx + y_2 - y_1 + \frac{V^2_2}{2\,g} - \frac{V^2_1}{2\,g}$$

or

$$y_1 + \frac{V^2_1}{2\,g} + I = y_2 + \frac{V_2^2}{2\,g} + h_f \qquad (14)$$

in which the subscripts 1 and 2 refer to two sections, distance dx apart; $I = S_o\,dx$ is the difference in elevation between the inverts at section 1 and section 2; and $h_f = S\,dx$ is the friction head lost. Eq. 14 may also be rewritten as,

$$E_1 + I = E_2 + h_f \qquad (15)$$

in which E_1 and E_2 are the specific energies at sections 1 and 2, respectively.

The only correct way to construct the water-surface curve is by integration. However, cases where integration can be used are rather limited because the channel is not a prism of uniformly varying cross sections. It may be quite irregular and can change abruptly from section to section.

The approximation proposed by the author involves constructing the water-surface profile by means of tangents connecting two consecutive depths in such a way that these tangents meet at the midpoint of the depth increment. It may be noted, that the standard mathematical procedure for constructing a curve is to mark equal horizontal increments dx, with varying vertical increments dy.

The tangents will thus intersect at the vertical ordinates or, if the ordinates are translated a distance ½ dx they will intersect at the midpoint between the ordinates.

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In Example 1, if the ordinates 2.5 and 3.0 are selected, the horizontal distance between them will be: $dx = \frac{0.25}{0.00275} + \frac{0.25}{0.00303} = 173.4$ ft. The same dx, computed by the Bakhmeteff method,[2] and using his notation, is obtained as follows: $y_o = 6.12$ ft; $S_o = 0.001$; the hydraulic exponent $n = 2.98$; σ (the critical slope) $= 0.00244$; $\beta = \frac{S_o}{\sigma} = 0.41$; and

$$dx = \frac{6.12}{0.001}\left[\frac{3.00}{6.12} - \frac{2.50}{6.12} - (1 - 0.41)(0.5059 - 0.4155)\right] = 176.0 \text{ ft}$$

in which the varied flow functions are $0.5059 = B\ (\eta_2)$ and $0.4155 = B\ (\eta_1)$. The curve in this example[5] is type M_3 and the flow is rapid and retarded. To use the Bernoulli equation in determining distance dx without trial, Eq. 15 must be solved, from which

$$dx = \frac{E_1 - E_2}{S - S_o} \qquad \text{(16)}$$

The numerical values entering Eq. 16 are as follows: $E_1 = 2.50 + 7.25 = 9.75$; $E_2 = 3.00 + 5.02 = 8.02$; $S_o = 0.001$; and $S = 0.01095$—determined on the basis of an average hydraulic radius $R_{av} = 2.155$ and an average velocity $V_{av} = 19.8$ ft per sec, with $n = 0.013$. Then, $dx = \frac{9.75 - 8.02}{0.01095 - 0.001} = \frac{1.73}{0.00995}$ $= 173.6$ ft.

If an average value of the friction slopes computed for the end section is used, the distance dx would differ more from the foregoing values. The following numerical data would be used: $V_1 = 21.6$ ft per sec; $R_1 = 2.00$ ft; $V_2 = 18.0$ ft per sec; and $R_2 = 2.31$ ft. Furthermore, $S_1 = 0.01455$, $S_2 = 0.00815$, and $S_{av} = 0.0117$, so that $dx = \frac{1.73}{0.0117 - 0.001} = 162$ ft. In general, the friction loss should be computed by introducing R_{av} and V_{av} for end sections and determining the friction slope on the basis of their values. This may be proved by taking the general expression for friction loss and comparing the results. Thus, integrating the theoretical friction loss with the aid of the Bakhmeteff formulas[6]:

$$\begin{aligned} h_{f1} &= \int S\,dx = \int \frac{(\mathrm{K}_o)^2}{(\mathrm{K})} S_o\,dx = \int \frac{(y_o)^3}{(y)} S_o\,dx \\ &= S_o\, y_o^3 \int_0^L \frac{dx}{\left(y_1 + \frac{y_2 - y_1}{L} x\right)} = S_o \frac{L}{2} \frac{(y_o)^3}{(y_1)} \left[\frac{y_1}{y_2} + \frac{(y_1)^2}{(y_2)}\right] \\ &= \text{Constant}\left[\frac{y_1}{y_2} + \frac{(y_1)^2}{(y_2)}\right] \qquad \text{(17)} \end{aligned}$$

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[5] "Hydraulics of Open Channel," by B. A. Bakhmeteff, McGraw-Hill Book Co., Inc., New York, N. Y., 1932, p. 76.

[6] *Ibid.*, p. 82.

In Eq. 17, $K = A\,C\,\sqrt{R}$ = the Bakhmeteff "conveyance" of a cross section, K_o being the conveyance at the normal depth y_o; and, L is the distance along the line of flow from an initial section.

If friction-loss computation is based on the data referred to the average cross section, the expression for friction loss will be:

$$h_{f2} = \int S\,dx = \int \frac{(K_o)^2}{(K)} S_o\,dx = \int y^3{}_o\,S_o \frac{dx}{y^3{}_{av}} = y^3{}_o\,S_o \int^L \frac{dx}{\left(\dfrac{y_1 + y_2}{2}\right)^3}$$

$$= \left(\frac{y_o}{y_1}\right)^3 \frac{L}{2} S_o \frac{16}{\left[1 + \dfrac{y_2}{y_1}\right]^3} = \text{Constant} \frac{16}{\left(1 + \dfrac{y_2}{y_1}\right)^3} \dots\dots\dots (18)$$

If the friction loss is computed by taking the average friction slope, the expression then will be

$$h_{f3} = S_{av}\,L = \frac{S_o L}{2}\left[\left(\frac{y_o}{y_1}\right)^3 + \left(\frac{y_o}{y_2}\right)^3\right] = \frac{S_o L}{2}\left(\frac{y_o}{y_1}\right)^3 \left[1 + \left(\frac{y_1}{y_2}\right)^3\right]$$

$$= \text{Constant}\left[1 + \left(\frac{y_1}{y_2}\right)^3\right] \dots\dots\dots\dots\dots\dots\dots\dots\dots (19)$$

Table 3 indicates that friction computations by the Bernoulli equation should be based on average area and average hydraulic radius, and not on average friction slope. In general, the method proposed by the author gives results within the desired degree of accuracy—that is, within the ordinary range of variation of depths. However, with rapidly varying depths, all approximation methods (which includes the author's method and the Bernoulli method when finite distances dx are used) are likely to give incorrect results, the degree of error being established only after integration is performed.

TABLE 3.—COMPARISON OF FRICTION COMPUTATIONS

Friction loss	Equation	VALUES OF y_2/y_1	
		0.7	1.3
h_{f1}	17	Constant [3.42]	Constant [1.36]
h_{f2}	18	Constant [3.25], or − 5%	Constant [1.32], or −3%
h_{f3}	19	Constant [3.93], or +15%	Constant [1.45], or +7%

It is important to classify the water-surface curves properly so that the type involved is evident at a glance. In Fig. 4, three major classes are noted, first: flow with $y_o > y_c$ (class A), with $y_o = y_c$ (class B), and with $y_o < y_c$ (class C).

The slope of invert in class A is adverse (class A–I), zero (class A–II), and gentle (class A–III); class B is critical; and in class C it is greater than critical. In any horizontal row the first figure shows the water depth y greater than both

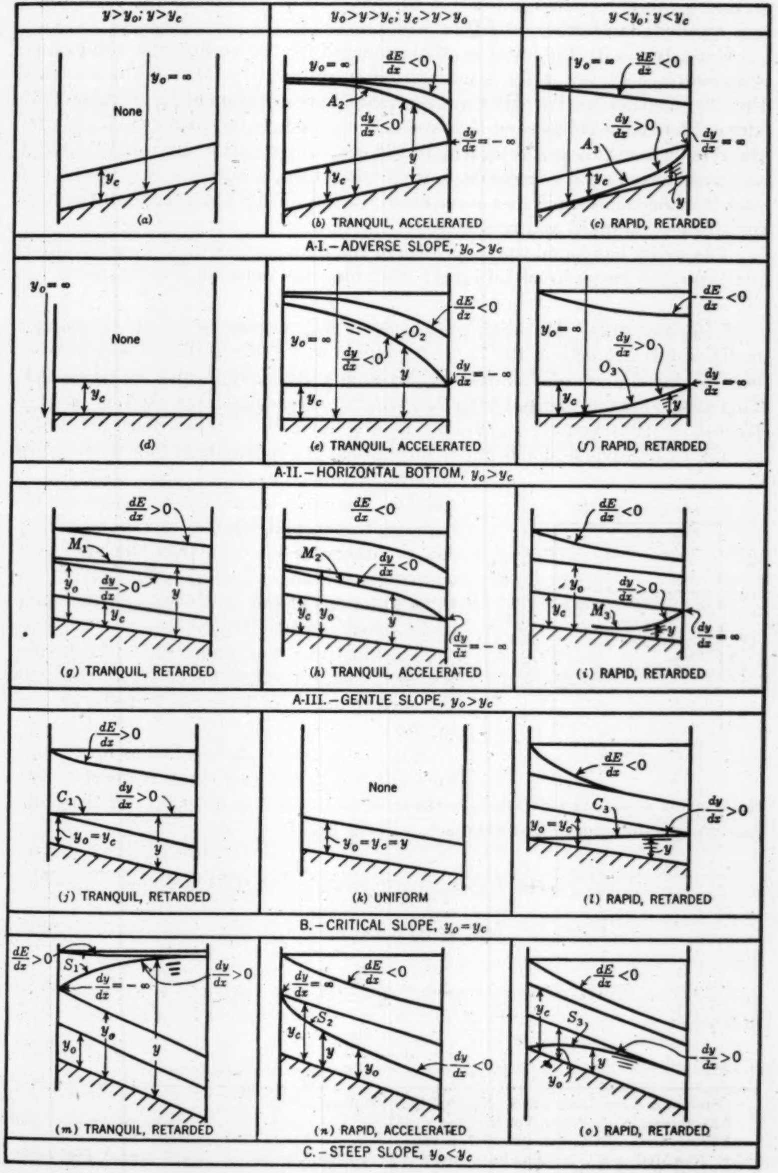
y>y₀; y>y_c
y₀>y>y_c; y_c>y>y₀
y<y₀; y<y_c
y₀=∞
None
y_c
(a)
dE/dx<0
A₂
dy/dx<0
dy/dx=−∞
y
(b) TRANQUIL, ACCELERATED
A₃
dy/dx>0
dy/dx=∞
(c) RAPID, RETARDED
A-I.—ADVERSE SLOPE, y₀>y_c
(d)
O₂
(e) TRANQUIL, ACCELERATED
O₃
(f) RAPID, RETARDED
A-II.—HORIZONTAL BOTTOM, y₀>y_c
dE/dx>0
M₁
dy/dx>0
y₀
(g) TRANQUIL, RETARDED
M₂
(h) TRANQUIL, ACCELERATED
M₃
(i) RAPID, RETARDED
A-III.—GENTLE SLOPE, y₀>y_c
C₁
y₀=y_c
(j) TRANQUIL, RETARDED
y₀=y_c=y
(k) UNIFORM
C₃
(l) RAPID, RETARDED
B.—CRITICAL SLOPE, y₀=y_c
S₁
dy/dx=−∞
(m) TRANQUIL, RETARDED
dy/dx=∞
S₂
(n) RAPID, ACCELERATED
S₃
(o) RAPID, RETARDED
C.—STEEP SLOPE, y₀<y_c

FIG. 4.—CLASSIFICATION OF WATER-SURFACE CURVES IN STEADY NONUNIFORM FLOW

y_o and y_c; in the second figure y is between y_o and y_c; and in the third figure y is smaller than both y_o and y_c.

Consistent with the letter symbols adopted for the author, the notation for water-surface curves is the same as that introduced by Mr. Bakhmeteff and the classification itself is only a more graphical presentation of Mr. Bakhmeteff's ideas. As far as the type of the flow is concerned, in the first vertical column the flow is tranquil and retarded; in the second vertical column it is tranquil and accelerated until it becomes flow at critical slope, where it is uniform, and past it, it becomes rapid and accelerated; in the third vertical column the flow, for all cases, is rapid and retarded.

The writer has found that this form of classification is helpful in determining the type of a curve involved before beginning the detailed computations.

J. C. STEVENS,[7] M. ASCE.—Engineers still persist in trying to solve a really simple problem in the hardest possible way. Since 1925, the writer has been urging the use of a simple step-by-step method applicable to all cases.[8,9] This method is not original with the writer. It was first presented by Alva G. Husted.[10] It is strictly applicable to any form of water-surface profile on sustaining or adverse slopes under sub-critical or super-critical flow on rough or smooth channels provided only they are in uniform channels—that is, rectangular, triangular, trapezoidal, circular, parabolic, elliptical, or otherwise—just so that the area and, consequently, the perimeter, radii, etc., are the same functions of the depth through the reach under consideration.

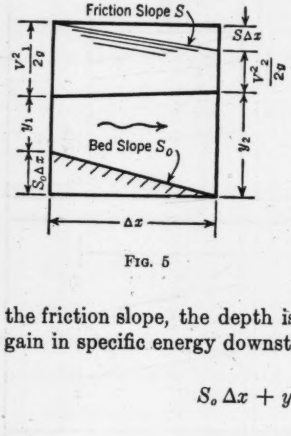

FIG. 5

To illustrate the point that the writer wishes to make it will be necessary to compute the surface curves of Example 2 by the Husted method.

Fig. 5 illustrates the conditions for this example in which the bed slope is greater than the friction slope, the depth is super-critical and super-normal, and there is a gain in specific energy downstream. From Fig. 5,

$$S_o \Delta x + y_1 + \frac{V^2_1}{2g} = y_2 + \frac{V^2_2}{2g} + S \Delta x \quad \text{(20a)}$$

and

$$(S_o - S) \Delta x = e_2 - e_1 \quad \text{(20b)}$$

$$\Delta x = \frac{\Delta e}{S_o - S} \quad \text{(20c)}$$

in which

$$e = y + \frac{V^2}{2g} \quad \text{(21)}$$

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[7] Stevens & Koon, Cons. Engrs., National Bldg., Portland, Ore.

[8] *Engineering News-Record*, Vol. 95, 1925, p. 550.

[9] *Transactions*, ASCE, Vol. 102, 1937, p. 666.

[10] "New Method of Computing Backwater and Drop-Down Curves," by Alva G. Husted, *Engineering News-Record*, Vol. 92, 1924, p. 719.

the specific energy of 1 lb of water measured as a head above stream bed as datum. Dimension Δx is a finite length of channel over which the surface of the stream is to be considered a straight line; S_o is the bed slope; and S is the friction slope. Table 4 shows the computations for this example. The flow is in the direction of increasing depths. There is a gain in specific energy downstream as shown in Fig. 5 due to the accelerating force of gravity. Fig. 6 shows the surface curve taken from Fig. 3. The circles are points given in Table 4. The agreement is practically perfect.

TABLE 4.—COMPUTATIONS FOR EXAMPLE 2, BY THE HUSTED METHOD

y = depth; A = area; p = wet perimeter; R = hydraulic radius; V = mean velocity; and e = specific energy. In Col.[a] 9, $\frac{1}{(n\,v)^2} = \frac{1}{2.2082\,R^{4/3}}$. In Cols. 10 and 11, S = friction slope and S_o = bed slope; and in Cols. 12 and 13, Δx = a finite length of channel, and $x = \Sigma\,\Delta x$, the distance from the starting point of the surface curve.

y (1)	A (2)	p (3)	R (4)	V (5)	$\frac{V^2}{2\,g}$ (6)	e (7)	Δe (8)	$\frac{1}{(n\,v)^2}$ (9)	S (10)	$S_o - S$ (11)	Δx (12)	x (13)
4.0	20.0	13.0	1.54	2.02	0.063	4.063	....	....	....	....	....	0
3.5	17.5	12.0	1.46	2.30	0.082	3.582	0.481	0.273	0.00052	0.00248	194	194
3.0	15.0	11.0	1.36	2.69	0.112	3.112	0.470	0.301	0.00079	0.00221	212	406
2.5	12.5	10.0	1.25	3.23	0.162	2.662	0.450	0.336	0.00126	0.00174	258	664
2.0	10.0	9.0	1.11	4.04	0.254	2.254	0.408	0.394	0.00230	0.00070	583	1,247
1.85	9.25	8.70	1.06	4.37	0.297	2.147	0.107	0.419	0.00288	0.00012	890	2,137

[a] "Handbook of Hydraulics," by Horace W. King, McGraw-Hill Book Co., Inc., New York, N. Y., 3d Ed., 1939, p. 309.

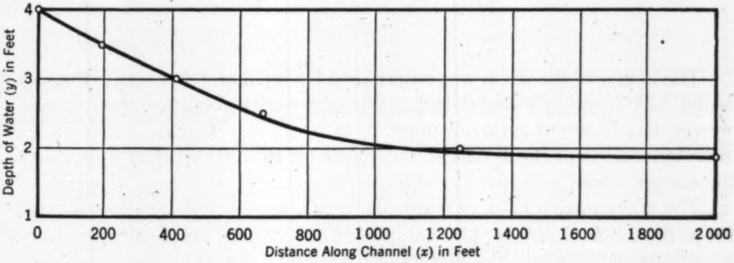

FIG. 6

Example 1 is the same as that presented by Mr. Bakhmeteff[2] in 1932, who recognized that this surface curve includes a hydraulic jump but did not define it.

For this example the bed slope is less than the friction slope, the depths are sub-critical and sub-normal, and specific energy diminishes downstream, resulting in the formula,

$$\Delta x = \frac{\Delta e}{S - S_o} \qquad (22)$$

Fig. 7 shows the actual surface curve. The computations began at the critical depth 4.49 and extended upstream using depth decrements of 0.10 ft on account of the rapidly changing curvature. The circles are points given by Mr. Bakhmeteff.

To locate the position of the hydraulic jump it was necessary to compute the surface curve downstream from the critical depth where super-critical depths obtain. At points upstream from the critical depth the values of $P + \frac{Q}{g} V$ were computed from both the sub-critical and super-critical curves, where P is the hydrostatic pressure in the channel cross section and $\frac{Q}{g} V$ is the momentum of the water, as set forth by Julian Hinds,[11] M. ASCE.

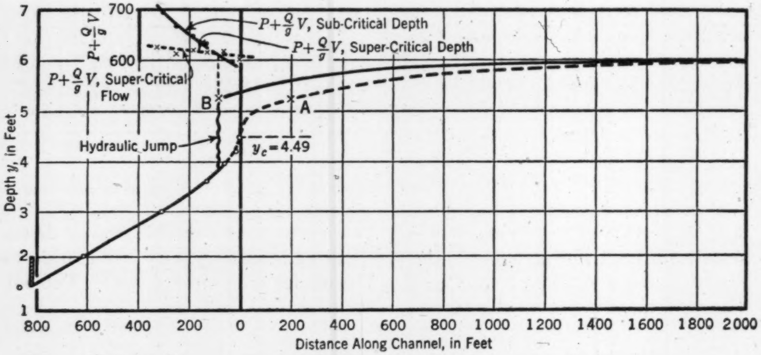

Fig. 7.

The height of the jump was determined by the method given by the writer[12] to be 4.24 ft above channel bed. The super-critical surface curve was then moved bodily upstream from point A to point B. The solid lines in Fig. 7 therefore represent the actual surface curve for the author's Example 1 neglecting energy losses.

The computation of surface curves from a varied-flow function, as used by the author, J. A. C. Bresse, Mr. Bakhmeteff, and others, practically precludes the possibility of taking all the factors into account. On the other hand, they are readily applied in the step-by-step method. For example, if the writer were trying to find the surface curve for Example 1 he would compute it from an equation something like

$$\Delta x = \frac{\Delta(y + 1.05\, h_V) - 0.20\, \Delta h_V}{S - S_o} \quad \text{.............. (23)}$$

in which h_V equals the velocity head of the mean velocity.

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[11] "The Hydraulic Jump and Critical Depths in the Design of Hydraulic Structures," by Julian Hinds, *Engineering News-Record*, Vol. 85, 1920, p. 1034.

[12] "The Hydraulic Jump in Standard Conduits, by J. C. Stevens, *Civil Engineering*, October, 1933, p. 565.

A much closer approximation to the true surface curve of Example 1 from the sluice gate to the foot of the hydraulic jump would thus be obtained, that is, by adding 5% to the kinetic energy of the mean velocity and then assuming only 80% velocity head recovery in expanding areas.

MAURICE BISHAY[13] AND H. E. BABBITT,[14] M. ASCE.—An illustrative problem involving the computation of a backwater curve has been solved by the author's method and five other methods. Results of this computation are shown in Table 5 and in Fig. 8. Comments are given on the relative accuracy of the various methods and on the relative tediousness of the work involved in each method.

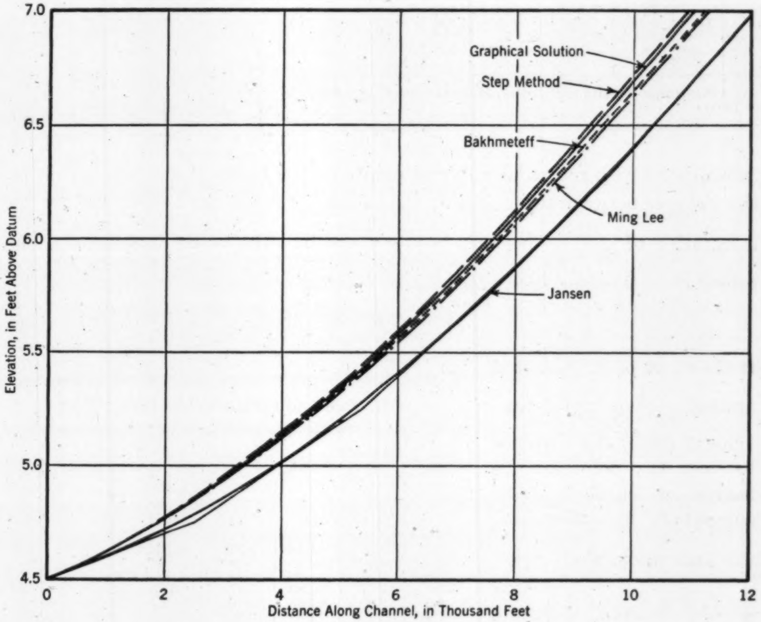

FIG. 8

Problem.—The problem selected for solution requires the construction of the backwater curve for a stream flowing under a sluice gate into a rectangular channel whose bottom width is 10 ft. The slope of the bottom of the channel is 0.0004 and the value of Manning's n is 0.01486. The sluice gate is regulated so that the rate of discharge is 136 cu ft per sec, with a depth at the vena contracta of 7.0 ft.

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[14] Prof. of San Eng., Univ. of Illinois, Urbana, Ill.

Preliminary Steps in Solution.—The depth at uniform flow, computed by the Manning formula, is found to be 4.0 ft. Critical depth is found to be 1.79 ft. Hence, the surface curve will be of the M-1 type of gradually varied flow in a rectangular channel.

TABLE 5.—COMPUTED DISTANCES (IN FEET) ALONG THE CHANNEL, BELOW A DEPTH OF 4.5 FT COMPARED BY SIX METHODS

Depth (ft)	Graphical (1)	Step (2)	Bakhmeteff (3)	Lee (4)	Mononobe (5)	Jansen (6)
5.0	3,281	3,250	3,336	3,344	....	....
5.5	5,683	5,580	5,723	5,737	....	....
6.0	7,638	7,542	7,708	7,724	....	....
6.5	9,386	9,299	9,469	9,483	....	....
7.0	11,025	10,933	11,204	11,230	10,170	12,088
Deviation[a]	0	−0.83	+1.62	+1.8	−7.75	+9.6

[a] Percentage deviation from the graphcial method at a depth of 7.0 ft.

Solution by Graphical Integration.—As explained by Hunter Rouse,[15] M. ASCE, but using the American Standards Association hydraulic symbols (ASA—Z10.2—1942):

$$\Delta x = \frac{\Delta E_s}{S_o - S_f} \ldots (24)$$

or

$$\int_a^b dx = \int_a^b \frac{dE_s}{S_o - S_f} \,. (25)$$

in which a and b represent any two points on the backwater curve. That is, the distance between two points on the backwater curve can be obtained by the integration of $\frac{dE_s}{S_o - S_f}$ between points a and b. The value of S_f can be obtained from the expression,

$$V = C\, R^{2/3}\, S^{1/2}{}_f \ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

If the curve of E_s is plotted against $\frac{1}{S_o - S_f}$ to any convenient scale, the distance between two points on the backwater curve can be computed by measuring the area under the $\left(E_s\text{-to-}\frac{1}{S_o - S_f}\right)$-curve. Such a curve has been drawn and the results of the computation by the graphical method are shown in Table 6.

TABLE 6.—COMPUTATION OF THE BACKWATER CURVE BY GRAPHICAL INTEGRATION

Depth (ft)	Hydraulic radius (ft) (1)	Specific energy[a] (2)	Slope ($S_f \times 10^{-4}$) (3)	$\frac{1}{S_o - S_f} \times 10^{+3}$ (4)	Area under curve (sq ft) (5)	Distance along channel[b] (6)
4.5	2.37	4,643	2.92	9.26	0	0
5.0	2.50	5,115	2.18	5.50	3,281	3,281
5.5	2.62	5,595	1.70	4.35	2,402	5,683
6.0	2.72	6,080	1.35	3.775	1,955	7,638
6.5	2.82	6,568	1.10	3.45	1,748	9,386
7.0	2.92	7,059	0.902	3.23	1,639	11,025

[a] In Col. 2, specific energy is $y + \frac{V^2}{2g}$. [b] Distance, in feet, along the channel below a depth of 4.5 ft.

Solution by Step Method.—The expression of the distance between two points on a backwater curve can be derived from the Bernouilli theorem and expressed in standard nomenclature as

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[15] "Elementary Mechanics of Fluids," by Hunter Rouse, John Wiley & Sons, Inc., New York, N. Y., 1946, pp. 80–82.

$$\Delta x = \frac{\Delta \epsilon_s}{S_o - \frac{S_{f1} - S_{f2}}{2}} \quad \text{.....................} (27)$$

Results of the computation of the backwater curve by this expression applied to the illustrative problem are shown in Table 7.

TABLE 7.—COMPUTATION OF THE BACKWATER CURVE BY THE STEP METHOD

Depth (ft)	Specific energy ϵ_s (ft) (1)	$\Delta \epsilon_s$ (ft) (2)	Friction slope[a] (3)	Mean $S_f \times 10^{-4}$ (4)	$\frac{1}{S_o - S_f} \times 10^{-3}$ (5)	Δx (ft) (6)	Distance along channel[b] (7)
4.5	4,643		2.92				0
5.0	5,115	0.472	2.18	2.55	6.9	3,250	3,250
5.5	5,595	0.480	1.70	1.94	4.86	2,330	5,580
6.0	6,080	0.485	1.35	1.525	4.05	1,962	7,542
6.5	6,568	0.488	1.10	1.225	3.60	1,757	9,299
7.0	7,059	0.491	0.902	1.001	3.33	1,634	10,933

[a] Friction slope in Col. 3 is $S_f \times 10^{-4}$. [b] Distance in feet along the channel below a depth of 4.5 ft.

Solution by Direct Integration, Using the Bakhmeteff Method[16].—Mr. Bakhmeteff has derived the expression,[3]

$$x_b - x_a = \frac{y_o}{S_o} \{\gamma_b - \gamma_a - (1 - \beta) [\beta(N, \gamma_b) - \beta(N, \gamma_a)]\} \quad \text{..} (28)$$

which can be solved by the aid of tables for N and for γ computed by Mr. Bakhmeteff, proceeding by small increments of depth so that β may be assumed to be constant within each interval. Results of computations by Mr. Bakhmeteff's method are compiled in Tables 8 and 9.

The Mononobe Method[17].—In this method the assumption is made that both the area A and the wetted perimeter P_w may be expressed as monomial functions of the depth y. For example, P_w is equal to a constant times y^l and A is equal to a constant times $y^{M/2}$. The expression of Nagho Mononobe, based on the preceding assumptions, is

$$x_2 - x_1 = \frac{y_o}{S_o} \left\{ \phi_1 (r_2) - \phi (r_1) - \frac{V^2_o M}{2 g y_o} [\phi_2 (r_2) - \phi_2 (r_1)] \right\} \quad \text{..} (29)$$

TABLE 8.—HYDRAULIC PROPERTIES OF A RECTANGULAR CHANNEL

Depth (ft)	Area (sq ft) (1)	Wetted perimeter (ft) (2)	Hydraulic radius (ft) (3)	$R^{2/3}$ (4)	Conveyance K (5)	Constant β (6)
4	40	18	2.22	1.71	6,840	0.0912
5	50	20	2.50	1.85	9,250	0.0854
6	60	22	2.72	1.95	11,760	0.0796
7	70	24	2.92	2.05	14,350	0.0746

in which

$$r = \frac{y}{y_o}; \phi_1 (r) = \int_{1.001}^{r} \frac{r^{C_o} dr}{r^{C_o} - 1}; \phi_2 (r) = \int_{1.001}^{r} \frac{r^{C_o - M - 1} dr}{r^{C_o} - 1} \quad \text{......} (30)$$

and

$$C_o = \frac{5}{3} M - \frac{4}{3} l \quad \text{.......................} (31)$$

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[16] "Hydraulics of Open Channels," by B. A. Bakhmeteff, McGraw-Hill Book Co., Inc., New York, N. Y., 1932, Chapter VIII.

[17] "Backwater and Drop-Down Curves for Uniform Channels," by Nagaho Mononobe, *Transactions*, ASCE, Vol. 103, 1938, p. 950.

The work involved in the computation of the complete backwater curve for the illustrative example is so tedious that only the distance between the depths of 4.5 ft and 7.0 ft has been located. It was found by means of the equation

TABLE 9.—COMPUTATION OF THE BACKWATER CURVE BY THE BAKHMETEFF METHOD

Depth (ft)	$\gamma = \frac{y}{y_o}$ (1)	$\gamma_b - \gamma_a$ (2)	$1 - \beta$ (3)	$1 - \beta$ (average) (4)	$\beta(N, \gamma)$ (5)	$\Delta\beta(N, \gamma)$ (6)	Δx (7)	Distance[a] (ft) (8)
4.5	1.125	0.125	0.9121		0.8045			0
5.0	1.250	0.125	0.9146	0.9133	0.5760	−0.2285	3,336	3,336
5.5	1.375	0.125	0.9174	0.9160	0.4520	−0.1240	2,387	5,723
6.0	1.500	0.125	0.9204	0.9190	0.3720	−0.0800	1,985	7,708
6.5	1.625	0.125	0.9229	0.9216	0.3165	−0.0555	1,761	9,469
7.0	1.750	0.125	0.9254	0.9242	0.264	−0.0525	1,735	11,204

[a] Distances in feet, along the channel, below a depth of $y = 4.5$ ft.

and charts that

$$x_2 - x_1 \frac{4.0}{0.004}\left[3.00 - 1.93 - \frac{(3.4)^2\,(2)}{64.4\,(4)}\,(2.42 - 1.83)\right] = 10{,}170 \text{ ft}$$

Method of Ming Lee,[18] J. M. ASCE.—Mr. Lee's basic expression is in the form,

$$\Delta x = \frac{y_o}{S_o}\left[\Delta r - \Delta\beta\,(N, \gamma) + \frac{V^2_o}{2\,g\,y_o}\,\Delta\beta\left(\frac{N}{M}, \frac{A^2_o}{A^2}\right)\right] \quad \text{........(32)}$$

in which all the nomenclature is "standard," except values of $\beta\left(\frac{N}{M}, \frac{A^2_o}{A^2}\right)$

TABLE 10.—COMPUTATION OF BACKWATER CURVE BY THE METHOD OF MING LEE

Depth (ft)	$\Delta\gamma$ (1)	$\Delta\,\beta(N, \gamma)$ (2)	$\frac{A^2_o}{A^2}$ (3)	$\beta\left(\frac{N}{M}, \frac{A^2_o}{A^2}\right)$ (4)	$\Delta\,\beta\left(\frac{N}{M}, \frac{A^2_o}{A^2}\right)$ (5)	Δx (6)	Distance[a] (ft) (7)
4.5			0.790	1.308			0
5.0	0.125	−0.2285	0.640	0.884	−0.424	3,344	3,344
5.5	0.125	−0.1240	0.530	0.667	−0.217	2,392	5,737
6.0	0.125	−0.0800	0.445	0.527	−0.140	1,987	7.724
6.5	0.125	−0.0555	0.378	0.427	−0.100	1,760	9,483
7.0	0.125	−0.0525	0.330	0.363	−0.064	1,746	11,230

[a] Distances in feet along the channel below a depth of $y = 4.5$ ft.

which are given by Mr. Lee in a table. Using Mr. Bakhmeteff's and Mr. Lee's tables, values of the coordinates of points on the curve have been computed, with the results shown in Table 10.

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[18] "Steady, Gradually Varied Flow in Uniform Channels on Mild Slopes" by Ming Lee, thesis presented to the University of Illinois, at Urbana, Ill., in 1947, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

The Jansen Method.—Results of computations of the slopes of the curve at various points by Mr. Jansen's method are shown in Table 11. In sketching curves by this method the tangents are made to intersect at the midpoint between contiguous depths. The backwater curve resulting is shown in Fig. 8.

Summary.—Theoretically, the graphical method yields the most accurate results because no assumptions are made concerning the conditions in the integration procedure, other than those made in the derivation of the equation of gradually varied flow. However, the work involved in this method is tedious. The work involved in the step method is relatively simple and the results are of sufficient accuracy for most purposes. The methods of direct integration proposed by Mr. Bakhmeteff and by Mr. Lee give similar results and are less tedious than the graphical method, but may be less accurate because of assumptions made in the computation of the integral functions. In the Bakhmeteff method, the empirical relationship between the conveyance and the depth holds for a wide range of depths. However, there may be appreciable changes in the velocity heads that are significant. Hence, the limitations imposed by assuming β to be constant make it necessary to restrict the computations to small increments of depth. In Mr. Lee's method β is regarded as a function of the depth, for practical purposes, so that the integration limits may be taken over wider ranges of depth. Under such conditions the Lee method may be convenient to use.

TABLE 11.—COMPUTATION OF BACKWATER CURVE BY THE JANSEN METHOD

Depth, y (ft)	Factor, $\mu \times 10^{-6}$ (1)	Factor, $i \times 10^{-15}$ (2)	Slope (3)
4.5	15.60	11.0	0.000101
5.0	11.82	8.0	0.000177
5.5	9.17	6.03	0.000288
6.0	7.29	4.635	0.000260
6.5	5.91	3.64	0.000280
7.0	4.92	1.92	0.000306

In the Mononobe method the computations are more direct in that the distance along the channel between any two sections may be determined in one step. The integration of the gradually varied flow equation is simplified by making the assumptions that both the wetted perimeter and the area can be expressed as monomial functions of the depth. The drawback to the Mononobe method is the difficulty of using the graphical charts prepared by him, and the inherent errors resulting from interpolations in the reading of the chart.

Mr. Jansen's method and Mr. Mononobe's method differ from each other and the other four methods investigated, in the results obtained in the illustrative problem. The difficulties with these two methods lie mainly in the reading of the intersection of lines and curves. In the Jansen method, in order to obtain a uniform curve the computed tangents are made to intersect at the midpoint between contiguous depths. The accuracy with which the location of such points is determined fixes the degree of approximation in the shape of the resulting curve.

K. C. Wu,[19] A. M. ASCE.—The determination of water-surface profiles in open channels carrying steady nonuniform flow was first solved by Mr.

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[19] Lecturer in Civ. Eng., Univ. of Hong Kong, Hong Kong.

Bresse[20,21] for an infinitely wide rectangular channel as early as 1860. In 1912 Mr. Bakhmeteff first suggested a generalized solution for channels of any shape which, in 1932, he published[2] giving many numerical solutions. In 1948 Phillip Z. Kirpich,[22] A. M. ASCE, proposed a dimensionless formula with a set of curves for the solution of the hydraulic exponent, n_B, for rectangular and trapezoidal channels. In 1949 M. E. Von Seggern,[23] A. M. ASCE, introduced another exponent m, in order to facilitate the integration of the equation of steady nonuniform flow. Mr. Jansen's paper is a further extension, and, at the same time, a new and ingenious approach to the same problem. As stated by the author, the method is applicable only to rectangular and trapezoidal channels of symmetrical shapes.

Eqs. 10, 11, and 12 can be extended so that values of the u-functions and j-functions are in dimensionless forms. Substituting $\alpha = y/b$ and simplifying, Eq. 10 becomes

$$u = \frac{(1 + P\,\alpha)^{4/3}}{b^{16/3}\,[\alpha\,(1 + \phi\,\alpha)]^{10/3}} \qquad (33a)$$

and Eq. 11 becomes

$$j = \frac{1 + 2\,\phi\,\alpha}{b^{5}\,[\alpha\,(1 + \phi\,\alpha)]^{3}} \qquad (33b)$$

Next, assume that $u\,b^{16/3} = U$ and $j\,b^{5} = J$ (both of which are dimensionless functions depending only on α and ϕ since P is also a function of ϕ). Eqs. 33 then become

$$U = \frac{(1 + P\,\alpha)^{4/3}}{[\alpha\,(1 + \phi\,\alpha)]^{10/3}} \qquad (34a)$$

and

$$J = \frac{1 + 2\,\phi\,\alpha}{[\alpha\,(1 + \phi\,\alpha)]^{3}} \qquad (34b)$$

Eq. 12 can then be written as

$$\frac{dy}{dx} = \frac{S_o - \dfrac{N}{b^{16/3}}\,U}{1 - \dfrac{G}{b^{5}}\,J} = \frac{S_o - N'\,U}{1 - G'\,J} \qquad (35)$$

in which

$$N' = \frac{N}{b^{16/3}} = \frac{n^2\,Q^2}{2.208\,b^{16/3}} \qquad (36a)$$

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[20] "Mechanique appliquée," by J. A. C. Bresse, Mallet-Bachelier, Paris, France, 1860.

[21] "Hydraulics of Steady Flow in Open Channels," by S. M. Woodward and C. J. Posey, John Wiley & Sons, Inc., New York, N. Y., 1941.

[22] "Dimensionless Constants for Hydraulic Elements of Open Channel Cross-Sections," by Phillip Z. Kirpich, *Civil Engineering*, October, 1948, p. 47.

[23] "Integrating the Equations of Nonuniform Flow," by M. E. Von Seggern, *Transactions*, ASCE, Vol. 115, 1950, p. 71.

and

$$G' = \frac{G}{b^5} = \frac{Q^2}{g\,b^2} \qquad (36b)$$

All terms on the extreme right side of Eq. 35 are dimensionless. For any prismatic trapezoidal channel (including rectangular channels), both N' and G' can be easily calculated, and U and J can be obtained either from curves (such as Fig. 9) or from prepared tables; and the slope dy/dx between the desired sections can be computed by Eqs. 35 and 36.

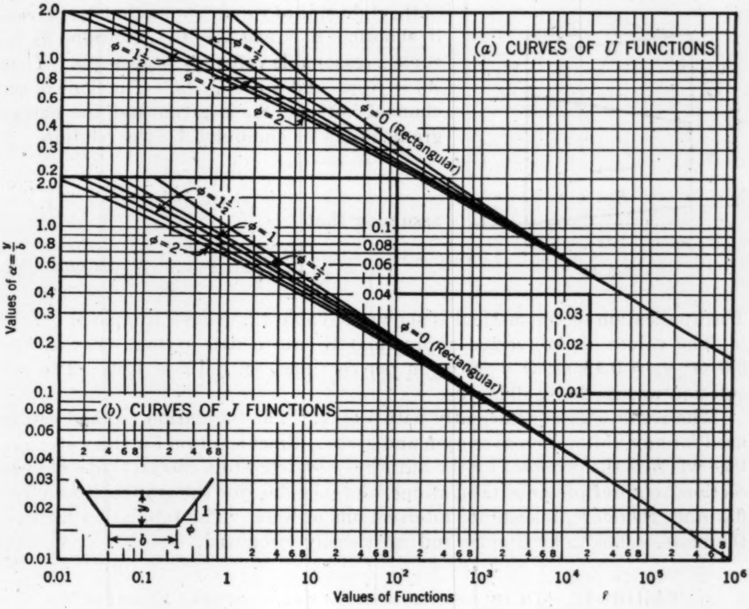

FIG. 9

J. PIETRKOWSKI[24].—An equation for the slope of the water surface in a trapezoidal channel for steady nonuniform flow has been developed by the author. The practical solution of the equation requires the use of special tables. The surface curve itself must be found from various values of the slope either graphically or numerically. In order to test the advantage of the proposed method, it seems logical to compare it with the usual tabulation method.

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[24] Director of Water Section, Public Works Dept., Tel-Aviv, State of Israel.

The basic relation (Fig. 10) is expressed by

$$dx = \frac{h_1 - h_2}{S - S_o} \qquad (37)$$

With h_1 and S_o known, textbooks usually assume a definite value for dx to find values h_2 and S that satisfy Eq. 37. The method thus becomes subject to trial and error, which can be avoided by solving Eq. 37 for dx. An inspection of the right side of Eq. 37 shows that h_1 is given, if shooting flow exists. To determine h_2 assume a reasonable value, $y_2 = y_1 + \Delta y$. Then $A_2 = f(y_2)$, and $V_2 = Q/A_2$. The slope of the channel bottom, S_o, is given and the energy gradient can be computed by the relation

$$S = \frac{V^2_m\, n^2}{(R_m)^{4/3}} \qquad (38)$$

assuming that

$$V_m = \sqrt{\frac{V^2_1 + V^2_2}{2}} = \frac{1}{n}\,(R_m)^{2/3}\, S^{1/2} \qquad (39)$$

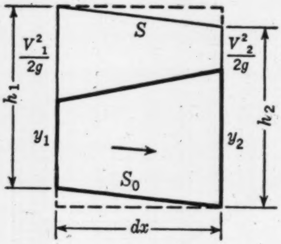

FIG. 10

The application of the method is shown in Table 12, using Example 1.

The values are converted approximately into metric system, that is, $b = 6.0\ m$; $y_1 = 0.45\ m$; $Q = 30$ cu m per sec; and, $n^2 \times 10^4 = 1.69$. The calculation was made by slide rule.

The result checks sufficiently with Fig. 2. The tabulation method can be used for any uniform cross section and gives a direct solution for the water surface whereas the proposal by the author is restricted to a special shape of cross section, requires previous tabulations, and includes (for the graphical solution) flat and, therefore, inaccurate intersections of lines. The tabulation method, therefore, seems to be simpler and most easily visualized.

TABLE 12.—DETERMINATION OF SURFACE SLOPE, EXAMPLE 1 (METRIC UNITS[a])

y_1 (1)	V_1 (2)	h_1 (3)	y_2 (4)	V_2 (5)	h_2 (6)	$h_1—h_2$ (7)	V^2_m (Eq. 39) (8)	R_m (9)	$(R_m)^{4/3}$ (10)	S (Eq. 38) (11)	$S—S_o$ (12)	x (Eq. 37) (13)
0.45	11.1	6.75	0.60	8.35	4.15	2.60	96.3	0.44	0.33	0.0495	0.0485	53.5
0.60	8.35	4.15	0.75	6.65	3.00	1.15	57	0.54	0.44	0.0220	0.0210	55.0
0.75	6.65	3.00	0.90	5.55	2.47	0.53	37.5	0.64	0.55	0.0115	0.0105	50.5

[a] 1 meter (Cols. 1, 3, 4, 6, 7, 9, and 13) = 39.37 in.; 1 meter per sec (Cols. 2 and 5) = 196.85 ft per min = 2.23693 miles per hr; and 1 meter raised to the four-thirds power (Col. 10) = 133.8 in.$^{4/3}$

In the case of the more complicated problems actually encountered in practice, the idea of finding the surface line by calculating the slope first, can be used with advantage in a tabulation method. For example, the writer has been required to calculate the backwater profile for a trapezoidal river channel with gradually increasing width and dikes for a double profile. The water-filled cross section in that case is not a function of the depth alone. It is also necessary to know the place along the channel at which this depth occurs. The distance between control sections is assumed and, for each of these sections, the elevation and slope of the energy grade line is determined, which will result in a continuous line without steps at the intersections.

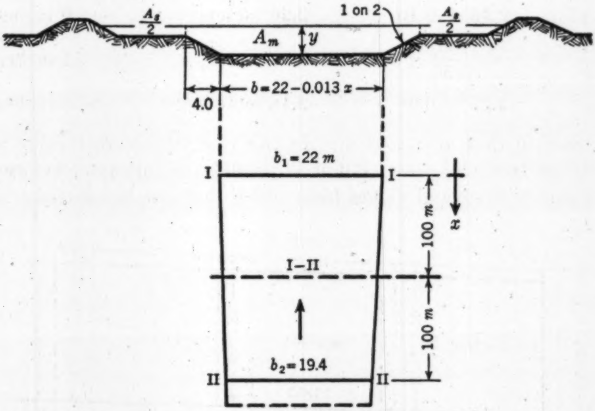

FIG. 11.—DOUBLE PROFILE WITH CHANGING WIDTH

As an illustration, assume a cross section such as Fig. 11 in which $S_o = 1{:}1250$; $Q = 90$ cu m per sec (263 acre-ft per hr); $n = 0.025$; the flow conditions at section 1 are assumed known, with $y_1 = 200$ m; and $S_1 = 1{:}1{,}000$. The discharge conditions at control sections are examined over a distance of $\Delta x = 200$ m, with the energy grade lines intersecting at $\Delta x\ 2$; thus:

Symbol	Computation	Metric units	English units
A_1	$2.0\ (22.0 + 4.0)$	52 square meters	560 square feet
$\frac{1}{n}$	$\frac{1}{0.025}$	40	40
$\frac{1}{n^2}$	. . .	1600	1600
R_1	. . .	1.68 meters	66 inches
$(R_1)^{2/3}$	. . .	1.41 meters$^{2/3}$	16.4 inches$^{2/3}$
S_1	. . .	1/1000	1/1000
$\sqrt{S_1}$	. . .	1/31.6	1/31.6
V_1	$\frac{40 \times 1.41}{31.6}$	1.78 meters per second	351 feet per minute
Q	1.78×52	93 cubic meters per second	272 acre-feet per hour

Symbol	Computation	Metric units	English units
$\frac{V^2_1}{2\,g}$	...	0.16 meter	6.3 inches
y_1	...	2.00 meters	78.7 inches
$H_1 - h_1$	$0.16 + 2.00$	2.16 meters	85.0 inches
$S_1 \frac{dx}{2}$	$\frac{100}{1000}$	0.10 meter	3.9 inches
H_{1-2}	$2.16 + 0.10$	2.26 meters	88.9 inches
$S_o \frac{dx}{2}$	$\frac{100}{1250}$	0.08 meter	3.1 inches
h_{1-2}	$2.26 - 0.08$	2.18 meters	85.8 inches

It is assumed that $y_2 = 2.03$ m. In this case, the correct value is found most easily by trial and error; but it is possible to calculate, beforehand, a table or diagram of several values from which that one is selected which will fulfill the required conditions. The influence of the side channels is neglected, thus (see Fig. 12):

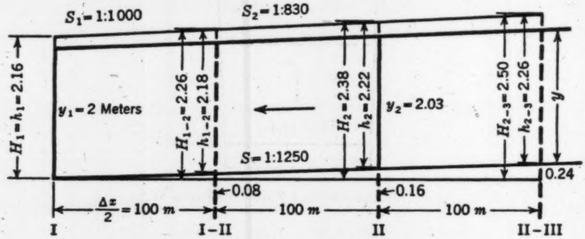
$S_1 = 1{:}1000$
$S_2 = 1{:}830$
$H_1 = h_1 = 2.16$
$y_1 = 2$ Meters
$H_{1-2} = 2.26$
$h_{1-2} = 2.18$
$S = 1{:}1250$
$H_2 = 2.38$
$h_2 = 2.22$
$y_2 = 2.03$
$H_{2-3} = 2.50$
$h_{2-3} = 2.26$
y
0.24
0.08
0.16
$\frac{\Delta x}{2} = 100\ m$
100 m
100 m
I
I-II
II
II-III

FIG. 12.—DETERMINATION OF WATER-SURFACE FLOW

Symbols	Computations	Metric units	English units
A_2	$2.03\ (19.4 + 4.0)$ 0.03×4.0	47.6 square meters	512 square feet
R_2	...	1.67 meters	65.7 inches
$(R_2)^{2/3}$	...	1.41 meters$^{2/3}$	16.4 inches$^{2/3}$
$(R_2)^{4/3}$	...	1.99 meters	78.4 inches
V_2	$\frac{93}{47.6}$	1.95 meters per second	384 feet per minute
V^2_2	...	3.82	...
$\frac{V^2_2}{2\,g}$	...	0.19 meter	7.5 inches
y_2	...	2.03	80.0 inches
h_2	$0.19 + 2.03$	2.22 meters	87.5 inches
$\frac{1}{S_2}$	$\frac{(R_2)^{4/3}}{V^2_2\,n^2}$	830	...

Symbols	Computations	Metric units	English units
$S_o\,\Delta x$	$\frac{200}{1250}$	0.16 meter	6.3 inches
H_2	2.22 + 0.16	2.38 meters	93.8 inches
$-\,S_2\,\frac{\Delta x}{2}$	$\frac{100}{830}$	0.12 meter	4.7 inches
H_{1-2}	2.38 − 0.12	2.26 meters	89.1 inches
H_{2-3}	2.38 + 0.12	2.50 meters	98.4 inches
h_{2-3}	2.50 − 0.24	2.26 meters	89.1 inches

The value of $H_{1-2} = 2.26$ meters (89 in.) agrees with that found in the preceding computation.

With the continuation of the calculation to sections upstream it will be necessary to take the flow in the side channels into account. It will be sufficient to distribute the discharge between the main channel and the side channels in accordance with the relation,

$$\frac{Q_m}{Q_s} = \frac{A_m \times (R_m)^{2/3} \times n_S}{A_S \times (R_S)^{2/3} \times n_m} \quad \ldots\ldots\ldots\ldots\ldots\ldots (40)$$

and then to continue the calculation for the main channel only.

Separation losses and eddy losses can be accounted for by a suitable selection of the friction value *n* for each stretch *dx*, separately.

To summarize: The basic idea offered in this paper should not be wasted on simple problems for which the existing methods are good enough, but should be used for more complicated cases, where the existing trial-and-error methods are inadequate.

ROBERT B. JANSEN,[25] J.M. ASCE.—A lucid verification of the results yielded by the suggested graphical procedure has been presented by Mr. Stevens. It should not be surprising that curves plotted by this method conform with those computed by the equation of Mr. Husted, since both methods of solution are based on the classic law of the conservation of energy, and both make use of weighted energy gradients to define the water slope in a finite increment of channel. Any discrepany observed between the two curves should be primarily a reflection of differences in assumptions of energy loss and of channel increment.

If Eq. 2 be considered in finite terms, it can be written as

$$\frac{\Delta h}{\Delta x} = \frac{\Delta z}{\Delta x} + \frac{\Delta y}{\Delta x} + \frac{\Delta}{\Delta x}\left(\frac{V^2}{2\,g}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (41)$$

Multiplying Eq. 41 by Δx, and substituting $S\,\Delta x = \Delta h$ and $S_o\,\Delta x = \Delta z$, it takes the form:

$$S\,\Delta x = S_o\,\Delta x + \Delta y + \Delta\left(\frac{V^2}{2\,g}\right) \quad \ldots\ldots\ldots\ldots\ldots (42)$$

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Eq. 42 can be transformed to

$$\Delta x = \frac{\Delta y + \Delta\left(\frac{V^2}{2\,g}\right)}{S - S_o} \qquad (43)$$

which is the same as Eqs. 20 and 22.

In deriving Eq. 43, friction was the only cause of energy loss included. In order to take eddy loss also into account, the term $K\Delta\left(\frac{V^2}{2\,g}\right)$ should be subtracted from the right side of Eq. 42 so that the final equation becomes

$$\Delta x = \frac{\Delta y + \Delta\left(\frac{V^2}{2\,g}\right)(1 - K)}{S - S_o} \qquad (44)$$

It is lamentable that relationships for energy loss in an open channel have not been developed for nonuniform flow. The use of the Chezy equation in this case is pardonable only because no satisfactory substitute is known. Although it is generally accepted that the loss in diverging flow is greater than that in uniform flow, no reliable mathematical statement of the difference is available in the technical literature. The application of the eddy loss coefficient, K, is a rough attempt to compensate for the lack of a proper formula. Mr. Stevens' selection, $K = 0.20$, is a value frequently applied in practice.

To inject an eddy-loss factor such as this into the writer's equations requires only a simple modification. If the velocity head differential in Eq. 2 is multiplied by the quantity $(1 - K)$ as suggested, it can be shown that Eq. 5 will become

$$\frac{dy}{dx} = \frac{S_o - \frac{Q^2}{A^2\,C^2\,R}}{1 - (1 - K)\,\frac{Q^2\,T}{A^3\,g}} \qquad (45)$$

and Eq. 12 will be changed to

$$\frac{dy}{dx} = \frac{S_o - N\,u}{1 - (1 - K)\,G\,j} \qquad (46)$$

Since both the coefficient K and the factor α, as applied by Mr. Stevens, are dimensionless, their introduction into the writer's equations calls for a very minimum of effort. This is contradictory to Mr. Stevens' opinion that "The computation of surface curves from a varied-flow function as used by the author * * * practically precludes the possibility of taking all the factors into account."

The writer does not consider it necessary here to defend the merits of the Manning formula, particularly since, in this case, it yields results close to those given by the Kutter relation. However, Mr. Stevens may find some argument from other quarters on his assertion that the Manning expression is dimensionally incorrect since the dimension of the roughness factor n is a topic for debate.

Contrary to the opinion of Mr. Stevens and Mr. Pietrkowski, the tables of the depth functions, u and j, are not essential to the method presented in the paper—no more so, in fact, than they would be to the Husted procedure or to methods advocated by others. It will be noted that these functions are simply

$$u = \frac{p^{4/3}}{A^{10/3}} = \frac{1}{A^2 R^{4/3}} \qquad (47)$$

and

$$j = \frac{T}{A^3} \qquad (48)$$

The Manning value of the energy gradient is obtained by multiplying u by the term N, thus,

$$S = N u = \left(\frac{n^2 Q^2}{2.208}\right)\left(\frac{1}{A^2 R^{4/3}}\right) \qquad (49)$$

The significance of the function j in defining the change in velocity head has already been indicated in Eq. 3. That relation can be rewritten to give

$$d\left(\frac{V^2}{2 g}\right) = - G j \, dy \qquad (50)$$

It is apparent from Eqs. 49 and 50 that the application of the depth functions u and j to the computation of surface curves by step procedures, such as those recommended by Mr. Stevens, would be more than a minor convenience.

In order to illustrate the general applicability of these functions, reference is made to a graphical method described by S. M. Woodward, Hon. M. ASCE, and C. J. Posey,[21] M. ASCE. Their general equation can be written

$$dx = f(y) \, dy \qquad (51a)$$

in which

$$f(y) = \frac{1 - G j}{S_o - N u} \qquad (51b)$$

The relation between Eqs. 51 and Eq. 12 is obvious.

In the graphical solution outlined by Messrs. Woodward and Posey, values of $f(y)$ are obtained for the range of depths covered by the water-surface profile. These values of $f(y)$ are then plotted as ordinates, versus depths as abscissas. The area under the resulting curve between y_1 and y_2 is the distance between the corresponding stations.

It should be emphasized that the functions u and j are of general purpose, and can be applied to advantage not only in the writer's solution but in the Woodward-Posey method and the Husted method as well. This point can best be clarified by considering a typical problem.

Example 3.—Water is flowing from a reservoir into a trapezoidal spillway channel whose bottom width is 15 ft. The slope of the spillway floor is $S_o = 0.0530$ and the roughness of the lining is given by $n = 0.025$. The rate of flow is 1,000 cu ft per sec. The side slopes of the channel are both 1 on 1.5.

The depth of the control section is given by $y_c = 4.43$ ft. The constant terms in the equations are: $N = \frac{n^2\,Q^2}{2.208} = \frac{(0.025)^2\,(1{,}000)^2}{2.208} = 283$ and $G = \frac{Q^2}{g} = \frac{(1{,}000)^2}{32.2} = 31{,}100$.

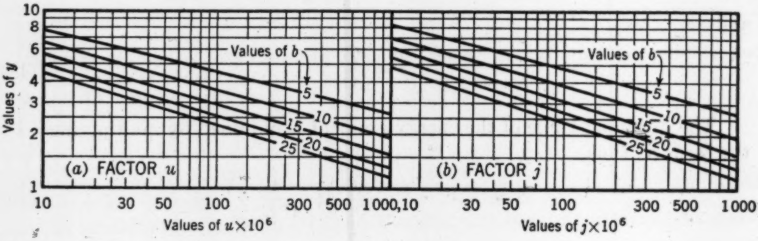

FIG. 13.—FACTORS FOR SIDE SLOPES OF 1 ON 1.5

The values of the functions u and j are obtained from Fig. 13 which was derived from the writer's tables of the depth factors for side slopes of 1 on 1.5. When $y = 4.0$ ft, $\frac{dy}{dx} = \frac{S_o - N\,u}{1 - G\,j} = \frac{0.0530 - 283\,(35.0 \times 10^{-6})}{1 - 31{,}100\,(45.6 \times 10^{-6})} = -\,0.1032$.

The surface slopes at other depths are obtained similarly. A summary of the calculations required for direct plotting is presented in Table 13(*a*). The curve of the water surface in the spillway is drawn in Fig. 14.

TABLE 13.—COMPUTATIONS FOR EXAMPLE 3

Depths, y or y_1	(*a*) DETERMINATION OF SURFACE SLOPES[a]				(*b*) SOLUTION BY THE HUSTED METHOD			
	Factor[b] u	Factor[b] j	Slope dy/dx	$f(y)$	Depth y_2	Average Factors[b]		Δx
						u	j	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
4.43	...	32.1	∞	0	4.00	29.6	38.9	2.0
4.00	35.0	45.6	−0.1032	9.68	3.50	45.9	58.6	10.3
3.50	56.7	71.6	−0.0301	33.2	3.00	77.5	95.8	31.8
3.00	98.2	119.9	−0.0092	108.6	2.75	115.9	139.9	41.5
2.75	133.6	159.9	−0.0038	261	...	...	...	...
2.50	186.6	218.5	0.0000	∞	Summation, in feet............			85.6

[a] Values of $f(y)$ in Col. 5 are by C. J. Posey and S. M. Woodward. [b] These values to be multiplied by 10^{-6}.

In order to solve this problem with the graphical device suggested by Messrs. Woodward and Posey, the values of $f(y)$ are plotted against the depths in Fig. 15. The area (A) under the curve is equal to the distance from the control section to the station where the depth is 4.00 ft; the area (B) equals the distance from where $y = 4.00$ to where $y = 3.50$ ft, etc. The water-surface profile determined by this method is superimposed on Fig. 14 for comparison.

For finding the water surface by use of the Husted method, Eqs. 51*a* and 51*b* can again be advantageous. As is customary in the step methods, the values of the variables will be averaged for each length of channel. The factors u and j in Eq. 51*b*, therefore, will be taken as the averages of the functions at the two stations that define the increment. The results summarized in Table 13(*b*) were obtained by applying Eq. 51*a* in finite terms.

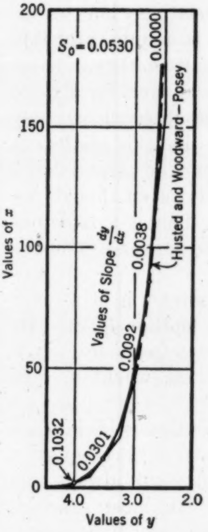

FIG. 14.—DETERMINATION OF WATER-SURFACE SLOPE, EXAMPLE 3

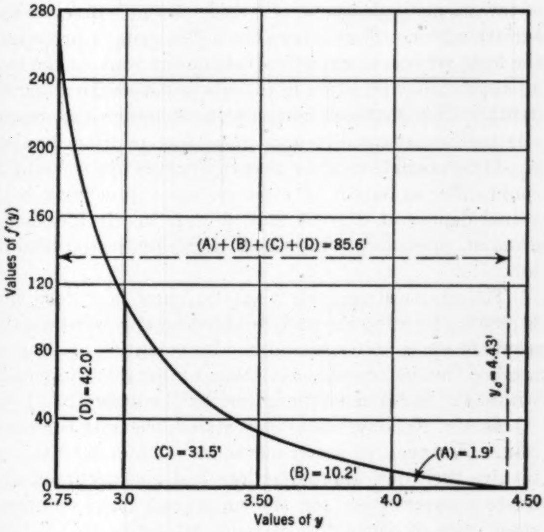

FIG. 15.—SOLUTION BY THE WOODWARD-POSEY METHOD, EXAMPLE 3

The lengths shown in Table 13(*b*) are seen to be practically the same as those in Fig. 15, the latter having been computed as suggested by Messrs. Woodward and Posey.

This typical problem has thus been solved by the Husted method, by the Woodward-Posey graphical procedure, and by direct plotting of the water-surface slopes. In each of these three approaches, the general depth functions u and j have been used to reduce the labor of computation. The close conformity of the resulting water profiles, illustrated in Fig. 14, indicates that any of these solutions provides acceptable accuracy. Whether one or another method will best solve a given case will depend upon the length of the profile and upon the facilities available.

These closing statements serve to show that Mr. Stevens and the writer are not widely separated in their lines of attack. The two methods stem from the same basic relationships and produce practically identical results. In each case the calculations are started from a known depth, and the depth functions for each increment are substituted into a formula such as Eq. 12. The effects of velocity distribution and eddy loss, if predictable, can be taken into account with equal ease by either method.

Mr. Stevens' distaste for lengthy formulas and laborious calculations is shared by the writer and by the multitude. The merits of the suggestions offered in this paper and its discussions will be found where there is a saving in the time spent by the designer.

Messrs. Bishay and Babbitt have contributed an interesting comparison of several methods by use of a single example involving a relatively long water-surface curve. They have shown the writer's procedure to require probably the least volume of numerical calculation; but, at the same time, they indicate an appreciable deviation in the computed length along the channel for a given depth. This type of comparison is misleading because it considers only relative horizontal distances instead of relative depths. By this method it would be possible to show that two curves approaching near the normal depth could differ in length by a percentage approaching infinity, even though the actual depths at a given station were nearly identical. This is a ludicrous example, of course, but it should illustrate the uncertainties of such a comparison.

The practical engineer wants to know how deep the stream is at a point 11,000 ft down the channel, not how far it is to where the depth is 7.0 ft. He wants to know how high to build his sidewalls and how much freeboard he will need. The percentage deviation of depths computed here would be well within the limits usually required by the designer.

As Mr. Nelidov has so ably stated, the only correct solution is by integration. However, common methods of integrating the water-surface equation involve their own approximations and give accuracy which is not always adequate compensation for the additional labor required. In most practical cases, ease of calculation is more attractive than laborious and unnecessary refinement.

In the introduction of the method of direct plotting of the water-surface curve, the tangents were arbitrarily made to intersect at the midpoints of the depth increments in order to attain uniform results for illustration. It was stated that this procedure can be modified in practice to give more accuracy in that part of the curve where the slope is varying rapidly. In problems solved by the writer through use of the unmodified procedure, the discrepancy in depth has not been greater than 4%, the average being about 2%. If more accuracy is required, smaller depth increments can be taken.

Mr. Pietrkowski states that direct plotting is restricted to a special shape of cross section. The method is in fact free of any such restriction. Although the final equations presented here are limited to trapezoidal channels, the potential applications of the basic idea are manifold.

The thought-provoking international responses to this paper have enhanced its value immeasurably.

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Paper No. 2528

RIVER CHANNEL ROUGHNESS

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Associate Members ASCE

With Discussion by Messrs. T. Blench; James J. Doland and Ven Te Chow; Robert B. Banks; L. Bajorunas; and Sir Claude Inglis

Synopsis

Solution of the problem of determining the frictional losses in natural streams usually depends on the judgment and experience of the engineer. A rational approach to this problem is offered in this paper. Existing theory on friction in open channels is reviewed, and a logical derivation leads to the development of formulas and charts to determine the relationship between stage and discharge in a given reach of river. An interdependent treatment of the hydraulic characteristics of flow and the sedimentary characteristics of the stream is presented. Friction losses due to grain roughness, channel irregularities, and other factors are studied and formulated, and suggestions are made for practical application of a proposed rational method of friction evaluation.

Introduction

In dealing with natural streams with movable beds, the engineer is confronted with the problem of evaluating frictional losses. Unless adequate and complete data are available to guide him, too often not the case, his evaluation of frictional effects is based almost entirely upon judgment. The element of doubt inevitably present forces the engineer to be conservative. The degree of conservatism exercised is reflected in project costs. If the element of doubt and confusion concerning frictional losses in natural streams could be reduced to the lowest practicable limit, substantial savings in the costs of many hydraulic projects undoubtedly would result.

Although many data concerning the roughness values of natural streams have been accumulated and published, the fact remains that, in estimating the roughness value of a particular stream for which no roughness data are available, there are likely to be as many different estimates as there are estimators. Even when

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roughness data are available for a particular stream, often these data are applicable only within a very narrow range of discharge and for a particular location on the stream. It is not uncommon for many engineers to think of a natural river channel as having a single roughness value for all discharges. Even some of those engineers who feel that the roughness value n (as applied to Manning's or Kutter's formula) varies with the discharge Q are compelled, by lack of data concerning the variation of n with Q, to assign a single-valued n to a particular stream.

It is evident that a new and more rational approach to the evaluation of frictional losses in natural channels is needed. The writers believe that a realistic appraisal of frictional losses in streams with movable beds is possible only when the sedimentary characteristics of the stream and the usual hydraulic characteristics of flow are treated interdependently. With regard to the sedimentary characteristics, frictional losses in natural streams depend on the size and gradation of sediments composing the stream bed surface and on the shape, size, and spacing of irregularities within the boundaries of flow. It is because of the lack of these irregularities that laboratory roughness values derived under hydrodynamically similar conditions often do not agree with the roughness values of natural streams. It is probable that these irregularities are a function of the sediment transport which is certainly a function of flow conditions. The interdependence of the sedimentary and hydraulic conditions of flow is obvious. If the functional relationship between the irregularities of the bed and the sediment transport can be expressed in dimensionless form and evaluated by means of experimental data, it will be possible then for the practicing engineer to use this relationship as a means of evaluating the frictional losses of natural streams with considerably more confidence than in the past.

EXISTING THEORY

Turbulence theories have substantiated beyond doubt that the distribution of flow velocities in a real fluid (with internal friction) is very closely related to the shear stresses in the fluid. Since the flow of water in an open stream channel is influenced by friction along its solid boundaries, the bed and the banks, the frictional effect on the flow may be measured by the shear stresses along these boundaries.

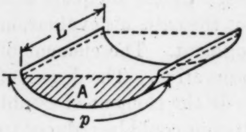

FIG. 1.—CROSS SECTION OF FLOW

In Fig. 1 the total shear force along the wetted boundary surface $L\,p$ is $L\,p\,\tau_o$ in which τ_o is defined as the average unit shear stress, parallel to the direction of flow, within the body of water of length L and wetted perimeter p. This body of water, of cross-sectional area A, has a weight component in the direction of flow of $A\,L\,w\,S$, in which w is the unit weight of water and the slope S of the water surface is small. For the case of uniform motion the algebraic sum of the forces acting parallel to the direction of flow must equal zero since no acceleration of flow takes place. Neglecting air friction at the open surface and noting that the total shear force acts in a retarding direction, it follows that

$$A\,L\,w\,S - L\,p\,\tau_o = 0 \quad \ldots\ldots\ldots\ldots (1)$$

If $\rho = w/g$ is the unit mass density, in which g is the acceleration due to gravity, Eq. 1 may be rewritten as

$$\tau_o = \frac{w\,S\,A}{p} \qquad (2a)$$

and

$$\frac{\tau_o}{\rho} = \frac{g\,S\,A}{p} \qquad (2b)$$

Both forms of Eq. 2 are equally valuable. The dimensions of the channel appear only in the ratio $A/p = R$, commonly known as the hydraulic radius. In the derivations of this paper it will prove most helpful, however, to think of the hydraulic radius as the ratio $\frac{A\,L}{p\,L}$, the ratio of the volume of the flow segment to the corresponding frictional surface area.

Fluid mechanics teaches that on a hydrodynamically smooth wall the shear forces are transmitted entirely by the tangential shearing stresses within the fluid, whereas on a hydrodynamically rough wall part of the shear forces are transmitted by unsymmetrically distributed normal stresses on the individual roughness protrusions of the wall. In the latter case the degree of resistance to flow depends to a great extent upon the size, shape, and spacing of the individual irregularities. This type of resistance is commonly called form resistance. The actual flow pattern in the immediate vicinity of a rough wall is basically different from that near a smooth wall. However, at a distance away from the wall of one or two roughness lengths K_s this difference disappears entirely, and the two flow patterns become identical.

Roughness of a Natural Stream Bed

Derivations.—The rough wall discussed so far is a wall with a surface characterized by straight alinement and practically uniform roughness. The pipes which J. Nikuradse[3,4] roughened by gluing sand grains to the inside are a good example of such a straight, uniformly rough surface. The individual grains of bed sediment of a natural stream also provide a grain roughness which is approximately comparable to that of Mr. Nikuradse's roughened pipes, and the resulting frictional losses due to the grain roughness alone may be calculated in much the same way as are the frictional losses for pipes or artificial uniform channels of immovable beds. However, the natural stream bed surface, especially that of a sediment-carrying alluvial stream, is basically different from that of Mr. Nikuradse's experimental pipes because the surface is not straight. It is often irregular due to sediment bars of various scales.

The foremost problem confronting engineers who deal with natural streams is to evaluate the influence of the irregularities of the stream bed surface upon the grain roughness. It is very likely that these surface irregularities are a source of additional friction losses over and above those caused by the grain roughness alone. The question is can these additional friction losses be described physically. A clue to the answer may be found from a consideration of

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[3] "Gesetzmässigkeiten der turbulenten Strömung in glatten Rohren," by J. Nikuradse, *Forschungsheft 356*, Verein Deutscher Ingenieur, Berlin, Germany, 1932.

[4] "Strömungsgesetze in rauhen Rohren," by J. Nikuradse, *Forschungsheft 361*, Verein Deutscher Ingenieur, Berlin, Germany, 1933.

the problem involving the evaluation of friction losses in a rough pipe line which has irregularities such as elbows, enlargements, contractions, or any other type of transitional fittings. It is common knowledge that, despite the very serious theoretical objections due to the basic nonlinearity of the problem, one may with a reasonable degree of accuracy determine individually the friction losses in each element of the pipe line and simply add them to obtain the total loss. By neglecting the mutual interference of flow patterns within the different elements of the pipe line, the problem is thus reduced to that of calculating the individual losses. The same type of approach is suggested in this paper for the determination of the additional friction loss due to irregularities of natural streams with moving sediment beds.

The friction loss due to grain roughness may be evaluated by use of the Manning equation or by any other suitable open channel formula. In the Manning equation with the average velocity

$$V = \frac{1.486}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

the roughness factor n may be replaced by the grain roughness diameter K_s by means of the empirical formula[5] derived by K. Strickler:

$$n = \frac{\sqrt[6]{K_s}}{29.3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which K_s is measured in feet. By means of this substitution and by the use of Eqs. 2, Eq. 3 may be rearranged into the form:

$$\frac{V}{S^{\frac{1}{2}} R^{\frac{1}{2}} g^{\frac{1}{2}}} = \frac{V}{\sqrt{\tau_0/\rho}} = 7.66 \left(\frac{R}{K_s}\right)^{1/6} \ldots\ldots\ldots\ldots\ldots (5)$$

It follows from the way Eqs. 2 were derived that in Eq. 5 R is the hydraulic radius pertaining to the grain roughness. Stated another way R is the ratio of the cross-sectional area A (contributing the energy that is converted into turbulence along the perimeter p of representative grain roughness K_s) and the perimeter p. Call this area A' and the corresponding hydraulic radius R'. If the abbreviation V'_f is introduced for $\sqrt{S\,R'\,g} = \sqrt{\tau'_0/\rho}$, Eq. 5 becomes

$$\frac{V}{V'_f} = 7.66 \left(\frac{R'}{K_s}\right)^{1/6} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

This is the Manning-Strickler equation. It may be replaced by a logarithmic equation that is better supported by theory:

$$\frac{V}{V'_f} = 5.75 \log_{10} \left(12.2 \frac{R'}{K_s}\right) \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

This equation gives for a very wide range of conditions ($4 < R'/K_s < 2{,}000$) practically the same results as Eq. 6.[6] Eq. 7 seems to be very reliable in describing all flows on hydrodynamically rough surfaces. For such flows the ratio of the roughness diameter K_s to the theoretical thickness δ' of the laminar sublayer is[7]

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[5] "Beiträge zur Frage der Geschwindigkeitsformel und der Rauhigkeitszahlen für Strom Kanäle und geschlossene Leitungen," by K. Strickler, Mitt. No. 16 des Eidgenössische Amtes für Wasser wirtschaft, Bern, Switzerland, 1923.

[6] "Flow on a Movable Bed," by H. A. Einstein, *Proceedings*, 2d Hydraulic Conference, Iowa Inst. of Hydr. Research, State Univ. of Iowa, Iowa City, Iowa, 1942, p. 332.

$$\frac{K_s}{\delta'} = \frac{K_s V'_f}{11.6 \nu} > 5 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

in which ν is the kinematic viscosity.

If K_s/δ' is smaller than 5, Eq. 7 must be corrected. The correction may be made by introducing a factor x into the equation which then becomes

$$\frac{V}{V'_f} = 5.75 \log_{10} \left(12.2 \frac{R'}{K_s} x \right) \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

in which the correction x is a function of K_s/δ' and is given by Fig. 2.[8] This relationship is derived from Nikuradse's measurements,[4] of flows over sand surfaces. (In some practical applications the factor x may be neglected since its effect upon the ratio V/V'_f can never be more than about 20%.) Besides these usual friction losses attributable to the grain roughness in the natural stream, there are additional friction losses whose origin is connected in some manner with the presence of sand bars, sand waves, and other irregularities of the channel. For the easiest understanding of how these irregularities generate additional friction losses, imagine them to be uniformly sized and uniformly spaced protrusions of the stream bed, such as piles or single rocks. Assuming N to be the number of protrusions on the surface area $L\ p$ (Fig. 1),

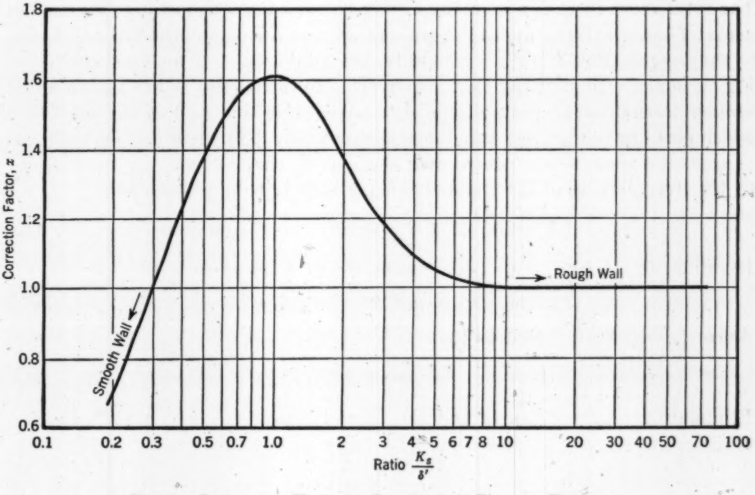

FIG. 2.—CORRECTION X IN THE LOGARITHMIC FRICTION FORMULA

each protrusion having a cross-sectional area a diverting the flow, the drag force f on each protrusion may be expressed as

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[7] "Fluid Mechanics for Hydraulic Engineers," by Hunter Rouse, *Engineering Societies Monographs*, McGraw-Hill Book Co., Inc., New York, N. Y., 1938, p. 243, Eq. 171.

[8] *Ibid.*, Fig. 123.

$$f = c_D\, a\, w \frac{V^2}{2\,g} \qquad (10)$$

in which c_D is the drag coefficient of the protrusion, depending on its shape and size. The total drag force, in the direction of flow, on N protrusions is then

$$F = N\, c_D\, a\, w \frac{V^2}{2\,g} \qquad (11)$$

The additional shear stress τ''_o on the boundary surface $L\,p$ due to this total drag force is

$$\tau''_o = \frac{F}{L\,p} = \frac{N\,a}{L\,p} c_D\, w \frac{V^2}{2\,g} \qquad (12)$$

The total shear force $L\,p\,\tau_o$ of Eq. 1 may be considered to include two components: The shear force due to the grain roughness $L\,p\,\tau'_o$ and the shear force due to the irregularities $L\,p\,\tau''_o$. Similarly, the total area A of the cross section in Fig. 1, contributing energy that is converted to turbulence at the bed, may be broken down into two parts A' and A''. Area A' is that part of the total area which pertains to the grain roughness as previously defined, and A'' is that part of the total area which contributes the energy that is spent in friction due to the irregularities of the channel. Eq. 1 may now be written as

$$(A' + A'')\, L\, w\, S - L\, p\, (\tau'_o + \tau''_o) = 0 \qquad (13)$$

Dividing Eq. 13 by $L\,p$ and designating R'' as A''/p gives

$$w\, S\, R' + w\, S\, R'' = \tau'_o + \tau''_o \qquad (14)$$

Since $\tau'_o = w\,S\,R'$, it follows that

$$\tau''_o = w\, S\, R'' \qquad (15)$$

Eliminating τ''_o from Eq. 12 and Eq. 15 and letting $V''_f = \sqrt{g\,S\,R''}$, then

$$\frac{V}{V''_f} = \sqrt{\frac{2\,L\,p}{N\,a\,c_D}} \qquad (16)$$

It will be noted that the derivation of Eq. 16 depends on the basic assumption that the grain roughness and the irregularities are uniformly distributed over the boundary surface $L\,p$.

The entire expression on the right side of Eq. 16 is a function of the shape, size, and frequency of the irregularities along the surface area $L\,p$. Now, thinking of the irregularities (or protrusions) as sand bars, it is most natural to expect them to be a function of the sediment transport along the channel bed. This sediment transport is itself a function of the shear stress τ'_o on the sediment grains and of the representative grain diameter D. The effect of the variables τ'_o and D is reflected in the dimensionless variable[9]

$$\psi' = \frac{\rho_s - \rho_f}{\rho_f} \frac{D}{R'\,S} \qquad (17)$$

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[9] "Formulas for the Transportation of Bed Load," by H. A. Einstein, *Transactions*, ASCE, Vol. 107, 1942, p. 561.

in which ρ_s is the unit mass density of the sediment in air and ρ_f is the unit mass density of water. Therefore, V/V''_f is a function of ψ'. This relationship must be determined from actual river measurements.

Determination of Additional Friction Losses Due to Channel Irregularities.—These losses are reflected in the friction velocity ($V''_f = \sqrt{g\,S\,R''}$) of Eq. 16. In an absolutely uniform channel these losses are obviously equal to zero, and, therefore, V''_f is never zero in a real channel but varies, depending upon the degree of the irregularities. The effect of these irregularities is to distort the flow pattern. When the discharge is least, the distortion of the flow pattern is greatest (as witness the meandering of natural stream at low flows). As the discharge increases (and hence the sediment transport along the bed also increases) the distortion of the flow pattern becomes less and less because the alinement of flow becomes progressively straighter. Consequently, one may expect that the additional friction loss (or V''_f) diminishes as the discharge Q increases. Since both V and R' become larger with increasing Q, it follows that the ratio V/V''_f also increases, but the ratio ψ' from Eq. 17 decreases. (Note that an increasing transport of sediment means that ψ' decreases.) The relationship between V/V''_f and ψ' has been determined from actual measurements for a number of rivers, most of which are in the Missouri River Basin. The results obtained are indicated in Fig. 3. Except for heavily vegetated streams, there appears to be a fairly well-defined relationship between V/V''_f and ψ'. A tentative curve for the relationship sought has been drawn in Fig. 3.

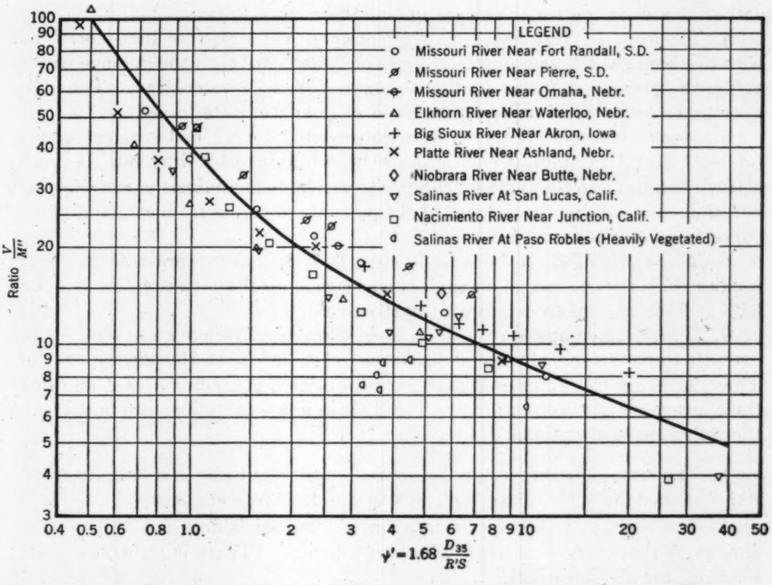

FIG. 3.—FRICTION LOSS DUE TO CHANNEL IRREGULARITIES AS A FUNCTION OF SEDIMENT TRANSPORT

The supporting computation for the results shown in Fig. 3 are based on the following procedure:

1. A comparatively long reach of river was selected within which there exists a gaging station where the relationship between discharge and water surface elevation is known;

2. The over-all slope of the reach was determined as the total drop divided by total distance;

3. Surveyed cross sections of the river channel spaced more or less uniformly throughout the reach were then superimposed upon one another at the gaging station, using the over-all slope as a means of bringing the cross sections together to a common water surface line at the gaging station;

4. The average river width for this common water surface line (correlated with discharge at the gaging station) was obtained, and then the average widths at depth intervals referred to the common line were calculated. Thus, the average cross section for the entire reach of river was determined.

5. Mechanical analyis curves were constructed for all bed surface samples (at least 3 or 4 samples at each cross section obtained by drag bucket in the active channel) and an average, composite mechanical analysis curve was then determined by taking the average of the percentage values from each individual curve for each chosen grain size.

6. The value of K_s to be used in Eqs. 6, 7, 8, and 9 represents the grain roughness. The grain size on the average mechanical analysis curve at which 65% is finer was taken as a representative value of K_s.[9]

7. Hydraulic computations were made using the average cross section, the over-all slope S, and the stage-discharge relationship at the gaging station. Thus, for every chosen discharge the cross-sectional characteristics A and R of the average section were known. The hydraulic computations are based upon the following assumptions:

a. The water surface profile is essentially a straight line in a long reach and remains straight with unvarying slope as the discharge changes.

b. The average cross section in a fairly long reach does not substantially change in shape and size as the discharge varies. (There is some supporting evidence for this assumption.)

c. Because of *a* and *b* the uniform flow equations may be used with negligible error.

8. Eqs. 7, 8, and 9 were then used to determine R', with all other quantities known in these equations.

9. The total R was determined as A/p, and the R'' was determined as $R - R'$. The latter step is permissible because both R' and R'' are derived from the same wetted perimeter p, and A equals $A' + A''$ as defined.

10. With R' and R'' known, the dimensionless values of V/V''_f and ψ' were finally evaluated and plotted in Fig. 3. By definition $V''_f = \sqrt{g\,S\,R''}$ and $\psi' = 1.68\,D/R'S$ from Eq. 17. The ratio $\frac{\rho_s - \rho_f}{\rho_f}$ which represents the density of the bed sediment under water with respect to the density of water may be

assumed to have a value between 1.65 and 1.70 for most natural sediment mixtures, with 1.68 a fair average. The representative grain diameter D of the sediment that is transported along the stream bed was chosen as that grain size on the average mechanical analysis curve that is 35% finer.[9]

11. In making computations for comparatively narrow streams with steep banks, the friction loss on these banks was calculated separately as friction on a rough wall, using Manning's formula (with V and S known and n assigned) to determine the hydraulic radius R_w pertaining to these banks. Then the area $A_w = R_w \times$ wetted perimeter of the banks was deducted from the total cross-sectional area A to give $A' + A''$. With $A' = R' \times$ total wetted perimeter less wetted perimeter of the banks, A'' was determined as $A - A' - A_w$. R'' was then computed as the quotient of A'' divided by the same wetted perimeter that was used to evaluate A'.

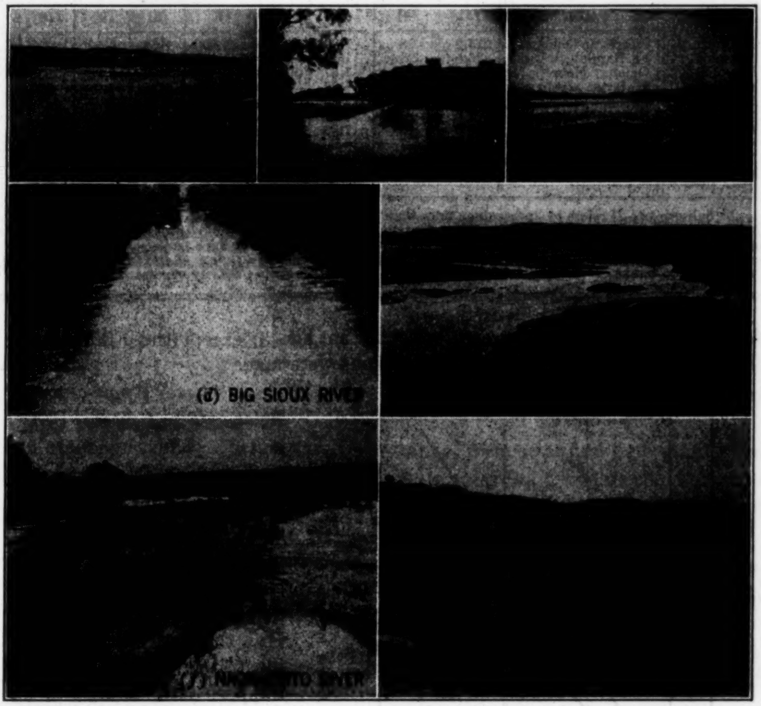

FIG. 4.—TYPICAL RIVER REACHES STUDIED

Description of River Reaches.—A brief description of the river reaches that were selected for study and pertinent data are given in Table 1. The data furnished in Table 1 are complete enough to permit a computational check of at

TABLE 1.—SUMMARY OF DATA

River	Test Reach				Discharge (cu ft per sec)		Average Velocity (ft per sec)		Water Surface Width (T) (feet)	
	Approximate location	Length (miles)	No. of sections	Water surface slope $(\times 10^4)$	Minimum	Maximum	Minimum	Maximum	Minimum	Maximum
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Missouri....	Pierre, S. Dak.[b].....	15.2	28	1.69	5,200	145,000	1.64	5.50	970	3,000
Missouri....	Ft. Randall, S. Dak.[c].	35.5	23	1.49	5,200	145,000	1.11	6.12	1,038	2,130
Missouri....	Omaha, Nebr.[d].....	10.9	14	1.67	30,800	30,800	3.74	3.74	839	839
Elkhorn.....	Waterloo, Nebr.[e]...	9.0	8	5.28	450	14,800	1.32	7.67	225	291
Big Sioux....	Akron, Iowa[f]	29.5	5	1.89	550	8,550	1.14	3.46	132	184
Niobrara....	Butte, Nebr.[g]......	1.0	5	12.57	1,045	1,045	1.95	1.95	804	804
Platte.......	Ashland, Nebr.[h]....	23.0	13	7.38	2,000	78,000	1.64	9.14	725	1.541
Salinas......	San Lucas, Calif.[i]...	7.0	7	11.60	55	43,500	0.61	6.67	130	2,050
Nacimiento..	Junction, Calif.[j]....	3.0	5	13.20	290	38,300	0.84	7.72	272	1,081
Salinas......	Paso Robles, Calif.[k].	6.0	8	17.20	21	6,000	0.14	3.00	110	595

[a] Subscript to D refers to "percentage finer than" on average mechanical analysis curve. [b] Fairly straight
many large sand bars. [d] River works for navigation have materially narrowed natural stream; sand bars not as
correction made. [f] Winding stream; banks vegetated; no sand bars visible; bank correction made. [g] Very
stream; many split channels; winding stream; great number of sand bars; range of stage for large change in Q is
vegetation on banks. [k] Fairly straight channel divided by wood strips; heavily vegetated.

least one point for each stream indicated in Fig. 3, except the Elkhorn and Big Sioux rivers for which bank corrections were made.

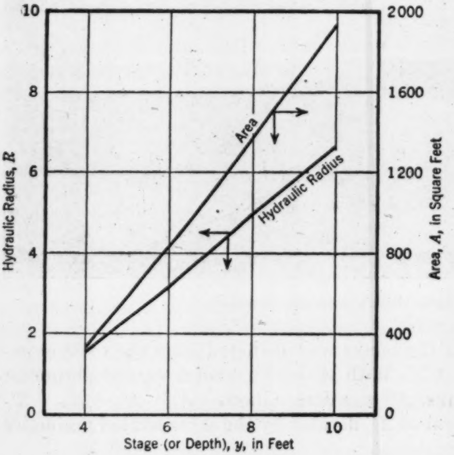

FIG. 5.—STAGE RELATIONSHIPS FOR PROBLEM STREAM

Practical Application of Relationship Between V/V''_* and ψ'.—When adequate hydraulic data are lacking for close determination of the stage-discharge relationship of a particular stream, the curve in Fig. 3 may be used to determine this relationship. How this may be done is best illustrated by an example. Suppose it is desired to determine the rating curve for a stream whose characteristics (having been determined by surveys and by application of procedures previously described) are as follows: (1) Slope $S = 5.00 \times 10^{-4}$; (2) cross-sectional dimensions of the average cross section of the channel in a fairly long reach as a function of stage y, as indicated in Fig. 5; (3) the grain size from the average mechanical analysis curve within the chosen reach at which 65% is finer is K_s = 0.50 mm, and the grain size from the same

ON RIVERS SELECTED FOR STUDY

PROPERTIES OF AVERAGE SECTION										SEDIMENT CHARACTERISTICS				
HYDRAULIC RADIUS (R) (FEET)		ROUGHNESS FACTOR (MANNING'S n)		RATIO T/R		R' (FEET)		δ' AT 60° F (FEET)		$K_s = D_{65}$ (feet)[a]	D_{35} (feet)	RATIO K_s/δ'		Photo Fig. 4
Minimum	Maximum	Minimum	Maximum	Minimum	Maximum	Minimum	Maximum	Minimum	Maximum			Minimum	Maximum	
(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)
3.26	8.79	0.015	0.026	297	341	0.880	6.73	0.00073	0.00201	0.000820	0.000590	0.41	1.12	....
4.53	11.13	0.014	0.045	191	229	0.520	8.28	0.00070	0.00280	0.000722	0.000525	0.26	1.03	(*a*)
9.83	9.83	0.024	0.024	85	85	3.420	3.42	0.00102	0.00102	0.001100	0.000918	1.08	1.08	....
1.52	6.62	0.016	0.034	44	148	0.240	4.26	0.00052	0.00218	0.000852	0.000657	0.39	1.63	(*b*)
3.64	13.40	0.033	0.042	14	36	0.460	2.87	0.00105	0.00263	0.001510	0.001050	0.57	1.44	(*d*)
0.67	0.67	0.021	0.021	1,200	1,200	0.210	0.21	0.00151	0.00151	0.000820	0.000602	0.54	0.54	....
1.68	5.54	0.014	0.034	278	432	0.280	5.16	0.00040	0.00171	0.001900	0.001050	1.11	4.75	(*c*)
0.69	3.18	0.016	0.065	189	644	0.050	2.14	0.00049	0.00320	0.002200	0.001300	0.69	4.50	(*e*)
1.27	4.59	0.019	0.076	214	236	0.150	3.95	0.00034	0.00170	0.025000	0.003300	15.00	73.00	(*f*)
0.45	3.36	0.046	0.262	177	244	0.015	0.55	0.00080	0.00480	0.006100	0.002300	1.30	7.60	(*g*)

reach; channel flow divided at places; numerous large sand bars. [e] Considerable winding of channel; split flow; prevalent. [g] Winding stream; banks vegetated; sand bars visible; probably slack water areas in channel; bank shallow stream; sinuous; great number of sand bars; split channels; vegetation negligible. [h] Very shallow, wide small. [i] Fairly straight channel; bed and banks all sand. [j] Fairly straight channel; bed sand and gravel;

curve at which 35% is finer is $D_{35} = 0.40$ mm. In feet, $K_s = 16.4 \times 10^{-4}$, and $D_{35} = 13.1 \times 10^{-4}$; (4) the bank friction is comparatively negligible, and the active channel (even though split by sand bars and possibly islands) is free of debris and vegetation.

The computations are carried out in tabular form after first determining the constants in Eqs. 6 and 17. For practical purposes Eq. 6 is more convenient to use than Eqs. 7 or 9 because solutions are direct. Using the data given above for the problem stream, Eq. 6 yields

$$V = \frac{7.66 \sqrt{g\,S}}{(K_s)^{1/6}} (R')^{\frac{2}{3}} = \frac{7.66 \times 5.67 \times 2.24}{10^{4/3} \times 16.4^{1/6}} = (R')^{\frac{2}{3}} \ldots\ldots (18a)$$

or

$$V = 2.83\ (R')^{\frac{2}{3}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18b)$$

From Eq. 17

$$\psi' = 1.68 \frac{D_{35}}{R'\,S} = 1.68 \frac{13.1 \times 10^{-4}}{5.00 \times 10^{-4}\,R'} = \frac{4.40}{R'} \ldots\ldots\ldots\ldots (19)$$

The computations for three points on the desired rating curve are shown in Table 2. The procedure is: (*a*) Assume R' such that Eqs. 18 will yield velocity values that reasonably may be expected for the given stream; (*b*) determine V and ψ' from Eqs. 18 and 19; (*c*) with the known value of ψ' read off the value of V/V''_f from the curve in Fig. 3; (*d*) then calculate V''_f and R''; (*e*) R is now determined as $R' + R''$, and the corresponding values of stage y and A are read from Fig. 5; (*f*) finally, Q is computed as $A\ V$. The relationship between Q and y is the desired rating curve, provided that all flow is confined to the active channel. If overbank flow occurs, then an estimated allowance for overbank flow must be made. In the last column of Table 2, Manning's n values, as computed, are shown as a matter of interest. These, of course, are not required for determination of the relationship between Q and y by the method described. The obvious advantage of this method is that the results do not depend upon individual judgment for selection of the appropriate n value.

TABLE 2[a].—TYPICAL COMPUTATIONS TO DETERMINE RELATIONSHIP BETWEEN STAGE AND FLOW

R' (1)	$(R')^{2/3}$ (1a)	V Eq. 18 (2a)	ψ' Eq. 19 (2b)	V/V''_f Fig. 3 (3)	V''_f (4a)	$R'' = (V''_f)^2/gS$ (4b)	$R = R' + R''$ (5a)	Stage y Fig. 5 (5b)	A Fig. 5 (5c)	$Q = AV$ (6)	Manning's n Eq. 3 (7)
1.00	1.000	2.83	4.40	13.3	0.213	2.81	3.81	6.7	1,010	2,860	0.029
2.00	1.587	4.49	2.20	20.4	0.220	3.00	5.00	8.1	1,380	6,200	0.022
3.00	2.080	5.89	1.47	27.5	0.214	2.85	5.85	9.0	1,640	9,630	0.018

[a] All dimensions in the foot-pound-second system.

ACKNOWLEDGMENTS

The cooperation of the Omaha District of the Corps of Engineers, United States Department of the Army, in permitting the use of river data taken from its files is gratefully acknowledged. The writers also wish to express their appreciation of the assistance rendered by B. T. Mitchell, J. A. Constant, J. M. Forns, J. M. ASCE, and all other Omaha District personnel who helped make the necessary computations.

DISCUSSION

T. Blench,[10] M. ASCE.—The extension of Dr. Einstein's methods[9] to deduce the gage-discharge relation of a regime-type[11,12] river seems over optimistic. The following phenomena, that seem to be well established, illustrate the reasons for the writer's belief:

1. In laboratory flume experiments[13] with a bed of natural regime-type river sand, constant discharge, and various bed-sand injection rates, observations after steady regime depths had been attained showed that:

a. Rugosity was greatly altered by changes of injection rate within the practical large-river range;

b. Bed-wave full amplitude was on the order of one-fifth water depth;

c. After a moderate displacement from regime, a 30-ft flume, carrying about one-fifth cu ft per sec, requires some 80 hr to settle down to regime; and

d. Bed-wave conformation changes markedly during attainment of regime, and bed waves have greatly enhanced amplitude when degradation is occurring.

2. In a large regime-type river[14] daily sonic observations of bed waves during a month including a high flood peak showed that:

a. Bed-wave length is closely related to discharge.

b. Bed-wave full amplitude is up to 40% water depth at high discharge.

c. Mean size of bed material in a reach between diversions is very closely correlated with estimated formative depth. (Samples were taken daily on various verticals in which formative depth did not vary appreciably with time.)

d. Discharge division between two branches is found to agree reasonably with the regime theory slope formula in terms of bed-sediment size.

3. In the same river it was found that distribution of sand sizes within bed samples follow a simple universal law.

4. In river models generally[15] it has been found that successful models have width and depth scales related in consonance with regime theory derived from the study of self-formed, sediment-bearing canals of controlled discharges.

In terms of the preceding facts the following phenomena might be expected in rivers: (*a*) Rugosity dependent on rate of change of discharge, that is, on hydrograph form; (*b*) rugosity dependent on bed-load discharge intensity, which can be affected by conditions upstream; (*c*) rugosity affected by meander pattern; (*d*) at some average discharge, conditions explicable in terms of regime theory for steady discharge; (*e*) Eq. 6, which pertains to noncorrugated boundaries, will be inapplicable except after severe distortion of its "constant"; and (*f*) for the conditions at which regime theory applies, only one parameter should be needed to define the bed-sediment size.

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[11] "Regime Theory for Self-Formed Sediment-Bearing Channels," by Thomas Blench, *Transactions*, ASCE, Vol. 117, 1952, p. 383, May, 1951.

[12] "Hydraulics of Sediment-Bearing Canals and Rivers," by Thomas Blench, Evans Industries Ltd., Vancouver, B. C., Canada, 1951.

[13] "Experiments for Fraser River Model," National Research Council of Canada, 1950–1951, Fraser River Model Office, Univ. of British Columbia, Vancouver, B. C., Canada (unpublished).

[14] "Final Report on Special Observation at Ladner Reach during 1950 Freshet," by E. S. Pretious and T. Blench, National Research Council of Canada, 1951.

[15] "Scales for Moving-Bed River Models," by Thomas Blench, London Dock and Harbour Authority, London, England, March, 1951.

Therefore, the points in Fig. 3 should be expected to scatter about a mean line representing regime conditions. The coordinates are nondimensional numbers containing the principal pertinent variables with the exception of bed-load discharge intensity. The line cannot be expected to have a simple equation since it has to correct for the misuse of Eq. 6; the two values of D are chosen in an arbitrary manner; and the use of any kind of D-term presumably hides the settlement velocity that does not follow a simple law in the size range of river sands.

Fig. 3 is of considerable value to the writer. It shows that, by choosing suitable river sites and analyzing data scientifically, a regime-type relation is demonstrated, just as such relations were shown by the much simpler data of controlled canals. Considering the scatter of points in canal analyses when regime theory was being established, the scatter on Fig. 3 is quite small. But, if instead of using the points of Fig. 3 to establish the existence of a regime relation, the smoothing curve is used to indicate where the point for a given stage of a given river is likely to be, then the scatter is very bad; for example, where the curve indicates a value of V/V''_f of 10 the correct answer appears to be anywhere between about 8 and 14 depending on a host of circumstances of which there is no measure.

The writer emphasizes that his criticism is of an application; he holds Dr. Einstein's basic work, from which the present paper springs, in the highest regard and has used it in practical applications.

JAMES J. DOLAND,[16] M. ASCE, and VEN TE CHOW,[17] A. M. ASCE.—In applying the Manning formula to a natural stream, engineers as well as engineering students often ignore two important facts:

1. In natural streams the flow is rarely uniform because of the effect of the inevitable change in discharge due to the varying shape and size of the channel cross section and the unavoidable irregularities in channel alinement;
2. In natural streams the roughness value n is not constant, varying with the discharge and the depth of water in the channel. This can be best illustrated by a typical case, such as is shown in Fig. 6, indicating how the roughness varies with the depth of water in the channel and with the corresponding variation in discharge. The n-value is high at high stages because of rough and grassy banks and increases rapidly near the bottom where debris has accumulated.

Engineers and students usually learn the Manning formula from college hydraulics, in which the theoretical cases are treated (for which cases the formula is designed only for uniform flow and a single-valued n is generally assigned), so it often happens that in practical problems they apply the Manning formula indiscriminately to natural streams in the same way as to theoretical or artificial channels. The condition of uniform flow and constant roughness is unconsciously assumed without apparent reason. The writers are glad that the paper points out that in the hydraulic computations for roughness in natural channels, the assumption of uniform flow may be made with negligible error. In practical problems the variation of n with Q is so evident that there is a need to develop a new and more rational procedure for estimating the varying roughness in natural channels.

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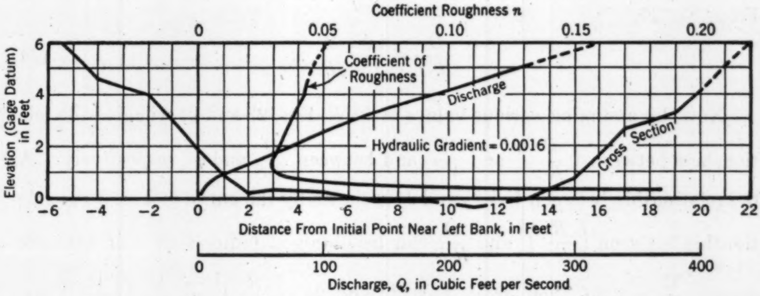

FIG. 6.—HYDRAULIC CHARACTERISTICS OF BONEYARD CREEK AT GAGING STATION, UNIVERSITY OF ILLINOIS, URBANA, ILL.

It seems that the procedure of hydraulic computation described in the paper may be simplified with practical advantages so that the most familiar form of the Manning formula is still retained in use and that several terms unfamiliar to practicing engineers are eliminated.

Relationship Between $\left(\frac{R}{K_s}\right)^{1/3}$ *and* $\frac{D}{R\,S}$ *for Given Values of* $\frac{R'}{R}$.—In the paper the definition of V'_f and V''_f and the equation for V/V'_f are given as follows:

$$V'_f = (g\,S\,R')^{1/2} \qquad (20)$$

$$V''_f = (g\,S\,R'')^{1/2} \qquad (21a)$$

and

$$\frac{V}{V'_f} = 7.66\left(\frac{R'}{K_s}\right)^{1/6} \qquad (21b)$$

Eliminating V'_f and S from the equations and simplifying,

$$\frac{R''}{R'} = \frac{58.68\left(\frac{R'}{K_s}\right)^{1/3}}{\left(\frac{V}{V''_f}\right)^2} \qquad (22)$$

Introducing $R'' = R - R'$ in Eq. 22 and solving,

$$\left(\frac{R}{K_s}\right)^{\frac{1}{3}} = \frac{1}{58.68} \frac{\left(\frac{V}{V''_f}\right)^2 \left[1 - \left(\frac{R'}{R}\right)\right]}{\left(\frac{R'}{R}\right)^{4/3}} \qquad (23)$$

From Eq. 17,

$$\frac{D}{R\,S} = \frac{\psi'}{1.68} \frac{R'}{R} \qquad (24)$$

Now, by assuming various values of R'/R, Eqs. 23 and 24 will give the relationships between $\left(\frac{R}{K_s}\right)^{\frac{1}{3}}$ and $\frac{V}{V''_f}$ and between $\frac{D}{R\,S}$ and ψ', respectively. As the relationship between V/V''_f and ψ' is shown by the curve in Fig. 3 the relationship between $\left(\frac{R}{K_s}\right)^{\frac{1}{3}}$ and $\frac{D}{R\,S}$ can be obtained indirectly. For example, assume $\frac{R'}{R} = 0.60$, then, from Eqs. 23 and 24, $\left(\frac{R}{K_s}\right)^{\frac{1}{3}} = \frac{1}{73.8}\left(\frac{V}{V''_f}\right)^2$ and $\frac{D}{R\,S} = \frac{\psi'}{2.80}$. Take $\psi' = 5.60$. Then, from Fig. 3, $\frac{V}{V''_f} = 11.5$. The corresponding values of $\left(\frac{R}{K_s}\right)^{\frac{1}{3}} = 1.79$ and $\frac{D}{R\,S} = 2.00$. The writers have computed a number of pairs of the values of the item $\left(\frac{R}{K_s}\right)^{\frac{1}{3}}$ and $\frac{D}{R\,S}$ for various values of R'/R. The results are plotted in the form of curves as shown in Fig. 7.

Equation for Roughness Factor n.—Since the Manning formula is

$$V = \frac{1.486}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} \qquad (3)$$

and Eq. 6 gives

$$V = 7.66\, V'_f \left(\frac{R'}{K_f}\right)^{1/6} \qquad (25)$$

in which

$$V'_f = (g\,S\,R')^{\frac{1}{2}} \qquad (20)$$

The roughness factor n may be then expressed by eliminating V, V'_f, and S from Eqs. 3, 20, and 25:

$$n = \frac{\frac{K^{1/6}_s}{29.2}}{\left(\frac{R'}{R}\right)^{\frac{2}{3}}} \qquad (26)$$

Procedure of Hydraulic Computation.—By means of the curves in Fig. 7 and Eq. 26 the practical procedure for computing the rating curve may be performed as follows:

a. From given data for y, S, K_s, and D, compute R, $\left(\frac{R}{K_s}\right)^{\frac{1}{3}}$, and $\frac{D}{R\,S}$;

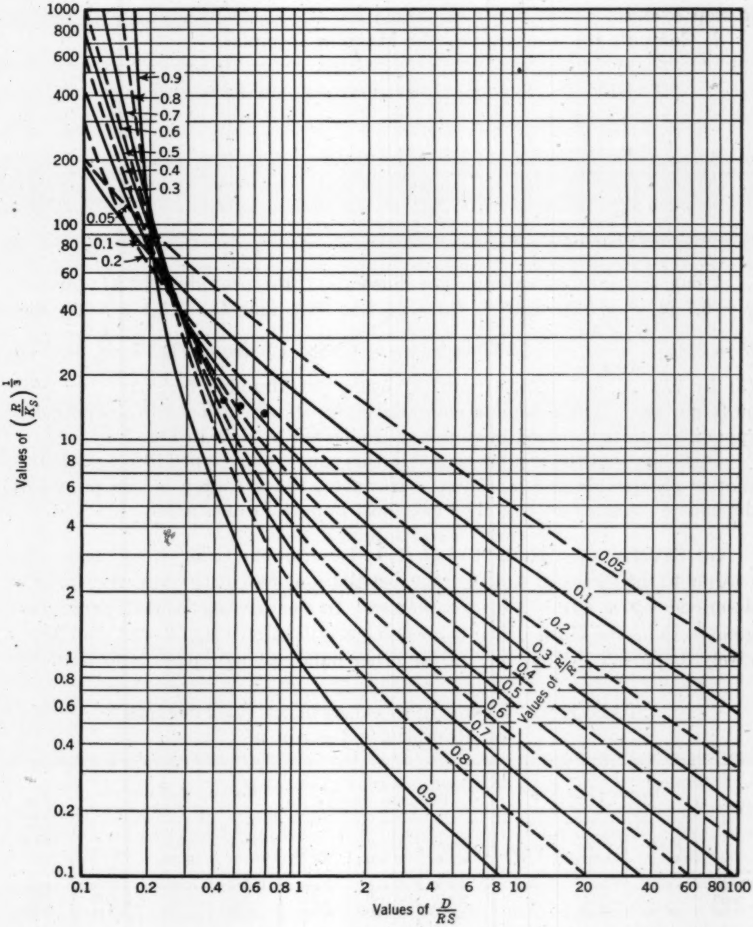

FIG. 7.—RELATIONSHIP CURVES

b. Using the computed values of $\left(\frac{R}{K_s}\right)^{1/6}$ and $\frac{D}{R\,S}$, find the value of $\frac{R'}{R}$ from Fig. 7;

c. Compute n by Eq. 26, using the given K_s and the value of R'/R obtained in step 2; and

d. Compute the discharge by the Manning formula.

TABLE 3.—TYPICAL COMPUTATIONS FOR RATING CURVE

y	R	$\left(\frac{R}{K_s}\right)^{1/6}$	$\frac{D}{R\,S}$	$\frac{R'}{R}$	n	A	Q
6.7	3.81	13.24	0.688	0.260	0.0286	1,010	2,860
8.1	5.00	14.50	0.524	0.366	0.0229	1,380	5,850
9.0	5.85	15.25	0.448	0.513	0.0178	1,649	9,630

Using the authors' example for illustration, the procedure is: (1) Assume $y = 6.7$ ft, $S = 5.00 \times 10^{-4}$, $K_s = 16.4 \times 10^{-4}$, and $D = 13.1 \times 10^{-4}$; then $R = 3.81$, $\left(\frac{R}{K_s}\right)^{1/6} = 13.24$, and $\frac{D}{R\,S} = 0.688$; (2) from Fig. 7, $\frac{R'}{R} = 0.26$; (3) by Eq. 26, $n = 0.0286$; and (4) using the Manning formula, $Q = 2{,}860$ cu ft per sec. The complete computation is shown in Table 3. The corresponding points involved in step 2 are indicated by dots in Fig. 7.

It is clear that the idea presented in this paper is developed into a procedure that is practicable and workable. However, the accuracy of the procedure depends on many factors. The two most important ones are:

(*a*) Satisfaction with the Use of K_s to Describe the Grain Roughness.—According to the theory of statistics the distribution of an observed variate may be described by a number of parameters. In engineering works, parameters greater than the third order (skewness) are usually insignificant, and hence rarely employed. However, on the other hand, a single parameter is too indefinite to describe a distribution. It seems that the distribution of grain size is so irregular that it cannot be described completely by any single value of representative diameter unless the pattern of distribution is of the same type and shape for all possible cases.

TABLE 4.—RANGE OF VARIATION IN COMPUTATION OF ROUGHNESS FACTOR n

Value of R'	Depth y, in Feet			Discharge Q, in Cubic Feet per Second			Roughness Factor n			Percentage variation
	Maximum	Minimum	Curve	Maximum	Minimum	Curve	Maximum	Minimum	Curve	
1.00	8.25	5.40	6.70	4,020	1,415	2,860	0.044	0.023	0.029	72.4
2.00	9.60	7.00	8.10	8,160	5,030	6,200	0.025	0.019	0.023	26.1
3.00	10.00	8.10	9.00	11,600	8,120	9,630	0.020	0.017	0.018	20.0

(*b*) The Accuracy of the Theory upon Which the Curve of Fig. 3 Is Based.—The authors have obtained the working relationship between the values of V/V''_f and ψ' from actual measurements for a number of rivers. This relationship is represented by a smooth curve fitted to the observed data. However, it appears that the plotted observed points scatter within a band of certain width rather than along a narrow determinable path that could be well represented by a line. It is interesting to see what the effect of this scatter is on the computed roughness factor n. Table 4 gives a computation in which the extreme values taken from the band of the plotted points in Fig. 3 are used. The range of variation in n amounts to as high as 72.4%. The resulting rating curves are shown in Fig. 8.

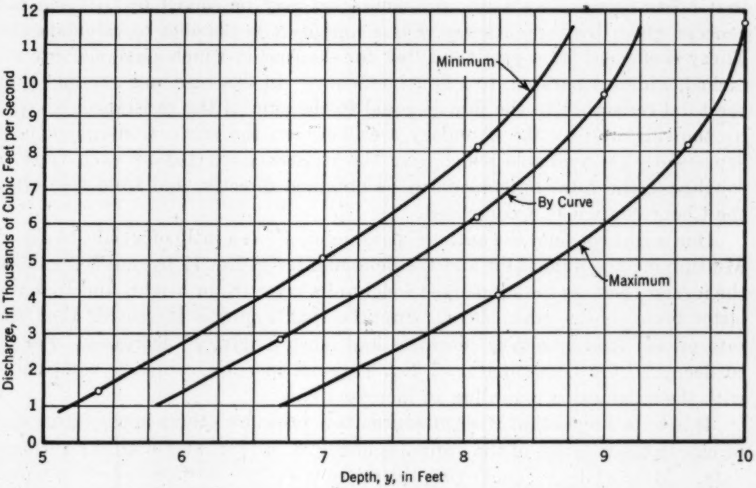

FIG. 8.—VARIATION IN RATING CURVES

ROBERT B. BANKS,[18] J. M. ASCE.—With rare exception, the historical problem of hydraulic roughness has been concerned with the development of empirical expressions for determining fluid resistance. However, in the case in which one type of roughness may be considered to be superimposed over another type, empirical formulas expressing the friction loss of the total roughness either do not exist, as in the authors' problem, or the formulas introduce such vague characteristics as an "equivalent sand roughness." Consequently, as the authors remark, the determination of friction losses in this case is necessarily based entirely upon judgment. Present-day knowledge of the mechanics of fluid flow should certainly warrant more precise statements "* * * a tortuous river with considerable vegetation possesses a Manning coefficient of 0.04." The rational approach proposed by the authors to the problem of river channel roughness thus permits the engineer to calculate the Manning coefficient for this kind of "composite" roughness.

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[18] Fulbright Fellow, Imperial College of Science and Technology, London, England.

The writer, in conjunction with Mr. Einstein, conducted an experimental investigation concerned with the resistance exerted by a boundary composed of two or more types of friction-producing elements acting over the same bed area.[19,20] Results obtained from this investigation have led to the conclusion that other types of hydraulic problems also may be solved by utilizing the concept given by the authors. For example, it is possible to calculate the Chezy coefficient for a particular flow for the case in which small sills are attached, widely separated, to a flow boundary. In this case, one assumes that the total resistance to the flow is equal to the sum of the resistances exerted by the sills, and by the boundary area between the sills. Consequently, if expressions are available which give the resistance exerted by each type of roughness, the total resistance can be obtained directly, and from the latter the Chezy coefficient is computed.

One might consider the authors' parameter, ψ', as a ratio of weight to shear. Multiplying the numerator and the denominator of Eq. 17 by g, it is seen that the former becomes the submerged weight of a layer D_{35} in height, and that the latter becomes the grain shear. Suppose that a flow is increased; then the rate of bed load transport increases and, accordingly, ψ' decreases. For ψ' to decrease, the denominator of Eq. 17 must increase, which is compatible with the assumption regarding an increased flow.

It is to be hoped that river measurements taken by others in the future can be calculated in terms of the ordinates of Fig. 3 and included with the authors' data. Meanwhile, by employing the curve which the authors regard as tentative and by utilizing their concept of resolution of shears, the problem of predicting river channel friction losses can be solved.

L. BAJORUNAS[21].—Available "open-channel flow" formulas do not permit accurate computation of discharges for varying stages. There are two ways of adjusting such formulas to produce accurate results. One is to adjust the roughness factor with changing stage and the other is to change the river cross section. Since no rules have existed for making such adjustments, the proposal in this paper will be very helpful.

The writer concurs with the authors (discussion preceding Eqs. 18) that, for practical purposes, the Manning-Strickler equation (Eq. 6) is more convenient to use than Eq. 7 or Eq. 9, because solutions are direct. However, if it is desired to use Eq. 6 in a practical application, the data used to develop the relationship (as the curve in Fig. 3) should be computed by use of Eq. 6 and not by Eqs. 7, 8, and 9, as was done by the authors.

The writer suggests the following procedure to evaluate the necessary relationships for the Manning equation: Assume that the Manning roughness

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[19] "Fluid Resistance of Composite Roughness," by H. A. Einstein and R. B. Banks, *Transactions*, Am. Geophysical Union, August, 1950, pp. 603–610.

[20] "Linearity of Friction in Open Channel Flow," by R. B. Banks, dissertation presented to the Univ. of California, at Berkeley, Calif., in June, 1951, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

[21] Hydr. Engr., Garrison (N. Dak.) Dist., Corps of Engrs., U. S. Dept. of the Army.

factor is composed of two parts:

$$n = n' + n'' \qquad (27)$$

in which

$$n' = \frac{K^{1/6}_s}{29.3} \qquad (28)$$

is a roughness factor depending on channel bed grain diameter and n'' is the roughness factor depending on channel bed irregularities.

The Manning equation is expressed by Eq. 3 and the Manning-Strickler equation is

$$V = \frac{1.486}{n'} R'^{2/3} S^{1/2} \qquad (29)$$

which is identical with Eq. 6. The expression for n given in Eq. 4 is properly an expression for n', the grain roughness, and the roughness factor n in Eq. 3 may be replaced by $K^{1/6}_s/29.3$ only if R is replaced by R'. It follows from Eqs. 3 and 29 that

$$R = R'\left(1 + \frac{n''}{n'}\right)^{3/2} \qquad (30a)$$

or

$$R = R'\left(\frac{n}{n'}\right)^{3/2} \qquad (30b)$$

By reasoning similar to that set forth by the authors in the discussion preceding Eq. 17, n'' is assumed to be a function of the sediment transport ψ'. To determine the relationship between n'' and ψ' the data from actual river flow measurements were used—namely, all data on which Fig. 3 is based supplemented by those for the Missouri River, near Williston, N. Dak. The following line of computations was used: n' from Eq. 28, n from Eq. 3, with V, R, and S from flow measurements, n'' from Eq. 27, R' from Eq. 30, and ψ' from Eq. 17. No adjustments were made for friction losses on banks.

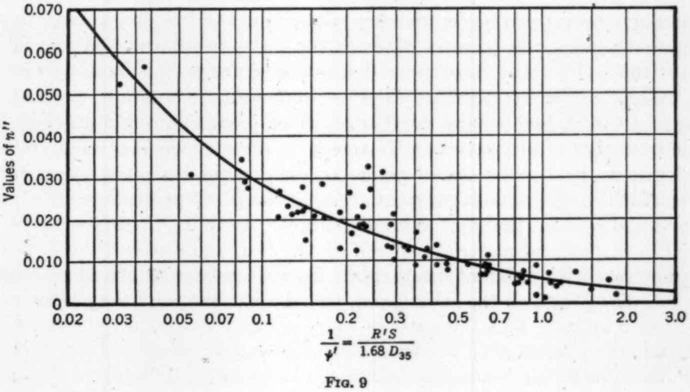

FIG. 9

The results are indicated in Fig. 9, with n'' as the ordinate and $1/\psi'$ as the abscissa, indicating the increase of sediment transport from left to right in the same manner as that proposed by Carl B. Brown,[22] M. ASCE. In Fig. 9 the roughness factor that reflects the channel irregularities n'' decreases with increasing flow and has a minimum value approaching zero.

The procedure for a practical application could be the following: Compute n' from Eq. 28; assume R'; determine $1/\psi'$ from Eq. 17; read off n'' from Fig. 9; and calculate n from Eq. 27, R from Eq. 30, and V from Eq. 3. For repeated use, it is advisable to draw a curve with n as ordinate and R as abscissa.

Acknowledgment.—The co-operation of Mr. Einstein in providing the flow data used in constructing Fig. 3, and of H. K. Pratt, M. ASCE (Chief, Reports and Hydraulics Branch of Garrison District of the Corps of Engineers), for suggestions and for permitting the use of the data for the Missouri River, near Williston, is gratefully acknowledged.

Sir Claude Inglis[23] M. ASCE.—The physical approach to the subject of channel roughness and its relationship to sand transport is sound as long as the authors restrict it to two-dimensional flow; but it is an over simplification to assume that all energy losses except those due to grain roughness were caused by shear force resulting from irregularities of the periphery. In a laboratory investigation of energy losses in one-way (river) and two-way (estuary) flow, at the Hydraulics Research Station of the Department of Scientific and Industrial Research at Wallingford, Berkshire, England, the energy losses have been divided into three categories: (1) Those due to textural roughness; (2) those due to ripple roughness; and (3) those caused by form drag resulting from the major irregularities of the banks, bed, islands, and sandbanks—all of which induce curvature of flow and tend to produce meanders. The authors cannot justifiably group these losses with irregularities of the bed due to sand ripples, bars, and sand dunes because, whereas the bed pattern and bed movements adjust themselves fairly rapidly as the discharge varies, this does not occur in the case of major irregularities such as promontories, outcrops, or gradually changing banks, islands, and sandbanks, which have an entirely different time scale from that of bed pattern adjustment by the movement of sand grains.

Next, as regards the size of grains exposed on the bed—the writer cannot agree that the size remains constant and that the 35:65 percentage distribution of material remains the same at all discharges. In India, the writer found material in suspension, material exposed on the bed, and also bed levels to vary considerably with different discharges. To collect samples of bed material at various discharges, on the other hand, would make the work very laborious.

It also appears that the authors have assumed not merely the same drop between the upstream and downstream observation points but also the same uniform slope for all discharges. Actually, however (as the authors themselves state), the curvature of flow and, hence, the distance the water travels, decreases as the discharge increases. In other words, the slope increases; and in many rivers the slope is far from uniform, at lower stages consisting of a series of pools and rapids.

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[22] "Engineering Hydraulics," edited by H. Rouse, John Willey & Sons, Inc., New York, N. Y., 1950, p. 796.

[23] Director, Hydr. Research Station, Dept. of Scientific Industrial Research, Wallingford, Berkshire, England.

Rather than a rational approach, the writer considers that linking the flow with a grain size which is assumed to be unaffected by discharge, as advanced in this paper, is extreme empiricism, in which variables are assumed to be constants.

The next question concerns the degree of accuracy attained. E. S. Crump (senior hydraulic engineer on the writer's staff) has analyzed the data. Beginning with the first item of Table 1 (Missouri River, Pierre, S. Dak.), he found by trial and error the results shown in Table 5. The labor involved in obtaining these results was so great that Mr. Crump adopted a simpler method for obtaining the probable error, which had the added advantage that it represented average performance of the several channels in the same manner as does the standard line in Fig. 3 which the authors recommend for solving specific problems. Disregarding results for the Salinas River at Paso Robles, Calif., the scatter of results has a standard deviation of ± 20% (very approximately) in the ordinate $\phi'' = V/V''_f = V/\sqrt{g\,S\,R''}$ for a given value of the abscissa $\psi' = \frac{(1.68\,D_{35})}{(S\,R')}$ of Fig. 3. For a given site where the constants K_s, D_{35}, and S are known, and the values of A and R for various values of the gage y have been established by survey, the authors' method of calibrating the gage for discharge $Q = V\,A$ is to assume successive values of the component R'. For each value assumed, the velocity V is determined from Eq. 9; and for the known value of $\psi' = \text{constant} \div R'$, the corresponding value of $R'' = \frac{V^2}{g\,S}\left(\frac{1}{\phi''}\right)^2$ is found by reading the value of ϕ'' from the curve of Fig. 3. Calling R''_E the value so obtained, it is seen that with a standard deviation in ϕ'' of ± 20% values of R'' may range from $R''_{0.8} = \left(\frac{1}{0.80}\right)^2 R''_E = 1.562\,R''_E$ to $R''_{1.2} = \left(\frac{1}{1.2}\right)^2 R''_E = 0.694\,R''_E$. The effect of this range of error on $R = R' + R''$ obviously depends on the ratio of R'' to R or R'.

TABLE 5.—ANALYSIS OF MISSOURI RIVER DATA (PIERRE, S. DAK.; TABLE 1) BY E. S. CRUMP

Stage	R'		R''		$R' + R''$		R''/R'		Q		$\frac{Q_c}{Q_m}$	Error
	Computed	Measured	Computed	Measured	Computed	Measured	Computed	Measured	Computed, Q_c	Measured, Q_m	(%)	(%)
Low....	0.300	0.880	2.960	2.380	3.26	3.26	9.87	2.70	2,616	5,200	50	−50
High....	5.103	6.730	3.687	2.060	8.79	8.79	0.72	0.31	128,700	145,000	89	−11

Following this method Mr. Crump found that the average value of R''/R was 5.80 for low discharges, 2.00 for intermediate discharges, and 0.66 for high discharges; and, accepting the foregoing average values of R''/R as typical, he found that the effect on $R = R' + R''$ of standard error in ϕ'' were as shown in

Table 6. The effect of these standard deviations on values of R is shown by the ratios in Cols. 8 and 9. The difficulty of expressing the effect in terms of relative discharges will be apparent from Fig. 10 which indicates the relationship between V and R for three values of ϕ''/ϕ''_E—namely, 0.8, 1.0, and 1.2, respectively. The curves are drawn for the case of medium flow, for which the abscissas of such points as C, B, and A have a constant ratio of 1.375:1.000: 0.796. Being based on a common value of the component R', all such points have the same value of V and therefore lie on a horizontal line. It is required to know the ratio of the ordinates of points D, B, and E which lie on the same vertical and therefore have the same value of R, y (gage reading) and A (sectional area) so that the ratio of the discharges $Q_D:Q_B:Q_E$ is the same as the ratio

TABLE 6.—ANALYSIS OF THE EFFECT OF STANDARD ERROR IN ϕ''

Stage of flow (1)	$\frac{R''_E}{R'}$ (2)	$\frac{R_E}{R'}$ (3)	$\frac{R''_{0.8}}{R''_E}$ (4)	$\frac{R''_{1.2}}{R''_E}$ (5)	$\frac{R_{0.8}}{R'}$ (6)	$\frac{R_{1.2}}{R'}$ (7)	$\frac{R_{0.8}}{R_E}$ (8)	$\frac{R_{1.2}}{R_E}$ (9)
Low...........	5.80	6.80	1.562	0.694	10.062	5.025	1.480	0.739
Medium.......	2.00	3.00	1.562	0.694	4.124	2.388	1.375	0.796
High..........	0.658	1.658	1.562	0.694	2.028	1.456	1.223	0.877

Explanation:

In the headings, subscript E denotes values given by the standard curve, Fig. 3, and subscripts 0.8 and 1.2 denote values that are, respectively, 20% less than, and 20% greater than, those given in Fig. 3.

Col. 3 = 1.0 + Col. 2
Col. 6 = 1.0 + Col. 2 × Col. 4
Col. 7 = 1.0 + Col. 2 × Col. 5
Col. 8 = Col. 6 ÷ Col. 3
Col. 9 = Col. 7 ÷ Col. 3

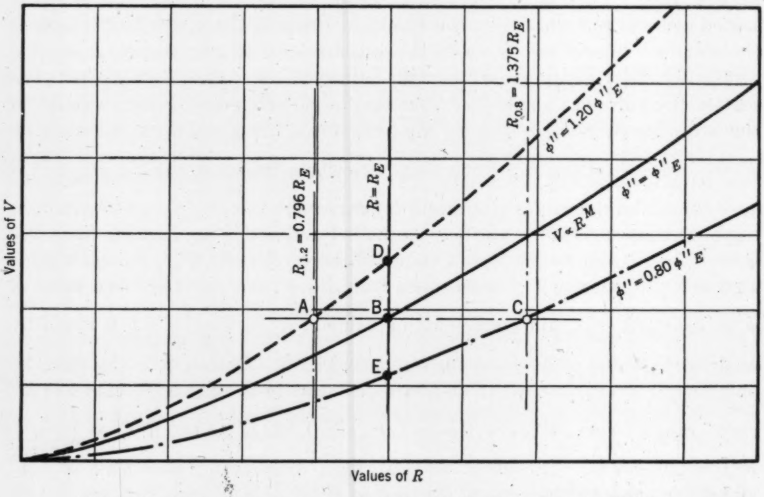

FIG. 10.—AVERAGE RELATIONSHIP BETWEEN V AND R FOR AUTHORS' DATA

of the velocities $V_D : V_B : V_E$. To know these ratios it is necessary to know the value of the index m in the assumed relationship $V \alpha R^m$. Fortunately, in Table 1 the authors have given two values of V and R in seven of the nine sites taken into consideration. With $\frac{V_1}{V_2} = \left(\frac{R_1}{R_2}\right)^m$ the value of m is given by $m = \log \frac{V_1}{V_2} \div \log \frac{R_1}{R_2}$. Similarly, if the ratio of the roughness factors, $\frac{N_2}{N_1} = \left(\frac{R_1}{R_2}\right)^n$, the value of the index n is given by $n = \log \frac{N_2}{N_1} \div \log \frac{R_1}{R_2}$.

TABLE 7.—COMPUTATION OF m AND n FROM DATA IN TABLE 1

Legend Fig. 3	R_1	R_2	V_1	V_2	N_1	N_2	$\frac{R_1}{R_2}$	$\frac{V_1}{V_2}$	$\frac{N_2}{N_1}$	m	n
ϕ	8.79	3.26	5.50	1.64	0.015	0.026	2.698	3.352	1.736	1.218	0.556
0	11.13	4.53	6.12	1.11	0.014	0.045	2.460	5.510	3.216	1.895	1.298
Δ	6.62	1.52	7.67	1.32	0.016	0.034	4.355	5.810	2.126	1.160	0.512
+	13.40	3.64	3.46	1.14	0.033	0.042	3.680	3.034	1.273	0.852	0.185
×	5.54	1.68	9.14	1.64	0.014	0.034	3.298	5.575	2.434	1.440	0.740
∇	3.18	0.69	6.67	0.61	0.016	0.065	4.610	10.940	4.060	1.566	0.918
□	4.59	1.27	7.72	0.84	0.019	0.076	3.616	9.190	4.000	1.724	1.079
Average	....	....	....	....	....	....	....	....	....	1.408	0.755

Computed values of m and n for the seven cases considered are given in Table 7. For the authors' results, Table 7, these computations establish the average relationship,

$$V \alpha R^{1.41} S^{0.5} \quad \text{(31)}$$

Comparison with Manning's formula—

$$V \alpha \frac{R^{0.67}}{N} S^{0.5} \quad \text{(32)}$$

—requires that

$$\frac{R^{0.67}}{N} \alpha R^{1.41} \quad \text{(33a)}$$

or

$$\frac{1}{N} \alpha R^{0.74} \quad \text{(33b)}$$

Eq. 33*b* yields $n = 0.74$ as against $n = 0.755$ found by analysis.

Reverting to Fig. 10, with $V \alpha R^{1.41}$, it is now possible to find the ratio of the velocity ordinates (which is also the ratio of relative discharges) of the three points E, D, and B having the same sectional area. In moving along the lower curve from points C to B (Fig. 10) the abscissa R is reduced in the ratio 1.375:1, so that the ordinate V is reduced in the ratio $(1.375)^{1.41}$:1—that is, in the ratio 1:0.639. This is also the ratio of the discharges, $Q_{0.8}:Q_{1.0}$, the percentage error in discharge being − 36.1%. Similarly in moving from point A to point D along the upper curve, the ordinate V is increased in the ratio $(0.796)^{1.41}$:1—that is, in the ratio 1:1.381; so that $Q_{1.2}/Q_{1.0} = 1.381$ giving an error of + 38.1%. This gives the result for medium stage flow. Results for the three stages of flow are shown in Table 8.

In the foregoing analysis, Mr. Crump has shown that the errors resulting from following the authors' method are far in excess of the ± 10% error to which experienced river engineers in India were able to work in assessing the discharges of selected short, straight reaches, and the claim (see sentence preceding "Acknowledgments") that "The obvious advantage of this method is that the results do not depend upon individual judgment for selection of the appropriate *n* value"—is shown to be unjustified; because the errors due to the authors' assumptions far exceed the errors to be anticipated in assessing the appropriate value for *n* in a selected reach.

TABLE 8.—COMPARISON OF ERRORS IN COMPUTED VALUES OF Q, FROM FIG. 10

Stage of flow	$\phi'' = 0.80\,\phi''_E$			$\phi'' = 1.20\,\phi''_E$		
	$\frac{R_{0.8}}{R_E}$	$\frac{Q_{0.8}}{Q_{1.0}} = \left(\frac{R_E}{R_{0.8}}\right)^{1.41}$	Percentage error in Q	$\frac{R_{1.2}}{R_E}$	$\frac{Q_{1.2}}{Q_{1.0}} = \left(\frac{R_E}{R_{1.2}}\right)^{1.41}$	Percentage error in Q
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Low........	1.480	0.575	−42.5	0.739	1.532	+53.2
Medium.....	1.375	0.639	−36.1	0.796	1.381	+38.1
High........	1.223	0.753	−24.7	0.877	1.203	+20.3

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TRANSACTIONS

Paper No. 2529

EXPERIMENTAL INVESTIGATION OF FIRE MONITORS AND NOZZLES

By Hunter Rouse,[1] M. ASCE, J. W. Howe,[2] M. ASCE, and D. E. Metzler[3]

With Discussion by Messrs. G. Halbronn; Pierre Oguey, Marcel Mamin, and François Baatard; John H. Arnold; and Hunter Rouse, J. W. Howe, and D. E. Metzler

Synopsis

Since the inefficiency of fire streams is directly attributable to the initial turbulence of the free jet, attention was given at the Iowa Institute of Hydraulic Research to the design of a fixed monitor, a portable monitor, and a nozzle that will reduce the turbulence of the flow to a practicable minimum. The underlying principles upon which the designs were based are discussed in this paper, the test facilities are described, the efficiencies of existing and improved units are compared, and design details and performance curves are presented for the recommended forms.

Introduction

In 1888 John R. Freeman published a prize-winning paper on fire streams[4] that is still the most widely accepted treatise on this subject, despite great advances in nearly every other phase of hydraulics. Lack of further progress in this particular field has been due in part to the excellence of Mr. Freeman's experiments and in part to apparent—but fallacious—corollaries of perfectly sound experimental findings.

The most frequently quoted of Mr. Freeman's results are his coefficients of nozzle discharge, determined with a precision that would be acceptable even under present-day standards. For well-faired nozzles yielding a negligible contraction of the free jet, he found the coefficient C in the customary discharge equation to be within less than 2% of unity. Repeated efforts during the intervening years to improve flow conditions by changing the boundary curvature,

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Note.—Published in October, 1951, as *Proceedings-Separate No. 92.* Positions and titles given are those in effect when the paper or discussion was received for publication.

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[4] "Experiments Relating to Hydraulics of Fire Streams," by John R. Freeman, *Transactions*, ASCE, Vol. 21, 1889, p. 303.

lengthening the taper, or polishing the inner surface have resulted in no significant increase in the discharge coefficient.

Unfortunately, two related (and equally false) presumptions have been the outgrowth of this situation. First, the most efficient nozzle is generally considered to be that for which the discharge coefficient is nearest unity—despite the facts that a contracting jet is known to yield a somewhat higher efficiency, even though the coefficient C is then well below unity, and an expanding jet yields a considerably lower efficiency, even though the value of C may then be greater than unity. Second, since nozzles that are now considered the best have coefficients close to unity, their high efficiency as conduit outlets is taken as an indication of an equally high efficiency as fire-stream producers—despite the fact that both good and bad jets are known to result from nozzles having identical values of C.

A stream from any standard nozzle is, needless to say, a notoriously poor counterpart of the ideal jet that has its parabolic trajectory based upon the physics of free fall, particularly in a cross or adverse wind. Discrepancies between theory and practice are attributed almost entirely to the unavoidable drag of the air upon the water that has left the nozzle. Indeed, it was not until 1943, when it was discovered that lengthening the barrel of a fire-boat monitor increased the carry of the stream some 30%, that attention was finally focused upon the true source of fire-stream inefficiency: The turbulence of the flow.

Since the investment of the United States Coast Guard for fire-fighting equipment during World War II involved a very considerable sum, it was decided before the end of hostilities to invest as well in a study of possible improvements in monitor and nozzle design. The Iowa Institute of Hydraulic Research (hereinafter called "the Institute") was commissioned in 1944, as one phase of its contract with the Navy Bureau of Ships, to conduct an extensive series of investigations of both fixed and portable monitors and nozzles, culminating in complete performance characteristics of the recommended designs.

UNDERLYING PRINCIPLES OF FLOW

Basic Flow Equations.—For a clear understanding of fire-monitor performance, reference must be made to the following elementary principles of flow, the development of which is to be found in most textbooks.[5] The simple equation of continuity,

$$Q = A_1 V_1 = A_2 V_2 \qquad (1)$$

states that the rate of flow Q, equal to the product of the cross-sectional area A of a conduit and the mean velocity V at that section, must be the same at all successive sections. The equally simple equation of energy (known as the Bernoulli theorem),

$$H_1 = H_2 + H_L \qquad (2)$$

states that the total head H_1 at one section is equal to the total head H_2 at the next section plus the intervening losses H_L. The total head H at any section consists of velocity head $V^2/2\,g$ and piezometric head h

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[5] "Elementary Mechanics of Fluids," by Hunter Rouse, John Wiley & Sons, Inc., New York, N. Y., 1946.

$$H = \frac{V^2}{2\,g} + h \qquad (3)$$

in which g is the acceleration of gravity. More specifically, the total head H represents the height to which water will rise in an open manometer column connected to the tip opening of a Pitot tube, the piezometric head h the height in a column connected to the side opening of a Pitot tube (or to a piezometer in the pipe wall), and the velocity head $V^2/2\,g$ the difference between these two. The piezometric head is in turn equal to the sum of the pressure head p/γ and the elevation z:

$$h = \frac{p}{\gamma} + z \qquad (4)$$

in which p is the pressure intensity relative to that of the atmosphere (as read on a Bourdon gage at the level z of the point in question) and γ is the unit weight of the water. The datum above which z is measured is arbitrary but must be the same for all sections. Likewise the same consistent units of measure (such as feet, seconds, and pounds) must be used to express all quantities involved.

The loss in head H_L between any two neighboring sections may invariably be expressed in terms of a coefficient C_L and a velocity head as follows:

$$H_L = C_L \frac{V^2}{2\,g} \qquad (5)$$

The magnitude of the coefficient C_L at moderate to high velocities depends almost entirely upon the geometrical form of the intervening conduit that produces the loss, and hence may be considered a constant for a given conduit form, such as a fixed contraction ratio or fixed proportions of an elbow, regardless of the conduit dimensions. It follows at once that the loss H_L through a given type of fitting under a given discharge may be reduced by increasing the size of the fitting and thereby reducing $V^2/2\,g$ in accordance with Eq. 1 or by streamlining the fitting to reduce C_L in Eq. 5 as far as possible toward its ideal limit of zero.

In a free jet the pressure intensity is the same as that of the atmosphere ($p = 0$), so the Bernoulli equation becomes

$$\frac{(V_1)^2}{2\,g} + z_1 = \frac{(V_2)^2}{2\,g} + z_2 + H_L \qquad (6)$$

If the air is assumed to have no effect upon the jet, the loss H_L will equal zero, and application of the physical laws of free fall,[5]

$$x = V_1 \cos \alpha\, t \quad \text{and} \quad z = V_1 \sin \alpha\, t - \tfrac{1}{2} g\, t^2 \qquad (7)$$

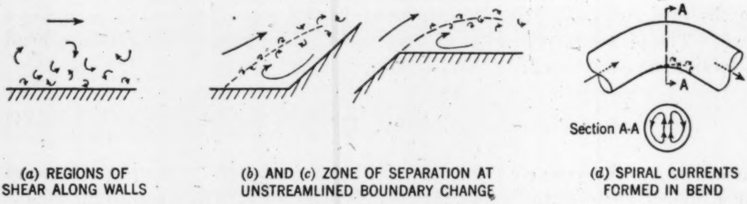
in which t is the time of fall, will yield for the nozzle inclination α and the efflux velocity V_1, a parabolic trajectory having the maximum height

$$z_{\max} = \frac{(V_1)^2}{2\,g} \sin^2 \alpha \qquad (8)$$

and the horizontal range at nozzle level

$$x_{\max} = 2\frac{(V_1)^2}{2g}\sin 2\alpha \qquad (9)$$

Since, as will be shown in the following paragraphs, C_L for a free jet depends upon far more than simply the efflux velocity, the actual losses and hence the true form of the jet cannot be expressed in any simple manner.



(*a*) REGIONS OF SHEAR ALONG WALLS (*b*) AND (*c*) ZONE OF SEPARATION AT UNSTREAMLINED BOUNDARY CHANGE (*d*) SPIRAL CURRENTS FORMED IN BEND

Fig. 1.—Sources of Turbulent Eddies

Effects of Turbulence.—Fluid turbulence[6] is generally defined as a haphazard secondary motion superposed on the mean translatory motion of a flowing fluid. It may thus be visualized as countless eddies of different shapes, sizes, orientations, and rotational speeds that are carried along with the flow. The sources of such eddies (see Fig. 1) are of several types: (1) They develop on a very small scale in regions of high shear along the walls of even very smooth conduits; (2) they are produced on a larger scale in zones of separation at boundary changes that are not fully streamlined; and (3) they result from major spiral currents formed in any conduit transition, such as a bend, that is not symmetrical about a linear axis. Practically all major losses of head, in fact, may be attributed to such eddy formation, and the magnitude of C_L for any portion of a conduit is a relative indication of the turbulence that will exist in the flow just downstream.

Turbulence in general is looked upon as an unwanted evil because of the power loss that it entails. In the case of fire streams, however, the loss of power accompanying the formation of turbulence within the monitor and nozzle is of small consequence compared with the subsequent loss that results from the presence of turbulence in the free jet. The latter is due to the fact that the secondary motion within each eddy of turbulent flow has transverse velocity components that, although usually small in comparison with the mean velocity, are nonetheless of finite magnitude. Within a conduit such eddies are laterally confined, as is the flow itself, by the conduit walls. A free jet is not confined, however, except by the rather slight effect of surface tension. Therefore, any lateral motion of the fluid due to an eddy will continue practically unabated past the original jet surface, with the result that only a short distance beyond the efflux section a turbulent jet will display a multitude of irregular surface eruptions.

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[6] "The Mechanics of Turbulent Flow," by Boris A. Bakhmeteff, Princeton University Press, Princeton, N. J., 1936.

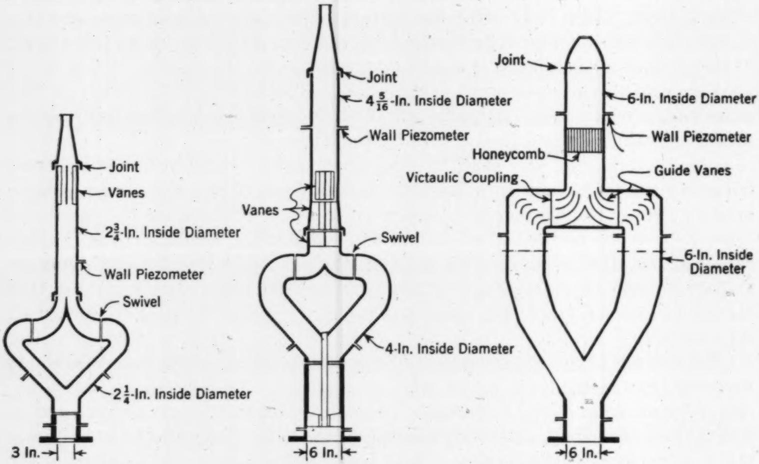
These liquid protuberances, readily visible in high-speed photographs (see Fig. 2), are not produced by the action of the air, for they would appear just as readily in a vacuum. In fact, the density of the air is so low (1/800 that of water) that its effect upon a smooth jet would be quite small in comparison to other factors. It is only after the surface of the jet has become sufficiently disrupted to produce an appreciable form resistance that the action of the air begins. Such resistance is roughly proportional to the square of the water velocity and to the cross-sectional area of the expanding jet. Thus, as ever more eddies carry water laterally out of the central stream, they are rapidly retarded in their longitudinal course by the surrounding air, but they continue to spread laterally at only slightly diminished speed. Although the outermost fringes of the jet at once form droplets that fall as a spray, the central portion appears simply to disintegrate in midair. In other words, since the equal and opposite reaction to the retardation of the water is the acceleration of the surrounding air, the originally intact but turbulent jet is transformed along its trajectory into an expanding mixture of dispersed water drops and air that travels at an ever-decreasing speed but brings an ever-increasing volume of air into motion.

Turbulence of the water emerging from a fire nozzle is hence seen to embody three distinct detriments to its efficiency as a fire-fighting tool. First, the original formation of the turbulence in monitor and nozzle represents a reduction in flow efficiency. Second, because of the lateral spread of the eddies in the free jet, air resistance reduces both the concentration and the carry of the stream. Third, entrainment of the surrounding atmosphere by the diffusing jet produces a powerful flow of air directly toward the fire.

Improvement of Fire Streams.—From the foregoing discussion it can be concluded that the only way to increase the efficiency of a fire stream is to decrease the turbulence of the flow before it leaves the nozzle. Although a wholly laminar (that is, nonturbulent) jet would be ideal, this is unfortunately out of the question for water jets having diameters and velocities of practical interest. Laminar flow seldom occurs at a Reynolds number greater than 2,000. With a water viscosity $\nu = 10^{-5}$ sq ft per sec, for example, flow in a 1-in. hose

FIG. 2.—HIGH-SPEED PHOTOGRAPH OF TURBULENT JET, 1/20,000-SECOND EXPOSURE

could not exceed a velocity of ¼ ft per sec without becoming turbulent. The fact remains, however, that the turbulence of a jet may be controlled to a considerable degree. Since the control of turbulence has not been the specific goal of monitor and nozzle designers in the past, it stands to reason that the incorporation of the principles of fluid mechanics in such design should effect a marked gain in fire-stream efficiency.


Joint
Vanes
2¾-In. Inside Diameter
Wall Piezometer
Swivel
2¼-In. Inside Diameter
3 In.
Joint
4 5/16-In. Inside Diameter
Wall Piezometer
Vanes
Swivel
4-In. Inside Diameter
6 In.
Joint
6-In. Inside Diameter
Wall Piezometer
Honeycomb
Victaulic Coupling
Guide Vanes
6-In. Inside Diameter
6 In.

(*a*) STANDARD 3-INCH U. S. COAST GUARD

(*b*) STANDARD 6-INCH U. S. COAST GUARD

(*c*) IOWA INSTITUTE OF HYDRAULIC RESEARCH

FIG. 3.—FIXED MONITORS

Wind-tunnel[7] and water-tunnel[8] requirements are closely allied with those of fire monitors and nozzles, for the flow at the test section of either type of tunnel must be of high velocity and yet as free from turbulence as possible. Nearly all such tunnels include: (*a*) Miter bends with curved vanes to guide the flow smoothly around every turn; (*b*) a very gradual expansion into a large approach section containing fine screens or honeycombs, in which the flow is finally allowed to "still" at low velocity; and (*c*) a well-formed nozzle, through which the fluid is accelerated continuously to the desired maximum speed.

In a fire monitor the passages should be free from cross-sectional discontinuities and should be of maximum dimensions commensurate with practicable over-all size—on the one hand to reduce the power loss and on the other hand to hold the energy of the turbulence to a minimum. The bends should not have the sweeping curves of the familiar rams-horn style, for these introduce secondary spiral currents (Fig. 1(*d*)) that persist far into the jet. Rather, miter bends with vanes should be used at all turns beyond the monitor base. In the approach section or barrel, a series of fine screens would be desirable to break remaining large-scale eddies into small ones that rapidly decay; however, such screens would be turbulence generators rather than dissipators as they became clogged with debris, and a coarse honeycomb is the only stilling device that should be considered.

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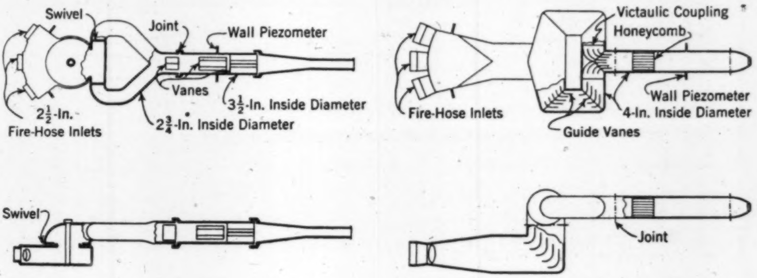
[7] "Experimental Methods—Wind Tunnels," by A. Toussaint, published in *Aerodynamic Theory*, Ed. by W. F. Durand, Julius Springer, Berlin, Germany, Vol. III, 1935, p. 252.

[8] "Cavitation and Pressure Distribution—Head Forms at Zero Angle of Yaw," by Hunter Rouse and John S. McNown, *Bulletin No. 32*, State Univ. of Iowa Studies in Engineering, Iowa City, Iowa, 1947.

The exact form of the nozzle is not critical so long as it does not in itself introduce turbulence. Standard nozzles are usually made with a 7° convergence, although it is only in diverging sections that such small angles are essential in the control of turbulence and separation. In fact, the shorter the nozzle (that is, the larger the angle of convergence), the lower the degree of turbulence generated along the walls. If the angle is too great, however, the zone of partial stagnation at the base (Fig. 1(*b*)) will shed eddies of its own production. Hence the nozzle should not be either too short or too long. Standard nozzles generally have cylindrical tips for final stability, frequently with abrupt angles between cone and cylinder (Fig. 1(*c*)) that again shed eddies and may conceivably result in cavitation (the formation of vapor pockets). Proper rounding in this zone will eliminate the larger eddies, but the length of boundary in contact with the flow again becomes excessive. As a matter of fact, a jet emerging from a conical outlet will contract and become parallel in a perfectly stable manner without boundary guidance, and hence the most efficient nozzle should have no cylindrical tip at all. A high polish of the inner surface should not be necessary, provided appreciable surface irregularities are not present, particularly near the outlet. The projection of gaskets into the flow passage (an evil all too frequently ignored) can wholly offset gains otherwise achieved, and hence should be avoided by proper mechanical design.

EXPERIMENTAL EQUIPMENT AND PROCEDURE

Monitors and Nozzles.—Supplied by the Coast Guard as a basis for test comparison were standard 3-in. and 6-in. fixed monitors (Fig. 3), and by the United States Navy a standard portable monitor (Fig. 4), each with a series of standard nozzles. The 3-in. monitor, originally proposed for improvement, had a 24-in. barrel with radial vanes, and could be directed manually through 360° horizontally and 70° vertically. Nozzles were of the usual 7° cone-

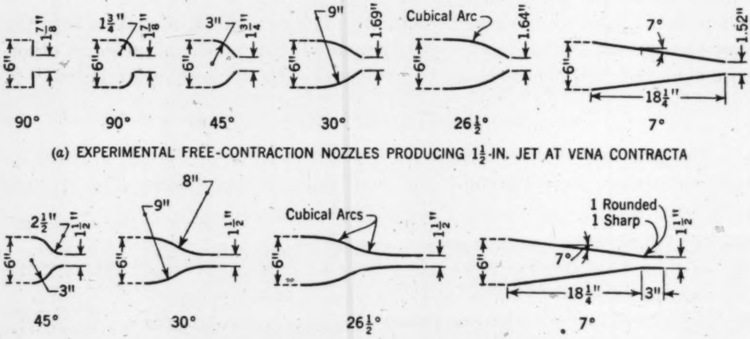


(*a*) U. S. NAVY (*b*) IOWA INSTITUTE OF HYDRAULIC RESEARCH

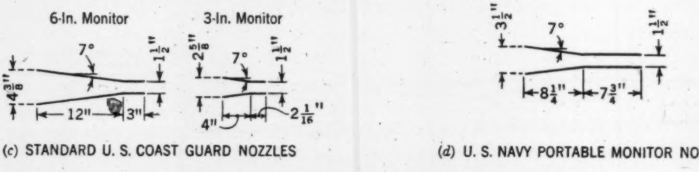
FIG. 4.—PORTABLE MONITORS

and-cylinder type with polished inner surfaces, abrupt boundary angles, and inwardly protruding gaskets, and ranged from $1\frac{1}{2}$ to $2\frac{1}{2}$ in. in outlet diameter. The 6-in. monitor shown in Fig. 3(*b*) had a 30-in. vaned barrel, and could be directed by worm drives through 360° horizontally and 120° vertically. The nozzles were quite similar to those of the smaller unit, and ranged from $1\frac{1}{2}$ in. to 3 in. in outlet diameter. The portable monitor shown in Fig. 4(*a*) had three hose inlets with check valves and a 15-in. barrel. It was designed for lashing in position but could be swiveled manually through 360° horizontally and 90° vertically. Nozzles ranged from $1\frac{1}{2}$ in. to 2 in. in outlet diameter and differed from the others primarily in greater length of cylinder.

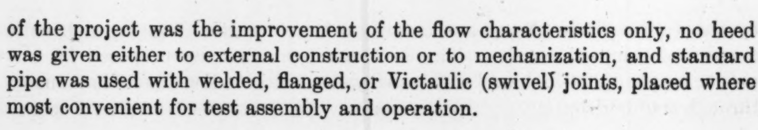
Only one fixed monitor and one portable monitor (also shown in Figs. 3 and 4) were built by the Institute as recommended forms. Other forms and sizes were not investigated, since significant variation in design is not permitted by the results to be accomplished and since essentially the same design may be adapted to different monitor sizes by the principle of geometric similarity. As indicated previously, the essential design criterion was the reduction of eddy losses and secondary currents to a practicable minimum in order to attain flow in the barrel with as low a degree of turbulence as possible. The flow passages were, therefore, enlarged and straightened, and the sweeping curves of the usual rams-horn were replaced with vaned miter bends. Since the purpose of the project was the improvement of the flow characteristics only, no heed was given either to external construction or to mechanization, and standard pipe was used with welded, flanged, or Victaulic (swivel) joints, placed where most convenient for test assembly and operation.



(*a*) EXPERIMENTAL FREE-CONTRACTION NOZZLES PRODUCING $1\frac{1}{2}$-IN. JET AT VENA CONTRACTA



(*b*) EXPERIMENTAL CONFINED-CONTRACTION NOZZLES



(*c*) STANDARD U. S. COAST GUARD NOZZLES (*d*) U. S. NAVY PORTABLE MONITOR NOZZLE

FIG. 5.—SCHEMATIC DRAWINGS OF $1\frac{1}{2}$-IN. NOZZLE FORMS

In order to verify the predicted criteria for nozzle design, a systematic series of contraction profiles, varying from a plate orifice to a long taper, both with and without cylindrical tips, were constructed of brass (see Fig. 5). The external form was again determined by expediency rather than ultimate use. In all cases, however, not only was the inner surface carefully machined, but a smooth, gasket-free, metal-to-metal joint was obtained between barrel and nozzle or nozzle parts. Supplementary nozzle forms included several with roughened surfaces and other internal obstructions, as well as one with a profile consisting of two cubical curves that electrical-analogy measurements had shown to yield a continuous pressure drop from base to tip. Each nozzle of this series was constructed and tested in the $1\frac{1}{2}$-in. size only, but the form finally recommended was repeated in diameters of $1\frac{1}{2}$, 2, $2\frac{1}{2}$, and 3 in. for the fixed monitor and $1\frac{1}{2}$, $1\frac{3}{4}$, and 2 in. for the portable monitor.

Pumping System.—Since the requirements of the monitor test program (maximum stream diameter of 3 in. and maximum base pressure of 200 lb per sq in.) were far in excess of the high-capacity, low-head pumps of the Institute laboratory, a battery of 8 standard Hale fire-boat pumps, driven by Chrysler gasoline engines, were supplied for the project by the Coast Guard. These pumps were housed in a temporary shelter at the south end of the laboratory (Fig. 6), and so arranged that they could either be connected in parallel to a 12-in. suction line leading from a sump in the 10-ft channel or drawn individually through standard 4-in. suction hoses from the 16-ft channel below. All pumps

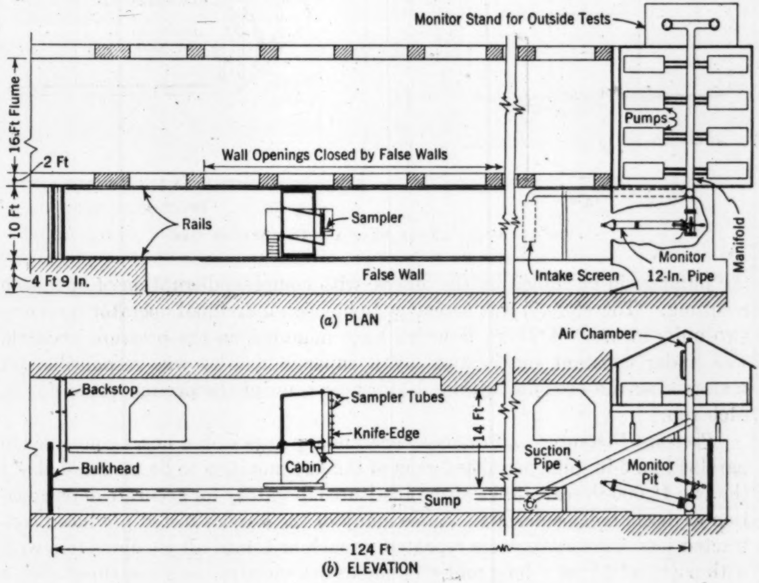

Monitor Stand for Outside Tests
16-Ft Flume
2 Ft
Wall Openings Closed by False Walls
Pumps
10 Ft
Rails
Sampler
Manifold
Monitor
4 Ft 9 In.
False Wall
Intake Screen
12-In. Pipe
(a) PLAN
Air Chamber
Backstop
Sampler Tubes
Knife-Edge
14 Ft
Bulkhead
Cabin
Suction Pipe
Monitor Pit
Sump
124 Ft
(b) ELEVATION

FIG. 6.—LAYOUT OF TEST GALLERY

discharged into a common 12-in. pressure line leading both to the 10-ft channel and to a platform on the outside of the shelter.

Unless each of the centrifugal pumps in a parallel system of this type operates at such a speed as to yield the same change in head, one or more of the pumps will be stalled by the others, with complete disruption of operation resulting. Therefore, it was necessary to have an individual operator for every two units in use. A 15-in. Bourdon gage mounted on the pressure manifold was under constant observation by a supervisor, who was responsible for maintaining the pressure within ±1 lb per sq in. of the desired value during each run.

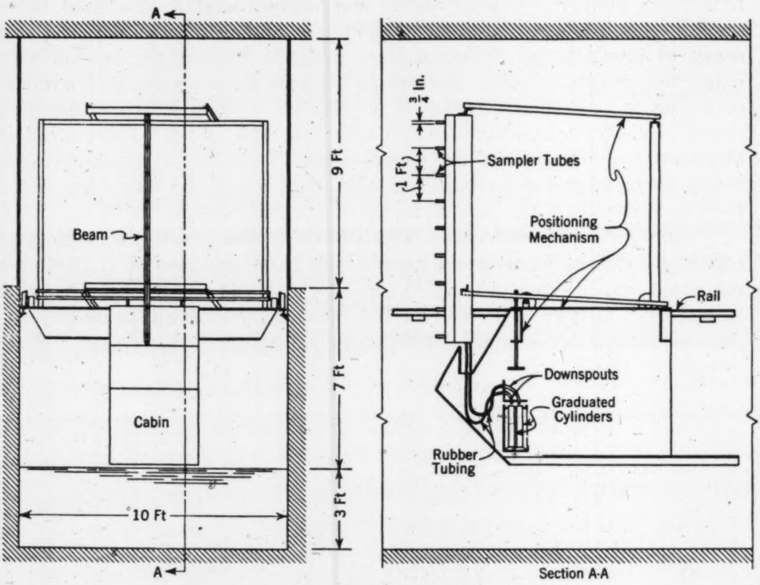


Fig. 7.—Sampler-Tube Beam and Observation Car

Pressure-Discharge Calibrations.—Secondary lines of fire hoses connected in parallel to a 6-in. pipe permitted each of the five monitors to be mounted above the two 18,000-lb scale tanks of the Institute laboratory for accurate determination of discharge coefficients. Since many of the nozzles tested produced contracted jets, the outlets were repeatedly machined until all jet diameters were within $\frac{1}{2}$% of $1\frac{1}{2}$ in. Jet profiles in the outlet vicinity were measured with a point gage that could be moved about 3 in. longitudinally and radially in any axial plane. A similar mounting permitted velocity traverses of the contracted jets to be made with a Pitot tube formed of a 0.05-in. hypodermic needle connected to a large Bourdon gage.

Wall piezometers were installed in each of the five monitors at key points (see Figs. 3 and 4), so that both the over-all losses and the relative losses at each transition could be determined. For comparative operation of the various monitors, the same pressure in the supply pipe at the monitor base was maintained in all cases.

Gallery Tests.—In order to compare both nozzle and monitor performance under laboratory conditions free from atmospheric disturbance, a test gallery was established in the 10-ft channel running under the Institute laboratory (Fig. 6). The ends of the channel were bulkheaded, and screens were installed along the sides, thus producing simultaneously a catch and storage basin for a 3-ft depth of clear water and an unobstructed test space 100 ft long, 10 ft wide, and 14 ft high. The south end of the gallery, containing the lower end of the discharge manifold from the pumps, was only partially screened, and the north end was fully vented to both the 16-ft channel on the one side and the 4-by-9-ft walkway on the other. Although this confined space could not be considered to simulate conditions in the open except as a first approximation, it nevertheless permitted operation of each monitor and nozzle under identical conditions, which would otherwise have been a practical impossibility.

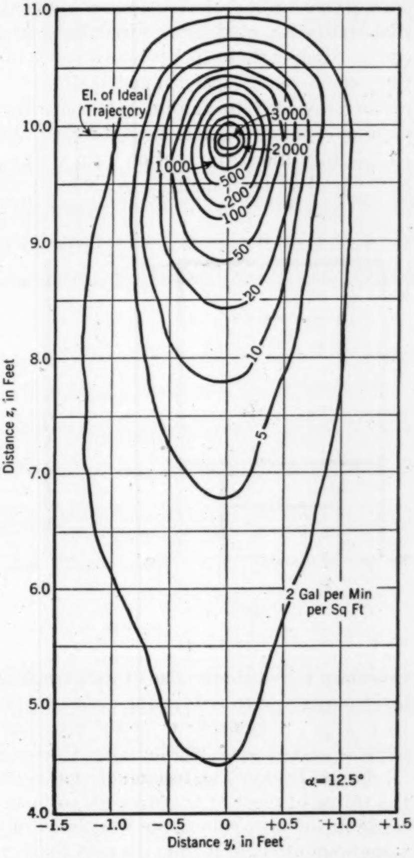


FIG. 8.—TYPICAL PLOT OF JET CONCENTRATIONS 90 FT FROM A 30° NOZZLE

Observations of the jets could be made in this gallery in three different ways: (1) Conditions of jet disruption could be studied by means of high-speed photography, a pair of flash lamps operating from a condenser permitting exposures of 1/20,000 sec; (2) the mean trajectory and jet thickness could be determined by means of a special profilometer operating by means of a horizontal beam of light; and (3) the discharge concentration of the jet at each and every point in any cross section up to 90 ft from the nozzle could be measured through use of a point sampler.

Since the sampler apparatus provided the basis for many performance ratings, it is described here in further detail. A small sampling car, with a cabin barely large enough for two observers, could be moved on the rails at either side of the channel to any section of the gallery. Mounted on the front of this car (see Fig. 7) was a vertical wedge-shaped beam that could be moved through 1 ft vertically and 4 ft either side of the centerline. From the leading edge of this beam projected a series of nine $\frac{3}{4}$-in. sampling tubes, 12 in. on center, that were connected by rubber tubing to a battery of downspouts within the cabin. Below these spouts was a series of graduated cylinders so framed that they could be abruptly alined with the downspouts, abruptly disalined, and rapidly inverted to empty. Concentration runs consisted of measuring the volume of inflow against time for each of the sampling tubes, the beam being shifted to section after section until a representative traverse (see Fig. 8) of the entire jet had been made. In zones of maximum concentration it was found necessary to reduce the size of the sampling tube to $\frac{3}{8}$ in. by means of a streamlined insert and to reduce the distance between samples both horizontally and vertically. A typical run at the common distance of 90 ft from the point of efflux involved some 900 individual measurements.

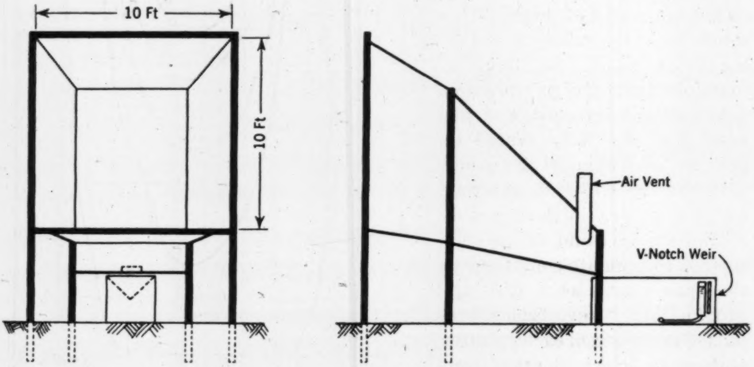


FIG. 9.—OUTDOOR TARGET

Target Tests.—The test gallery had a maximum length that was far less than the throw of most of the streams, so further comparisons of performance were made in the open, along the west bank of the adjoining river, through use of a target mounted 200 ft from the monitor stand. The target (Fig. 9) had an opening 10 ft by 10 ft in vertical section, and the rate of flow entering the opening was determined by means of a suitably baffled **V**-notch weir directly behind and shielded by the target. Owing to the pronounced effect of wind upon the streams, tests could be made only on relatively calm days. Each stream was so directed that the maximum quantity entered the target, the same base pressure of 150 lb per sq in. and the same jet diameter of $1\frac{1}{2}$ in. being used in all cases. Since slight disturbances were almost immediately reflected in the head on the weir, repeated observations were made at intervals of ½ min and the results were analyzed statistically to eliminate obviously erroneous values.

Trajectory Tests.—Initial comparison of the jet trajectories for the various monitors[9] was made by means of photographs of the nozzle streams taken from a point on the opposite side of the river, using a fixed-focus camera. The monitors were directed north at an inclination of 20°, with the laboratory building as a combined shield and background. This method could not be used for final performance tests, however, since at high pressures the spray from the streams interfered with traffic on the bridge 350 ft from the test stand. Moreover, at higher jet inclinations the stream could not be seen distinctly on the photographs against the background of sky above the laboratory.

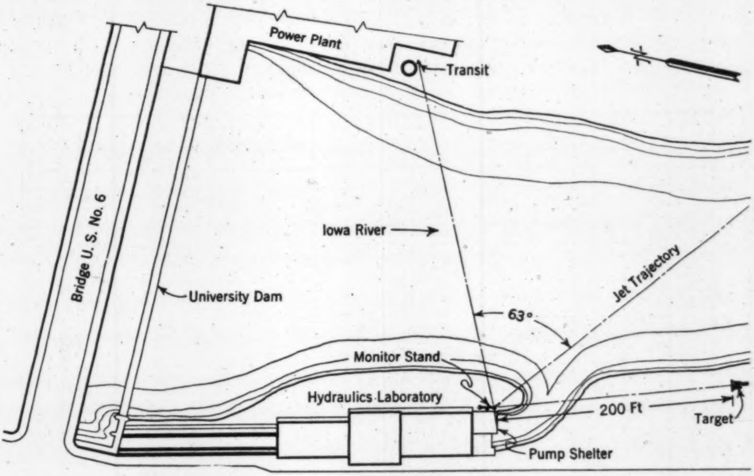


Fig. 10.—Plan of Outdoor Testing Area

The method of observation finally adopted involved the use of a surveyor's transit, on the telescope of which a nonmagnifying sight with cross wires and a large field of view had been mounted. The monitors were directed diagonally southeast across the river (see Fig. 10), and the absolute trajectories were calculated trigonometrically from readings of the horizontal and vertical circles of the transit and predetermined base-line characteristics. Measurements were carried as far along each trajectory as the stream retained a readily visible core. A very long period was required to complete these observations, because of the fact that satisfactory measurements could be made only under calm or nearly calm conditions. These conditions prevailed, unfortunately, only about 1% of the available time.

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[9] "Characteristics of High-Velocity Jets," by Joseph W. Howe and Chesley J. Posey, *Proceedings, Third Hydraulics Conference; Bulletin No. 31*, State Univ. of Iowa Studies in Engineering, Iowa City, Iowa, 1947, p. 315.

Discussion of Test Results

Relative Performance of Nozzles.—Gallery tests on each of the nozzle forms shown in Fig. 5 resulted in contour plots of discharge concentration similar to those of Fig. 8 By integration of the successive contour areas, each of these plots was reduced to a single characteristic curve of percentage of total nozzle discharge against cross-sectional area. A typical family of such curves, corresponding to the nozzle forms at the top of Fig. 5, can be seen in Fig. 11. The logarithmic abscissa scale is used to permit convenient study of each portion of the field. If the jet from the nozzle remained absolutely intact throughout the 90-ft distance to the sampler, the resulting curve would have the form indicated by the broken line labeled "intact jet" in Fig. 11. The extent to which the actual curve falls below this ideal limit is an indication of the diffusion that the jet has actually undergone. Unless a considerable amount of the spray from the jet has fallen into the reservoir before reaching the sampler, the curve should approach the 100% line as the cross-sectional area increases; this portion of the curve is otherwise of little interest, since the jet at this point is dispersed over such a large area that a great quantity of air travels with the water at fairly high speed. The initial portion of the curve, on the other hand, is of great significance, as it is a measure of the relative jet concentration at the axis.

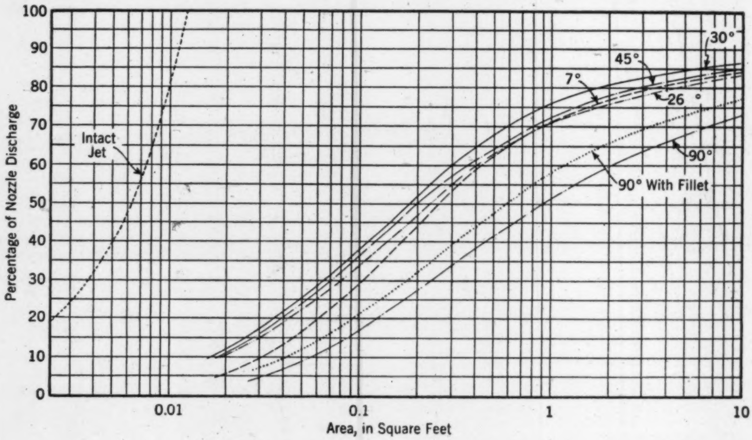

Fig. 11.—Comparative Area-Concentration Curves for Free-Contraction Nozzle Forms

With regard to the several curves shown in Fig. 11, it will be seen that the poorest performance (the lowest curve) corresponds to the 90° orifice plate. That this is due to disruption of the jet by eddies formed in the stagnation zone at the base of the plate is shown by the improvement in performance resulting from rounding the plate in this zone. Decreasing the angle of convergence is likewise seen to improve the performance, largely because of the corresponding reduction in eddy formation in the steadily diminishing stagnation zone. Eventually, however, this effect is not only minimized, but at the same time the increased length of boundary begins in turn to augment the surface turbulence, so that optimum performance is found at an intermediate angle considerably greater than the standard 7°.

A similar family of curves for the 7° nozzles showed graphically the relative effects of the three types of nozzle tip. The standard type with abrupt intersection between cone and cylinder was very nearly comparable in performance to the 90° plate with fillet. Careful rounding of the intersection produced a marked improvement, owing to the elimination of the eddies (and possibly vapor bubbles) that are formed at such a discontinuity. The maximum concentration was produced by the cone without any cylindrical tip whatever, since omission of the cylinder shortened the zone of boundary contact with the high-velocity flow. Velocity traverses just beyond the efflux section revealed a significant reduction in the thickness of the boundary layer from one case to the next. Moreover, jets emerging from nozzles without tips invariably tended to appear crystal clear in comparison with the milky whiteness of jets from cylindrical tips with abrupt discontinuities.

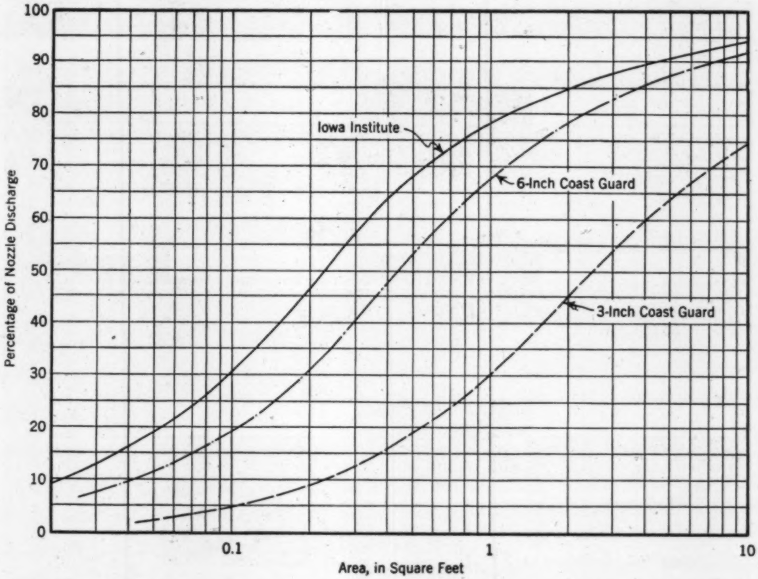

Fig. 12.—Comparative Area-Concentration Curves for Fixed Monitors

Supplementary tests with the nozzles included artificial production of the three types of eddy formation leading to jet diffusion. Vanes and baffles in the approach barrel simulated the effects of upstream bends or obstructions, resulting in large-scale disruption of the stream. Boundary irregularities comparable to protruding gaskets or surface accretions caused dispersion to a smaller but still serious degree. Finally, increased length of cylindrical tip produced a marked tendency of the stream to surround itself with a fine mist that appeared to protect the central stream to some extent from the surrounding air but rapidly fell below the mean trajectory. It is noteworthy that minute nicks at the very tip of a nozzle yielded perceptible furrows in the jet which gradually expanded into regions of appreciable dispersion. Although the nozzles constructed by the Institute were all given a sharp 90° edge at the end of the contraction, it was subsequently found that a very short (⅛- to $\frac{3}{16}$-in.) cylindrical tip could be used for contraction angles as low as 26.5° without affecting the free jet. Not only is such an outlet easier to fabricate, but it also serves to protect the critical portion from damage.

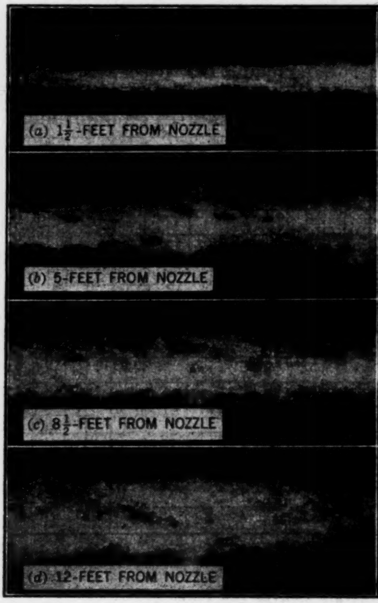

FIG. 13.—HIGH-SPEED PHOTOGRAPHS OF 1⅛-IN. JET FROM 3-IN. COAST GUARD MONITOR

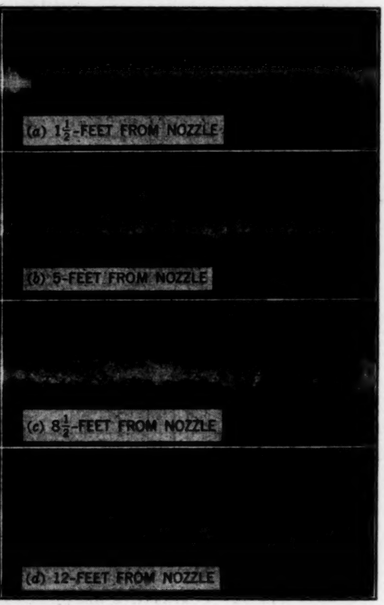

FIG. 14.—HIGH-SPEED PHOTOGRAPHS OF 1⅛-IN. JET FROM INSTITUTE FIXED MONITOR

The foregoing analysis of comparative tests on the nozzles themselves leads to the conclusion that, aside from the absence of boundary discontinuities, the best form of nozzle is one that results in a minimum length of contact between wall and fluid and yet produces a minimum eddy effect from the stagnation zone at the base. Although these factors tend in opposite directions, there is a considerable intermediate range of form that satisfies the requirements to an acceptable degree. The form chosen by the Institute was the 30° profile with well-rounded base and free contraction, shown in Fig. 5(a).

Relative Performance of Fixed and Portable Monitors.—Owing in part to the larger flow passages and in part to the reduced losses through the vaned bends, the flow efficiency of the Institute monitor with the recommended nozzle was appreciably higher than that of the 6-in. Coast Guard monitor (with a rounded rather than a standard nozzle) and considerably higher than that of the 3-in. Coast Guard monitor with a standard nozzle. If the rate of discharge is expressed in terms of the base pressure head h_b in the supply main and the area a of the contracted jet as

$$Q = C_d\, a \sqrt{2\, g\, h_b} \quad \ldots\ldots (10)$$

the discharge coefficient C_d for the three monitors with 1½-in. jets is as follows:

Monitor	C_d
3-in. Coast Guard	0.917
6-in. Coast Guard	0.955
Institute	0.982

In other words, the Institute monitor will discharge about 3% more water than the 6-in. Coast Guard monitor and 7% more than the 3-in. monitor at the same base pressure. Similarly, if the over-all loss in head H_L is expressed in terms of the jet velocity head as

$$H_L = C_L \frac{(V_j)^2}{2\, g} \quad \ldots\ldots (11)$$

the following values result for the three monitors with 1½-in. jets:

Monitor	C_L
3-in. Coast Guard	0.211
6-in. Coast Guard	0.104
Institute	0.049

Evidently, the power requirement of the Institute monitor is 5% below that of the 6-in. Coast Guard and 15% below that of the 3-in. one at the same discharge.

Gallery tests for jet concentration produced by the three monitors with 1½-in. jets operating at the same base pressure of 100 lb per sq in. resulted in the three curves plotted in Fig. 12. The performance of the 3-in. standard monitor is obviously quite inferior. Although the Institute monitor does not at first glance seem much superior to the larger Coast Guard monitor, it should be noted that the peak concentration, 90 ft from the nozzle, was actually 60% greater than that for the larger and some 14 times as great as that for the smaller one. High-speed photographs of the jets from the 3-in. Coast Guard and the Institute monitors over the first 14 ft of their course (Figs. 13 and 14) show graphically the comparative degrees of turbulence producing the diffusion.

Essentially the same relative performance was indicated by the target tests over a 200-ft range with 1½-in. jets at a base pressure of 150 lb per sq in. The 3-in. Coast Guard monitor fell short of this throw by some 15 ft and hence was omitted from further study. The Institute monitor showed an improvement of 26% over the 6-in. Coast Guard monitor, based on the proportion of discharged water striking the target, and an improvement of 32% based on the total amount of water caught in a given time while operating at the same base pressure.

Final comparisons involved the measured trajectories of streams from the three monitors with the same jet diameter of 1½ in., for base pressures of 50 and 150 lb per sq in., and at 20° and 50° inclinations. The results are shown in Fig. 15, together with the ideal trajectories for the discharge of the Institute monitor under the four conditions. The Institute monitor obviously leaves much to be desired when compared with the ideal. On the other hand, its throw is some 20% better than that of the larger Coast Guard monitor and 40% better than that of the smaller. The relative performance is essentially independent of base pressure and inclination.

Although the portable Navy monitor was comparable in size and weight to the 3-in. Coast Guard unit, it was generally superior in performance. As a result, the improvement obtained with the portable model constructed by the Institute was considerably less than that with the fixed model. The discharge coefficients of the two portable monitors were as follows:

Monitor	C_d
Navy portable	0.900
Institute portable	0.950

At the same base pressure, the Institute monitor will thus discharge about 6% more water than the Navy monitor (under the same conditions, the Navy portable will deliver 2% less than the small Coast Guard unit). The loss coefficients were likewise as follows:

Monitor	C_L
Navy portable	0.288
Institute portable	0.118

Hence, at the same discharge, the Navy portable monitor will require 17% more power than the Institute monitor (it will, on the other hand, require 7% more than the small Coast Guard unit). Gallery measurements, the results of which are plotted in Fig. 16, showed the Institute portable to produce an appreciably lower rate of jet dispersion, with a peak concentration about 2½ times that of the Navy portable monitor (which was, in turn, 4 times that of the small Coast Guard monitor). In the target tests, the Institute portable monitor showed improvements of 12% and 17% in proportion of discharged water and in total amount of water caught, respectively. Records of trajectory tests, reproduced in Fig. 17, indicate that the throw of the Institute model is about 15% greater than that of the Navy portable (the latter being some 10% greater than that of the small Coast Guard monitor). The performance of both portable units was essentially independent of the number of supply hoses used.

Mention should be made at this point of tests on honeycombs conducted with the larger Institute monitor. Since the passages of this monitor were all of the same diameter, the same honeycombs could be placed in either the barrel or the base riser. Two geometrically similar honeycombs, 4 in. and 6 in. long, were made of $\frac{1}{16}$-in. steel plate by slotting each set of plates over half

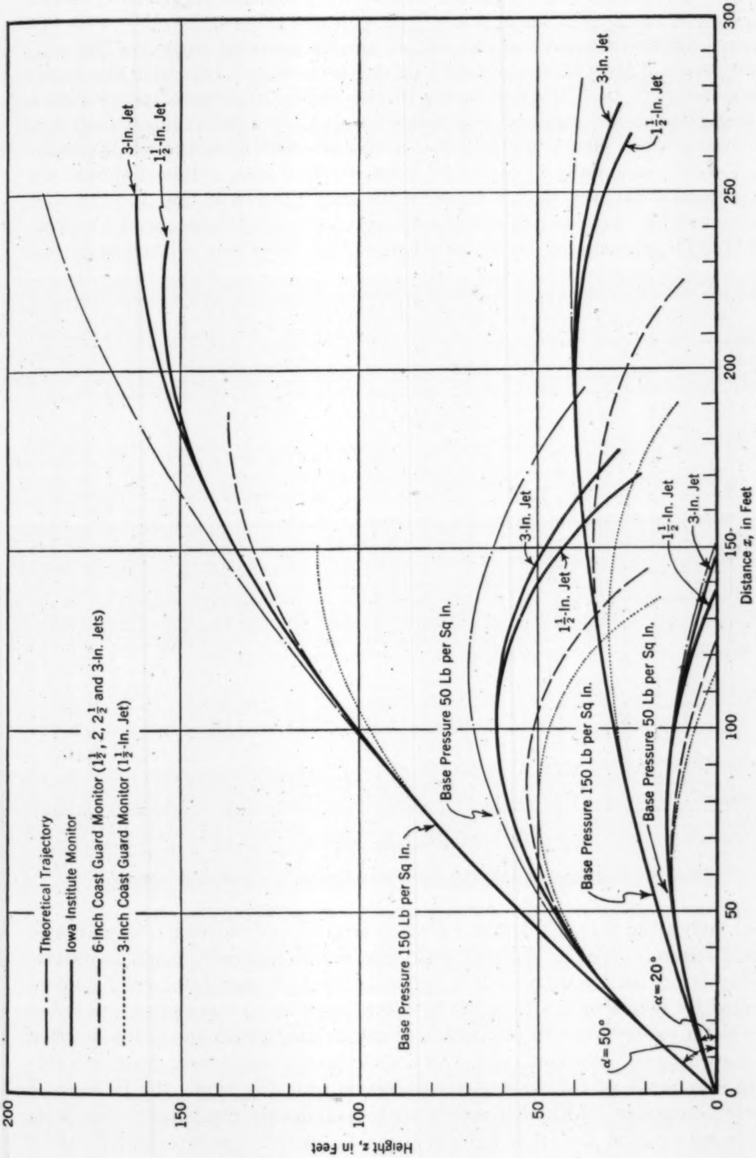

FIG. 15.—COMPARATIVE PERFORMANCE CURVES FOR FIXED MONITORS

their length and joining the respective sets in the familiar "egg-crate" fashion. The square passages of one unit were ½ in. wide and those of the other were ¾ in. wide. Anti-swirl devices of this nature can be expected to reduce the scale (eddy size) of the turbulence only to a degree commensurate with the size of the passages. As in the case of the turning vanes, finer openings than those investigated were considered impracticable, since even the slightest fouling of the coarse units with debris produced a marked effect upon the jet dispersion. As a result, essentially the same jet characteristics were obtained either with or without a honeycomb in a barrel of the length shown in Fig. 3. Although shortening the barrel to half this length made a honeycomb necessary, doubling the length produced no noticeable change whether or not a honeycomb was used, indicating that the flow was already as good as honeycombs of this scale would ensure. That a relatively poor flow, on the contrary, would be considerably improved by even such a coarse honeycomb was graphically demonstrated by reversing the Institute monitor—that is, by connecting the barrel end to the supply line and placing the nozzle directly on the end of the inverted **Y**-shaped base. Without a honeycomb the jet was extremely poor, whereas with a honeycomb a short distance upstream from the nozzle the jet became fairly acceptable. Although nearly all measurements described herein were made with one or the other honeycomb in the barrel, the subsequent studies just described showed this to be neither necessary nor generally desirable with a barrel of normal length.

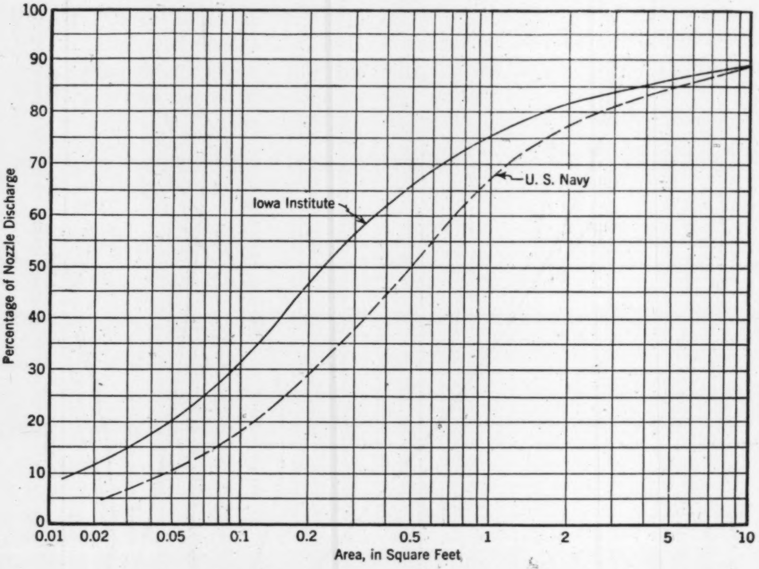

FIG. 16.—COMPARATIVE AREA-CONCENTRATION CURVES FOR PORTABLE MONITORS

Equally illuminating was another supplementary test in which a nozzle was mounted at one end of the largest stilling chamber available—a pressure tank 2½ ft in diameter and 6 ft long—in the hope of further reducing the turbulence. As is common practice in hydraulics, the discharge from the supply hoses at the upstream end was baffled by a 1-ft layer of ¾-in. crushed rock extending over the full cross section of the tank and held in place by wire mesh, thus leaving an additional 4 ft of unobstructed passage before the rounded entrance to the nozzle. The result was disappointingly poor, the jet being comparable to that from the small Coast Guard monitor. Later tests made by the Institute on the characteristics of flow through screens[10] pointed clearly to the source of the trouble: Unless the ratio of open to closed area is greater than unity, a baffle may do more harm than good. The crushed rock, in other words, very likely produced nearly as many disturbances as it eliminated, and the subsequent length of tank was not sufficient to damp the large-scale eddies remaining in the flow.

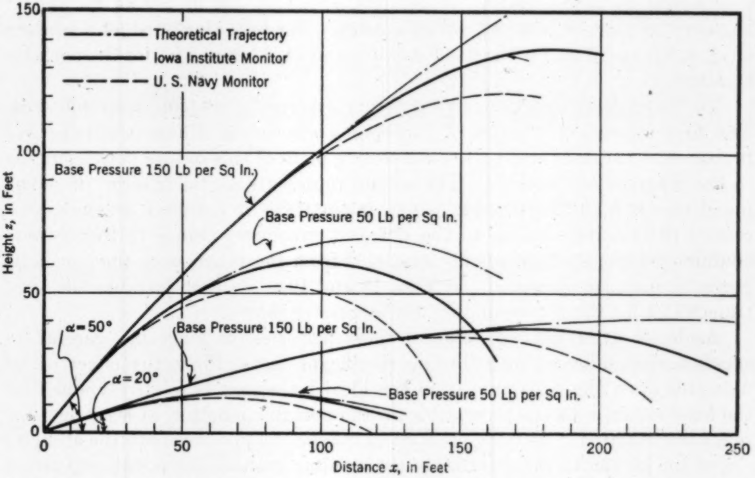

FIG. 17.—COMPARATIVE PERFORMANCE CURVES FOR PORTABLE MONITORS WITH 1½, 1¾, AND 2-IN. JETS

While the concentration and throw of a fire stream still can be vastly improved, it appears from the foregoing tests that this could be accomplished only at the expense of such other practical requirements of the monitor as lightness and maneuverability. If the approach passage were of the same relative diameter, length, and gradual taper as that of a stationary wind or water tunnel, the turbulence (and hence the rate of jet dispersion) could probably be reduced to a small percentage of that of the Institute monitors; such a system would, of course, be both prohibitively heavy and very difficult to mechanize. Although a shorter approach might be fully as effective if it were possible to make the turning vanes as small and numerous as in a wind tunnel and to include a series of fine stilling screens of low solidity ratio, an arrangement of this nature would be practicable only with very clean water. In view of these circumstances, the two monitors developed by the Institute were assumed to embody the optimum improvement possible without undue change in size or difficulty of manufacture. Hence, these units were used as the basis of the final performance tests.

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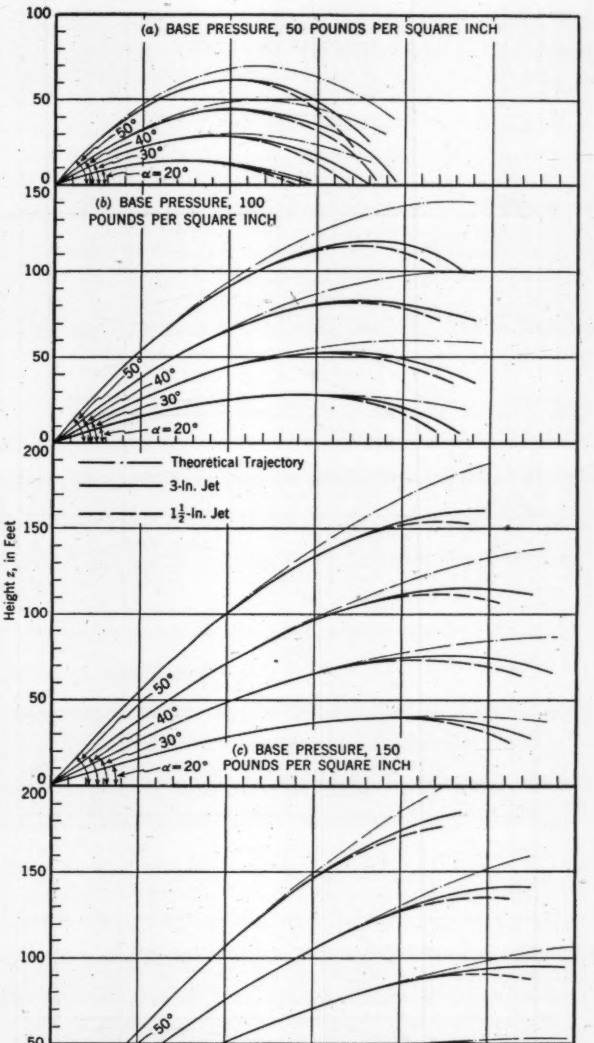
[10] "An Investigation of Flow Through Screens," by W. D. Baines and E. G. Peterson, *Transactions*, A.S.M.E., Vol. 73, July, 1951, p. 467.

Performance Tests of Recommended Monitors.—Since three independent variables—jet diameter, base pressure, and inclination—were involved in the final tests of the Institute monitors, the effect of any one variable upon the stream could be studied only if the other two were at the same time held constant. The series of tests, therefore, involved measurement of the stream coordinates at base pressures of 50, 100, 150, and 200 lb per sq in. and at angles of 20°, 30°, 40°, and 50° for all nozzles. The latter included jet diameters of 1½, 2, 2½, and 3 in. for the fixed monitor and 1½, 1¾, and 2 in. for the portable monitor.

To facilitate comparison, two different systems of plotting were followed. The first represents a series of superposed curves for different nozzles and pressures at constant inclination, successive plots of this nature corresponding to the different inclinations. The second represents a similar series of superposed curves for different nozzles and inclinations at constant pressure, successive plots corresponding to the different pressures. Since neither system contains information not readily available from the other, only the constant-pressure plots are presented (see Figs. 18 and 19). In each instance the ideal trajectories for the corresponding discharges are shown.

Aside from the general value of these performance diagrams, careful inspection will indicate the following significant facts: First, the effect of jet diameter upon the departure from the ideal trajectory is relatively small for the fixed monitor and imperceptible for the portable monitor, at least over the range investigated. Although this effect necessarily involves both the absolute size of the jet and its ratio to that of the monitor, it should be possible to utilize these curves with good approximation for monitor and nozzle dimensions a reasonable fraction larger or smaller than those investigated. Second, since the degree of jet disruption at a given pressure depends primarily upon distance traveled, the horizontal length of throw of the actual jets tends to reach a maximum sooner and then to decrease more rapidly with increasing angle of inclination than does the ideal trajectory. Except for the lowest pressure, the maximum horizontal throw is seen from Fig. 20 to occur at about 30° rather than at the ideal 45°. Finally, whereas the coordinates of the ideal trajectory increase with the square of the efflux velocity (that is, with the first power of the base pressure), the dependence of the turbulence upon the mean velocity tends to make the angle of diffusion independent of the velocity. However, the latter still controls the intensity of droplet dispersion by the air. As a result, continued increase in the pressure does not produce a comparable increase in length of throw but eventually even leads to an appreciable reduction. The summary plot of Fig. 20 indicates that a point of diminishing returns is reached



(a) BASE PRESSURE, 50 POUNDS PER SQUARE INCH

(b) BASE PRESSURE, 100 POUNDS PER SQUARE INCH

(c) BASE PRESSURE, 150 POUNDS PER SQUARE INCH

(d) BASE PRESSURE, 200 POUNDS PER SQUARE INCH

Fig. 18.—Performance Curves for Recommended Fixed Monitor with 1½ and 3-In. Jets

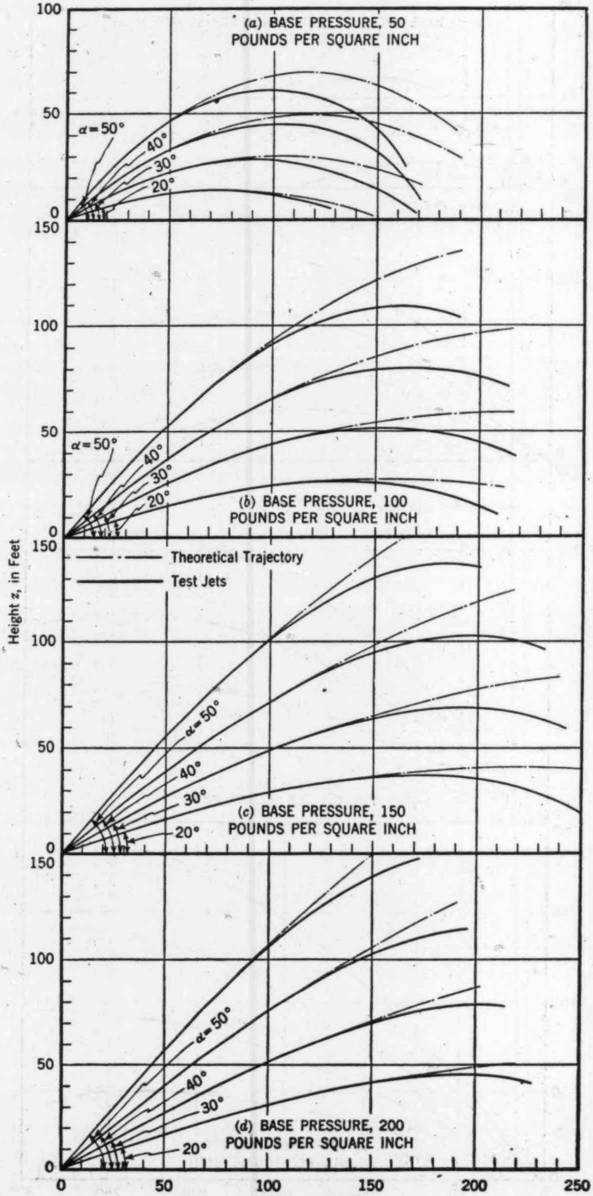


FIG. 19.—PERFORMANCE CURVES FOR RECOMMENDED PORTABLE MONITOR WITH 1½, 1¾, AND 2-IN. JETS

at a pressure not much above 150 lb per sq in. Although this point may be expected to advance somewhat with increasing nozzle and monitor size, higher pressures than those investigated should be of little practical use.

Conclusions and Recommendations

Investigations made by the Institute have indicated that the considerable difference that exists between the actual and the ideal trajectories of fire streams is due primarily to the turbulent eddies present in the flow as it leaves the nozzle. Complete elimination of turbulence would not necessarily result in a perfect stream, since the action of the air on the surface of the high-velocity flow could still form capillary waves whose variable resistance would eventually lead to disruption of the stream as a whole. A turbulent jet, on the contrary, would spread rapidly even in a vacuum. Only after the eddies break the jet surface does the resistance of the air become pronounced, the velocity of the diffusing water thereby being rapidly decreased as an ever greater volume of air is brought into motion. The extent to which fire streams can actually be improved through reduction of turbulence is governed almost entirely by practical considerations. Since the long approach passage, the large stilling section, and the fine turning vanes and screens of low-turbulence wind and water tunnels cannot be reproduced well in a fire monitor, considerable turbulence must still be expected under even the best conditions. Nevertheless, the foregoing studies indicate that its intensity may be reduced to an appreciable degree through standard principles of streamlining.

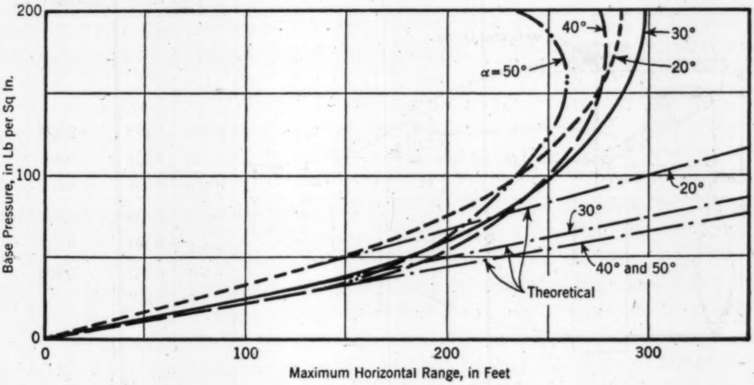
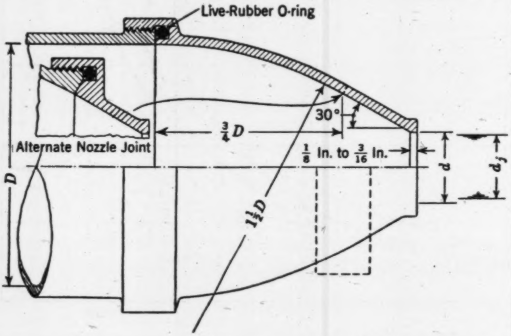


Fig. 20.—Summary Plot for Throw of Recommended Fixed Monitor

The form of nozzle recommended by the Institute on the basis of these studies has a well-rounded base, a relatively rapid convergence, and an unconfined zone of jet contraction. Although a 30° angle between wall and centerline was chosen as the most desirable, this angle is not the critical factor in nozzle performance. The essential features are the elimination of: (*a*) A zone of eddy formation at the base that would exist in very short nozzles; (*b*) the excess surface resistance and formation of fine-scale turbulence in very long nozzles; and (*c*) the region of separation and possible cavitation at an abrupt juncture between the usual nozzle cone and cylinder. Of comparable importance but of somewhat different category is the elimination of a gasket that could (and frequently does) project into the flow at the nozzle base.

Table 1 shows schematically the general details of the recommended nozzle profile. The diameter of the barrel is taken as the reference dimension, and all other dimensions are given as ratios thereto. Although the relative jet diameters could conceivably be made either greater or smaller without undue change in performance, the range shown is judged to cover practical requirements. The table gives the relative diameter of outlet that will produce fully contracted jets of the relative diameters indicated, and other sizes may readily be obtained through interpolation. Also indicated are the coefficients of nozzle discharge based upon the equation,

TABLE 1.—CHARACTERISTICS OF RECOMMENDED FIRE NOZZLE

Schematic diagram	Ratio d_j/D	Ratio d/D	Coefficient of discharge C_d
	0.250	0.279	0.805
	0.300	0.335	0.806
	0.350	0.391	0.807
	0.400	0.446	0.811
	0.450	0.501	0.817
	0.500	0.557	0.825

$$Q = C_d\, a \sqrt{2\, g\, h_n} \qquad (12)$$

in which a is the area of the nozzle outlet and h_n is the piezometric head in the barrel approximately a diameter upstream from the nozzle.

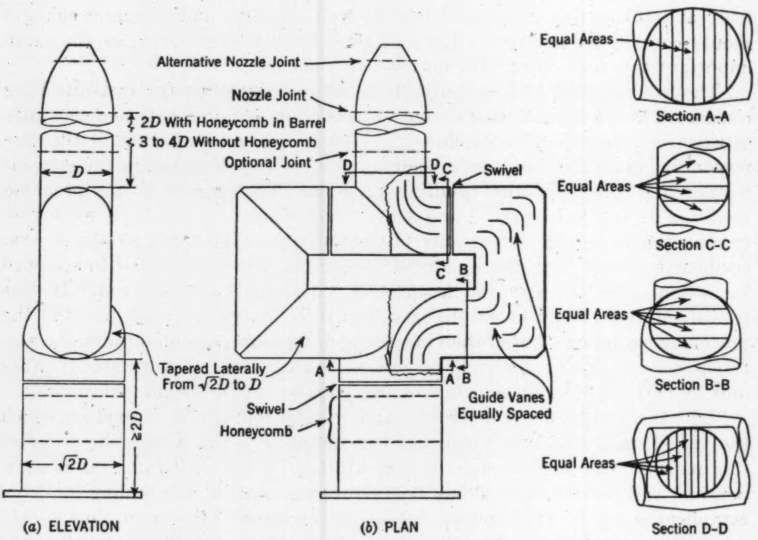
Two alternative locations of the nozzle joint are shown in the figure—one at the base and one along the contraction cone. That at the base is preferable so far as the flow is concerned, but the resulting nozzles would be heavier and more costly. That along the cone will require greater precision of fabrication, but the nozzles would be far lighter and less expensive. In either instance the nozzle seats tightly against the barrel without a gasket between the surfaces; if a gasket is needed for an absolute seal, it should have the form of an external soft-rubber O-ring as shown. Although fire nozzles are usually turned painstakingly (but not always accurately) out of brass castings piece by piece, it would appear quite possible that far cheaper, lighter, and more accurate units could be formed of plastic from a single, carefully made die and mold. Under the latter circumstances, the base joint would be preferable and quite feasible, and the nozzles could be considered expendable if and as the tips became damaged. The latter tendency should be minimized by the very short tip cylinder shown in the drawing; this is merely for protection during handling since the stream springs completely free at the inner edge.

The monitor itself is subject to three general requirements: (1) The passages should be as long and large in cross section as possible; (2) the cross-sectional area should either be constant or increase very gradually in the direction of flow; and (3) section changes should be well rounded, and direction changes should contain guide vanes. Each of these requirements becomes the more important the more closely the nozzle is approached.

The fixed monitor and the portable monitor designed by the Institute were considered at the time of their design to meet these requirements to a generally satisfactory degree, and comparison of their operation with that of the corresponding standard monitors revealed a marked improvement in the concentration and throw of the resulting streams. The general forms of these monitors as shown in Figs. 3 and 4 are recommended as the basis for future specifications. Their performance characteristics as indicated by the several coefficients and jet trajectories already presented may be expected to apply to the extent that the essential features of the designs are followed. In this regard the fact should be emphasized that it is the requirements listed in the preceding paragraph, rather than incidental dimensions, that determine the performance. Moreover, it should be possible to change the absolute size of either unit by perhaps 30% without serious variation in any of the characteristics.

Like the design of the horizontal and vertical joints, the method by which the vaned units could be constructed in quantity is not within the scope of this paper. The welded construction utilized by the Institute was not expensive, and present-day casting techniques are capable of handling far more complicated units. The actual details of the vanes have been shown only schematically, since their number and fineness are governed by structural as well as hydraulic considerations. That is, the smaller and more numerous the vanes, the finer the scale of the eddies that they induce—but the costlier they are to fabricate and the greater their tendency to foul with debris. The number shown should be regarded as the minimum, the optimum being that commensurate with practicability. Although their arrangement is also somewhat flexible (two methods of subdivision being shown), the spacing of their leading and trailing edges must be such as to maintain constant proportions between the areal subdivisions at the beginning and end of each flow deflection.

One feature of the fixed monitor designed by the Institute that is subject to modification for several reasons is found in the initial Y-transition from the base pipe to two passages of the same cross-sectional area. This not only adds to the height of the unit but also results in an unguided deceleration of the flow with accompanying eddy formation. Although the distance from the Y to the nozzle appears to minimize the effect of the eddies upon the jet, it is probable that replacement of this **Y** by a vaned transition of essentially constant total cross-sectional area (similar to that of the portable unit) would improve the performance to at least an appreciable degree and would at the same time reduce the over-all height of the unit. A schematic indication of the resulting monitor form is shown in Fig. 21. The upper **T** containing the joints for vertical sweep remains the same as before, but the enlarged passage containing the joint for horizontal sweep ends in a vaned **T** with two tapered arms of noncircular section. These transitions terminate in vaned **L**'s with circular sections just prior to the original design. This improved design is obviously more complex than that of the original recommendation, but casting should still be quite feasible. In this regard it is suggested that consideration be given to the use of stainless steel rather than brass, the vaned portions of the monitor being cast in symmetrical half sections that are ultimately welded together externally.


Alternative Nozzle Joint
Nozzle Joint
2D With Honeycomb in Barrel
3 to 4D Without Honeycomb
Optional Joint
D
D
D
C
Swivel
C
B
A
A
B
Tapered Laterally
From $\sqrt{2}D$ to D
Swivel
Honeycomb
Guide Vanes
Equally Spaced
≥2D
$\sqrt{2}D$
Equal Areas
Section A-A
Equal Areas
Section C-C
Equal Areas
Section B-B
Equal Areas
Section D-D

(a) ELEVATION (b) PLAN (c) DETAILS

FIG. 21.—IMPROVED FORM OF RECOMMENDED FIXED MONITOR

ACKNOWLEDGMENTS

The project described was conducted under Contract NObs-24084 with the Bureau of Ships, United States Navy, and Contracts N7onr-495 and N8onr-500 with the Office of Naval Research. The present paper summarizes the more detailed information presented under the same title in November, 1949, as Institute Report No. 3 to the Office of Naval Research. Additional copies of

this report have been filed with the Engineering Societies Library and the Library of Congress for reference purposes.

Practically the entire staff of the Institute participated in the investigation in one manner or another. Particular credit is due Messrs. Chesley J. Posey, M. ASCE, who conducted many of the preliminary tests; E. M. Laursen, A. M. ASCE, who detailed the test monitors; and Dale C. Harris, who supervised the construction and operation of the experimental facilities. Lt. W. S. McConnell, of the Coast Guard, provided valuable assistance during his summer assignment to the project.

DISCUSSION

G. **Halbronn**[11].—Current observation shows that jets emerging from circular orifices into still air do not remain cylindrical and coherent but diverge and break up into drops. This phenomenon, which is of theoretical interest, plays an important part in various technical problems, such as that of carburetor injection and that of the structure of Pelton nozzle or fire nozzle jets. The last two examples concern jets of comparable size and velocity and therefore it is advantageous to compare them so that any improvements which are likely to be made in one of these branches may be used to the benefit of the other.

Although the problem of jet diffusion has been the subject of many investigations, one cannot consider the problem as having been solved. The number of intervening factors is great—design and size of the nozzle, nature of the fluid, and head at the orifice. In addition, the factors which are likely to govern diffusion are numerous and complex—the influences of vibration, surface tension, flow turbulence, and air resistance. Moreover, research workers have not always succeeded in isolating the factors that have a major influence in the case investigated.

The first experimental research work on this subject was that of F. Savart,[12] with thin-walled orifices of 1.2-in. maximum diameter and 3.3-ft-per-sec maximum velocity. J. Plateau[13] gave the first theoretical explanation, based on his investigations on the stability of liquid cylinders, of the effects observed by Mr. Savart, which he attributed to the action of surface tension. This process of disintegration can thus be qualified as a capillary dissolution regime.

Mr. Plateau's calculations were completed and made more accurate first by Lord Rayleigh[14,15] and then by C. Weber.[16] Roughly speaking, the length of a nondisintegrated jet in this case is proportional to its velocity and to the cube root of the orifice diameter.

Some research workers, of which A. Haenlein[17] was one, observed a new disintegration process when the velocity was increased. This consists in the transverse oscillation of the jet and in a very great reduction in its nondisintegrated length. According to Mr. Haenlein, this phenomenon is caused by the effect of the surrounding air. From this hypothesis, Mr. Weber endeavored to interpret Mr. Haenlein's experiments theoretically but without much success.

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[11] Ingénieur des Ponts et Chaussées, Détaché au titre de la Recherche Scientifique auprès des Etablissements Neyrpic à Grenoble, France.

[12] "Mémoire sur la constitution des veines liquides lancées par des orifices circulaires en mince paroi," by F. Savart, *Annales de Chimie et de Physique*, Paris, France, Vol. 53, 1833, p. 337.

[13] "Statique expérimentale et théorique des liquides soumis aux seules forces moléculaires," by J. Plateau, Gauthier-Villars, Paris, France, 1873, p. 326.

[14] "Instability of Jets," by Lord Rayleigh, from "Scientific Papers," Vol. I, 1899, pp. 361–368.

[15] "On the Instability of a Cylinder of Viscous Liquid under Capillary Forces," by Lord Rayleigh, *ibid.*, Vol. III, 1899, p. 585.

[16] "Zum Zerfall eines Flüssigheitsstrahles," by C. Weber, *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. II, No. 2, 1931, p. 136.

[17] "Über den Zerfall eines Flüssigheitsstrahles," by A. Haenlein, *Forschung auf dem Gebiete des Ingenieurwesens*, Vol II, 1931, p. 139.

G. Littaye[18] then undertook further experimental studies of the phenomenon. He showed that with an absolutely circular, thin-walled orifice, having no burrs on the upstream edge, the limits given by Mr. Haenlein are easily exceeded and the capillary dissolution of the jet is maintained. Therefore, this phenomenon is not due to air action. If it were, it would occur just as well with a jet from a thin-walled orifice as with one from a thick-walled orifice or a capillary tube. However, Mr. Littaye was able to reproduce the phenomenon only with the latter two orifices, and he attributed it to the process of transformation of the parabolic distribution of the velocities in a cross section of the tube into their uniform distribution in a cross section of the jet.

When the velocity continues to increase, the transverse oscillation decreases and even disappears. However, the surface of the jet then becomes irregular on leaving the orifice and the drops formed are very much smaller than in the two former regimes.[19] This latter regime, therefore can be termed the "pulverization regime." The experimental criteria of the existence of the three different regimes were given by W. Ohnesorge[20] in a nondimensional form. When these criteria are extrapolated, it is seen that industrial jets are practically always pulverized.

Hydraulic engineers today agree that pulverization is caused by the inner turbulence of the jet. As mentioned by the authors, tests performed in a vacuum chamber should therefore give rise in this regime, to jets having the same structure as those in air; according to present knowledge, however, such experiments have never been published. On the contrary, in the course of tests conducted at the Neyrpic Laboratory, the vacuum chamber was filled with a fine mist, which indicated an even greater disintegration of the jet and made observation impossible. This phenomenon is probably due to the sudden release of the air dissolved in the water as the jet enters the vacuum. Therefore, the experiment should be made with a liquid from which dissolved gases have been removed and would consequently be exceedingly difficult to perform. However, there is a simpler way of making sure that air action cannot be the initial cause of pulverization for, if this were the case, the phenomenon would re-occur in an identical manner with jets of the same diameter and velocity, regardless of the design of the nozzle. Now, the tests made by the authors have shown that all the devices, which were known to reduce turbulence, also cause a reduction in the diffusion of the jet. This fact alone shows that, in this case, turbulence is without doubt the initial cause of jet diffusion.

The hypotheses made by the authors to explain the mechanism of the action of this turbulence can be compared to those which were made recently in France, to account for the occurrence of "white water" in high velocity, free surface flow. It has been shown[21] that air entrainment cannot occur upstream from a critical point at which the boundary layer, generated by the contact of the water and the face of the spillway apron, attains the same height as the flow, and turbulent action consequently begins to make itself felt on the surface. At this point, a criterion for which the writer has endeavored to give an explicit expression must be found, such that transverse kinetic energy which the turbulence transmits to the surface droplets is sufficient to overcome the opposing stresses due, in particular, to surface tension and gravity. When this is the case, the droplets detach themselves from the body of water and entrain air as they fall back in, giving the flow its characteristic aspect.

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[18] "Contribution à l'étude des jets liquides," by G. Littaye, *Publication Scientifique et Technique du Secretariat d'État à l'Aviation*, Paris, Franc , 1942.

[19] "The Break-up of Liquid Jets," by A. C. Merrington and E. G. Richardson, *Proceedings*, Physical Soc., January, 1947, Vol. 59, No. 331, p. 1.

[20] "Die Bildung von Tropfen an Düsen und der Zerfall flüssiger Strahlen," by W. Ohnesorge, *Zeitschrift des Vereines Deutscher Ingenieure*, No. 16, Vol. 81, 1937, p. 465.

[21] "Étude de la mise en régime des écoulements sur les ouvrages à forte pente-application au problème de l'entraînement d'air," by G. Halbronn, Thèse de l'Université de Grenoble, France, 1951.

These views are identical with those given by the authors to explain the turbulent disintegration of jets and it should be possible, therefore, to apply the same method of calculation. Unfortunately, however, even with the simplest type of nozzle, it is still difficult to evaluate the intensity of the turbulent fluctuations at the nozzle outlet and a choice between the various possible hypotheses can be made only after systematic experiments. Nevertheless, from the qualitative point of view, the effect of surface tension makes it possible to explain the experimentally verified fact that jets can remain clear and translucid (when not in the pulverized state) even when the Reynolds number in the nozzle exceeds 2,400, at which value turbulence occurs. The calculation also makes questionable the veracity of the authors' statement (under the heading, "Discussion of Test Results: Performance Tests of Recommended Monitors") that

> "* * * the dependence of the turbulence upon the mean velocity tends to make the angle of diffusion independent of the velocity * * *."

This fact would doubtless be correct if the turbulence characteristics at the outlet remained the same as within the body of the nozzle, but the effect of contraction (defined as the ratio between the pipe-line diameter and the outlet diameter) is likely to change these characteristics and to invalidate the conclusion. The writer is inclined to believe that diffusion increases when the velocity of the jet increases, and it decreases, with a constant outlet cross section, when contraction increases. This confirms the statement (under the heading, "Underlying Principles of Flow: Approvement of Fire Streams") that "* * * the passages * * * should be of maximum dimensions commensurate with practicable over-all size."

This contraction effect is not shown by the experiments mentioned herein since jets flowing from the same pipe line through nozzles having different outlet diameters (and, therefore, different degrees of contraction) all have practically the same range for the same initial velocity. However, even though turbulence accounts for the diffusion on the jet in the first part of its trajectory and the occurrence of the surface protuberances which are subject to the relative wind action of the air, this latter goes on to play a predominant part in the dispersion of the jet and in the shortening of its range. This fact is clearly shown in the experiments made by A. C. Merrington and E. G. Richardson,[19] in which it is seen that the diameter of the drops created does not depend on the absolute velocity of the jet but on its realtive velocity in relation to the surrounding air. A thinner, and therefore more contracted, jet would undergo air action through to its center more rapidly than would a thick jet, and it is possible that this effect may counterbalance the effect of the former's good initial performance. Some of the conclusions given here may therefore no longer be valid when use is made only of a short length of the jet, such as is the case with Pelton nozzles.

These remarks show the difficulty of the problem. The systematic study of a schematic case would be desirable but, in the more complex, practical case, only tests which are similar to those described by the authors are likely to show the constructive steps that should be taken to reduce the intensity of the turbulence, of which the occurrence is inevitable. The systematic introduction of curved guide vanes is certainly very favorable. The determination of the nozzle's optimum angle of convergence and the physical explanation of its existence are of great value. It is of interest to note that Pelton nozzle designers have also been led along different lines to adopt convergence angles of approximately 30°. It would also be useful if the action of the guide vanes upstream from the orifice, considered by the authors as being unfavorable, were studied in greater detail since they are always used in Pelton nozzles. P. Oguey[22,23] even reached the conclusion that an increase in the number of vanes reduces the angle of diffusion of the jet. There is thus a contradiction for which an explanation would be desirable.

PIERRE OGUEY,[24] MARCEL MAMIN,[25] AND FRANÇOIS BAATARD.[26]—Since 1942 the writers have studied carefully all scientific publications concerning hydraulics and this paper is the first to offer an explanation of jet dispersion that seems valid. In their research, the authors studied the means of augmenting the range of fire streams by improvement of nozzles and monitors. They have demonstrated for the first time in America that jet dispersion is due to turbulence caused by the structural details preceding the jet.

The writers have undertaken a theoretical and experimental study of the high-speed water jet to clarify the hydraulic phenomenon of dispersion and, particularly, to improve the performance of Pelton turbines. To this effect a theory of jet dispersion resulting from turbulence was established on the basis of numerous observations. This theory was then largely verified by tests made with the most varied types of nozzles, functioning under very different conditions. The conclusions obtained are valid for any kind of jet, whatever liquid is used. As a matter of fact, the calculation method recommended permits a rational study of the improvements that can be effected in jet-producing units, more particularly in fire monitors.[27,28] This application to fire-stream engines had already been demonstrated distinctly in 1944.

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[22] "Etude théorique et expérimentale de la dispersion du jet dans la turbine Pelton," by P. Oguey and M. Mamin, *Bulletin Technique de la Suisse Romande*, Nos. 21 and 22, 1944, pp. 265, 277, and 281–291.

[23] "Etude théorique et expérimentale de la dispersion du jet dans la turbine Pelton," by P. Oguey, M. Mamin, and F. Baatard, *ibid.*, Nos. 4 and 5, 1951, pp. 37-47 and 53-64.

[24] Conseiller d'Etat, professeur à l'Ecole polytechnique de l'Université de Lausanne, Lausanne, Switzerland.

[25] Chef de travaux à l'Ecole polytechnique de l'Université de Lausanne, Lausanne, Switzerland.

[26] Chef de travaux à l'Ecole polytechnique de l'Université de Lausanne, Lausanne, Switzerland.

[27] "La dispersion du jet d'eau à grande vitesse: Part I, Etude théorique et expérimentale de la dispersion sous l'effet de la turbulence," by Pierre Oguey and Marcel Mamin, *Publication No. 16*, Polytechnic School of Lausanne University, Lausanne, Switzerland, 1944. (A theoretical and experimental study of dispersion due to the phenomenon of turbulence including general equations, laws of similitude, and applications to Pelton turbines.)

[28] "La dispersion du jet d'eau à grande vitesse: Part II, Nouvelle étude expérimentale," by Pierre Oguey, Marcel Mamin, and François Baatard, *Publication No. 17*, Polytechnic School of Lausanne University, Lausanne, Switzerland, 1951. (A new experimental study; transformation of small jets into drops

Structure of a Dispersed Jet; Hypotheses and Fundamental Equations.—On leaving the nozzle, a jet undergoes—from its contracted section on—the phenomenon of dispersion. Instead of keeping its cylindrical shape as the laws of hydrodynamics would require, it reveals two distinct areas—(1) a central, convergent, homogeneous zone composed of water and (2) a divergent, heterogeneous zone, surrounding the first, and consisting of a mixture of air and water. It is this mixture (area 2) which gives the jet its dispersed appearance and, at a certain distance from the outlet, makes it look like a steam jet. It can be mathematically demonstrated (and experimentally verified) that the outer limit line of the jet and the inner limit line that separates the homogeneous zone from the heterogeneous one are symmetrical with the theoretical limit line of the cylindrical jet, with which they form an angle α.

The basic hypotheses of the theory (as applied by the writers and the authors) are as follows:

1. The air resistance is negligible.

2. The axial velocity V_1 of all the water particles is constant in intensity and direction and equal to velocity V_0 in the contracted section. Velocity V_1 is constant for a horizontal jet. In a slanting jet, V_1 diminishes with the altitude. In this connection, it must be mentioned that the Bernoulli equation is valid when it is applied to each jet particle considered as a projectile. On the contrary, the equation is not valid for the jet considered as a whole because the equation of continuity is not applicable between two sections crossed by a mixture of air and water, the density of which is unknown.

3. Furthermore, the writers maintain that at the periphery of the jet, a water particle must submit to:

(*a*) A centripetal action caused by atmospheric pressure and by a molecular attraction which creates a surface tension T_s and

(*b*) A centrifugal action from the inside, caused by the liquid turbulence characterized by the "Reynolds number for the apparatus" **R**. If the centrifugal forces are superior to the centripetal ones, the particle in question escapes from the jet surface at a radial speed V_2 that is constant (it is constant, at any rate, to the moment when it is distant enough to be exposed to the action of the surrounding air).

Thus the angle α, which is characteristic of the jet dispersion, is indicated by the following basic equation:

$$\tan\alpha = \frac{V_2}{V_1} = \sqrt{F(\mathbf{R}) - \frac{2\,T_s}{\rho\, d_o\, V^2_1}} \dots\dots\dots\dots\dots (13)$$

in which d_o is the diameter of the jet in contracted section and $\rho = \gamma : g$ is the specific mass of liquid.

Transformation of a Jet into Drops, Without Dispersion.—In Eq. 13, if the quantity under the radical is negative or equal to zero, it denotes that surface tension is prevalent and that no jet dispersion will occur. The stream will be

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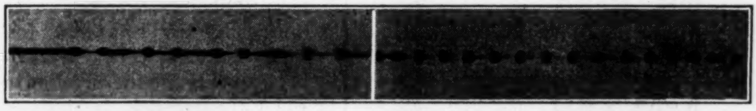
without dispersion including dispersion of big jets, influence of structural details preceding the jet, and applications to Pelton turbines.)

transformed into drops according to a process which is entirely visible[29] in Fig. 22. The liquid cylinder (which must submit to a surface tension) being unstable, forms dilated, periodical zones that are separated by a liquid cylinder with a diminishing diameter. This cylinder can free itself from the dilated zones and, after yielding to its own molecular attraction, can contract itself into a droplet which is much smaller than adjacent drops. Then the air sucks these droplets out of the track of the large drops, which explains the vapor that may be observed around big long-distance jets. Clouds were visible on the photographs of big jets, in the files of the writers and the authors alike. However, the contrary phenomenon may occur if one of the two dilating zones has more attracting power than the other. In this case, the small cylinder connects itself with the stronger zone without forming an intermediary droplet.



(a) JET VELOCITY, 15 METERS PER SECOND (2,067 FT PER SEC) (b) JET ENTIRELY TRANSFORMED, WITHOUT DISPERSION

FIG. 22.—TRANSFORMATION OF A JET INTO DROPS WITHOUT DISPERSION (CAMERA EXPOSURE ABOUT 0.000001 SEC)

Dispersion of Big Jet (Fire Stream or Jet of Pelton Turbine).—In any section where dispersion is already strong, the jet surface is very irregular. Thus, it is legitimate to admit that the effect of surface tension has become negligible and, therefore, that the phenomenon of dispersion is caused by turbulence only. The expression of tan α is then reducible to

$$\tan \alpha = \sqrt{F(\mathsf{R})} \quad \ldots\ldots (14)$$

If the axial velocity V_1 is constant, the specific weight γ of the air and water mixture at a distance y from the axis will be lower than the specific weight γ_o of the water, and

$$\frac{\gamma}{\gamma_o} = f\left(\frac{y}{y_o}\right) = f\left(\frac{2y}{d_o}\right) \quad \ldots\ldots (15)$$

A theoretical calculation based on the foregoing three hypotheses, and the study of the appearance of the experimental curves allowed the writers to develop the general equation of this curve.

The curves of Eq. 15 are presented in Fig. 23 for successive sections of jet.[30] Each section is characterized by the ratio $\beta = \frac{a_{\min}}{y_o}$ in which $a_{\min}$ is the minimum radius of the homogeneous-remaining zone. The theory, tested by experiments easy to perform, determines the distance from the outlet where these curves must be placed in order to have the complete image of any jet, whatever its diameter (d_o) and the generating net head H.

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[29] "La dispersion du jet d'eau à grande vitesse: Part II, Nouvelle étude expérimentale," by Pierre Oguey, Marcel Mamin, and François Baatard, *Publication No. 17*, Polytechnic School of Lausanne University, Lausanne, Switzerland, 1951, p. 65, Figs. 26 and 27.

[30] *Ibid.*, p. 40, Fig. 19.

The process of dispersion is illustrated[31] by the photographs in Fig. 24. On leaving the nozzle outlet, the jet first shows streaks, then protuberances which move away from the axis more and more, and gradually transform themselves into drops and droplets on which the surrounding air may then have a braking action. The action of the air begins only at a distance which is equal to 70 or 80 times the jet diameter at the outlet; but protuberances are not waves caused by air resistance (on many photographs it is impossible to distinguish the direction in which the stream is circulating), rather they are waves provoked by internal turbulence, which confirms the fundamental hypothesis of the theory. The observations reported by the authors fully confirm both the processes of dispersion thus described, and the effect of air resistance.

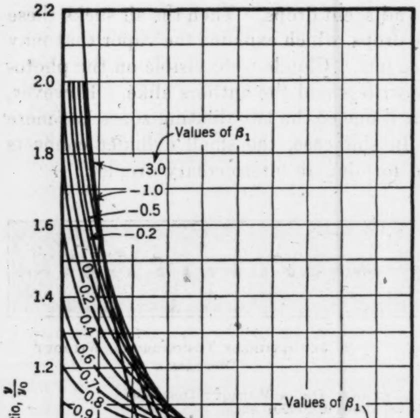

FIG. 23.—RELATION BETWEEN THE SPECIFIC WEIGHT RATIOS, $\frac{\gamma}{\gamma_o}$, OF THE AIR AND WATER MIXTURE, AND THE RADIUS RATIOS $\frac{y}{y_o}$ AT DIFFERENT SECTIONS OF A DISPERSED JET

Conditions for the Similarity of Jets.—The authors have presented true propositions for the similarity of apparatus. It is easy to demonstrate that two units which are geometrically similar in all their parts will produce streams of diameters d'_0 and d''_0 which will have the same dispersion characteristics, under the condition that their Reynolds number is equal; that is, that

$$\mathsf{R}' = \mathsf{R}'' \; \ldots\ldots (16)$$

or that

$$\frac{V'_o \, d'_o}{\nu'} = \frac{V''_o \, d''_o}{\nu} \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

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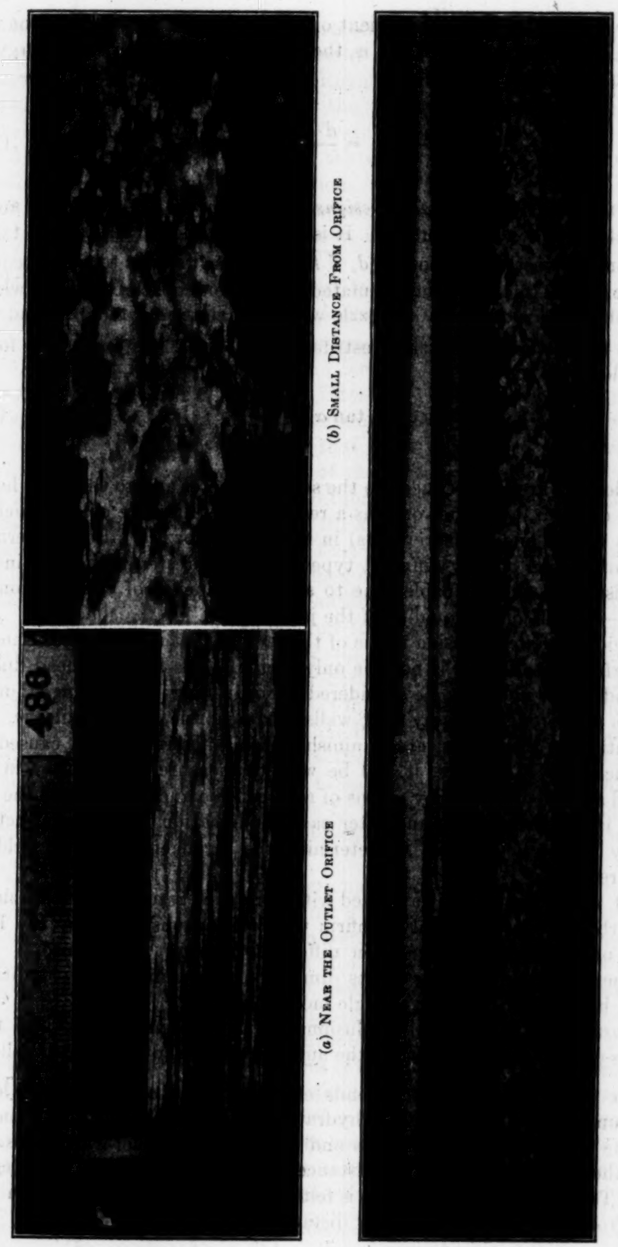
[31] "La dispersion du jet d'eau à grande vitesse: Part II, Nouvelle étude expérimentale," by Pierre Oguey, Marcel Mamin, and François Baatard, *Publication No. 17*, Polytechnic School of Lausanne University, Lausanne, Switzerland, 1951, Figs. 31, 35, and 36.



(b) Small Distance From Orifice

(a) Near the Outlet Orifice

(c) Large Distance from Orifice (Jet Entirely Dispersed)

Fig. 24.—Photographs of a Jet Moving at a Rate of Thirty-Six Meters per Second (118 ft per sec) under a Head of Sixteen Meters (197 ft)

$\nu = \mu/\rho$ being the kinematic coefficient of viscosity and ν being the dynamic viscosity. If H is the net head (that is, the total hydraulic generating energy of the jet), Eq. 17 can also be written

$$\frac{d'_o \sqrt{H'}}{\nu'} = \frac{d''_o \sqrt{H''}}{\nu''} \quad \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

Comparison of Different Nozzle Designs As to Dispersion of Jet.—The angle α is characteristic of the dispersion. It is possible to demonstrate that $\tan \alpha$ is a function of (construction nozzle) $d_o \sqrt[4]{H}$. The next step is to try to reduce the value of $\tan \alpha$ (measured or calculated for any one nozzle) to those which might be obtained with the same nozzle with a diameter d_{oc} under a head H_c. Since α_c is a function of (nozzle construction) $d_{oc} \sqrt[4]{H_c}$, the expression for a given nozzle is

$$\tan \alpha_c = \tan \alpha \frac{d_{oc}}{d_o} \sqrt[4]{\frac{H_c}{H}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

If all nozzle tests are thus reduced to the same diameter and to the same head, the values of $\tan \alpha_c$ can differ only as a result of the influence of construction differences (shape and measurements) in the tested nozzles. Eq. 19 permits an easy comparison of very different types of nozzles tested with any kind of head or discharge and it enables one to see the influence of construction on dispersion—that is, on the quality of the jet.

It is evident that dispersion is one of the factors that cause the lowering of the jet performance, but it is not the only factor. The losses in head due to friction and eddies must also be considered. For instance, by placing a honeycomb with numerous and very long walls into a nozzle, the turbulence, and consequently the dispersion, are diminished, while losses in head caused by friction increase. Therefore, it will be wise to measure the velocity in the contracted section of the jet by means of a Pitot tube and to calculate the loss of energy in the unit before and after each modification in the construction. Thus only will it be possible to determine the arrangements that yield an optimum result.

Eq. 19, which the writers have used with very good results on the nozzles of Pelton turbines, could certainly confirm the comparisons made by the Iowa Institute of Hydraulic Research on different types of fire monitors. The improvement of a unit, whatever its dimensions and functioning conditions, may then be planned by using a single model unit.

Conclusions.—The general conclusions of the writers' study, and their comments on the work reported by the authors may be summed up as follows:

1. The dispersion of a jet depends essentially on turbulence, the latter being a function of the head (total hydraulic energy before the nozzle outlet) and of the construction (dimensions and form) of the structural details that precede the outlet orifice. Air resistance has no direct effect on dispersion proper. The effect of air resistance is felt only when the droplets, which have escaped from the jet, are sufficiently distant to be braked by the air.

2. The laws of similitude, once established, permit the planning of a unit to be built from a model which is geometrically similar in all its parts and which operates under such a head that the Reynolds numbers can be equated (Eq. 16). Besides, a convenient criterion for the comparison of different units will be provided by introducing the specific values tan α_e (Eq. 19).

3. The conclusions advanced by the authors afford an interesting means of confirming that method of calculation. Besides it proves obviously the care with which they have made their experimental study of jets, their sagacity in the interpretation of their observations, and the great value of their work.

John H. Arnold[32].—During 1940 and 1941 the National Board of Fire Underwriters conducted a series of experiments on the subject of this paper. The results were reported to the International Association of Fire Chiefs, in 1943, in an unpublished paper by Clarence Goldsmith, M. ASCE, entitled "The Effective Reach of Powerful Streams." The research described in the present paper is in accord with suggestions made by Mr. Goldsmith. Like the authors, Mr. Goldsmith concluded that there was a need for a redesign, and an increase in the waterway areas of the current types of monitors. Under the heading, "Introduction," the authors report an increase in the reach of streams obtained with a modified tip and a lengthened nozzle. In similar observations, Mr. Goldsmith found that the increase ranged from 2% to 30%, the latter being the maximum obtained on one test.

In the test following Eq. 5, the authors state that C_L "* * * may be considered a constant for a given conduit form, such as a fixed contraction ratio or fixed proportions of an elbow, regardless of the conduit dimensions." Possibly, this conclusion was adopted because variations may not have much practical significance. The theories of fluid turbulence seem to apply logically, and it may be a fair assumption that liquid protuberances would appear as readily in a vacuum, although no practical methods of proving this assumption seem feasible. It may well be that there is adequate proof of the introduction of serious secondary spiral currents in the sweeping curves of the bends in conventional monitors. It is on this assumption that the entire new design, proposed in the paper, is based.

In preliminary work by the National Board of Fire Underwriters, much difficulty was encountered in deciding at what point the stream ceased to be effective. The same general criteria were used as in the present paper. However, varying judgment and lack of clear visibility by day, even with the use of a transit (as in the present work), gave unsatisfactory results and led to the development of a system of night observations with lights.

The writer believes that the authors' comparisons of results showing an advantage for the proposed new types of monitors are not a true measure of the relative merits. In each instance, the waterway areas of the new types are considerably larger than those in either of the old types which were used for comparison. It would be valuable to know if there is any assurance that better results would be obtained from the new type if comparisons were made with monitors of the old design, but having equivalent waterway areas.

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[32] Asst. Chf. Engr., National Board of Fire Underwriters, New York, N. Y.

The paper offers theories that may be of much value in reconsidering current practice on the subject, and it has made a start toward putting these theories into practice. However, it is felt that much further work and much more accurate comparisons of the old and the new designs are needed before any practical superiority can be proved.

Hunter Rouse,[33] M. ASCE, **J. W. Howe,**[34] M. ASCE, and **D. E. Metzler**[35].—To Messrs. Halbronn, Oguey, Mamin, Baatard, and Arnold the writers are greatly indebted for the considerable background, breadth, and practical scrutiny which their discussions have given to the original investigation. The international aspect of these contributions, moreover, supports the writers' belief as to the general importance of the subject. It is not only heartening that their findings have been verified in other countries, but also stimulating that the profession most closely concerned with fire-stream efficiency shows a healthy interest in advancement and yet a cautious insistence upon full verification of their conclusions.

Mr. Halbronn's excellent documentation of the topic more than compensates for the meagerness of the writers' bibliography. His comparison of the mechanism of jet dispersion with that of air entrainment by high-velocity flow in an open channel is extremely apt. Although there are essential differences due to the continuous generation of turbulence and the restoring force of gravity in the latter phenomenon, in both it is the disruption of the free surface by the turbulent filaments which is the basic factor.

The writers' statement (under the heading, "Discussion of Test Results: Performance Tests of Recommended Monitors") that " * * * the dependence of the turbulence upon the mean velocity tends to make the angle of diffusion independent of the velocity * * * " tacitly assumes constancy of the boundary geometry. As Mr. Halbronn correctly believes, an increase in the relative size of the approach section will decrease the angle of diffusion, provided that the accompanying reduction in flow stability is otherwise offset. The writers' statement is also a rough approximation rather than a precise generalization because of the fact that the phenomenon involves viscous and capillary effects as well as those of geometric form. Any general law, therefore, must be expressed in terms of not only a series of length ratios describing the monitor, nozzle, and jet form, but also the Reynolds and Weber numbers—and the Froude number as well, if characteristics of the relative trajectory are to be included.

Mr. Oguey and his colleagues have had the courage to undertake the first approach to such a formulation, although the result is of practical necessity somewhat less than general. To the extent that unpredictable upstream effects can be eliminated, their method of analysis is without doubt a useful one for the immediate vicinity of the nozzle. Unfortunately, it is still the monitor itself which produces the major disturbances, and the essential compactness of fire-monitor design makes the prediction as well as the elimination of these disturbances at best a difficult matter.

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[33] Director, Iowa Inst. of Hydr. Research, State Univ. of Iowa, Iowa City, Iowa.

[34] Head, Dept. of Mechanics and Hydr., State Univ. of Iowa, Iowa City, Iowa.

[35] Associate Prof., Dept. of Mechanics and Hydr.; Research Engr., Iowa Inst. of Hydr. Research, State Univ. of Iowa, Iowa City, Iowa.

The mechanism of droplet formation without dispersion surely plays a predominant role in small jets, but the writers do not believe that this is the phenomenon which gives rise to the fine mist surrounding the jets from certain types of nozzles. The source of the mist seems to lie instead in the small-scale boundary-layer turbulence near the jet surface, for it was most prominent in streams from very long nozzles and from those which had a slight break in profile near the tip. Large droplets, on the contrary, appeared to result from turbulence of larger scale generated upstream from the nozzle.

The writers are in wholehearted accord with the similarity criteria which Messrs. Oguey, Mamin, and Baatard propose—except for their belief that the Froude and Weber numbers are fully as important as the Reynolds number. The performance of Pelton nozzles and that of fire nozzles are, to be sure, judged from somewhat different points of view, but—except for range—the basic factors involved in both are just the same. The writers strongly recommend that engineers interested in either aspect of the problem refer to the two papers[27,28] cited in the discussion—not only for the authors' extensive analysis, but also for the superb photographic evidence of the dispersive action of turbulence which they contain.

The writers must confess to a complete ignorance of the Goldsmith report[36] prior to the receipt of Mr. Arnold's discussion. This report contains a wealth of experimental data on the reach of streams. Contrary to the writers' findings, however, Mr. Goldsmith concluded that an increase in pressure will invariably increase the effective reach. Although the writers' method of observation may well have been less precise than that of Mr. Goldsmith, they do not believe that either inaccuracy of measurement or the admittedly arbitrary definition of effective reach could account for the distinct and systematic reversal of the curves shown in Fig. 20.

Mr. Arnold's question as to the constancy of C_L necessarily involves the hydraulics of all closed conduits rather than that of fire monitors alone. At high Reynolds numbers the coefficients of any conduit transition tend toward constant values—particularly if the transition is not well streamlined. The writers agree that the testing of fire streams in a vacuum is not practicable, especially in view of Mr. Halbronn's account of just such an attempt; nevertheless, their basic hypothesis does not appear subject to challenge on any other grounds. The generation of spiral flow by conduit bends[37] is hardly an "assumption" in view of the experimental evidence which is available in many related fields.

Of primary importance, in the writers' opinion, is the point raised by both Mr. Arnold and Mr. Halbronn as to the effect of enlarging the approach passage. Enlargement of cross section actually has two effects which are quite distinct. On the one hand, the reduction in mean velocity is accompanied by a reduction in the turbulence intensity. On the other hand, however, the increase in boundary spacing permits the scale of the turbulence to increase. Only if the same degree of stabilization is maintained by the insertion of guide vanes as the section is enlarged will the jet be appreciably improved. This was clearly demonstrated by two supplementary tests described in the paper. First, enlargement of the approach section to 2.5 ft without guide vanes actually resulted in greater jet dispersion, for the large spacing of the walls permitted the formation of equally large eddies by the crushed-rock baffle. Second, the Institute monitor produced an inferior jet when inlet and outlet ends were reversed, for it was not the enlargement of passage which had increased the range of the stream but rather the proper guidance of the flow just before the barrel. From these observations it appears reasonable to conclude that a rams-horn monitor of large diameter would be better than one of small diameter only from the standpoint of internal loss in head.

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[36] "The Effective Reach of Powerful Streams," by Clarence Goldsmith, published and distributed by the Educational Committee, International Assn. of Fire Chiefs, 1943.

[37] "Engineering Hydraulics," Ed. by Hunter Rouse ("Steady Flow in Pipes and Conduits," by V. L. Streeter, Chapter VI), John Wiley & Sons, Inc., New York, N. Y., 1950, pp. 419–423.

With regard to Mr. Halbronn's question about the efficacy of guide vanes or a honeycomb just before the nozzle, the writers can only rephrase their previous statements. If the approaching flow is very turbulent, vanes of the proper form and spacing will assuredly result in a marked improvement of the jet. However, if the flow is already relatively free from turbulence, the insertion of vanes may well increase the rate of jet dispersion because of the additional boundary-layer and wake turbulence produced by each vane.

The writers agree with Mr. Arnold that further comparative tests are needed, if only to convince engineers beyond a doubt that modern principles of fluid motion are useful practically as well as theoretically. To the other discussers the writers are grateful for much corroborative evidence of this sort.

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Paper No. 2530

SEWAGE RECLAMATION BY SPREADING BASIN INFILTRATION

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SYNOPSIS

To study the feasibility of recharging the ground water with effluent from sewage treatment plants, experimental spreading basins were established in Los Angeles County, California. Two locations were used to evaluate the effect of climate, soil characteristics, type of spreading fluid on the suitability of the percolated product, and rates of infiltration.

It was found that Biochemical Oxygen Demand (B.O.D.) and Dissolved Oxygen (D.O.) content of the sewage effluent profoundly influenced the quality of the fluid sampled at various depths in the soil beneath the spreading basin. Additional data are given concerning the chemical and bacteriological data observed at the two test sites. The effect of algae in replenishing the oxygen in the basins is also discussed.

Conclusions relating to the successful operation of these types of basins are given and are based on an analysis of the experimental data.

INTRODUCTION

The replenishment of the ground-water supply in Los Angeles County is an increasingly urgent project. Almost 66%[3] of the water used in the area comes from the underground supplies, and the accelerated industrial and agricultural demands have resulted in a serious depression of the water table. Thus, problems such as salt water intrusion, local or spot shortages, increased pumping costs, and the need for deeper wells have become evident.

The reclamation of sewage effluents has long been recognized as a means of conserving the water resources of arid regions. In fact, cesspools and septic

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[3] "South Coastal Basin Investigation; Overdraft on Ground Water Basins," *Bulletin No. 63*, State of Calif. Dept. of Public Works, Div. of Water Resources, 1947.

tanks in Los Angeles County have been returning sewage effluent to the ground water for many years. Realizing the possibilities of large-scale reclamation, the Board of Supervisors of Los Angeles County, on January 27, 1948, appointed a Board of Engineers to investigate the feasibility of conserving water reclaimed from sewage. In the course of the investigation the Los Angeles County Flood Control District constructed test spreading basins to percolate effluents from the Whittier, Calif., and Azusa, Calif., sewage treatment plants.

Scope of the Experiments.—The test spreading basins presented areas of known dimensions into which precisely measured amounts of fluids could be introduced and in which percolation rates could be closely determined. Therefore, experimental studies were made to consider the following problems:

1. The development of a method of obtaining samples suitable for testing the bacteriological, chemical, and biochemical characteristics of fluids infiltrated through the soils;
2. To make an evaluation of the hydrologic characteristics of earth basins treated with sewage effluents;
3. To make an evaluation of operating difficulties, such as odors that may arise from spreading sewage effluents;
4. To determine the dissolved solids increment in the cycle linking potable water to sewage effluent, and the probable effect of the infiltration of such effluent into the ground waters of the test area;
5. To make an evaluation of infiltration through earth beds as a means of removing the bacterial pollution of the spread waters.

The Whittier test program was undertaken as an exploration into the feasibility of reclaiming sewage effluent. After the encouragement of satisfactory performance of the spreading basin at Whittier, the percolation studies were continued at a second location at Azusa in order to evaluate the results that might be obtained under varying conditions of operation.

This paper is divided to conform with the chronological performance of infiltration studies. The discussion of the first installation at Whittier follows, with the Azusa data forming the second part of the paper.

TESTS AT THE WHITTIER SEWAGE TREATMENT PLANT

The Experimental Test Basin.—A small experimental spreading basin was constructed near the Whittier Sewage Plant. The 26-ft by 50-ft inside dimensions provided 0.028 acres of wettable area. The basin was excavated by removing 1 ft of cover from the surface of the ground; this earth was then built into levees approximately 3 ft high. The fluid to be spread was brought into a stand pipe from which it entered a weir box and passed through a V-notch weir into the basin. A hook gauge was installed in the weir box for measuring the rate of flow into the test basin, and, in addition, a continuous stage recorder took 24-hr records. A similar 24-hr record was made of the elevation of the fluid in the test basin. The fluids spread included potable well water and sewage effluents from the Whittier Plant. This plant is a 10-yr-old trickling filter plant, operating on an average daily loading of 2.8 mgd and normally delivering its effluent to irrigation or to an open water course. The sewage plant process varies from the usual only in the fact that the raw sewage is aerated before primary sedimentation.

Sampling Pans.—Samples of the percolated fluid were obtained in open-top, water-tight collection pans buried at 1-ft intervals from 4-ft to 7-ft depths below the floor of the basin. Details as to the placement and construction of the sampling pans are outlined in Fig. 1. Originally the pans at the 4-ft, 5-ft, and 6-ft levels were filled with finer sands, but difficulties with permeability led to the adoption of the coarser material originally employed in the pan at the 7-ft level as shown in Fig. 1.

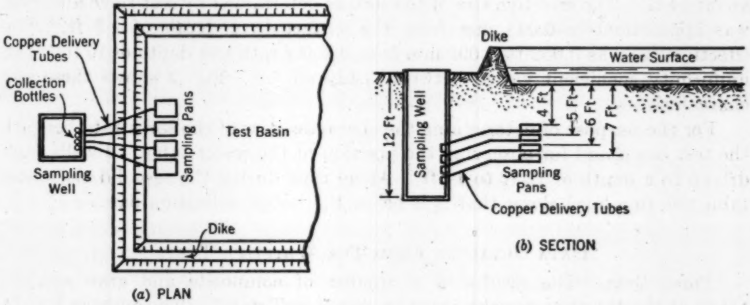
Copper Delivery Tubes
Collection Bottles
Sampling Well
Sampling Pans
Test Basin
Dike
(a) PLAN
Dike
Water Surface
Sampling Well
12 Ft
4 Ft
5 Ft
6 Ft
7 Ft
Sampling Pans
Copper Delivery Tubes
(b) SECTION

FIG. 1.—PLACEMENT OF SAMPLING PANS IN WHITTIER TEST BASIN

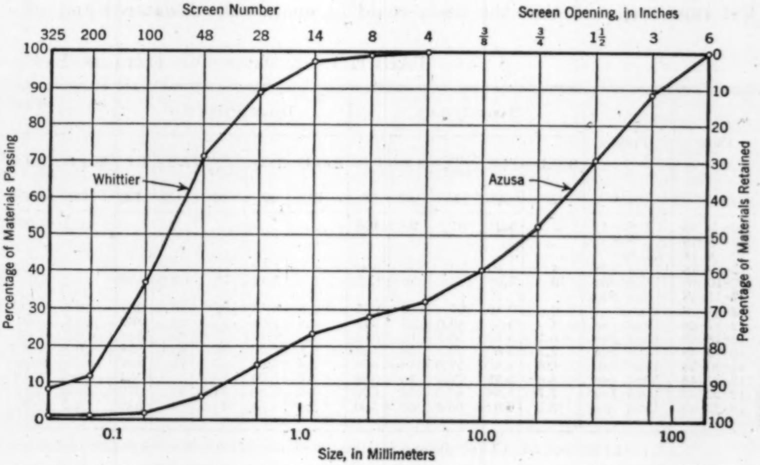

FIG. 2.—ANALYSIS OF TEST BASIN SOILS BEFORE SPREADING

Subbasin Conditions.—The type of soil upon which the Whittier Basin was constructed consists of a top layer of 4.5 ft of sandy loam varying from fine to coarse as the depth increases. From the 4.5-ft to 7-ft depth there is a sand stratum that increases in coarseness in proportion to depth. Below the 7-ft level the material is finer and test-well logs show red clays at a depth of about 14 ft. The effective size of the soil, as determined by wet sieve analysis, was approximately 0.044 mm from the surface to a depth of 4.5 ft. The effective size was 0.052 to 0.092 mm from 4.5-ft depth to a depth of 10 ft. The uniformity coefficient varied between 3.42 and 5.0. Fig. 2 shows these soil characteristics.

For the purpose of determining the characteristics of the soil strata beneath the test basin and for observing the position of the water table, 12 wells were driven to a depth of 12 ft to 14 ft. At no time during the test did the water table rise to a level closer than 3 ft below the lowest collection pan.

DATA OBTAINED FROM THE WHITTIER TESTS

Plant Data.—The results of a number of composite and grab samples taken at the Whittier Sewage Plant are listed in Table 1. The average B.O.D. of the unchlorinated secondary effluent was 14 ppm.

Hydraulic Data.—Several test runs, using various fluids, were made in the basin prior to a final 7-day test using a single fluid. The interval between test runs was such that the basin could be completely dewatered and runs prior to the final 7-day test did not exceed 28 hr in length. Potable water, chlorinated secondary effluent, and unchlorinated secondary effluent were used during the short tests; but only unchlorinated secondary effluent was used during the 7-day run. A fluid depth of 0.5 ft was maintained in the basin throughout most of the test, and the rate of infiltration varied between the upper limit of 460,000 gal per acre per day to 582,000 gal per acre per day

TABLE 1.—SUMMARY OF DATA IN PARTS PER

Date	Fluid	BASIN INFLOW					BASIN EFFLUENT					TEST WELL FOOT		
		B.O.D. (3)	D.O. (4)	Chlorine (5)	S.S.[b] (6)	T.S.[c] (7)	B.O.D. (8)	D.O. (9)	Chlorine (10)	S.S.[b] (11)	T.S.[c] (12)	B.O.D. (13)	D.O. (14)	Chlorine (15)
(1)	(2)[a]													
12- 6-48	Cl. Sec	43	5.24	81	20	408	...	...	...	...	...	<0.5	4.04	112
12- 8-48	Cl. Sec	...	...	...	...	...	...	...	...	...	...	...	...	...
12- 8-48	Cl. Sec	...	...	...	...	...	...	...	...	...	...	...	...	...
12- 9-48	Uncl. Sec	4	4.40	92	52	560	...	7.52	...	...	...	...	...	...
12-10-48	Uncl. Sec	10	3.96	142	50	622	6	5.44	71	0+	480	...	...	...
12-11-48	Uncl. Sec	...	...	...	...	...	...	...	...	...	...	...	...	...
12-14-48	Water	0.5	0.16	107	0	965	<0.5	5.16	107	16	967	<0.5	...	122
12-17-48	Uncl. Sec	4	5.64	89.5	20	542	2.5	7.04	75.8	32	462	<0.5	6.20	68
12-18-48	Uncl. Sec	6.1	3.12	58.5	12	512	4.6	6.12	82.8	52	548	<0.5	6.00	92.6
12-19-48	Uncl. Sec	7.3	5.60	63.1	48	482	6.1	6.20	86.0	40	558	<0.5	...	98.5
12-20-48	Uncl. Sec	6.3	4.88	57.9	48	456	2.3	5.60	67.9	76	492	<0.5	5.52	60.6
12-21-48	Uncl. Sec	8.6	5.88	76.1	16	588	3.5	5.24	86.3	...	504	<0.5	5.24	86.3
12-22-48	Uncl. Sec	7.7	1.64	88.3	32	636	4.6	6.80	81.2	12	572	<0.5	5.24	68.6
12-23-48	Uncl. Sec	8.5	2.67	66.1	60	560	3.4	4.68	91.2	4	580	<0.5	5.36	81.3

[a] In this column, Cl. Sec denotes chlorinated effluent of secondary treatment process and Uncl.

for chlorinated secondary effluent and the lower limit of 236,000 gal per acre per day to 378,000 gal per acre per day for the unchlorinated effluent. The effect of evaporation was ignored in the hydraulic measurements.

The final run indicated the feasibility of spreading the Whittier unchlorinated secondary effluent on the test soil at an average percolation rate of 0.5 cu ft per sec per acre or 323,000 gal per acre per day (0.99 ft per day vertical travel) without sealing the test basin or causing odor nuisances. The average (0.5 cu ft per sec per acre) percolation rate developed compares favorably with prior experience.[4]

Bacteriological Data

The Most Probable Number of Coli-Aerogenes Bacteria in Test Fluids.— The sanitary quality of water is conveniently measured by the determination of the Most Probable Number (MPN) of coli-aerogenes bacteria per cubic centimeter of sample. The unchlorinated secondary effluent from the Whittier Plant was grossly polluted, having a MPN of 110 + coli-aerogenes bacteria per cubic centimeter in all samples from the basin.

It is important to note that, after 304 hr of spreading polluted secondary effluent (168 hr continuously) at an average rate of 0.5 cu ft per sec per acre, there was no evidence of coli-aerogenes pollution in the percolated sewage samples. The tabular data show minor contamination in several of the percolated samples, but this is probably due to outside pollution resulting when the 4-ft-depth collection pan was dug up for inspection. Also, it was necessary to collect samples of percolated fluid slowly in sterile bottles open to the atmosphere for several hours.

Million—Whittier, Calif., Spreading Basin

—Four-Level		Test Well—Five-Foot Level					Test Well—Six-Foot Level					Test Well—Seven-Foot Level				
S.S.[b]	T.S.[c]	B.O.D.	D.O.	Chlorine	S.S.[b]	T.S.[c]	B.O.D.	D.O.	Chlorine	S.S.[b]	T.S.[c]	B.O.D.	D.O.	Chlorine	S.S.[b]	T.S.[c]
(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)
48	816	<0.5	5.80	137	32	944	<0.5	3.80	122	22	1,020	<0.5	7.24	117	18	672
...	...	...	...	...	...	...	...	5.12	...	...	...	...	5.64	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	...	<0.5	8.80	152	...	...	<0.5	3.64	112	...	...	<0.5	6.60	122	8	600
...	...	...	...	...	...	...	<0.5	...	107	...	1,032	<0.5	5.64	122	...	432
...	...	...	...	...	...	...	<0.5	...	103	...	1,054	...	7.04	93	0	584
20	1,020	...	...	...	...	...	...	...	...	...	...	<0.5	5.52	99	4	663
0	988	...	...	...	...	...	...	...	...	...	...	<0.5	6.28	101	0	854
0	888	...	...	...	...	...	...	...	...	...	...	<0.5	4.88	89.5	0	780
0	700	...	...	...	...	...	...	...	...	...	...	<0.5	5.04	84.8	0	710
0	534	...	...	...	...	...	...	...	...	...	...	<0.5	6.48	94.4	0	660
0	504	...	...	...	...	...	<0.5	...	82.8	0	904	<0.5	6.00	73.6	0	528
0	708	...	...	...	...	...	...	...	...	...	...	<0.5	6.00	87.4	0	584
0	504	...	...	...	...	...	<0.5	...	91.4	0	904	<0.5	6.04	86.2	0	610

Sec denotes unchlorinated effluent of secondary treatment process. [b] Suspended solids. [c] Total solids.

Soil samples from a test hole dug in the basin at the conclusion of the tests seemed to confirm the foregoing findings, for the soil showed coli-aerogenes pollution ranging from 100 to 6,000 organisms per cu cm at the surface to 40 per cu cm at the 3-ft depth, and negative results for coli-aerogenes bacteria below the 3-ft level.

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[4] "Filtering Materials for Sewage Treatment Plants," *Manual of Engineering Practice No. 13*, ASCE, 1937, p. 16.

CHEMICAL AND PHYSICAL DATA

Dissolved Oxygen Content and Biochemical Oxygen Demand.—The D.O. content of the treated effluents received at the test basin averaged 4.2 ppm. After exposure to the atmosphere the D.O. had increased to an average of 5.95 ppm at the far end of the basin, and the samples of percolated fluid obtained at the 7-ft level averaged 5.6 ppm. The B.O.D. of the spread waters varied inversely with the D.O., and the inflowing sewage effluents averaged 6.9 ppm of B.O.D. and percolated fluid at the 7-ft depth averaged less than 0.5 ppm B.O.D.

These D.O. and B.O.D. values, when combined with the bacteriological results, seem to be of great importance in formulating an explanation of the mechanisms involved in the purification of the percolated sewage.

From these data it was postulated that the purification process was analogous to trickling filter operation. As long as an adequate D.O. content was maintained, aerobic zoogloeal bacteria would develop in and on the surface of the basin and perhaps to some depth in the soil. These would act, as in trickling filters, to adsorb quickly the organic matter from the percolating fluids and, in addition, from a mechanical filter mat where bacteria would be trapped and gradually die out.

From the experimental results it was empirically decided that a satisfactory percolating fluid should have 0.5 ppm of D.O. and less than 0.5 ppm of B.O.D. at the sampling pan level that was to be used for control.

The B.O.D. of the soil samples obtained at the end of the tests seemed to confirm this hypothesis. A substantial B.O.D. was obtained in the top soil, but at the 4-ft sampling point this B.O.D. decreased to less than 0.5 ppm and no B.O.D. was found at lower depths.

Suspended Solids Present in the Percolated and Unpercolated Effluents.—The quantity of suspended solids in the fluid introduced into the spreading basin influences the percolation rate. These solids settle initially and form a mat upon the surface of the bed, acting to seal off the percolation of the fluid. In addition, the organic matter present in the mat undergoes decomposition. Hence, in order to maintain aerobic conditions, the B.O.D. of the organic matter must be satisfied.

If aerobic conditions are maintained in the bed, decomposition of the organic matter in the suspended solids mat is apparently quite rapid. In the experimental test basin a relatively constant percolation rate was reached. However, inorganic materials and relatively stable undigested organic matter, together with zoogleal slimes, algae, and so forth, in time accumulate within the surface soil and necessitate drying and scarifying of the surface of the basin.

If the D.O. to B.O.D. relationship is such that no D.O. is found at the test level and a B.O.D. in excess of 0.5 ppm is obtained, the solids mat builds up quite swiftly, reducing the percolation rate and forcing early drying and scraping of the basin surface. Furthermore, the Azusa tests indicate that when these conditions prevail anaerobic environment exists and coli-aerogenes bacteria are carried by the percolating fluid.

The suspended solids content of the unchlorinated effluent entering the Whittier test basin averaged 34 ppm, but at all sampling depths the percolated fluid showed an absence of suspended solids.

The pH, total solids, and other characteristics of the Whittier sewage were found to be normal for domestic sewage in the southern California area (see Table 1).

Temperature and Weather Data.—The studies described were made between the dates of November 29, 1948, and December 23, 1948. During this period the temperature of the influent into the basin ranged from 48° F to 72° F and the basin temperature ranged between 43° F and 71° F. The average air temperature during December, 1948, was 53.4° F. Thus the tests were made during a period when temperatures favored oxygen absorption and minimum odors and biochemical action was at a low rate. No nuisance arose from the operation of the bed, and the loading was such that sparkingly clear effluent of good sanitary quality was produced at the 7-ft level.

The experimental data from the Whittier test are summarized in Table 1 and Fig. 3.

Test at the Azusa Sewage Treatment Plant

The Experimental Test Basin.—A second experimental spreading basin was established adjacent to the Azusa Sewage Treatment Plant in order to: (*a*) Check the results of the original Whittier Studies; (*b*) evaluate the effect of the coarser soils upon the spreading phenomenon; (*c*) evaluate the results of spreading in a larger basin; (*d*) determine the effects of continuous long-term spreading operations throughout a period of more than 60 days; (*e*) observe the effects of warm weather operation of the beds with emphasis on odor development; (*f*) determine the dissolved mineral pickup in the normal domestic use of water as a refuse carrier and from this to estimate the number of cycles of possible reuse of the reclaimed water; (*g*) test the effect of spreading effluents with varying values of B.O.D.; (*h*) observe the effect of algae action upon the spreading phenomena; and (*i*) evaluate the effects of an anaerobic environment on the spread fluid.

At the Azusa treatment plant an unused percolation basin was reconstructed to provide a bed 50 ft by 70 ft with 3-ft fluid depth. The inflow structure and flow and depth recording mechanisms were similar to those employed at the Whittier site.

The Azusa treatment plant is a 9-yr-old trickling filter plant handling primarily domestic sewage and operating at the designed capacity of 475,000 gal of raw sewage a day. Unusual features in plant operation include: (1) The primary settled sewage is chlorinated before discharging onto the trickling filter; and (2) the plant is operated by one man with a minimum of maintenance difficulties.

Throughout the entire operating history of this plant the effluent has been disposed of in percolation basins. These basins are operated by discharging the total final effluent flow of the plant into a single basin for a 7-day period, after which it is diverted to another basin. The flooded basin is allowed to dry and is then reused. In time a mat of inorganic and organic matter builds up on the bottom of the basin reducing a percolation rate and necessitating the removal of a few inches of top soil in order to restore the percolation rate.

The experimental basin was similarly operated although the amount of fluid spread was closely controlled. The period of application was for much longer than 7 days, and for a one-week period an admixture of primary and secondary effluents was spread.

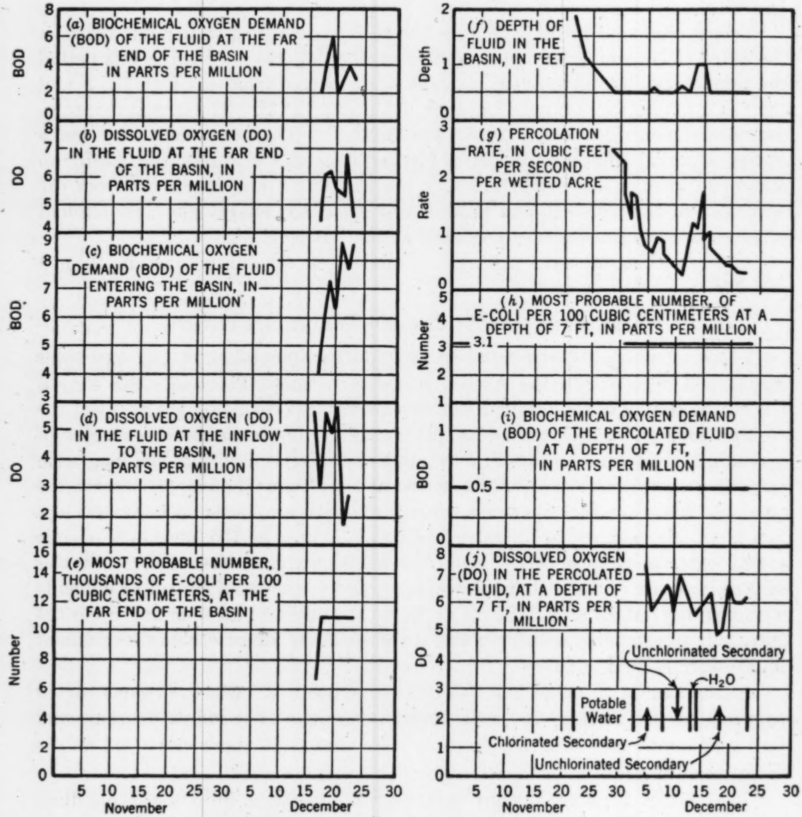

FIG. 3.—OPERATIONAL RECORD, WHITTIER, CALIF., SPREADING BASINS

Sampling pans similar to those developed during the Whittier tests were used, with the recommended filling material graded from $\frac{1}{4}$-in. gravel at the bottom to 1/30-in. sand at the top. Only two sampling points were used, with 3 pans placed at the 7-ft depth to facilitate rapid sampling, and one placed at the 2½-ft depth to provide a check on the rapidity of purification of the percolating fluids. As at Whittier, a sampling well was employed to obtain samples of the percolated fluids.

The Azusa spreading basins are underlain by coarse sand and gravel. A test hole dug to the depth of 10 ft indicated an effective size varying between 0.36 mm and 0.45 mm, with a uniformity coefficient between 40.5 and 91.0 (see Fig. 2). The log of the Glendora Consolidated Irrigation Company Vosburg Well, located adjacent to the sewage plant, shows the underground formation to consist of gravel and boulders to a depth of 414 ft. The water table stands at depths exceeding 200 ft below the ground surface, and on September 1, 1949, was at 264 ft.

DATA OBTAINED FROM THE AZUSA TESTS

Plant Data.—Throughout most of the testing period a supply of D.O. was present in the sewage at all points in the treatment process including the plant influent. The secondary effluent D.O. varied from 1.87 ppm to 3.68 ppm. The secondary effluent B.O.D. averaged 33 ppm. A listing of the complete test data can be found in Tables 2 and 3.

Hydraulic Data.—The Azusa reclamation tests started with the spreading of final treated effluent for 5 days. After this time a mixture of primary and secondary effluents was introduced into the spreading basin for a period of 1 week. The test basin was then returned to the spreading of final treated effluent. After 1 month of continuous percolation the basin was dried up. After a subsequent 1-week drying period soil samples were taken and secondary effluent was again percolated for a 2-month continuous period.

By the admixture of primary effluent to the final effluent a fluid with 100 ppm of B.O.D. was obtained. When this fluid was introduced into the basin to a depth of 2 ft, the available oxygen supply was quickly depleted and B.O.D.'s as high as 24 ppm were obtained at the 7-ft level. A suspended solids content was also found in the percolated fluid at all test levels, and the percolation rate dropped sharply. The percolation rate for this first 30-day test started at approximately 10 cu ft per sec per wetted acre, but at the end of the period it had dropped to 0.9 cu ft per sec per wetted acre. During a continuous run of 2 months the predominant percolation rate was 0.6 cu ft per sec per wetted acre although the rate fluctuated from 0.19 cu ft per sec per wetted acre to greater than 2 cu ft per sec. The fluid depth averaged 2 ft.

The extended Azusa test indicated that in a soil with the characteristics shown in Fig. 2 a fluid with 33 ppm of B.O.D., 70 ppm of suspended solids, and 2.2 ppm of D.O. would achieve a percolation rate (0.6 cu ft per sec per acre) that would remain relatively constant for a long period of time; and soil of this nature would deliver an effluent of a satisfactory sanitary quality that would cause no odor or operating difficulties. It is of interest to note in Fig. 4 the inverse variation in percolation rate to the observed D.O. content as the bed condition changed from the time at which dissolved oxygen and a B.O.D. of less than 0.5 ppm were found in the percolated fluid at the 7-ft level to the time when no D.O. and a substantial B.O.D. existed, and then back to the original condition.

The fine Whittier soil gave a maximum percolation rate that could be varied little, if at all, by measures such as increasing the head on the basin whereas the percolation rate in the coarse soil of the Azusa basin could initially be varied by changing the head. Thus, practically, the organic loading on the Whittier basin could be controlled by the B.O.D. alone, but both the initial percolation rate and the B.O.D. were possible variables at Azusa. However, after continuous spreading over a considerable period a surface mat layer is built up in any type of soil. Hence this interface becomes a controlling factor in the ultimate rate of percolation regardless of the soil coarseness.

TABLE 2.—MINERAL ANALYSES IN PPM—AZUSA SPREADING TEST

Sample	Total Solids	COMPONENT												pH
		Sodium (Na^+)	Potassium (K^+)	Calcium (Ca^{++})	Magnesium (Mg^{++})	Sulfate ($SO_4^{-}+$)	Bicarbonate (HCO_3^-)	Chloride (Cl^-)	Nitrate (NO_3^-)	Ammonia (NH_3)	Nitrogen Organic (N)	Boron (B)	
(*a*) AZUSA PLANT, MAY 19, 1949														
Raw Influent	550	61.0	12.4	44.4	14.4	19.8	369	61.5	0.1	34.6	14.5	0.5	7.1
Primary Effluent	554	64.0	13.6	47.2	15.0	3.3	370	70.1	0.08	33	12.7	<0.1	7.1
Trickling Filter Effluent	506	64.9	15.6	43.5	12.6	42.8	182	68.6	1.4	5.4	7.8	0.2	7.0
Influent Percolation Basin	494	64.0	15.6	49.3	12.0	47.3	182	67.0	2.0	4.6	6.4	0.2	7.0
Far End of Percolation Basin	430	58.0	20.8	32.0	18.3	42.4	176	67.6	1.6	5.6	6.3	0.1	7.0
(*b*) AZUSA WATER SYSTEM, MAY 17, 1949														
Azusa Top Water	381	9.6	3.1	54.4	14.4	27.2	206	11.2	0.2	...	...	0.1	7.4
Well No. 1	391	9.3	3.1	52.0	14.6	28.0	206	9.7	0.56	...	...	0.1	7.3
Well No. 2	224	...	...	...	...	...	205	7.6	...	...	...	...	7.3
Well No. 3	200	9.2	4.5	48.0	13.2	22.6	193	7.6	1.2	...	...	0.2	7.3
Well No. 4	400	19.6	1.6	76.8	16.4	37.4	283	21.3	1.6	...	...	0.1	7.3
(*c*) AZUSA PLANT, JUNE 17, 1949														
Raw Influent	720	82.1	12.4	...	...	...	383	103.7	0.13	29	7.7	1.3	...
Primary Influent	532	61.0	12.4	...	...	...	390	51.8	0.8	23.4	9.1	1.6	...
Trickling Filter Effluent	512	62.6	13.1	...	...	...	200	45.8	0.4	3.2	3.8	<0.1	...
Influent Percolation Basin	480	64.8	12.8	...	...	...	193	45.8	0.4	5.6	3.9	1.7	...
Far End of Percolation Basin	488	64.0	13.3	...	...	...	176	50.8	0.28	0	7.3	1.2	...
(*d*) AZUSA PLANT, JUNE 20, 1949														
Influent Percolation Basin	606	...	...	...	...	...	220	62.8	0.8	9.4	8.0	...	...
Far End of Percolation Basin	480	...	...	...	...	...	194	41.6	1.0	3.9	5.2	...	...
Percolation Fluid 2½-ft Depth	400	53	12.2	51.4	15.5	56.4	205	40.5	1.6	0.3	2.4	0.5	...
Percolation Fluid 7-ft Depth	420	53	12.5	51.2	15.5	48.1	228	43.7	0.4	0.4	2.4	0.3	...
(*e*) METROPOLITAN WATER DISTRICT 1947 TO 1948														
...	727	190	3	30	14	319	99	98	0.01	...	...	0.14	...

TABLE 3.—SUMMARY OF DATA—AZUSA, CALIF., SPREADING BASIN

Location (1)	Time (2)	B.O.D. (ppm) (3)	D.O. (ppm) (4)	MPN coli-aerogenes (per cc) (5)	pH (6)	Temperature (degrees F) (7)
(*a*) JULY 1, 1949						
Inflow to basin	....	38.3	2.80	....	....	....
Far end of basin	....	27.4	1.60	700	....	....
Percolated fluid 2½-ft depth	....	2.0	0.75	2.40	....	....
Percolated fluid 7-ft depth	....	<0.5	0.30	0.62 2.40	....	....
(*b*) JULY 3, 1949						
Percolated fluid 2½-ft depth	....	....	....	70	....	....
Percolated fluid 7-ft depth	....	....	....	70	....	....
(*c*) JULY 5, 1949						
Inflow to basin	....	....	2.4	....	....	....
Far end of basin	....	....	2.4	....	....	....
Percolated fluid 2½-ft depth	....	....	1.6	24	....	....
Percolated fluid 7-ft depth	....	....	0	24 7 2.4	....	....
(*d*) JULY 7, 1949						
Inflow to basin	....	....	2.0	....	....	....
Far end of basin	....	....	3.6	....	....	....
Percolated fluid 2½-ft detph	....	....	1.6	0.031	....	....
Percolated fluid 7-ft depth	....	....	0	0.031	....	....
(*e*) JULY 11, 1949						
Inflow to basin	....	81.9	....	....	....	....
Far end of basin	....	14.5	....	....	....	....
Percolated fluid 2½-ft depth	....	4.3	....	....	....	....
Percolated fluid 7-ft depth	....	1.4	....	....	....	....
(*f*) JULY 14, 1949 PERCOLATION RATE = 0.51 CU FT PER SEC PER ACRE						
Inflow to basin	0945	....	2.4	....	7.3	78
Far end of basin	0855	....	5.9	....	7.1	76
Percolated fluid 2½-ft depth	1120	....	0.9	0.06	7.3	76
Percolated fluid 7-ft depth	1010	....	0	0.03	7.2	74
(*g*) JULY 19, 1949 PERCOLATION RATE = 0.53 CU FT PER SEC PER ACRE						
Inflow to basin	1125	62.8	3.32	....	7.4	78
Far end of basin	1110	14.8	19.20	....	8.0	85
Percolated fluid 2½-ft depth	1100	0.7	0.28	....	7.2	78
Percolated fluid 7-ft depth	1100	<0.5	0.20	....	7.2	77
(*h*) JULY 21, 1949 PERCOLATION RATE = 0.73 CU FT PER SEC PER ACRE						
Inflow to basin	0900	....	4.0	....	7.5	77
Far end of basin	0855	....	7.0	....	7.6	76
Percolated fluid 2½-ft depth	0825	....	0.2	....	7.2	76
Percolated fluid 7-ft depth	0835	....	0	....		

TABLE 3.—*Continued*

Location (1)	Time (2)	B.O.D. (ppm) (3)	D.O. (ppm) (4)	MPN coli-aerogenes (per cc) (5)	pH (6)	Temperature (degrees F) (7)
(*i*) JULY 26, 1949 PERCOLATION RATE = 0.73 CU FT PER SEC PER ACRE						
Inflow to basin	0945	29.30	2.88	120+	7.2	76
Far end of basin	0925	36.10	2.80	0.62	7.3	73
Percolated fluid 2½-ft depth	0820	0.77	0	0.23	7.2	75
7-ft depth	0850	0.50	0	0.23	7.2	75
(*j*) JULY 28, 1949 PERCOLATION RATE = 0.63 CU FT PER SEC PER ACRE						
Inflow to basin	0945	....	2.80	....	7.2	73
Far end of basin	0930	....	2.32	....	7.4	72
Percolated fluid 2½-ft depth	1040	....	2.38	0.23	7.2	74
7-ft depth	0915	....	0	1.03	7.2	75
(*k*) AUGUST 2, 1949 PERCOLATION RATE = 0.58 CU FT PER SEC PER ACRE						
Inflow to basin	1030	42.1	4.44	13	7.4	80
Far end of basin	1040	15.4	7.56	120+	7.4	83
Percolated fluid 2½-ft depth	0850	6.3	0	0.61	7.2	77
7-ft depth	0855	9.9	2.20	0.29	7.2	76
(*l*) AUGUST, 4, 1949 PERCOLATION RATE = 0.58 CU FT PER SEC PER ACRE						
Inflow to basin	1030	....	5.48	24	7.4	82
Far end of basin	1015	....	9.56	24	7.6	83
Percolated fluid 2½-ft depth	0900	....	0	0.031 −	7.2	79
7-ft depth	0910	....	0.28	0.060	7.2	77
(*m*) AUGUST 9, 1949 PERCOLATION RATE = 0.63 CU FT PER SEC PER ACRE						
Inflow to basin	0935	28.7	4.20	120+	7.2	76
Far end of basin	0945	18.0	5.20	120+	7.4	75
Percolated fluid 2½-ft depth	0910	15.5	4.00	....	7.2	78
7-ft depth	0920	<0.5	0.88	0.62	7.2	76
(*n*) AUGUST 11, 1949 PERCOLATION RATE = 0.24 CU FT PER SEC PER ACRE						
Inflow to basin	0950	....	5.60	120+	....	75
Far end of basin	1000	....	6.04	70	....	74
Percolated fluid 2½-ft depth	0925	....	3.36	....	....	75
7-ft depth	0940	....	0	0.06	....	78
(*o*) AUGUST 17, 1949 PERCOLATION RATE = 0.19 CU FT PER SEC PER ACRE						
Inflow to basin	1110	36.1	3.20	120+	....	75
Far end of basin	1105	10.0	5.92	70	....	74
Percolated fluid 2½-ft depth	1030	0.8	0	0.031 −	....	76
7-ft depth	1045	10.1	0.08	0.031 −	....	75
(*p*) AUGUST 19, 1949 PERCOLATION RATE = 0.87 CU FT PER SEC PER ACRE						
Inflow to basin	1135	....	3.8	120	....	76
Far end of basin	1130	....	4.8	70	....	83
Percolated fluid 2½-ft depth	1115	....	0	0.060	....	80
7-ft depth	1120	....	0	0.031 −	....	77

TABLE 3.—*Continued*

Location (1)	Time (2)	B.O.D. (ppm) (3)	D.O. (ppm) (4)	MPN coli-aerogenes (per cc) (5)	pH (6)	Temperature (degrees F) (7)
(q) August 23, 1949 Percolation Rate = 0.53 cu ft per sec per acre						
Inflow to basin	1020	9.4	3.60	120+	....	....
Far end of basin	1030	7.1	1.56	120+	....	....
Percolated fluid 2½-ft depth	0945	6.4	0	0.06	....	77
Percolated fluid 7-ft depth	0955	1.3	0.68	0.06	....	77
(r) August 25, 1949 Percolation Rate = 1.16 cu ft per sec per acre						
Inflow to basin	0945	30.3	3.96	....	....	78
Far end of basin	0935	2.5	7.28	....	....	76
Percolated fluid 2½-ft depth	0925	....	4.32	....	....	78
Percolated fluid 7-ft depth	0940	<0.5	0.64	....	....	76
(s) August 30, 1949 Percolation Rate = 0.29–0.19 cu ft per sec per acre						
Inflow to basin	0820	11.5	2.16	....	....	78
Far end of basin	0810	8.6	6.30	....	....	76
Percolated fluid 2½-ft depth	....	....	....	....	....	....
Percolated fluid 7-ft depth	0825	<0.5	0.40	....	....	76
(t) September 1, 1949 Percolation Rate = 0.19–0.24 cu ft per sec per acre						
Inflow to basin	1005	12.0	3.40	120+	....	79
Far end of basin	1000	9.5	14.52	2.40	....	78
Percolated fluid 2½-ft depth	....	....	....	....	....	....
Percolated fluid 7-ft depth	1020	1.9	2.48	0.03−	....	76

Recommended Test Basin Operations.—The experimental work and available data do not warrant conclusions as to the operation of the beds. It is probable that operational schedules would have to be fixed for each soil and each spreading location. In the Los Angeles area the Azusa sewage plant has successfully used the procedure already mentioned, but the Polaris Flight Academy, at Lancaster, Calif., in a very tight soil, has used 1 day of filling followed by drying and harrowing of the basin. It seems certain, then, that local conditions such as soil, type of effluent, available land and equipment, and similar factors will decide operational procedure.

Bacteriological Data.—As in the Whittier tests, the secondary plant effluent entering the basin was grossly polluted, having 120 + coli-aerogenes organism per cu cm in the incoming fluid and at the far end of the basin. During the first week of the tests, when only secondary plant effluent was spread, the bacterial samples at the 7-ft depth were all negative. However, when the mixed primary and secondary effluents were added to the basin and anaerobic conditions developed in the soil, the coli-aerogenes counts in the percolated fluid at the 2½-ft and 7-ft levels rose to 1,200 + per cu cm. When secondary effluent was again used and aerobic conditions were gradually re-established, the coli-aerogenes counts dropped from 0.62 to 0.31 − per cu cm. This approaches drinking water quality. Thus it seems probable that when anaerobic conditions exist in the basin and in the soil, bacterial pollution can be carried to some depth by the percolating waters, but if aerobic conditions prevail the coli-aerogenes group is largely eliminated.

Soil samples were taken in two of the dry operating basins and at the test basin before it was placed in use. These samples showed bacterial pollution ranging between 100 and 6,000 coli-aerogenes groups per cu cm in the surface soil to 40 per cu cm in the soil at the 3-ft depth. Below this depth no coli-aerogenes were found. However, when soil samples were taken after the test basin had become anaerobic, coli-aerogenes were found at all test depths. The following 2 months of aerobic operation gradually eliminated the bacterial contamination.

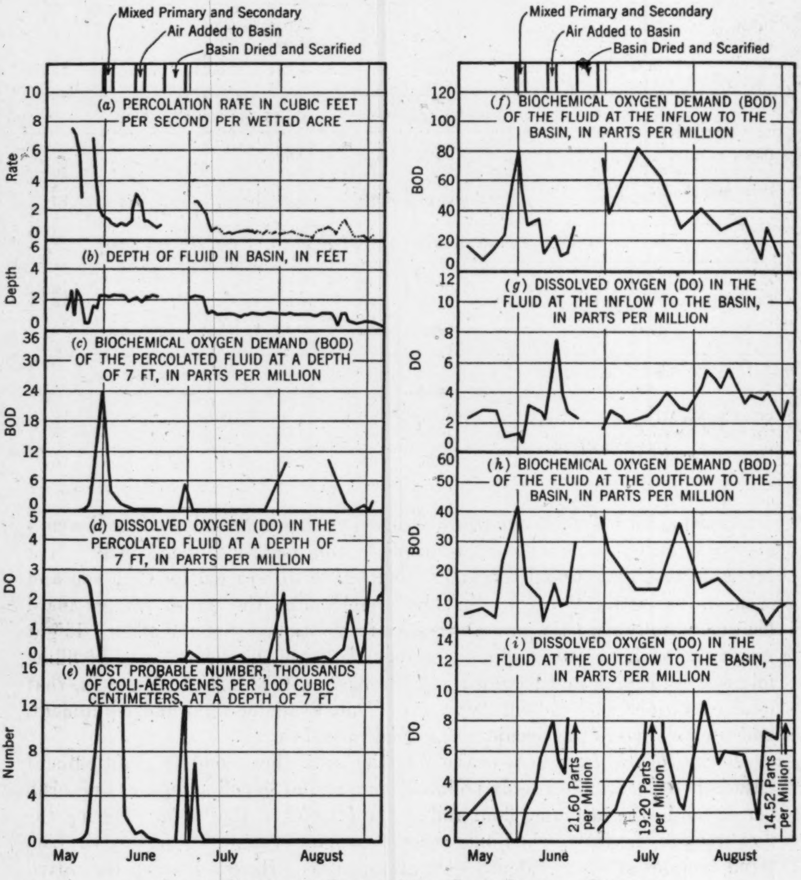

FIG. 4.—OPERATIONAL RECORD, AZUSA, CALIF., SPREADING BASINS

Bio-Chemical Data.—The D.O. content of the secondary effluent entering the basin averaged 2.2 ppm, and the D.O. of the percolated fluid at the 2½-ft depth averaged 1.5 ppm, and at the 7-ft depth the fluid was generally depleted of oxygen. When primary effluent was added to the basin no oxygen was found in any of the percolated samples.

The B.O.D. of the secondary effluent entering the test basin at Azusa averaged 23 ppm. At the overflow end of the basin the B.O.D. had dropped to 15 ppm, and at the 7-ft depth the percolated fluid showed a B.O.D. of 1.6 ppm. The mixture of primary and secondary effluents had a B.O.D. of approximately 100 ppm, and after this fluid had been added to the basin for 5 days, the percolated fluid at the 7-ft depth showed a B.O.D. of 25 ppm. When secondary effluent was again used, the B.O.D. at the 7-ft level gradually dropped to the 1.6-ppm average given previously.

Conclusions From Data

From the bacterial, D.O., and B.O.D. data on the percolated fluid at the 7-ft level, certain conclusions can be drawn. The Whittier test basin had delivered a satisfactory effluent at the 7-ft level during the whole test period. Certain factors such as soil characteristics and weather were very different in the two tests, but the major differences seemed to be in the total organic loading and in the action of algae. It was assumed during the Whittier test that a good working criterion would be 0.5 ppm of D.O. and less than 0.5 ppm of B.O.D. at the test level. The test level would of course be dependent upon the ground-water level. This limit still seems to be an acceptable working rule since the sanitary quality of the Azusa effluent throughout the aerobic test averaged just a little poorer than that acceptable for drinking water. The Azusa plant effluent averaged 2.2 ppm of D.O. and 33 ppm of B.O.D. and was percolated at a rate of 0.6 cu ft per sec per acre. The Whittier waters, with 14 ppm of B.O.D. and 5.95 ppm of D.O., were spread at a rate of 0.5 cu ft per sec per acre. The organic loading at Azusa through hydraulic control averaged about twice that at Whittier. Knowing both the oxygen demand and the D.O. of the influent to the basins and the D.O. or the oxygen demand of the percolated fluid at the sampling level (or both), the approximate quantity of additional oxygen needed can be calculated. It seems probable that the permissible loading will approach a constant on all types of soil except for factors such as temperature, the action of algae in supplying extra oxygen, and the relation between the B.O.D. value and reoxygenation. The method used in these experiments to calculate the amount of oxygen supplied to the basin by the atmosphere is given here:

The amount of oxygen supplied by the atmosphere to a fluid percolated from earth basins can be estimated by difference. Thus if the oxygen demand, original D.O. of the fluid, or the D.O. and oxygen demand of the fluid at the testing depth is known, a calculation can be made.

Assuming no oxygen is present in the top soil and the spread fluid temperature averaged 75° F, let O.L. equal the oxygen load of the spreading basin in pounds of oxygen absorbed in 1 day by 1 wetted acre of spreading basin. Then, O.L. is the rate of percolation in millions of pounds of spread fluid per day times the quantity given by adding the 5-day B.O.D. in ppm of the spread fluid to the D.O. of the percolated fluid at the testing depth subtracting the D.O. at the inflow to the basin and also subtracting the 5-day B.O.D. in ppm of the percolated fluid at the testing depth.

Using the data obtained from the Whittier and Azusa tests, calculations of the actual amount of oxygen absorbed can be made.

	Whittier	Azusa
B.O.D. of basin influent (ppm)	14	33
B.O.D. of fluid at 7-ft depth (ppm)	0	1.6
D.O. of basin influent (ppm)	4.3	2.2
D.O. of fluid at 7-ft depth (ppm)	5.6	0.2
Percolation rate (cu ft per sec per acre)	0.5	0.6
Percolation rate (lb per acre per day)	2.695	3.234
Oxygen load (lb per acre per day)	41	95

On the basis of the above figures, it is indicated that 41 lb of oxygen were supplied per wetted acre at the Whittier basin and 95 lb were supplied at the Azusa basin. The temperature and the oxygen in the soil would make some difference, but the action of algae and increased rate of reoxygenation in the higher B.O.D. Azusa waters seem to offer the most logical explanation. The Azusa tests were made when temperature and weather conditions were at an optimum for the growth of algae, while the Whittier tests were made when algae action was at a minimum. Previous experiments,[5,6,7,8,9] as well as 24-hr D.O. tests made at Azusa (see Fig. 5) show that algae can introduce very large quantities of D.O. into waters in which they are present.

The calculation is not exact and test plots such as those described here would be desirable, if not necessary, at most locations where the spreading of treated sewage effluents is proposed. However, the calculation when used in connection with known test plot data will give an accurate estimation of the organic loading permissible at any proposed location. Thus, if the factor O.L. is calculated for a certain test plot, this value can be reinserted into the equation along with the desired standards of 0.5 ppm of D.O. and less than 0.5 ppm of B.O.D. in the percolated fluid at the test depth. The factors left to determine are the B.O.D. of the basin influent and the percolation rate. Since the influent B.O.D. is usually known, the safe percolation rate can then be calculated. Also if a limited percolation rate is possible the maximum permissible influent B.O.D. can be determined.

Solids Present in the Percolated and Unpercolated Effluents

Suspended Solids.—In the Azusa test basin the secondary effluent added an average of 70 ppm of suspended solids. At the overflow end the suspended solids content had dropped to 32 ppm, and at the 2½-ft level the percolated fluid showed 9 ppm. At the 7-ft level 3 ppm of suspended solids were present during the aerobic period then when mixed primary and secondary effluents were being added and anaerobic conditions existed, the suspended solids in the

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[5] "The Role of Algae in Waste Treatment," by A. J. Espinosa, *Public Works*, Vol. 79, August, 1948, p. 36.

[6] "Sewage Lagooning," by Jack Meyers, *ibid.* Vol. 80, January, 1949, p. 45.

[7] "Oxidation Ponds," Report of the Committee on Sewage Disposal, Eng. Section, Am. Public Health Assn., *Sewage Works Journal*, Vol. 20, 1948, p. 1025.

[8] "Sewage Oxidation Ponds—Performance Operation and Design," by D. H. Caldwell, *ibid.*, Vol. 18, 1946, p. 433.

[9] "Rate of Production of Oxygen by Freely Developing Algae," by W. E. Abbot, *Water and Sewage Works*, Vol. 96, 1949, p. 141.

percolated fluid at the 7-ft depth rose to 24 ppm. When aerobic conditions were re-established, the solids in the percolated samples quickly dropped to the average of 3 ppm. Illustrative of the effect of oxygen on the solids mat and thus percolation rate, is the fact that, when large quantities of air were diffused into the basin, the percolation rate increased 1 cu ft per sec per wetted acre. It seems apparent then that the maintenance of aerobic conditions is important both from the standpoint of delivering a percolated effluent of a satisfactory sanitary quality and from that of maintaining satisfactory percolation rates.

Total Solids.—The secondary plant effluent received at the test basin averaged 510 ppm of total solids, and the percolated fluid at the sampling depths averaged 435 ppm (see Table 3). There were 3 ppm of suspended solids and 432 ppm of dissolved solids in the percolated fluid. The total and dissolved solids loading of the Azusa water supply averaged 260 ppm during the test period. A well subsequently added to the system has increased this somewhat. Thus it can be seen that there is a dissolved solids increase of 172 ppm as the water passes from a potable water supply to sewage effluent. The total dissolved solids content of the unsoftened Metropolitan Water District water is 737 ppm,[10] and the upper limit for the permissible dissolved solids content of irrigation water, assuming allowable chemical characteristics, is 1,400 ppm.[11]

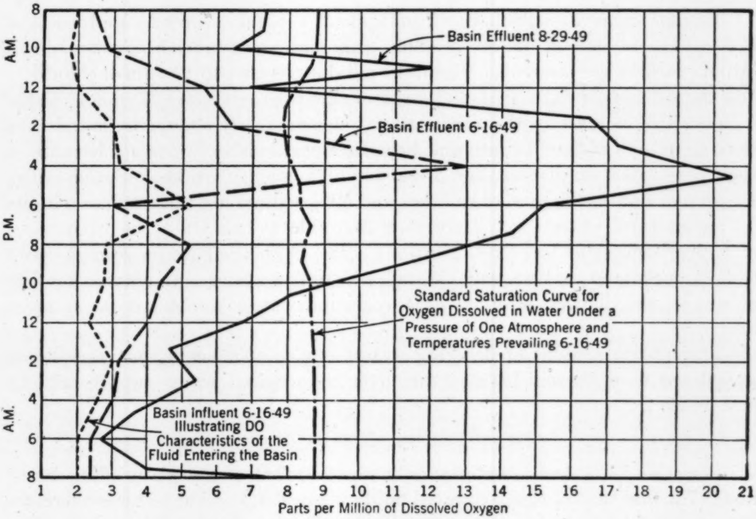

FIG. 5.—EFFECT OF ALGAE GROWTH ON D.O. CONTENT OF FLUID

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[10] From Table of "Chemical Analyses of Los Angeles Water—Fiscal Year 1947–48," Los Angeles Department of Water and Power, Los Angeles, Calif.

[11] Report Upon the Reclamation of Water from Sewage and Industrial Wastes in Los Angeles County, California, by C. E. Arnold, H. E. Hedger, and A M Rawn, Los Angeles County Sanitation Districts, Los Angeles, Calif., April, 1949.

The Azusa sewage is representative of the domestic sewage of the Los Angeles area, so the 172-ppm pickup of dissolved solids might be considered as an average usable figure for this region. If, for example, the Azusa percolated effluent is used with 432 ppm of dissolved solids and it is assumed there is a standard pickup of about 172 ppm of nontoxic minerals each time the water is reused, the percolated effluent must be used twice more to reach the total solids content of the Metropolitan Water District water and 3½ times more to reach the limits for irrigation water. Rain-water and ground-water dilution provide an additional safety factor. Hence, it is probable that if the sanitary quality and content of undesireable minerals is controlled, treated domestic sewages similar to those reported here can be percolated into the underground with safety.

To determine the ultimate effect upon the potable water supply of the 172-ppm increment of dissolved solids, this increment must be broken down into its various components. Thus, the potable waters had 12 ppm of chlorides, and the percolated fluids had 49 ppm, or a chloride increase of 37 ppm. Table 2 shows the increase in the other critical elements. It should be noted that the amounts of sodium sulfate and boron are particularly important because of their toxic effect on plants when present in high concentrations. For instance, 1.5 ppm of boron, 400 ppm of sulfates, and 50% sodium are the maximum limits for agricultural water. Similarly, it is desirable to maintain the carbonates and bicarbonates at less than 250 ppm. The percolated Azusa sewage could thus be reclaimed continuously through two to five cycles. In actual practice, of course, the dilution afforded by the ground waters would cut the mineral load.

The nitrate content of drinking waters is also of concern, but samples of percolated fluid showed little pickup over the original water supply content (see Table 2).

THE pH OF THE FLUIDS TESTED

The pH of the Azusa water supply was 7.5, that of the plant effluent was about 7.2, and that of the percolated fluid averaged 7.3. The pH was therefore close enough to neutral to cause little or no effect on the ground waters. The pH of the spread fluids rose as high as 8.3 when profuse light algae growths developed.

THE EFFECT OF ALGAE ON THE SPREADING OF EFFLUENT

Algae will grow in an aerobic fluid when sunshine is present and temperature conditions are favorable. They are particularly sensitive to temperature,[8] so the Whittier basin, operating during the winter, received very little oxygen from the action of algae, while the Azusa basin received large quantities of oxygen. The algae supply oxygen to the fluids they grow in as a result of their life processes, wherein carbon dioxide is synthesized to starch, using chlorophyll and sunlight, and oxygen is liberated as a by-product.

In the experiment, the algae seemed to have little effect on the coli-aerogenes count of the fluid in the percolation bed, but the D.O. they produced helped satisfy the B.O.D. and maintain aerobic conditions.

Since the algae obtain their energy from the sunlight, the D.O. they produce follows a characteristic 24-hr curve, and they are usually not effective at fluid depths of over 2 ft.[7] Therefore, although the algae do contribute organic matter to the basin, the amount of D.O. they produce makes their presence in percolation beds for sewage effluents quite desirable. These algae may also appear unsightly.

TEMPERATURE AND WEATHER DATA

Between May 13, 1949, and June 30, 1949, the temperature of the fluid in the Azusa basin ranged from 60° F to 83° F. Later the water temperatures averaged 89° F. Thus, maximum warm conditions were approximated in the spreading basin. The warm weather did not create any odor problem although in the one-week period during which 100 ppm of B.O.D.fluid was spread and no D.O. existed there were slight odors.

It is probable that, as long as aerobic conditions are maintained in the basins and no noxious chemical wastes are allowed, serious odor problems will not be encountered.

The experimental data obtained in the Azusa experiment are summarized in Table 3 and in Fig. 4.

THE CONSERVATION OF FERTILIZER VALUES

The data obtained were not sufficient to allow valid conclusions on this point. The spread effluents averaged about 30 ppm of organic nitrogen, and percolated fluids showed about 7 ppm of nitrate nitrogen. However, the sludge that collects on the floor of the spreading basin contains 3.1% of organic nitrogen and can be collected by scraping.

CONCLUSIONS

1. A method for successfully collecting percolated fluid samples from the subsoils was developed. Collection pans buried within the spreading grounds successfully carried the percolated fluid into a sampling well.

2. It was found that it is possible to percolate satisfactorily sewage effluents at rates as high as 0.6 cu ft per sec per acre for periods of several months. Initially very high percolation rates are obtainable in coarse soils but the bed will eventually seal off and the rates approach 0.2 cu ft per sec per acre. Within the limits of the rates used in these tests and dependent upon an adequate distance between the basin and the ground-water level, initial percolation rates on porous soils are controlled by the depth of the fluid in the basin; after continuous spreading an impervious mat builds up on the surface of the coarse soil and the fluid depth has little effect reacting the same as a fine soil. Even when aerobic conditions are present, the percolation rate gradually decreases, and the bed must be dried and cultivated. The actual operating schedule must be decided by the local conditions.

3. No coli-aerogenes pollution resulted when aerobic conditions were maintained in the percolated fluid. Conversely anaerobic conditions allowed the bacteria to penetrate at least 7 ft into the ground.

4. Bacteriological examination of the soil beneath the test spreading beds seemed to confirm the assumption that bacterial pollution was carried by fluids percolated under anaerobic conditions.

5. Experience at the Whittier and Azusa test basins indicates that if the arbitrarily determined standard of 0.5 ppm of D.O. and less than 0.5 ppm of B.O.D. in the percolated fluid at the testing depth is maintained, water of a satisfactory sanitary quality can be produced.

6. The organic loading placed on the basin by the spread fluid is determined by the D.O., B.O.D., and percolation rate. Hence, initial high percolation rates must be carefully evaluated when spreading on coarse soils. It seems probable that the permissible organic loading on any soil approaches a constant that can be calculated but that the large amounts of D.O. contributed by algae can cause variation. Safe practice would be to apply a base loading such as obtained in the winter when algae are not active. In these experiments this was calculated to be 40 lb of B.O.D. per wetted acre per day.

7. The physical characteristics of the soil in the percolation basins are of some importance in the process, so the depth of soil above the water table should be evaluated. However, water of good quality was produced at the 7-ft depth in both the tight Whittier soil and loose Azusa soil. Impervious soils obviously cannot be used for percolation basins.

8. If the basin was kept in an aerobic condition, the suspended solids of the spread fluid had little effect on the operation and were rapidly oxidized. If the organic loading increased and anaerobic conditions prevailed, the basin was rapidly sealed off and percolation rates decreased. Odor difficulties did not occur if aerobic conditions were maintained although basin temperatures reached 89° F.

9. It was found that the dissolved solids increment in the cycle connecting potable water to secondary sewage effluent at the Azusa plant was 172 ppm. It was also suggested that this be considered as normal for domestic sewages of the Southern California area. Such a pickup would allow the water to be used continuously two to five times unless undesireable wastes were not permitted in the sewers. The Azusa effluent showed a pickup of 23 ppm of sulfates, 30 ppm of chlorides, 14% sodium, and 0.1 ppm of boron per single use cycle and could, therefore, be used as suggested. Rain water and ground water provide a dilution factor for the reclaimed sewage.

10. No odor problems appeared, although fluid temperatures ranging from 67° F to 89° F were found. However, the basins were somewhat unsightly in appearance because of the profuse algae growths. The percolated fluids were sparkling clear as long as aerobic conditions were maintained.

ACKNOWLEDGMENTS

As officials of the Los Angeles County Flood Control District, H. E. Hedger and Paul Bauman, Members, ASCE, authorized the sewage reclamation tests at Whittier and Azusa. Finley B. Laverty, M. ASCE, handled the general direction of the reclamation test program, and L. W. Jordan, A. M. ASCE, was the immediate supervisor of the work in the field. A M Rawn, M. ASCE, acting for the Los Angeles County Sanitation Districts, authorized the chemical and bacterial testing that was necessary.

DISCUSSION

R. B. Krone,[12] **J. F. Thomas,**[13] **and Harvey F. Ludwig,**[14] **A. M. ASCE.**—The method of sampling described in this paper has also been used by the writers in a field testing program[15] at Lodi, Calif., in which collection pans were placed at varying depths in the soil in shafts dug from a well placed at the center of each test plot. Extensive use of the pan method showed it to have the following disadvantages:

(1) It is difficult to place the pans without disturbing the overlying soil, and even more difficult to secure a good bond between the pan and the overlying soil;

(2) The method fails when the percolating water is under tension (less than the atmospheric pressure), a condition often found, especially in stratified soils;

(3) The flow path and perhaps the characteristics of the percolating water are seriously altered; and

(4) The size of the pans, if large enough to collect significant samples, limits the number that may be used, and hence makes it difficult to collect samples from enough locations to assure representative results.

An improved and more convenient sampling laboratory method[16] involves the use of a porous tube or "probe," inserted in the soil at the desired sampling depth, to which negative pressures or tensions are applied. The porous tube is comparable in size and porosity to a coarse bacteriological filter. It extracts water from the soil by forming a hydrodynamic sink. The configuration of the flow and the pressures around the probe are similar to those existing around the perforated zone of a well into which underground water is flowing. However, the magnitude of the tension applied to the probe is so slight that the volume or region of soil affected is very small, extending only about 10 cm from the probe for applied probe tensions of about 100 cm of water. Thus the disturbance caused by the sampling scarcely affects the general pattern of the flow of water through the soil.

The proper tension to be applied to the probe depends upon the soil permeability, the frictional resistance of the probe material and of the connecting tube, and the initial soil solution pressure. The necessary probe tension is readily applied by means of a hanging water column. With a probe of proper dimensions and porosity, the feasible yield per probe is about 20 milliliters per hr.

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[15] "Reclamation of Sewage Water," by A. E. Greenberg and H. B. Gotaas, *American Journal of Public Health*, Vol. 42, April, 1952, p. 401.

[16] "Porous Tube Device for Sampling Soil Solutions During Water Spreading Operations," by R. Krone, H. F. Ludwig, and J. F. Thomas, *Soil Science* (publication pending).

A M Rawn,[17] M. ASCE, and F. R. Bowerman,[18] J. M. ASCE.—The case for reclamation of sewage effluents by recharge of ground-water gains added support from the excellent performance data on operating percolation ponds presented in this paper. Reclaimed sewage effluent has been directly re-used for industrial and agricultural purposes in many places and on many occasions, and the records of such re-use abound in the literature.[11,19,20,21] The Stone-Garber paper, however, presents a practical method whereby sewage effluent percolated into the ground may be rendered wholly acceptable for any and all uses to which normal ground waters may be placed.

Direct re-use of sewage effluents is highly restricted, but the infiltration of such sewage effluent through the soil and into ground waters permits unrestricted use of such waters upon subsequent pumping. A preliminary report presents part of these data on reclamation by spreading-basin infiltration.[11] The authors have re-analyzed and amplified this information and have shown more precisely what results were obtainaed.

It is of particular interest to note the widely differing character of the soil at the two spreading areas under consideration. At Whittier the soil was tight; cursory examination would suggest that it was not appropriate for recharge purposes. In referring to the Azusa basins, the term, soil, was almost a misnomer in that once the thin surface layer was pushed aside, the material remaining was a very coarse mixture of sand and gravel—a highly permeable medium. Consideration of the excellent performance recorded at both sites increases one's confidence in the general thesis of sewage reclamation by spreading basin infiltration.

Some significant questions are posed by this paper. First: What effect does long-term spreading have upon the underground geological formations that receive the effluents? It is logical to anticipate that the structural and chemical nature of the soil will be changed by the pickup or loss of inorganic constituents which may be deposited in the soil or leached from it into the percolating water. An operation extending over several years might yield results somewhat different from those observed during the short period of the test runs. The chemical or physical changes occurring in the soil can be observed by suitable testing, although long periods of time might elapse before such changes appear. The Azusa basins have been operated as sewage effluent percolation ponds since 1943 and it would be expected that any changes occurring underground would already have taken place. However, the alluvial deposits at Azusa may not be representative of other conditions as they are stream-washed and, consequently, contain little organic or other chemical components.

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[19] "Industrial Uses of Reclaimed Sewage Effluents," by N. T. Veatch, *Sewage Works Journal*, Vol. XX, January, 1948, pp. 3-11.

[20] "Agricultural Uses of Reclaimed Sewage Effluent," by L. V. Wilcox, *ibid*, pp. 24-33.

[21] "The Utilization of Sewage Water for Agriculture," by A M Rawn, *Proceedings*, 18th International Cong. of Agri., Dresden, 1938.

A second question: What effect does the continual discharge of sewage effluents through a percolation pond have on the receiving underground waters? The chemical content of the underground waters immediately underlying an infiltration basin must reflect the chemical nature of the sewage effluent introduced above it. Although the authors clearly indicate that bacteria and organic matter are filtered out within a few feet, the chemical nature of the effluent is little changed, except possibly through exchange with the soil. An examination of ground waters surrounding existing percolation ponds should indicate the extent to which chemicals are added to the underground waters.

In an investigation, it is seldom possible to study all phases of a problem, and it is not with the thought of criticism, but merely as a suggestion, that these particular questions are presented. The transfer of chemical constituents between the soil and the water will exert some effect on the final condition of the effluent as it reaches the underground waters. However, this will probably prove to be of secondary importance, and it is doubted if the over-all picture will be greatly affected. The importance of reclaiming, for re-use, a steady and reliable source of water, suitable for return to the underground aquifers—particularly in semi-arid areas such as Southern California—makes it necessary that all aspects of this problem be resolved as quickly as possible.

Ralph Stone,[22] A. M. ASCE, and William F. Garber[23].—The investigators (Messrs. Krone, Thomas, and Ludwig) at the University of California have reviewed the effectiveness of subsurface sampling of percolating waters with collection pans in contrast to the use of a porous probe, which is essentially a tube with a porous ceramic membrane tip, operated with negative heads induced by a vacuum pump or a hanging column of water. It is true that it may be difficult to place collection pans without disturbing the overlying soil and even more difficult to secure a good bond between the pan and the overlying soil. However, with the full comprehension of the factors involved in achieving continuity between the collection pan media and the overlying soils, an effective percolation pattern was achieved in fact, both at the Azusa and Whittier test basins since satisfactory large volumes of percolated waters were collected at a rate varying between 1 gal and 0.1 oz per 10 min of sampling period, depending on the concurrent velocities of the surface percolation. The University of Southern California investigation of leaching from dumps[24] has successfully utilized a type of collection pan placed into undisturbed soils for their leaching studies. Similarly, the Lodi test conducted by the University of California (at Berkeley) employed collection pans for the subsurface sampling device.

In the "Waste Water Reclamation and Utilization Project" (University of California at Los Angeles), the investigators are continuing to employ the same installation of collection pans described for the Azusa test site. After four years of utilization at the test basin site, these Azusa collection pans still operate efficiently, as judged by the consistency of chemical and bacteriological data obtained from these sources. At both the Whittier and Azusa test basins, the collection pans were installed carefully by the physical removal of the surface material and then backfilled carefully to assure continuity between the liquid phase of the covering soils and that of the interior of the collection pans. With the passage of time, superior consolidation of this backfill has taken place at the Azusa installation thereby approaching the undisturbed soil condition. Still, there has not been a noticeable change in the quality of the samples collected.

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[23] Lab. Director, Hyperion Sewage Treatment Plant, Los Angeles, Calif.

[24] "Investigation of Leaching from Dumps," Dept. of Eng., San. Eng. Lab., Univ. of Southern California, Los Angeles, Calif., *Monthly Progress Reports Nos. 5 to 8*, July to October, 1951.

Soil Moisture Tension.—Under conditions of high soil moisture tension, there is little movement of free capillary water through the soil. Soil moisture tension can be defined as the molecular attraction or "wetting" between the water molecules and the soil particle surfaces. Before appreciable gravity flow can take place in the soil interstices, these soil particle surfaces must be "wetted" and the capillary soil pores filled. This moisture demand is often called the field capacity of a soil. Only after the surface tension forces have filled the capillary pores can the smaller forces of gravity act to allow water to percolate in appreciable quantities through the open soil interstices. It is possible to measure, actually, the hydraulic gradient of the water in a soil by use of devices called "tensiometers." These tensiometers are similar to manometers except that they have a ceramic membrane top so that they record the direct positive pressures of the liquid phase at the surface of a soil column as well as the negative pressures resulting from surface tensions in the water covering the capillary pores and particle surfaces. Under conditions of evaporation or plant transpiration of the soil moisture, these tensiometers (located at different depths in the soil) actually can measure the hydraulic gradient in the liquid phase of the soil moving the moisture toward the top of the soil column. The negative surface tension of liquid phases, with percolation rates in the range of 0.2 ft per day, is -2 ft to -4 ft of water head, whereas the tensiometer readings, when evaporation and transpiration result in upward movement of the soil moisture, are as low as -5 ft to -10 ft of water head. Hence, it can be stated that, when the collection pan sampling method fails, little if any downward movement of recharge water into the underground aquifers occurs. The rate of flow of percolated water can be equated as follows:

$$V = K\,\Delta H \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which the velocity of flow is V; the permeability coefficient is K; and the difference in hydraulic gradient between two points a and b is ΔH. Hence, if the rate of percolation or flow is measured and the hydraulic gradient between two points a and b in the soil column are determed through the use of tensiometers, then the permeability of the soil column (a–b) can be calculated.

$$K = \frac{\Delta H}{V} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Porous Probe.—The porous tube or probe actually alters the hydraulic movement pattern of the percolated water by causing a hydrodynamic sink with negative hydraulic gradient in the liquid phase. Thus, capillary water will move toward the porous probe operating under vacuum. The porous probe, therefore, withdraws the surface tension waters while the collection pan intercepts free water moving through the interstices as a result of the action of gravity.

The reason that the Berkeley investigators have not adopted the porous probe for field sampling is that a probe of proper dimensions and porosity will collect only a feasible "yield" per probe of 20 milliliters per hr. However, the collection pan, under conditions of high percolation, discharges much greater volumes of percolated effluent and, therefore, is much more convenient for field sampling. Finally, field samples collected for bacteriologic, B.O.D., turbidity, and possibly other analyses would be appreciably affected by filtration through the ceramic membrane. Therefore, such samples could not be obtained satisfactorily with a probe.

Rawn-Bowerman Discussion.—Messrs. Rawn and Bowerman have quite properly stated that considerable field experience has been obtained empirically about industrial and agricultural re-use of waste waters. Furthermore, the county sanitation districts cooperated in the performance of the Whittier and Azusa studies by making available the services of the laboratory and the laboratory personnel of the districts. A preliminary report covering a part of the data has been published.[11]

Performance of Intermittent Sand Filters.—It is well known that the effluent from intermittent sand filters and fine grain contact beds produces a clear, well nitrified, and stabilized effluent.[25] The spreading basins represent, in essence, natural filters that are generally of even a finer particle size than sand filters or fine-grain contact beds. Hence, it would be obvious that sewage spreading can be considered as a legitimate type of sewage treatment. Initially, a continuously operated spreading basin acts as an oxidation pond which reduces both the organic load and the pathogenic organisms content by (1) storage, (2) the activity of either aerobic or anaerobic biological action, and (3), of course, the filtration phenomenon which can also be considered both physical and biochemical action.

The Whittier and Azusa tests clearly indicate the following: (1) The remarkable effectiveness of such sewage filtration in stabilizing organic matter and removing pathogenic organisms, particularly under an aerobic environment, with a B.O.D. loading of from 40 lb to 100 lb per wetted acre; (2) the recharge percolation rate is largely controlled by the spreading-basin surface condition and the manner of spreading-basin operation; (3) the major effect of algae was that of providing additional oxygen during the warm daylight hours; (4) the importance of drying out the surface of spreading basins in order to redevelop high percolation rates; and (5) no mosquito or fly nuisances were noted at Azusa in the regularly operated old spreading basins. The old basins had evidently developed a balanced biological cycle wherein predatory insects and other organisms controlled the presence of mosquitoes, whereas the new test basins allowed the propagation of mosquito larvae.

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[25] "Sewerage and Sewage Disposal," by L. Metcalf and H. P. Eddy, McGraw-Hill Book Co., Inc., New York, N. Y., p. 567.

Whittier.—The Whittier sandy loam soil maintained effective recharge rates averaging more than 1 ft of water per day for one week of continuous operation of the spreading basin. These initial Whittier and Azusa tests indicate the importance of maintaining fluid depths of more than 1 ft for effective filtration rates. At the 0.5-ft depths of spreading in these basins, the percolation rates were much less than when the normal 1-ft depth basin head was employed (Figs. 3 and 4). The Whittier well waters fed by the recharged sewage effluent were chemically and bacteriologically satisfactory to the adjacent farmers. No complaints by health or other authorities on water quality have ever been placed against these Whittier wells.

Anaerobic Disposal Operation.—The Whittier-Azusa tests quite clearly indicate that, under anaerobic environmental conditions, there can be considerable noxious odor in the spreading basins. In addition, with the anaerobic environment, the samples collected at various depths in the soil had reduced effluent quality when measured in terms of (1) high counts of MPN coliform, (2) presence of increased B.O.D., and (3) reduced spreading-basin percolation rates (see Fig. 4). Nevertheless, these studies indicated the fact that there was considerable treatment even under anaerobic environment of the spread effluent and that the B.O.D. and MPN counts for the spread sewage, after anaerobic percolation, were considerably reduced. As stated, the efficiency of the aerobic percolation is much higher, however.

Soil Influence.—The point has been raised about the type of soils satisfactory for percolation or recharge purposes. The controlling factor, as evidenced by the Whittier-Azusa tests, are that any soil is satisfactory which is permeable enough to allow a satisfactory entrance and percolation of water for effective recharge. In fact, the finer grained soils provide greater surface area and thereby insure more satisfactory biological and physical filtration. Of course, very fine clay soils of very low permeability would be practically impervious to the movement of percolating water and therefore, would be unsatisfactory for spreading basins.

The Effect of Long-Term Spreading on Soils.—Waste waters spread into the soil can be classified in their effect exactly as any other similar mineral type water. Waste waters can be employed feasibly for leaching alkali soils and, thereby, reclaiming such soils that would otherwise be uneconomical to reclaim with other water sources. Spreading basins have been used to recharge aquifers at Azusa since before 1940 and there has not been any noticable reduction in the capacity of soil to percolate the effluent. Since before 1927, numerous other farm locations including Pomona, Fresno, Bakersfield, and Ontario—in California—have been operating without noticeable deleterious effects on the receiving soil strata. Hence, the effect of infiltration of spread water on the underground geological formations can be summarized as follows: (1) The infiltrating sewage will leach out and drain alkalis present in the surface soil so that a continuous steady-state permeable condition prevails; (2) where a satisfactory percentage-sodium relation exists in the spread effluent, the clay and other colloids remain flocculated and allow maximum permeability (however, with a high-percentage-sodium, chemical deflocculation of the clay, colloids may appear which will reduce the permeability of the porous media);

(3) the organic material and organic growths in the soil under proper cycles of resting and desiccation will not interfere with percolation and, in fact, may assist in maintaining high percolation rates;[26] (4) fines—that is, small, inert particles introduced into the surface of the spreading basin—will reduce percolation rates, of course, but these can be removed by scarification of the basin surface; and (5) in the operation of cesspools and septic tanks, the Azusa and other spreading basins observed by the authors indicate that there is no unpredictable change in soil characteristics and that breakdown of the geological formations resulting from percolation of effluents is similar in nature to that of

TABLE 4.—PERMISSIBLE LIMITS FOR DISSOLVED SOLIDS, BORON, CHLORIDE (IN PARTS PER MILLION) AND PERCENTAGE OF SODIUM

Factor	CLASS 1	CLASS 2	CLASS 3
	Excellent to good	Good to injurious	Injurious to unsuitable
Percentage of sodium[a]	< 60	60 to 75	> 75
Parts per million			
Total dissolved solids (D.O.)	<700	700 to 2,100	>2,100
Boron	> 0.5	0.5 to 2.0	> 2.0
Chloride	178	178 to 355	> 355

[a] $\frac{\text{Sodium} + 100}{\text{Calcium} + \text{magnesium} + \text{sodium} + \text{potassium}}$.

TABLE 5.—MINERAL PICKUP RESULTING FROM WATER SUPPLY UTILIZATION

Analysis[a] (1)	CITY OF AZUSA[b]				CITY OF LOS ANGLEES[c]			
	Average potable water (2)	Average percolated effluent[d] (3)	Mineral pickup (4)	Cycles of re-use[e] (5)	Average potable water (6)	Average hyper-ion effluent (7)	Mineral pickup (8)	Cycles of re-use[e] (9)
Sodium, Na^+	9.6	53.0	43.4	...	45.2	200	154.8	...
Potassium, K^+	3.1	12.5	9.4	...	4.0	20	16.0	...
Calcium, Ca^{++}	54.4	51.2	0	...	31.2	92	60.8	...
Magnesium, Mg^{++}	14.4	15.5	0	...	7.2	21	13.8	...
Sodium (%)	11.8	40	28.2	three to five	51.8	60	8.2	one to eight
Chloride, Cl^-	11	44	32	five to eight	23	236	213	one to two
Boron, B	0.1	0.3	0.2	two to seven	0.43	0.42	0	many
Total solids[f]	260	432	172	three to six	262	769	507	one to three

[a] Units are parts per million except sodium (%) = $\frac{\text{sodium} \times 100}{\text{calcium} + \text{magnesium} + \text{sodium} + \text{potassium}}$.

[b] Primarily domestic utilization of water.

[c] Domestic, industrial waste, and salt brines introduced into sewage.

[d] Average percolated effluent at a depth of 7 ft.

[e] Refer Table 4 for "good to injurious" permissible limits of certain ions.

[f] Total dissolved solids.

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[26] "Research in Water Spreading," by Dean C. Muckel, *Proceedings–Separate No. 111*, December, 1951, p. 9.

water infiltration providing that the recharged waters are comparable in mineral quality.

Chlorides.—The chloride analyses at the Whittier test plot indicate that the leaching of the soluble salts occurred in the top soil. This leaching should have stopped when the salts were completely washed out. The percolated effluents at the Azusa tests did not show evidence of either leaching or soil breakdown. These Azusa soils that have been percolated continuously for many years show no evidence of breakdown.

The Effect of the Sewage Effluents on the Irrigation of Plants.—The paper indicates that, even disregarding the dilution effect of underground aquifer waters from other sources, the reclaimed effluent still might be recycled two to five times before mineral pickup would be great enough to present a problem.

TABLE 6.—CHEMICAL ANALYSES OF WELL WATER IN THE AZUSA AREA[a]

Date (1)	Well No. (2)	$K \times 10^5$ (3)	Total solids (4)	Hardness as $CaCO_3$ (5)	CO_3 (6)	HCO_3 (7)	Cl (8)	SO_4 (9)	NO_3 (10)	Ca (11)	Mg (12)	Na+K (13)
Aug. 21-31	4217	43	...	...	14.05	0	0.52	1.22	0.23	3.44	0.99	0.99
Aug. 30-50	4236	...	352	328	212	0	12	34	Tr.	72	36	None
Aug. 30-50	4239A	...	490	464	216	0	32	120	5	120	40	None
Nov. 14-50	4248	...	330	207	216	0	16	21	Tr.	60	14	7
Nov. 8-50	4255C	...	210	178	144	0	12	169	None	60	7	None
Aug. 30-49	4257	...	414	393	224	0	44	148	30	80	47	3
Nov. 9-50	4259	pH6.9	360	331	216	0	12	98	10	88	27	None
Nov. 18-50	4267[b]	...	872	481	264	0	232	168	15	132	37	109
Feb. 28-50	4268	...	430	338	212	0	20	77	10	96	24	None
Nov. 9-50	4278	...	396	373	228	0	28	52	10	92	35	None
Nov. 8-51	4284B	pH6.4	...	...	216	0	16	67	...	...	...	...
Aug. 5-51	4285	...	420	390	208	0	8	86	None	84	44	None
Nov. 14-50	4286	...	250	111	128	0	8	34	Tr.	28	10	19
Aug. 30-51	4287A	...	492	458	252	0	28	213	40	136	29	4
Sept. 1-49	4287A	...	334	279	248	...	20	33	20	84	17	None
Aug. 21-31	4289	48.7	...	...	13.52	0	0.52	1.57	1.57	3.38	1.05	1.34

[a] Units are in parts per million except first and last lines which are grains per gallon. Except the data in the first line, these tests were observed at the laboratory of the Los Angeles County Flood Control District. The first line was reported by the Division of Water Resources, State of California. The boron content was less than 0.5 parts per million in each test reported. [b] Cyanamide plant.

Perhaps it would be well to review the mineral standards, developed at the Rubidoux Regional Salinity Laboratory, United States Department of Agriculture, that are generally accepted as fair criteria for irrigation water quality. Using the standards described in Table 4, the mineral analyses for the increment of mineral pickup of the Azusa and Los Angeles sewage over the potable water supplies are enumerated in Table 5. By reviewing these tables, it is evident that it is feasible to re-use these waste waters from one to five times before reaching mineral classifications that would be unsatisfactory for agricultural purposes. The Hyperion sewage treatment plant is introduced into this discussion because of the large (averaging 220,000,000+ gal per day) volume of water discharged into the ocean from the plant. Considerable quantities of industrial wastes and salt brines are introduced into the plant influent and, under a program for control of industrial wastes and other high mineral water, the Hyperion effluent would be of better mineral quality. The Azusa mineral pickup can be used as a yardstick for minimum mineral pickup. Table 5 classifies the Azusa and Los Angeles effluents in terms of the agricultural standards. The Azusa sewage does not contain appreciable brine or industrial wastes and may be considered a representative domestic Southern California sewage effluent. Industrial and potable water quality standards, in general, are in the same approximate range as the agricultural quality standards with certain exceptions which need not be discussed in this paper.

Analyses of Surrounding Ground Water.—The Los Angeles County Flood Control District has taken samples in the neighborhood of the Azusa sewage treatment plant as well as in other surrounding areas for many years. Table 6 compiles the results of these and other analyses. It will be noted from Table 6 that there is no consistent pattern of degradation of the ground water that can be attributed to the Azusa sewage discharging into the underground aquifers. Table 2 indicates quite clearly that, contrary to obvious considerations, all the mineral constitutents present in the spread effluent do not remain in the percolated liquid. For example, the total nitrogen present in the samples collected at the 7-ft-depth collection pans at Azusa is much less than that present in the spread waste waters. Evaporation of ammonia gas, loss of nitrogen (as N) to the atmosphere, fixation of nitrogen in the organic matter in the surface soil—are factors causing this nitrogen removal. Similarly, base exchange relationships between the percolating effluent and the soil, flocculation phenomenon, and other physical-chemical-biological factors may influence certain ions.

Water percolating through the soil into the underground aquifer, of course, will dissolve considerable mineral content. Hence even rain water, after recharging to the underground, contains the various categories of anions and cations. With this thought in mind, the feasibility of direct re-utilization of properly controlled and treated waste waters for cropping, industry, or recharge gains new validity.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 2531

UPLIFT IN MASONRY DAMS

FINAL REPORT OF THE SUBCOMMITTEE ON UPLIFT IN MASONRY DAMS OF THE COMMITTEE ON MASONRY DAMS OF THE POWER DIVISION, 1951

SCOPE OF REPORT

Herewith is submitted a report by the Subcommittee on Uplift in Masonry Dams. The Subcommittee has been functioning for several years under an instruction "to study the possibility of clarifying the uplift problem."

The available information has been assembled and studied. The data applicable to the question of foundation uplift are held to be adequate to warrant a report on this phase. There are numerous unanswered questions and a scarcity of field data as to uplift effects in the concrete of the dam structure, and prospects for obtaining such information in the near future are not very promising. Hence, this report is presented as a statement on foundation uplift. Since the dimensions of a dam depend on the assumptions for foundation uplift, these are of primary importance.

Assumptions upon which any dam is designed are the responsibility of the designer and cannot be waived by him. Therefore, neither this Subcommittee nor anyone else can arrive at rules for such assumptions of general or universal applicability. A few conclusions will be presented at the end of this report, but they must not be considered as rules. The report is essentially a digest of information and practice which, when considered in connection with the particular circumstances of each project, should aid the designer in arriving at adequate assumptions. Also this report should not be considered as establishing any rules by which the adequacy of existing dams should be determined. Here again, every existing structure is a special case, and any determination of its adequacy is a matter for individual expert study.

The uplift question is only one of a number of factors entering into the design of masonry dams. Others are ice pressure, assigned to another ASCE committee; earthquake effects, temperature effects, and other factors causing volume change; and the specification of materials and processes to secure concrete of adequate quality as to strength and permeability. The task of this Subcommittee has been to study uplift, and others presumably will investigate additional phases.

NOTE.—Published in June, 1952, as *Proceedings-Separate No. 133.* Positions and titles given are those in effect when the paper was received for publication.

1. FAILURES OF DAMS

The most noteworthy failures of concrete or masonry gravity dams in the United States have been those of Austin, Tex.; Austin, Pa.; the St. Francis Dam in California; and Dam No. 26 on the Ohio River. In all these cases failure was due to inadequate resistance in the foundation to horizontal movement—that is, to shearing failure in the foundation. Also, in all these cases there was evidence that uplift contributed to the failure by reducing the effective weight of the structure, and, consequently, the frictional resistances dependent upon weight.

At least in the United States, no important failure has been due to overturning of a dam; nor have any failures been ascribed to the development of internal pore pressures in the concrete or masonry itself.

These considerations lead to the conclusion that uplift effects, especially in the foundation, are always to be reckoned with and are an essential element of design.

2. EXISTING DAMS

While modern practice has come to include an uplift allowance, many existing structures, designed without any allowance for uplift—perhaps without any conception of it—have been for many years, and are still, functioning satisfactorily. Similar attitudes have existed with reference to design against ice pressures. Some of these are in the category of high structures, although most of them are probably less than 100 ft in height and may be classed as low dams.

Among the reasons which may be assigned for the apparent stability of these structures, even though no allowance for uplift forces entered into their design, may be listed the following:

a. The presence of joints or other openings in the foundation which facilitate downstream flow in the rock and discourage the building up of any great uplift intensity. Coupled with this is possible sedimentation or other means of sealing the entrance to water passages upstream of the dam which would still lower the pressure gradient further. The practice of providing a cutoff wall through the upper strata of the foundation, whatever may be its disadvantages from other points of view, has had at times beneficial effects upon the uplift intensity. The principal objection to the use of a cutoff wall in rock, aside from its cost, appears to be that the necessary blasting will open passages or joints which otherwise would remain closed and thus facilitate the entrance of water.

b. While tensile strength in the foundation and in the concrete or masonry, as well as in the connecting bond, is generally neglected in design, such strength does exist. In dams of moderate height, this factor may have great significance. Particularly when the foundation rock is of massive, and steeply dipping character, tensile and shearing strength may provide material anchoring forces at the "heel" of the dam, directly contributing to stability. In horizontally stratified and thinly bedded rocks such resistances are less significant. In any case, these effects become less important as the height of the dam increases. While the destructive forces increase as the square of the height of the dam, the resistance due to tensile strength increases much less rapidly. Tensile strength will help to account for the performance of many light dams of considerable age which were built on strong foundations.

c. While dams are customarily designed upon the "middle third" theory, assuming a straight-line distribution of the load stress across the foundation, there is actually some uncertainty about this distribution. Some mathematical analysis and some model tests have shown departures from the straight-line distribution.[1]

An illuminating illustration of departure from the straight line is found in the stress distribution determined for Shasta Dam in California.[2] In this case stresses much less than the maximum were determined at the toe of the dam, even at a time when the reservoir was almost full. Temperature effects were moderate. While this determination was made upon a very large dam, there seems to be no reason why material departure from the straight-line assumption may not exist in much smaller structures.

d. Related to Section 2*c* are the effects of temperature load transfer and swelling of masonry at the face exposed to water, which have the result of modifying the pressure gradient from a straight line and increasing the intensity of pressure at the heel and toe of the dam above that given by the straight-line distribution. These effects may not be significant in low dams, where tensile strength is relatively important, but they can be important in high structures.[3,4]

e. The resistance to horizontal movement as expressed by the shear friction factor is actually much greater than any resistance related only to frictional factors, such as the sliding factor.

f. In some sites there are peculiarities favorable to stability, such as the shape of abutments in narrow canyons. In such cases arch effects may be realized which offset any deficiency in a gravity section.

3. Uplift Pressure

There are two constituent elements in uplift pressure—the area factor and the intensity factor.

The area factor, A, is the proportion of a horizontal section at the base of the dam which is assumed to be subject to uplift pressure. It has a maximum value of 100%, or 1. Smaller values have been assumed in numerous dams, according to the judgment and experience of the designer.

The intensity factor, I, is the assumed ratio which intensity bears to an intensity gradient extending from maximum headwater to maximum tailwater. It is commonly expressed as a gradient extending from maximum tailwater to some assumed higher elevation at the upstream face of the dam, below the maximum headwater elevation. Thus, a 50% intensity factor indicates a gradient, the upstream end of which is at an elevation midway between maximum headwater and maximum tailwater.

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[1] "Stress Studies for Boulder Dam," *Bulletin No. 4*, Part V, Boulder Canyon Project Final Repts., U. S. Bureau of Reclamation, U. S. Govt. Printing Office, Washington, D. C.

[2] "The Development of Stresses in Shasta Dam," by J. M. Raphael, *Proceedings-Separate No. 117*, February, 1952.

[3] "Measurements of the Structural Behavior of Norris and Hiwassee Dams," *Technical Monograph No. 67*, TVA, Knoxville, Tenn., 1951.

[4] "Hiwassee Project Report," Part I, TVA, U. S. Govt. Printing Office, Washington, D. C., 1948.

In Fig. 1 is shown a break in the intensity diagram at the line of drains if such are used. The diagram with a "break" conforms more nearly to observed data as cited later whereas the straight-line assumption simplifies computation somewhat. In either case the intensity factor expresses the relation of the intensity assumed to that which would occur if the gradient extended between maximum headwater and tailwater in a straight line. An intensity factor of 100% or 1 would mean that the assumed gradient extends from maximum headwater to maximum tailwater.

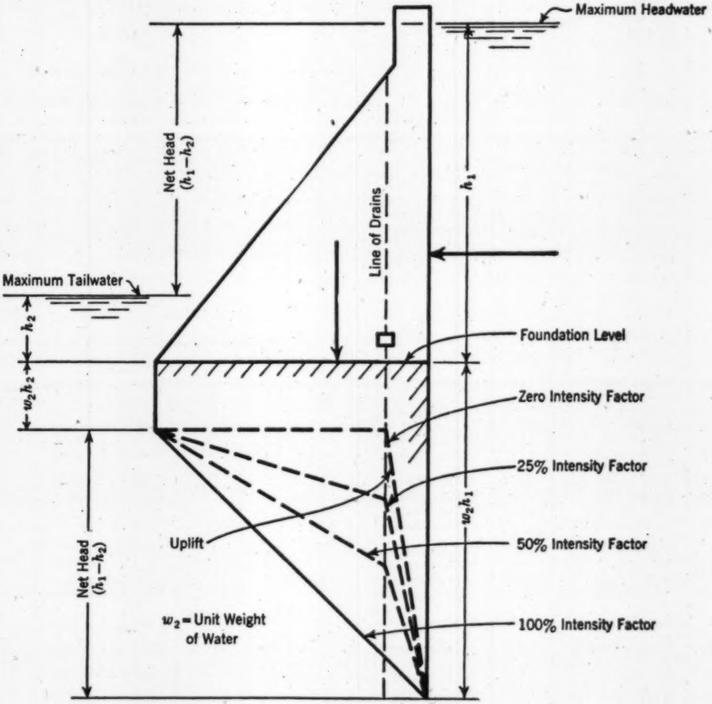
Maximum Headwater
Net Head (h_1-h_2)
Line of Drains
h_1
Maximum Tailwater
h_2
Foundation Level
w_2h_2
Zero Intensity Factor
25% Intensity Factor
w_2h_1
Uplift
50% Intensity Factor
Net Head (h_1-h_2)
w_2 = Unit Weight of Water
100% Intensity Factor

FIG. 1.—DIAGRAM OF UPLIFT INTENSITY WITH VARIOUS FACTORS

There has been some confusion in the past as to the use of these factors. They are not interchangeable. In a high dam with low tailwater it does not make much difference whether, for example, I be taken as 1.0 and A as 0.50, or I be assumed as 0.50 and A as 1.0; but in a low dam with high tailwater such interchange makes a great difference in the computed uplift effect.[5] It is

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[5] "Engineering for Dams," by W. P. Creager, J. D. Justin, and Julian Hinds, John Wiley & Sons, Inc., New York, N. Y., 1945, Vol. II, p. 269.

TABLE 1.—FOUNDATION DATA,

Line No.	Name of dam (1)	Height of dam (ft) (2)	Type of rock (3)	Variety of rock (4)	Bedding thickness (5)
					(a) BUREAU OF RECLAMATON
1	American Falls	...	Igneous	Columnar basalt	Solid and massive
2	Hoover	...	Metamorphic	Andesitic tuff breccia	Massive
3	Friant	...	Metamorphic	Quartz-biotite schist	Massive
4	Grand Coulee	...	Igneous	Fine-grained granite	Varies with shearing
5	Gibson	...	Sedimentary	Crystalline-limestone	Hard massive
6	Marshall Ford	...	Sedimentary	Austin and glenrose limestone	Thin
7	Owyhee	...	Igneous	Porphyrite or glassy rhyolite	Hard and durable
8	Seminoe	...	Igneous	Dark gray granite[d]	Blocky, with thin mica seams
					(b) TENNESSEE VALLEY AUTHORITY
9	Wheeler	72	Sedimentary	Argillaceous limestone	Thin, 3 in. to 6 in.
10	Fontana	480	Metamorphic	Quartzite[e]	Massive
11	Douglas	202	Sedimentary	Dolomite	Massive
12	Cherokee	173	Sedimentary	Shale and limestone	Thin
13	Hiwassee	307	Metamorphic	Schist and quartzite	Variable
					(c) CORPS OF ENGINEERS
14	Clark Hill	200	Igneous[g]	Granite or granite gneiss	Varies (thin to massive)
15	Allatoona	190	Metamorphic	Schist, quartzite, dolomite	Schist foliated; quartzite massive
16	Buggs Island	144	Igneous[g]	Granite and biotite gneiss	None
17	Philpott	220	Metamorphic	Schist	Foliated
18	Tygart	207	Sedimentary	Spillway sandstone	Thin to medium
19	Bluestone	165	Sedimentary	Abutment shale	Thin to massive
20	Conemaugh	137	Sedimentary	Sandstone, shale, fire-clay, etc.[k]	Thin to medium
21	Wolf Creek	240	Sedimentary	Limestone	Thin to massive
22	Center Hill	240	Sedimentary	Limestone	Thin to medium
23	Dale Hollow	185	Sedimentary	Limestone	Thin to massive
24	Stewarts Ferry	145	Sedimentary	Limestone with shale intercalations	Thin to massive
25	Narrows	190	Sedimentary	Sandstone and shale	Thin bedded
26	Norfork	220	Sedimentary	Dolomitic limestone	Thin to medium
27	Bull Shoals	258	Sedimentary	Dolomitic limestone	Thin to medium
28	Fort Gibson	110	Sedimentary	Limestone with shale intercalations	Thin to medium bedded
29	Fall River	66	Sedimentary	Shale, sandstone, and limestone[m]	Thin to medium
30	Harlan County	106	Sedimentary	Chalk with bentonite intercalations	Thin to medium
31	Conchas	235	Sedimentary	Shale and sandstone	Thin to medium
32	Caddoa	130	Sedimentary	Sandstone	Thin to massive
33	Whitney	155	Sedimentary	Limestone	Thin to medium
34	Chief Joseph	220	Igneous	Granodiorite	Master joint system in granite[r]
35	McNary	158	Volcanic	Basalt	Massive
36	Bonneville	170	Igneous[t]	Diabase (lock and power house)	Medium to massive tuff (main dam)
37	Detroit	390	Volcanic[u]	Andesite and breccia diorite	Medium to massive
38	Meridian[w]	250	Volcanic[t]	Andesite tuff breccias	Massive flows
39	Cottage Grove	95	Pyroelastic	Tuffs	Medium
40	Dorena	145	Volcanic[t]	Tuffs, agglomerates, andesite flows	Flows of andesites[s]
41	Folsom	268	Igneous[g]	Schists and metabasite[aa]	Foliation[bb]
42	Pine Flat	440	Metamorphic	Amphibelite	Debatable[dd]

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[a] With reference to the axis of the dam. [b] At the maximum section. [c] Between centers. [d] Broken by seams and fracture. [e] Phyllite in left abutment. [f] Maximum depth. [g] Igneous and metamorphic. [h] Local solution in dolomite. [i] Each of the dams in Table 1(c) has a drainage gallery and, with two excep-

Dams in the United States

Line No.	Dip and Strike Angle[e] Dip (6)	Strike (7)	Jointing (spacing and tightness) (8)
(USBR) Dams			
1	Upstream about 90 ft per mile	...	Tight
2	Downstream	North and South	In many directions
3	Downstream	N 55° W	Few and tight
4	Irregular in direction		Average
5	West 70° to 86°	N 5° W	Numerous seams and joints
6	Northward 5 ft in 1,000 ft.	...	Numerous seams and parting planes
7	Westward 5° to 10°	...	Visibly fissured
8	Irregular in direction		Tight
(TVA) Dams			
9	Horizontal		Moderate
10	Downstream 10°	Transverse to river, 30°	Frequent
11	Transverse to river, 10° to 25°		Moderate
12	Downstream 15° to 45°	...	Frequent
13	Transverse to river, 50° to 80°		Frequent
(USED) Dams			
14	Upstream 45° (varies)	Almost parallel to axis	Wide, tight
15	Upstream 40° to 70°	Parallel to axis	Wide, mostly tight[k]
16	Not bedded		Most wide, tight
17	Downstream 40° to 60°	Almost parallel to axis	Wide, mostly tight
18	5° into left abutment	Almost normal to axis	Medium to wide, tight
19	Nearly horizontal		Close and open[j]
20	Nearly horizontal		Wide and tight
21	Nearly horizontal		Wide and tight, some channeling[l]
22	Slight dip to right abutment	...	Medium to wide, tight
23	Downstream slightly	Almost parallel to axis	Tight to open, some channeling[l]
24	5° toward right abutment	45° to axis	Wide, numerous open and weathered
25	70° downstream	11° to axis	Close jointed, tight
26	Nearly horizontal		Medium to wide, some channeling[l]
27	Nearly horizontal		Medium to wide, solution channels
28	Nearly horizontal		Wide and tight, some channeling[l]
29	Nearly horizontal		Wide and tight
30	Nearly horizontal		Wide and tight
31	Nearly horizontal		Wide and tight
32	Nearly horizontal		Medium and wide
33	Nearly horizontal		Wide and tight
34			Close to medium[o]
35	Horizontal flows		Wide and tight
36	Nearly horizontal		Wide, tight to open
37	Nearly horizontal		Variably open[p]
38	Steep downstream[x]	Diagonal to axis	Wide and tight[y]
39	East 7°	Diagonal to axis	5 ft to 20 ft apart, open
40	20° to 30° upstream	Parallel to axis	Columnar and platy[z]
41	Variable foliation in Mormon Dam, diagonal to axis		Close to medium, weathering[cc]
42	Upstream 40° to 70°[ee]	...	Close to medium, mostly tight

tions, the curtain grouting is done from the gallery. At Tygart Dam (line 18) and at Conchas Dam (line 31), the grout curtain was placed through the heel of the dam. [j] A fault at the right end of the spillway. [k] Also thin limestone and coal. [l] Solution channeling. [m] Interbedded. [n] Across fault zone only. [o] On

TABLE 1.—(*Continued*)

Line No.	Name of dam	Compressive strength (lb per sq in.) (9)	Depth of grout curtain[b] (ft) (10)	Depth of drainage (ft) (11)	Spacing of drains[e] (ft) (12)
		(*a*) BUREAU OF RECLAMATION (USBR) DAMS			
1	American Falls	Not available	20	20	5
2	Hoover	15,000	480	30 to 50	20
3	Friant	High	100	50	20
4	Grand Coulee	High	300	50	20
5	Gibson	Not available	40	40	4 to 7
6	Marshall Ford	Not available	125	40	20
7	Owyhee	400	270	110	10 to 20
8	Seminoe	Not available	100	100	5 to 10
		(*b*) TENNESSEE VALLEY AUTHORITY (TVA) DAMS			
9	Wheeler	High	30	...	...
10	Fontana	High	150	50	10
11	Douglas	High	200[f]	40	8
12	Cherokee	Low	100	40	8
13	Hiwassee	Variable	80	40	8
		(*c*) CORPS OF ENGINEERS (USED) DAMS			
14	Clark Hill	...	80	30	12
15	Allatoona	1,900 to 44,000	75	20	5
16	Buggs Island	6,721 to 19,118	50	40	10
17	Philpott	3,210 to 7,018	100	To be determined	...
18	Tygart	200 to 5,000	160[i]	100	2.5
19	Bluestone	1,095 to 8,810	150	100	10
20	Conemaugh	1,095 to 8,810	To be determined	...	...
21	Wolf Creek	2,840 to 13,100	75	65	10
22	Center Hill	6,180 to 13,860	100	80	10
23	Dale Hollow	3,110 to 9,640	100	50	10
24	Stewarts Ferry	6,132 to 7,050	...	...	...
25	Narrows	913 to 34,900	100	40	10
26	Norfork	2,100 to 30,200	150	53	8
27	Bull Shoals	25,000 average	200	60	8 and 9
28	Fort Gibson	813 to 13,200	75	40	10
29	Fall River	70 to 5,000	30	20	10
30	Harlan County	1,227 to 1,400	85[n]	50 to 75[o]	5 and 10[p]
31	Conchas	1,067 to 9,717	56[q]	84	15
32	Caddoa	2,510 to 6,150	52	35	10
33	Whitney	4,000	150	60	7
34	Chief Joseph	12,288 to 34,960	To be determined	To be determined	7
35	McNary	3,900 to 44,600	Varies from 60 to 100	50	10
36	Bonneville	370 to 1,250	55	50	60
37	Detroit	6,000 to 48,000	150	To be determined	To be determined
38	Meridian[w]	4,050 to 7,250	150	100	10
39	Cottage Grove	1,100 to 4,700	50	50	20
40	Dorena	None available	75	40	10
41	Folsom	Not available	50	40	10
42	Pine Flat	9,350 to 31,500	150	To be determined	10

the upstream side alternating between 50 ft and 75 ft; downstream depth, 45 ft. [p] On the upstream side of the grout gallery, 5 ft and 10 ft on the downstream side. [q] The grout curtain for the main dam was placed through the fillet at the heel of the dam. Provision for future grouting was made by constructing a grouting gallery along the axis of the main dam approximately 15 ft above the foundation. This gallery was provided with 3½-in. pipe casings set on 5-ft centers and extending from foundation rock to the gallery floor. [r] Close and tight. [s] Mostly tighty with some weathering. [t] Pyroelastic. [u] With igneous dikes. [v] Variably open and weathered abutment, tight in valley bottom. [w] Lookout Point. [x] Flows tilted 5° to 20° into right abutment. [y] Fault to the left of the concrete section. [z] Over fragmentary beds of tuffs, agglomerate. [aa] Main dam to Mormon Island Dam. [bb] Foliation in Mormon Dam, none in the main dam. [cc] Close to medium and extensive weathering from 20 ft to 30 ft, moderate weathering to 105 ft. [dd] Upstream dip. Joints most pronounced. [ee] Most pronounced dip, secondary dip is set normal to these.

sought in this report to adhere to the definitions previously stated, and to express all the ideas presented in these terms.

This report is considering primarily the matter of foundation uplift. In order to secure consistency of treatment through the entire length of the structure—that is, in the abutments as well as in the maximum section—the same design assumption should be made for all levels in the dam above the foundation. Modifications may be made for differences in tailwater conditions which would lead to steeper surface slopes on abutment sections, but designers will generally prefer to use uniform slopes in order to minimize construction difficulties and to secure satisfactory appearance.

4. Importance of Exploration and Geological Study

The proper determination of these factors, especially I, must be based on the most thorough possible study of the foundation and geological conditions. It should include determination of the geologic structure and its significance, the composition, absorptive capacity, and permeability of the rocks. The prevailing bedding and joint systems; the thickness and relative tightness of strata; and the presence, magnitude, and course of solution passages and of any other systematic or unsystematic openings are significant. The ground-water table as it exists, and as it may be expected to be modified by the proposed construction, is important. If there is evidence of underground flow in a direction parallel to the dam axis, the proper uplift assumption may be affected. The quality of water to be impounded should be studied—for instance, whether it is hard or soft—to determine the possible effect of the water on drains and grouted cutoffs. In short, it is not possible to know too much about subsurface conditions.[6,7] (See case history of Center Hill Dam in Section 7.) Tables 1(*a*), 1(*b*), and 1(*c*) present considerable condensed data on foundation conditions, but individual sites require much more detailed investigation than these tables indicate.

5. Evidence as to Area Factor

The value of the area factor, A, in foundations or in structures has not been determinable *in situ* by any known method. There is some experimental evidence believed to be pertinent, and there are certain arguments and theoretical discussions by authorities which constitute additional evidence.

The most recent experiments upon the effective area in concrete include those of Karl Terzaghi, Hon. M. ASCE, and S. Leliavsky Bey and Douglas McHenry, Members, ASCE. The first investigation, by Mr. Terzaghi, published in 1936, reached a value of 0.98 as the area factor in concrete.[8] Leliavsky Bey, after an elaborate series of experiments, obtained a mean value of 0.91 from his work on concrete.[9] When the concrete was considered to be in a saturated condition, he corrected this value to 0.85. Mr. McHenry, reported to the Third Large Dams Congress in 1948 a value of 1.00± for the area

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[6] "More and Better Geological Data for Use in Hydraulic Projects," by G. H. Matthes, published by the Am. Inst. of Mining and Metallurgical Engrs., 1929.

[7] "Geology and Foundation Treatment for TVA Structures," TVA, U. S. Govt. Printing Office, Washington, D. C., 1949.

[8] *Engineering News-Record*, June 18, 1936, p. 872.

[9] "Experiments on Effective Uplift Area in Gravity Dams," by Serge Leliavsky Bey, in "Uplift Pressure In and Beneath Dams: A Symposium," *Transactions*, ASCE, Vol. 112, 1947, p. 444.

factor,[10] after experiments on concrete with the triaxial compression machine at Denver, Colo. The mean of these values is 0.94.

As to theoretical evidence, L. F. Harza,[11] M. ASCE, has presented an argument by deductive reasoning that the area factor must be 1.0. His reasoning is applied primarily to concrete, but it appears to be equally applicable to rocks in which there is defined porosity, and this includes a large number of such rocks. The essence of this paper was also presented to the Third Congress on Large Dams at Stockholm, Sweden. It has been widely discussed.

In a previously cited work[5] a strong argument is made for the use of an area factor of 1.0 in foundation rocks, particularly when these are horizontally bedded. Mr. Terzaghi argues[12] that conditions similar to those which he found in concrete can be expected in rocks which are measurably porous, and that these rocks in their own structure will be subject to a high area factor. This is aside from the influence of defined openings in the rock, whether they be joints, openings along beds, or the results of solution. The presence of these defined openings facilitates access to rock which is porous and promotes the saturation thereof and the development of a high area factor. The defined openings may themselves occur in such a position as to present an area factor of approximately 1.0.

In 1933 W. A. Zisman experimented on the compressibility of rocks by applying hydraulic pressures to jacketed and unjacketed specimens by a method similar to that followed by Mr. Terzaghi in his experiments on concrete.[13] The compressibility of most of the rocks examined was markedly greater in the jacketed specimens than in the unjacketed ones. These results point to a high value of the area factor in these rocks. The Chairman of this Subcommittee has estimated from the given data that a specimen of marble had an area factor of 0.90, while specimens of three different granites had a factor of approximately 0.75. An exception was found in the case of obsidian, which, because of its glassy nature, had no porosity and showed no difference between the performance of jacketed and unjacketed specimens. This confirms Fig. 2 as mentioned later. Mr. Zisman's data also indicate that, when the pressure intensity reached 720 kg per sq cm (10,200 lb per sq in.), the difference in compressibility between unjacketed and jacketed specimens very markedly diminished, indicating that pores and cracks were closed by the high pressure.

In the discussion and closure of Mr. Harza's paper, previously cited, it is pointed out that the existence of pore pressure area up to values of 100% can occur together with a concurrent high degree of structural strength which will resist the disruptive effects of pore pressures. Similarly, the body of unweathered foundation rock will be characterized usually by even higher qualities of structural strength. However, if weathered rock exists in the foundation, as it frequently does along joints, the weathered material can be characterized by a high area factor with little or no structural strength. Whereas weathered material is usually removed before construction, the possibility that some residuals will still remain in the foundation is strong and must be recognized.

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[10] Discussion by Ross M. Riegel of "The Significance of Pore Pressure in Hydraulic Structures," by L. F. Harza, *Transactions*, ASCE, Vol. 114, 1949, p. 250.

[11] "The Significance of Pore Pressure in Hydraulic Structures," by L. F. Harza, *ibid.*, p. 193.

[12] *Proceedings*, A.S.T.M., Vol. 45, 1945, p. 777.

[13] *Proceedings of National Academy of Sciences*, Vol. 19, 1933, p. 666.

An interesting statement was made by W. Schürter, general reporter of the Third Large Dams Congress at Stockholm, in the report of that body, namely:

> "The ratio between the area effectively submitted to uplift pressure and the total area considered is an essentially empirical value, which may be determined only by experiments.
>
> "Investigations, communicated to the Third Large Dams Congress by different authors, have supplied for this coefficient, values included between 0.85 and 1.0.
>
> "If special steps are not taken against uplift pressure, the action of the aforesaid is, in principle, to be felt in the substratum of the foundation area and in the body of the dam."

In the same congress, E. Reinius stated: "In Sweden the effective uplift area is always assumed to be 100 percent of the total area."

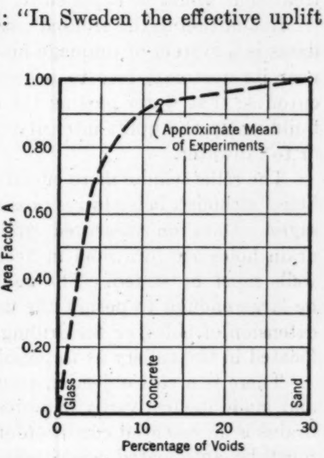

FIG. 2.—AREA FACTOR RELATED TO VOIDS, ACCORDING TO W. P. CREAGER

An interesting diagram (Fig. 2) has been submitted by W. P. Creager, Hon. M. ASCE, of this Subcommittee. Based on an admission of 1.0 as the area factor in sand, and a factor of 0 in an absolutely nonporous material such as glass, the area factor would vary roughly as shown in the diagram, giving the general position of experimental values.

The further fact may be cited that near the base of high dams, where tailwater is normally present and may have a high value, a condition of ultimate saturation must be regarded as attainable. With it is a tendency to a high area factor.

In the light of this experimental evidence and argument, most of the members of the Subcommittee believe that as a practical matter all new dams should generally be designed with an assumed area factor of 1.0.

6. METHODS OF REDUCING INTENSITY FACTOR *I*

The first method of reducing *I* is that of the grouted cutoff. The object, of course, is the prevention or retardation of the passage of water beneath the dam and the production of a reduced uplift. The depth, frequency of holes, and pressure employed will be determined by the geological conditions, as revealed by the exploration. A great deal of trial drilling and grouting is generally advisable before a final program is arrived at. Useful preliminary information may be obtained from the accounts of similar operations elsewhere and from Tables 1(*a*), 1(*b*), and 1(*c*). Wherever a definitely impervious stratum exists in the foundation, it is usually advantageous to connect the grout curtain with this stratum. (However, see discussion of Center Hill Dam in Section 7.) Where definitely pervious strata are known to exist the curtain should cut off these formations. Where there is little variation in the strata, approximate flow nets may be utilized to determine the desirable depth of cutoff. It will be seen that no rule can be given for this depth.

The grouted cutoff is regarded by most engineers as permanent. However, at the Third Large Dams Congress, Mr. Reinius of Sweden expressed some doubt as to this permanency. In any event if a gallery is provided near the foundations, as is usual in the larger American dams, additional grouting of the cutoff can be performed from this gallery if it is ever required. In the case of lower dams, such grouting can be performed either initially or later through the water of the reservoir. Evidence as to the advisability of later treatment would be supplied by observation of leakage.

In addition to the grouted cutoff, an additional provision in many American dams is a system of drainage holes which is drilled from a gallery in the dam near its upstream face to penetrate the foundation rock downstream of the cutoff so that water passing the cutoff area can escape to the gallery without building up the uplift intensity. This water escapes to the downstream face or to tailwater.

The relief will be more effective and reliable with closer spacing of the drain holes, although it is always possible that the drains will not reach certain passages. Data on measured uplift pressure and on spacing and depth of drain holes are furnished in Section 7. Certain requirements as to drainage wells must be stated. The gallery from which the holes are drilled should be large enough to permit the use of machinery for the cleaning, reaming, or extension of holes, or for drilling additional holes, and the holes should be so located in the gallery as to permit such operations.

There is a real objection to drain holes in that they may become clogged and made ineffective by deposits. Regular inspection of the condition of the drains is an essential condition of their use, and cleaning, scraping, or reaming must be anticipated when necessary. The drilling of a new set of drains may possibly be necessary if other measures are ineffective. Several instances have come to the attention of the Subcommittee where measured uplift pressures have been reduced by drilling additional drain holes, notably at Hoover Dam (in Arizona and Nevada) and Norris Dam (in Tennessee).[3,14]

The clogging of drains may arise from the following causes:

a. Evaporation, which may at times leave deposits of calcium carbonate or other minerals upon the exposed surface of the drain hole.

b. Reduction in the capacity of the water to carry solids because of its loss of pressure on escaping into the drains.

c. Increase of alkalinity by the solution of salts contained in the rocks.

d. Collection of trash or debris in the drains.

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[14] "Uplift Pressures in Concrete Dams," by Kenneth B. Keener, *Transactions*, ASCE, Vol. 116, 1951, p. 1218.

Swedish engineers are opposed to the use of drains because of troubles experienced in their country. On the other hand, Tables 1(*a*), 1(*b*), and 1(*c*) show a wide use of drains in the United States, and the Subcommittee has not been informed of troubles similar to those reported in Sweden. It is suggested that Swedish waters in general have much lower alkalinity than those of the United States and, therefore, are much more active in their effects upon concrete and rocks with soluble elements. As suggested in Section 4, the quality of reservoir water is one of the factors requiring examination in the preliminary investigations.

The most satisfactory check on the performance of the drains is furnished by observations of the uplift pressure occurring under the structure (supplemented by measurements of the actual flow), and these observations are most permanently reliable when made from transverse galleries in the dam. In structures of any size when reliance is placed on the effectiveness of the drains, the provision for uplift measurement should be an essential feature of the design.

Members of the Subcommittee have called attention to the wide buttress-type dam constructed in Sweden and Scotland. In this type drains can readily be installed between the buttresses and a very complete drainage of the entire foundation can be secured. The same result can be obtained by a system of galleries transverse to the axes of solid dams with drains drilled in the floor thereof. Subsurface conditions may make such drainage advisable. The selection of a type of structure is not, however, the function of this Subcommittee, and is a question of economy of design. It is held to be the Subcommittee's function to deal generally with so-called gravity dams.

7. EVIDENCE AS TO INTENSITY FACTOR IN FOUNDATIONS

Herewith is summarized the available data on intensity of actual foundation uplift as measured in the structures themselves.

a. The first publication which may be classed as giving modern information on the subject is that of Julian Hinds,[15] M. ASCE. (The data given on two masonry dams, Willwood (in Wyoming) and American Falls (in Idaho), are included in the following paragraph.)

b. Another reference is to the collection made by Ivan E. Houk,[5,16] M. ASCE, and reproduced as Fig. 3. In this diagram are shown measured intensity diagrams for structures with data as to cutoff grouting and drainage provisions. It is to be noted that the highest intensity is shown for a structure having neither grouting nor drainage and that those structures in which both grouting and drainage have been provided show the lowest range of intensities.

c. In Fig. 4 are shown separately data for Neye (in Germany) and Willwood dams listed by Mr. Houk, for which cutoff grouting was performed but no drainage was provided. To these have been added a record from Wheeler Dam of the Tennessee Valley Authority (TVA) compiled by the Chairman of this Subcommittee and hitherto unpublished. The record of Wheeler Dam (in Alabama) is a composite of readings at nine stations over a period of ten years. The foundation characteristics at Wheeler are given in Table 1(*b*).

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[15] "Upward Pressure Under Dams," by Julian Hinds, *Transactions*, ASCE, Vol. 93, 1929, p. 1527.

[16] "Measurements of Existing Structures," by Ivan E. Houk, *Civil Engineering*, September, 1932, p. 578.

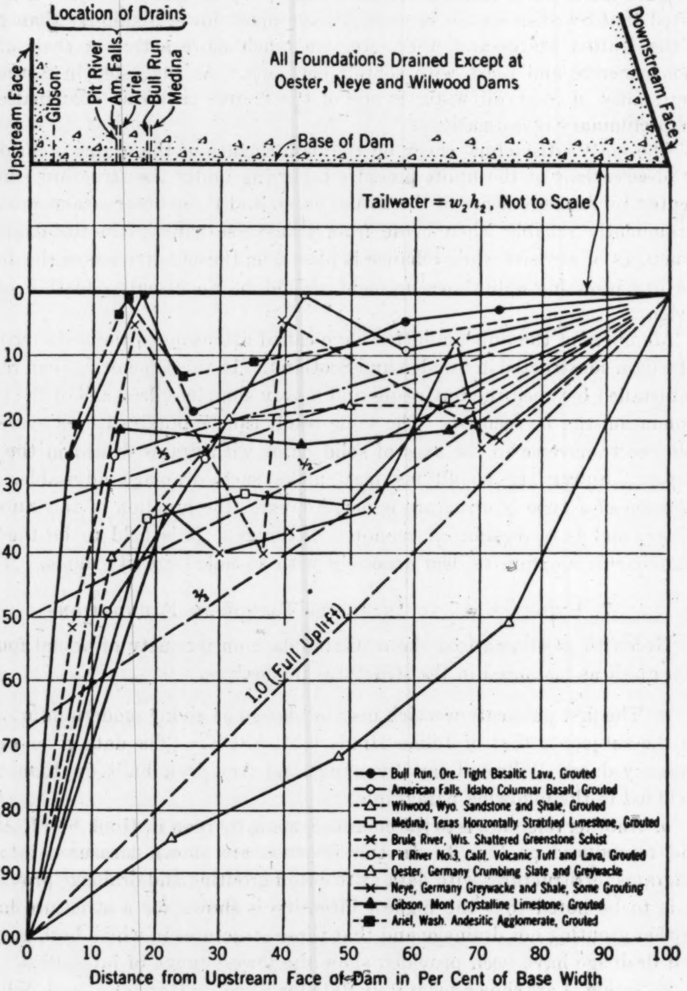

FIG. 3.—MAXIMUM OBSERVED HYDROSTATIC PRESSURE UNDER EXISTING GRAVITY DAMS ON ROCK FOUNDATIONS

This dam is a relatively low structure, being subject to only about 50 ft of net head. In these dams there is a marked reduction from 100% intensity as previously defined. For convenience the average line of the Bureau of

Reclamation (USBR), United States Department of the Interior (from Fig. 9), is included in this diagram.

There is much more information available as to dams which are provided with both grouting and drainage, particularly from the USBR and the TVA.

d. Information for eight dams of the Bureau of Reclamation has been given by K. B. Keener, M. ASCE, in a paper before the Society's Power Division in November, 1949, at Washington,[14] from which Fig. 5 is reproduced. Foundation data for these dams, furnished by Mr. Keener, are presented also in Table 1, together with data as to depth of cutoff and drainage.

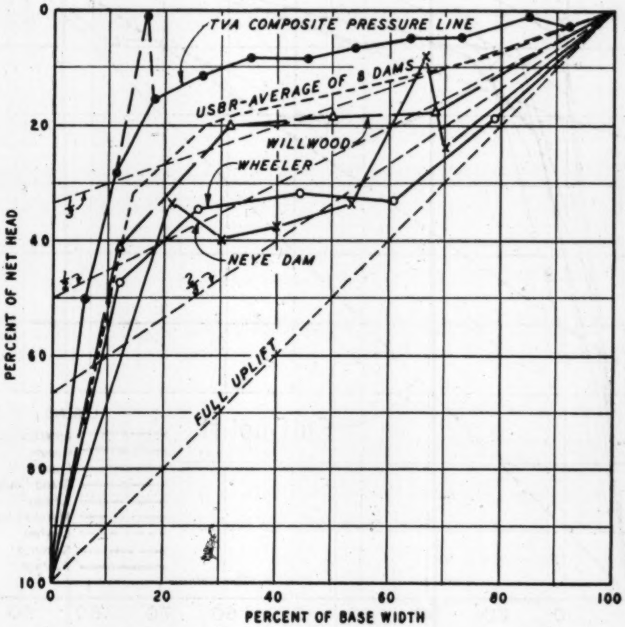

Fig. 4.—Uplift Intensity for Undrained Dams

Certain features of Fig. 5 require further explanation. It shows an average figure for Hoover Dam, extending over the period from 1935 to 1948. However, in about 1939 additional grouting and drainage were performed at Hoover Dam. Subsequent to this treatment the intensity diagram has been measured to have the form shown in Fig. 6. Obviously the additional work had a pronounced effect. The form of the intensity diagram for Owyhee Dam in Oregon is explained in Section 7*l*.

It is to be noted with reference to Mr. Keener's average curve that, if the later values for Hoover Dam had been used as obtained after the supplementary drilling and grouting, and if Owyhee Dam with its rather exceptional foundation condition had been omitted, the average would have been closer to the TVA composite curve, referred to later, than appears in Fig. 5.

The uplift diagrams for Grand Coulee (in Washington) and Shasta dams are shown in Figs. 7 and 8,[17] respectively, and indicate a high degree of effectiveness in the drainage system. The Grand Coulee exhibit is given because it seems to represent the lowest intensity condition for the eight listed dams. The Shasta diagram is not included in Mr. Keener's general average. As to the Shasta diagram, Mr. Keener advises that the foundation rock is a solid and massive meta-andesite, very hard, with few and tight joints, although there are some crushed zones. The grout curtain is 150 ft deep at the maximum section with drain holes 10 ft on center extending 50 ft into the rock at the maximum section and 10 ft at the abutments.

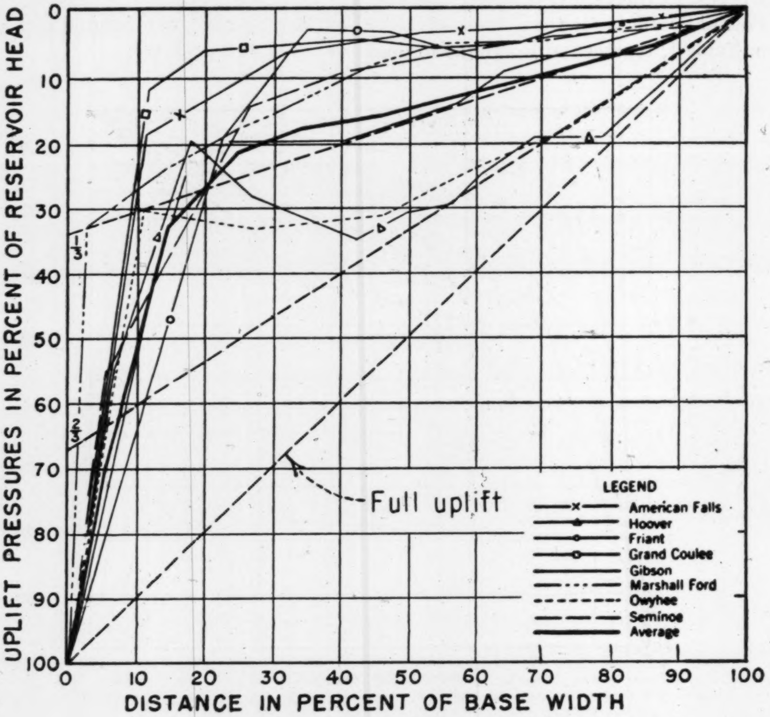

Fig. 5.—Average Uplift Pressure at Base of Several Dams

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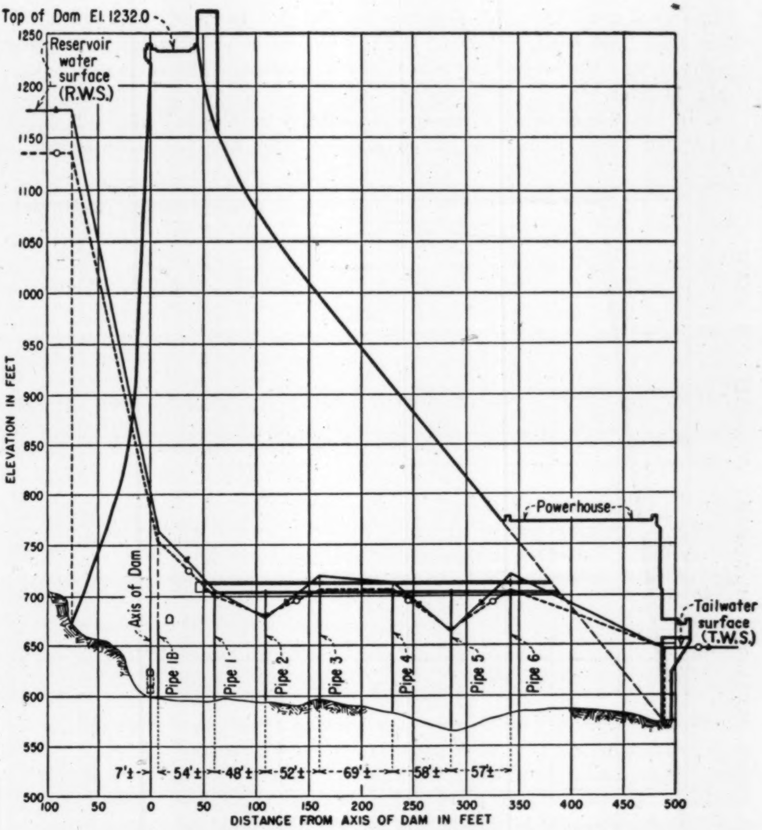
[17] "Uplift Pressures in Concrete Dams," by Kenneth B. Keener, *Transactions*, ASCE, Vol. 116, 1951, p. 1227.



DATE	R.W.S. EL.	EL. OR EQUIVALENT EL. OF W.S. IN PIPE							T.W.S. EL.	LEGEND
		1B	1	2	3	4	5	6		
JAN. 15, 1947	1146.22	754	700	682	707	706	666	707	647.00	
FEB. 14,	1140.40	754	701	682	707	707	665	707	647.00	
MAR. 14,	1136.08	755	700	681	707	707	665	707	647.00	
APR. 15, 1947	1135.12	755	700	681	707	707	665	707	647.00	--○--
MAY 15,	1141.03	755	700	681	707	707	665	707	647.00	
JUNE 16,	1160.52	758	702	680	707	707	664	707	647.00	
JULY 15,	1175.65	762	704	680	707	707	664	707	647.00	
AUG. 15,	1177.23	766	706	680	719	714	665	721	647.00	
SEPT. 15,	1179.40	766	707	680	720	714	665	722	647.00	
OCT. 15,	1176.78	767	707	680	720	714	664	722	647.00	
NOV. 14, 1947	1177.01	766	707	679	720	714	664	722	647.00	—△—
DEC. 15, 1947	1173.55	766	707	679	720	714	664	722	647.00	

LINE B ALONG 10-FOOT GALLERY FROM ELEVATOR TO POWERHOUSE-NEVADA SIDE

FIG. 6.—HOOVER DAM (IN ARIZONA-NEVADA)—UPLIFT PRESSURES AT FOUNDATION; MEASUREMENTS DURING 1947

In Figs. 5 and 7 the uplift pressures are stated in terms of "reservoir head" rather than "net head." For these dams the effect of tailwater is not very significant, and the inaccuracy is not very important. The same can be said of Fig. 11 later.

e. In Fig. 9 is shown a composite of intensity profiles on four dams of the TVA.[18] In these dams observation sections were established as shown in Table 2. The intensity profiles used in constructing the composite were the maximum ones in the most recent year. Data on the character of foundation rock and on the drainage and cutoff provisions are given in Table 1(*b*).

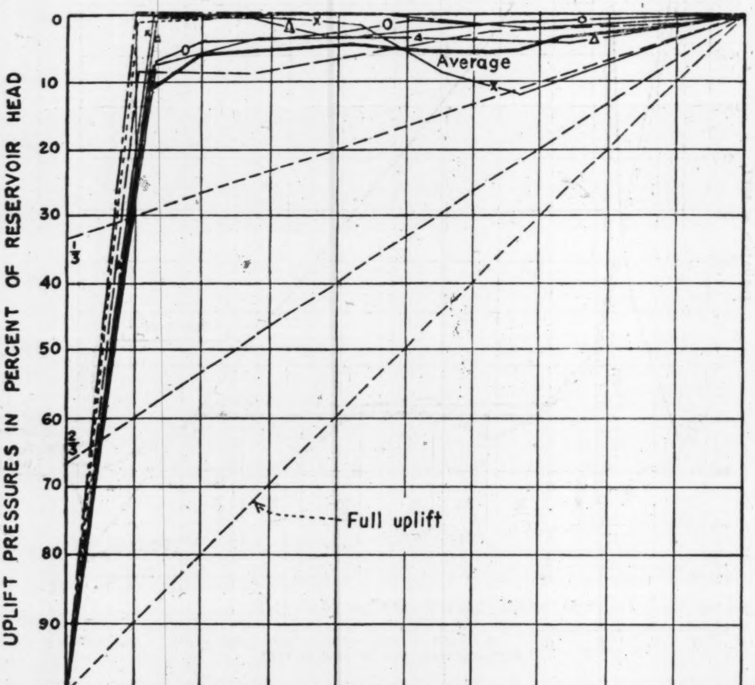


FIG. 7.—GRAND COULEE DAM (IN WASHINGTON)—UPLIFT PRESSURE AT BASE OF DAM, 1942–1945, ALONG VARIOUS LINES NORMAL TO AXIS. (HEAVY LINE IS THE AVERAGE)

The solid line as shown in Fig. 9 represents composite readings of intensity of pressure actually made in the area of contact between concrete and foundation or within the top 2 ft to 3 ft of foundation. Dotted lines are shown from the composite pressure line to a point representing the mean position of drainage outlets in a gallery. In the four dams there is perceptible flow from about 90% of the holes and in the remainder water stands at some level below the gallery floor. Although the drain holes do not fall exactly on the 14 measuring sections, there is justification for the view that the pressure gradient in the composite is no higher than the outlet of the drains in the gallery floor, and the dotted lines represent a gradient corresponding to this view. While there is considerable variation in the measurements as between individual sections of the total of 14, in no section is any measured pressure importantly higher than the assumed design gradient, as checked by the effect on the overturning moment. (This statement is on the authority of the Chairman, member of the staff of the TVA.) This diagram is furnished through the courtesy of C. E. Blee, M. ASCE, chief engineer, TVA.

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[18] Discussion by Ross M. Riegel of "Uplift Pressures in Concrete Dams," by Kenneth B. Keener *Transactions*, ASCE, Vol. 116, 1951, p. 1239.

TABLE 2.—DATA ON FOUR TVA DAMS

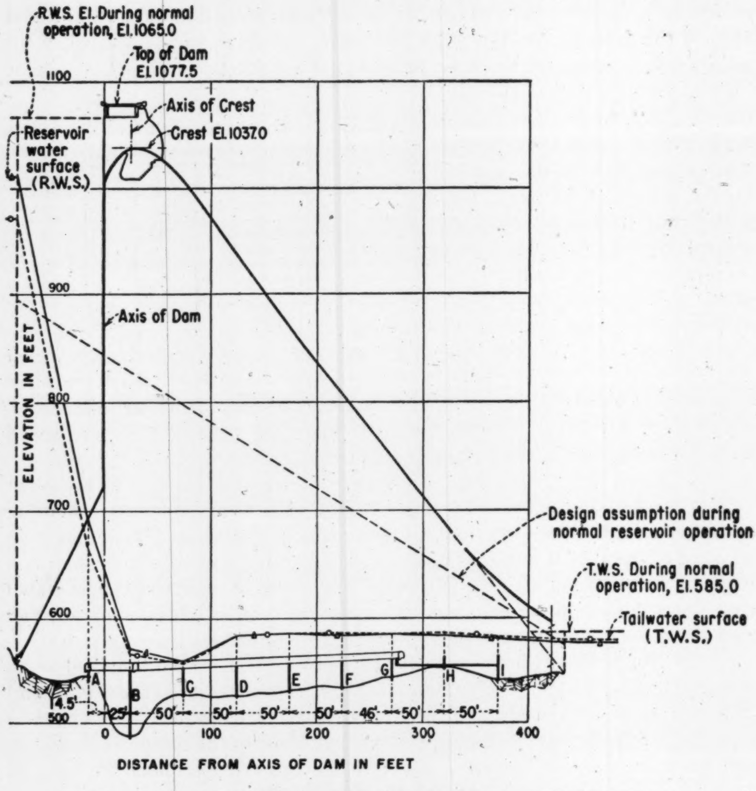
Dam	Maximum height, in feet	Number of sections observed
Hiwassee in North Carolina......	307	4
Fontana in North Carolina.......	480	3
Douglas in Tennessee............	202	3
Cherokee in Tennessee...........	175	4

One reason for the difference between the average curves for the eight USBR dams and the four TVA dams (Fig. 9) has been explained. Another is that the drain holes are somewhat more closely spaced on TVA dams than on most of the USBR dams. On the other hand, discussion by the Chairman of this Subcommittee with the Bureau of Reclamation engineers has indicated that the accuracy of Bureau measurements has probably been superior, and that the TVA intensities may have been slightly under-read.

f. At Norris Dam (in Tennessee) of the TVA, drains were provided at an average spacing of 34 ft. (This statement is on the authority of the Chairman.) This dam does not classify as either well drained or undrained. As might be expected, the uplift intensities are greater than those of the composite in Fig. 5, and less than those at Wheeler. Both Norris and Wheeler dams are founded upon calcareous rocks with nearly horizontal bedding.

g. As to dams provided with neither grouting nor drainage there are cited only two instances of measured uplift intensity, for example, the Oester Dam in Germany and Holtwood Dam on the Susquehanna River. Data as to measurements are given in Figs. 3 and 11, respectively. As to foundation conditions at Oester, little is known, but those at Holtwood are described in Section 7*m*. There is so little resemblance in these records that no important conclusions can be drawn.

h. Table 1(*c*) shows the characteristics of foundations and similar information to that in Tables 1(*a*) and 1(*b*) for a large number of dams constructed by the Corps of Engineers, United States Department of the Army. Unfortunately, uplift intensity measurements do not appear to be available to accompany this information. It is evident that the three major public organizations engaged in the construction of dams practice the systematic drainage of foundations, at least in recent years, and presumably have faith in its usefulness.



DATE	R.W.S. EL.	EL. OR EQUIVALENT EL. OF W.S. IN PIPE									T.W.S. EL.	LEGEND
		A	B	C	D	E	F	G	H	I		
JAN. 22, 1947	964.0	668	569	559	583	586	585	586	586	583	580.9	
FEB. 19,	974.1	673	569	559	585	587	586	587	586	583	577.8	
MAR. 18,	995.2	680	571	560	587	587	586	586	586	582	577.8	
APR. 16, 1947	1010.6	689	572	560	584	586	585	586	585	582	577.8	——o——
MAY 28,	1006.1	690	572	560	584	587	586	586	586	583	581.4	
JUNE 25,	1006.4	689	572	560	584	587	587	587	586	584	581.8	
JULY 23,	996.4	680	571	560	584	586	585	586	586	583	582.1	
AUG. 20,	984.9	673	570	559	583	586	586	586	586	583	581.7	
SEPT. 17,	976.1	668	569	559	582	585	585	585	585	583	579.8	
OCT. 15, 1947	972.3	665	568	559	584	586	586	586	586	583	579.7	----o----
NOV. 26,	973.1	664	569	559	583	586	583	585	585	582	579.3	
DEC. 23, 1947	972.8	664	569	559	583	585	585	586	585	582	578.2	

LINE 4 - STA. 26 + 06.75 ON ℄ OF TRANSVERSE GALLERY

FIG. 8.—SHASTA DAM (IN CALIFORNIA)—UPLIFT PRESSURES AT FOUNDATION; MEASUREMENTS DURING 1947

i. In connection with drainage, it may be noted that in Fontana Dam (in North Carolina) of the TVA a portion of the dam was provided with a secondary drainage gallery and drain holes at about the downstream third point, because the foundation conditions appeared to require this. The Bureau of Reclamation has also used a secondary gallery at times. This arrangement may sometimes be advantageous in other proposed structures if the geological study so indicates. It is also generally advisable to provide drainage under spillway aprons. When drainage is provided, the possibility of the drains becoming ineffective through deposits on the surface of the holes or in pores adjoining the holes must be recognized when active waters are circulating through the rocks. Regular inspection of the drained dams is a necessity.

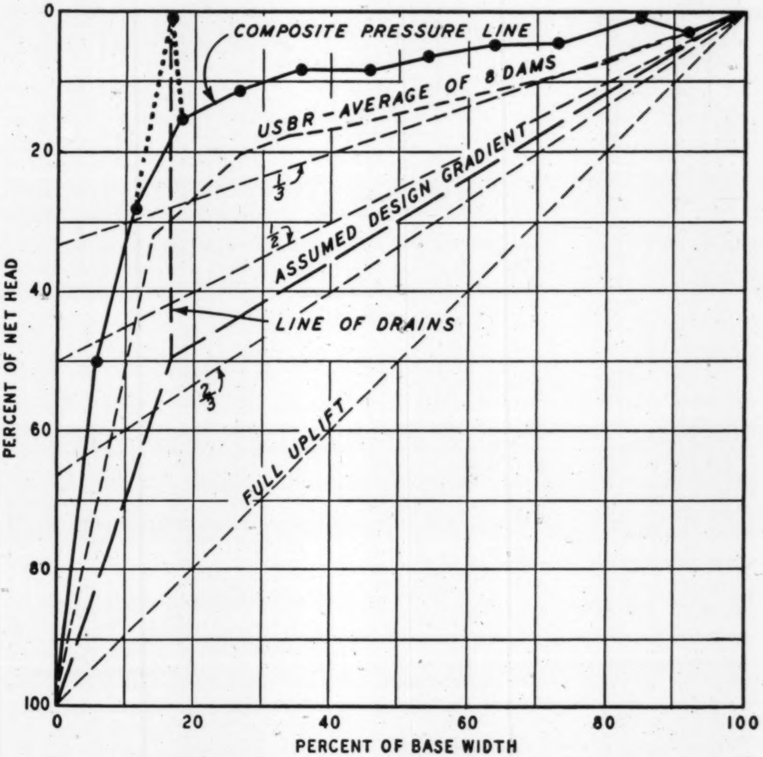


FIG. 9.—COMPOSITE FOUNDATION UPLIFT INTENSITY FOR FOUR TVA DAMS

When dams are constructed without drains but when grouted cutoffs are relied upon to reduce uplift (as would be justified by the three pertinent cases cited), a gallery should be provided in high dams in order to facilitate the drilling of additional grout holes, if evidence should arise that the curtain is losing its effectiveness. Such evidence would be provided by an observed increase in leakage, or by an observed increase in uplift pressure. In low dams such grouting could be done through the water of the reservoir, but, if reservoir depths are much more than 50 ft, the difficulties of drilling are likely to be considerable.

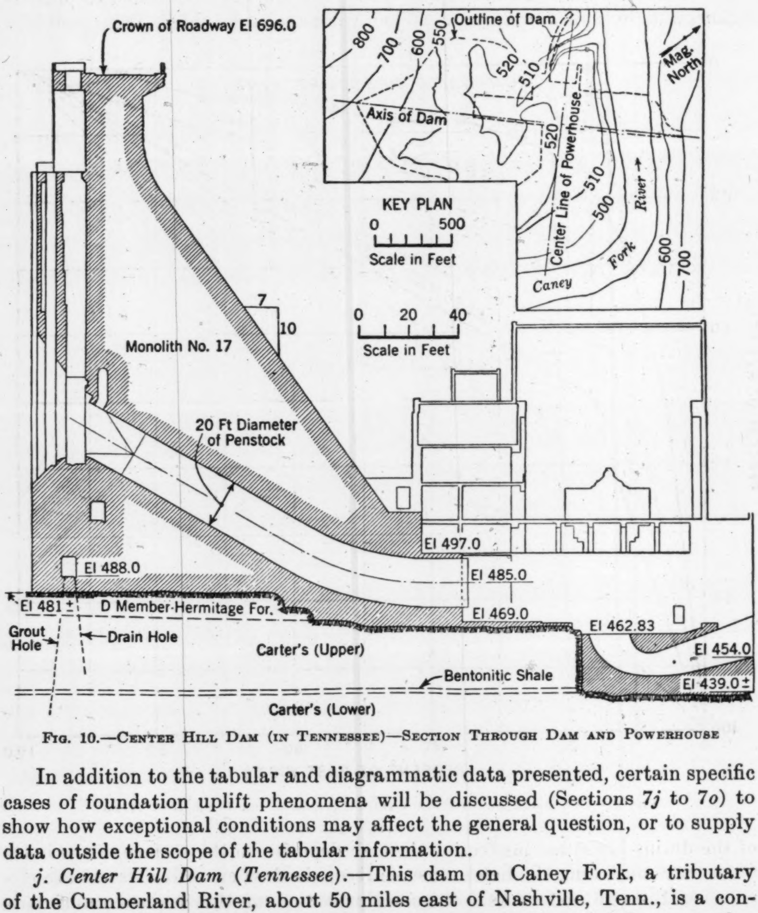

Crown of Roadway El 696.0
Outline of Dam
800
700
600
550
520
510
Axis of Dam
520
Center Line of Powerhouse
510
500
River
Fork
Caney
600
700
Mag. North
KEY PLAN
0 500
Scale in Feet
7
10
Monolith No. 17
0 20 40
Scale in Feet
20 Ft Diameter of Penstock
El 497.0
El 488.0
El 485.0
El 481 ±
D Member-Hermitage For.
El 469.0
El 462.83
Grout Hole
Drain Hole
Carter's (Upper)
El 454.0
Bentonitic Shale
El 439.0 ±
Carter's (Lower)

Fig. 10.—Center Hill Dam (in Tennessee)—Section Through Dam and Powerhouse

In addition to the tabular and diagrammatic data presented, certain specific cases of foundation uplift phenomena will be discussed (Sections 7*j* to 7*o*) to show how exceptional conditions may affect the general question, or to supply data outside the scope of the tabular information.

j. Center Hill Dam (Tennessee).—This dam on Caney Fork, a tributary of the Cumberland River, about 50 miles east of Nashville, Tenn., is a concrete gravity and earth-fill type for flood control, navigation, and the generation of power—the cross section of the powerhouse section being shown in Fig 10. The dam itself was completed in December, 1949, including some foundation excavation for the powerhouse.

The concrete dam and powerhouse are founded on nearly horizontally bedded limestone containing certain impervious members. The strata dip moderately downstream and toward the right abutment (that is, approximately due north) so that the impervious members are exposed in the reservoir upstream from the dam but not in the river valley downstream from the dam. The foundation area thus overlies an artesian system several miles long.

Specifically, a layer of bentonitic shale, approximately 18 in. thick, was found in the exploration, lying about 40 ft below the base of the dam and 25 ft below the base of most of the powerhouse. It was penetrated, however, by the draft tube excavation (Fig. 10), the foundation concrete at this point being placed immediately below the bentonite bed.

In Fig. 10 it will be noted that there is a grouting gallery about 10 ft above the foundation and closely adjacent to the upstream face of the dam; this gallery is continuous through the concrete structure. Curtain grouting was performed from this gallery for a depth of 100 ft below the base of the dam and approximately 35 ft below the bentonitic shale. Relief drain holes were then drilled to a depth of 35 ft below the base of the dam and 10 ft above the bentonite.

During the winter and spring of 1948 and 1949, and prior to placing any appreciable weight of concrete in the draft tube excavation, the reservoir was filled to an elevation about 149 ft above the base of the dam and 189 ft above the bottom of the draft tube excavation. It was noted during this period that there was some upward movement of the rock in the zone of deep excavation for the draft tube, and a system of checks was set up to follow this movement and determine the cause. During the late spring the reservoir was drawn down to a level approximately 100 ft above the base of the dam when it was noted that the zone in which there had been upward movement then settled with reasonably close relationship to the lower elevation of the reservoir.

After further checks it was decided in the ensuing fall to drill test holes under the dam gallery and powerhouse area to discover whether or not there were any water bearing or pressure zones in the foundation rock at lower elevations. Two water pressure zones were developed under the dam and powerhouse area at levels of approximately 50 ft and 60 ft, respectively, under the dam and 10 ft and 20 ft, respectively, under the bentonitic shale. These test holes initially had a gage pressure at top of the hole of 47 lb per sq in. in the dam gallery upstream from the power intake section, and 42 lb per sq in. at top of the hole in the powerhouse area where upward movement had been noted.

It was then decided to drill additional drain holes on 20-ft centers in the dam gallery through the bentonitic shale to a level 20 ft above the bottom of the grout curtain. In addition, about 30 holes were drilled in the powerhouse area, below the powerhouse, along the channel side of the powerhouse, and at the downstream edge of the spillway bucket. The holes first drilled initially had high flows and pressures; but both flow and pressure were rapidly reduced with lapse of time and as other holes were drilled until all flow from holes in the dam gallery ceased and flow in the powerhouse area holes was reduced to a negligible amount for the usual reservoir levels. It should be noted that the foundation rock, which had risen, had settled back again.

It was the opinion of consultants that the upward movement resulted from reservoir pressures superimposed on the artesian system and that, on relief of these artesian pressures, the drainage from the permanent relief holes would be negligible. It is to be noted that no water bearing or pressure levels were encountered in the concrete or in the foundation rock above the bentonitic shale. The Center Hill example illustrates the importance of geological study.

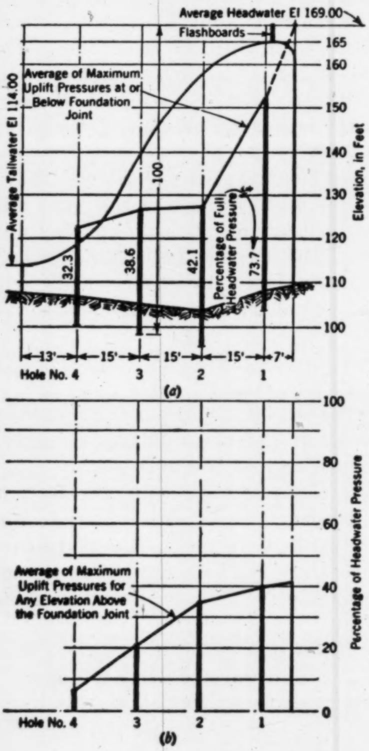


FIG. 11.—HOLTWOOD DAM (IN PENNSYLVANIA) —(a) AT FOUNDATION JOINT (b) ABOVE FOUNDATION JOINT

k. Chickamauga Dam (*Tennessee*).—At the Chickamauga Dam of the TVA there were several beds of bentonite in the limestone formations constituting the foundation of the dam. These beds dipped upstream but were broken somewhat by minor faults not far above the dam. The bentonite beds were 6 in. or less in thickness. During exploration artesian flows in drill holes were observed when the holes penetrated the bentonite beds, in a manner similar to that developed at Center Hill.

The bentonite also provided a floor against solution effects and in this respect was very useful from the standpoint of construction. A system of drain holes downstream of the grouted cutoff, penetrating the bentonite seams, was designed to minimize uplift intensity under the structures. The importance of geological relationships is again illustrated.

l. Owyhee Dam (*Oregon*).—With regard to the uplift diagram for Owyhee Dam (Fig. 5), Mr. Keener has furnished the following comment:

> "The uplift pressures at Owyhee Dam may be ascribed to the nature of the foundation rock, a porphyritic or glassy rhyolite, full of joints and fractures which are nearly vertical. The innumerable joints and fissures formed a widespread system of channels through which water could readily travel. Thus the drains were relatively effective only a short distance downstream from their locations as may be surmised from the shape of the diagram."

m. Holtwood Dam (*Pennsylvania*).—This dam (Fig. 11[19]), on the Susquehanna River, is an overflow ogee spillway about 70 ft in height above the lowest foundation. The underlying rock is a metamorphosed granitic schist containing many quartz veins and occasional fault seams. No grouting or drainage was provided. The design assumed that pressure would vary from 50% of headwater to 50% of tailwater, applied on the full area ($A = 1.0$). The dam was constructed from 1906 to 1910.

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[19] "Uplift Measurements at Holtwood Dam," by P. E. Gisiger, *Civil Engineering*, July, 1938, p. 447, Fig. 1.

In 1936, seventeen vertical holes were drilled through the dam, of which thirteen penetrated the foundation rock. Observations of pressures occurring at various levels were determined, making use of packers to isolate 5-ft sections of hole. The principal conclusions stated in the reference paper are as follows:

1. The four holes drilled through to rock next to the upstream face of the dam all showed substantial amounts of uplift at or near the foundation joint, varying from 51% to 84.5% of full head of forebay water at joint elevation, and averaging 73.7%.
2. The average maxima of uplift measured at or below the foundation joints, in percentage of full headwater pressure, are as shown in Fig 11(*a*).
3. The maximum uplift measured (in any of the four holes next to the upstream face drilled through to rock) for any horizontal joint above the foundation joint is 72.7% of full headwater pressure at joint elevation. The average of the four holes, however, is only 39.9%.
4. The average maxima of uplift measured for planes above the foundation joint are as shown in Fig 11(*b*).

It will be noted that measurements were also made in the concrete of the dam, this being almost the only case found where intensity of uplift was actually measured in these circumstances.

n. Stewartville and Aguasabon Dams (Ontario, Canada).—In "The Review of Research Activities of The Hydro-Electric Power Commission of Ontario for 1949," there are diagrams of observed foundation uplift for the Stewartville and Aguasabon dams of that organization (Fig. 12). The apparatus for measurement was of the same type as that used by the USBR and the TVA, as previously described. The observation points were in the inspection galleries, indicated on the diagrams.

These dams differ from those previously discussed in that in both cases there is a continuous surface drain at the rock surface parallel to the axis of the dam. Correspondence with the director of engineering of the commission, J. R. Montague, has furnished the following information:

> Uplift measuring facilities have been provided for a number of the commission's more recent structures including the dams at Stewartville, Aguasabon, Des Joachims, LaCave, Pine Portage, and Tunnel projects, all in Ontario. The conditions at Stewartville and Aguasabon may be representative of the group. The rock underlying the Stewartville Dam is primarily crystalline limestone. Over a wide section of the foundation, steeply inclined faulting and subsequent solution along the fault zones have greatly altered the rock. As insurance against leakage a grout curtain was carried to a maximum depth

of 140 ft adjacent to the upstream face of the dam, which, in places, exceeded 200 ft in height with a net hydrostatic head of approximately 150 ft.

The rock under the Aguasabon Dam is mainly competent red granite. Here grouting was carried out along the face of the dam to a depth of approximately 70 ft. The maximum height of the dam itself was 120 ft. Most of the leakage here, which was of small proportions, originated through fine seams near the excavated rock surface and was readily gathered in the foundation drain.

The drain in both of these structures is continuous throughout the length, has a width of 15 in. and a minimum depth of 12 in. At Stewartville the drain leads directly to the powerhouse sump, the elevation of which is considerably lower than the tailrace. A by-pass to tailwater was also provided in case leakage should become excessive. At Aguasabon drainage is directly to the downstream side of the dam. In all cases the inspection tunnel and foundation drain are connected by a series of vertical 6-in. pipes spaced at from 20-ft to 38-ft centers. These provide emergency relief in case of blockage of the main drain and will facilitate drilling into the foundation rock should this be found desirable. Uplift along horizontal construction joints is obviated by the provision of drains leading to the inspection tunnel. Similar drains intercept leakage in the vertical construction joints.

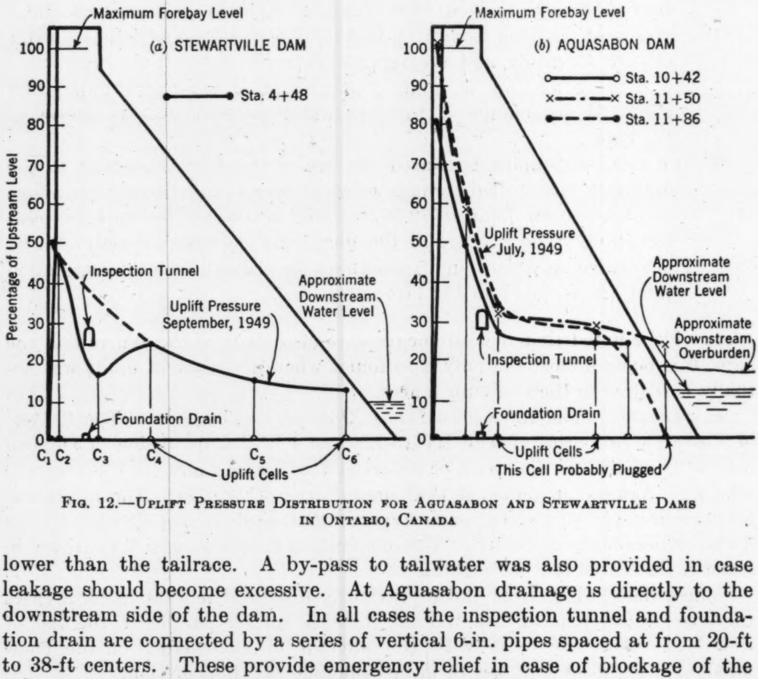

Fig. 12.—Uplift Pressure Distribution for Aguasabon and Stewartville Dams in Ontario, Canada

It was considered that sufficient access to the drain was provided by the fractures and joints in the rock and that relief would thus be provided against any excessive pressure which might build up back of the grout curtain. Mr. Montague further states that any structure which might be placed on horizontally stratified rock would be provided with vertical wells to considerable depth to relieve uplift on the various strata affected.

o. Bartlett's Ferry Dam (Georgia).—In a paper before the Hydraulics Division, ASCE,[20] some interesting data on uplift in this dam on the Chattahoochee River in Georgia are given. The following information has been taken from the paper.

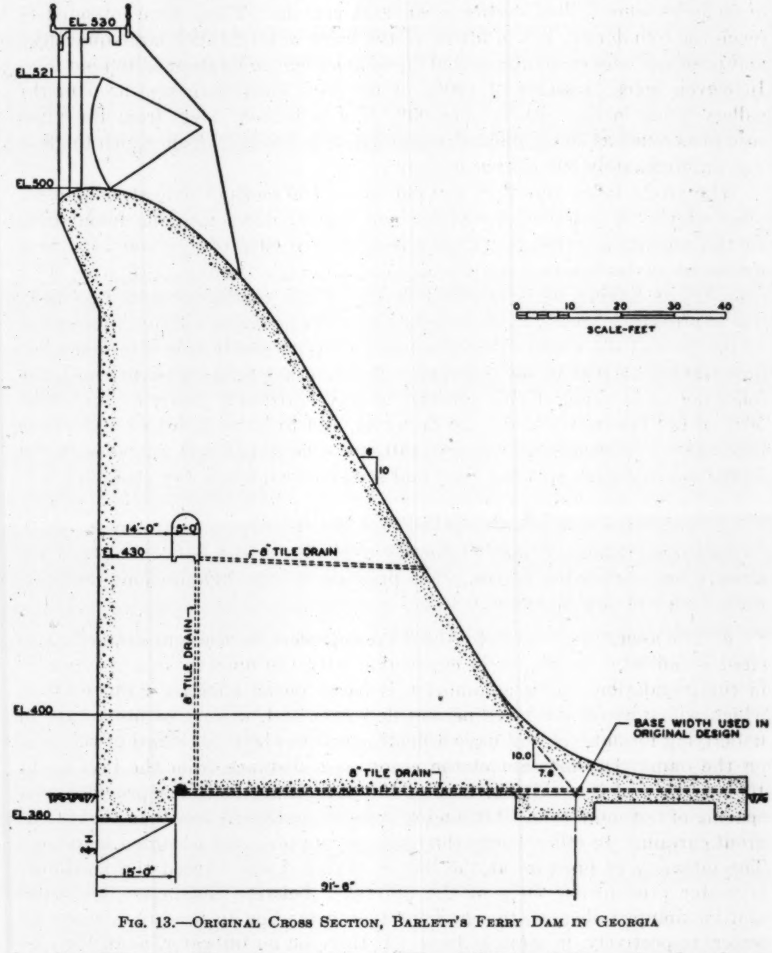
EL 530
EL 521
EL 500
0 10 20 30 40
SCALE-FEET
8
10
14'-0"
5'-0"
EL 430
8" TILE DRAIN
8" TILE DRAIN
EL 400
BASE WIDTH USED IN ORIGINAL DESIGN
10.0
7.8
8" TILE DRAIN
EL 380
15'-0"
91'-6"

FIG. 13.—ORIGINAL CROSS SECTION, BARLETT'S FERRY DAM IN GEORGIA

This dam (Fig. 13) was built in 1925, and was acquired by purchase in 1929 by the Georgia Power Company, its present owner. The foundation uplift assumption is shown in the figure. Mr. Harrison advises by letter that the assumed intensity was applied over 100% of the area, and that the longitudinal drain was assumed to prevent any uplift pressure downstream thereof.

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[20] "Remedial Modifications to 25-Year-Old Bartlett's Ferry Dam, Checked by Hydraulic Models," by E. S. Harrison and C. E. Kindsvater, *Proceedings-Separate* ASCE (publication pending).

The Georgia Power Company engineers came to believe that uplift pressures existed in the concrete above the rock. Accordingly, thirty-one 3-in. drain holes were drilled vertically at 20-ft centers. These were intended to reach the foundation, but in fifteen of the holes water, in such large quantities and pressures, was encountered that these holes had to be stopped temporarily. In eleven holes pressures of 100% of reservoir head were measured in the gallery while in four others over 90% was measured. Jets from the pipes rose to as much as 59 in. above the galley floor in one hole, from which the flow was approximately 390 gal per min.

The fifteen holes were then grouted, about 560 sacks of cement being used, after which the holes were redrilled and carried down into the foundation. In this condition, pressures ranging from 2% to 69% of reservoir head were measured in ten of the thirty-one holes although twenty-eight holes were recorded as having seeps or small flows. These conditions may be chiefly due to pressure intensity in the foundation. The pertinence of this information to the present discussion is that foundation uplift was in this case much less important than that in the concrete. The dam has been redesigned with the following assumption: Uplift pressure at any horizontal plane, to vary from 50% of full headwater to 0%, or tailwater, and to be applied to 100% of the base area. To secure satisfactory stability, a large buttress was constructed downstream of each spillway pier, and secured to the spillway structure.

8. Assumptions for Design

The observations of uplift intensity available to the Subcommittee have already been presented herein. The practices of the organizations reporting these data will now be stated.

a. The assumption made by the TVA engineers for the four drained dams cited is indicated in Fig. 9, in connection with the observations of intensity in the foundation. This assumption is based on an effective grout curtain, which will shut off the more permeable strata and, in case of uniformity in underlying formations, will have a depth equal to about one third of the head on the dam. Downstream of the cutoff at a distance from the heel about 10% to 15% of the width of base of the dam, drain holes are provided at a spacing of not more than 10 ft, and of a depth materially less than that of the grout curtain. In other words, the uplift assumption may be stated as follows: The intensity of pressure at the line of drains should exceed the maximum tailwater pressure by 50% of the difference between maximum headwater and maximum tailwater, the gradient then extending to headwater and tailwater, respectively, in straight lines. If there be no tailwater, as in the case of abutment sections of high dams, the downstream end of a similar diagram should be located at the surface of foundation rock. The area factor was assumed as two thirds, except for Fontana Dam where it was 1.0.

In further explanation of Fig. 9, the line designated "assumed design gradient" is not the real design gradient; Fig. 9 is based on the tailwater existing at the time when the field observations were made, and is drawn to show how the field observations compare with an uplift diagram of the type suggested. The true design diagram is based on the maximum tailwater and headwater to be expected under design flood conditions, for which condition, of course, there are no field observations. It is difficult to express the composite of the design flood conditions. The essential purpose of the diagram is to show the effectiveness of the drainage system.

A similar treatment of intensity has been favored by other engineers than those of the TVA.[21] The Owyhee Dam of the USBR was designed on a similar intensity assumption, combined with the 100% area assumption. This treatment has the advantage that the gradient assumed for design is similar in form to the actual gradients noted in the TVA and USBR observations.

b. Inasmuch as the designer's judgment must determine the intensity factor, he may prefer, for simplicity, to assume the intensity factor I in terms of a straight-line variation, as shown in Fig. 3. There is no reasonable objection to such a procedure. For a discussion of some suggested values of this factor, reference may be made to a previously cited work.[5]

In Mr. Keener's paper[14] all the Bureau of Reclamation dams discussed are stated to be grouted and drained. The uplift intensity assumption assumed in design in all cases discussed (except Owyhee Dam) was that of a straight gradient from headwater to tailwater, herein designated as a 100% uplift, that is, $I = 1.00$ (Fig. 1). In all cases except Owyhee the area factor A was assumed at two thirds. This assumption was also used in the design of the Norris and Wheeler dams, referred to previously, these being designed for the TVA by the Bureau of Reclamation.

It appears in the light of the various figures presented that the 100% intensity factor ($I = 1$) is decidedly conservative, particularly if an area factor A of 1.00 be assumed, and that the intensity factor of the TVA diagram would be substantially supported by the USBR data.

c. The depth of the grouted cutoff near the heel of concrete dams designed by the Corps of Engineers is governed by foundation conditions that may affect seepage of water out of the reservoir. The depth of this cutoff bears no direct relation to the height of the dam. If impervious strata are available at reasonable depths, the cutoff grouting is carried to the impervious strata. Drain holes downstream from the grout holes are generally carried to about 80% of the depth of the grout holes. The grout and drain holes are generally drilled from a drainage gallery located in the concrete near the heel of the dam and a few feet off the foundation.

In the past the uplift was assumed as reservoir head at the upstream face, varying uniformly to tailwater head at the downstream face, applied to a percentage of the area of the base. Recent practice has assumed a percentage of reservoir head at the upstream face, varying uniformly to tailwater head at the downstream face, applied to the entire base area. Results obtained from these two uplift criteria are not materially different except in the case of unusually great tailwater depth, as noted in Section 3. (Statement of B. W. Steele, A. M. ASCE, member of the Subcommittee.)

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[21] Discussion by William P. Creager of "The Significance of Pore Pressure in Hydraulic Structures," by L. F. Harza, *Transactions*, ASCE, Vol. 114, 1949, p. 215.

d. If drainage be omitted and reliance for the reduction of uplift be placed only on the grouted cutoff, the available evidence is shown in Fig. 4. This shows a pattern of intensity favoring an assumption of less than 1.0 for the intensity factor *I*, but not so favorable as the USBR and TVA diagrams. Inasmuch as the undrained group comprise only three structures (as compared with twelve structures in the drained classification), a more conservative attitude toward this condition would seem to be required, that is, that a higher intensity factor should be assumed than would be assumed if drainage were provided in addition to cutoff. The designer's judgment must determine the proper assumption. A study of flow nets would be valuable in this connection. Those who would install drains but take no credit for their presence can still logically assume an intensity factor materially less than 1.0 if they use the grouted cutoff.

e. It may possibly be established by the geological exploration that the downstream portion of the dam rests on relatively tight strata whereas the upstream portion is founded upon relatively open structure. None of the examples cited herein is known to be in this category. For such a foundation, the uplift pressure is likely to equal reservoir pressure at the upstream face, to decrease slowly through the porous foundation, and to decrease rapidly through the tight portion. This would result in an uplift intensity greater than 1.0. Special foundation treatment methods might justify a reduction in the factor.

f. As to dams built without any cutoff grouting or drainage provisions, the following comments appear to be in order:

(1) It might appear that a 100% intensity factor should be applied. However, if foundation conditions are so unfavorable as to seem to justify such an assumption, they should be remedied by grouting, of the consolidation or the cutoff type, or both, by draining and by "dental work," and then an appropriate factor of intensity determined. These provisions apply chiefly to major structures.

(2) In minor structures it may be more economical to increase the intensity factor (and, consequently, the dam section) rather than undertake the cost of protective measures. By "minor structures" are meant those of less than, say, 100 ft in height. However, an intensity factor as high as 1.00 would seem to be only rarely justified.

g. Certain European engineers prefer to construct a gravity dam without drains or other openings which would provide shortened paths of seepage. They consistently employ an intensity factor of 1.0, even though the foundation be grouted. Such procedure is, of course, in accordance with their best judgment; but usually American engineers, including most of this Subcommittee, believe that considerations of economy generally dictate a different attitude.

h. Care should be taken to determine the maximum possible tailwater conditions, as they may be affected by ice jams or similar occurrences.

9. DESIGN OF DAM ABOVE FOUNDATION

The dimensions of the dam will be determined by the uplift assumption in the foundation. If drainage of the foundation be adopted, similar or equivalent provisions for relief of uplift or pore pressure may then properly be made in the dam itself. This may be done by wells similar in spacing to those in the foundation, by open contraction joints, by horizontal drainage of lift joints, or by other expedients, all designed to insure that no greater uplift effects occur in the dam structure than occur in the foundation. The same precautions as to location of drainage wells and ability to clean or supplement them should be observed as for the foundation.

The question of uplift in concrete is not covered in this report although some pertinent data thereon have been given. Certain investigations by the Corps of Engineers promise to provide an important contribution to the subject and to be fruitful material for any future committee study.

10. CONCLUSIONS AND RECOMMENDATIONS

The value of this report lies chiefly in the data as to measurements and practice presented. Adequate design assumptions are the responsibility of the designer. However, a few conclusions or recommendations appear to be warranted, although they should not be taken as "codifying" design procedures.

a. The Subcommittee finds it inadvisable to recommend definite, or even minimum, values for the intensity factor I. The circumstances peculiar to each project must guide the designer in making the proper assumption. His own experience will be a factor in his judgment. The structural and geological characteristics of foundation rock are of fundamental importance, as evidenced by the case histories cited. The treatment of the foundation rock, as by cutoff grouting only, or by grouting combined with drainage, is equally important. The precedents and measurements cited in this report should be considered.

b. Most of the Subcommittee join in the conclusion that, for the design of new structures, it is advisable to assume an area factor of 1.0, for the reasons given in the body of the report.

c. The systematic inspection and maintenance of drainage systems are of great importance. In the opinion of this Subcommittee, if there is not adequate assurance that drains will be inspected and maintained, no credit for their performance should be assumed in design. Where the dams in question are in the custody of large organizations which maintain their own engineering staffs, it would appear that these organizations will automatically attend to such inspections and maintenance. Such are the USBR, the TVA, the Corps of Engineers, and the public utility systems which operate a number of hydroelectric projects. The projects operating under Federal Power Commission license receive such inspection from engineers of the commission. The Subcommittee believes that there is a duty upon any governmental authorities which may issue charters or licenses, or which may have the responsibility for approving plans for construction, to insure that such inspections are systematically made, either by the owners or by their own agents.

d. In a dam which is large enough to permit the installation it is advantageous to have an inspection gallery or galleries close to the foundation, whether or not it be used for drainage. In this gallery it is advisable to install devices for measuring foundation uplift. Should unduly high uplift pressures develop, the gallery will facilitate their correction.

e. With the presentation of this report, the present Subcommittee should be considered as discharged. However, since the consideration of pore pressure or uplift in concrete has not been treated because of inadequate present information, it is recommended that a new committee or subcommittee be constituted to study and report upon this phase of the uplift question. Such a committee will probably have to function for several years before it will be able to report.

11. Need for Information

This report will not be satisfactory to those who are looking for finality in its conclusions. It cannot be made so in view of the lack of conclusive data in certain directions and in view of the fact that reliance for design assumptions must rest upon individual judgment. Necessary additional data include the following:

a. Additional observations of actual pore pressure intensity in concrete structures, including both those having provision for relief of such pressures and those lacking such provision.

b. Determinations of permeability of the concrete in such structures.

c. Similar determinations for foundation rocks, as and when feasible.

d. Determination of effective area in structures and foundations, if found possible.

e. Determinations of the intensity of uplift in existing dams and their foundations, particularly in those of considerable age. The methods employed at Holtwood are pertinent.

The accumulation of such data requires much time and considerable money, but it may be hoped that in the course of some years data will be accumulated. The public organizations, such as the USBR, the Corps of Engineers, and the TVA, have been trying, and may be expected to continue trying, to accumulate data. The owners of other dams, particularly those of light section, should be interested in answers to point "*e.*" As and when such information becomes available to an important extent, this report should be supplemented to incorporate the results. Meanwhile, the report represents the accomplishment of the Subcommittee to date.

Respectfully submitted,

John B. Alexander
William P. Creager
L. F. Harza
Ivan E. Houk

A. V. Karpov*
Gerard H. Matthes
Byram W. Steele*
James H. Stratton
Thomas H. Wiggin*

Ross M. Riegel, *Chairman*

Subcommittee on Uplift in Masonry Dams of the Committee on Masonry Dams of the Power Division

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* Submitted individual comments, see Appendices. Mr. Wiggin did not sign the report.

APPENDIX A. EXCEPTION BY A. V. KARPOV

The writer believes the Subcommittee is not justified in specifying a 100% pressure area (conclusion 10*b*). In some conditions, where the rock stratification is horizontal or nearly so, the 100% area may be justified. When, however, good rock is encountered with vertical or close to vertical stratification, there is no justification for assuming a 100% area.

After all, the 100% area theory is based on pure speculation which, in the writer's opinion, is contrary to the available evidence. There is no use in increasing arbitrarily the cross section of a dam if that does not contribute to its safety.

APPENDIX B. SUPPLEMENTARY STATEMENT BY BYRAM W. STEELE

The writer concurred in Section 10*b* because there is no other logical method of preparing a structural analysis of a section of a gravity dam except to assume 100% area and then apply to this area an intensity factor obtained from a thorough analysis of the geology and foundation conditions obtained at the dam site in question. Uplift is a force that must be recognized as a potential hazard in any water retaining structure and hence, must be provided for in the design.

It is obvious that there will be differences of opinion among designers relative to the shape of the uplift intensity curve proposed for any design. It should be equally obvious in the case of Center Hill Dam cited herein that a curve in the form of a straight line between the headwater and the tailwater, as used so often in the past, would not have been adequate since the pressure actually developed was 100% of reservoir head over 100% of area. Foolproof drainage will cost only a fraction of what the assumption of full uplift costs.

To ignore drainage is to ignore an economic factor that is not in keeping with present-day engineering standards of design and economy. In addition to the system of drainage, designed and built as a part of the dam construction program, natural drainage through cracks and crevices in the rock and through construction joints in the concrete constitute important factors in molding the shape of the uplift curve.

Horizontally stratified rocks are potential sources of high uplift intensity unless relieved by proper and thorough drainage, whereas igneous rocks and columnar basalts provide a degree of drainage that is too often improperly evaluated in assuming an uplift value to use in the design of a dam with the result that the cost of the dam is excessive and in many cases the type of dam should have been changed from concrete gravity to one of the less expensive but equally acceptable types.

APPENDIX C. MINORITY REPORT BY THOMAS H. WIGGIN

The report of the Subcommittee is a monument to the arduous labors, engineering knowledge, and experience of Chairman Riegel and it contains many valuable data and sound engineering opinion on uplift and related questions. Nevertheless, the writer, after much thought, refrained from signing the report because such signing would seem to indicate his belief that the problem of uplift in masonry dams was now solved and the Society could forget it and pass on to other unsolved matters.

The writer was in favor of calling the report a progress report and of having the Subcommittee, after infusion of some younger blood, continue its studies and particularly extend them to the behavior of typical older dams which, with smaller base widths and without aid of grouted cutoffs or artificial drainage, continue to stand without apparent distress. If one's scientific conclusions appear not to square with human experience, it seems always safe to carry the research further.

Only three failures of masonry dams in the United States are generally mentioned as worthy of note and in each case foundation conditions were flagrantly bad and seemed sufficiently so to account for the failures. The dam at Austin (Pa.) which failed was not only on a poor foundation but had a base only 60% of its height.

The writer actively advocated the formation of an ASCE committee on uplift pressure and ice pressure by communication to the Society more than thirty years ago, again somewhat later when he was a member of the Society's Research Committee, and, finally, at the Chattanooga meeting in 1939 when he made two accepted motions for committees on uplift and ice pressure.

Lest the impression should be gained that the writer is in favor of skimpy dam sections, he would add that early in his technical schooling he noted what was apparently an inconsistency between factors of safety for framed structures and those for gravity masonry dams, and he has been responsible for increasing the factor of safety in a considerable number of older dams and is still frequently so engaged, sometimes by use of prestressed steel rods with specially designed anchorages. Provisions for factors of safety can never be wholly satisfactory, however, until the forces involved are more surely known.

It may be remarked, incidentally, that most dams in regions of cold winters would be rocked forward on their foundations or rocked at some upper joint if anything like the 30,000 or 40,000 lb per lin ft generally considered proper for ice pressure for monumental dams were actually developed. This statement may be considered as irrelevant to uplift pressure, but the writer has always associated the two problems of uplift and ice pressure and has long been disturbed that the national civil engineering society has in a century produced no complete and convincing answer as to the forces to be withstood by so important and potentially dangerous a type of structure as the gravity masonry dam. It may be added that the situation with respect to spillway provisions is in a similar state of uncertainty as between the provisions ordinarily made and the extraordinary provisions made in recent years where money was available.

The main conclusions of the report are that substantially 100% of the areas of foundation and joints receive uplift pressure, that the intensity of such pressure can be reduced materially below a straight pressure grade line from headwater level to tailwater level by grouting and drainage, but that such drainage should not be relied on unless systematic inspection at proper intervals over the life of the dam can be guaranteed. (It may be noted in passing that there is a possibility that pressures might be above the straight line, although friction head from flow through a fine-grained porous medium, provided it is uniform, follows the first power.)

The fact is stated that Swedish engineers do not count on drainage. The wide buttress type of dam developed in Sweden with space between buttresses for drainage to the uncovered surface of the ground is noted. Some possible reasons are given for the continued stability of older dams designed and built without benefit of modern knowledge as to uplift pressures.

Apart from such corroboration as may be drawn from certain ingenious laboratory tests on small specimens subjected to combined compression and water pressure, proof of the existence of uplift pressure over nearly, if not quite, 100% of the base of masonry dams rests on the finding of pressure in dams of the USBR, TVA, and some others in practically all drill holes which cross the base joint. It is recognized in the noteworthy paper by Mr. Keener on tests of USBR dams that such drill holes cover only a microscopic proportion of the foundation area. At first glance, however, such tests would seem certainly to be equal in probability to a Gallup poll. The possibility seems to exist, however, that the drill holes, which extended 3 ft or more into the rock below the foundation joint, might pierce some minute passages in the rock not at the base joint. Furthermore, why should a base joint behave differently from a construction joint in the masonry on which experiments were also tried by the USBR, as noted in the following paragraph.

As to the pressure at construction joints in concrete, Mr. Keener states that no such pressure was found. This would seem to be contrary to the conclusions of the experiments by Mr. Terzaghi, Leliavsky Bey, and others who found proof in the behavior of concrete laboratory specimens under stress that water pressure existed throughout the specimens. The USBR experiments on joints were given up after about eleven years without finding uplift pressures.

An interesting but unpublished paper (entitled "Hydrostatic Pressure in Concrete") by T. C. Powers, manager of basic research, Portland Cement Association, treats of the microstructure of concrete and evolves the conclusion that water permeates nearly but not quite the whole of concrete. However, he finds that the pressure is not disrupting but on the contrary is negative, except where the concrete lies below a continuously existing water level so that water can soak back into the concrete and keep its water-demand satisfied. As the joints under greatest stress are not unlikely to be below tailwater elevation, this theory does not afford much comfort with respect to the stability of the dam as a whole.

The report mentions the adhesion of mortar joints as one reason for the stability of light, ungrouted, and undrained dams. The writer recalls that years ago the late Howard A. Carson, Hon. M. ASCE, made many informative pioneer tests on unreinforced concrete beams 6 in. by 6 in. in section and about 6 ft long. A goodly number of these were further tested as follows: Halves of broken beams were put back in the forms and built out with fresh concrete to full length. A coating of neat cement was used at the joint. The beams were then retested with the joint in the middle and gave strengths nearly, and in some cases quite, equal to that of the original 6-ft beams. The strength which these joints had would resist perhaps 400 ft of head without aid from weight.

Whatever the shortcomings of the old dam builders, most of them were careful with joints so that doubtless many of the older dams profit greatly by adhesion of joints.

In the repair and alteration of older dams the writer has sometimes seen strong stone break under the force of steel wedges, leaving the mortar intact; but the reverse is more often true and repointing and grouting are often necessary. Bottom joints however are seldom uncovered. They are better protected from the effects of leakage and do not suffer from frost action. Leaks in joints or elsewhere tend to enlarge by solution of the cement but fortunately the areas of joint affected are generally small in proportion to the whole, although sometimes a stalagmitic surface will be formed over the whole of the lower part of the downstream face of an older dam from well-distributed leakage higher up. Grouting cannot hope to restore strength of adhesion equal to the original strength of a well-made mortar or construction joint.

The writer is conscious of the inadequacy of this presentation, touching as it does on so few of the pertinent papers on the subject. He is, however, firmly of the opinion that the Society should press for further fundamental research as to uplift pressure, including investigations of typical older dams. Those who have conceived and carried out the tests considered in the Subcommittee's report are deserving of every commendation and Chairman Riegel likewise, not only for his share in the TVA tests but for his arduous labor in assembling the test data and putting so much information pertinent to uplift in form for study and reference.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 2532

SPECIFICATIONS FOR HEAVY DUTY STRUCTURES OF HIGH-STRENGTH ALUMINUM ALLOY

FINAL REPORT OF THE COMMITTEE OF THE STRUCTURAL DIVISION ON DESIGN IN LIGHTWEIGHT STRUCTURAL ALLOYS

SYNOPSIS

These specifications cover allowable stresses, design rules, and fabrication procedures for riveted heavy duty structures built of the high-strength aluminum alloy known commercially as 14S–T6. The basic allowable tensile working stress is 22 kips per sq in. based on a minimum yield strength of 53 kips per sq in. and a minimum tensile strength of 60 kips per sq in.

PART I. GENERAL

INTRODUCTION

These specifications cover the allowable stresses, the design rules, and the fabrication procedures for structures built of the high-strength aluminum alloy most commonly used for heavy duty structural purposes. In the preparation of these specifications the Committee has made use of the available theoretical and experimental work relating to this subject and especially to previous specifications by O. H. Ammann, Shortridge Hardesty, and the late Leon S. Moisseiff,[1] Members, ASCE.

These specifications are confined to allowable stresses, design rules, and fabrication. No attempt has been made to cover the loading, erection, inspection, or nontechnical provisions included in many specifications, since such provisions are fairly well established in current good structural practice. Furthermore, no attempt has been made to include design rules which cover every detail of construction but rather those which are different from steel practice or which are needed for the sake of completeness. It is intended, of

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NOTE.—Published as *Proceedings-Separate No. 22* in June, 1950.

[1] "Design Specifications for Bridges and Structures of Aluminum Alloy 27S-T," by Leon S. Moisseiff, Aluminum Co. of America, Pittsburgh, Pa., 1940.

course, that structures built under these specifications will be designed, constructed, and erected by following the current good practice already well established for steel structures, except as modified herein.

MATERIAL

The principal material considered in these specifications is a high-strength aluminum alloy having the following nominal chemical composition:

Composition	Percentage by weight
Copper	4.4
Silicon	0.8
Manganese	0.8
Magnesium	0.4
Aluminum	93.6
Total	100.0

This material is covered by the American Society for Testing Materials (ASTM) Specifications Nos. B221–49T(CS41A), B235–50T(CS41A), B211–49T(CS41A), B247–50T(CS41A), and B209–50T (Clad CS41A). It is produced by several manufacturers under the commercial designation 14S–T6 and is available in the form of shapes, tubes, rods, bars, and forgings. It is also produced in the form of sheet and plate covered on both surfaces with an integral coating, or "cladding," of a corrosion-resistant aluminum alloy. In the latter form it is identified commercially by the designations, Alclad 14S–T6 and R–301. All these products are given a solution heat treatment and a precipitation heat treatment before being shipped.

The specified minimum tensile properties of this material are not the same in all the various products (plate, shapes, etc.). The specified minimum tensile strengths vary from 60 kips per sq in. to 68 kips per sq in., and the specified minimum yield strengths vary from 53 kips per sq in. to 58 kips per sq in. The following are the lowest of the various specified minimum properties and have been used as a basis for the selection of allowable stresses in these specifications (in kips per square inch):

Description	Stress
Tensile strength	60
Yield strength (offset 0.2%)	53

In addition to the specified minimum tensile properties, the engineer will be interested in some of the other mechanical properties not covered by specifications. The following are typical mechanical properties of this alloy and may be considered applicable to "nonclad" products, such as shapes, and to "clad" plate:

Shear strength, in kips per square inch	41
Modulus of elasticity in tension and compression, in kips per square inch	10,600
Modulus of elasticity in shear, in kips per square inch	4,000
Poisson's ratio	1/3
Coefficient of expansion per degree Fahrenheit	0.000012
Weight, in pounds per cubic inch	0.101

(The foregoing value of shear strength is typical as determined with steel shearing tools. The value determined by torsion tests is greater.)

Fig. 1 shows tensile and compressive stress-strain curves and the compressive tangent modulus curve for 14S–T6 material having the minimum properties listed in the second paragraph of this section.

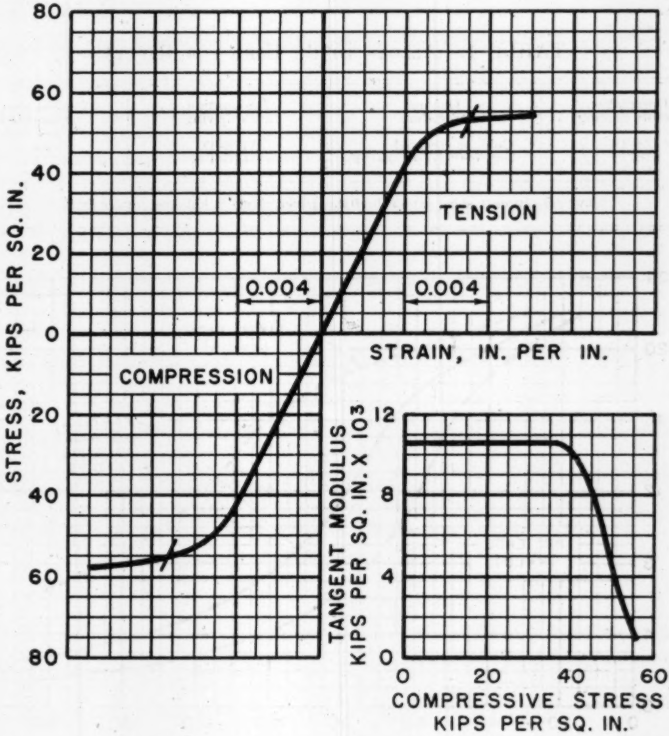

Fig. 1.—Stress Strain and Tangent Modulus Curves

Alloy 14S–T6 is the one principally considered in the preparation of these specifications and the one to which the allowable stresses for parts other than rivets and bolts apply. However, these specifications may be applied to structures built of other suitable aluminum alloys, provided such alloys meet the specified strengths and elongations listed in the ASTM specifications mentioned in the first paragraph of this section.

Rivets used in fabricating structures designed in accordance with these specifications shall be of aluminum alloy and may be either cold driven or hot driven. The alloys used are indicated in Table 1. Supplementary information on riveting was published[2] by E. C. Hartmann, M. ASCE, G. O. Hoglund, and M. A. Miller in 1944.

Permanent bolts used in structures designed in accordance with these specifications shall be of the aluminum alloy known commercially as 24S–T4. Such bolts have a specified minimum ultimate shear strength of 37 kips per sq in.

TABLE 1.—ALLOYS TO BE USED FOR RIVETS

Designation before driving	Driving procedure	Designation after driving	Typical shear strength[a]
A17S-T4........	Cold, as received	A17S-T3	33
61S-T4........	Hot, 990° F to 1,050° F	61S-T43	24

[a] Typical ultimate shear strength of the driven rivet, in kips per square inch.

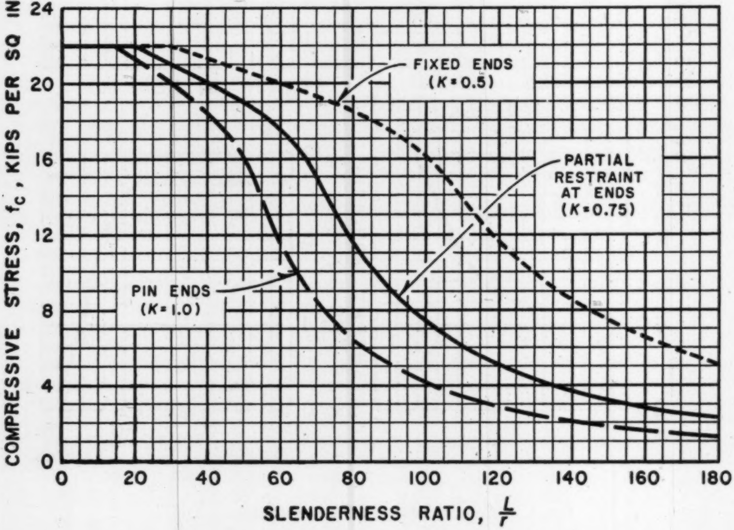

FIG. 2.—ALLOWABLE COMPRESSIVE STRESSES FOR AXIALLY LOADED COLUMNS (GROSS SECTION)

The materials covered by these specifications are heat treated for maximum strength. They cannot be welded without a considerable loss in strength. Structures designed under these specifications shall be assembled by riveting or bolting.

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[2] "Joining Aluminum Alloys," by E. C. Hartmann, G. O. Hoglund, and M. A. Miller, *Steel*, August 7, 1944, p. 84.

PART II. SPECIFICATIONS

SECTION A. SUMMARY OF ALLOWABLE STRESSES

The allowable stresses to be used in proportioning the parts of a structure shall be as follows:

Specification	Description	Stress in kips per square inch
A-1	Axial tension, net section (see Specification H-4)......	22
A-2	Tension in extreme fibers of shapes, girders, and built-up members subject to bending, net section (see Specification H-4).........................	22
A-3	Axial compression (see Section B)..................	....
A-4	Compression in extreme fibers of shapes, girders, and built-up members subject to bending (see Section C)	....
A-5	Compression in plates, legs, and webs (see Section D)	....
A-6	Stress in extreme fibers of pins......................	34
A-7	Shear in plates and webs (see Section E).............	....
A-8	Shear in aluminum alloy A17S-T3 rivets, cold driven (see Tables 4 and 5)...........................	10
A-9	Shear in aluminum alloy 61S-T43 rivets, driven at temperatures of from 990° F to 1,050° F (see Tables 4 and 6)...................................	8
A-10	Shear in turned bolts of aluminum alloy 24S-T4 in reamed holes (see Table 4)......................	12
A-11	Shear in pins....................................	16
A-12	Bearing on pins..................................	30
A-13	Bearing on hot-driven or cold-driven rivets, milled stiffeners, turned bolts in reamed holes, and other parts in fixed contact (see Section G)..................	36

SECTION B. COLUMN DESIGN

B-1. Allowable Compressive Stress in Columns.—The allowable compressive stress on the gross section of axially loaded columns shall be determined from the curves in Fig. 2. Let k be a factor describing end restraint. Ordinarily the curve for partial restraint ($k = 0.75$) shall be used. The curves for pin-ended and fixed-ended columns, also shown in Fig. 2, may be used as a guide in the selection of allowable compressive stresses for those cases in which the degree of end restraint is known to be different from that represented by $k = 0.75$. It is important, however, that no allowable stresses higher than those given for the case of $k = 0.75$ be used in actual design unless a detailed analysis of the structure demonstrates convincingly that a value of k smaller than 0.75 is justified for the member in question.

Columns having cross sections involving webs and outstanding legs of such proportions that local buckling may control the design shall be checked by the method outlined in Section D.

B-2. Maximum Slenderness Ratio.—The ratio of unsupported length to least radius of gyration for compression members shall not exceed 120.

B-3. Connections.—Compression members shall be so designed that the main elements of the section will be connected directly to the gusset plates, pins, or other members.

B-4. Compression Splices.—Members designed for compression, if faced for bearing, shall be spliced on four sides sufficiently to hold the abutting parts true to place. The splice shall be as near a panel point as practicable and shall be designed to transmit at least one half of the stress through the splice material. Members not faced for bearing shall be fully spliced for the computed stress. In either case, adequate provision shall be made for transmitting shear.

B-5. Stay Plates.—On the open sides of compression members, the flanges shall be connected by lacing bars, and there shall be stay plates as near each end as practicable. There shall be stay plates at intermediate points where the lacing is interrupted. In main members the length of the end stay plates shall not be less than one and one-fourth times the distance between rivet lines. The thickness of stay plates shall not be less than one fortieth of the distance between rivet lines.

B-6. Diagonal Lacing.—The slenderness ratio of the part of the flange between the lacing bar connections shall be not more than two thirds of the slenderness ratio of the member.

The angle between the center line of the lacing bar and the center line of the column shall be about 60° for single lacing and about 45° for double lacing.

B-7. Combined Compression and Bending.—The allowable bending stress in a member which carries bending moment in addition to uniform compression (as, for example, an eccentrically loaded column) shall be determined from one of the following two formulas—whichever gives the lower value:

a. Failure by Bending in Plane of Bending Forces.—The maximum bending stress (compression) which may be permitted at or near the center of the unsupported length, in addition to uniform compression, P/A, equals

$$f_b = \left(f_1 - \frac{P}{A}\right)\left(1 - \frac{P/A}{fc_p}\right) \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which (in kips per square inch):

P/A is the average compressive stress on the gross cross section (A) of a member, produced by a column load, P;

f_1 is the compressive stress taken from Figs. 4, 5 or 6 for the compression flange of a member when considered as a beam; and

fc_p is the allowable compressive working stress for a member considered as an axially loaded column tending to fail in the plane of the bending forces.

b. Failure by Buckling Normal to Plane of Bending Forces.—The maximum bending stress (compression) which may be permitted at or near the center of the unsupported length, in addition to the uniform compression, P/A, equals

$$f_b = f_B\left(1 - \frac{P/A}{fc_n}\right)\left(1 - \frac{P/A}{fc_E}\right) \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which (in kips per square inch):

f_B is the allowable compressive working stress for a member considered as a beam; f_{C_n} is the allowable working stress for a member considered as an axially loaded column tending to fail in a direction normal to the plane of bending forces; and

$$f_{CB} = \frac{74{,}000}{\left(\frac{L}{r}\right)^2}.$$

In the equation for f_{CB}, $\frac{L}{r}$ is the slenderness ratio for a member considered as a column tending to fail in the plane of the bending forces.

B–8. Transverse Shear in Columns.—In designing lacing or shear webs for columns, the maximum shear on the column shall be computed from the formula:

$$V = P\,\frac{4.5\,r^2\,(f_B - P/A)}{f_C\,L\,c} + V_t \quad \text{.................(3a)}$$

but shall not be taken less than

$$V = 0.02\,P + V_t \quad \text{.......................(3b)}$$

in which:

V is the maximum transverse shear on any transverse section of a column in the outer eighth of the length at each end, in the direction of assumed bending, in kips;

r is the radius of gyration, in inches;

f_C is the allowable compressive stress taken from Fig. 2, in kips per square inch;

L is the length of the member, in inches;

c is the distance from the centroidal axis to the extreme fiber, in inches; and

V_t is the shear due to any transverse loads on a column, in kips.

The values of f_B, f_C, L, r, and c must be consistent with the direction of bending assumed.

SECTION C. ALLOWABLE COMPRESSIVE STRESSES IN FLANGES OF BEAMS AND GIRDERS

C–1.—The allowable compressive stress in the extreme fiber (gross section) of single-web rolled shapes, extruded shapes, girders, and built-up sections, subject to bending, shall be determined from the curve in Fig. 3. The terms used in Fig. 3 are defined as follows:

L is the laterally unsupported length of compression flange (clear distance between supports at which the beam is prevented from lateral displacement), in inches;

S_c is the section modulus for the beam about the axis normal to the web (compression side), in inches;

$$B = I_1\,d\sqrt{11.7 + \frac{J}{I_1}\left(\frac{L}{d}\right)^2};$$

I_1 is the moment of inertia for the beam about the axis parallel to the web, in inches to the fourth power;

J is the torsion factor, in inches[4]; and

d is the depth of beam, in inches.

In the case of beams having top and bottom flanges of different lateral stiffness, I_1 should be calculated as if both flanges were the same as the compression flange. Values of the torsion factor J are published for many standard shapes.[3]

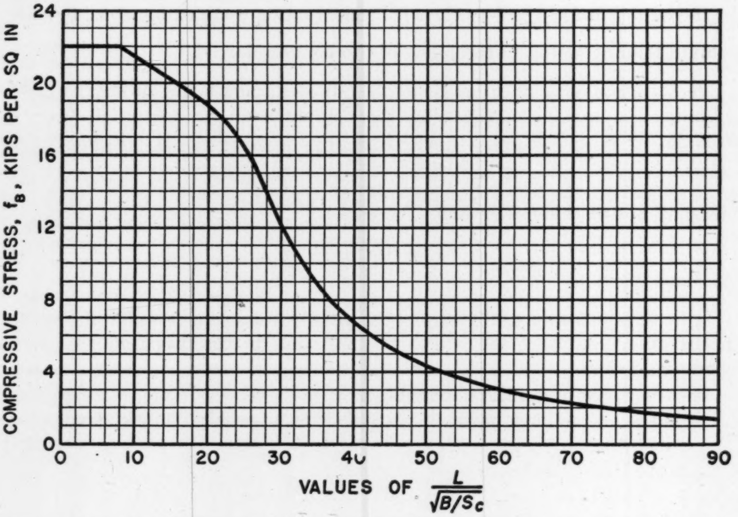

FIG. 3.—ALLOWABLE COMPRESSIVE STRESSES IN BEAM AND GIRDER FLANGES (GROSS SECTION)

Values of J for plates and shapes not published may be calculated by assuming the section to be composed of rectangles and taking the sum of the terms $b\,t^3/3$ for each rectangle, in which b equals the length and t, the thickness of the rectangle, both in inches. The value of J for a built-up member is the sum of the individual values of J of the sections of which it is composed.

The allowable stresses from Fig. 3 provide a safe margin against the lateral buckling type of failure. The outstanding compression flanges of the beams and girders should be checked for local buckling by the method outlined in Section D.

Table 2 lists values of allowable stress determined from Fig. 3 and Section D for various laterally unsupported lengths of a number of standard I-beams and H-beams. Table 3 lists similar values for standard channels.

Because of their tube-like cross section, double-web box girders are very stiff in torsion compared with single-web girders of comparable size, and hence, lateral buckling failures such as are considered in Fig. 3 do not occur in such girders. For double-web box girders it is necessary only to check for local buckling of the flanges by the method outlined in Section D.

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[3] "Alcoa Structural Handbook," Aluminum Co. of America, Pittsburgh, Pa., 1950, pp. 87–112.

SECTION D. ALLOWABLE COMPRESSIVE STRESS FOR PLATES, LEGS, AND WEBS

D–1.—For struts consisting of a single angle or a T-section, the compressive stress on the gross area shall not exceed the values given by the curves in Fig. 4 or Fig. 2, whichever is smaller.

TABLE 2.—ALLOWABLE COMPRESSIVE STRESS IN BEAM FLANGES FOR VARIOUS VALUES OF LATERALLY UNSUPPORTED LENGTH OF COMPRESSION FLANGE, *L*, IN INCHES

Procedure.—Maximum allowable bending moments are found by multiplying the allowable compressive stresses (in kips per square inch) by the gross section modulus of the beam. The stress on the net section of the tension flange must also be kept within allowable limits.

Depth (in.)	Weight (lb per ft)	Section modulus (in.^{3})	VALUES OF L									
			16	32	64	96	128	160	192	256	352	480
(a) I-BEAMS												
2	0.804	0.481	20.6[a]	18.8	10.2	6.1	4.4	3.5	2.9	2.1	1.6	....
2	1.473	0.782	21.6	19.9	17.0	12.2	9.1	7.2	6.0	4.5	3.3	2.4
2.5	1.850	1.162	21.8	20.3	18.3	15.6	12.0	9.6	8.0	6.0	4.4	3.2
3	2.02	1.68	21.4	19.8	15.8	10.2	7.5	5.9	5.0	3.7	2.7	2.0
3	2.67	1.95	21.7	20.3	17.9	14.1	10.4	8.3	6.9	5.2	3.8	2.8
4	2.72	3.03	21.7	20.0	15.7	9.7	7.1	5.5	4.6	3.4	2.5	1.8
4	3.74	3.59	21.8[a]	20.4	17.8	13.6	10.0	7.9	6.5	4.9	3.5	2.6
5	3.53	4.90	21.8[a]	20.2	16.2	9.9	7.1	5.5	4.5	3.4	2.4	1.8
5	5.25	6.09	21.9[a]	20.6	18.2	14.7	10.8	8.5	7.0	5.2	3.8	2.8
6	4.43	7.36	21.8[a]	20.5	16.9	10.5	7.3	5.6	4.6	3.4	2.4	1.8
6	6.13	8.77	21.8[a]	20.7	18.0	13.6	9.8	7.6	6.3	4.6	3.4	2.5
7	5.42	10.48	21.8[a]	20.8	17.5	11.2	7.7	5.9	4.7	3.4	2.5	1.8
7	7.12	12.12	21.8[a]	20.9	18.2	13.4	9.4	7.3	5.9	4.4	3.1	2.3
8	6.53	14.39	21.8[a]	21.0	18.0	12.2	8.2	6.1	5.0	3.6	2.5	1.8
8	9.07	17.18	21.8[a]	21.1	18.7	15.0	10.5	8.0	6.6	4.8	3.4	2.5
9	7.72	19.09	21.8[a]	21.2	18.5	13.2	8.8	6.5	5.2	3.7	2.6	1.9
9	10.68	22.75	21.8[a]	21.2	18.9	15.6	10.9	8.3	6.7	4.9	3.5	2.5
10	9.01	24.68	21.8[a]	21.3	18.8	14.5	9.4	6.9	5.5	3.9	2.7	2.0
10	12.45	29.41	21.8[a]	21.4	19.2	16.3	11.5	8.7	7.0	5.1	3.6	2.6
12	11.31	36.35	21.8[a]	21.5	19.1	15.2	9.8	7.1	5.6	3.9	2.7	1.9
12	17.78	50.81	22.0	21.7	19.8	17.6	14.0	10.4	8.4	6.0	4.2	3.1
(b) H-BEAMS												
4	4.85	5.36	21.7[a]	21.4[a]	19.6	17.7	15.1	11.8	9.7	7.2	5.2	3.8
5	6.63	9.53	21.5[a]	21.5[a]	20.1	18.5	16.4	13.1	10.6	7.7	5.5	4.0
6	8.04	14.69	21.3[a]	21.3[a]	20.5	19.0	17.2	14.4	11.4	8.0	5.6	4.0
6	9.40	15.81	21.3[a]	21.3[a]	20.6	19.2	17.7	15.6	12.7	9.0	6.3	4.6
8	11.51	28.23	20.7[a]	20.7[a]	20.4[a]	19.7[a]	18.6	16.7	13.7	9.1	6.0	4.2
8	13.32	30.23	20.7[a]	20.7[a]	20.4[a]	19.8[a]	18.8	17.2	14.8	10.0	6.7	4.7

[a] These values are governed by local buckling (see Section D). All other values are determined from Fig. 3, Section C.

D–2.—For compression members other than those consisting of a single angle or a T-section, the following procedure shall be followed to provide a suitable margin of safety against the weakening effects of local buckling of flat plates, legs, and webs:

a.—Compute the compressive stress f_c on the flat plate, leg, or web in question, based on the design loads and the gross area, without regard to local buckling. This stress must be within allowable limits as defined in Sections B and C.

b.—Find the limiting value of b/t corresponding to the stress, f_c, by the use of Fig. 5 or Fig. 6. If the flat plate, leg, or web has a ratio of unsupported width to thickness not exceeding this limiting value, local buckling is not a problem and the full gross area of the plate, leg, or web may be considered effective.

TABLE 3.—ALLOWABLE COMPRESSIVE STRESS IN CHANNEL FLANGES FOR VARIOUS VALUES OF LATERALLY UNSUPPORTED LENGTH OF COMPRESSION FLANGE, L, IN INCHES

Procedure.—Maximum allowable bending moments are found by multiplying the allowable compressive stresses (in kips per square inch) by the gross section modulus of the beam. The stress on the net section of the tension flange must also be kept within allowable limits.

Depth (in.)	Weight (lb per ft)	Section modulus (in.^{3})	Values of L									
			16	32	64	96	128	160	192	256	352	480
3	1.46	1.10	21.2	19.3	13.2	8.4	6.2	4.9	4.1	3.1	2.2	1.6
3	2.13	1.38	21.6	20.0	17.4	13.2	9.9	7.8	6.5	4.8	3.5	2.6
4	1.90	1.92	21.4	19.2	12.3	7.6	5.5	4.4	3.6	2.7	2.0	1.4
4	2.58	2.29	21.6[a]	19.7	15.7	10.2	7.4	5.9	4.9	3.6	2.6	2.0
5	2.38	3.00	21.6[a]	19.4	12.3	7.4	5.3	4.2	3.4	2.8	1.8	1.3
5	4.09	4.17	21.7[a]	20.2	17.4	12.9	9.5	7.5	6.2	4.6	3.4	2.5
6	2.91	4.37	21.5[a]	19.7	12.9	7.4	5.2	4.1	3.3	2.4	1.8	1.3
6	4.63	5.80	21.7[a]	20.2	16.7	11.1	8.0	6.3	5.2	3.9	2.8	2.0
7	3.47	6.08	21.6[a]	20.0	14.0	7.8	5.4	4.1	3.4	2.5	1.8	1.3
7	6.13	8.64	21.7[a]	20.4	17.5	12.5	9.0	7.0	5.8	4.3	3.1	2.2
8	4.38	8.46	21.7[a]	20.2	15.3	8.4	5.7	4.3	3.5	2.6	1.8	1.3
8	6.99	11.34	21.7[a]	20.5	17.5	12.2	8.7	6.7	5.5	4.1	2.9	2.1
9	4.74	10.60	21.6[a]	20.4	16.0	8.8	5.8	4.4	3.5	2.5	1.8	1.3
9	8.90	15.75	21.7[a]	20.8	18.1	13.7	9.7	7.4	6.2	4.5	3.2	2.4
10	5.43	13.47	21.6[a]	20.6	16.6	9.3	6.1	4.5	3.6	2.5	1.8	1.3
10	10.67	20.69	21.7[a]	21.0	18.5	14.8	10.4	8.0	6.5	4.8	3.4	2.5
12	7.63	21.97	21.6[a]	21.0	17.9	11.3	7.2	5.2	4.1	2.9	2.0	1.4
12	12.45	29.94	21.7[a]	21.1	18.6	14.6	9.9	7.5	6.0	4.4	3.1	2.2

[a] These values are governed by local buckling (see Section D). All other values are determined from Fig. 3, Section C.

c.—If the flat plate, leg, or web has a ratio of unsupported width to thickness greater than the limiting $\frac{b}{t}$-ratio found in step b, only a part of its unsupported width shall be included in computing its effective area. The part of the unsupported width of any individual flat plate, leg, or web which may be considered effective shall be found as follows:

$$b_e = b\frac{f_1}{f_c} \quad \text{(4)}$$

in which:

b_e is that part of the unsupported width considered effective, in inches;

b is the unsupported width, in inches;

f_c is the compressive stress based on gross area from step (a), in kips per square inch; and

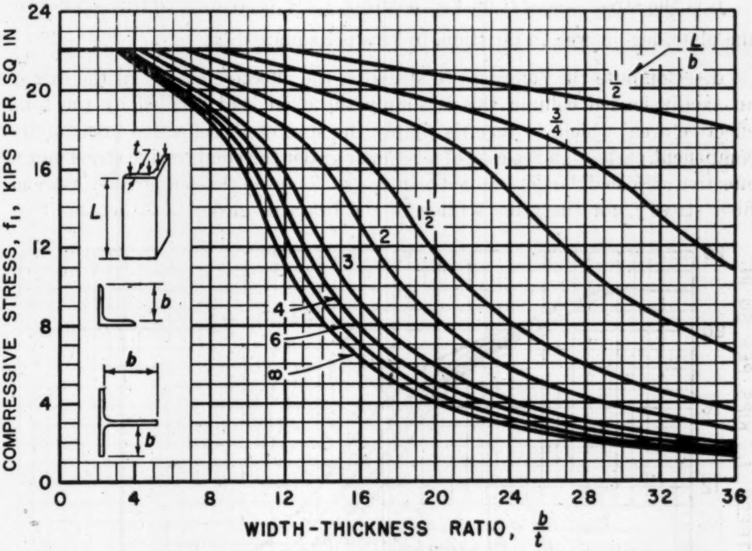

FIG. 4.—ALLOWABLE COMPRESSIVE STRESSES IN OUTSTANDING LEGS OF SINGLE-ANGLE AND T-SECTION STRUTS (GROSS SECTION)

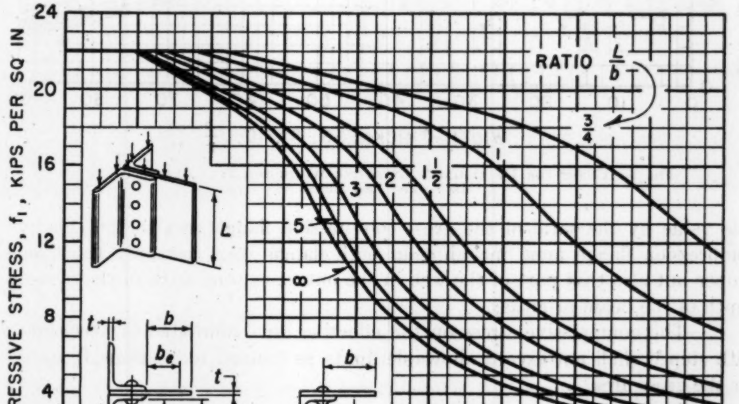

FIG. 5.—CHART FOR DETERMINING EFFECTIVE WIDTH FOR OUTSTANDING LEGS OF ANGLES BUILT INTO OTHER PARTS AND FOR PLATES BUILT IN ALONG ONE EDGE

f_1 is the stress found from Fig. 5 or Fig. 6 corresponding to the $\frac{b}{t}$-value for the plate, leg, or web in question, in kips per square inch.

d.—Compute the compressive stress on the effective area. In the case of an axially loaded column this is simply the axial load divided by the total effective area, which, in turn, is simply the sum of the effective areas of the component parts. In the case of a beam or girder the compressive stress on the effective area shall be determined as follows: Compute the compressive extreme fiber stress f_c for the gross section of the beam or girder and then multiply this value by the ratio of the gross compression flange area to the effective compression flange area, including in both flange areas not only the flange proper but also that part of the web in the outermost one sixth of the over-all depth of the beam or girder.

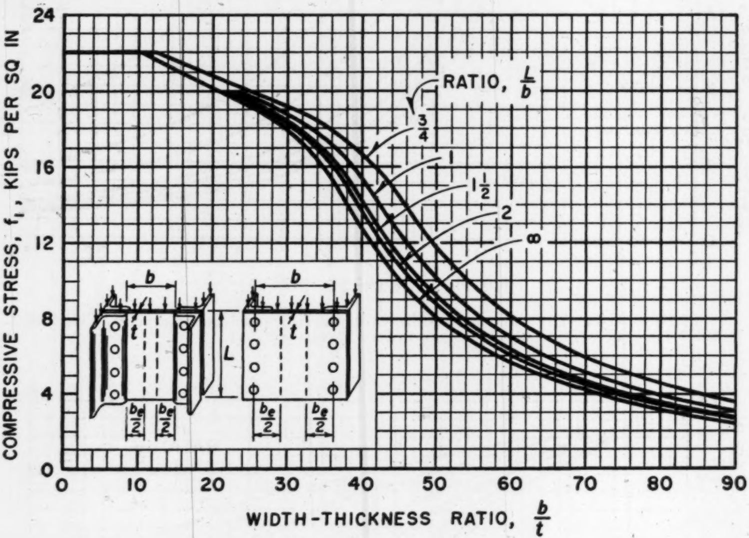

FIG. 6.—CHART FOR DETERMINING EFFECTIVE WIDTH FOR FLAT PLATES BUILT IN ALONG TWO EDGES

e.—The compressive stress on the effective area computed in accordance with step d shall not exceed allowable limits as defined in Sections B and C for the gross area.

f.—Step c provides a suitable factor of safety against the collapse of the member as a whole but does not necessarily provide complete protection against the local buckling of individual flat surfaces at the design load. Where local buckling at the design load cannot be tolerated because of appearance, or for other reasons, the compressive stress on the gross section of the member shall be less than 1.5 times the value given in Fig. 5 or Fig. 6 for the $\frac{b}{t}$-ratio in question.

SECTION E. ALLOWABLE SHEAR STRESSES IN PLATES AND WEBS

E-1.—The allowable shear stress on flat webs shall not exceed the values given by the curves in Fig. 7. The values in Fig. 7 apply to the gross area of the web, but the shear on the net area shall not exceed 15 kips per sq in.

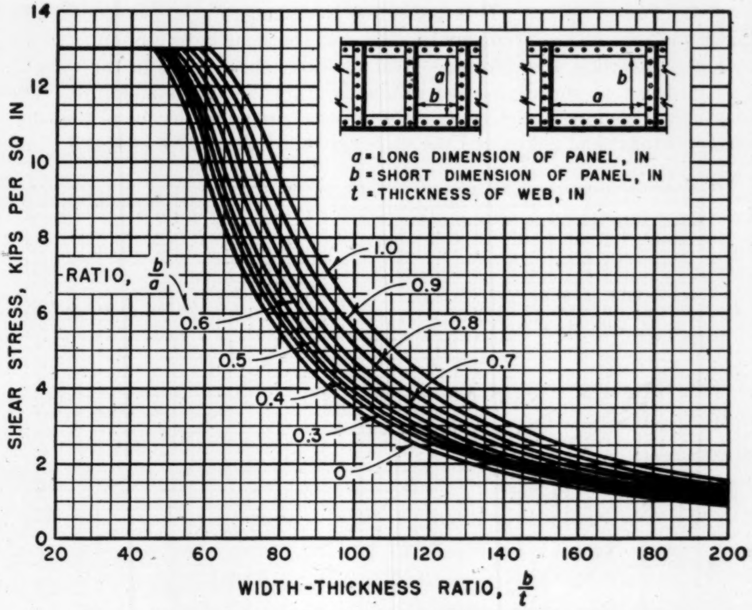

FIG. 7.—ALLOWABLE SHEAR STRESSES ON WEBS; PARTIAL RESTRAINT ASSUMED AT EDGES OF RECTANGULAR PANELS (GROSS SECTION)

SECTION F. PLATE GIRDER DESIGN

F-1. Proportioning Plate Girders.—Plate girders shall be proportioned by the moment of inertia method, with the gross section used to determine the moment of inertia.

The stress on the net area of the tension flange shall be found by multiplying the stress on the gross section by the ratio of the gross area of the tension flange to the net area. In determining this ratio the tension flange shall be considered to consist of the flange angles and cover plates plus that part of the web included in the outermost one sixth of the over-all height of the girder.

F-2. Allowable Flange Stress.—The allowable compressive stress in the extreme fiber of plate girders shall be determined as outlined in Sections C and D. The numerical value of the term $\sqrt{B/S_c}$, used in Fig. 3, is rarely less than one half of the width, in inches, of the compression flange for a plate girder. This fact is useful in preliminary design.

F-3. Flange Cover Plates.—Cover plates shall extend far enough to allow at least two extra rivets at each end of the plate beyond the theoretical end, and the spacing of the rivets in the remainder of the plate shall be such as to develop the required strength of the plate at any section.

F-4. Flange Rivets.—The flanges of plate girders shall be connected to the web with enough rivets to transmit the longitudinal shear at any point together with any load that is applied directly on the flange.

F-5. Flange Splices.—It is preferable that flange angles be spliced with angles and that no two members be spliced at the same cross section.

F-6. Allowable Web Stresses.—The allowable shear stress in the webs of plate girders shall not exceed the values given by the curves in Fig. 7. The longitudinal compressive stress in webs of plate girders at the toe of the compression flange shall not exceed the values given by the curves in Fig. 8.

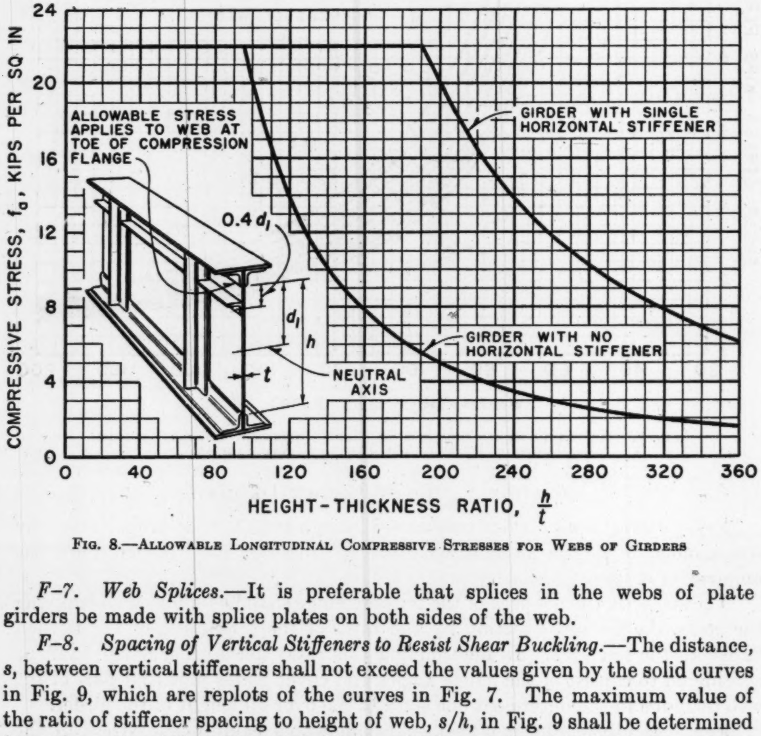

FIG. 8.—ALLOWABLE LONGITUDINAL COMPRESSIVE STRESSES FOR WEBS OF GIRDERS

F-7. Web Splices.—It is preferable that splices in the webs of plate girders be made with splice plates on both sides of the web.

F-8. Spacing of Vertical Stiffeners to Resist Shear Buckling.—The distance, *s*, between vertical stiffeners shall not exceed the values given by the solid curves in Fig. 9, which are replots of the curves in Fig. 7. The maximum value of the ratio of stiffener spacing to height of web, *s/h*, in Fig. 9 shall be determined from the ratio of clear height to thickness, *h/t*, and the computed shear stress on the girder web. Where a stiffener is composed of a pair of members, one on each side of the web, the distance *s* shall be the clear distance between the stiffeners. Where a stiffener is composed of a member on one side of the web only, the distance *s* shall be the distance between rivet lines. In determining the spacing of vertical stiffeners to resist shear buckling in panels containing a horizontal stiffener located as shown in Fig. 8, the distance *h* in Fig. 9 may be taken as 90% of the clear height between flanges.

F-9. Size of Vertical Stiffeners to Resist Shear Buckling.—Stiffeners applied to plate girder webs to resist shear buckling shall have a moment of inertia not less than the values given by the dotted curves in Fig. 9. The minimum value of the ratio of the stiffener moment of inertia to the fourth power of the web thickness, I_s/t^4, in Fig. 9, shall be determined from the ratio of height of web to thickness of web, h/t, and the computed shear stress on the girder web.

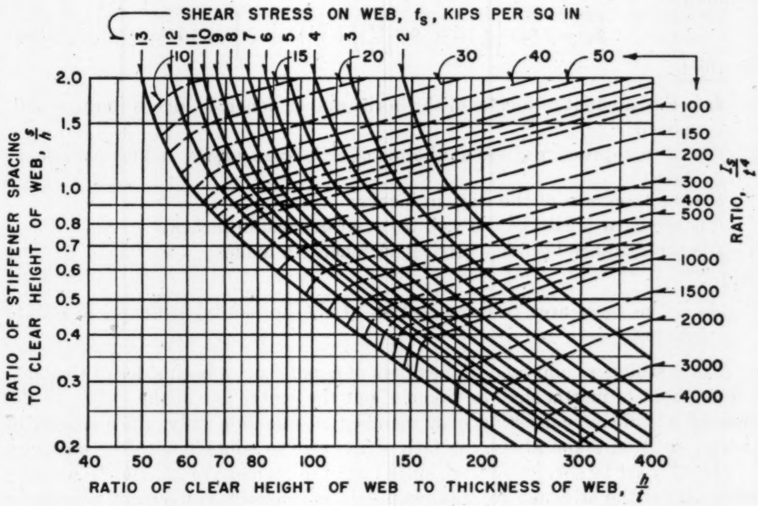

FIG. 9.—SPACING AND MOMENT OF INERTIA OF VERTICAL STIFFENERS TO RESIST SHEAR BUCKLING ON WEBS OF PLATE GIRDERS

For a stiffener composed of members of equal size on both sides of the web, the moment of inertia shall be taken about the center line of the web. For a stiffener composed of a member on one side only, the moment of inertia shall be taken about the face of the web in contact with the stiffener. In determining moment of inertia of stiffeners, the term *h* shall always be taken as the full clear height between flanges, regardless of whether or not a horizontal stiffener is present.

F-10. Vertical Stiffeners at Points of Bearing.—Stiffeners shall be placed in pairs at end bearings of plate girders and at points of bearing of concentrated loads. They shall be connected to the web by enough rivets to transmit the load. Such stiffeners shall have a close bearing against the loaded flanges. Only that part of the stiffener cross section which lies outside the fillet of the flange angle shall be considered effective in bearing.

The moment of inertia of the stiffener shall not be less than that given by the formula:

$$I = I_s + \frac{P h^2}{74{,}000} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which:

I_s is the moment of inertia, in inches to the fourth power, required to resist shear buckling (Fig. 9);
P is a local load concentration on the stiffener, in kips; and
h is the clear height of the web between flanges, in inches.

F–11. Horizontal Stiffeners.—A horizontal stiffener of the type shown in Fig. 8 shall have a moment of inertia not less than that given by the following formula:

$$I_h = f\,t\,h^3 \left[\left(16 + 90\,\frac{A_h}{h\,t}\right)\left(\frac{s}{h}\right)^2 + 6\right] \times 10^{-7} \dots\dots\dots\dots (6)$$

in which:

I_h is the moment of inertia of the horizontal stiffener, in inches to the fourth power;
f is the compressive stress at the toe of the flange angles, in kips per square inch;
t is the thickness of the web, in inches;
h is the clear height of the web between flanges, in inches;
s is the distance between vertical stiffeners, in inches;
and
A_h is the gross area of cross section of the horizontal stiffener, in square inches.

For a stiffener composed of members of equal size on both sides of the web, the moment of inertia shall be taken about the center line of the web. In the case of a stiffener consisting of a member on one side only, the moment of inertia shall be taken about the face of the web in contact with the stiffener.

Eq. 6 must be solved by trial, since both the moment of inertia, I_h, and the area, A_h, of the stiffener are unknown. It is generally convenient to assume as a first approximation that the ratio $\frac{A_h}{h\,t}$ has the value of 0.1.

SECTION G. RIVETED AND BOLTED CONNECTIONS

G–1. Allowable Loads.—The allowable loads on rivets and bolts shall be calculated using the allowable shear and bearing stresses listed in Section A with the following exceptions:

a.—If a rivet or a bolt is used in relatively thin plates or shapes the allowable shear stress shall be reduced in accordance with the information given in Table 4.

b.—If the distance from the center of a rivet or bolt to the edge of a plate or shape toward which the pressure of the rivet or bolt is directed is less than twice the diameter of the rivet or bolt, the allowable bearing stress shall be reduced in accordance with the following:

Ratio of edge distance to rivet or bolt diameter	Allowable bearing stress, in kips per square inch
2 or more	36
1¾	33
1½	30

The allowable loads calculated for cold-driven A17S–T3 rivets are given in Table 5 and those for 61S–T43, hot-driven rivets, are given in Table 6.

G–2. Effective Diameter.—The effective diameter of rivets shall be taken as the hole diameter but shall not exceed the values of hole diameter given in Table 5 for cold-driven rivets and in Table 6 for hot-driven rivets. The effective diameter of pins and bolts shall be the nominal diameter of the pin or bolt.

G–3. Bearing Area.—The effective bearing area of pins, bolts, and rivets shall be the effective diameter multiplied by the length in bearing; except that for countersunk rivets, half of the depth of the countersink shall be deducted from the length.

TABLE 4.—PERCENTAGE REDUCTION IN SHEAR STRENGTH OF ALUMINUM ALLOY RIVETS RESULTING FROM THEIR USE IN THIN PLATES AND SHAPES[2]

Ratio,[a] $\frac{D}{t}$ (1)	Loss in double shear[b] (3)	Ratio,[a] $\frac{D}{t}$ (1)	Loss in double shear[b] (3)	Ratio,[a] $\frac{D}{t}$ (1)	Loss in:		Ratio,[a] $\frac{D}{t}$ (1)	Loss in:	
					Single shear (2)	Double shear (3)		Single shear (2)	Double shear (3)
1.5	0	2.2	9.1	2.9	0	18.2	3.5	2.0	26.0
1.6	1.3	2.3	10.4	3.0	0	19.5	3.6	2.4	27.3
1.7	2.6	2.4	11.7	3.1	0.4	20.8	3.7	2.8	28.6
1.8	3.9	2.5	13.0	3.2	0.8	22.1	3.8	3.2	29.9
1.9	5.2	2.6	14.3	3.3	1.2	23.4	3.9	3.6	31.2
2.0	6.5	2.7	15.6	3.4	1.6	24.7	4.0	4.0	32.5
2.1	7.8	2.8	16.9	....	....	....	....	....	....

[a] Ratio of the rivet diameter, D, to the plate thickness, t. The thickness used is that of the thinnest plate in a single shear joint or of the middle plate in a double shear joint. [b] The percentage loss of strength in single shear is zero for D/t less than 3.0.

G–4. Arrangement and Strength of Connections.—Connections shall be arranged to minimize the eccentricity of loading on the member. Members and connections shall be proportioned to take into account any eccentricity of loading introduced by the connections.

G–5. Net Section.—The net section of a riveted tension member is the sum of the net sections of its component parts. The net section of a part is the product of the thickness of the part multiplied by its least net width. The net width for a chain of holes extending across the part in any straight or broken line shall be obtained by deducting from the gross width the sum of the diameters of all the holes in the chain and adding $\frac{s^2}{4g}$ for each gage space in the chain. In the correction quantity $\frac{s^2}{4g}$:

s is the spacing parallel to direction of load (pitch) of any two successive holes in the chain, in inches; and

g is the spacing perpendicular to direction of load (gage) of the same holes, in inches.

TABLE 5.—ALLOWABLE DESIGN LOAD, IN KIPS PER RIVET, (SHEAR, 10 KIPS PER SQ IN. AND

Dimensions, in Inches								
Rivet diameter	3/8		7/16		1/2		9/16	
Hole diameter	0.386		0.453		0.516		0.578	
Drill size	W		29/64		33/64		37/64	
Thickness of plate, or shape, in inches:	RIVET IN SINGLE SHEAR (ss)							
	ss	ds	ss	ds	ss	ds	ss	ds
1/8	1.17	1.74[b]	1.58[c]	2.04[b]	2.01[c]	2.32[b]	2.52[c]	2.60[b]
3/16	1.17	2.19[c]	1.61	2.88[c]	2.09	3.48[b]	2.62	3.90[b]
1/4	1.17	2.34	1.61	3.12[c]	2.09	3.91[c]	2.62	4.74[c]
5/16	1.17	2.34	1.61	3.22	2.09	4.13[c]	2.62	5.04[c]
3/8	1.17	2.34	1.61	3.22	2.09	4.18	2.62	5.25
7/16	....	....	1.61	3.22	2.09	4.18	2.62	5.25
1/2	....	....	....	....	2.09	4.18	2.62	5.25
9/16	....	....	....	....	....	....	2.62	5.25
5/8	....	....	....	....	....	....	....	....
3/4	....	....	....	....	....	....	....	....
7/8	....	....	....	....	....	....	....	....
1	....	....	....	....	....	....	....	....

[a] Assuming distance from center of rivet to edge of member toward which the pressure of the rivet is directed is by bearing. [c] These values are governed by reduced shear strengths as indicated in Table 4. All other values

TABLE 6.—ALLOWABLE DESIGN LOADS, IN KIPS PER RIVET, (RIVETS DRIVEN AT 990° F TO 1,050° F; SHEAR, 8 KIPS

Dimensions, in Inches:								
Rivet diameter	3/8		7/16		1/2		9/16	
Hole diameter	0.397		0.469		0.531		0.594	
Drill size	X		15/32		17/32		19/32	
Thickness of plate, or shape, in inches:	RIVET IN SINGLE SHEAR (ss)							
	ss	ds	ss	ds	ss	ds	ss	ds
1/8	0.99	1.60[b]	1.35[b]	2.05[b]	1.70[b]	2.39[c]	2.08[b]	2.67[c]
3/16	0.99	1.85[b]	1.38	2.47[b]	1.77	3.01[b]	2.22	3.57[b]
1/4	0.99	1.98	1.38	2.68[b]	1.77	3.31[b]	2.22	4.00[b]
5/16	0.99	1.98	1.38	2.76	1.77	3.50[b]	2.22	4.26[b]
3/8	0.99	1.98	1.38	2.76	1.77	3.54	2.22	4.43
7/16	....	....	1.38	2.76	1.77	3.54	2.22	4.43
1/2	....	....	....	....	1.77	3.54	2.22	4.43
9/16	....	....	....	....	....	....	2.22	4.43
5/8	....	....	....	....	....	....	....	....
3/4	....	....	....	....	....	....	....	....
7/8	....	....	....	....	....	....	....	....
1	....	....	....	....	....	....	....	....

[a] Assuming distance from center of rivet to edge of member toward which the pressure of the rivet is directed is by reduced shear strengths as indicated in Table 4. [b] These values are governed by bearing. All other values

The net section of the part is obtained from that chain which gives the least net width. The hole diameter to be deducted shall be the actual hole diameter for drilled or reamed holes.

For angles, the gross width shall be the sum of the widths of the legs less the thickness. The gage for holes in opposite legs shall be the sum of the gages from the back of the angle, less the thickness.

For splice members, the thickness shall be only that part of the thickness of the member that has been developed by rivets beyond the section considered.

for Cold-Driven A17S–T3 Rivets in 14S–T6 Structures
Bearing, 36 Kips per Sq In.[a])

5/8 0.641 41/64		3/4 0.766 49/64		7/8 0.891 57/64		1 1.016 1 1/64		Dimensions, in Inches: Rivet diameter Hole diameter Drill size
or in Double Shear (ds)								Thickness of plate, or shape, in inches:
ss	ds	ss	ds	ss	ds	ss	ds	
2.88[b]	2.88[b]	....	....	....	....	....	....	1/8
3.18[c]	4.33[b]	4.42[c]	5.17[b]	5.82[c]	6.02[b]	....	....	3/16
3.23	5.62[c]	4.61	6.89[b]	6.11[c]	8.02[b]	7.78[c]	9.14[b]	1/4
3.23	6.03[c]	4.61	8.14[c]	6.24	10.02[b]	8.04[c]	11.43[b]	5/16
3.23	6.31[c]	4.61	8.62[c]	6.24	11.12[c]	8.11	13.72[b]	3/8
3.23	6.45	4.61	8.97[c]	6.24	11.66[c]	8.11	14.58[c]	7/16
3.23	6.45	4.61	9.22	6.24	12.07[c]	8.11	15.16[c]	1/2
3.23	6.45	4.61	9.22	6.24	12.39[c]	8.11	15.64[c]	9/16
3.23	6.45	4.61	9.22	6.24	12.47	8.11	16.00[c]	5/8
....	....	4.61	9.22	6.24	12.47	8.11	16.22	3/4
....	....	....	....	6.24	12.47	8.11	16.22	7/8
....	....	....	....	....	....	8.11	16.22	1

not less than twice the nominal rivet diameter (see Specification G-1, exception *b*). [b] These values are governed are governed by basic allowable shear stress.

for Hot-Driven 61S–T43 Rivets in 14S–T6 Structures
per Sq In.; and Bearing, 36 Kips per Sq In.[a])

5/8 0.656 21/32		3/4 0.781 25/32		7/8 0.922 59/64		1 1.063 1 1/16		Dimensions, in Inches: Rivet diameter Hole diameter Drill size
or in Double Shear (ds)								Thickness of plate, or shape, in inches:
ss	ds	ss	ds	ss	ds	ss	ds	
2.49[b]	2.95[c]	....	....	....	....	....	....	1/8
2.66[b]	4.12[b]	3.68[b]	5.17[b]	4.98[b]	6.22[c]	....	....	3/16
2.70	4.71[b]	3.83	6.17[b]	5.23[b]	7.91[b]	6.82[b]	9.59[b]	1/4
2.70	5.06[b]	3.83	6.77[b]	5.34	8.88[b]	7.04[b]	10.88[b]	5/16
2.70	5.29[b]	3.83	7.17[b]	5.34	9.53[b]	7.10	12.04[b]	3/8
2.70	5.41	3.83	7.46[b]	5.34	9.99[b]	7.10	12.77[b]	7/16
2.70	5.41	3.83	7.67	5.34	10.34[b]	7.10	13.28[b]	1/2
2.70	5.41	3.83	7.67	5.34	10.61[b]	7.10	13.69[b]	9/16
2.70	5.41	3.83	7.67	5.34	10.68	7.10	14.02[b]	5/8
....	....	3.83	7.67	5.34	10.68	7.10	14.20	3/4
....	....	....	....	5.34	10.68	7.10	14.20	7/8
....	....	....	....	....	....	7.10	14.20	1

not less than twice the nominal rivet diameter (see Specification G-1, exception *b*). [b] These values are governed are governed by basic allowable shear stress.

G–6. Effective Sections of Angles.—If an angle in tension is connected on one side of a gusset plate, the effective section shall be the net section of the connected leg plus one half of the section of the outstanding leg, unless the outstanding leg is connected by a lug angle. In the latter case the effective section shall be the entire net section of the angle, and there shall be at least two extra rivets in the lug angle beyond the gusset plate.

G–7. Grip of Rivets.—If the grip of rivets carrying calculated stress exceeds four and one-half times the diameter the allowable load per rivet shall be reduced. The reduced allowable load shall be the normal allowable load divided by $\left(\frac{1}{2} + \frac{G}{9\,D}\right)$, in which G is the grip and D the nominal diameter of the rivet. If the grip exceeds six times the diameter, special care shall be taken in driving the rivets to insure that the holes will be filled completely.

G–8. Pitch of Rivets in Built-Up Compression Members.—The pitch in the direction of stress shall be such that the allowable stress on the individual outside plates and shapes, treated as columns having a length equal to the rivet pitch in accordance with Fig. 2, exceeds the calculated stress. In no case, however, shall the pitch in the direction of stress exceed six times the diameter of the rivets; and for a distance of one and one-half times the width of the member at each end, the pitch in the direction of stress shall not exceed three and one-half times the diameter of the rivets.

G–9. Stitch Rivets.—Where two or more web plates are in contact, there shall be stitch rivets to make them act in unison. In compression members, the pitch of such rivets in the direction of stress shall be determined as outlined in Specification G–8. The gage at right angles to the direction of stress shall not exceed twenty times the thickness of the outside plates. In tension members the maximum pitch or gage of such rivets shall be twenty times the thickness of the outside plates; and in tension members composed of two angles in contact, the pitch of the stitch rivets shall not exceed 10 in.

G–10. Minimum Spacing of Rivets.—The distance between centers of rivets shall not be less than three times the diameter of the rivets.

G–11. Edge Distance of Rivets.—The distance from the center of a rivet to a sheared, sawed, rolled, or planed edge shall be not less than one and one-half times the diameter, except in flanges of beams and channels, where the minimum distance may be one and one-fourth times the diameter. For rivets under computed stress, the distance from the center of the rivet to the edge of the plate or shape toward which the pressure of the rivet is directed should normally be at least twice the nominal diameter of the rivet. In cases where a shorter edge distance must be used, the allowable bearing stress shall be reduced in accordance with Specification G–1, exception b.

The distance from the edge of a plate to the nearest rivet line shall not exceed six times the thickness of the plate.

G–12. Sizes of Rivets in Angles.—The diameter of the rivets in angles whose size is determined by calculated stress shall not exceed one fourth of the width of the leg in which they are driven. In angles whose size is not so determined, 1-in. rivets may be used in 3½-in. legs; 7/8-in. rivets, in 3-in. legs; and 3/4-in. rivets, in 2½-in. legs.

G–13. Extra Rivets.—If splice plates are not in direct contact with the parts which they connect, there shall be rivets on each side of the joint in excess of the number required in the case of direct contact, to the extent of two extra lines for each intervening plate.

If rivets carrying calculated stress pass through fillers, the fillers shall be extended beyond the connected member and the extension secured by enough additional rivets to distribute the total stress in the member uniformly over the combined section of the member and filler.

SECTION H. MISCELLANEOUS DESIGN RULES

H-1. Reversal of Load.—Members subject to reversal of load under the passage of live load shall be proportioned as follows: Determine the tensile load and the compressive load and increase each by 50% of the smaller; then proportion the member and its connections so that the allowable stresses given in Sections A to G, inclusive, will not be exceeded by either increased load.

H-2. Slenderness Ratio of Tension Members.—The ratio of unsupported length to least radius of gyration for tension members shall not exceed the value given by the following formula:

$$\frac{L}{r} = 150 + 10f_t \qquad (7)$$

in which f_t is the lowest net section tensile stress to which the member will be subjected in actual service, in kips per square inch.

H-3. Stay Plates for Tension Members.—Segments of tension members not directly connected to each other shall be stayed together. The length of the stay plate shall be not less than three fourths of the distance between rivet lines of the segments. Stay plates shall be connected to each segment of the tension member by at least three rivets. The distance between stay plates shall be such that the slenderness ratio of the individual segments does not exceed that given by Eq. 7, Specification H-2.

H-4. Fatigue.—Tests indicate that riveted members designed in accordance with the requirements of these specifications and constructed so as to be free from severe reentrant corners and other unusual stress raisers will safely withstand at least 100,000 repetitions of maximum live load without fatigue failure regardless of the ratio of minimum to maximum load. Where a greater number of repetitions of some particular loading cycle is expected during the life of the structure, the calculated net section tensile stresses for the loading in question shall not exceed the values given by the curves in Fig. 10. When using the curves in Fig. 10 the reversal-of-load rule in Specification H-1 should be ignored. The final member and connections selected, however, shall be strong enough to satisfy the requirements of Specification H-1.

In considering fatigue action on structures it is well to bear in mind the following points:

a.—The most severe combination of loadings for which a structure is designed (dead load, maximum live load, maximum impact, maximum wind, etc.) rarely occurs in actual service and is of little or no interest from the standpoint of fatigue.

b.—The loading of most interest from the fatigue standpoint is the steady dead load with a superimposed and repeatedly applied live load having an intensity consistent with day-to-day normal operating conditions.

c.—The number of cycles of load encountered in structures is usually small compared with those encountered in fatigue problems involving machine parts. It takes many years of service to accumulate even 100,000 cycles of any significant stress application in most structures as is indicated by the following examples: 100,000 cycles represent 10 cycles every day for 27 years; 10,000,000 cycles represent 20 cycles every hour for 57 years. Care must be taken not to overestimate grossly the number of cycles for any given load condition.

d.—Careful attention to details in design and fabrication pays big dividends in fatigue life. When a fatigue failure occurs in a structure it is usually at a point of stress concentration where the state of stress could have been improved with little or no added expense.

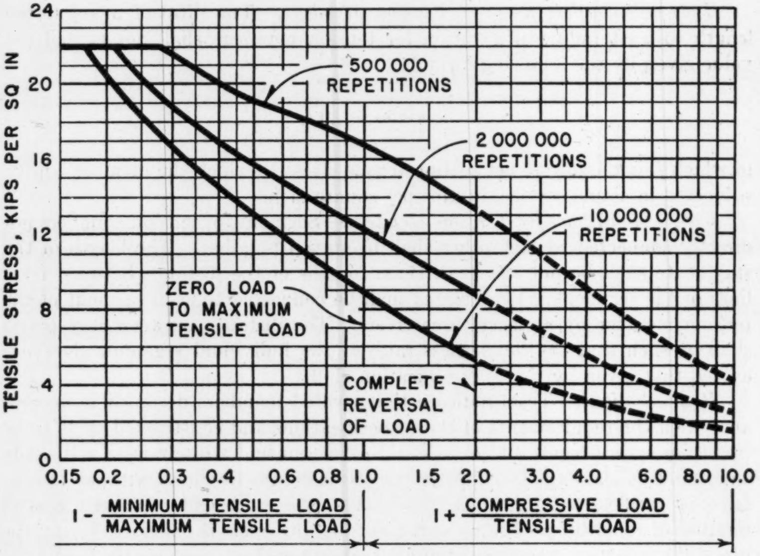

FIG. 10.—ALLOWABLE TENSILE STRESSES ON NET SECTION FOR VARIOUS NUMBERS OF REPETITIONS OF LOAD APPLICATION

SECTION I. FABRICATION

I-1. Laying Out.—

a.—Hole centers may be center-punched and cutoff lines may be punched or scribed. Center-punching and scribing shall not be used where such marks would remain on fabricated material.

b.—A temperature correction shall be applied where necessary in the layout of critical dimensions. The coefficient of expansion shall be taken as 0.000012 per degree Fahrenheit.

I-2. Cutting.—

a.—Material 1/2 in. thick or less may be sheared, sawed, or cut with a router. Material more than 1/2 in. thick shall be sawed or routed.

b.—Cut edges shall be true and smooth, and free from excessive burrs or ragged breaks.

c.—Edges of plates carrying calculated stresses shall be planed to a depth of 1/4 in. except in the case of sawed or routed edges of a quality equivalent to a planed edge.

d.—Reentrant cuts shall be avoided wherever possible. If used they shall be filleted by drilling prior to cutting.

e.—Flame cutting of aluminum alloys is not permitted.

I-3. Heating.—Structural material shall not be heated, with the following exceptions:

a.—Material may be heated to a temperature not exceeding 400° F for a period not exceeding 15 min to facilitate bending. Such heating shall be done only when proper temperature controls and supervision are provided to insure that the limitations on temperature and time are carefully observed.

b.—Hot-driven rivets shall be heated as specified in Section I-5.

I-4. Punching, Drilling, and Reaming.—Rules for punching, drilling, and reaming are as follows:

a.—Rivet or bolt holes in main members shall be subpunched or subdrilled and reamed to finished size after the parts are firmly bolted together. The amount by which the diameter of a subpunched hole is smaller than that of the finished hole shall be at least one-quarter the thickness of the piece and in no case less than 1/32 in. If the metal thickness is greater than the diameter of the hole, punching shall not be used.

b.—Rivet or bolt holes in secondary material not carrying calculated stress may be punched or drilled to finished size before assembly.

c.—The finished diameter of holes for cold-driven rivets shall be not more than 4% greater than the nominal diameter of the rivet.

d.—The finished diameter of holes for hot-driven rivets shall be not more than 7% greater than the nominal diameter of the rivet.

e.—The finished diameter of holes for unfinished bolts shall be not more than 1/16 in. larger than the nominal bolt diameter.

f.—Holes for turned bolts shall be drilled or reamed to give a driving fit.

g.—All holes shall be cylindrical and perpendicular to the principal surface. Holes shall not be drifted in such a manner as to distort the metal. All chips lodged between contacting surfaces shall be removed before assembly.

I-5. Riveting.—

a.—The driven head of aluminum alloy rivets preferably shall be of the flat or the cone-point type, with dimensions as follows:

(1) Flat heads shall have a diameter not less than 1.4 times the nominal rivet diameter and a height not less than 0.4 times the nominal rivet diameter.

(2) Cone-point heads shall have a diameter not less than 1.4 times the nominal rivet diameter and a height, to the apex of the cone, not less than 0.65 times the nominal rivet diameter. The included angle at the apex of the cone shall be approximately 127°.

b.—Rivets shall be driven hot or cold as called for on the plans, provision for heating being as follows:

(1) Hot-driven rivets shall be heated in a hot air type furnace providing uniform temperatures throughout the rivet chamber and equipped with automatic temperature controls.

(2) Hot-driven rivets shall be held at from 990° F to 1,050° F for not less than 15 min and for not more than 1 hour before driving.

(3) Hot rivets shall be transferred from the furnace to the work and driven with a minimum loss of time.

c.—Rivets shall fill the holes completely. Rivet heads shall be concentric with the rivet holes and shall be in proper contact with the surface of the metal.

d.—Defective rivets shall be removed by drilling.

I–6. Welding.—Welding is not permitted.

I–7. Cleaning and Treatment of Metal Surfaces.—

a.—Surfaces of metal shall be cleaned immediately before painting by a method which will remove all dirt, oil, grease, chips, and other foreign substances.

b.—Either of the two following methods of cleaning may be used on exposed metal surfaces:

(1) Chemical Cleaning.—Parts may be immersed in, or swabbed with, a solution of phosphoric acid and organic solvents diluted with water in the ratio of 1:3. The solution temperature shall be between 50° F and 90° F. The solution shall remain in contact with the metal not less than 5 min. Residual solution shall be removed with clear water.

(2) Sandblasting.—Standard mild sandblasting methods may be used on sections more than 1/8 in. thick.

c.—For contacting surfaces only, the metal may be cleaned in accordance with Specification I–7b, or with a solvent such as mineral spirits or benzine.

d.—Flame cleaning is not permitted.

I–8. Painting.—Specifications to control painting operations are as follows:

a.—Metal parts shall be painted as described in Specifications I–8b to I–8h except where the plans specifically permit a deviation.

b.—Contacting metal surfaces shall be painted before assembly with one coat of zinc chromate primer in accordance with United States Navy Department Specification 52P18 or the equivalent, or with one coat of suitable aluminum pigmented calking compound (brushing consistency with chromate pigment added). Zinc chromate paint shall be allowed to dry before assembly of the parts.

c.—In all cases where aluminum work is to be fastened to steel members or other dissimilar metal parts, the aluminum shall be kept from direct contact with such parts by painting the aluminum surface as described in Specification I–8b and by painting the dissimilar metal with a suitable metal priming paint.

d.—Aluminum surfaces to be placed in contact with concrete or masonry construction shall, before installation, be given a heavy coat of an alkali-resistant bituminous paint. The quality of the bituminous paint used shall be equal to that called for in the Army-Navy Aeronautical Specification AN–P–31. The paint shall be applied as it is received from the manufacturer without the addition of any thinner.

e.—All other surfaces shall be given one shop coat of zinc chromate primer made in accordance with Navy Department Specification 52P18, or one giving equivalent performance.

f.—All surfaces, except those covered by Specifications I–8b, I–8c, and I–8d, shall be given a second shop coat of paint consisting of 2 lb of aluminum paste pigment (ASTM Specification D962–48T, Type II, Class A) per gallon of varnish meeting Federal Specification TTV81a, Type II, or the equivalent. Sufficient Prussian blue shall be added to permit detection of an incomplete application of the subsequent paint coat.

g.—After erection, bare spots shall be touched up with zinc chromate primer followed by a touch-up coat of aluminum paint as specified in Specifications I–8e and I–8f.

h.—The completed structure shall be finished according to one of the following methods:

(1) One field coat of aluminum paint as specified in Specification I–8f, except that Prussian blue shall be omitted from the field coat.

(2) One or more field coats of alkyd base enamel pigmented to meet a desired color scheme.

PART III. EXPLANATION OF SPECIFICATIONS

SECTION A. SUMMARY OF ALLOWABLE STRESSES

A–1. Basic Tensile Design Stress.—The basic tensile design stress of 22 kips per sq in. represents a factor of safety of 2.41 based on the specified tensile yield strength. This is a larger factor of safety with respect to yield strength than is ordinarily encountered in specifications for structural steel. In selecting this rather large factor of safety on yield strength, the committee was influenced to a considerable extent by the fact that there is a smaller spread between yield strength and tensile strength in this aluminum alloy than is commonly encountered in structural steels.

A–8, A–9, and A–13. Allowable Stresses on Rivets.—The allowable shearing and bearing stresses on rivets were selected on the basis of the results of numerous shearing and bearing tests. The factors of safety used are greater than those used for most of the other allowable stresses.[2]

A–6, A–11, and A–12. Allowable Stresses on Pins.—The allowable bending, shearing, and bearing stresses on pins were selected to have about the same relation to the corresponding properties of the material as is the case in standard steel specifications. It is not anticipated that any wide use of pins will be made in aluminum alloy structures but it is assumed that where they are used they will be of the same material as the structural members themselves, and that they would probably be obtained in the form of rolled rod, ASTM Specification B211–49T(CS41A–T6).

SECTION B. COLUMN DESIGN

B–1. Curves for Allowable Compressive Stresses in Axially Loaded Columns.—The curves in Fig. 2 are the tangent-modulus column curves with a factor of safety of 2.5 and with a cutoff at the basic allowable design stress of 22 kips per sq in. The formulas for all three curves can be written[4,5]

$$f_C = \frac{\pi^2 E_t}{2.5\left(\frac{k\,L}{r}\right)^2} \qquad \text{(8)}$$

in which:

f_C is the allowable compressive stress on the gross cross-sectional area, in kips per square inch;

E_t is the tangent modulus taken from Fig. 1 at stress corresponding to $2.5(f_C)$, in kips per square inch;

L is the length of the column, in inches;

r is the least radius of gyration of the column, in inches; and

k is a factor describing the end conditions as defined in Fig. 2.

For values of slenderness ratio, L/r, greater than 72, the formula for the partial restraint curve in Fig. 2 reduces to

$$f_C = \frac{74{,}000}{\left(\frac{L}{r}\right)^2} \qquad \text{(9)}$$

B–7. Formulas for Combined Compression and Bending.—Eq. 1, which applies to bending in the direction of the applied bending moment, takes into account the additional bending moment due to the deflection of the column.[6]

Eq. 2, covering failure by buckling normal to the plane of the bending forces, is a simplified design formula which is conservative compared to test results and to the theoretical solution for this case.[7] A modification of this solution has been suggested by H. N. Hill, Assoc. M. ASCE, and J. W. Clark, Jun. ASCE.[8]

B–8. Formula for Transverse Shear on Columns.—Eq. 3*a* is based on the transverse component of the column load at the point of maximum slope of the column in its deflected position. A derivation by Mr. Hartmann has been published elsewhere.[6]

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[4] "Column Strength of Various Aluminum Alloys," by R. L. Templin, R. G. Sturm, E. C. Hartmann, and M. Holt, *Aluminum Research Laboratories Technical Paper No. 1*, Aluminum Co. of America, Pittsburgh, Pa., 1938.

[5] "Inelastic Column Theory," by F. R. Shanley, *Journal of the Aeronautical Sciences*, Vol. 14, 1947 pp. 261–268.

[6] Discussion by E. C. Hartmann of "Rational Design of Steel Columns," by D. H. Young, *Transactions*, ASCE, Vol. 101, 1936, pp. 475–481.

[7] "Torsional and Flexural Buckling of Bars of Thin-Walled Open Section Under Compressive and Bending Loads," by J. N. Goodier, *Transactions*, A.S.M.E., Vol. 64, 1942, pp. A-103-A-107.

[8] "Lateral Buckling of Eccentrically Loaded I- and H-Section Columns," by H. N. Hill and J. W. Clark. *Proceedings*, First U. S. National Congress of Applied Mechanics (publication pending).

SECTION C. CURVE FOR ALLOWABLE COMPRESSIVE STRESS IN BEAM AND GIRDER FLANGES

C–1.—The curve in Fig. 3 is based on the theoretical solution for the critical bending moment in I-beams as given by S. Timoshenko.[9] It represents a factor of safety of 2.5 applied to the theoretical solution for beams subjected to a uniform bending moment. It is assumed that at the ends of the laterally unsupported length there is partial restraint against rotation about a vertical axis and complete restraint against lateral displacement or rotation about a horizontal axis parallel to the web. The part of the curve for values of $\frac{L}{\sqrt{B/S_c}}$ greater than 27.5 is based on elastic action, whereas the remainder is simply an extension of the same formula using tangent modulus rather than initial modulus. The curve has a cutoff at the basic allowable design stress of 22 kips per sq in. It is important to note that the term L is defined as "laterally unsupported length of compression flange," which is not necessarily the same as the span of the beam or the girder. The case of uniform bending moment has been used in setting up Fig. 3 because it is a good approximation of conditions frequently encountered in actual design, and because it is somewhat more conservative than many of the other cases that might have been selected.

For values of $\frac{L}{\sqrt{B/S_c}}$ greater than 27.5, the curve in Fig. 3 may be represented by the formula:

$$f_B = \frac{10{,}900}{\left(\frac{L}{\sqrt{B/S_c}}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

The curve in Fig. 3 is based on a theoretical solution applicable only to I-beams having cross sections symmetrical about both axes. The modified interpretation of the term I_1 indicated in specification C–1, however, permits the curve to be used without serious error for beams and girders having one flange differing in lateral stiffness from the other. It should be used with caution in cases of beams and girders which are unsymmetrical by a considerable margin. (In connection with this subject several supplementary references [7,10,11,12,13] will be of interest.)

SECTION D. CURVES FOR DESIGN OF FLAT PLATES, LEGS AND WEBS

The curves of Figs. 4, 5, and 6 are based on values of critical stress compiled by Mr. Hill in 1940.[14] Partial restraint along the supported edges and loaded edges was assumed in all cases except for the supported edge in Fig. 4, which was considered simply supported. Parts of the curves that represent critical buckling stresses above the elastic range are computed by using the tangent modulus instead of the modulus of elasticity, a procedure which is known to be conservative when applied to problems of plate buckling.[15] A factor of safety of 2.5 against critical buckling has been used in all three charts and in all cases the curves have a cutoff at the basic allowable design stress of 22 kips per sq in.

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[9] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1936.

[10] "The Lateral Instability of Unsymmetrical I-Beams," by H. N. Hill, *Journal of the Aeronautical Sciences*, Vol. 9, 1942, pp. 175–180.

[11] "The Lateral Stability of Equal-Flanged Aluminum-Alloy I-Beams Subjected to Pure Bending," by C. Dumont and H. N. Hill, *Technical Note No. 770*, National Advisory Committee for Aeronautics, Washington, D. C., 1940.

[12] "Lateral Stability of Unsymmetrical I-Beams and Trusses in Bending," by George Winter, *Transactions*, ASCE, Vol. 108, 1943, pp. 247–268.

[13] "Strength of Beams as Determined by Lateral Buckling," by Karl de Vries, *ibid.*, Vol. 112, 1947, pp. 1245–1320.

[14] "Chart for Critical Compressive Stress of Flat Rectangular Plates," by H. N. Hill, *Technical Note No. 773*, National Advisory Committee for Aeronautics, Washington, D. C., 1940.

When a flat plate, leg, or web is built in along one or both edges to other parts of a compression member which offer partial edge restraint, the local buckling of the plate, leg, or web does not precipitate collapse of the member as a whole as it probably would in the case of a single-angle strut. For this reason it is proper to permit a decreased factor of safety against local buckling in such cases if suitable precautions are taken to avoid collapse. Step c of Specification D–2 provides a simple method for accomplishing this result by introducing the well-known "effective width" concept. After a plate, leg, or web buckles, a part of its area is considered to be ineffective in supporting load, whereas a strip along each supported edge is considered still fully effective in working with the supporting material to which it is attached. The formula (Eq. 4) for effective width used in step c of Specification D–2 is generally more conservative than other accepted methods of calculating effective width.[16,17,18,19]

Section E. Curves for Allowable Shear Stress in Webs

E–1.—The values of allowable stress in Fig. 7 are obtained by applying a factor of safety of 2 to the critical shear buckling stresses for flat plates with the edges about halfway between the fixed and hinged conditions.[9,20,21]

Those parts of the curves of Fig. 7 which represent critical buckling stresses above the elastic stress range are computed from formulas for elastic buckling with the tangent modulus substituted for the modulus of elasticity. For a given value of critical shear stress, the tangent modulus is that corresponding to an axial stress equal to $\sqrt{3}$ times the shear stress.[22] As in the case of compressive buckling of flat plates, the tangent modulus is conservative.

For values of allowable stress below 10.4 kips per sq in., the curves of Fig. 7 may be represented by the formula:

$$f_v = \frac{35{,}000}{(b/t)^2}\left[1 + 0.75\left(\frac{b}{a}\right)^2\right] \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

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[15] "Buckling Stresses for Flat plates and Sections," by Elbridge Z. Stowell, George J. Heimerl, Charles Libove, and Eugene E. Lundquist, *Proceedings*, ASCE, Separate No. 77, July, 1951.

[16] "The Strength of Thin Plates in Compression," by Theodor von Kármán, Ernest E. Sechler, and L. H. Donnell, *Transactions*, A.S.M.E., Vol. 54, 1932, pp. 53–57.

[17] "The Apparent Width of the Plate in Compression," by Karl Marguerre, *Technical Memorandum No. 833*, National Advisory Committee for Aeronautics, Washington, D. C., 1937.

[18] "Strength of Thin Steel Compression Flanges," by George Winter, *Transactions*, ASCE, Vol. 112, 1947, pp. 527–576.

[19] "Performance of Thin Steel Compression Flanges," by George Winter, preliminary publication, 3d Cong. of the International Assn. for Bridge and Structural Engrs., Liege, Belgium, 1948.

[20] "Formulas for Stress and Strain," by Raymond J. Roark, McGraw-Hill Book Co., Inc., New York, N. Y., 1938.

[21] "Observations on the Behavior of Aluminum Alloy Test Girders," by R. L. Moore, *Transactions*, ASCE, Vol. 112, 1947, pp. 901–920.

[22] "Critical Shear Stress of an Infinitely Long Plate in the Plastic Region," by Elbridge Z. Stowell, *Technical Note No. 1681*, National Advisory Committee for Aeronautics, Washington, D. C. ,1948.

SECTION F. PLATE GIRDER DESIGN

F–6. Curves for Allowable Longitudinal Compressive Stress in Webs of Girders.—The curve in Fig. 8 for girders with no horizontal stiffeners is based on the critical buckling stress for rectangular flat plates under pure bending in the plane of the plate. Partial restraint is assumed at the toes of the flanges (about halfway between the solution given by Mr. Timoshenko for the case of a plate simply supported on all four edges[9] and the solution of K. Nolke for a plate with the loaded edges simply supported and the other two edges fixed).[23]

The curve in Fig. 8 for girders with a single horizontal stiffener is based on the critical buckling stress given by Mr. Timoshenko for plates simply supported on all four edges under combined bending and axial stress in the plane of the plate.[9] The simple support condition is used for this case because the horizontal stiffener would provide comparatively little restraint against rotation. The location of the horizontal stiffener shown in the sketch in Fig. 8 is chosen so that the parts of the plate above and below the stiffener will buckle at approximately the same load.

A factor of safety against buckling of 1.5 was used for the curves of Fig. 8. Although this factor of safety is not as large as some used elsewhere in these specifications, it is considered adequate in this instance since tests have shown that the critical bending stress for girder webs may be considerably exceeded without affecting the load-carrying capacity of the girder.[21,23] Use of Fig. 8, however, will prevent buckling from occurring at design stresses.

The curves of Fig. 8 may be represented by the following formulas: No horizontal stiffener—

$$f_a = \frac{200{,}000}{\left(\frac{h}{t}\right)^2} \qquad \text{(12}a\text{)}$$

and single horizontal stiffener—

$$f_a = \frac{800{,}000}{\left(\frac{h}{t}\right)^2} \qquad \text{(12}b\text{)}$$

The curves are cut off at the basic allowable design stress of 22 kips per sq in.

F–8 and F–9. Curves for Spacing and Moment of Inertia of Vertical Stiffeners.—The curves for determining stiffener spacing, in Fig. 9, are merely replots of the data of Fig. 7. The curves of I_s/t^4 in Fig. 9 represent the following formula:

$$I_s = 8 \times 10^{-5} \frac{f_s h^3 t \left(\frac{s}{h}\right)}{1 + 5\left(\frac{s}{h}\right)^3} \qquad (13)$$

in which f_s is the average shear stress on the web in kips per square inch.

Eq. 13 is designed to fit the theoretical solution of M. Stein and R. W. Fralich[24] for values of s/h between 0.2 and 1.0. This solution does not cover values of s/h greater than 1.0. In this range, however, Eq. 13 is conservative in comparison with the recommendations of L. S. Moisseiff.[1]

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[23] "Buckling of Webs in Deep Steel I-Girders," by Georg Wastlund and Sten G. A. Bergman, rept. of investigation made at the Royal Inst. of Technology, Stockholm, Sweden, 1947.

[24] "Critical Shear Stress of Infinitely Long, Simply Supported Plate with Transverse Stiffeners," by Manuel Stein and Robert W. Fralich, Technical Note No. 1851, National Advisory Committee for Aeronautics, Washington, D. C., 1949.

F–10. Formula for Moment of Inertia of Stiffeners at Points of Bearing.—Eq. 5 simply states that the moment of inertia of a stiffener at a point of bearing should be equal to the sum of the moment of inertia required to resist the tendency of the web to buckle and the moment of inertia required for the stiffener to carry the bearing load as a column with length equal to the height of the web.

F–11. Formula for Radius of Gyration of Horizontal Stiffeners.—Eq. 6, for the moment of inertia of horizontal stiffeners, is based on the theoretical work of C. Dubas, reported by F. Bleich.[25]

SECTION H. MISCELLANEOUS DESIGN RULES

H–2. Formula for Slenderness Ratio of Tension Members.—Eq. 7 is designed to yield slenderness ratios in agreement with values generally accepted for tension members, at the same time taking into account the fact that the higher the minimum tensile stress on the member the less tendency there will be for the member to bend or sway.

H–4. Curves of Allowable Tensile Stress on Net Section for Various Numbers of Repetitions of Load Application.—The curves in Fig. 10 are plotted from the results of fatigue tests conducted at the Aluminum Research Laboratories of the Aluminum Company of America at New Kensington, Pa., on 14S–T6 butt joints with double straps joined with eight cold-driven 5/8-in. A17S–T3 rivets. The type of testing equipment and specimen (Type M1) used are illustrated in a paper by R. L. Templin,[26] M. ASCE, in 1939, and a paper by Mr. Hartmann, J. O. Lyst, and H. J. Andrews,[27] Jun. ASCE, in 1944.

A factor of safety of 1.2 has been applied to the test data and all curves are cut off at the basic allowable design stress of 22 kips per sq in. The right-hand part of the diagram is largely based on extrapolation of the data, but this is not considered to be a serious matter since the design of most members in this range will be governed primarily by Specification H–1 rather than by fatigue considerations.

Respectfully submitted,

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R. Ebenbach
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F. J. Tamanini

E. C. Hartmann, *Chairman*

Committee of the Structural Division on Design in Lightweight Structural Alloys

December 6, 1951

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[25] "The Buckling Strength of Metal Structures," by Friedrich Bleich, McGraw-Hill Book Co., Inc., New York, N. Y., 1952, p. 422.

[26] "Fatigue Machines for Testing Structural Units," by R. L. Templin, *Proceedings*, A.S.T.M., Vol. 39, 1939, pp. 711–722.

[27] "Fatigue Tests of Riveted Joints," by E. C. Hartmann, J. O. Lyst, and H. J. Andrews, *Wartime Report W55*, National Advisory Committee for Aeronautics, Washington, D. C., 1944.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

TRANSACTIONS

Paper No. 2533

PROFESSIONAL STATUS AND THE THREAT OF STATISM

ADDRESS AT THE SUMMER CONVENTION DENVER, COLO., JUNE 18, 1952

BY CARLTON S. PROCTOR,[1] PRESIDENT, ASCE

The President's annual address is required by our By-Laws. Because that address is placed in the Society's archives for posterity, it is frequently a most erudite and formidable document, setting forth the results of weeks of diligent research, designed more for the enlightenment of posterity than for the interest of its listener audience; but 1952 is a most unusual year in our Society's history, one in which precedent must give way to obligation. For the engineer, this is the year of great decision.

The temptation will be strong, of course, to bask in the reflection of the glory that is ours in this great Centennial Year, to accept complacently the plaudits that will come with public recognition of the compelling part the engineer has played in the one hundred years of engineered progress in this great country of ours—to be content with the magnificent professional record behind us.

If that is the net result of our Centennial Year and its celebrations, then the efforts and the money that have gone into it will have been wasted and we will have tragically missed the greatest opportunity in our profession's long history. But if we accept our centennial as the year of the engineer coming-of-age, the celebration of his maturity, when it is encumbent on him to take stock of his inheritance and of his equipment to face the engineering world of tomorrow, with its rapidly increasing demands of expanding technological development, then this will be a proud year for us and for our country. Then we will justify our pride in our heritage.

Looking at the opportunities and demands ahead of us, one must sense that we stand on the threshold of an engineering age, where mechanization and technological advancements dominate a materialistic world; where all life, thought, and action are overshadowed by the fear of a third world war; where conflicting, confused ideologies and the indigestion of materialistic philosophies have undermined our spiritual life.

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[1] Cons. Engr., Moran, Proctor, Mueser & Rutledge, New York, N. Y.

We look into the future to see two worlds; one is dimly lit and but vaguely defined, leading to a prosperous, happy world at peace, utilizing the products of engineered science for the universal benefit of mankind, and the other road is brightly lit and gayly festooned, readily defined by its signboards advertising an easy dissipation and by its milepost of progressive degeneration, leading to a world in chaos.

To plot the course of the road to peace and prosperity and to expose to public view the mirage of the tinsel-fringed road to chaos, we will require a much improved transit, designed for employment by dedicated professional hands, to replace our outworn "theodolite" of short vision and subprofessional range.

No longer may we remain indifferent to the social impacts, the economic implementations, and the cultural repercussions of our work and our inventions. No longer can we bury our heads in the sands of our chosen branch of engineering technology to escape the accusing eye of those who look to us for guidance and accurate road directions.

No longer may we hide behind our slide rule and expect to be able to maintain the stewardship of our God-given privilege to utilize and direct the forces of nature to the benefit of mankind, and no longer may we shirk our responsibilities for direction of the educational training of the young men coming into our profession, to meet the pressing demand for the truly professional man equipped to carry through in his expanding responsibilities for public leadership.

The engineer of tomorrow faces a radically changed set of conditions in a drastically altered atmosphere. The classic fears of man, influencing all past progress of civilization, those fears of the ravages of nature—storm, famine, cold, and flood—have been dispelled by modern science, technology, and engineering. But while disposing of those fears, engineered science has created the current atmosphere in which man dreads, more desperately than he has ever feared before, the consequences of the inventions of science, technology, and engineering. So today our western world is grasping at the straws of pagan, Godless ideologies in a panicked attempt to defend itself against its fear that engineered science has invented and discovered beyond its capacity to control its forces of destruction.

Because of our direct responsibilities for the growth of this atmosphere of dread, we must accept our obligations for its immedate correction; the more so since our public leaders, either honestly misled or irresponsibly seeking personal power, too frequently see the solution in agencies and trappings of the superstate.

No engineer is immune to the enervating conditions and constricted opportunities that such statism is sure to impose. I am convinced that this is as true for engineers in the employ of government, federal or local, as it is for those in the employ of public utilities, of industry, and of consulting engineers in private practice.

Each one, regardless of what type of engineering endeavor he finds himself, faces a deterioration of his standing in the public's estimation unless he himself assumes the responsibility of guiding and influencing public leadership at every opportunity. This may be in group action, such as through the activities of this great Society of ours. It may be in personal conversations with friends. It will frequently be in speeches before lay audiences or in appearances before legislative committees.

In whatever form it takes, our responsibility for public leadership and for the maintenance and growth of a genuine professional status for ourselves is the challenge that this Centennial Year brings to each of us. It is my hope that I can emphasize to you the compelling nature of this challenge and present a few of the ways in which it can be met.

Let us remember that our responsibilities for public leadership, and for the maintenance of a genuine professional status, have not been suddenly thrust upon us. We have seen the inevitability of expanding responsibilities for years and have done little if anything about it; so that today we face a cumulative responsibility.

At the Annual Meeting of the Society in January, 1939, there was held a symposium by representatives of the American Institue of Architects, the American Bar Association, the American Medical Association, and our own Society. This symposium[2] had grown out of three years of study by the Society's Committee on Professional Objectives and was held for the purpose of determining how our sister professions had obtained their advanced degrees of professional status and as to what was necessary for the obtainment of improved status for the engineer.

Notwithstanding that our Society was the oldest of all national professional organizations, we recognized that the advancement of our prestige as professional men had not kept pace with the other represented professions. So we set out to find out why.

Many of the answers were readily apparent. In each case the national professional societies had demanded and obtained such steadily increasing standards of education that their professional training was gradually placed on the post-graduate level, and each national society had steadily upgraded its requirements for its accrediting of schools for professional training. I represented out Society and closed that symposium with a summarization of the experiences of the other professions as they applied directly (and still apply) to our situation.

In my paper I referred to the editorial statement[3] to the effect that for all that the engineer is necessary to the maintenance of modern civilization, he still lacks status by comparison with members of older professions and with the administrators who control the undertakings that he has made possible. Status is something that can be conferred but cannot be commanded.

In concluding my paper, I had this to state[4]:

> "If we are thoroughly honest with ourselves, I think we will admit that * * * we are but tilting at windmills when we place the responsibility for our present lack of status on any but ourselves * * *.

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[2] "Professional Status of Architecture, Medicine, Law, and Engineering: A Symposium," *Civil Engineering*, March, 1939, p. 145.

[3] *Engineering* (London), November 4, 1938, p. 538.

[4] "Professional Aims of the Civil Engineer," by Carlton S. Proctor, *Civil Engineering*, March, 1939, p. 151.

* * *

> "With the training for architects, lawyers, and doctors requiring a basic undergraduate academic course in the humanities followed by a post-graduate graduate professional training; * * * are not the engineers obtaining but a glorified trade school training instead of a real professional education?
>
> * * *
>
> "It requires plenty of determination and fortitude to challenge the institutions of professional education; to force the abolition of competition for students between engineering schools; and to impose a strict self-discipline on the profession. But the fruits of such efforts are the objectives we seek."

A glance at the status of engineering education today should convince us that engineers as a professional group have failed to exercise that degree of leadership and direction so long needed. We have failed in still greater degree to accept our special responsibilities in public affairs generally. Today our profession and our country have reached the "road block" where we can no longer evade our responsibilities for active leadership in public affairs. In the past much of the impetus for such leadership by engineers was derived from our obligations as good citizens. But today those obligations devolve upon us also in defense of our continued professional life and in the abolition of a prevailing atmosphere of confusion, loss of respect for our political institutions, and agnostic defeatism—for all of which our profession must accept its full share of responsibility.

The political exploitation of men's elemental fear of the destructive power of engineered science has created a widespread doubt as to the ability of our democratic processes to combat the threat of that pagan, Godless, and sterile police state that we so loosely call communism. Our leaders in public life should have been exposing the obvious fact that—so long as this great nation of ours maintains its cherished principles of individual dignity and liberty, its atmosphere of mutual confidence and respect inherent to our competitive enterprise system, and our reliance on divine guidance through our working partnership with the Creator—we have nothing to fear from the empty, embittered, soul-destroying philosophy of a system, so torn by suspicions, dissension, and dread of the concentration camp that it can be maintained only by a ruthless police state dictatorship.

Instead, our leaders have encouraged men to turn back the clock, degenerating to a dependence on the superstate as the best defense to the straw man called communism—completely concealing the fact that the totalitarian state toward which we are moving so fast is exactly what we have fought two terrible world wars to defeat and that, in its end result, the centralized paternal state, must embrace all the evils, the moral and financial bankruptcy and the retrogression to paganism, of its alleged enemy, communism.

For wherein is the end result different? Both statism and communism require state control of enterprise, demand the cancellation of individual freedoms, and deny man's hope of immortality. They both discredit the value of creative effort and inventive intelligence, preach the agnosticism that ideas and incentive can be socialized, and require our acceptance of the unethical and the expedient. They both deify mediocrity. If there is a difference it is that statism attempts to outbid communism by offering state subsidized indolence and abundance, something for nothing—in other words, increasing return for decreasing effort. Only statism preaches that a man is a "sucker" who works any harder than he has to because the taxpayer should take care of him anyway; and statism along with communism must presuppose the ever-continuing existence and availability of supermen. These supermen must have superknowledge, experience, and integrity to direct, control, and distribute equitably all the products of their own genius.

The job of guiding men's minds back to the simple precepts so fundamental to the growth of this republic is ours, since statism's clever play of group against group, class against class, and succeeding sectors of industry against competitor sectors, has resulted in the gagging, through threat of reprisal, of most of our industrial leaders, who should now be in the vanguard of this struggle for the survival of democracy.

It devolves upon us, in that postion of respect and trust in which we have always been held, standing as we do as arbiters and impartial agents between labor and management, to throw immediately the full weight of our leadership to public guidance in exposing to public view the fallacious precepts of statism and in restoring man's confidence in man's works.

When efforts are being made to socialize medicine in complete disregard of its abject failure and sterilizing results in England, we should be the first to unite with our sister profession in determined opposition. Socialization of engineering would quickly follow that of medicine, when invention and technological advancement would be stultified and the practice of engineering as a profession completely emasculated, as is inevitable to all creative effort under the sterile influence of paternalism.

When power-mad labor leaders drive headlong for "feather bedding" of jobs, enforced reductions in labor's productivity, and a merry-go-round of wage inflation, as the price of their support of an American superstate, we should point out to labor that this has been the pattern in the projection of every totalitarian government to date, and that once firmly established, every superstate has immediately relegated labor to the level of serfdom.

When in the name of the American superstate, an attempt is made to take over a national industry, such as steel, we should immediately point out that this again is in the pattern of statism's development—that Lenin, Hitler, Mussolini, and Peron all expropriated industry, segment by segment, under the guise of a national defense emergency.

As our liberties and individual freedoms are being canceled out and as the powers of law and regulation are being progressively and steadily shifted from community, county, and sovereign state into a centralized all-powerful paternal state, it is high time that we stand up and be counted as keeping faith with the souls of our sons and brothers who laid down their lives in two dreadful wars, waged for the single purpose of defending our liberties and of maintaining freedom's dignity of the individual—wars bitterly fought to defeat the very totalitarianism toward which we are being dragged.

Today we face a politico-economic ideology more insidious and more destructive than any that has so far challenged our progress. Far too many accept the false theses that moral lassitude in public office; public subsidy of indigence and tax-supported "boondoggling"; the trading of the dignity and integrity of the individual for a superstate; and the sacrifice of our priceless national heritage of morality, the dignity of work, and the essential place of God in our daily life are but a fair price to pay for the new, abundant, indolent, life.

Having played our decisive role in the development of the greatest social structure the world has yet known, imperfect and still inequitable though it undeniably is, we have watched this challenge to our basic standards, as spectators. Notwithstanding the abject failures of the economy in every nation that has grasped the dead hand of socialism, ignoring the stark evidence of want and deprivation, the failures of industry, and the cancellation of individual freedoms in all countries in the lethal embrace of statism, we have failed to offer that leadership in which men have hope and confidence.

For statism, in its insidious usurpation of the rights, liberties, and incentives of the individual, may in its end result be more permanently destructive to human rights than communism, for the more subtle reason that statism is being approached through a perversion of the democratic process, whereas communism has only been enforced on any people through a police state supporting a ruthless dictatorship.

Of all the groups in our entire economy, none in their daily work and in their essential attitude to the accomplishment of their jobs, so completely refutes these present-day politico-economic doctrines as does the engineer.

As engineers, let us stand up and be counted now, if we are to lay any claim to the plaudits coming to our profession for its important place in our centennial's depiction and celebration of one hundred years of engineered progress, and if we aspire to the hope of an extension of that progress through the engineer's continued creative and cooperative effort.

We have talked of engineering leadership for years but the time has come when the profession must replace talk by action. The high status which engineering should hold in the public mind will then be conferred on the profession. This will come to pass only when we have demonstrated the quality of leadership in engineering education and in public affairs of which our profession is so eminently capable.

The hour is late, but not too late, if we will but do our essential part and if we will remember that great injunction of 1900 years ago, never more timely than it is today—"What does it profit a man if he gain the whole world but lose his own soul."

MEMOIRS OF DECEASED MEMBERS

CHARLES HENRY PURCELL, HON. M. ASCE[1]

DIED SEPTEMBER 7, 1951

Charles Henry Purcell, the son of John and Mary (Gillis) Purcell, was born on January 27, 1883, at North Bend, Nebr. He attended Stanford University at Stanford, Calif., for one year in 1902 and was graduated a civil engineer from the University of Nebraska at Lincoln in 1906.

For a short time Mr. Purcell worked for the Union Pacific Railroad in Wyoming and as resident engineer built his first bridge across Bitter Creek. He was engaged in several engineering positions for private firms in Nevada, Peru, and Marysville, Calif., until he became bridge engineer for the Oregon State Highway Department in 1912. In 1920 he was appointed district engineer of the Bureau of Public Roads in charge of District I with headquarters at Portland, Ore.

Mr. Purcell was appointed state highway engineer of California in February, 1928. He was a member of the Hoover-Young San Francisco-Oakland Bay Bridge Commission, later becoming chief engineer, and was instrumental and responsible for securing funds for building the Bay Bridge from the Reconstruction Finance Corporation.

In 1938 he served as president of the American Association of State Highway Officials and was a member of the Executive Committee of that organization. Mr. Purcell was a representative of the United States on the Permanent International Commission of the Permanent International Association of Road Congresses.

He held honorary degrees of Doctor of Laws from the University of California and Doctor of Engineering from the University of Nebraska. His college fraternity was Chi Phi.

He belonged to the Family and Press clubs of San Francisco and to the Sutter and Del Paso Country clubs of Sacramento, Calif.

On February 14, 1914, in Portland, he was married to Minnie Pullen, who survives him.

Mr. Purcell was elected an Associate Member of the American Society of Civil Engineers on March 14, 1916; a Member on November 15, 1943; and an Honorary Member on July 17, 1945.

NOTE.—Many of these memoirs have been abbreviated for printing in *Transactions*. Complete manuscripts, including photographs, have been deposited in the Engineering Societies' Library, 29 West 39 St., New York 18, N. Y., and are available for consultation.

[1] Memoir prepared by R. M. Gillis, A.M. ASCE, and F. W. Panhorst and Fred Grumm, Members, ASCE.

ANDREW WEISS, HON. M. ASCE[1]

DIED SEPTEMBER 2, 1951

Andrew Weiss, the son of Johann and Anna Weiss, was born in Wels, Austria, on July 18, 1867. After receiving a common school training, he came to America and in St. Louis, Mo., attended business college. He completed studies at the Colorado School of Mines, at Golden, receiving the degree of Engineer of Mines in 1899, and there he remained as assistant professor of mathematics and surveying until 1903 when he joined the newly created Reclamation Service.

Mr. Weiss worked on the Salt River Project in Arizona and the North Platte Project in Nebraska and Wyoming, serving as project manager between 1907 and 1923 on the 250,000-acre North Platte Project. By promotion he was then appointed assistant director of reclamation economics to study projects experiencing difficulties in operation and to recommend adjustments.

In 1926 Mr. Weiss began work on reclamation in Mexico that was to continue for the remainder of his long life. The first six years were spent as resident engineer on the Don Martin Project (150,000 acres) in the states of Coahuila and Nuevo Leon, with part responsibility also for the Conchos Project in the State of Chihuahua. Then in 1932 he was appointed consulting engineer for the National Commission of Irrigation covering technical and economic planning of all projects.

Finally, from 1938 onward, again by promotion, he was chief of the Technical Consulting Department of the commission. In addition to reclamation and irrigation, his problems included flood control and sanitary water supplies, as well as multi-purpose projects involving hydroelectric power. During these periods, he was also consultant on the North Platte and the Santee-Cooper (South Carolina) projects in the United States. Mr. Weiss will long be remembered for having found a way to pass floods over earth dams under construction without injuring their stability, which was previously felt to be impracticable.

Mr. Weiss was a member of the American Society of Agricultural Engineers, the American Concrete Institute, and the Association of Engineers and Architects of Mexico. In 1949, on the fiftieth anniversary of his graduation, the Colorado School of Mines conferred on him its medal for distinguished achievement. Similarly, he was granted the highest token of esteem in the Society—election to Honorary Membership. In its resolution following his death, the Board of Direction properly characterized him as

> "a noted consultant * * * for a quarter of a century, a most effective ambassador of good will from the United States, the land of his first adoption, to Mexico, his final home country * * * and outstanding leader in the profession."

In 1903, Mr. Weiss was married in Denver, Colo., to Rebecca Minnie Hessel. She died in 1922. He was survived by two daughters, Lela H. and June H.

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[1] Memoir prepared by Gerard H. Matthes, Hon. M. ASCE, and Charles T. Craig, Denver, Colo.

Weiss, both of Denver. He was buried in Mexico City, after memorial services and eulogies by leaders of engineering and government there. Similar services were held in Denver.

Mr. Weiss was elected an Associate Member of the American Society of Civil Engineers on January 8, 1908; a Member on April 18, 1917; and an Honorary Member on October 11, 1948.

FRANK SEYMOUR BAILLIE, M. ASCE[1]

DIED MARCH 27, 1951

Frank Seymour Baillie, the son of William F. Baillie, was born on August 11, 1869, in Flint, Mich., and there completed his early education. He was graduated from the University of Michigan at Ann Arbor in 1890, having majored in civil engineering.

For the next six years he had varied civil engineering work—as surveyor and draftsman for W. W. Duffield & Son in Pineville, Ky.; as draftsman and engineer for the American Bridge Works in Chicago, Ill., designing subways in Jackson Park for the Chicago World's Fair; and as assistant engineer for the Gillette Herzog Manufacturing Company in Minneapolis, Minn. Mr. Baillie then went to Oregon, where for nineteen years (1896 to 1915) he devoted his attention to mining, first as vice-president and general manager for the Columbia Gold Mining Company and later, in a similar position, with the Baker Mines Company in Cornucopia, Ore.

During World War I, as manager of the Portland (Ore.) office of the Grant Smith Porter Ship Company, he supervised the building and launching of forty-eight ships. He continued with this firm until 1925 when he came to Sacramento, Calif., where he acquired controlling interest in the East Lawn Cemetery. As chief engineer and general manager, he continued the development of the cemetery and the construction of its mausoleum until his death at the age of 81.

Always maintaining an active interest in engineering, Mr. Baillie was a regular attendant at the meetings of the Sacramento Section of the Society throughout his twenty-five-year residence in that city. He was a proponent of licensing for civil engineers and, while in Oregon, had been a member of the State Board of Civil Engineering Examiners.

He was associated with many fraternal and civic activities, being a Past-Grand Master of the Free and Accepted Masons in Oregon, a member of the Mystic Shrine, Rotary Club, University Club, Sutter Club, and Del Paso Country Club. He also belonged to the American Society of Mining and Metallurgical Engineers.

On November 21, 1905, Mr. Baillie was married to Jessie Ellen Perkins who survives him.

Mr. Baillie was elected a Member of the American Society of Civil Engineers on May 12, 1919. He became a Life Member in January, 1944.

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[1] Memoir prepared by Joseph W. Gross, Carl Hoskinson, and Frank Reynolds, Members, ASCE.

FRED ASA BARNES, M. ASCE[1]

Died April 5, 1950

Fred Asa Barnes was born in Stockbridge, Mass., on June 17, 1876. His parents were Albert W. and Una M. (Thompson) Barnes. In 1897 he was graduated from Cornell University at Ithaca, N. Y., with the degree of Civil Engineer and in 1898 he received the degree of Master of Civil Engineering. Before entering his real life work of teaching, he spent four years in a variety of engineering positions—first, in the District Engineer's Office at Washington, D. C., and, later, in Cuba, working, successively, for the Hydrographic Office at Santiago, for the Ponupo Mining Company, and for Hugh Reilly, consulting engineer.

In September, 1902, he was appointed instructor in what was then the College of Civil Engineering at Cornell University, thus beginning a teaching and administrative career that continued until his retirement in 1944. He was promoted to the postion of assistant professor in 1905 and was made full professor of railroad engineering in 1915. In 1920 when the College of Civil Engineering became the School of Civil Engineering, Professor Barnes became the school's first director, a position he held until 1930.

Throughout his many years of service he played a prominent part in solving educational and administrative problems and was responsible for initiating and organizing many new courses. Professor Barnes was a joint author with Prof. Charles Lee Crandall, A.M. ASCE, of two textbooks entitled "Railroad Surveying" and "Railroad Construction." He also wrote many articles on transportation. He was an enthusiastic and active member of numerous professional and honorary societies, including Sigma Xi, Tau Beta Pi, Phi Kappa Phi, and Chi Epsilon. He was an honorary member of the Steuben Area Chapter of the New York State Society of Professional Engineers, and a member of the American Railway Engineering Association, the American Society of Engineering Education, and the American Association for the Advancement of Science. Through his personal efforts the Ithaca Section of the Society was founded in 1932. He was one of its early presidents and his enthusiasm and wise counsel contributed in a large measure to its continued successful operation.

A remarkable characteristic of Professor Barnes was his ability to remember the names and faces of civil engineering alumni. For many years he kept up-to-date a file of their addresses and occupations. No member of the faculty did more to create and maintain a strong bond between the alumni group and the School of Civil Engineering.

In the passing of Professor Barnes, Cornell alumni have lost a real friend, and the faculty, a highly regarded colleague. He will be kindly and gratefully remembered for his genial personality, his warm sympathy, his willingness to advise and help all who came to consult him, and his strong loyalty to the school and to the university.

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[1] Memoir prepared by Romeyn Y. Thatcher, M. ASCE.

Leaving Ithaca in 1949, Professor Barnes moved to Pleasant Valley, Conn. He died there on April 5, 1950.

On April 15, 1903, he was married to Bertha Lula Blaisdell. He is survived by Mrs. Barnes; by a daughter, Mary Louise Hall, also of Pleasant Valley; and by two grandsons, Nicholas B. and Stephen M. Eddy.

Professor Barnes was elected an Associate Member of the American Society of Civil Engineers on December 7, 1904, and a Member on June 20, 1922. He became a Life Member in January, 1939.

THOMAS RUPE BEEMAN, M. ASCE[1]

DIED SEPTEMBER 27, 1951

Thomas Rupe Beeman, the son of Gerald Alonzo and Susan Cornelia (Rupe) Beeman, was born in Comanche, Tex., on May 11, 1881. In 1903 he was graduated from Texas Agricultural and Mechanical College, at College Station, with the degree of Bachelor of Science in Civil Engineering.

His early experience was in railroading, including surveying, office work, and outside responsibilities for location, construction, and maintenance of way. He was first with the Southern Pacific Railroad and then with the "Milwaukee" Road, advancing to district engineer. From 1914 to 1917 he was construction engineer for King County, Washington.

In June, 1917, on the entrance of the United States into World War I, Mr. Beeman was commissioned Captain in the Engineer Officer Reserve Corps. His active service overseas included all types of heavy construction and maintenance, which involved him in the following major operations: Champagne-Marne (defense); Aisne-Marne (offense, including Chateau Thierry); St. Mihiel (drive); Meuse-Argonne (offense); and Defense Sector. He was also in the Army Reserve Corps through World War II, as Lieutenant-Colonel.

From 1919 to 1937, Mr. Beeman served, successively, as resident engineer, Washington State Highway Department; county engineer, King County; consulting engineer to the Washington State Highway Commission; chief engineer, Regional Planning Commission of King County; and city engineer, Seattle, Wash.

In 1937, he moved to Pittsburgh, Pa., where he was employed by the Pittsburgh District, Corps of Engineers, United States Army, as chief of the Relocations Branch, handling the design, negotiations, and settlements for alterations to railroads, highways, and public utilities for the several flood control reservoirs constructed by the Pittsburgh District.

Mr. Beeman was a member of the United Presbyterian Church of Mt. Lebanon, Pa., of the Society of American Military Engineers, and of three branches of the Free and Accepted Masons, in Seattle.

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[1] Memoir prepared by Conrad P. Hardy, M. ASCE.

On August 23, 1917, he was married to Byrnina M. Gebhart. He is survived by his widow, a sister, and two brothers.

Mr. Beeman was elected a Member of the American Society of Civil Engineers on October 1, 1912. He became a Life Member in January, 1942.

REUBEN FRANCIS BROWN, A.M. ASCE[1]

Died November 24, 1950

Reuben Francis Brown was born on June 14, 1891, in Bay City, Mich., and there attended the public schools. Following a year's study at Oberlin College (Oberlin, Mich.), majoring in engineering courses, he attended the Y.M.C.A. Training School at Chicago, Ill., where for three years he majored in physical training and secretarial work. He then enrolled in the International Correspondence Schools, studying municipal engineering for several years.

On completion of his studies at Chicago, Mr. Brown returned to Bay City and for several years acted as physical training director of the Y.M.C.A. He later filled a similar position at Jackson, Mich., for the Board of Education. During World War I he held the rank of Master Engineer in the United States Army and saw considerable action in France.

Following the war he obtained valuable construction and inspection experience working for construction and mining companies in and near Los Angeles, Calif. Later, in September, 1922, Mr. Brown became affiliated with the City of Los Angeles, in the Engineering Department, as a construction inspector. He was subsequently promoted to assistant sewer maintenance superintendent (1934) and superintendent of the Sewer Maintenance Division (1943), the position he held at the time of his death.

His enthusiasm and energy soon found him active in city employees' association affairs as an editor of, and a constant contributor to, the employees' magazine. He also presented many enlightening talks before civic and technical groups. An outstanding accomplishment was his inspection trip through the North Outfall Sewer in 1936, which created national interest and led to the formulation of better planned repairs and improved operations.

Mr. Brown was recognized for his knowledge of sewer maintenance work, being a member of several national committees of the Federation of Sewage Works Associations. He was also very active in the California Sewage Works Association, heading various committees.

A long-time member of the Free and Accepted Masons, he was a charter member of the Atwater Lodge 622, being Master of the lodge in 1926. He was a Knight Templar, a member of the Al Malaikah Temple of Los Angeles, and was well versed in Masonic history.

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[1] Memoir prepared by W. A. Schneider, A.M. ASCE.

On September 12, 1923, he was married to Anna Irene Hunrich, at San Jose, Calif. Besides his widow, he leaves a son, Donald W., and a daughter, Joyce C. Brown.

Mr. Brown was elected an Associate Member of the American Society of Civil Engineers on August 13, 1945.

EDWIN FOSTER CODDINGTON, M. ASCE[1]

DIED DECEMBER 21, 1950

Edwin Foster Coddington was born a farmer's son at Conover, Ohio, on June 24, 1870. His parents were James and Jane (Morris) Coddington. To secure his education at Ohio State University, at Columbus, he helped around the college barn and worked as janitor at the observatory. He received a degree in civil engineering (1896) as well as the degree of Master of Science (1897).

Following a strong interest in astronomy, Mr. Coddington became a Fellow at Lick Observatory, University of California, at Berkeley, continuing at the Friedrich-Wilhelm University in Berlin, Germany, where he received the degree of Doctor of Philosophy two years later, in 1902. For the following thirty-nine years he served at Ohio State University, in the departments of mechanics and civil engineering. After fifteen years as professor in charge of geodetic engineering, he was made professor emeritus. During the period from 1915 to 1920 he was acting dean of the College of Engineering.

From astronomy to mathematics, to mechanics and geodetic surveying, he proved himself a master. Students, practicing engineers, and colleagues found in him capable and willing assistance for intricate mathematical problems. Many of his summers were spent in surveying or geodetic work, for the government, for the Muskingum Valley Conservancy District, and for engineers in private practice. He demanded of his students, and of himself, devotion to duty and conscientious attention to thoroughness and detail. In his home and social life he was the same self-effacing gentleman as in the classroom. His devotion to his family and justifiable pride in his foster daughter were characteristic.

Predominant among the bases of his character was faith—in his profession, his ideals, his friends. With this was a determination verging at times toward a crusader's zeal for the educational ideals of the university. Then too he had an open-mindedness and an intense loyalty to his principles, his friends, and his church. Even his infinite patience could not tolerate mediocrity or intellectual disinterest.

When most men would have been content to live in the memories of past successes, he helped to create and then to serve in the Mapping and Charting Research Laboratory at the university. With his death on December 21, 1950, he joined the ranks of those, all too few, who leave the world a little better off because they were a part of it.

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[1] Memoir prepared by Clyde T. Morris, M. ASCE, George H. Harding, A.M. ASCE, and H. E. Nold.

Dr. Coddington was a member of Acacia Fraternity and a long time faithful member of the First Community Church of Columbus. He is survived by his wife, who was Cora Stoner before their marriage in 1899, and by their daughter, Dorothy.

Dr. Coddington was elected a Member of the American Society of Civil Engineers on August 29, 1927.

WILLIAM HENRY CODE, M. ASCE[1]

DIED MARCH 28, 1951

William Henry Code was born in Saginaw, Mich., on November 22, 1865. He received his engineering education at the University of Michigan at Ann Arbor and was later awarded the degree of Bachelor of Science in Civil Engineering in 1892.

From 1891 to 1892 he was assistant state engineer of Wyoming. He then went to Arizona as chief engineer of the Consolidated Canal Company in the Salt River Valley, and later served as superintendent of that company. From 1902 until 1911 Mr. Code was chief engineer of irrigation for the United States Indian Service with headquarters in Arizona. He was also special agent on irrigation investigations in Arizona for the United States Department of Agriculture.

In 1912 Mr. Code joined John H. Quinton to form the partnership of Quinton and Code for the practice of consulting civil engineering in Los Angeles, Calif. Two years later Louis C. Hill joined the firm and its name was changed to Quinton, Code and Hill. Mr. Code continued as a member of this partnership and also its successor, Quinton, Code and Hill-Leeds and Barnard, until 1936 when he retired from active practice. During those years the firm handled many projects in Canada and Mexico as well as in the United States, mainly in the field of hydraulics. Mr. Code continued to specialize in irrigation works.

Although Mr. Code resided in Los Angeles, he never lost touch with the irrigation problems of Arizona. Primarily an engineer, he was interested in the arts and civic affairs. He was socially inclined and his sense of humor was keen. He was also interested in finance and was a director of the California Bank of Los Angeles during the late twenties and early thirties.

In 1893, in Bay City, Mich., Mr. Code was married to Martha E. Devlin, who survives him.

Mr. Code was elected a Member of the American Society of Civil Engineers on June 5, 1907. He became a Life Member in January, 1936.

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[1] Memoir prepared by Raymond A. Hill, M. ASCE.

CLERMONT C. COVERT, A.M. ASCE[1]

DIED DECEMBER 11, 1950

Clermont C. Covert, the son of William R. and Frances (Genung) Covert, was born at Townsendville, Seneca County, New York, on January 25, 1872. After his elementary education in district schools, he attended Cazenovia Seminary at Cazenovia, N. Y., for his secondary education, and for his professional training he attended Ohio Northern University in Ada, where he received the degree of Civil Engineer in 1894.

Mr. Covert was in general practice of his profession from 1895 to 1902. His various activities included: Chainman, city engineers' office, Syracuse, N. Y.; general engineer and railroad surveyor, Chenango Engineering Company, Binghamton, N. Y.; surveyor for the Penn Yan, Lake Keuka and Southern Railroad in New York; and railway engineer for the Choctaw, Oklahoma and Gulf, and also the Erie, and the Delaware, Lackawanna and Western railways. During the Spanish-American War, he acted as transitman on military road surveys with Company B, First United States Volunteer Engineers, in Puerto Rico. In 1902 he began a private practice in Binghamton.

From 1902 to 1922 Mr. Covert was with the Water Resources Branch of the United States Geological Survey. Later (1908) he was in Alaska, investigating water supply in relation to mining. From 1909 to 1922 he was hydraulic engineer with the department at Albany, N. Y., serving as district engineer of the New York District and, for part of that period, of the New England District also.

In August, 1922, he became hydraulic engineer with W. and L. E. Gurley, instrument makers of Troy, N. Y. From May, 1926, until his retirement in 1948, he was in charge of the company's New York office. After leaving that position he traveled extensively for the same company in the New York-New England and southern area, until his death.

Mr. Covert was the author of Geological Survey government publications on "Surface Water Supply" in New York, New England, and Alaska.

He belonged to the New York State Society of Professional Engineers. He was also a member of the First Presbyterian Church of Albany and Lodge No. 177 of the Free and Accepted Masons in Binghamton.

On August 7, 1901, he was married to Mildred Edith Birdsall, who survives him.

Mr. Covert was elected an Associate Member of the American Society of Civil Engineers on July 1, 1909. He became a Life Member in January, 1943.

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[1] Memoir prepared by Paul H. Underwood, M. ASCE.

WILLIAM CANTRELL CURD, M. ASCE[1]

DIED JULY 20, 1951

William Cantrell Curd, the son of Will and Belle (Saunders) Curd, was born on May 28, 1880, at Somerset, Ky., and attended public school there. For three years (1899 to 1902) he pursued engineering studies at Purdue University in West Lafayette, Ind.

After four years of railroad work, Mr. Curd was employed by the Missouri Pacific Lines in St. Louis, Mo., and served as assistant engineer and drainage engineer from 1906 to 1917. Following this period, he was engaged as contracting engineer on the sales of water development services and equipment with headquarters at Memphis, Tenn. In 1919 he moved to Chicago, Ill., and opened an office as consulting engineer, specializing in water supply, sewage disposal, drainage, flood control, and related projects. This work was discontinued in 1924 when he organized a construction company, serving as vice-president and chief engineer until 1933, when he again entered private practice.

Mr. Curd subsequently served as public relations officer and engineer during the construction of the Grand River Dam, near Vinita, Okla. (1939–1941); general office manager and disbursing officer on the construction of the St. Louis Ordnance Plant (1941–1942); and assistant secretary and resident corporate officer on the Milwaukee Ordnance Plant (1942–1943).

He was research engineer and analyst for the Association of American Railroads at New York, N. Y., from 1943 to 1945. In 1946, he returned to Chicago to accept an appointment as railroad specialist for the Chicago Plan Commission where he was active in handling railway terminal problems prior to his death.

Mr. Curd was a registered professional engineer in Illinois, a member of the Western Society of Engineers, and the American Railway Engineering Association, on whose roadway committee he served from 1909 to 1922.

His death was a great loss to his many friends. He is survived by his widow, Jonella Z. Curd.

Mr. Curd was elected a Member of the American Society of Civil Engineers on June 11, 1917. He became a Life Member in January, 1951.

FRANK HENRY FELLER, M. ASCE[2]

DIED OCTOBER 19, 1950

Frank Henry Feller, the son of John and Margaret (Tuttle) Feller, was born in Boone, Iowa, on March 31, 1875. After being graduated from the local high school he became interested in engineering field work, and studied under special tutors in civil and hydraulic engineering.

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[1] Memoir prepared by Charles P. Richardson, M. ASCE.

[2] Memoir prepared by Carl Crandall, M. ASCE.

Early in his career Mr. Feller held positions in Mexico and Alaska in railroad work. In 1906 he settled in Spokane, Wash., where he operated his own engineering company, covering numerous bridge and water works projects. Following service in the United States Army during World War I, in which he served overseas as Captain of Engineers, he was supervising engineer for Spokane County for a brief time and then he became construction engineer for the Iowa Highway Commission at Sac City. Then, again, he returned to Spokane County as bridge engineer for four years.

For the following nine years Mr. Feller served the City of Ithaca, N. Y., first as construction engineer and for the last five years, 1930 to 1935, as superintendent of the Department of Public Works. The first part of his superintendency was marked by a large program of capital improvements and the latter part by the introduction of Temporary Emergency Relief Administration projects, including construction of the Municipal Airport and transformation of the old city dump into the Newman Municipal Golf Course.

After leaving the employ of the city, he opened an office for private engineering practice in Trumansburg, near Ithaca. Later, he was attached to the Corps of Engineers, U. S. Army, engaged in flood control studies and installation, with headquarters at Syracuse, N. Y. For the three years immediately preceding his death, Mr. Feller served the Finger Lakes State Parks Commission as engineer in charge of construction work at Fair Haven Beach State Park, Fair Haven, N. Y.

On January 7, 1920, he was married to Ruth Marjorie Townsend. He is survived by his widow; a son, John (of Ithaca); a daughter, Mrs. J. Ralph Walker (of Parma, Idaho); and three grandchildren.

Mr. Feller was elected a Member of the American Society of Civil Engineers on September 2, 1914. He became a Life Member in January, 1946.

ELMER LEROY FERGUSON, A. M. ASCE[1]

DIED MARCH 28, 1951

Elmer Leroy Ferguson was born in Elkton, S. Dak., on January 3, 1888. Early in his career he taught school in South Dakota after attending the University of North Dakota at Grand Forks. He attended Iowa State College at Ames in 1914–1915 and later became assistant county engineer at Sioux City.

Mr. Ferguson then worked for several well-known engineering firms in Sioux City and Kansas City, Mo., designing and supervising the construction of drainage, highway, and municipal projects from 1916 to 1928. This work was interrupted for a period of one and a half years during World War I while he served with the United States Army in the Field Artillery, advancing to the rank of Second Lieutenant.

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[1] Memoir prepared by J. K. Alewine, A. M. ASCE.

From 1928 to 1934 Mr. Ferguson practiced engineering independently in Sioux City, specializing in drainage work, municipal projects, and appraisals.

He entered the service of the federal government in November, 1934, when he became associated with the Public Works Administration at Topeka, Kans., beginning as a resident engineer and inspector on water works and power construction. He later held positions of examining engineer and squad chief in the regional office at Fort Worth, Tex.

During the emergency period prior to World War II Mr. Ferguson was engaged as office engineer for Brown-Bellows-Columbia Contractors on the Naval Air Base project at Corpus Christi, Tex. He was in charge of estimators and draftsmen on the engineering construction for this base.

In July, 1941, he returned to the Federal Works Agency at Fort Worth where he became chief of the engineering section in the regional office. He continued to serve in this capacity through the several reorganizations of the agency until his death.

Mr. Ferguson was a member of the First Methodist Church in Fort Worth and sang in the men's choir. He was active in the local affairs of the Society and was a Past-President of the Fort Worth branch of the Texas Section.

He was married on September 9, 1939, to Carol McDowell, who survives him, together with a son, Keith.

Mr. Ferguson was elected an Associate Member of the American Society of Civil Engineers on October 15, 1923.

RAYMOND FRANCIS FLETCHER, M. ASCE[1]

DIED DECEMBER 26, 1950

Raymond Francis Fletcher was born on February 17, 1901, in Philadelphia, Pa., where he spent the first two thirds of his life. He attended the public schools and the Philadelphia Trade School. His engineering training was received at Drexel Institute of Technology, from which he was graduated a structural engineer in 1928.

Between 1917 and 1934 he worked for the following architectural and engineering design firms: Ballinger Company, Clarence E. Wunder, Ritter and Shay, Gravell and Duncan, and Fred W. Abbott. He then left Philadelphia and, until his death, was employed in the Design Division, Engineering Department of E. I. du Pont de Nemours and Company in Wilmington, Del. Most of his design work was on chemical plants and buildings, but it also included site development, transportation facilities, sanitation, and process-waste neutralization plants. For the last six years of this service, he held the title of senior architectural and civil engineer, conducting all the corresponding design work for the Rayon Department of the du Pont Company. Notable among the projects he handled were the new Nylon Plant at Chattanooga, Tenn., the Olin Industries Cellophane Plant at Pisgah Forest, N. C., and phases of the Atomic Energy Plant at Hanford, Wash.

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[1] Memoir prepared by George W. Dutcher, A.M. ASCE.

Mr. Fletcher was a registered engineer in the states of Delaware, New Jersey, Pennsylvania, and North Carolina. He was a deacon on the Church Council of Holy Trinity Lutheran Church, in Wilmington, and a member of the Free and Accepted Masons.

Surviving are his wife, who before her marriage was Janet Weir; a son, William Weir, who is a divinity student; and three brothers.

Mr. Fletcher was elected a Member of the American Society of Civil Engineers on October 20, 1947.

WILLIAM McLAURINE HALL, M. ASCE[1]

DIED NOVEMBER 25, 1951

William McLaurine Hall, the son of David Cowan and Mary (McLaurine) Hall, was born in Fayetteville, Tenn., on March 1, 1860. He was a member of the class of 1881 at the United States Military Academy at West Point, N. Y., but returned to civilian life as an inspector on the building of the West Shore and Buffalo Railroad. For a period he was chief engineer on the New Croton Aqueduct for New York City, but largely he did construction work for the West Shore and Buffalo, the South Penn, and the Norfolk and Western railroads; then, in 1893, he joined the United States Engineers and remained in federal employ until his retirement in 1932.

On the Rough River in Kentucky he built what is believed to be the first monolithic concrete river lock in the world and at New Bedford, Mass., the first forts of monolithic concrete.

Among the notable innovations introduced by Mr. Hall during the construction of Lock and Dam No. 18 on the Ohio River were the use of a core drilling to locate bed rock; the anchoring of concrete masonry to bed rock; the design of a new bear-trap weir for controlling the pool stage above the dam; and the effecting of a new type of lock power plant. These methods are now standard practice. Mr. Hall had full field charge of building sixteen Ohio River locks and dams. By an act of Congress he was retained in service beyond retirement age and served as chairman of a commission whose report was enacted into a law under which government appropriations are made for flood control.

While Mr. Hall was a member of the Parkersburg (W. Va.) Water Works Commission in 1907, the controversial experimental river bed sand infiltration system was built. He wrote a complete treatise on the subject[2] which was only one of several papers and many discussions he contributed to the Society's *Transactions*.

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[1] Memoir prepared by Samuel B. Settle, M. ASCE.

For notable service in the field of engineering Mr. Hall received the honorary degree of Doctor of Science at Marietta College, Marrietta, Ohio, in June, 1947.

He was a charter member of the Parkersburg chapter of the West Virginia Society of Professional Engineers and a member of the First Presbyterian Church of Parkersburg for fifty years.

In June, 1886, he was married to Jean Agnew Crawford at Chambersburg, Pa. Mrs. Hall died in October, 1908. Surviving are a brother, David Hall; a daughter, Eleanor S. Hall; two sons, Allen C., and William M. Hall, Jr.; six grandchildren; and six great-grandchildren.

Mr. Hall was elected a Member of the American Society of Civil Engineers on December 6, 1893. He became a Life Member in January, 1928.

ALBERT PRESTON HARNESS, JR., M. ASCE[1]

DIED FEBRUARY 7, 1951

Albert Preston Harness, Jr., was born on May 28, 1895, at Henderson, Ky. He was the elder of two sons born to Albert Preston Harness, Sr., and Camilla (Alves) Harness.

His early life was spent in Henderson, where he received his elementary and high school training. For his technical education he attended the University of Kentucky at Lexington and the University of Wisconsin at Madison.

For a short time in 1918, Mr. Harness was employed by the Tennessee Iron and Coal Company at Birmingham, Ala., and in the company's shipyards at Mobile, Ala., prior to entering military service. He served in the Quartermaster Corps of the United States Army during World War I.

Deciding to make Columbus, Ohio, his future home, he became associated with The Jennings-Lawrence Company immediately after his discharge from army service and stayed until his death with this organization, whose business included surveying and civil and municipal engineering. He was first employed as a draftsman, becoming vice-president in 1926, general manager in 1938, and president in 1943.

Mr. Harness was highly regarded personally. His judgment and his advice as an engineer was often sought. However, it was in the field of public affairs that he was best known and respected. Mr. Harness was active in many organizations—he was a member of the National Society of Professional Engineers, the Young Men's Business Club, the Brookside Golf and Country Club, and the Board of Trustees of Mt. Carmel Hospital; past-president of the Engineers' and Athletic clubs of Columbus; director of the Columbus Chamber of Commerce, Columbus Builders' Exchange, and Columbus Town Meeting; and an organizer of the original Franklin County Planning Commission (for which he was secretary) and of the Joint City-County Planning Commission which succeeded it. He was a devout member of St. Stephen's Episcopal Church, having served the church in many official positions.

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[2] "The Water Supply of Parkersburg, W. Va.," by William M. Hall, *Transactions*, ASCE, Vol. LXXI, 1917, p. 749.

[1] Memoir prepared by C. C. Walker, M. ASCE.

He not only belonged to these many organizations but he gave freely of his time and efforts. Because of his intense interest in community planning, he appeared on many radio programs to speak on its behalf. His passing was recognized by half-column-long editorials in each of the three newspapers of Columbus.

Mr. Harness used his prestige to advance public welfare, never for his own personal honor. He believed that service to his community as a layman was more effective than filling an elective office with its resulting public acclaim.

On August 15, 1918, he was married to Hattie Belle Spoehr. He is survived by his widow; one son, Albert Preston Harness, III (J. M. ASCE), who served in World War II; and a grandson, Stephen Dallam Harness. Also surviving are his mother, an aunt, and two nieces.

Mr. Harness was elected a Member of the American Society of Civil Engineers on January 12, 1948.

FRED WILLIS HASELWOOD, M. ASCE[1]

DIED OCTOBER 27, 1951

Fred Willis Haselwood was born in Milford, Nebr., in May, 1880, and moved to Kansas eleven years later. After being graduated with the degree of Bachelor of Science from Kansas State Agricultural College at Manhattan in 1901, he did graduate work, with special emphasis in hydraulics, at Stanford University at Stanford, Calif., until 1903.

His early work, beginning in 1903, was mostly railroading for the Western Pacific, the Santa Fe, and again, from 1904 to 1909, for the Western Pacific Railroad on location and as resident engineer. Following an interval of private practice and one of investigating water rights for the California State Engineer Office, he started his real life work early in 1912, with the California Highway Commission.

First Mr. Haselwood was assistant engineer in District I at Willets, then in 1923 in the bridge department on investigation and construction. In August, 1924, Mr. Haselwood became division engineer of District III at Sacramento and in 1929 returned to District I at Eureka, until February, 1932, when he was transferred to Redding as district engineer of District II, which comprises the seven northeastern counties of California. He remained until his retirement in May, 1950.

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[1] Memoir prepared by F. S. Saunders, Asst. District Engr., Administration, California State Div. of Highways.

In spite of increased duties involved in directing these multiple activities, Mr. Haselwood found time for one of his chief interests, the location and design of hundreds of miles of modern highways which serve as a monument to his efforts. His progressive thinking and inherent ability to foresee the needs of the future, coupled with the strength of his convictions, made his work outstanding.

Typical of his accomplishments was the Oregon Hill location in Trinity County, involving a 200-ft cut through gold-bearing gravel at the site of an old mine. Ordinary methods being futile, he made a unique solution using two 8-in. giant monitors. Even this required five years of work to move ten million yards. Another famous exploit was a seventy-mile location along the Feather River, ten miles of which was in solid rock canyon requiring three tunnels. These and many other long stretches of modern highways remain as constant reminders of his service to the people of California, not only as an engineer but as an individual vitally interested in progress.

Mr. Haselwood was elected a Junior of the American Society of Civil Engineers on January 3, 1907; an Associate Member on September 6, 1910; and a Member on April 17, 1923. He became a Life Member in January, 1945.

JOHN LESLIE HEMMERT, A.M. ASCE[1]

DIED JUNE 28, 1951

John Leslie Hemmert was born on December 1, 1895, in Thayne, Wyo. He lived there until 1917 when he entered the United States Army.

Overseas Mr. Hemmert served as a member of the famous Rainbow Division. Having been wounded in one leg by shrapnel he was discharged from the army in 1919. Later, he served as post commander of his local Disabled American Veterans, Post No. 6.

In 1920, he moved to Idaho and there entered the University of Idaho at Moscow, receiving the degrees of Bachelor of Science in 1924 and Master of Science in 1927. In addition to the usual engineering course, he became interested in materials testing and gained considerable experience in such work. During vacation periods he was employed on various engineering projects in Idaho and Washington, and from 1924 to 1927 served as materials engineer for the State of Idaho and as instructor in civil engineering at the university. In 1927 he became a junior highway engineer in District No. 12 of the United States Bureau of Public Roads at Ogden, Utah. Part of this period was spent on projects in Zion National Park in southern Utah.

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[1] Memoir prepared by F. N. Hveem, A.M. ASCE.

In April, 1930, Mr. Hemmert reported for work with the California Division of Highways in the Materials and Research Department at Sacramento. He was placed in charge of all testing and inspection on steel and other metals and also timber and piling used in highway bridges and structures, including much of the material used on the San Francisco-Oakland Bay Bridge. Until June, 1950, he remained in this work, attaining the position of supervising materials and research engineer.

In his daily work, a variety of problems constantly arose and these Mr. Hemmert handled with skill and diplomacy to the satisfaction of both the manufacturers of materials and the engineers with whom he came in contact. He left many friends.

He was a member of the Society's Sacramento Section; the honorary engineering society, Sigma Tau; the American Road Builders' Association; and the Military Order of the Purple Heart. He also held membership in the Rio Del Mar Country Club.

Mr. Hemmert was elected an Associate Member of the American Society of Civil Engineers on December 11, 1939.

LEROY LEMAYNE HIDINGER, M. ASCE[1]

DIED NOVEMBER 11, 1951

Leroy Lemayne Hidinger, the son of John and Lydia (Walters) Hidinger, was born on a farm near Prescott, Adams County, Iowa, on May 19, 1879. He attended Iowa Business College in Des Moines and Iowa State College at Ames, where he was graduated in 1906 with the degree of Bachelor of Science in Civil Engineering and from which college he received the degree of Civil Engineer in 1911.

On being graduated he was appointed drainage engineer in the United States Department of Agriculture, participating in surveys, investigations, and reports on drainage and flood control in half a dozen states and on reclamation of the alluvial lands in the Mississippi Valley.

In 1910 Mr. Hidinger and Arthur E. Morgan, M. ASCE, organized the Morgan Engineering Company with headquarters in Memphis, Tenn. He was, successively, partner, vice-president, and president of the company, and in 1915 he became senior executive officer in charge of operations embracing several branch offices. In 1943 he became sole owner of the company. He was also vice-president of the Dayton-Morgan Engineering Company.

During his lifetime Mr. Hidinger was actively engaged in flood control, drainage, irrigation, and highway projects in twenty four of the states involving more than 150 projects and expenditures of more than $200,000,000.

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[1] Memoir prepared by Garner W. Miller, M. ASCE.

He was frequently retained as a member of special boards or as a consultant to investors, bond underwriters, and railroads, as well as the Tennessee Valley Authority, the National Resources Planning Board, and the U. S. Department of Agriculture. Mr. Hidinger was a member of Tau Beta Pi, the American Institute of Consulting Engineers, the National Panel of Arbitrators, and the Engineers' Club of Memphis since its organization in 1914. He was active in St. John's Methodist Church of Memphis, and served as chairman of the Finance Committee, as a member of the Board of Stewards there for more than twenty years, and as a member of the Board of Trustees since 1941.

Until his latter years he owned and operated a large cotton plantation in Arkansas near Memphis, near a community center bearing the abbreviation of his name, Lehi. He was a member of the Chickasaw Country Club, devoting his spare time to golfing and travel.

In the Society he was a member of the Constitutional Committee (1920–1921), the Committee for Registration of Engineers, the Executive Committee of the Waterways Division, the Joint Committee on Floods, and the Committee on Cost Allocation for Multiple Purpose Water Projects. He was a member of the Board of Direction from 1936 to 1938, and served on the National Water Policy Panel of the Engineers' Joint Council and as chairman of the Committee on Land Drainage. He was a charter member and Past-President of the Mid-South Section of the Society. In 1942 he was awarded the Marston Medal of Iowa State College.

In 1911 Mr. Hidinger was married to Anna E. McKinney. She died in 1921. He was married in 1925 to Olla V. Johnson who, together with a son, Leroy L. Hidinger, Jr., survives him.

He was regarded as the dean of engineers in the Mid-South Section, a Christian gentleman with an uncompromising fidelity to principle, fearless in the discharge of his duties, high in the ranks of his adopted profession, and a sincere friend and valued counselor to all those who sought his advice. He was active in both professional and civic affairs up to the day of his death.

Mr. Hidinger was elected an Associate Member of the American Society of Civil Engineers on October 4, 1910, and a Member on January 18, 1916.

MYRON TUTTLE JONES, M. ASCE[1]

DIED JUNE 11, 1950

Myron Tuttle Jones, the son of Myron Germain and Susan (Foley) Jones, was born on October 13, 1897, at Ithaca, N. Y. He attended Cleveland "Y Tech." (later Fenn College) in Cleveland, Ohio, from 1915 to 1916 and was graduated from Cleveland Law School in 1932 with the degree of Bachelor of Law. He was a member of Delta Theta Phi.

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[1] Memoir prepared by Paul A. Uhlmann, M. ASCE.

From 1919 to 1921 he practiced municipal and railroad engineering in Cleveland and Akron, Ohio. He started his own private practice at Cleveland in 1921, specializing in land development, planning, and engineering and property surveys until 1934.

During the following three years Mr. Jones acted as resident engineer inspector for the Public Works Administration (PWA) in Ohio and Michigan. From 1937 to 1938 he was consulting engineer for the Bellevue Flood Relief Commission and in 1939 was appointed associate engineer of the PWA at Columbus, Ohio, and Chicago, Ill. He left the PWA in 1940 to act as engineer for the sewer and water facilities at Camp Grant and at Scioto Ordnance Plant for Holabird and Root, architect-engineers of Chicago, and as central engineer during the Camp McCoy planning survey for Mead, Ward and Hunt of Madison, Wis.

In 1943 Mr. Jones established his own office at Columbus, specializing in municipal planning and engineering, flood control, and drainage. He also specialized in aerial, tax, and topographic maps, including property surveys which were the basis for several articles and papers in connection with surveying and aerial mapping.

Mr. Jones was always interested in civic problems as well as in furthering the standing of the engineer. He conducted his work in a highly efficient and ethical manner, and will long be remembered by everyone of his large circle of acquaintances as a dignified representative of the engineering profession.

Mr. Jones held membership in the American Water Works Association, the National Society of Professional Engineers, the American Congress on Surveying and Mapping, and the Columbus Chamber of Commerce. He also belonged to the Ohio Society of Professional Engineers and was president of the Franklin County Chapter in 1948–1949.

On July 17, 1937, he was married to Jean Logue Parker, who, with their children, Sandra Allerton, Michael Germain, and Mark Tuttle, survives him.

Mr. Jones was elected a Member of the American Society of Civil Engineers on February 13, 1945.

THEODORE TEMPLE KNAPPEN, M. ASCE[1]

DIED MARCH 20, 1951

Theodore Temple Knappen was born in Minneapolis, Minn., on November 21, 1900. He was the son of Theodore M. and Nellie (Cross) Knappen. After being graduated from the United States Military Academy at West Point, N. Y., in 1920, he served as Second Lieutenant and First Lieutenant in the Corps of

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[1] Memoir prepared by Eugene E. Halmos, M. ASCE.

Engineers and did graduate work at Rensselaer Polytechnic Institute at Troy, N. Y., receiving the degree of Civil Engineer in 1922.

Leaving military service in March, 1923, Mr. Knappen devoted the next few years to a number of projects in California, one of which included the position of resident engineer for the Nevada Irrigation District. In 1928, after five years of such valuable hydraulic design and construction experience, he rejoined the Corps of Engineers in a civilian capacity—first as area engineer on Mississippi River flood control, Memphis District; then as chief of the Engineering Division for the Muskingum Flood Control Project in Ohio; and, finally (1936), as chief of the Flood Control Section for the North Atlantic Division.

In August, 1937, he joined Parsons, Klapp, Brinckerhoff, and Douglas, consulting engineers of New York, N. Y., as engineer-manager of their Caracas, Venezuela, office, designing many port, water supply, irrigation, and flood control projects. He became a partner in January, 1941, and for the firm directed the design of United States military bases in the Caribbean.

Mr. Knappen resigned in November, 1942, to establish his own consulting office which later became the firm of Knappen Tippetts Abbett McCarthy, Engineers. Of the firm's numerous design projects, all over the world, typical were port improvements in Greece, Venezuela, Israel, and Sumatra, as well as at Philadelphia, Pa., Baltimore, Md., Buffalo, N. Y., and Miami, Fla., in this country; hydroelectric and irrigation projects in Turkey and Haiti; the Pine Flat Dam in California and flood control works on Frankford Creek, Philadelphia; a vehicular tunnel in Buenos Aires, Argentina; a subway in São Paulo, Brazil; and a highway and airports in Bolivia. In all these Mr. Knappen was the guiding spirit.

He was brilliant and foresighted. His writings on soil mechanics and flood control were widely read. Only too well he realized that American engineers should meet the challenge of engineering service abroad and for his untiring efforts and extensive travels he was called an engineering ambassador-at-large. His splendid personality and convincing expression of ideas, together with his remarkable business sense and tireless energy, were the secrets of his great professional success. He was a member of numerous organizations including the American Institute of Consulting Engineers, the New York State Society of Professional Engineers, the American Geophysical Union, the American Railway Engineering Association, and the American Concrete Institute. His social organizations were the Metropolitan (N. Y.) and Cosmos (Washington) clubs, and Theta Chi Fraternity.

Surviving are his widow, the former Georgianna Ferguson; three minor children, Theodore Compton, Georgianna Cross, and Nathaniel Ferguson Knappen; also, a sister, Phoebe M.; and two brothers, Judson and Andrew Knappen.

Mr. Knappen was elected a Junior of the American Society of Civil Engineers on January 16, 1928; an Associate Member on June 10, 1929; and a Member on April 13, 1936.

THOMAS DOTTERER LEBBY, M. ASCE[1]

DIED SEPTEMBER 1, 1951

Thomas Dotterer Lebby, the son of T. D. and Caroline H. Lebby, was born on June 23, 1896, in Summerville, S. C. He received an academic degree from the College of Charleston (Charleston, S. C.) in 1915 and engineering degrees from the Massachusetts Institute of Technology and Harvard University (of Boston and Cambridge, Mass.) in 1917. He was a member of Alpha Tau Omega Fraternity.

After serving for eighteen months in World War I as a commissioned officer, he was employed by the Bethlehem Shipbuilding Corporation. Later he embarked on a construction career in Mississippi under the name of Williams and Lebby Engineering Service. Mr. Lebby then entered the firm of Morgan and Tate (in Knoxville, Tenn.), which also specialized in construction work. Following this employment the partnership of Steel and Lebby was formed for the design and construction of bridges, and for eleven years this was a highly successful venture.

In 1935 with construction at a low ebb, Mr. Lebby worked for the Tennessee Valley Authority (TVA) as a construction and maintenance superintendent in the Wheeler Reservoir area. His executive ability was soon recognized and with the retirement of the director of the construction and maintenance division, Mr. Lebby was appointed to head this department. He held this position at his death.

In golf he was in his avocational element and found one big energy outlet. In fact, it was while playing this favorite game that he suffered a fatal heart attack. He lived fifty-five years of a rich and abundant life. His South Carolina chivalry, in the ballroom or on the meanest construction project, never forsook him. His booming voice and jovial laugh were synonymous with gatherings of the Society or the TVA. His ability as a raconteur was inevitably put to the test, and never found wanting. He was sociable; he worked hard and he played hard.

Mr. Lebby belonged to the Chattanooga Engineers Club, the Free and Accepted Masons, the Shrine, the Chattanooga Golf and Country Club, Lookout Mountain Fairyland Club, and Lookout Mountain Presbyterian Church.

Always active in Society affairs, Mr. Lebby was President of the Tennessee Valley Section in 1945 and of its Chattanooga Subsection in 1944.

On December 18, 1923, he was married to Betty Moore Jordan of Lexington, Miss. He is survived by his widow, his mother, a brother, and a sister.

Mr. Lebby was elected a Member of the American Society of Civil Engineers on April 10, 1940.

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[1] Memoir prepared by Lewis A. Schmidt, Jr., M. ASCE.

STERRY JENCKS MAHAFFEY, A.M. ASCE[1]

DIED JULY 1, 1950

Sterry Jencks Mahaffey, the son of Robert and Jessie (Cassidy) Mahaffey, was born on June 12, 1904, in the small town of Mahaffey in Central Pennsylvania, founded by and named for his grandfather. There he attended the public schools. After a brief experience in accounting work, he spent three years at Pennsylvania State College, at State College, studying civil engineering.

He then accepted a position in the offices of the Pennsylvania Department of Highways, the organization with which he had spent summer vacations as inspector on construction. Yielding to the urge to do construction work he spent eleven and a half years, beginning in June, 1926, with the Virginia Highway Department, advancing through the successive stages from inspector to resident engineer, in charge of construction and maintenance for a number of counties.

During this period he grasped the opportunity of night study offered by the city of Richmond, Va.—at the University of Richmond, the Richmond Professional Institute, and the Virginia Mechanics Institute. He was an avid student bent on bettering his engineering knowledge.

In the fall of 1937 he accepted the position of town engineer of Franklin, Va., and after four years of successful work was called to Henrico County, Virginia, as county manager. This work near Richmond also enhanced his reputation, so that in 1943 he was offered and accepted the attractive post of city manager of Durham, N. C., which he held for three years. Mr. Mahaffey then (1946) returned to Virginia and achieved his lifelong ambition of purchasing a farm, settling in Chesterfield County. This enabled him to become engineer director of the Virginia Road Builders' Association (an affiliate of the American Road Builders' Association), which position he filled with marked success until his death.

Mr. Mahaffey's cheerful personality, generosity, and cooperation brought him a host of friends. He was a member of the International City Managers Association, Rotary Club, Free and Accepted Masons (32nd degree), Shrine, Theta Xi Fraternity, and a member of First Church of Christ, Scientist of Richmond. He was married in 1931 at Warrenton, Va., to Frances Hurst who survives him.

Mr. Mahaffey was elected an Associate Member of the American Society of Civil Engineers on August 18, 1941.

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[1] Memoir prepared by C. S. Mullen, M. ASCE.

WILLIAM LINDSAY MALCOLM, M. ASCE[1]

DIED JANUARY 18, 1948

William Lindsay Malcolm, the son of George and Margaret (Milligan) Malcolm, was born at Mitchell, Ont., Canada, on February 2, 1884. When completing his studies at Stratford Collegiate Institute, of which his father had been vice-principal, he received a scholarship to Queens University, Kingston, Ontario, from which institution he received the degree of Master of Arts in 1905, and the Bachelor of Science degree in 1907, with a gold medal in mathematics.

He remained at the university as assistant professor of surveying, and following World War I (1919–1938) he was appointed professor of municipal engineering. Meawhile, as occasion offered, he was engaged in professional work at Stratford, and served as associate city engineer for Guelph, Ontario, and also in a professional capacity under the firm name of Malcolm and Rudd at Guelph. For the latter he designed and built the stadium and skating rink at the university, and planned and supervised the erection of an up-to-date sanitary engineering laboratory there.

With the outbreak of World War I, Professor Malcolm volunteered with the Canadian Expeditionary Forces, being commissioned Captain of Engineers. During service he was promoted to Lieutenant-Colonel and was twice cited for gallant and distinguished field service. He remained in the Canadian Engineers Reserve until 1944.

While still at Queens University, he undertook graduate work at Cornell University, at Ithaca, N. Y., receiving the degrees of Master of Civil Engineering in 1934 and Doctor of Philosophy in 1937. In 1938 he accepted the appointment of director of the School of Civil Engineering at Cornell, which position he held at the time of his death.

As an undergraduate Professor Malcolm was a member of the football team and later was coach for intercollegiate football and hockey. He excelled in tennis and badminton, and was twice Ontario Senior Doubles Champion. He served as president of the Ontario and Canadian Badminton Association and was a member of the Ithaca Golf and Country Club, and the Ithaca Badminton Club.

Scholastically he was honored by the election to Chi Epsilon (civil engineering), Phi Kappa Phi ("superior scholarship and character"), Sigma Xi (research), and Tau Beta Pi (engineering) fraternities. He constantly exemplified their ideals throughout his professional life.

Professor Malcolm was always professionally minded—he qualified early for Ontario land surveyor and Dominion land surveyor. He advanced through all the grades of the Engineering Institute of Canada, and was chairman of the Kingston Branch and represented it on the Institute's Council in 1930. He was actively interested in Society work, the Cornell Student Chapter, and the Ithaca Section of which he was President in 1944. He also held membership in the

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[1] Memoir prepared by Charles L. Walker, M. ASCE.

American Society of Engineering Education, the New York State Sewage Works Association, the American Waterworks Association, the Ontario Land Surveyors Association of Professional Engineers, and the Society of American Military Engineers.

On September 2, 1933, he was married to Margaret Elizabeth Murray, who survives him.

Professor Malcolm was elected a Member of the American Society of Civil Engineers on November 14, 1938.

EDWARD BROMLEY MAYER, M. ASCE[1]

DIED APRIL 5, 1951

Edward Bromley Mayer was born in Hartford, Conn., on December 5, 1890. His early life was spent in City Island, N. Y., and San Diego, Calif., his family having moved to San Diego in 1906.

After being graduated from high school in 1910, he worked as a rodman, and finding engineering work to his liking, he pursued a correspondence course in civil engineering from 1911 to 1914, and later a course on reinforced concrete design. He followed surveying work on subdivisions and industrial development in San Diego and the Los Angeles (Calif.) harbor area.

Mr. Mayer entered the employ of the water system of the Department of Water and Power of the City of Los Angeles in 1916, serving as instrumentman and draftsman until 1923 when he was appointed chief draftsman. In 1925 he organized a new engineering cost division, designing budget control and expenditure authorization systems and reclassifying accounts. This work continued until 1932 when he transferred to the Executive Office as an executive assistant, in which position he assumed increasing responsibilities until his death.

Mr. Mayer was a member of the American Water Works Association since 1929 and was a registered civil engineer in California. In 1929 he supplemented his education with a course in law at Southwestern University at Los Angeles.

On June 28, 1917, he was married to Edith Morgan in Los Angeles. His widow and two sons, Morgan and Donald, survive him.

Mr. Mayer was elected an Associate Member of the American Society of Civil Engineers on April 18, 1927, and a Member on February 10, 1936.

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[1] Memoir prepared by Laurance E. Goit, M. ASCE.

RICHARD HUBERT PENNARTZ, A.M. ASCE[1]

Died November 16, 1951

Richard Hubert Pennartz was born on May 21, 1887, at Fort Madison, Iowa. He received his engineering education at the University of Iowa at Iowa City. His entire lifetime was devoted to training and professional work in Iowa, Illinois, Kansas, and Oklahoma.

Following survey and construction work on hydroelectric projects at Keokuk, Iowa, for Stone and Webster (1911–1912) and surveying in Lee County, Iowa, and Henderson County, Illinois, he spent the succeeding years, 1915 to 1932, on various phases of highway work for a number of employers: The Illinois Highway Department in Springfield; Vermillion County, Illinois; the Kansas Highway Commission in Topeka; the Barrett Company (consultation work); the Federal Aid Roads Project in Shawnee County, Kansas (as resident engineer); and the Kansas Highway Commission (as construction engineer and right-of-way agent).

From 1933 to 1941 Mr. Pennartz was superintendent of the Civilian Conservation Corps dam projects at Reading, Marysville, Quinter, and Ashland in Kansas, and then from 1941 to 1945 was a construction engineer for the Long-Manhattan Construction Company, working in Fort Riley, Kans., and Tulsa, Okla. Beginning in 1945 he was assistant engineer for Wyandotte County, Kansas, making his headquarters in Kansas City, Kans., where he died.

Mr. Pennartz was a registered professional engineer, a member of the Kansas Engineering Society, and a charter member of the engineers' clubs of Topeka and Kansas City. In 1924 he was president of the Topeka club and in 1928 he was President of the Society's Kansas State Section.

His genial personality will be greatly missed by a host of friends among engineers and others in the construction industry. He is survived by his wife, Lola H. Pennartz, and by a daughter, Mrs. Donald E. French, of Manhattan, Kans.

Mr. Pennartz was elected an Associate Member of the American Society of Civil Engineers on October 11, 1920.

FRANK ALFRED RANDALL, M. ASCE[2]

Died December 2, 1950

Frank Alfred Randall, the son of Samuel Benjamin and Anna Louise Randall, was born at Cambridge, Ill., on November 1, 1883. He was graduated from the University of Illinois at Urbana in 1905, as valedictorian, with the degree of Bachelor of Science in Civil Engineering and received the degree of Civil Engineer in 1909.

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[1] Memoir prepared by F. W. Epps, M. ASCE.

[2] Memoir prepared by Verne O. McClurg, M. ASCE.

His professional life work in Chicago, Ill., was devoted to structural engineering. He was an employee of the American Bridge Company, of the "Milwaukee" Railroad, and of the Chicago Sanitary District. He was a partner of Randall and Warner and of Berlin, Swern and Randall. From 1923 he conducted his own consulting office, known since 1947 as Frank A. Randall and Sons. His office prepared structural plans for monumental buildings in Chicago, Minneapolis (Minn.), and Oklahoma City (Okla.).

Locally he was consultant for the Chicago Park District; from 1935, for the Department of Subways and Superhighways; and from 1938, for the Outer Drive Improvement. He served as building commissioner for the Century of Progress Exposition and as chairman of the following: The Structural Committee for the Chicago Building Code, a Joint Committee on Soil Mechanics and Foundations, and the Underground Construction and Dispersion Sub-Committee of the Chicago Civil Defense Corps. Mr. Randall was a member of the Column Research Council and of the Board of Education of Wilmette, Ill.

Besides many articles in the technical press, Mr. Randall wrote "History of the Development of Building Construction in Chicago" and a genealogy of 20,000 names entitled "Randall and Allied Families." He held membership in more than twenty-five engineering, genealogical, patriotic, social, and fraternal organizations, among them the Western Society of Engineers, the Chicago Building Congress, the American Concrete Institute, the American Soicety for Testing Materials, and the Chicago Engineers' Club. In addition to three Free and Accepted Masonic orders, he belonged to Alpha Delta Phi, Tau Beta Pi, Sigma Xi, and Phi Kappa Phi.

Especially conspicuous was Mr. Randall's contribution on the strength of steel beams fully encased in concrete, on lightweight concrete for floors in building construction, on proportioning of building foundations, and on design of building frames to resist lateral forces due to wind. In spite of his large practice as a consulting engineer, he always found time for outstanding service in engineering committee work and in civic affairs.

He was married to Mabel Madeline Morris on February 1, 1908, and is survived by Mrs. Randall and their children: Ruth Louise, Helen Anna, Frank Alfred, Jr., and John Deacon.

Mr. Randall was elected an Associate Member of the American Society of Civil Engineers on December 5, 1911, and a Member on April 18, 1917. He became a Life Member in January, 1947.

JOHN REIMANN, A.M. ASCE[1]

DIED MAY 10, 1951

John Reimann was born at New Haven, Conn., on October 12, 1888. After attending the public schools of New Haven, he entered Yale University and was graduated in 1909 with the degree of Bachelor of Science in Civil Engineering.

Following graduation he was employed for a short time with the maintenance-of-way forces of the Missouri Pacific Railroad Company and later with the New York Central Railroad Company. In 1910 Mr. Reimann went to Panama with the Panama Railroad Company and for one year he was engaged in location surveys for the railroad across Panama. On his return to the United States he worked for a short time with the Western Maryland and the New York, New Haven and Hartford railroad companies, respectively.

In 1914 he entered the service of the United States government and for ten years was affiliated with the Bureau of Valuation of the Interstate Commerce Commission. His work consisted of computing the valuations of the railroads of the United States. This work took him to many states and railroads, especially those in the eastern section of the country.

He resigned in 1924 from the Bureau of Valuation to accept a position with the Richmond, Fredericksburg and Potomac Railroad Company in Richmond, Va. His first job with this railroad consisted of the necessary valuation work to present the case of the railroad before the Interstate Commerce Commission. This work was under the direction of Edwin F. Wendt, M. ASCE, consultant for the railroad company, under the direct supervision of E. M. Hastings, Past-President, ASCE, then chief engineer of the Richmond, Fredericksburg and Potomac Railroad. Gradually his work with the railroad company was expanded, and he was engaged in handling construction and surveys in addition to valuation on the railroad. This comprised many small jobs and a number of rather extensive projects, including the building of new diesel facilities in Richmond, new freight and passenger stations in Quantico, Va., and a new bridge across Aquia Creek, Virginia, this last being a project costing more than a million dollars.

Upon his sudden death after a short illness, Mr. Reimann left a host of friends. His unfailing good humor and wit made him beloved by all who knew him. He was a member of the Central Virginia Engineers' Club and the Yale Club of Virginia.

On December 26, 1916, he was married to Marie Barker of Washington, D. C. He is survived by his widow and by a sister Louise Reimann of New Haven.

Mr. Reimann was elected an Associate Member of the American Society of Civil Engineers on January 18, 1926.

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[1] Memoir prepared by P. A. Rice, A.M. ASCE.

ANDREW ROSEWATER, M. ASCE[1]

DIED APRIL 17, 1909

Andrew Rosewater, the son of Herman and Roselia (Kohn) Rosenwasser, was born of German parentage in Bohemia, on October 30, 1848. The family, six boys and three girls, settled in Cleveland, Ohio, in 1854. There Andrew received a public school education.

At 16, following in the footsteps of an older brother, he got into railroad work, being flagman, inspector, rodman, and assistant engineer on the original construction of the Union Pacific Railway between Nebraska and Utah. Beginning in 1868, and, except for a few years in the newspaper publishing business with the *Omaha Bee,* and another brief term as assistant engineer on what is now the Chicago and Northwestern Railroad (between Tekemah and Oakland in Nebraska), his entire professional lifetime was spent in a splendid service to the City of Omaha, Nebr. He was city engineer from 1871 to 1874 and again from 1881 until his death.

As a consulting engineer he helped plan public improvements in Colorado and South Dakota; he had charge of construction at the Homestake Mine, Lead, S. Dak.; and he gave expert counsel on sewerage for more than twenty-five American cities. His success in these fields was recognized by his appointment as chairman of an engineering commission in Washington, D. C., by President Harrison; by his being called to Mexico City by President Diaz to consult on public improvements; and by the University of Nebraska, at Lincoln, in conferring on him, a man without college training, its first degree of Doctor of Engineering.

Mr. Rosewater developed a series of flush tanks and of the more than three hundred installed in Omaha 90% were still in use in 1952. He was one of the early students of modern hydraulics, developing charts for the flow of water through pipes and sewers using the Kutter formula.

His life and work were characterized by his earnestness and foresight, by confidence in conclusions quickly but accurately reached, and by sincerity and probity as a citizen. He was a man of strong personality, of positive views and was not to be moved from what he believed to be right. He was conscientious and faithful in the devotion to his profession, and was loyal to employer, friends, family, and community.

On October 18, 1883, he was married to Frances Meinrath of Boston, Mass. He is survived by his widow; one son, Stanley M., an attorney; four brothers; and two sisters.

Mr. Rosewater was elected a Member of the American Society of Civil Engineers on October 3, 1883.

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[1] Memoir prepared by the late Frank T. Darrow, M. ASCE.

BLAIR ARTHUR ROSS, M. ASCE[1]

DIED APRIL 30, 1951

Blair Arthur Ross, the son of Albert L. and Mary E. (Clifford) Ross, was born on December 16, 1887, of English parentage in Novelty, Mo. He attended the local schools and then the University of Missouri, at Columbia, receiving the degree of Bachelor of Science in Civil Engineering in 1912.

After a few years of experience in structural drafting, drainage, and flood work, World War I intervened. He served with distinction in important engagements overseas as Lieutenant and Captain, 29th Engineers, being made an "Officier d'Académie" in recognition by the Republic of France.

Starting in 1912 and taking up again after the war, he devoted many years to levee and revetment work in the 1st and 2nd districts of the Mississippi River Commission at Memphis, Tenn., as junior engineer, assistant engineer, and associate engineer, successively. As chief assistant to the chief of the Memphis District, he had a prominent part in studies for the formulation of the "Jadwin Plan." Still later, as chief of the area embracing the Arkansas, White, and St. Francis basins, he distinguished himself by his success in closing many levee breaks and preventing others. He also helped in developing concrete matresses for bank protection.

His later years, beginning in March, 1940, were devoted to the National Park Service, first at Shiloh National Park, near Corinth, Miss., and in 1945 at Smoky Mountain National Park with headquarters at Gatlinburg, Tenn. As superintendent in both locations he was responsible for extensive improvements as well as the operations. His love of nature and natural friendliness were important assets in these positions until his health forced him to resign in December, 1949. The family then moved to Jackson, Tenn.

Mr. Ross was a former member of the Memphis Engineers' Club, a member of Acacia fraternity, the Free and Accepted Masons (32nd degree), and the Shrine. He served the Methodist Church at Gatlinburg enthusiastically as a member of its building committee.

In 1925, he was married to Alice Murray of Jackson. His wife and son, Blair A. Ross, Jr., a graduate of the United States Military Academy at West Point, N. Y., class of 1950, survive him.

Mr. Ross was elected a Junior of the American Society of Civil Engineers on March 13, 1917; an Associate Member on October 14, 1919; and a Member on June 16, 1924.

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[1] Memoir prepared by Herbert S. Gladfelter, M. ASCE.

NED HENSEL SAYFORD, M. ASCE[1]

DIED FEBRUARY 5, 1952

Ned Hensel Sayford, the son of Harry Peddle and Katherine Myers (Hensel) Sayford, was born in Harrisburg, Pa., on March 8, 1884. Two years before being graduated from Lehigh University, in Bethlehem, Pa., and one year afterward, he worked in the engineering department of the City of Camden, N. J. Then he broadened his experience on a variety of projects, including locations in Montreal, Que., Canada, Schenectady, N. Y., Grand Rapids, Mich., Altoona, Pa., and Atlanta, Ga.

In 1913 Mr. Sayford joined the Morgan Engineering Company in Memphis, Tenn., and in the following twenty years he planned, designed, and constructed seventy-five major projects for flood control, irrigation, and drainage. He resigned as vice-president in 1933 to join the Tennessee Valley Authority (TVA), of which organization he later became head of the Maps and Surveys Branch.

The TVA is indebted to his ingenuity for many of its successes. For example, in planning Norris Reservoir, one of the earliest TVA projects, about 190,000 acres, largely small and irregular parcels in rough terrain with poor boundaries, were involved. Ordinary survey methods being impracticable, Mr. Sayford developed the use of enlarged aerial photographs as plane table sheets. More advanced procedures with major savings in time and money were later adopted, including particularly stereo-photogrammetric methods for mapping large areas.

Under his guidance, and with the cooperation of the United States Geological Survey, a German invention, the multiplex aeroprojector, was adopted and this revolutionized the mapping profession. It led to the mapping personnel and facilities of his organization being practically taken over by the United States Department of the Army during World War II, for mapping almost 800,000 square miles.

With an insistence for accuracy and economy, Mr. Sayford had an amazing capacity for searching out and applying new ways. His intensity made failure one of those things that just could not happen.

He held membership in the American Institute of Consulting Engineers, the American Society of Photogrammetry, the Chattanooga and the Memphis engineers clubs, and the Knoxville Technical Society, also the Sigma Phi Epsilon Fraternity and Tau Beta Pi honor society. He was a life member of the American Congress on Surveying and Mapping.

On August 9, 1911, in Monroe, N. Y., Mr. Sayford was married to Helen Newland Newburgh. He is survived by Mrs. Sayford; a daughter, Mrs. Raybourne Thompson, of Houston, Tex.; a son, Alan N. Sayford, of Chattanooga, Tenn.; two brothers, Frank M. Sayford of Montclair, N. J., and Howard M. Sayford of Bloomfield, N. J.; and a sister, Mrs. Harold Rasmussen of Westfield, N. J.

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[1] Memoir prepared by Robert E. Frierson, M. ASCE.

Mr. Sayford was elected an Associate Member of the American Society of Civil Engineers on December 3, 1912, and a Member on June 24, 1916. He became a Life Member in January, 1947.

HERMAN SCHORER, M. ASCE[1]

DIED DECEMBER 20, 1950

Herman Schorer was born in Wangen a. Aare, Switzerland, on June 24, 1892. He studied at the Federal Institute of Technology in Zurich, Switzerland, where he received the degree of Civil Engineer in 1914. During the next few years he was engaged in hydroelectric surveys, field work, and design with the Schweizerische Wasserwirtschaft in Bern and the Elektrizitätswerk Lonza at Basel, Switzerland.

Mr. Schorer came to the United States in 1920 and held various positions during the following years: First as draftsman in New York, N. Y., and East Rochester, N. Y.; then hydraulic designer in Fresno and San Francisco in California; and structural designer on the Oakland-Alameda Tunnel project also in California. With this background Mr. Schorer became engaged in work which allowed full development of his special talents, serving (from 1926 to 1932) first as hydraulic engineer and then as chief engineer for Thebo, Starr, and Anderton, consultants and constructors, of San Francisco. He prepared reports on, and preliminary designs for, various hydroelectric and water supply projects; he designed the Glines Canyon arch dam and appurtenant structures near Port Angeles, Wash.; and he directed the design of the Guadalupe River Power Project near Medellin, Colombia, and the design and construction of several air-butane gas plants in California and Oregon.

For a short period Mr. Schorer was an engineer for the United States Bureau of Reclamation in Denver, Colo., and in 1934 he became manager and president of the Borsari Tank Corporation in New York, a position which he held until his death. He directed the design and construction of numerous concrete tank installations and large industrial structures while he was with the Borsari Corporation. Later he also formed a separate company for the utilization of his patents for prestressing reinforced concrete structures. His three papers[2,3,4] published by the Society, "Buttressed Dam of Uniform Strength," "Design of Large Pipe Lines," and "Line Load Action on Thin Cylindrical Shells," were authoritative and became standards for engineering practice. He also published several outstanding papers on general principles and special features of prestressed concrete structures, and prior to his death he was working on further developments of design and construction of such structures. In addition to complete and original mastery of theory, Mr. Schorer demonstrated conspicuous ability in directing the design and construction activities and the business management of a large engineering firm. He will be remembered by all who knew him for his loyalty to friends and for fairness and consideration in all his dealings with those who worked with him and for him.

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[1] Memoir prepared by Mikael Juul Hvorslev, M. ASCE.

[2] *Transactions*, ASCE, Vol. 96, 1932, p. 666.

[3] *Ibid.*, Vol. 98, 1933, p. 101.

[4] *Ibid.*, Vol. 101, 1936, p. 767.

He held membership in the American Concrete Institute and the Association des Anciens Eleves de L'Ecole Polytechnique Fédérale of Zurich.

Mr. Schorer was elected an Associate Member of the American Society of Civil Engineers on January 18, 1926, and a Member on December 9, 1942.

MORTIMER WILSON SMITH, JR., M. ASCE[1]

DIED APRIL 10, 1951

Mortimer Wilson Smith, Jr., the older son of Mortimer Wilson and Emma (Bartlett) Smith, was born in Clarksburg, W. Va., on October 2, 1890. From forbears in Virginia and Massachusetts he came of pioneer stock prominent in local and national life.

He attended the public schools of Clarksburg, the Fishburn Military Academy in Waynesboro, Va., and in 1914 he was graduated from the University of Virginia at Charlottesville with the degree of Bachelor of Science in Civil Engineering. Until World War I he enjoyed a varied private practice of engineering in connection with coal mines, railroads, land surveys, and other developments near Clarksburg.

Following service in World War I as a Lieutenant, Corps of Engineers, United States Army, for two years he was employed by the Hope Natural Gas Company of Clarksburg, then a subsidiary of the Standard Oil Company (New Jersey) on gasoline and coal mining developments. Then for five years he was city engineer for the rapidly developing City of Clarksburg, in charge of construction and maintenance of street paving, sewers, bridges, incinerator plant, and other municipal works.

In 1932 Mr. Smith was elected to the West Virginia Legislature where he served with distinction on important committees and was the author of several bills which are now state laws. During this period the State Road Commission was empowered to take over all highways, including 28,000 miles, formerly administered by the counties. Mr. Smith effected a complete reorganization and successfully administered it for the next four and a half years.

In 1937 Mr. Smith became director of the Motor Carrier Department of the Public Service Commission. In this connection he was, for five years, the West Virginia member of the Interstate Commerce Commission Joint Boards.

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[1] Memoir prepared by a Committee of the West Virginia Section consisting of Frank D. McEnteer, M. ASCE, Chairman, George E. Norris, A.M. ASCE, and George L. Coyle, M. ASCE.

In World War II he was commissioned a Major in the United States Army and proceeded to Fort Custer in Michigan for training under the American Military Government. Following training at Fort Custer he was hospitalized for four and a half months. When he had sufficiently recovered he returned to his former position with the Public Service Commission of West Virginia, where he served as special engineer until 1949.

Physical limitations and personal feelings led him to accept the management of two nonprofit organizations, the Monongahela Valley Hospital Service, Incorporated, and the Medical Surgical Service, Incorporated, which would permit residence in his old home in Clarksburg, and to which he brought the same ability, enthusiasm, sterling integrity, and unfailing geniality that characterized his whole life.

Mr. Smith was a lifelong member of the Presbyterian Church, a member of the Sons of The American Revolution, a Rotarian, a past-Post Commander of the American Legion, and a registered professional engineer.

On June 6, 1917, Mr. Smith was married to Pauline Musgrave in Clarksburg. He is survived by his widow and two brothers, Augustine and Ethelbert Smith.

Mr. Smith was elected an Associate Member of the American Society of Civil Engineers on October 2, 1922, and a Member on October 20, 1947.

WILLIAM HAIBER SPIRZ, A.M. ASCE[1]

DIED FEBRUARY 16, 1951

William Haiber Spirz, the son of Gustav and Frieda Spirz, was born in San Francisco, Calif., on May 21, 1902. He attended the city's public schools and San Mateo Junior College. In 1931 he received the degree of Bachelor of Science in Civil Engineering from the University of California at Berkeley.

Mr. Spirz began his engineering career with the National Park Service in Sequoia and General Grant parks, after which he was employed by the United States Bureau of Public Roads on road and bridge construction in Yosemite Park until 1933. The same year he joined the engineering staff of the San Francisco-Oakland Bay Bridge where for three years he was engaged on construction in the West Bay and in San Francisco. During 1937 he did construction work on buildings for the Golden Gate International Exposition in San Francisco.

In 1938 he was employed by the California State Division of Highways on bridge design and construction. He then (1941) entered the United States Army as a First Lieutenant, and served as post engineer at nine stations in the western United States during the four war years. In 1945 he returned to the San Francisco-Oakland Bay Bridge and he was a member of the engineering staff at the time of his death.

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[1] Memoir prepared by R. W. Hutchinson, I. O. Jahlstrom, and A. J. Meehan, Members, ASCE.

Although Mr. Spirz had traveled widely as a youth, he loved his native city and was proud of the part he had played in creating the San Francisco-Oakland Bay Bridge, which meant so much to the entire metropolitan community. He was a member of the Commonwealth Club, the retired Officers' Association, Park Lodge No. 449, Free and Accepted Masons—all of San Francisco, and the Society's Sacramento Section.

In 1940 he was married to Meta Corville, who survives him.

Mr. Spirz was elected an Associate Member of the American Society of Civil Engineers on August 18, 1941.

ERNEST OSGOOD SWEETSER, M. ASCE[1]

DIED JANUARY 18, 1951

Ernest Osgood Sweetser, the son of Frederick R. and Mary E. (Osgood) Sweetser, was born on October 19, 1883, in Cumberland Center, Me., and there received his preliminary education at Greely Institute. Entering the University of Maine at Orono, he gained the Bachelor of Science degree in Civil Engineering in 1905, and later, in 1912, the degree of Civil Engineer.

His entire professional life was spent at Washington University, in St. Louis, Mo.: As instructor from 1905 to 1910, assistant professor from 1910 to 1917, and professor of structural engineering from 1925 and of civil engineering from 1937. Following his special interests, he was the author of reports on research concrete and other structural materials, on impact and live load stresses in steel bridges, and the St. Louis tornado (1927).

Professor Sweetser's forty-five years of loyal service to the university was the subject of a glowing tribute by the faculty, which stated in part:

> "Ernest Sweetser earned the sincere respect and affection of the many students under his instruction. To his colleagues he was known as one always ready to enter wholeheartedly into the professional and social life of the University. His efficient service for many years as Grand Marshall at Commencement and at other academic processions will be gratefully remembered."

During World War I he served from 1917 to 1919 as Captain, 116th Engineers, American Expeditionary Forces.

He was a lover of music, possessor of a fine baritone voice, member and soloist of the glee club as an undergraduate, and choir member with Mrs. Sweetser at the Pilgrim Congregational Church, in St. Louis. He was also on the music committee of the church, president of its Men's Club, and member of its Board of Trustees.

Professor Sweetser belonged to many engineering organizations: In the American Railway Engineering Association he had been a member of the Committee on Masonry; in the American Concrete Institute, a member of the Committee on Research; in the American Society of Engineering Education, a member of the Committee on Education Methods, and president of the Missouri Section in 1935; and in the Engineers' Club of St. Louis, he served as president in 1935. He also belonged to Sigma Xi, Tau Beta Pi, Sigma Chi (social), and the Free and Accepted Masons.

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[1] Memoir prepared by F. C. Woermann, M. ASCE.

On August 7, 1909, he was married to Carrie L. Blanchard, also of Cumberland Center. Both fond of the out-of-doors, they made frequent trips to the Ozarks and spent summers at their cottage on the Maine coast. He is survived by his widow and by a brother, Herman P., of Cumberland Center.

In the Society Professor Sweetser had served as a member of the Committee on Aims and Activities, and the Committee on Public Information. In 1928 he was President of the St. Louis Section.

Professor Sweetser was elected an Associate Member of the American Society of Civil Engineers on November 27, 1917, and a Member on April 7, 1930.

ELLIS WING TAYLOR, M. ASCE[1]

DIED JANUARY 20, 1951

Ellis Wing Taylor, the son of Frank Wing and Minnie (Cray) Taylor, was born in Chicago, Ill., on October 2, 1887. The family moved to Los Angeles, Calif., in 1894. His father was in the banking business in both cities. After a year at Columbia University in New York, N. Y., he transferred to the College of Engineering, University of California at Berkeley, from which institution he received the degree of Bachelor of Science in 1912.

Almost immediately Mr. Taylor entered into partnership with his brother, Edward Cray Taylor, in the practice of architecture and engineering in Los Angeles. During World War I he attended the Submarine Officers' School at Annapolis, Md., being graduated in June, 1918, as Ensign and assigned to the F-2, on which he also served as Lieutenant (jg) and temporary Commander. He was later assigned as Commander of the F-3.

After the war, the brothers reopened their architectural office in Los Angeles, continuing until the death of Edward Taylor in 1946. The practice was then carried on under the name, Ellis Wing Taylor, Architect and Engineer.

Although the office was responsible for the design of many types of buildings such as schools, hospitals, and commercial buildings, Mr. Taylor was perhaps best known for the design of the Douglas Aircraft plants at Santa Monica, El Segundo, and Long Beach and Consolidated Aircraft's plants No. 1 and No. 2 at San Diego—all in California. These assignments led to his being engaged as a consultant for a large British aircraft plant in 1936. During World War II perhaps his most unusual project was the camouflage design for the aircraft plants.

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[1] Memoir prepared by S. B. Barnes, M. ASCE.

A lifelong yachtsman, Mr. Taylor skippered the heavy undercanvased *Trojan* to a victory in the 1915 San Pedro to San Francisco Race. He sailed in the Honolulu Race of 1928 and in the 1930 Honolulu Race on the winning schooner, *The Enchantress.* Besides being official measurer he was for a number of years on the Race Committee of the Southern California Yachting Association.

Greatly interested in the welfare and integrity of the engineering profession, he served on a number of engineering committees and in 1950 was chairman of the Building and Safety Sub-Committee of the Los Angeles Chamber of Commerce.

All who knew Mr. Taylor respected him and at the same time loved him for his merry kindliness. His impersonation of a hula dancer at a party was a work of art and he could recall the words to most of the sea chanties.

Mr. Taylor was a registered civil and structural engineer in California and was also a licensed architect. He belonged to the Structural Engineers' Association of Southern California and the American Institute of Architects. He was a member of the Los Angeles Yacht Club, the Santa Monica Swimming Club, and the Jonathan Club. His college fraternity was Phi Kappa Sigma.

He is survived by his widow, Elma Ficks Taylor, and two sons, Ellis Wing Taylor, Jr., and Peter Tracy Taylor, by a previous marriage.

Mr. Taylor was elected a Member of the American Society of Civil Engineers on May 19, 1941.

JOHN CRESSON TRAUTWINE, III, M. ASCE[1]

Died March 28, 1949

John Cresson Trautwine, III, the son of John Cresson, Jr., and Lucy Lane (Smith) Trautwine, was born in Philadelphia, Pa., on February 25, 1878. His grandfather, John Cresson Trautwine, was the originator of *Trautwine's Civil Engineer's Pocket-book.*

He was educated in the public schools of Philadelphia and the Cascadilla Preparatory School at Ithaca, N. Y. After being graduated from Cornell University at Ithaca in 1900 with the degree of Civil Engineer, he soon became associated as one of the editors of the family publication, *The Pocket-book,* and in time became the author of many of its sections. In 1909, he was made its editorial director. After his father's death, he took over the publication as owner as well as editor and continued in this position until his own death. Chapters in the book were frequently revised by him and he planned extensive revisions of the text prior to his death.

Besides his editorial duties, Mr. Trautwine was engaged over the years in a diversity of engineering work: Investigator of water power possibilities in New

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[1] Memoir prepared by John E. Perry, M. ASCE.

Jersey and Pennsylvania; engineer with the Union Railroad Company at Port Perry, Pa.; and mine surveyor for the Glen Iron Furnace Company. For more than two years (1912–1915), he was employed as a consultant in Philadelphia on the investigation of building vibrations and foundations, on water power tests at Manayunk, Pa., and on a mine tunnel survey at Monatawny, Pa. In this period, experimenting and inventing consumed some of his time: He developed a monopod for dispensing with instrument tripods in mine surveying; experimented with a sounding balloon to carry sighting light into inacessible portions of mines, and was the inventor of a shader for player pianos. During all these activities, however, he was closely associated with the business of publishing *The Pocket-book.*

For years, Mr. Trautwine made his home in Philadelphia and held membership in the engineers' club of that city. He moved to Ithaca, in 1935, so that he could carry on his editorial work near Cornell, his alma mater, which he loved so dearly. His chief outside interests were music and photography. While a resident of Philadelphia, he was a member of the Philadelphia Orchestral Society and for some time, its treasurer. For seventeen years, he held membership in the Franklin Institute of Philadelphia and served on its Science and Arts Committee. He enjoyed photography, specializing in motion pictures, and in his extensive travels he had assembled a very large film collection.

In 1930, he was married to Florence M. Bailey of Philadelphia. She died at Ithaca in 1949, six months prior to his death.

Mr. Trautwine was elected a Member of the American Society of Civil Engineers on August 9, 1920.

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